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Input varieties and growth: a micro-to-macro analysis

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Abstract

We investigate the effects of input variety creation and destruction on both micro- and macroeconomic outcomes using detailed data from Belgium. Our microeconomic analysis establishes that the elasticity of downstream firms' marginal cost to supplier separation captures the area under the input demand curve, and this elasticity can be utilized to calibrate love-of-variety and Schumpeterian models. Empirically, we estimate that marginal costs rise by 0.6% for every 1% of suppliers lost. Our macroeconomic analysis develops a growth-accounting framework that captures the role of supply chain churn for aggregate growth. Using firm-level production network data and estimated microeconomic elasticities, we show that supplier churn can plausibly account for a large portion of the trend component of growth in aggregate productivity. Our findings highlight the crucial role of input entry and exit in driving economic growth.

Keywords: growth, churn, exit, entry.

JEL Classifications: D21, D22, E10.

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Non-technical summary

The creation and destruction of products play a vital role in driving growth and trade. But how exactly do new producers and products benefit consumers, and how do they affect the overall economy? For a large class of modern growth and trade models, we show that the answer to these questions is shaped by the same sufficient statistic: consumer surplus due to additional suppliers per unit of expenditures.

This paper has two parts. First, we define this surplus for a general class of models and provide an identification strategy for estimating it using microeconomic data from Belgian firms. Second, we develop a growth accounting framework to quantify the contribution of supplier churn to aggregate productivity. We discipline our aggregation exercise using the microeconomic estimates from the first part of the paper and implement it using firm-to-firm production network data from value-added tax (VAT) filings.

We find that if 1 percentage point of a firm's suppliers (in terms of the firm's cost share) exit, then this raises the firm's marginal cost by around 0.6 percentage points. In a CES expanding varieties model, the "love-of-variety" corresponds to an elasticity of substitution of roughly 2.5. In a quality-ladder model with unitary elasticities between inputs, this corresponds to an innovation step-size of around 60 log points. In other words, at the microeconomic level, the destruction of supply linkages has strong effects on downstream marginal costs.

Our aggregation exercise computes how the addition and separation of suppliers affects the prices of downstream firms, and how these price changes are transmitted along existing supply chains from suppliers to customers, all the way down to final consumers. We find that almost the entirety of the trend growth component in the aggregate Solow residual in Belgium can plausibly be accounted for by churn in the supply chain. Overall, our paper shows that these effects are large at the micro and macro levels and can be used for structural growth and trade models.

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1 Introduction

The creation and destruction of products play a vital role in driving growth and trade. But how exactly do new producers and products benefit consumers, and how do they affect the overall economy? For a large class of modern growth and trade models, we show that the answer to these questions is shaped by the same sufficient statistic: consumer surplus due to additional suppliers per unit of expenditures.

We call this surplus the "inframarginal surplus" ratio, which represents the area under the input demand curve relative to expenditures. This statistic is crucial for the positive and normative implications of many growth and trade models, including expanding-variety models and quality-ladder models. In expanding-variety models, this statistic measures the "love-of-variety" effect, whereas in quality-ladder models, it measures the "step-size" of innovation.[1]

This paper has two parts. First, we define the inframarginal surplus ratio for a general class of models and provide an identification strategy for estimating it using microeconomic data from Belgian firms. Second, we develop a growth accounting framework to quantify the contribution of supplier churn to aggregate productivity. We discipline our aggregation exercise using the microeconomic estimates from the first part of the paper, and implement it using firm-to-firm production network data from value-added tax (VAT) filings. We describe the micro and macro parts of the paper in turn.

To estimate the inframarginal surplus ratio at the micro-level, we use firm-level data and an instrumental variable strategy to estimate the elasticity of firms' marginal costs to exogenous supplier separations. We show that this elasticity measures the inframarginal surplus ratio. Our approach is unique in that it allows us to estimate the integral of demand without specifying the demand system itself. Bypassing a fully specified demand system is useful because it would be extremely high dimensional in our dataset, and could only be estimated under very strong functional form assumptions. The integral of demand resulting from such an exercise could have large misspecification and extrapolation errors.

Our estimates of the integral of demand contribute to the goal of measuring higher derivatives of demand curves. Estimates of the first derivative of demand are common, since the first derivative of demand affects the price elasticity of demand (see, e.g., Berry and Haile, 2021). The second derivative of demand has also received considerable em-

[1] Expanding varieties models of growth and trade include Dixit and Stiglitz (1977), Krugman (1979), Romer (1987), and Melitz (2003). Ricardian models of growth and trade include Dornbusch et al. (1977), Aghion and Howitt (1992), and Eaton and Kortum (2002). For a synthesis of these models see Grossman and Helpman (1993), Acemoglu (2009), and Costinot and Rodriguez-Clare (2014).

pirical attention, since it determines the pass-through of marginal cost into the price (see, e.g., Burstein and Gopinath, 2014 for a survey on exchange rate pass-through). Even the third derivative of demand is an important statistic, because it disciplines the rate at which pass-through changes along the demand curve (e.g. Amiti et al., 2019). Our paper is one of the only attempts to directly measure the first anti-derivative – the integral – of demand.

To estimate the inframarginal surplus ratio, we use a detailed survey of manufacturing firms in Belgium called Prodcom. This survey contains sales and quantity information for most manufacturing firms in Belgium. We merge this data with firm-to-firm input-output linkage information from VAT returns. Using this tax information, we observe, at annual frequency, almost all suppliers of the firms in Prodcom. We calculate a measure of marginal cost for Prodcom firms, and we regress marginal cost on supplier separations. We provide assumptions under which the estimated coefficient identifies the area under the exiting input's demand curve.

Consistent estimation requires an instrument for separation from suppliers. The instrument ensures that separations are caused by a negative shock to the supplier and not other drivers of the downstream firms' marginal cost. To instrument for supplier exits, we consider two alternative identification strategies. In our preferred specification, we predict exits using a Bartik-type instrument of upstream firms' sales to non-manufacturing industries. That is, a supplier is more likely to exit if sales in non-manufacturing industries it sells to decline. Our second instrument uses supplying firms' short-term debt obligations interacted with changes in aggregate interest rates. That is, a supplier is more likely to exit if it has taken on a large amount of short-term debt and the aggregate interest rate rises. In either case, the identification requirement is that exits predicted by our instrument are not correlated with other reasons why downstream firms' marginal costs change, such as a change in the firm's own productivity or entry of better suppliers.

We find that if 1 percentage point of a firm's suppliers (in terms of the firm's cost share) exit, then this raises the firm's marginal cost by around 0.6 percentage points. In a CES expanding varieties model, the "love-of-variety" corresponds to an elasticity of substitution of roughly 2.5. In a quality-ladder model with unitary elasticities between inputs, this corresponds to an innovation step-size of around 60 log points. In other words, at the microeconomic level, the destruction of supply linkages has strong effects on downstream marginal costs.

On the macroeconomic front, we develop a growth-accounting framework that quantifies the importance of supplier churn for aggregate growth. Because new suppliers affect the downstream firms' marginal cost, the effects of supplier churn are captured by standard measures of real economic activity like real GDP. Our macroeconomic results, which extend growth accounting formulas to include an extensive margin (i.e. Solow, 1957; Domar, 1961; Hulten, 1978; Basu and Fernald, 2002; Baqaee and Farhi, 2019), can be disciplined using our microeconomic regression estimates.

Our aggregation exercise computes how the addition and separation of suppliers affects the prices of downstream firms, and how these price changes are transmitted along existing supply chains from suppliers to customers, all the way down to final consumers. Our accounting framework does not fully specify the structure of the economy, since it takes as given observed movements in endogenous quantities.

To perform our aggregation exercise, we extrapolate our microeconomic estimates to the whole economy. Under the assumption that this extrapolation is valid, we find that almost the entirety of the trend growth component in the aggregate Solow residual in Belgium can plausibly be accounted for by churn in the supply chain. Overall, our paper provides more direct estimates of the consumer surplus from new suppliers, shows that these effects are large at the micro and macro levels, and can be used as calibration targets for structural growth and trade models.

The structure of the paper is as follows. Section 2 contains theoretical microeconomic results. These results motivate our microeconomic empirical strategy, which we describe and report in Section 3. Section 4 introduces the aggregation framework and presents our theoretical macroeconomic results. We use these results, and our earlier microeconomic estimates, to decompose aggregate growth in our data in Section 5. We conclude in Section 6.

Related literature. We discuss how our paper is related to three different literatures. First, as discussed above, our analysis contributes to expanding-varieties and quality-ladder models of entry and exit. In these models, a key object of interest and source of welfare gains is either the love for product variety or the gap in quality between incumbents and entrants.

The love-of-variety effect is usually defined using the elasticity of the utility function.[2] In this paper, as in Baqaee et al. (2020), we relate love-of-variety to the area under the

[2] The love-of-variety effect has been theoretically studied extensively by Zhelobodko et al. (2012), Dhingra and Morrow (2019), Baqaee et al. (2020), Matsuyama and Ushchev (2020), amongst many others. The love-of-variety effect is sometimes viewed with suspicion since it is not easily measured and does not show up in conventional index number statistics. This may be exacerbated by the fact that in models where it plays a central role, it is often described using variables that are unobservable. For example, Vives (1999), Benassy (1996), Zhelobodko et al. (2012), and Dhingra and Morrow (2019) all use definitions that rely on the elasticity of the utility function with respect to quantity — an inherently unobservable object since utility is only defined up to monotone transformations.

demand curve instead. One reason to do this is that it demystifies the love-of-variety and makes clear that it simply corresponds to changes in marginal cost caused by large (non-marginal) changes in input prices. If one is comfortable with the idea that small input price changes have effects on costs and welfare, then one should also be comfortable with the love-of-variety effect. A second reason is that the area under the input demand curve is a versatile concept that can be used in a broader set of contexts, including in general specifications of demand and in quality-ladder models.

We contribute to the expanding-variety and quality-ladder literatures by directly estimating the inframarginal surplus lost when firms lose access to suppliers. We can do this because our data allows us to measure both marginal costs and track firms' suppliers. In lieu of this data, researchers have typically relied on very indirect evidence to discipline the consumer surplus from new suppliers in their models. For example, expanding-varieties models typically use a CES demand system, where the price elasticity of residual demand at any point on the demand curve also controls the love-of-variety effect. Similarly, in quality ladder models, researchers typically discipline the step-size between the best and second-best supplier by indirect inference via matching moments on firm employment dynamics, patents, and growth (see Garcia-Macia et al., 2019 and Akcigit and Kerr, 2018 for example).[3]

The second literature our paper is related to is the one on production networks, particularly those with an extensive margin. For example, Baqaee (2018) and Baqaee and Farhi (2020) show that cascades of supplier entry and exit in production networks change how aggregate output responds to microeconomic shocks. The response of aggregate output to a microeconomic shock, in turn, crucially depends on the same notion of surplus as discussed above. The importance of the extensive margin of firm-to-firm linkages has also been emphasized and studied by Lim (2017), Tintelnot et al. (2018), Oberfield (2018), Acemoglu and Tahbaz-Salehi (2020), Elliott et al. (2020), Taschereau-Dumouchel (2020), Kopytov et al. (2022), and Bernard et al. (2018). Empirical studies by Jacobson and Von Schedvin (2015), Barrot and Sauvagnat (2016), Carvalho et al. (2021), and Miyauchi et al. (2018) have shown that shocks and failures to one firm are transmitted across supply chains and affect the sales and employment of other firms in neighboring parts of the production network. Huneeus (2018) and Arkolakis et al. (2021) study adjustment costs in link-formation between firms and their aggregate consequences using a struc-

[3]There is a large literature that provides reduced-form evidence of how changes in policies (e.g. import tariffs) impact firm outcomes such as productivity, markups, and firm product-scope. See, for example, Amiti and Konings (2007), Brandt et al. (2017), Goldberg et al. (2010), and De Loecker et al. (2016). Although this literature provides suggestive evidence that input variety matters for firm-level outcomes, it does not provide an estimate of how large these gains are.

tural model. Our paper complements this literature by providing direct estimates about the value of link formation at the microeconomic level and a growth accounting exercise that quantifies the importance of link formation.

Third, our paper is also related to a deep literature on correcting price indices to account for the entry and exit of goods. The macroeconomic and trade literatures on the importance of entry and exit, which trace their origins to Hicks (1940), have been greatly influenced by Feenstra (1994) who introduced a methodology for accounting for product entry and exit, or other types of mismeasurement, under a CES demand system. This CES methodology owes its popularity to its simplicity and nondemanding information requirements. Broda and Weinstein (2006) apply it to calculate welfare gains from trade due to newly imported varieties, and Broda and Weinstein (2010) compute the unmeasured welfare gains from changes in varieties in consumer non-durables. Using a similar methodology, Jaravel (2016) calculates the gains from consumer product variety across the income distribution, while Gopinath and Neiman (2014), Melitz and Redding (2014), Halpern et al. (2015), and Blaum et al. (2018) study the welfare gains from trade in intermediate inputs.[4] Aghion et al. (2019) build on this methodology to correct aggregate growth rates for expanding varieties and unmeasured quality growth. Outside of the CES literature, Hausman (1996), Feenstra and Weinstein (2017), and Foley (2022) have provided alternative price index corrections that dispense with the CES assumptions.

A universal theme in this literature is to estimate or calibrate price elasticities of demand and infer the value of entering and exiting products by inverting or integrating demand curves under parametric restrictions (e.g. isoelastic, linear, or translog demand). Our approach differs from this literature in that we attempt to identify the area under the input demand curve directly through its effect on downstream marginal costs rather than via implicit or explicit integration of demand curves. Of course, our methodology cannot be applied to household demand since marginal utility, unlike marginal cost, is not observable.

[4]The methodology of Feenstra (1994) requires knowledge of the elasticity of substitution, which is typically estimated using data on expenditure switching. Blaum et al. (2018) instead uses changes in the buying firm's revenues (and parametric assumptions on the production function and demand for the buying firms' output) to estimate the elasticity of substitution between imports and domestic inputs. One of our regressions uses a similar approach, however we use quantity data to obtain a measure of marginal cost, so that we do not have to take a specific stance on the production function and the output demand elasticity.

2 Microeconomic Value of Link Formation: Theory

In this section, we derive expressions for how supplier entry-exit affects a downstream firm's marginal cost. We consider two approaches. The first approach, in Section 2.1, imposes minimal assumptions on the demand system. We then compare this to the more traditional approach that imposes CES input demand in Section 2.2. The partial equilibrium results in this section serve as the basis for our firm-level regressions in Section 3. We delay general equilibrium and aggregation to Sections 4 and 5.

2.1 Direct Approach Using Area Under Demand Curve.

Consider a *downstream* firm, indexed by i whose variable cost function is

$$\mathcal{C}_i(\boldsymbol{p}, A, y_i) = mc_i\left(\boldsymbol{p}, A_i\right) y_i,$$

where $\boldsymbol{p}$ is the vector of quality-adjusted input prices, A_i indexes technology, and y_i is the total quantity of output. We allow the firm to have fixed costs of operation, but assume that variable production has constant returns to scale. We allow for the possibility that the price of some inputs is equal to infinity (i.e. some inputs are not available).

Assume that there is a continuum of inputs that can be grouped into types. The cost function is symmetric in input prices that belong to the same type but not necessarily symmetric across types. More formally, two inputs belong to the same group if swapping their prices does not affect variable cost. This assumption ensures that the downstream firm's input demand curve for all varieties of a given type j are the same function $x_{ij}(\boldsymbol{p}, A, Y)$.

We do not restrict own-type or cross-type price elasticities. We assume without loss of further generality that inputs of the same price have the same initial price. We can do this by defining inputs that have different initial prices to be different types. To simplify notation, we assume that there is a countable number of types. Let M_{ij} denote the mass of inputs of type j used by firm i.

Almost all popular production technologies used in macroeconomics and trade feature a notion of "types." For example, for CES, we say two inputs have the same type if they have the same share parameter and price. More generally, for the Kimball (1995) demand system, the homothetic demand systems introduced by Matsuyama and Ushchev (2017), and the separable demand system introduced by Fally (2022), we say that two inputs have the same type if they share the same residual demand function and the same price.

Our paper focuses on the creation and destruction of buyer-supplier relationships.

These events are typically discrete in the sense that when suppliers are added or dropped, expenditures change discontinuously. To account for this phenomenon, we introduce the concept of a *jump* in the price of an input j, which is defined by the size of the jump or the *step-size* $z_{ij} = \Delta \log p_j$. This means that for each item j, there is a possibility of a discontinuous change in its price.

In quality-ladder and expanding-variety models, we use jumps to represent when suppliers enter or exit. In quality-ladder models, each input has a price that changes when a new supplier replaces an old one. If the new (quality-adjusted) price is greater than the initial price, this represents a move down the quality ladder, and if the new (quality-adjusted) price is less than the initial price, this represents a move up the quality ladder. In expanding-variety models, prices jump to infinity when a variety is dropped and become finite when a new variety is added. This means that in expanding-variety models, the step-size is plus or minus infinity.[5] We identify instances of price jumps in the data that can be attributed to exogenous supplier separations. We do not investigate price jumps that may be occurring within continuing buyer-supplier relationships, which could be caused by process innovation from continuing suppliers.

Define the *inframarginal surplus ratio* associated with a change in the price of input j (holding the price of all other inputs constant) to be

$$\delta_{ij}(p_j, p'_j) = \frac{\int_{p_j}^{p'_j} x_{ij}(\xi) d\xi}{p_j x_{ij}(p_j)} \geq 0, \tag{1}$$

where we define p_j to be the lower price and p'_j to be the higher of the two possible prices for input i. Equation (1) is the surplus to i from the jump in the price of input j per unit of expenditures. Since we define p_j to always be the lower of the two possible prices, δ_{ij} is always a non-negative number. As long as the demand curve is strictly downward sloping, δ_{ij} is strictly positive.[6] If the demand curve is perfectly horizontal, then $\delta_{ij} = 0$.

Denote the input share of each type-j variety purhcased by firm i to be Ω_{ij}:

$$\Omega_{ij} = \frac{p_j x_{ij}(\boldsymbol{p}, A)}{C_i(\boldsymbol{p}, A_i, y_i)}.$$

The next proposition loglinearizes the downstream firm's marginal cost.

[5] Technically, the price need only jump to/from the reservation or choke price (the price at which demand is zero). However, since demand is zero beyond the choke price, we can also think of the price as jumping to/from infinity.

[6] In equation (1), we suppress dependence of the conditional input demand x_{ij} on arguments other than the price of j since those other arguments are being held constant. We include the additional arguments when it helps the exposition.

Proposition 1 (Downstream Marginal Cost)**.** *Consider a change in the vector of input prices by type* $\Delta \boldsymbol{p}$*, the vector of the measure of inputs whose price jumps* $\boldsymbol{m}_i$*, and the technology parameter* ΔA_i*. To a first-order approximation in these primitives, the change in the downstream firm's marginal cost is*

$$\Delta \log mc_i \approx \underbrace{\sum_j \Omega_{ij} M_{ij} \Delta \log p_j}_{\textit{marginal changes}} + \underbrace{\sum_j \Omega_{ij} m_{ij} \delta_{ij} \left[\mathbb{1}(z_{ij} > 0) - \mathbb{1}(z_{ij} < 0) \right]}_{\textit{inframarginal jumps}} + \underbrace{\frac{\partial \log \mathcal{C}_i}{\partial \log A_i} \Delta \log A_i}_{\textit{technology}} . \tag{2}$$

In words, the marginal cost of the downstream firm depends on the costs of its inputs, captured by the first two summands, as well as its own technology, the last summand. The price of inputs can change on the margin or they can jump. If the change in input prices is small, then their effect on the downstream firm's marginal cost depends on the expenditures on the input. On the other hand, if input prices jump discretely, then their effect on the downstream firm's marginal cost depends on the area under the input demand curve, which is captured by δ_{ij}. Intuitively, movements along the quality ladder and variety creation generate surplus for the downstream producer according to the area under the input demand curve.

The intuition for δ_{ij} is depicted in Figure 1. The left panel depicts a jump along the quality ladder where the price jumpts from p_j to p_j'. The right panel depicts the case where the price p_j jumps to infinity. The former is a quality-ladder model and the latter is an expanding-variety model. In both cases, the inframarginal surplus ratio is given graphically by

$$\delta_{ij} = \frac{A}{B} \geq 0.$$

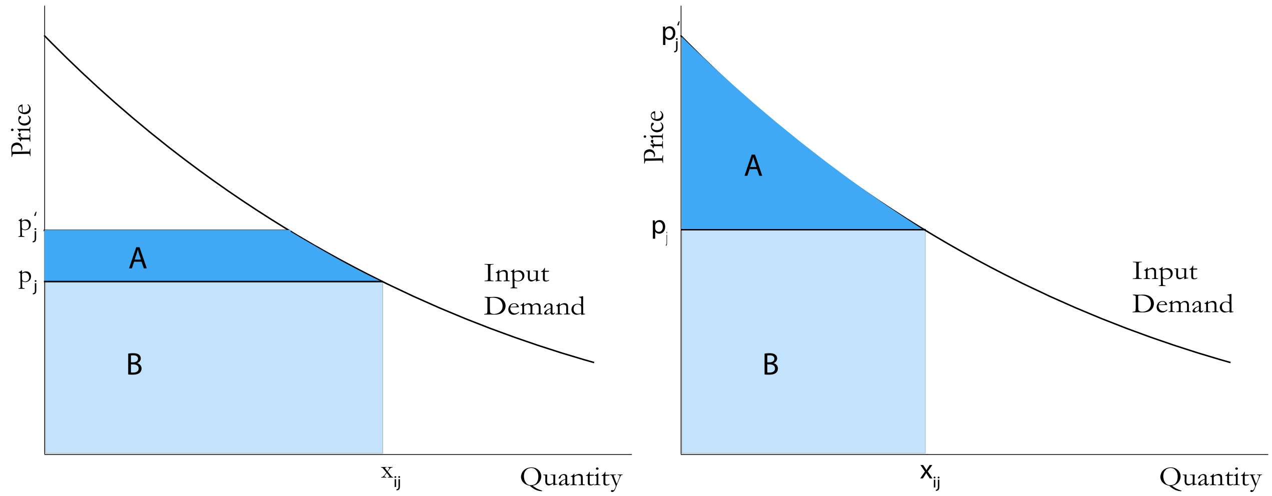
Either way, this jump in input price raises the costs of production by an amount commensurate with δ_{ij}, and this is weighted by the expenditures.

To better understand Proposition 1, we work through a series of simple examples.

Example 1 (CES with Quality Ladders)**.** Consider the CES special case, in which the demand for an input variety of type j takes the form

$$x_{ij} = \frac{b_{ij} p_j^{-\sigma} y_i}{\left(\sum_k b_{ik} p_k^{1-\sigma} M_{ik} \right)^{\frac{-\sigma}{1-\sigma}}}, \tag{3}$$

where b_{ij} and b_{ik} are exogenous parameters. Suppose that each time an input with price p_j is destroyed, it is replaced by a competitor whose quality-adjusted price is lower p_j'.



(a) Quality ladder (b) Expanding variety

Figure 1: Graphical illustration of input price jump. In both figures, the inframarginal surplus ratio δ_i is A/B.

The inframarginal surplus ratio is

$$\delta_{ij} = \frac{\int_{p_j}^{p_j'} x_{ij}(\xi) d\xi}{p_j x_{ij}} = \frac{1}{\sigma - 1} \left(1 - \left(\frac{p_j'}{p_j} \right)^{1-\sigma} \right) \geq 0.$$

Hence, Proposition 1 implies that the change in the downstream firm's marginal cost in response to the creative destruction of a mass m_{ij} of input j is

$$\Delta \log mc_i = -\Omega_{ij} m_{ij} \delta_{ij} = -\frac{1}{\sigma - 1} \Omega_{ij} m_{ij} \left(1 - \left(\frac{p_j'}{p_j} \right)^{1-\sigma} \right). \tag{4}$$

The negative sign in (4) is because $z_j = \log(p_j'/p_j) < 0$ and so the change in marginal cost is proportional to $-\delta_{ij}$ given our convention that we always integrate from the low to the high price.

Example 2 (CES with Expanding Varieties)**.** If the variety is entering, then the initial price is infinity $p_j = \infty$ in (4). Hence, in response to a change in the availability of some varieties of type j, the change in the downstream marginal cost is

$$\Delta \log mc_i = -\Omega_{ij} m_{ij} \delta_{ij} = -\frac{1}{\sigma - 1} \Omega_{ij} m_{ij}. \tag{5}$$

This is the so-called "love-of-variety" effect and is just the limiting case of quality-ladders where the step size is infinitely large.

Due to the near-ubiquitous use of the CES demand system, "love-of-variety" is sometimes conflated with the price elasticity of demand. However, as pointed out by Dixit and Stiglitz (1977), outside of the expanding-variety CES model, these two statistics are not the same. Under a plausible condition, we can show that the surplus produced by new varieties is maximized under the CES demand system.

Proposition 2 (Love-of-Variety with Marshall's Second Law)**.** *Denote the own-price elasticity of i's demand for input j by*

$$\sigma_{ij}(\boldsymbol{p}) = -\frac{\partial \log x_{ij}(\boldsymbol{p})}{\partial \log p_j} > 1.$$

Marshall's second law of demand *holds if* $\partial \sigma_{ij}/\partial p_j > 0$*. Under this condition,*

$$\delta_{ij}(\boldsymbol{p}, p_j') < \frac{1}{\sigma_{ij}(\boldsymbol{p}) - 1}\left[1 - \frac{p_j' x_{ij}(p_j')}{p_j x_{ij}(p_j)}\right], \tag{6}$$

as long as $\sigma_{ij}(\boldsymbol{p}) \geq 1$*.*

Note that the right-hand side of (6) is the infra-marginal surplus ratio implied by a CES demand system calibrated to match the initial price elasticity demand, the initial expenditure share, and the change in the expenditure share caused by the price jump.[7] Hence, the infra-marginal surplus ratio that is implied if one were to incorrectly impose CES input demand is strictly larger than the true one, as long as as Marshall's second law holds.[8] For a specific example, see Appendix B.

Proposition 1 motivates our regression specification in Section 3. Before discussing those results, however, we first compare Proposition 1 to the more traditional approach in the literature, following Feenstra (1994), which imposes a CES functional form.

2.2 Indirect Approach Exploiting CES

If we assume that technology is CES, then we can infer the value of supplier entry-exit using an alternative approach due to Feenstra (1994).

[7]See expression in (4) and use the fact that $(p_j'/p_j)^{1-\sigma} = (p_j' x_i(p_j'))/(p_j x_i(p_j))$.

[8]The proof builds on similar results in Matsuyama and Ushchev (2020) and Grossman et al. (2021). They prove a similar result assuming the input demand system belongs to the HSA/HDIA/HIIA class and the step size is infinite.

Proposition 3 (Feenstra, 1994)**.** *Suppose that the downstream firm has a CES technology with elasticity of substitution σ. Consider a change in the price of inputs by type $\Delta \boldsymbol{p}$, the measure of inputs whose price jumps $\boldsymbol{m}$, and the technology parameter ΔA_i. To a first-order approximation, the change in the downstream firm's marginal cost is*

$$\Delta \log mc_i \approx \sum_j \Omega_{ij} M_{ik} \Delta \log p_j + \frac{1}{\sigma - 1} \sum_j \Omega_{ij} M_{ij} \Delta \log \Omega_{ij} + \frac{\partial \log \mathcal{C}_i}{\partial \log A_i} \Delta \log A_i. \tag{7}$$

That is, as long as technology is CES, Proposition 3 allows us to infer the value of jumps by relying on the elasticity of substitution σ and the change in the share of non-jumping inputs.

Comparing equations (7) and (4) reveals the differences between Propositions 1 and 3, since (4) applies Proposition 1 under the additional assumption that input demand is CES. In equation (7) the coefficient of the change in the share of non-jumping inputs is always $1/(\sigma - 1)$ regardless of the size of the price jumps. On the other hand, in equation (4) the coefficient of the share of jumping inputs is equal to the inframarginal surplus ratio, which under CES is shaped both by σ and the size of the price jump. Another difference is that equation (7) uses the change in an expenditure share (of continuing suppliers) whereas (4) uses the level of an expenditure share (of entering/exiting suppliers). That is, both the right-hand side variable and the coefficient on the right-hand side variable associated with entry-exit are different in Propositions 1 and 3. A stark example is the case when $\sigma = 1$. In this case, Proposition 1 can still be used to recover the change in marginal cost induced by entry-exit (i.e. as in a quality-ladder model), but Proposition 3 cannot.

Under CES, both coefficients coincide only under expanding varieties (when the size of the jump is infinity). Furthermore, if the demand system is not CES, then Proposition 3 is no longer valid, whereas Proposition 1 continues to apply.

3 Empirical Microeconomic Results

Motivated by Propositions 1 and 3, we consider two different regressions aimed at identifying the benefits of inputs and the elasticity of substitution between continuing and non-continuing inputs.

3.1 Estimating Equations

Motivated by Proposition 1, we estimate the inframarginal surplus ratio, δ, by estimating the following regression

$$\Delta \log mc_{it} = \beta \times \text{separation share}_{it} + \text{controls}_{it} + \varepsilon_{it}, \tag{8}$$

where separation share$_{it}$ is the expenditure share of firm i in period t on those suppliers who ceased to be suppliers to firm i in period $t+1$. Following Proposition 1, the estimated coefficient $\hat{\beta}$ should reflect inframarginal surplus ratios if variation in the separation share is caused by jumps.

Proposition 1 enumerates some threats to identification if we rely on an OLS regression. First, the error term includes changes in prices of continuing suppliers and own technology shocks. These are plausibly correlated with the separation share. For example, it could be that exits are caused by changes in continuing suppliers' prices or shocks to the downstream firm's technology. Second, unconditionally, we do not know if a separation is due to an increase or a decrease in the input price. That is, a supplier could discontinue because the input price jumps up (i.e. the input becomes unavailable because the supplier ceases to sell the input) or because the input price jumps down (i.e. the supplier is replaced by a better alternative). To identify the inframarginal surplus ratios, we need to use supplier separations that are associated with input price jumps of a common sign rather than pooling all exits together.

If we impose the assumption that the downstream firms' technology is CES between continuing and non-continuing varieties, then following Proposition 3, we can identify the elasticity of substitution between continuing and non-continuing varieties by estimating the following regression:

$$\Delta \log mc_{it} = \beta \times \Delta \log \text{continuing share}_{it} + \text{controls}_{it} + \varepsilon_{it}, \tag{9}$$

where $\Delta \log \text{continuing share}_{it}$ is the log change in the expenditure share of firm i on continuing suppliers between period t and $t+1$.[9] Once again, the coefficient on the log change in continuing share should identify an average across downstream firms of $1/(\sigma-1)$, where σ is the elasticity of substitution between inputs, as long as CES is a valid assumption and the error term is uncorrelated with the log change in the continuing share. As explained in Section 2, regressions (8) and (9) estimate different objects even

[9] The continuing share at t equals one minus the separation share at t, and the continuing share at $t+1$ equals one minus the share on new suppliers at $t+1$. Hence, to a first order, $\Delta \log \text{continuing share}_{it}$ equals the share on new suppliers at $t+1$ minus the separation share at t.

if one assumes CES technology. Furthermore, regression (8) is motivated by Proposition 1 which holds under quite general technology, whereas regression (9) requires assuming CES technology between continuing and non-continuing suppliers. Of course, as with (8), endogeneity is a major concern since changes in the continuing share could be caused by changes in the prices of continuing suppliers or shocks to the downstream firms' technology.

To overcome the identification challenges, we use an instrumental variables strategy. We describe our instruments after describing the data sets we use.

3.2 Data

Our empirical analysis makes use of a rich micro-level data structure on Belgian firms in the period 2002-2018. The data structure brings together information drawn from six comprehensive panel-level data sets: (i) the National Bank of Belgium's (NBB) Central Balance Sheet Office (CBSO), which we refer to as the annual accounts; (ii) the Belgian Prodcom Survey, which covers firms that produce goods covered by the Prodcom classification and that have at least 20 employees or 5 million euros turnover in the previous reference year; (iii) the NBB Business-to-Business (B2B) Transactions data; (iv) International Trade data at the NBB; (v) VAT declarations; and (vi) the Crossroads Bank of Enterprises (CBE) which we use to identify Mergers and Acquisitions. Additional details are provided in Appendix C.

Below, we describe how we construct the different variables used in our analysis.

Network of Suppliers. We construct the network of domestic suppliers of Belgian firms using the confidential NBB B2B Transactions data set. This data set contains the values of yearly sales relationships among all VAT-liable companies for the years 2002 to 2018, and is based on the VAT listings collected by the tax authorities. At the end of every calendar year, all VAT-liable in Belgium have to file a complete listing of their Belgian VAT-liable customers over that year. An observation in this data set refers to the sales value in euro of enterprise j selling to enterprise i within Belgium, excluding the VAT amount due on these sales. The reported value is the sum of invoices from j to i in a given calendar year. As every firm in Belgium is required to report VAT on all sales of at least 250 euros, the data has universal coverage of all businesses active in Belgium.

We drop from the network those suppliers that produce capital goods, identified from the Main Industrial Groupings (MIG) Classification of the EU. To control for misreporting errors, we drop a transaction if its value is 10 times larger than the seller's aggregate sales or 10 times larger than the buyer's aggregate intermediaries (which is reported separately). We define a subset of suppliers composed of self-employed, government entities, and financial entities (SGF) which we exclude from our measures of exit and continuing suppliers, as described below.[10]

Sales. For each firm in Prodcom, we define firms' total sales as the highest value between sales reported in the annual accounts (reported mainly by large firms) and sales reported in the VAT declarations. We replace this measure of sales by the sum of exports reported in the international trade data set and sales to other Belgian firms reported in the B2B data set if the latter exceeds the prior. Our sample of "downstream" firms are firms in the Prodcom survey who file annual accounts and whose Prodcom sales are at least 30% of the firm's total sales (to ensure that Prodcom variables are representative of a firm's overall activities).

Costs. Firms' variable input costs consist of labor costs, the user cost of capital, and purchases of intermediates (excluding purchases of capital goods). Labor costs are reported in the annual accounts. The user cost of capital defined as the product of the capital stock reported in the annual accounts and the user cost of capital. The user cost of capital is the sum of a risk premium (set as 5 percent), the risk-free real rate (defined as the corresponding governmental 10 year-bonds nominal rate minus consumer price inflation at that time period), and the industry-level depreciation rate, $(1-d) \times g$, where d is the industry level depreciation rate (defined as consumption of fixed capital as a ratio of net capital stock) and g is the expected growth of the relative price of capital at the industry level (defined as the growth in the relative price of capital computed from the industry-specific investment price index relative to the consumer prices index in each year).

Purchases of intermediates are the sum of imports reported in the international trade data set and domestic intermediates purchased from other Belgian firms reported in the B2B data set. We do not include as part of intermediate consumption the goods purchased from other Belgian firms classified as capital goods providers, and we drop imported goods that are classified as capital goods in the Broad Economic Categories (BEC) classification (BEC codes 410 and 521), as these goods are not considered part of the vari-

[10]We exclude self-employed because of data-privacy considerations. We exclude government suppliers because a large part of their activities are not included in the VAT data and because our exit instruments are less relevant for public sector suppliers. We exclude financial entities because (i) banks fill special annual accounts that we do not have access to, (ii) interest receipts by banks are not included in the VAT data, and (iii) insurance premia receipts by insurance companies are not included in the VAT data. For similar reasons, from the set of non-financial suppliers we exclude financial activities (nace 64-66) and non-market services (nace 84 and higher, including education, health, art and entertainment).

able intermediate inputs bundle. We replace the sum of imports and domestic purchases by total sales minus value added reported in the annual accounts if the latter exceeds the former.[11]

For each firm i and period t, we identify in the B2B data set the set of continuing suppliers (excluding capital goods and SGF suppliers) from which firm i purchases intermediates both in period t and $t+1$. We measure $\Delta \log \text{continuing share}_{it}$ as the log change in firm i's intermediate purchases from its continuing links between t and $t+1$ minus the log change in i's total variable costs. We measure firm i's purchases of intermediates from its non-continuing (or separating) suppliers as the difference between i's purchases from all domestic suppliers and purchases from its continuing suppliers – both excluding capital and SFG suppliers. We calculate separation share_{it} as the ratio of i's purchases of intermediates from non-continuing suppliers relative to total variable costs.[12]

We trim the data for firm-year observations in which either total costs (the sum of inputs, labor, and capital) or total sales rise or fall by at least a factor of 5. Table C.8 in Appendix C reports summary statistics about the level and changes in the continuing share of suppliers, as well as basic information on the number of suppliers each firm has, and the share of intermediate materials as a share of total costs for our Prodcom sample.

Output prices. For each firm in Prodcom, we construct changes in output prices using sales and quantity data from Prodcom survey data. Products are identified at the 8-digit level of the Prodcom product code (PC) classification, which is common to all EU member states.[13] Sales values (in euros) and quantities are available at the firm-PC8-month level. Quantities are reported in one of several measurement units (over two thirds of observation are in kilograms; other units include liters, meters, square meters, kilowatt, and kg of active substance). We aggregate monthly observations to yearly values to match the

[11]For the few small set of suppliers with missing data on value added, we construct value added as the difference between sales (as defined above) and the maximum between intermediate purchases from the VAT declarations and sum of imports and domestic purchases of intermediates.

[12]We measure purchases on non-continuing suppliers as a residual — rather than directly from discontinued links in the B2B data set — because exiting suppliers tend to under-report B2B sales the year prior to disappearing. Table C.5 in the Appendix C shows that the share of B2B sales in total sales at t and the number of B2B costumers at t fall significantly for firms exiting at $t+1$ but not for firms exiting at $t+2$. Whereas sellers do not report B2B transactions reliably in the year prior to exit, buyers continue to report their total intermediate purchases. Table C.6 shows that buyers with an increase in the share of intermediate input purchases not reported in the B2B data tend to have a reduction in the number of their suppliers. This suggests that purchases of intermediates from suppliers that disappear in $t+1$ are unreliable at t in the B2B data set. We thus measure non-continuing purchases as a residual between total intermediates from the annual accounts (reported by the buyers) and those from the B2B data on continuing suppliers (reported by the sellers).

[13]As product codes tend to vary from year to year, we use the correspondence of 8-digit products in the Prodcom classifications that trace products over time used by Duprez and Magerman (2018).

other data sets. Changes in output prices are obtained as log differences in unit values from year $t-1$ to t for all PC8 products. For multi-product firms (defined as Prodcom firms that produce multiple PC8 products), changes in output prices for individual products are aggregated to the firm-level using a Divisia index, with weights given by the revenue share of each product in the corresponding year. This is valid if we assume that demand for multi-product firms in Prodcom is homothetic. In this case, a Divisia index reliably aggregates multiple products into a single product bundle. As output prices (and implied quantities) can be noisy, we trim the change in prices and quantities at the 5-95th percentile level.

Marginal cost and markups. We calculate the change in markup of each firm as the log change in the ratio of revenues to total variable costs. To measure the log change in the marginal cost of Prodcom firms, we subtract log changes in markups from log changes in output prices. We also report results where we measure changes in markups as the log change in revenues relative to purchases of intermediate inputs. The latter accurately measures changes in markups if the elasticity of output with respect to the bundle of intermediate inputs is constant.

Input prices. We observe a subset of input prices. We observe the price of suppliers who are themselves Prodcom firms (see Duprez and Magerman, 2018 and Cherchye et al., 2021), and we control for these input prices in our regressions. We also measure the price of labor by dividing total labor costs by total full time employed workers. We measure the price of capital services via the user cost of capital as described above. We measure the price of imported inputs using a firm-level Divisia index of changes in unit values faced by firm i at the CN8 product level. As unit values can be noisy, we trim the change in unit values at the 5th-95th percentile.

3.3 Identification Strategy and Results

To identify δ and σ in (8) and (9), we use an instrumental variables identification strategy. To identify δ, the instrument must induce variation in the separation share, must be associated with an increase in the input price that jumps (otherwise the sign is flipped), and it must not be correlated with own technology shocks or the prices of continuing suppliers.

We instrument the endogenous variable in (8) and (9) using a Bartik-type demand shock to the suppliers. For each downstream firm i at time t, we use the instrument

$$\text{Suppliers' Demand}_{it} = \sum_{j} \sum_{K} \Omega_{ij,t} \times r_{jK,t} \times \Delta \log \text{sales}_{K,t+1}, \tag{10}$$

where $\Omega_{ij,t}$ is the share of i's total variable costs spent on each supplier j, and $r_{jK,t}$ is the share of supplier j's sales to other domestic firms in each non-manufacturing industry K, and $\Delta \log sales_{K,t+1}$ is the change in total sales of industry K.[14] Intuitively, a reduction in the sales of i's suppliers, triggered by shocks to non-manufacturing industries, makes it more likely that i's suppliers shrink or shutdown operations (for example, due to the presence of overhead costs). This induces variation in the endogenous variable in equations (8) and (9) that is uncorrelated with technology shocks to i and continuing suppliers' prices.

The regression results for (8) are shown in Table 1. We start with the OLS results in Column (ii), which show that increases in the separation share are associated with small and statistically insignificant reductions in marginal cost. Of course, the OLS is subject to severe omitted variable bias. For example, exiting suppliers could be replaced by better suppliers, as in models of creative destruction, flipping the sign on the coefficient in front of the exit share in Proposition 1. Or, a positive productivity shock to the downstream firm may induce the firm to switch suppliers or perform some operation in-house. For these reasons, we instrument the separation share with demand shocks to suppliers.

As a preliminary step to understand how our instrument works, column (i) is an OLS regression of a $\{0, 1\}$ indicator of supplier exit on the Bartik-style demand instrument constructed for the supplier itself, and a 4-digit industry $\times$ year fixed effect. We see that an increase in demand for the supplier predicts a decline in supplier death.[15] That is, when suppliers get favorable demand shocks, they are less likely to cease operations. Our instrument, defined in (10) is, for each downstream firm, the average demand shock for this firm's suppliers. Hence, our instrument induces variation in the separation share by, at least partially, causing existing suppliers to exit due to unfavorable demand shocks.

Column (iii) is a reduced-form regression regressing changes in marginal cost directly on our instrument. This shows that increased demand for a firms' suppliers reduces that firm's marginal cost. Columns (iv) and (v) run regression (8) using our suppliers' demand instrument, first without and then with controls. All regressions include 6 digit product

[14] Results are very similar if $r_{jK,t}$ is calculated as the share of supplier j's total sales (rather than domestic sales) to each non-manufacturing industry K, or if $\Delta \log \text{sales}_{K,t+1}$ is the change in intermediate consumption of industry K (rather than the change in total sales).

[15] Table C.10 in Appendix D tabulates unconditional death rates for firms year by year in Belgium for small and large firms.

code by year fixed effects.[16] Column (vi) adds a firm fixed effects, allowing for the possibility of firm-level trends. Column (vii) weights observations by log sales. Column (viii) constructs the instrument using lagged sales shares. In all cases, the first-stage is strong (demand shocks to a downstream firm's suppliers help predict separation between the firm and those suppliers, conditional on other controls). Moreover, the second stage estimates are positive and significant. The point estimates imply that $\delta \approx 0.6$. If technology is CES and there are expanding varieties, then $\delta = 0.6$ corresponds to a CES elasticity of substitution of a little higher than 2.5. On the other hand, in a typical quality ladders model with unitary elasticity across inputs, the implied step size is roughly 60 log points. Either way, marginal costs of downstream firms react very strongly to separations with its suppliers.

Table 1: Estimating δ

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
	Supplier Exit	$\Delta \log mc$						
Separation share		-0.013		0.744***	0.677***	0.869***	0.661***	0.698**
		(0.008)		(0.167)	(0.161)	(0.194)	(0.159)	(0.157)
Supplier Demand	-0.54***		-0.999***					
	(0.003)		(0.199)					
F-stat				62	63	91	61	71
Specification	OLS	OLS	RF	IV	IV	IV	IV	IV
Controls	N	N	N	N	Y	Y	Y	Y
4 digit x year FE	Y	N	N	N	N	N	N	N
6 digit x year FE	N	Y	Y	Y	Y	Y	Y	Y
Firm FE	N	N	N	N	N	Y	N	N
Obs	5,088,039	39,123	39,123	39,123	39,123	38,351	39,123	39,123

Notes: Column (i) reports estimates of a regression of supplier death on the supplier's demand shock, and Columns (ii)-(viii) report estimates of regression (8). Demand shock is the instrument in the IV regressions and is defined by (10). Supplier death is an indicator for suppliers who ceased operations. Controls are log changes in the price of imported inputs, log changes in the price of inputs purchased from other Prodcom firms, changes in log wages, changes in the log user cost of capital, and a Bartik-type demand shock constructed for the downstream firm itself. All regressions are unweighted except (vii), which is weighted by log sales. Column (viii) uses lagged shares at $t-1$ instead of initial t shares in constructing the instrument. Standard errors are clustered at the firm-level.

We consider the following sensitivity checks of our baseline results, reported in Tables D.11, D.12, and D.13 in Appendix D. First, we vary the product disaggregation in the product $\times$ year fixed effects, considering 4 or 8 digit products rather than 6 digits. With more stringent fixed effects, estimates are fairly robust but point estimates are a bit smaller (Table D.11 reports more complete results with 8 digit fixed effects). Second, we drop

[16] For multi-product firms, we use the product code of the product with the greatest sales share.

downstream firms that switch their output product mix between years.[17] Third, when constructing the suppliers' demand instrument according to (10), we redefine $r_{jK,t}$ to be the share of supplier j's *non-manufacturing sales* to each non-manufacturing industry K (these share add up to one), and include a firm fixed effect to take into account the firm's average exposure to non-manufacturing sales. Fourth, we exclude from our separation share measure suppliers either in the utilities sector or in the wholesale/retail sector. This increases point estimates slightly. Fifth, we do not exclude SGF suppliers, as well as both SGF and capital input suppliers, from our separation share measure. This increases the point estimates. Sixth, we measure changes in markups by changes in the share of materials to sales. Seventh, we change our sample selection by varying the trimming of price, quantity, and cost changes and by altering the minimum threshold in the ratio of a firm's Prodcom sales to the firm's aggregate sales from the annual accounts. Across all of these sensitivities, we continue to find positive and significant point estimates.

Table 2: Other outcomes

	(i)	(ii)	(iii)	(iv)
	$\Delta \log mc$ no K	$\Delta \log P$	$\Delta \log \mu$	$\Delta_2 \log mc$
Separation share	0.690***	-0.061	-0.738***	1.013***
	(0.163)	(0.054)	(0.156)	(0.255)
Specification	IV	IV	IV	IV
Controls	Yes	Yes	Yes	Yes
F-stat	63	63	63	44
6 digit x year FE	Y	Y	Y	Y

Notes: This tables displays estimates of regression (8) for different outcome variables.The instrument in the IV regressions is the suppliers' demand shock defined by (10). Controls are log changes in the price of imported inputs, log changes in the price of inputs purchased from other Prodcom firms, changes in log wages, changes in the log user cost of capital, and a Bartik-type demand shock constructed for the downstream firm itself. All regressions are unweighted. Standard errors are clustered at the firm-level.

Table 2 shows results for other left-hand-side variables using the Bartik supplier's demand instrument. Column (i) uses a measure of marginal cost which leaves out the user cost of capital, and shows that the results are very similar. Column (ii) replaces marginal cost with the output price of the downsteam firm and shows that the pass-through of these marginal cost shocks is close to zero in our context. This pass-through estimate is reduced-form, and does not have a simple structural interpretation, since pass-through

[17]We also consider another sensitivity in which we measure price using the downstream firm's largest 8-digit product (rather than averaging across all products).

generically depends not just on technology, but also market structure and conduct. The very low reduced-form pass-through we estimate could be due to strategic complementarities in firms' pricing decisions and/or sticky prices.[18] Column (iii) shows that, predictably given the results in column (ii), it is the downstream firm's markup (defined as the ratio of price to marginal cost) that responds one-for-one to the marginal cost shock. Column (iv) uses two-year changes in marginal costs as the outcome and shows that for the types of supplier separations caused by our instrument, the effects are persistent.

Table 3 shows results for regression (9). Here, we instrument using the suppliers' demand shock for the change in continuing share rather than the separation share, and the coefficient identifies $1/(\sigma-1)$ under the assumption that the input technology is CES. Columns (i) and (ii) include 6 digit product code by year fixed effects, and columns (iii) and (iv) have 8 digit product code by year fixed effects. The point estimates are slightly smaller with 8 digit product fixed effects. On average our estimates suggest that $1/(\sigma-1) \approx 0.5$ or that $\sigma \approx 3$.

Table 3: Estimating $1/(\sigma-1)$

	(i)	(ii)	(iii)	(iv)
	$\Delta \log mc$			
Continuing share	0.579***	0.534**	0.448***	0.415***
	(0.125)	(0.123)	(0.113)	(0.113)
F-stat	79	79	74	74
Controls	N	Y	N	Y
FE	6 digit x year	6 digit x year	8 digit x year	8 digit x year
Obs	39,123	39,123	35,135	35,135

Notes: Estimates of regression (9). The instrument is the demand shock defined by (10). Controls are log changes in the price of imported inputs, log changes in the price of inputs purchased from other Prodcom firms, changes in log wages, changes in the log user cost of capital, a Bartik-type demand shock constructed for the downstream firm itself, and the firm's own short-term debt obligations interacted with interest rate changes. Standard errors are clustered at the firm-level.

Alternative instrument As a final robustness exercise, we consider an alternative instrument. Rather than using suppliers' demand shocks, we construct an instrument that induces variation in suppliers' financial health. For each downstream firm i in period t,

[18] Using the alternative instrument based on the financial position of suppliers, we find that pass-through is incomplete but positive (see Table 4). Of course, the degree of pass-through from costs to prices can vary with the nature of the shock, depending on the model of pricing, and our micro model is silent on this.

we construct the following variable

$$\text{Rate shock}_{it} = \sum_j \sum_K \Omega_{ij,t} \times d_{j,t} \times \Delta \text{R}_{t+1}, \tag{11}$$

where $\Omega_{ij,t}$ are the expenditures of firm i on supplier j as a share of i's total costs, $d_{j,t}$ are the short-term debt obligations of j as a share of total assets (from the annual accounts), and ΔR_{t+1} is the change in the 1-month money market interest rate for the euro area. An increase in this variable indicates a negative financial shock to i's suppliers.

Table 4: Estimating δ using alternative instrument

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
	Supplier Death			$\Delta \log mc$			$\Delta \log P$
Separation share		-0.013		0.617*	0.595**	0.628**	0.385**
		(0.008)		(0.322)	(0.290)	(0.291)	(0.176)
Rate shock	0.002**		0.052**				
	(0.001)		(0.022)				
F-stat				18	21	21	21
Specification	OLS	OLS	RF	IV	IV	IV	IV
Controls	N	N	N	N	Y	Y	Y
4 digit x year FE	Y	N	N	N	N	N	N
6 digit x year FE	N	Y	Y	Y	Y	Y	Y
Firm FE	N	N	N	N	N	N	N
Obs	4,324,156	39,123	39,123	39,123	39,123	39,123	39,123

Notes: Column (i) reports estimates of a regression of supplier death on the interest rate instrument, and Columns (ii)-(viii) report estimates of regression (8). Rate shock is the instrument in the IV regressions and is defined by (11). Supplier death is an indicator variable for suppliers who ceased operations. Controls are log changes in the price of imported inputs, log changes in the price of inputs purchased from other Prodcom firms, changes in log wages, changes in the log user cost of capital, and the firm's own short-term debt obligations interacted with interest rate changes. All regressions are unweighted except (vi), which is weighted by log sales. Standard errors are clustered at the firm-level.

The regression results are shown in Table 4. Column (i) shows that an increase in financial shock to suppliers makes it more likely that the supplier ceases operations. That is, when suppliers get unfavorable financial shocks, they are more likely to exit. As before, Column (ii) is the OLS showing that increases in the separation share are associated with reductions in marginal cost, albeit the magnitude is very small and statistically insignificant. Column (iii) is the reduced-form regression showing that worse financial conditions for suppliers predict an increase in the downstream firm's marginal cost. Column (iv) and (v) are the IV regressions (8) using the financial shock instrument, first without and then with controls. All regressions include 6 digit product code by year fixed effects. Column

(vi) weights by log sales. The estimated coefficients in the IV regressions are similar to those in Table 1, suggesting that $\delta \approx 0.6$. The final column, (vii), replaces marginal cost with the price as the left-hand side variable. Whereas the estimated δ is broadly similar for the two instruments, the estimated pass-through of the marginal cost shock into the price is different. For the financial instrument, we find that pass-through is positive but incomplete (roughly around 50%). Since we do not model firms' pricing decisions, these reduced-form estimates of pass-through do not have structural interpretations.

4 Macroeconomic Value of Link Formation: Theory

In the previous section, we estimated the area under the input demand curve and found that input suppliers generate a considerable amount of inframarginal surplus for their downstream customers. We now develop a growth accounting framework that can account for the value of supplier churn in aggregate growth. We discipline our macro growth accounting results using estimates from the micro sample. Our micro regressions are estimated using only the Prodcom sample of manufacturing firms. However, we apply our growth accounting formulas to a much larger sample of Belgian firms.

We specify minimal structure on the aggregative model and do not fully specify the environment. This is because we take advantage of the fact that endogenous variables, like changes in factor prices, are directly observable and capture whatever resource constraints the economy is subject to. Our goal, in this section, is to develop the theoretical apparatus for aggregation. We explicitly account for how changes in one firm's marginal cost, due to entry and exit of its suppliers, spill over to that firms' customers, customers' customers, and so on. This exercise allows us to decompose the fraction of aggregate productivity growth that can be accounted for by churn in supply chains.

4.1 Definitions and Environment

Consider a set of producers denoted by N, called the *network*. There is a set of *external inputs* denoted by F. An external input is an input used by producers in the network, N, that those producers do not themselves produce. In practice, the set F includes labor, capital, and intermediate inputs purchased from firms not in the network N. The firms in N collectively produce *final outputs*. Final output is the production by firms in N that firms in N do not themselves use. A stylized representation is given in Figure 2 showing the flow of goods and services.

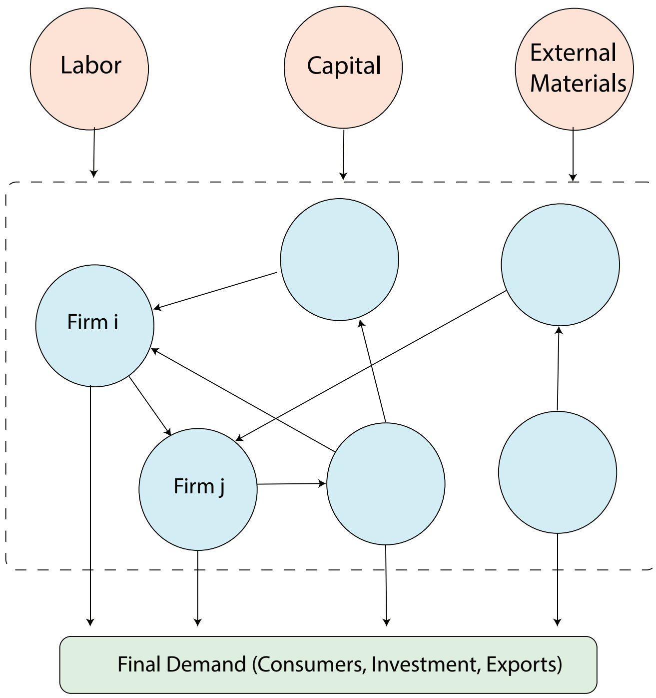


Figure 2: Graphical illustration of the economy. External inputs are red nodes and final output are green nodes. The set N is depicted by the dotted line.

Production. Each producer $i \in N$ has a constant-returns-to-scale production technology in period t given by

$$q_{i,t} = A_{i,t} F_{i,t} \left(\{x_{ij,t}\}_{j \in N}, \{l_{if,t}\}_{f \in F} \right).$$

In the expression above, $l_{if,t}$ is the quantity of external input f and $x_{ij,t}$ is the quantity of intermediate input j used by i at time t. The exogenous parameter $A_{i,t}$ is a technological shifter. There may be fixed costs that must be paid in addition to the variable production technology defined above, but we do not take a stance on these fixed costs for the time being. Importantly, we abstract from multi-product firms and associate each input with a specific supplying firm.

Each producer solves the cost-minimization problem

$$\min_{x_{ij,t}, l_{if,t}} \sum_{j \in N} p_{j,t} x_{ij,t} + \sum_{f \in F} w_{f,t} l_{if,t},$$

subject to their production technology, where $p_{j,t}$ and $w_{f,t}$ are the prices of internal and external inputs. Define the markup $\mu_{i,t}$ charged by each producer i is defined to be the ratio of its price $p_{i,t}$ and its marginal cost.

We say that good i is a *continuing* good between t and $t+1$ if $q_{i,t} \times q_{i,t+1} > 0$. Denote by C_t the set of all goods who are continuing at time t.

Resource Constraints. We construct a measure of net or final production by the set of continuing, C_t, firms. Let the total quantity of external inputs used by continuing firms be

$$L_{f,t} = \sum_{i \in C_t} l_{if,t} + \sum_{i \in C_t} l_{if,t}^{\text{fixed}},$$

where $l_{if,t}$ is used in variable production and $l_{if,t}^{\text{fixed}}$ are fixed costs. Firm i's final output is defined to be the quantity of its production that is not sold to other firms in C_t:

$$y_{i,t} = q_{i,t} - \sum_{j \in C_t} x_{ji,t}.$$

That is, final output of good $i \in C_t$, denoted by $y_{i,t}$, is the quantity produced of i that is not used by any $j \in C_t$ and is either consumed by households, used for investment, sold as exports, or sold to other suppliers that are not in the network of continuing producers.

The *final output price deflator* is defined to be the share-weighted change in the price of continuing goods

$$d \log P_t^Y = \sum_{i \in C_t} b_{i,t} d \log p_{i,t},$$

where

$$b_{i,t} = \frac{p_{i,t} y_{i,t}}{\sum_{j \in C_t} p_{j,t} y_{j,t}}.$$

Growth in *real final output* of the set of continuing goods, denoted by $d \log Y_t$, is the change in nominal final output deflated by the final output price deflator:

$$d \log Y_t = d \log \left(\sum_{i \in C_t} p_{i,t} y_{i,t} \right) - d \log P_t^Y. \tag{12}$$

To calculate growth in real final output between two time periods, we cumulate $d \log Y$ over time:

$$\log Y_{t+T} - \log Y_t = \sum_{s=t}^{t+T} d \log Y_s.$$

Our objective is to decompose the contribution of supplier churn to growth in real final output.

4.2 Theoretical Results

To state our decomposition result, we need to first set up some input-output notation. Define the $C_t \times C_t$ cost-based input-output network of continuing firms to have ijth element equal to:

$$\Omega_{ij,t} = \frac{p_{j,t} x_{ij,t}}{\sum_{k \in C_t} p_{k,t} x_{ik,t} + \sum_{f \in F} w_{f,t} l_{if,t}}.$$

Let Ω^F be the $C_t \times F$ matrix of external input usages, where the ifth element is

$$\Omega^F_{if,t} = \frac{w_{f,t} l_{if,t}}{\sum_{k \in C_t} p_k x_{ik} + \sum_{f \in F} w_f l_{if}}.$$

Group inputs of each continuing firm i into $\mathcal{J}_i$ types (similar to Section 4). Let $M_{iJ,t}$ be the mass of inputs of type $J \in \mathcal{J}_i$ used by firm i at time t. Firm i adds suppliers of type J if $\Delta M_{iJ,t} > 0$ and removes suppliers if $\Delta M_{iJ,t} < 0$. Denote the per-variety expenditure share on type J inputs by $\Omega_{iJ,t}$. The average infra-marginal surplus for entering suppliers is

$$\bar{\delta}^{\text{entry}}_{i,t} = \sum_{\substack{\Delta M_{iJ,t} > 0 \\ J \in \mathcal{J}_i}} \frac{\Omega_{iJ,t} \Delta M_{iJ,t}}{\sum_{\substack{\Delta M_{iK,t} > 0 \\ K \in \mathcal{J}_i}} \Omega_{iK,t} \Delta M_{iK,t}} \delta_{iJ,t}(p_{J,t}, \infty)$$

and the average infra-marginal surplus for exiting suppliers is

$$\bar{\delta}^{\text{exit}}_{i,t} = \sum_{\substack{\Delta M_{iJ,t} < 0 \\ J \in \mathcal{J}_i}} \frac{\Omega_{iJ,t} \Delta M_{iJ,t}}{\sum_{\substack{\Delta M_{iK,t} < 0 \\ K \in \mathcal{J}_i}} \Omega_{iK,t} \Delta M_{iK,t}} \delta_{iJ,t}(p_{J,t}, \infty).$$

This representation can capture both expanding variety models and quality-ladder models. To capture a movement along a quality ladder, we consider the simultaneous addition and removal of supplier-pairs. That is, if an input climbs the quality ladder, a low quality supplier is eliminated and a high quality supplier is added. See Appendix E for more details and an example.

Define the set of continuing suppliers for firm i by $C_{i,t}$. That is,

$$C_{i,t} = \{ j \in C_t : x_{ij,t} \times x_{ij,t+1} > 0 \}.$$

We assume that $C_{i,t}$ is non-empty and denote by

$$\Delta \mathcal{E}_{i,t} = - \left(\sum_{J \in \mathcal{J}_i} M_{iJ,t} \Omega_{iJ,t} \right) \log \left(\frac{\sum_{j \in C_{i,t}} p_{j,t+1} x_{ij,t+1} / \sum_{k \in N} p_{k,t+1} x_{ik,t+1}}{\sum_{j \in C_{i,t}} p_{j,t} x_{ij,t} / \sum_{k \in N} p_{k,t} x_{ik,t}} \right),$$

the negative log change in continuing inputs' expenditure share between t and $t+1$

weighted by the material share of total variable cost. Denote by

$$\Delta \mathcal{D}_{it} = \left(\sum_{J \in \mathcal{J}_i} M_{iJ,t} \Omega_{iJ,t} \right) \left(1 - \frac{\sum_{j \in C_{i,t}} p_{j,t+1} x_{ij,t+1}}{\sum_k p_{k,t+1} x_{ik,t+1}} \right)$$

the expenditure share on entering suppliers weighted by the material share of total variable cost. The following lemma, which is a consequence of Proposition 1, shows that the effect of supplier churn on the downstream firm's marginal cost can be written in terms of $\Delta \mathcal{E}_{i,t}$ and $\Delta \mathcal{D}_{it}$.

Lemma 1 (Decomposition of Marginal Cost)**.** *Consider a change in the price of inputs $\Delta \boldsymbol{p}_t$, the measure of inputs by type $\Delta \boldsymbol{M_{i,t}}$, and the technology parameter $\Delta A_{i,t}$. Let $\Delta \mu_{i,t}$ be the change in markups. To a first-order approximation, the change in the price of each continuing firm i is given by*

$$\Delta \log p_{i,t} \approx \Delta \log \frac{\mu_{i,t}}{A_{it}} + \sum_{j \in \mathcal{J}_i} \Omega_{ij,t} \Delta \log p_{j,t} + \sum_{f \in F} \Omega^F_{if,t} d \log w_{f,t} - \bar{\delta}^{exit}_{i,t} \Delta \mathcal{E}_{i,t} + (\bar{\delta}^{exit}_{i,t} - \bar{\delta}^{entry}_{i,t}) \Delta \mathcal{D}_{it}.$$

The first three summands are standard, reflecting changes in *i*'s own markup and technology as well as changes in the prices of *i*'s continuing suppliers and external inputs. The fourth summand reflects changes in *i*'s marginal cost due to churning of suppliers assuming that the average inframarginal surplus created by entering and exiting suppliers is the same. The final term accounts for the discrepancy between the average inframarginal surplus of entering and exiting suppliers.

Lemma 1 is a useful reformulation of Proposition 1 since it allows us to summarize heterogeneous extensive margin effects into two sufficient statistics: $\bar{\delta}^{\text{exit}}$ and $\bar{\delta}^{\text{exit}} - \bar{\delta}^{\text{entry}}$. These sufficient statistics are multiplied by observable statistics: changes in the share of continuing suppliers and the share of entering suppliers. If we calibrate $\bar{\delta}^{\text{exit}}$ and $\bar{\delta}^{\text{entry}}$, then using observational data on expenditures, from say VAT returns, we can infer the effect of extensive margin adjustments on every firm's price without needing to measure the price of every firm in the economy.

The following corollary specializes Lemma 1 to the CES special case.

Corollary 1 (CES Special Case)**.** *If i's production technology is CES with elasticity of substitution $\sigma > 1$, then*

$$\bar{\delta}^{exit} = \bar{\delta}^{entry} = \frac{1}{\sigma - 1}.$$

Hence,

$$\Delta \log p_{i,t} \approx \Delta \log \frac{\mu_{i,t}}{A_{it}} + \sum_{j \in \mathcal{J}_i} \Omega_{ij,t} \Delta \log p_{j,t} + \sum_{f \in F} \Omega^F_{if,t} d \log w_{f,t} - \frac{1}{\sigma - 1} \Delta \mathcal{E}_{i,t}.$$

CES input demand is a useful benchmark since it greatly simplifies the expression in Lemma 1. Under CES, the treatment effect associated with each entry or exit event is just the expenditure share of that supplier multiplied by $1/(1-\sigma)$ — there is no heterogeneity in inframarginal surplus and entry is as beneficial as exit is costly per dollar of spending. Furthermore, since inframarginal surplus is constant, if we know it, then changes in the continuing input share are all we need to measure over time to see the effect of the extensive margin on marginal cost.[19]

Lemma 1 is about a single firm, but we can build on it to decompose aggregate growth $d \log Y_t$. To do this, note that, in matrix notation, Lemma 1 can be rewritten as

$$d \log \boldsymbol{p}_t = d \log \boldsymbol{\mu}_t - d \log \boldsymbol{A}_t + \Omega_t d \log \boldsymbol{p}_t + \Omega^F_t d \log \boldsymbol{w}_t - \bar{\boldsymbol{\delta}}^{\text{exit}}_t \Delta \mathcal{E}_t + (\bar{\boldsymbol{\delta}}^{\text{exit}}_t - \bar{\boldsymbol{\delta}}^{\text{entry}}_t) \Delta \mathcal{D}_t.$$

Define the *cost-based continuing* Leontief inverse to be

$$\Psi_t = (I - \Omega_t)^{-1} = \sum_{s=0}^{\infty} \Omega^s_t.$$

Then, we can solve out for changes in the prices of continuing firms:

$$d \log \boldsymbol{p}_t = \Psi_t \left[d \log \boldsymbol{\mu}_t - d \log \boldsymbol{A}_t + \Omega^F_t d \log \boldsymbol{w}_t - \bar{\boldsymbol{\delta}}^{\text{exit}}_t \Delta \mathcal{E}_t + (\bar{\boldsymbol{\delta}}^{\text{exit}}_t - \bar{\boldsymbol{\delta}}^{\text{exit}}_t) \Delta \mathcal{D}_t \right]. \quad (13)$$

That is, changes in the price of continuing goods depend on changes in markups, $d \log \boldsymbol{\mu}_t$, productivity shifters, $d \log \boldsymbol{A}_t$, prices of external inputs, $d \log \boldsymbol{w}_t$, as well as the extensive margin terms. All of these effects are mediated by the forward linkages in the Leontief inverse Ψ_t.

[19] As long as input demand is CES, Lemma 1 applies, regardless of whether supplier churn occurs according to a quality-ladder or expanding-varieties model. This is because, as mentioned earlier, in Section 4, we model a movement along the quality-ladder as the simultaneous addition and subtraction of a supplier pair. In this case, both the entering and exiting supplier's inframarginal surplus per unit of expenditure is $1/(\sigma - 1)$, but the downstream firms' marginal cost will rise or fall depending on whether expenditures on the entering supplier are higher or lower than the exiting supplier. This corollary is a different perspective on Feenstra (1994). The derivation is different since in deriving it, we must assume that $\sigma < 1$, because $\bar{\delta}^{\text{exit}} = \bar{\delta}^{\text{entry}} = \infty$ when $\sigma \leq 1$. In contrast, the formula in Feenstra (1994) applies even when $\sigma < 1$.

Define the *revenue-based Domar weight* of $i \in C_t$ and $f \in F$ to be

$$\lambda_{i,t} = \frac{p_{i,t} q_{i,t}}{\sum_{j \in C_t} p_{j,t} y_{j,t}}, \quad \text{and} \quad \Lambda_{f,t} = \frac{\sum_{i \in C_t} w_{f,t} l_{f,t}}{\sum_{j \in C_t} p_{j,t} y_{j,t}}.$$

Define the *cost-based continuing Domar weights* for $i \in C_t$ and $f \in F$ to be

$$\tilde{\lambda}_{i,t} = \sum_{j \in C_t} b_{j,t} \Psi_{ji,t}, \quad \text{and} \quad \tilde{\Lambda}_{f,t} = \sum_{j \in C_t} \tilde{\lambda}_{j,t} \Omega^F_{f,t}.$$

This weights the exposure of each continuing firm j to each continuing supplier i, captured by $\Psi_{ji,t}$, by the importance of j in the final output price $b_{j,t}$. Substituting (13) into the definition of the final output price deflator yields

$$d \log P^Y_t = \sum_{i \in C_t} \tilde{\lambda}_{i,t} \left[d \log \frac{\mu_{i,t}}{A_{i,t}} - \bar{\delta}^{\text{exit}}_{i,t} \Delta \mathcal{E}_{i,t} + (\bar{\delta}^{\text{exit}}_{i,t} - \bar{\delta}^{\text{exit}}_{i,t}) \Delta \mathcal{D}_{i,t} \right] + \sum_{f \in F} \tilde{\Lambda}_{f,t} d \log w_{f,t}.$$

That is, shocks to i are transmitted into the final output price according to the cost-based Domar weight $\tilde{\lambda}_{i,t}$. Similarly, changes in the price of external input f affects the final output price deflator according to its cost-based Domar weight $\tilde{\Lambda}_{f,t}$.

Plugging this into the definition of real final output in (12) yields the following decomposition.

Proposition 4 (Growth-Accounting with Entry-Exit)**.** *The change in real final output is given, to a first-order, by*

$$\Delta \log Y_t = \underbrace{\sum_{i \in C_t} \tilde{\lambda}_{i,t} \Delta \log A_{i,t}}_{\textit{technology}} + \underbrace{\sum_{f \in F} \tilde{\Lambda}_{f,t} \Delta \log L_{f,t}}_{\textit{factor quantities}} - \underbrace{\sum_{i \in C_t} \tilde{\lambda}_{i,t} \Delta \log \mu_{i,t}}_{\textit{markups}} - \underbrace{\sum_{f \in F} \tilde{\Lambda}_{f,t} \Delta \log \Lambda_{f,t}}_{\textit{factor shares}} + \underbrace{\sum_{i \in C_t} \tilde{\lambda}_{i,t} \left(\bar{\delta}^{exit}_{i,t} \Delta \mathcal{E}_{i,t} + (\bar{\delta}^{entry}_{i,t} - \bar{\delta}^{exit}_{i,t}) \Delta \mathcal{D}_{i,t} \right)}_{\textit{extensive-margin}}.$$

Aggregate output growth can be broken down into different components.[20] We de-

[20]For counterfactuals, we need to be able to solve for changes in factor shares $d \log \Lambda$. This requires modelling the details of fixed costs and entry decisions. However, conditional on changes in factor shares, we do not need to specify these details.

scribe the different terms in sequence starting with the first line. The first term is exogenous productivity growth weighted by cost-based Domar weights. This accounts for how exogenous improvements in technology affect output, accounting for the fact that improvements in each firm's technology will mechanically raise production by its consumers, and its consumers' consumers, and so on. The second term captures a similar effect but for changes in factor quantities — if the quantity of factor f rises, then that raises the production of all firms that use factor f, which raises the production of all firms that use the products of factor f, and so on.

The second line captures the way changes in markups and factor prices affect output. An increase in i's markup will raise i's price, which raises the costs of production for i's consumers, and i's consumers' consumers, and so on. Similarly, if the Domar weight Λ_f of factor f rises more quickly than the quantity L_f of factor f, then this means that the relative price of factor f has increased. An increase in f's price will raise the costs of production for all firms.

The last line is what this paper is focused on and captures the effects of supplier churn. Churn at the level of each individual firm percolates to the rest of the economy through the input-output network and this effect is captured by weighing the extensive margin terms from Lemma 1 by the cost-based Domar weight of each firm and summing across all firms. This captures the idea that if one firm's marginal costs change from entry-exit of its suppliers, then those marginal cost changes will propagate to that firms' consumers, its consumers' consumers, and so on.

4.3 Special Cases of Growth Accounting

To better understand the intuition for Proposition 4, it helps to consider some special cases.

Corollary 2 (Neoclassical Economy without Entry-Exit)**.** *For an efficient economy with no markups and no entry-exit margin, the change in aggregate output is*

$$\Delta \log Y_t = \sum_{i \in N} \lambda_{i,t} \Delta \log A_{i,t} + \sum_{f \in F} \Lambda_{f,t} \Delta \log L_{f,t}.$$

To derive this from Proposition 4, note that there are no markups, cost-based and revenue-based Domar weights are the same. Furthermore, since all firms are continuing and there are no profits, $\sum_{f \in F} \tilde{\Lambda}_f \Delta \log \Lambda_f \approx \sum_{f \in F} \Delta \Lambda_f \approx 0$, where the final equality follows from the fact that expenditures on external inputs must equal total final output since firms earn no profits. Finally, since there is no extensive margin, $\Delta \mathcal{E}_{i,t} = \Delta \mathcal{D}_{i,t} = 0$.

In other words, under these assumptions, output growth is the sum of technology and external input growth weighted by sales. This is the neoclassical case considered by Solow (1957), Domar (1961), and Hulten (1978).

Corollary 3 (Markups without Entry-Exit)**.** *For an economy with no entry-exit, the change in aggregate output is*

$$\Delta \log Y_t = \sum_{i \in N} \tilde{\lambda}_{i,t} \Delta \log A_{i,t} + \sum_{f \in F} \tilde{\Lambda}_{f,t} \Delta \log L_{f,t} - \sum_{i \in N} \tilde{\lambda}_{i,t} \Delta \log \mu_{i,t} - \sum_{f \in F} \tilde{\Lambda}_{f,t} \Delta \log \Lambda_{f,t}.$$

This is the environment considered by Baqaee and Farhi (2019). The first two terms measure the increase in output growth due to the increase in technology and inputs, holding fixed the allocation of resources, and the latter two terms measure the effect of changes in the allocation of resources. Reallocations are beneficial if the reduction in factor prices, as measured by $-\sum_{f \in F} \tilde{\Lambda}_{f,t} \Delta \log \Lambda_{f,t}$, outpace the increases in prices caused by markups $\sum_{i \in N} \tilde{\lambda}_{i,t} \Delta \log \mu_{i,t}$. Intuitively, if factor shares fall by more than markups rise, then this indicates that resources are being reallocated to high-markup firms. Since those firms are initially too small from a social perspective, this reallocation boosts aggregate output.

Corollary 4 (Constant Non-Zero Markups and Zero Profits)**.** *For an economy with CES input demand, monopolistic competition, a single external input (labor), and a zero-profit condition, we have*

$$\Delta \log Y_t = \sum_{i \in C_t} \tilde{\lambda}_{i,t} \Delta \log A_{i,t} + \Delta \log L_{f,t} + \sum_{i \in C_t} \tilde{\lambda}_{i,t} \frac{1}{\sigma_i - 1} \Delta \mathcal{E}_{i,t},$$

where σ_i is the elasticity of substitution among input varieties in i's production function.

The economy above nests Melitz (2003) and the input-output model in Baqaee (2018). Mechanically, monopolistic competition with CES implies constant markups, so that $\Delta \log \mu_i = 0$. The zero-profit condition with a single factor implies that $\Delta \log \Lambda_f = 0$. Substituting these into Proposition 4 yields the result. That is, similar to traditional neoclassical models, exogenous technology growth $\Delta \log A$ and factor growth $\Delta \log L$ can raise final output. However, there is new term involving churn in the supply chain.

The final term measures the importance of supplier churn — if suppliers are added or discontinued in equilibrium in response to shocks, then these will affect marginal cost of downstream firms. These marginal cost shocks will then spill-over to other producers and the importance of these spill-overs is captured by the cost-based continuing Domar weight $\tilde{\lambda}_{it}$. Intuitively, supplier churn is especially powerful when the infra-marginal surplus ratio, $\bar{\delta}^{\text{exit}}_{i,t}$, is high, and the cost-based Domar weight, $\tilde{\lambda}_{it}$, is large. The extensive margin term would be zero in a counterfactual in which the change in the price of inputs that enter or exit is equal to the change in the price of continuing inputs.

5 Empirical Macroeconomic Results

In this section, we apply Proposition 4 to decompose aggregate growth for a large subset of the Belgian economy. We begin by describing how we map the data to the terms in Proposition 4 before showing the results.

5.1 Mapping to Data

To apply Proposition 4, we need to define the set of continuing firms C_t, the elasticity parameters $\bar{\delta}_{i,t}^{\text{exit}}$ and $\bar{\delta}_{i,t}^{\text{entry}}$, the matrices Ω_t and Ω_t^F for all continuing firms in Belgium, markups $\mu_{i,t}$, the growth in external input quantities (labor, capital, and imported materials), and the growth in final real output. We discuss these in turn.

Assigning the continuing network set. We calculate our output measure for continuing non-financial domestic Belgian firms. That is, we exclude financial corporations, government entities, and the household sector from the set of firms we track N. In addition, we also exclude firms with zero employment or firms that consist of self-employed owner operators. Within non-financial corporations, we exclude financial activities (NACE codes 64-66) and non-market services (NACE codes 84 and higher). This is because non-markets services, such as education, health, art and entertainment, are not well-covered by VAT data (for example hospitals and health centers are not required to submit VAT declarations). We define a firm in N to be continuing in t if its sales and intermediates purchases are larger than 1000 euros in t and $t+1$. This gives us the set C_t, which covers around 70% of both value-added and total employment in Belgium (see Table C.7). Crucially, our output measure is much broader than the Prodcom sample that we used in Section 3.

Calibrating $\bar{\delta}_{i,t}^{\text{exit}}$ and $\bar{\delta}_{i,t}^{\text{entry}}$. We calibrate $\bar{\delta}_{i,t}^{\text{exit}} = \bar{\delta}_{i,t}^{\text{entry}}$ for all i and t, and set this parameter to match our point estimates of δ based on separations (equation 8). If we assume CES input demand (with elasticity of substitution σ), then these requirements hold automatically because, by Corollary 1, $\delta_{iJ,t}(p_{J,t}, \infty) = \bar{\delta}_{i,t}^{\text{exit}} = \bar{\delta}_{i,t}^{\text{entry}} = \frac{1}{\sigma-1}$ for all i, J, and t. We experiment with different values of $\bar{\delta} \in \{0, 0.2, 0.4, 0.6\}$ and report the results. Outside of CES, if $\bar{\delta}^{\text{entry}}$ is greater than $\bar{\delta}^{\text{exit}}$, then the extensive margin's contributions to growth will be larger than what we report. On the other hand, if the reverse is true, the contributions will be lower. Since we do not estimate $\bar{\delta}^{\text{entry}}$, we assume this difference is zero, as in the CES benchmark.

Calibrating input-output shares and markups. As in Section 3, we construct the $C_t \times C_t$ network of domestic suppliers of Belgian firms using the NBB B2B Transactions data set. We exclude purchases of capital inputs from variable costs. As mentioned before, almost all firms in Belgium are required to report VAT on all sales of at least 250 euros, and the data has universal coverage of all businesses in C_t. There are four external inputs: labor, capital, imported materials, and materials from outside the set N (i.e. purchased from self-employed firms, government entities, etc.) We construct the $C_t \times F$ matrix of external input requirements using data from the annual accounts, B2B transactions, and customs declarations. For capital, as in Section 3, we multiply the industry-specific user cost of capital by firms' reported capital stocks. We measure firm-level markups by dividing sales by total variable costs, where total variable cost is the sum of all material purchases, domestic or foreign and from continuing or non-continuing firms, plus the wage bill and the cost of capital. Any other expenditures the firm incurs are treated as fixed, and not variable, costs.

Calibrating final output. Final output is defined to be the sales of C_t minus sales of materials to other firms in the production network. That is, final output are sales to households, exports, investment, and any other sales that are not considered to be intermediate purchases by firms in N.[21] We convert nominal final output into a real measure by deflating nominal growth in final output using the Belgian GDP deflator from the national accounts. That is, we assume that our final output measure's price deflator grows at the same rate as the Belgian GDP deflator.

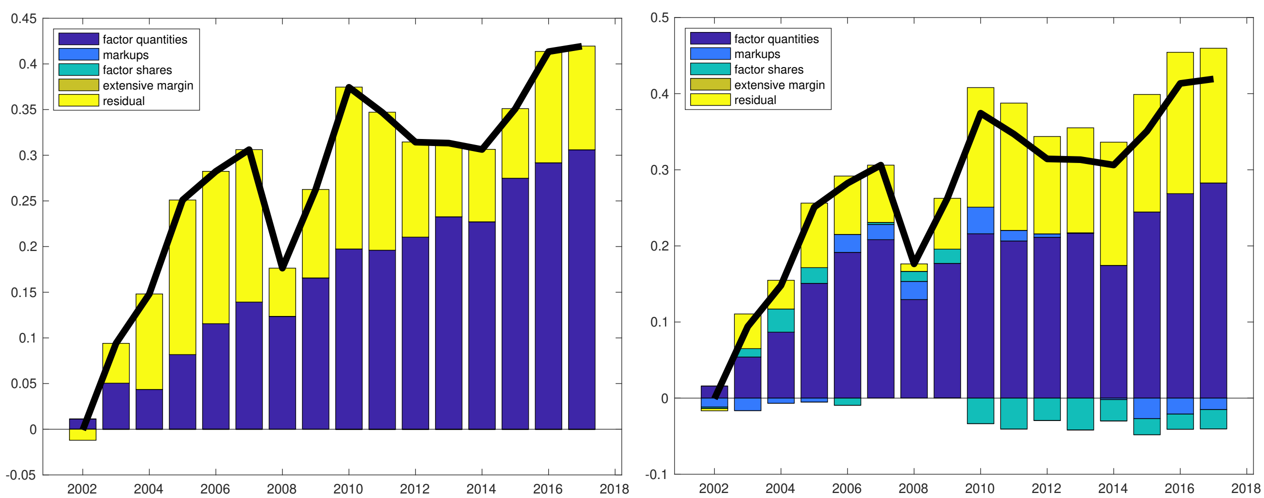
Calibrating external input quantities. We measure growth in labor quantity using total full time employees for firms in our sample. We measure growth in the capital stock of each firm by deflating the value of its plants, property, equipment, and intellectual property using the aggregate investment price deflator from the national accounts of Belgium. We measure the growth in imported materials by deflating the nominal imported

[21] Given data on sales for each firm $i \in C_t$, and the input-output matrix relative to sales, $\Omega^s_{ij} = \frac{p_j x_{ij}}{sales_i}$, we calculate total final output as $E = \begin{pmatrix} \vdots \\ sales_i \\ \vdots \end{pmatrix}'_{[i \in C_t]} (I - \Omega^s) \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$. Final demand shares are $b' = \lambda'(I - \Omega^s)$ where $\lambda_i = sales_i / E$.

material input growth with the import price deflator used for constructing the national accounts in Belgium. We cannot measure growth in the quantity of materials purchased from excluded domestic firms, so growth in the quantity of these materials are part of the residual.

Table C.7 in Appendix D reports information on the fraction of Belgian value-added in our sample and compares how aggregate growth rates in our sample compare to Belgian national accounts data. C.9 in Appendix D reports basic statistics (calculated using a sales-weighted distribution of each statistic across firms) on the level and changes in the continuing share of suppliers, as well as basic information on the number of suppliers each firm has, and the share of intermediate materials as a share of total costs for our sample.

5.2 Results

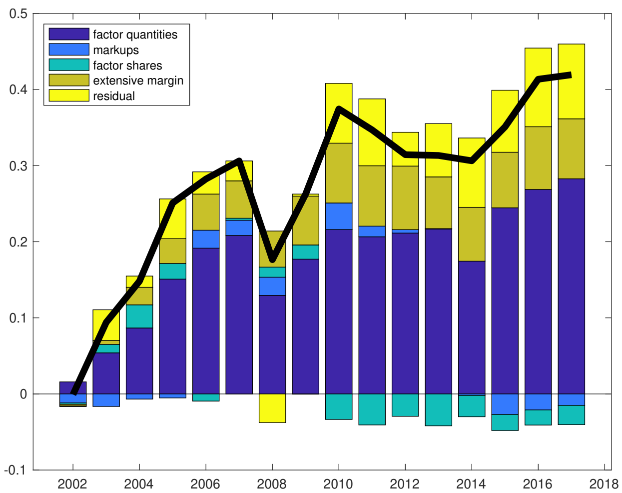


(a) Perfect competition and $\bar{\delta} = 0$.

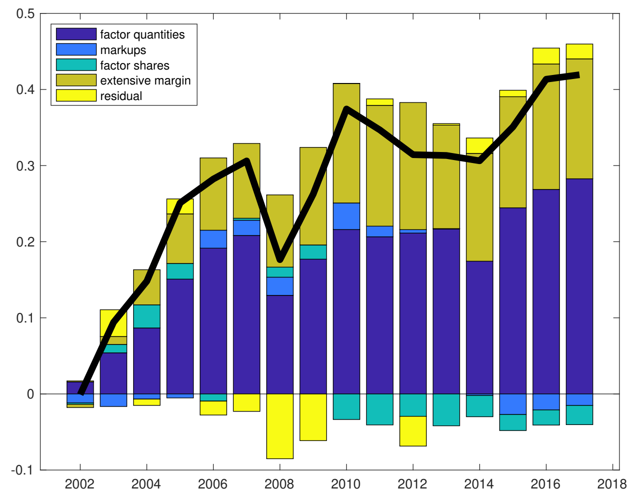
(b) Imperfect competition and $\bar{\delta} = 0$

Figure 3: Growth accounting special cases

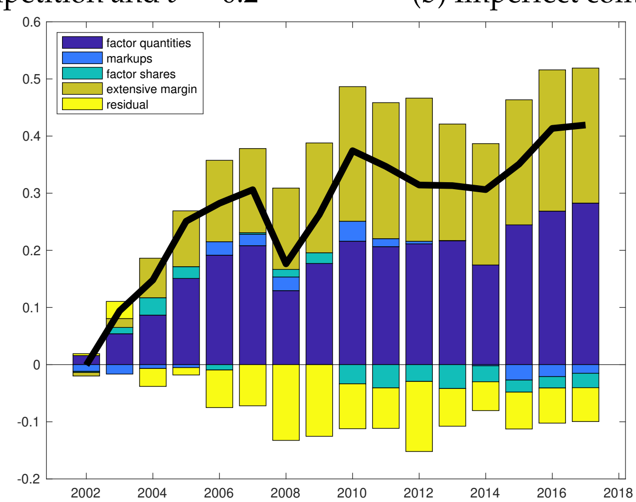
Before showing our benchmark results, we start with two special cases of Proposition 4. The left panel of Figure 3 assumes perfect competition and no extensive margin. To do this, we make two assumptions: (i) we set $\mu_{i,t} = 1$ for all i and t and assume that the cost of capital is such that the firm makes no profits, and (ii) we set $\bar{\delta} = 0$. In other words, the left panel of 3 is a traditional Solow-Hulten decomposition that breaks down overall growth into growth in the quantity of labor, capital, and imported materials (external inputs) and a residual term. In these figures, a substantial portion of aggregate growth, slightly less than half, is driven by unexplained technological growth. The right panel maintains the assumption that the extensive margin is irrelevant, $\bar{\delta} = 0$, but allows for firm-level markups. That is, it implements a Baqaee and Farhi (2019) style decomposition. This figure shows that small reductions in markups and factor shares over the sample have slightly increased aggregate output. Intuitively, the reduction in average markups and factor shares indicates that firms with initially high-markups are using more resources over time. This is beneficial for aggregate growth since these firms are inefficiently too small to begin with.



(a) Imperfect competition and $\bar{\delta} = 0.2$



(b) Imperfect competition and $\bar{\delta} = 0.4$



(c) Imperfect competition and $\bar{\delta} = 0.6$

Figure 4: Growth accounting baseline results

Figure 4 shows the role of supplier churn in growth. The first panel shows the results with $\bar{\delta} = 0.2$, which in a CES model corresponds to an elasticity of substitution of 6. The second panel shows $\bar{\delta} = 0.4$, which corresponds to an elasticity of substitution of

3.5. The final panel shows the results for $\bar{\delta} = 0.6$, which corrresponds to an elasticity of substitution of around 2.7.

The first two panels, with $\bar{\delta} \in \{0.2, 0.4\}$ are conservative compared to our point estimates in Table 3 in the sense that they assign a slightly smaller role to supplier churn than our point estimates suggest.[22] Nevertheless, even with these numbers, we find that the extensive margin of adding and subtracting suppliers can explain a substantial fraction of the residual. In the final panel, when $\bar{\delta} = 0.6$, the role of the residual, intensive margin improvements for existing firms, has all but disappeared from the calculation, showing that almost the entirety can be attributed to the supplier churn.

Whereas the supplier churn term is very important for long-run growth, and the residual is almost irrelevant when $\bar{\delta} = 0.6$, the picture is reversed for annual fluctuations. At annual frequency, the standard deviation of fluctuations in the residual are fifty percent larger than that of the supplier churn term. That is, unlike long-run growth, supplier-churn is not as important for explaining cyclical movements in aggregate output like the recession following the 2008 financial crisis.

Of course, these results are very speculative since they involve extrapolating estimates from the Prodcom manufacturing sample of firms to the a much broader subset of Belgian firms outside the manufacturing sector. In practice, the infra-marginal surplus ratio, δ, is likely highly heterogeneous and varies by both the characteristics of the suppliers being added or dropped as well as by the characteristics of the purchasing firm. However, our aggregation exercise suggests that the extensive margin of supplier entry and exit is plausibly a very important, if not the most important, driver of aggregate growth.

6 Conclusion

This paper analyzes and quantifies the microeconomic and macroeconomic importance of creation and destruction of supply linkages. Our analysis shows that downstream firms' marginal costs are greatly affected by supplier exits, which enables us to directly calculate the change in inframarginal surplus. This captures the love-of-variety effect in an expanding variety model and the innovation step-size in a quality-ladder model. Additionally, we demonstrate that supplier entry and exit can plausibly account for a major part of the growth component of the unexplained residual in a Solow (1957)-style growth accounting exercise. This paper provides a novel approach to estimate the area under the input demand curve using the elasticity of marginal cost to supplier exit. Future

[22] If we assume that supplier separations induced by our instrument in Section 3 do not result in simultaneous additions of suppliers, then we can interpret our point estimates, of around 0.6, as measuring $\bar{\delta}^{exit}$.

research can refine and replicate these estimates by exploring heterogeneity in δ, using other identification strategies, or data from other countries.

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Appendix A Proofs

Proof of Proposition 1. We suppress the index for the downstream firm throughout the proof since all variables are indexed by the identity of the downstream firm. Use Shephard's lemma to get

$$d\mathcal{C} = \sum_i x_i dp_i M_i + \frac{\partial \mathcal{C}}{\partial A} dA + \frac{\partial \mathcal{C}}{\partial y} dy.$$

Consider the change in costs due to a change in primitives. For any smooth path, indexed by $t \in [0,1]$, with end points given by $(\boldsymbol{p}^0, A^0, y^0)$ and $(\boldsymbol{p}^1, A^1, y^1)$ the change in costs is

$$\mathcal{C}(\boldsymbol{p}^1, A^1, y^1) - \mathcal{C}(\boldsymbol{p}^0, A^0, y^0) = \sum_i M_i \int_{p_i^0}^{p_i^1} x_i(\boldsymbol{p}(t), A(t), y(t)) \frac{dp_i}{dt} dt + \int_0^1 \frac{\partial \mathcal{C}}{\partial A} \frac{dA}{dt} dt + \int_0^1 \frac{\partial \mathcal{C}}{\partial y} \frac{dy}{dt} dt.$$

Given this exact representation of the cost function, we can now consider infinitesimal changes in the price of inputs by type dp, the mass of inputs of each type whose price jumps by a discrete amount dM, technology dA, and output dy. Omitting the dependence of conditional input, x_i, on its other arguments (which are held constant when we take the derivative), this results in the following expression

$$d\mathcal{C} = \sum_i M_i x_i dp_i + \sum_i \left(\int_{p_i^0}^{p_i^1} x_i(\xi) d\xi \right) dM_i + \frac{\partial \mathcal{C}}{\partial A} dA + \frac{\partial \mathcal{C}}{\partial y} dy.$$

This first-order approximation can be rewritten as

$$d \log \mathcal{C} = \sum_i M_i s_i d \log p_i + \frac{1}{\mathcal{C}} \sum_i \left(\int_{p_i^0}^{p_i^1} x_i(\xi) d\xi \right) dM_i + \frac{\partial \log C}{\partial \log A} d \log A + \frac{\partial \log \mathcal{C}}{\partial \log y} d \log y. \tag{14}$$

Next, by constant-returns, $\partial \log \mathcal{C} / \partial \log y = 1$ and $d \log mc = d \log \mathcal{C} - d \log y$. Hence, we can rewrite (14) as in (2) in Proposition 1 using the definition of δ_i, and noting that if $p_i^1 < p_i^0$, then $-\delta_i$ must be used.

□

Proof of Proposition 2. Observe that

$$x_i(p) = \frac{\frac{\partial (p_i x_i(p_i))}{\partial p_i}}{1 - \sigma_i(p_i)}.$$

Substitute this into the definition of δ_i to get

$$\delta_i = \frac{\int_{p_i}^{p'_i} x_i(\xi)d\xi}{p_i x_i(p_i)} = \frac{\int_{p_i}^{p'_i} \frac{\frac{\partial(\xi x_i(\xi))}{\partial \xi}}{1-\sigma_i(\xi)} d\xi}{p_i x_i(p_i)}.$$

Using Marshall's second law, and the fundamental theorem of calculus, we have

$$\delta_i < \frac{\int_{p}^{p'} \frac{\partial(\xi x_i(\xi))}{\partial \xi} d\xi}{p_i x_i(p_i)(1-\sigma_i(p))} = \frac{p'_i x_i(p'_i) - p_i x_i(p_i)}{p_i x_i(p_i)(1-\sigma_i(p_i))} = \frac{1}{\sigma_i(\boldsymbol{p})-1}\left[1 - \frac{p'_i x_i(p'_i)}{p_i x_i(p_i)}\right].$$

□

Proof of Proposition 3. To obtain equation (7), we invert the CES demand in equation (3) and express changes in marginal cost (for constant technology) as $d\log p_j + \frac{1}{\sigma-1} d\log\Omega_{ij}$ for any input j, where $d\log\Omega_{ij}$ is the log change in cost share for a non-jumping input of type j. Averaging over all input types using weight $\Omega_{ij}M_{ij}$ gives the first two terms in (7). The term $\sum_j \Omega_{ij}M_{ij}\Delta\log\Omega_{ij}$ is, up to a first-order, the log change in the cost share of non-jumping inputs. □

Proof of Lemma 1. To derive the last two terms in equation Lemma 1, write the second term in (2) as

$$- \sum_{\Delta M_{iJ,t}<0} \Omega_{iJ,t}\Delta M_{iJ,t}\delta_{iJ,t}(p_{J,t},\infty) - \sum_{\Delta M_{iJ,t}>0} \Omega_{iJ,t}\Delta M_{iJ,t}\delta_{iJ,t}(p_{J,t},\infty) =$$

$$- \sum_{\Delta M_{iJ,t}<0} \Omega_{iJ,t}\Delta M_{iJ,t}\bar{\delta}^{exit}_{i,t} - \sum_{\Delta M_{iJ,t}>0} \Omega_{iJ,t}\Delta M_{iJ,t}\bar{\delta}^{entry}_{i,t} =$$

$$\Omega_{i,t}(- \sum_{\Delta M_{iJ,t}<0} \frac{\Omega_{iJ,t}}{\Omega_{i,t}}\Delta M_{iJ,t} - \sum_{\Delta M_{iJ,t}>0} \frac{\Omega_{iJ,t}}{\Omega_{i,t}}\Delta M_{iJ,t})\bar{\delta}^{exit}_{i,t} - \Omega_{i,t}\sum_{\Delta M_{iJ,t}>0} \frac{\Omega_{iJ,t}}{\Omega_{i,t}}\Delta M_{iJ,t}(\bar{\delta}^{entry}_{i,t} - \bar{\delta}^{exit}_{i,t}),$$

where we omit the notation $J \in \mathcal{J}_i$ from all the summands. In the last line, the first term in brackets is, up to a first-order, the exit share minus the entry share of firm i's suppliers, which to a first order equals the log change in the continuing share, $\Delta\log\mathcal{S}^c_{i,t}$. The term $\sum_{\Delta M_{iJ,t}>0} \frac{\Omega_{iJ,t}}{\Omega_{i,t}}\Delta M_{iJ,t}$ is, up to a first-order, the entry share, which is equal to one minus the continuing share at $t+1$. □

Proof of Proposition 4. In the text, we showed that the final output price deflator is given by

$$d\log P^Y_t = \sum_{i\in C_t} \tilde{\lambda}_{i,t}\left[d\log\frac{\mu_{i,t}}{A_{i,t}} - \bar{\delta}^{\text{exit}}_{i,t}\Delta\mathcal{E}_{i,t} + (\bar{\delta}^{\text{exit}}_{i,t} - \bar{\delta}^{\text{exit}}_{i,t})\Delta\mathcal{D}_{i,t}\right] + \sum_{f\in F}\tilde{\Lambda}_{f,t}d\log w_{f,t}.$$

Substitute this into

$$d \log Y = d \log(\sum_{i \in C_t} p_{i,t} y_{i,t}) - d \log P_t^Y$$

and use the fact that $\sum_{f \in F} \tilde{\Lambda}_{f,t} = 1$ and the fact that $d \log w_{f,t} = d \log \Lambda_{f,t} - d \log L_{f,t} + d \log(\sum_{i \in C_t} p_{i,t} y_{i,t})$. □

Appendix B Additional Example

The following is a concrete example of Proposition 2.

Example 3 (Non-CES with expanding varieties)**.** Consider the HSA technology from Matsuyama and Ushchev (2017), and parameterize it in the following way. The expenditure share on each input type is given by

$$s_i = \max \left\{ 0, 1 - \frac{p_i}{D} \right\},$$

where D is a scalar that ensures the expenditure shares add up to one. As long as p_i is below its choke price (which is D), the price elasticity of demand is given by

$$\sigma_i = \frac{1}{1 - p_i / D} > 1.$$

Notice that the price elasticity of demand is increasing in the price, therefore satisfying Marshall's second law. In the limit, as the price approaches the choke price, the price elasticity goes to infinity. On the other hand, the inframarginal surplus ratio from new varieties is

$$\delta_i = \frac{-\log(p_i / D)}{1 - p_i / D} - 1.$$

The inframarginal surplus ratio is decreasing in the price and goes to zero in the limit as the price approaches the choke price. That is, a new variety appearing at the choke price produces no inframarginal surplus.

We can re-express the inframarginal surplus ratio in terms of the price elasticity of demand at each point. This gives the following inequality consistent with Proposition 2:

$$\delta_i = -\sigma_i \log(1 - \frac{1}{\sigma_i}) < \frac{\sigma_i}{\sigma_i - 1}. \tag{15}$$

Appendix C Additional Data Details

Mergers and acquisitions. One challenge with using data recorded at the level of the VAT identifier is the case of mergers and acquisitions, since this might blur our entry/exit analysis of suppliers.[23] When a firm stops its business, it reports to the Crossroads Bank of Enterprises (CBE) the reason for ceasing activities, one of which is merger and acquisition. In such cases, we use the financial links also reported in the Crossroads Bank of Enterprises (CBE) to identify the absorbing VAT identifier and we group the two (or more) VAT identifiers into a unique firm. We choose the VAT identifier with the largest total assets. We use this head VAT identifier as the identifier of the firm. Having determined the head VAT identifier, we aggregate all the variables up to the firm level. For variables such as total sales and inputs, we adjust the aggregated variables with the amount of B2B trade that occurred within the firm, correcting for double counting. For other non-numeric variables such as firms' primary sector, we take the value of its head VAT identifier. It is important to emphasize that we group VAT identifiers only for the corresponding cross-section (the year of the M&A and after), and not over the whole panel period.

Table C.10 provides the number of firms for every year t between 2002 and 2017, as well as the fraction of firms that exit rate between t and $t + 1$. The exit rate is much higher for small firms (those below the median size) and large firms.

[23] Another challenge is that VAT declarations are made at the unit level, which in some instances group more than one VAT identifier. In this case, we group the two (or more) VAT identifiers into a unique firm.

Table C.5: B2B sales and firm exit

Panel A: B2B reporting at t for firms exiting at t+1

	(i)	(ii)	(iii)	(iv)	(v)
		Share of sales		Indicator B2B	Number of
	B2B	exports	residual	not reported	B2B custom.
Firm dies at t+1	-0.116***	-0.000	0.116***	0.294***	-9.062***
	(0.001)	(0.000)	(0.001)	(0.001)	(0.397)
Controls	N	N	N	N	N
Firm FE	Y	Y	Y	Y	Y
Obs	4,646,592	4,646,592	4,646,592	4,646,592	4,646,592

Panel B: B2B reporting at t for firms exiting at t+2

	(i)	(ii)	(iii)	(iv)	(v)
		Share of sales		Indicator B2B	Number of
	B2B	exports	residual	not reported	B2B custom.
Firm dies at t+2	-0.006***	-0.002***	0.008***	0.044***	-3.385***
	(0.001)	(0.000)	(0.001)	(0.001)	(0.436)
Controls	N	N	N	N	N
Firm FE	Y	Y	Y	Y	Y
Obs	4,051,149	4,051,149	4,051,149	4,051,149	4,051,149

Notes: The outcome variable is an indicator of whether the firm ceases operation (firm death) in $t+1$ (Panel A) or $t+1$ (Panel B). The regressors are the period t share of the firm's sales to B2B, exports, and residual (defined by total sales - B2B sales - exports), an indicator of whether the firm reports B2B sales in t, and the number of B2B costumers in t. The sample includes firms with positive sales in t that report B2B sales at least one year. The regressor is the share of

Table C.6: Intermediate input purchases and number of suppliers

	(i)	(ii)
	Number of suppliers	Change in number of suppliers
Residual intermediate input share	-5.045***	
	(0.038)	
Change in residual intermediate input share		-2.535***
		(0.017)
Controls	N	N
Firm FE	Y	Y
Obs	5,212,288	5,109,323

Notes: Residual intermediate input purchases equals total intermediate input purchases minus B2B purchases minus import purchases.

Table C.7: Coverage of growth accounting sample of firms

	(i)	(ii)	(iii)	(iv)	(v)
year	count	value added	% of agg.	employment	% of agg.
2002	113,417	107,373	72%	1,738	74%
2003	117,497	113,034	73%	1,742	74%
2004	120,283	120,362	74%	1,729	73%
2005	121,875	103,765	61%	1,743	72%
2006	124,550	125,032	70%	1,797	73%
2007	126,164	112,715	59%	1,819	72%
2008	126,949	123,326	63%	1,837	71%
2009	127,997	124,391	66%	1,772	69%
2010	126,360	131,740	66%	1,745	67%
2011	127,259	121,718	59%	1,784	67%
2012	129,206	124,440	59%	1,804	68%
2013	127,848	141,614	67%	1,824	69%
2014	129,396	144,510	67%	1,815	69%
2015	129,347	147,725	66%	1,819	68%
2016	126,304	163,311	70%	1,904	70%
2017	126,506	169,721	71%	1,940	70%
avg. growth (%)		3.1	3.3	0.7	1.1

Notes: The sample of firms used in this table are those used in the growth accounting exercise (continuing corporate non-financial firms) in Section 5. % agg. is the share of value added and employment in the non-financial corporate sector reported in the national statistics.

Table C.8: Descriptive statistics

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
	Share in costs					# suppliers	Continuing supplier share	
	materials			labor	capital		level	dlog
	dom. cont.	imports	other					
mean	0.53	0.18	0.05	0.23	0.02	191	0.69	-0.03
p25	0.40	0.00	0.01	0.13	0.00	96	0.55	-0.14
p50	0.53	0.13	0.03	0.21	0.01	145	0.73	-0.02
p75	0.66	0.31	0.06	0.31	0.02	222	0.87	0.09
count	46,021	46,021	46,021	46,021	46,021	46,021	46,021	46,021

Notes: The sample of firms used in this table are those used in the micro regressions in Section 3 based on the Prodcom sample.

Table C.9: Descriptive statistics.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
	Share in costs					# suppliers	Continuing supplier share	
	materials			labor	capital		level	dlog
	dom. cont.	imports	other					
mean	0.59	0.18	0.10	0.12	0.01	699	0.47	0.03
p25	0.39	0.00	0.02	0.02	0.00	120	0.12	-0.14
p50	0.62	0.07	0.05	0.07	0.00	323	0.46	0.00
p75	0.82	0.31	0.12	0.16	0.01	880	0.81	0.12
count	1,996,407	1,996,407	1,996,407	1,996,407	1,996,407	1,996,407	1,995,921	1,995,921

Notes: The sample of firms used in this table are those used in growth accounting in Section 5. Summary statistics are calculated from sales-weighted distribution.

Table C.10: Exit rates

	(i)	(ii)	(iii)	(iv)
	Count	Exit rate		
	all	all	small	large
2002	251,966	0.061	0.101	0.022
2003	259,432	0.061	0.101	0.021
2004	268,276	0.063	0.103	0.022
2005	275,322	0.063	0.107	0.020
2006	284,535	0.058	0.099	0.019
2007	296,061	0.056	0.096	0.018
2008	307,938	0.060	0.101	0.019
2009	315,051	0.059	0.097	0.020
2010	326,375	0.058	0.097	0.019
2011	339,206	0.058	0.097	0.018
2012	348,103	0.065	0.109	0.022
2013	350,618	0.066	0.110	0.021
2014	357,898	0.074	0.124	0.023
2015	359,003	0.057	0.096	0.018
2016	369,106	0.055	0.093	0.016
2017	379271	0.056	0.095	0.017

Notes: Number of continuing firms at t and fraction of firms that exit between t and $t+1$. Small and large firms are those below and above the median sales firm.

Appendix D Additional Tables and Figures

Table D.11: Identifying $\delta - 1$ with more stringent fixed effects

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
				$\Delta \log mc$			
Exit share	-0.010		0.539***	0.490**	0.668***	0.480**	0.523**
	(0.009)		(0.140)	(0.135)	(0.175)	(0.135)	(0.133)
Demand Shock		-0.794***					
		(0.196)					
F-stat			62	64	80	63	74
Specification	OLS	RF	IV	IV	IV	IV	IV
Controls	N	N	N	Y	Y	Y	Y
8 digit x year FE	Y	Y	Y	Y	Y	Y	Y
Firm FE	N	N	N	N	Y	N	N
Obs	35,135	35,135	35,135	35,135	34,303	35,135	35,135

Notes: Columns (ii)-(viii) report estimates of regression (8) using 8-digit $\times$ year fixed effects. Demand shock is the instrument in the IV regressions and is defined by (10). Controls are log changes in the price of imported inputs, log changes in the price of inputs purchased from other Prodcom firms, changes in log wages, changes in the log user cost of capital, and a demand shock constructed for the downstream firm itself. All regressions are unweighted except (vi), which is weighted by log sales. Column (vii) uses lagged shares at $t-1$ instead of initial t shares in constructing the instrument. Standard errors are clustered at the firm-level.

Table D.12: Sensitivity analysis I

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
	Δ log mc						
	Baseline	4-digit FE	8-digit FE	No change in product mix	Price largest product	Alt. demand instrument	Alt. markups
Separation share	0.677***	0.490***	0.756***	0.576***	0.683***	0.443***	1.044***
	(0.161)	(0.135)	(0.167)	(0.156)	(0.163)	(0.127)	(0.216)
Specification	IV	IV	IV	IV	IV	IV	IV
Controls	Y	Y	Y	Y	Y	Y	Y
F-stat	62.669	64.367	73.377	57.910	62.669	175.784	62.669
N	39,123	35,135	42,103	36,586	39,123	38,359	39123

Table D.13: Sensitivity analysis II

	(i)	(ii)	(iii)	(iv)
	Δ log mc			
	Exclude utilities suppliers	Exclude wholesale & retail suppliers	Include SFG suppliers	Include capital & SFG suppliers
Separation share	0.738***	0.919***	0.725***	1.215***
	(0.179)	(0.283)	(0.153)	(0.268)
Specification	IV	IV	IV	IV
Controls	Y	Y	Y	Y
F-stat	53.763	21.467	67.468	40.031
N	39,054	38,091	39,222	39,206

Notes: These tables report sensitivity analysis of regression (8).

Appendix E First-Order Equivalence of Quality-Ladder and Expanding-Variety Models

In section 4, we say that firm i adds suppliers of type J if $\Delta M_{iJ,t} > 0$ and removes suppliers if $\Delta M_{iJ,t} < 0$. That is, each input is associated with an individual supplier and that input becomes unavailable when a supplier is dropped, as in expanding varieties models. However, as long $\delta(p,\infty) < \infty$, Lemma 1 also applies to quality-ladder models. To see this, notice that a quality-ladder model can be represented via the simultaneous addition and removal of suppliers. Suppose that a mass m of inputs of type j improve by climbing the quality ladder and reducing their price from p'_j to p_j. By Proposition 1, the effect on the marginal cost of i is

$$\Delta \log mc_i \approx \Omega_{ij}(p_j)\delta_{ij}(p_j, p'_j)m,$$

where for clarity we suppress the time subscript and we index the cost share by the input price. This equation can be re-written as the outcome of adding m suppliers who price at p_j and removing m suppliers who price at p'_j:

$$\Delta \log mc \approx \Omega_{ij}(p_j)\delta_i(p_j, p'_j)m = \Omega_{ij}(p_j)\delta_i(p_j, \infty)m - \Omega_{ij}(p'_j)\delta(p'_j, \infty)m.$$

That is, a quality-ladder model can be represented using an expanding-variety model, to a first order approximation. The following example applies this result in the case of CES input demand.

Example 4 (Equivalence of Quality-Ladders and Expanding-Varieties under CES)**.** Consider a downstream firm i with CES input demand with elasticity of substitution $\sigma > 1$. Suppose that some mass $m > 0$ of inputs climb the quality ladder from p_j to p'_j. Then by Proposition 1, the change in the marginal cost of i is given by

$$\Delta \log mc_i = \Omega_{ij}(p_j)\frac{1}{1-\sigma}\left[\left(\frac{p'_j}{p_j}\right)^{1-\sigma} - 1\right] m$$

as in Example 1. To show that this can be represented in our framework using an expanding-variety model, suppose there are two types of suppliers indexed by j and j' that price at p_j and p'_j. Now imagine that a mass m of j-type suppliers exit and a mass m of j'-type suppliers enter. Then, following Proposition 1, the change in marginal cost is given by

$$\Delta \log mc_i = \Omega_{ij'}(p'_j)\frac{1}{1-\sigma}m - \Omega_{ij}(p_j)\frac{1}{1-\sigma}m = \Omega_{ij}(p_j)\frac{1}{1-\sigma}\left[\left(\frac{p'_j}{p_j}\right)^{1-\sigma} - 1\right]m,$$

which is the same as the change caused by a shift along the quality-ladder.

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