Working Paper Research

March 2023 N° 435

BEMGIE: Belgian Economy in a Macro General and International Equilibrium model

by Gregory de Walque, Thomas Lejeune, Ansgar Rannenberg and Raf Wouters

NationalBank OF BELGIUM
Eurosystem

Publisher

Pierre Wunsch, Governor of the National Bank of Belgium

Statement of purpose

The purpose of these Working Papers is to promote the circulation of research results (Research Series) and analytical studies (Documents Series) made within the National Bank of Belgium or presented by external economists in seminars, conferences and conventions organised by the Bank. The aim is therefore to provide a platform for discussion. The opinions expressed are strictly those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

The Working Papers are available on the website of the Bank: http://www.nbb.be

ISSN: 1375-680X (print)
ISSN: 1784-2476 (online)

Abstract

This paper outlines the three-country New Keynesian Dynamic Stochastic General Equilibrium model of the National Bank of Belgium. The model is named BEMGIE for Belgian Economy in a Macro General and International Equilibrium model. It features imperfect market competition, standard real and nominal rigidities, local currency pricing, energy in consumption and oil and foreign inputs in production. The model is estimated using Bayesian econometric techniques on Belgian, euro area and US data. BEMGIE is designed to provide quantitative simulations of macroeconomic shocks and policies, and to be used in the context of the Eurosystem projection exercises.

Keywords: DSGE model, Open economy model, Multi-country model, International spillovers, Monetary policy, Exchange rate pass-through, Bayesian estimation.

JEL Classifications: E10, E17, E30, E40, E52, F41, F45, F47, C11, C32, C51.

Authors:
Gregory de Walque, Economics and Research Department, National Bank of Belgium
e-mail: Gregory.dewalque@nbb.be
Thomas Lejeune, Economics and Research Department, National Bank of Belgium
e-mail: Thomas.lejeune@nbb.be
Ansgar Rannenberg, Economics and Research Department, National Bank of Belgium
e-mail: Ansgar.rannenberg@nbb.be
Raf Wouters, Economics and Research Department, National Bank of Belgium
e-mail: Rafael.wouters@nbb.be

We thank Koen Burggraeve, Charles de Beauffort, Cédric Duprez, Catherine Fuss, Pelin Ilbas and Jolan Mohimont, for their helpful discussions on the model and comments on the paper. We would also like to thank participants to the various NBB internal seminars and to the Eurosystem Working Group on Economic Modelling where the model has been presented. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium or any other institution with which one of the authors are affiliated. We are fully responsible for any remaining error.

Non-technical summary

This paper details the present version of the multi-country macroeconomic model developed at the National Bank of Belgium. The model is named ’BEMGIE’ for Belgian Economy in a Macro General and International Equilibrium model. It belongs to the class of New-Keynesian Dynamic Stochastic General Equilibrium models, which are reference tools in modern macroeconomics. It is designed for use in simulations of macroeconomic shocks and policies and in the construction of macroeconomic scenarios. It can also be used in forecasting, in the context of the participation of the National Bank of Belgium to the Broad Macroeconomic Projection Exercises conducted twice a year by European Central Bank and Eurosystem staff.

BEMGIE features three fully-fledged blocs: Belgium, the euro area and the United States. Belgium is modelled as a small open economy inside a monetary union, sharing the same nominal exchange rate and monetary policy rule as (the rest of) the euro area. The international environment consists in two endogenous regions, the eurozone and the US, while the rest of the world is exogenous. The model is equipped with standard elements proved to be relevant in the literature when it comes to fit empirical dynamics: price and wage rigidities and indexation, habits in consumption and adjustment costs in investment and in imported quantities. The open economy dimension is enriched by the use of foreign inputs and energy commodities in the production function. This feature is not frequently present in open economy macro models, but turns to be useful to reconcile (i) a large, though incomplete, exchange rate pass-through at the border and a limited pass-through at the level of home final prices with (ii) a large import-to-GDP ratio as the one that characterizes the Belgian economy. The Belgian bloc of BEMGIE is more detailed than its euro area and US counterparts in the modelling of separate import contents for its national aggregates, and in the presence of housing services in consumption, of several direct and indirect taxes, and of automatic wage indexation.

The model is estimated on Belgian, eurozone and US data using Bayesian econometric techniques. It is especially successful in replicating the relative magnitude of fluctuations in real and nominal time series, as well as the correlation between GDP growth and its components and between consumption price inflation and the growth rate of other observed price series. The estimation of correlated innovations between demand shocks across countries helps the model to generate a co-movement between Belgian and euro area internal demands in line with the data. On the nominal side, the mechanisms limiting the exchange rate pass-through (and the energy pass-through) to final prices help the model to generate a high correlation between exchange rates and import and export price inflations, but a small and limited correlation between exchange rates (respectively crude oil prices) and final price inflation.

The open economy dimension of BEMGIE makes it possible to endogenise the evolution of international variables, such as the euro-dollar exchange rate, the euro area short-term interest rate and main trade flows. This paper illustrates the effects of main international shocks in the model, and underlines the resulting significant co-movements between Belgian and euro area macroeconomic variables.

BEMGIE is aimed to be an evolving model, whether it be through a regular reestimation of its parameters or through relevant extensions. The lessons learned for macro models after the Global Financial crisis and the tasks assigned to the National Bank of Belgium in terms of macroprudential supervision naturally put financial frictions on top of our agenda. We plan to equip the Belgian bloc of the model with corporate and mortgage credit and banks with sector-specific capital requirements. These financial additions would make the simulations of macroprudential policies possible in BEMGIE.

TABLE OF CONTENTS

1. Introduction 1

2. The model 4
2.1. Households 5
2.2. Production of domestic goods 9
2.3. Production of exported goods 13
2.4. International environment 14
2.5. Government 16
2.6. Closing the model 17

3. Linearisation and mechanics 18
3.1. Consumption 18
3.2. Investment 19
3.3. Real wage dynamics 20
3.4. Price Phillips curves 21
3.5. International trade 22

4. Estimation 24
4.1. Data and shocks 24
4.2. Calibration and priors 28
4.3. Posterior estimates 30

5. Moment analysis 37

6. Dynamics 40
6.1. A conventional monetary policy shock 41
6.2. A demand shock in the euro area 41
6.3. A productivity shock in the euro area 44
6.4. A euro depreciation shock 46
6.5. An oil price shock 48
6.6. Extra-euro area (and global) demand shocks 52

7. Shock decomposition 53

8. Conclusion 57

References 57

Appendix 62

National Bank of Belgium - Working Papers Series 66

1 Introduction

The New-Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model has become a reference tool in modern macroeconomics. Its micro-foundations with optimising agents provide a framework suited for a structural interpretation of the economic outlook and for the simulation of alternative economic scenarios. Moreover, the key role of agents' expectations and the non-trivial effect of monetary policy on real variables generated by this class of models makes them particularly attractive to central banks. In this paper, we give an overview of the current version of the open-economy NK-DSGE developed at the National Bank of Belgium (NBB). We name the model 'BEMGIE', for Belgian Economy in a Macro General and International Equilibrium model. It is designed for use in simulations of macroeconomic shocks and policies and in the construction of macroeconomic scenarios. It can also be used in forecasting, in the context of the participation of the NBB to the Broad Macroeconomic Projection Exercises (BMPE) conducted twice a year by ECB and Eurosystem staff.

BEMGIE is a multi-country model estimated on data for Belgium (BE), the euro area (EA) and the United States (US). The Belgian bloc is modelled as a small open economy inside a monetary union, sharing the same nominal exchange rate and monetary policy rule as (the rest of) the EA. The small open economy setup implies that EA and US blocs are not affected by BE dynamics. The international EA-US environment draws on the two-country model developed in de Walque et al. (2017; 2023b). The rest of the world is exogenous and affects the three modelled regions under the form of external economic shocks. The domestic core of the model builds extensively on the Smets and Wouters (2007) closed economy setup and includes nominal and real rigidities that are identified as empirically relevant in the NK-DSGE literature. In particular, prices and wages are sticky in the short run, and the equilibrium equations feature indexation to past nominal developments. Habits in consumption and adjustment costs in the changes of the investment path generate gradual and persistent responses of domestic demand to shocks. Agents are forward-looking, and their expectations play a key role in the dynamics of macro variables after policy and economic shocks. The three areas are linked to each other through bilateral trade, offering a comprehensive treatment of cross-border trade interdependence. BEMGIE is also equipped with relevant open economy elements such as adjustment costs in imported quantities, local currency pricing, energy distribution costs, and international trade in energy and non-energy intermediate inputs and in transit goods.

The last two mechanisms are particularly helpful to reconcile (i) a large, though incomplete, exchange rate pass-through (hereafter ERPT) at the border and a limited pass-through at the level of home final prices with (ii) a large import-to-GDP ratio for BE. In traditional open economy models, the weight of imported prices in the CPI is directly related to the import-to-GDP ratio (see Galí and Monacelli, 2005). A large ratio thus implies a large and direct effect of volatile international prices to consumer prices. As detailed in de Walque et al. (2023b), the introduction of imported inputs used in the production of domestic intermediate goods and imported goods that are directly re-exported (transit goods) helps to break this direct relationship as part of the imported quantities do not appear directly in the CPI. This smaller share of final imported goods in the basket of consumers has a first consequence of reducing the direct ERPT to final prices. This low direct ERPT is supplemented by an indirect one, which affects the price of domestically produced goods through the effect of the exchange rate on the price of foreign inputs. The combination of (generally strong) domestic nominal and real rigidities makes this indirect component of the CPI-ERPT limited in the short run though much more persistent. The mechanism described here for the exchange rate pertains also to energy prices such that the inflationary pressures of these two highly volatile drivers of the CPI are fully taken into account though one substantial part of it remains longer in the production chain before affecting the end user.

The Belgian bloc of the model is more sophisticated than its EA and US counterparts in several directions. First, we account for a separate import content of private consumption, investment and government spending. Investment in housing assets is modelled separately from business investment. In addition to (domestic or imported) final goods and energy products, households also consume housing services. Second, the government collects value-added taxes, labour income taxes and social contributions, taxes on capital income and lump-sum taxes, and issues debt to finance its expenditures. To replicate a key institutional feature of the Belgian labour market, wage setting is subject to automatic indexation to the inflation rate of a 'health' consumption index, which has a different composition –particularly regarding energy– compared to the consumption price index.[1]

The model is log-linearised around a deterministic steady-state and estimated using Bayesian techniques. The dataset starts in 1995Q1 and ends in 2019Q4. In each of the three fully modelled blocs, we observe real GDP and its main components, private consumption, investment, exports and imports, as well as hours worked and real wages. On the nominal side, the GDP deflator is taken as a measure of domestic prices, while we also include the private consumption deflator together with a proxy for its energy component,[2] the import price deflator and a short-term interest rate. The observation of an energy component of consumption price indices helps us to take onboard the evolution of non-oil energy prices in the model. Regarding international variables common to the three blocs, the euro-dollar exchange rate and an index of raw oil price are observed. On the top of these time series, traditionally used in the estimation of open-economy NK-DSGE, a larger set of variables are included to guide the dynamics of BE variables. The BE dataset also contains public consumption and investment, real housing investment and its deflator, an effective exchange rate, the export deflator, the implicit tax rate on value-added, on labour income and social contributions. We also observe the 'health' consumption index, used in the BE wage indexation mechanism, as well as its energy component. Moreover, variables from the 'Trade Consistency Exercise' (hereafter TCE, see Hubrich and Karlsson, 2010) are included and are particularly useful to implement the external assumptions provided by the ECB in the BMPE. This set of variables includes proxies for BE import prices of goods coming from inside and outside the eurozone, as well as measures for the intra- and extra-euro area demand for BE exports. Given the large scale of the model and the important number of observables, a two-step approach is implemented to ease the estimation of BEMGIE. Relying on our small open economy setup for the BE bloc, we first estimate a two-country EA-US model and collect posterior mode values for parameters and shocks associated with the eurozone and American regions. Next, we integrate BE into the EA-US environment. The resulting three-country model is calibrated with estimated values obtained in the first step and Belgian parameters and shocks are estimated in a second step.

A comparison of data-based and model-implied unconditional moments reported in this article delivers interesting information about the strengths and weaknesses of BEMGIE in fitting the data. The model is especially successful in replicating the relative magnitude of fluctuations in real and nominal time series, as well as the correlation between GDP growth and its components and between consumption price inflation and the growth rate of other deflators. The estimation of correlated innovations between demand shocks across

[1]See Section 2.2 for more details on this health consumption price index. In a nutshell, this health index excludes some 'unhealthy' items from its composition, in particular tobacco, alcoholic beverages and motor fuels (except LPG). Hence, it is characterized by a smaller weight associated with energy products compared to the private consumption deflator. Its composition also differs from the one of the private consumption price by the absence of the rental rate of housing services.

[2]That is, the energy component of the harmonised index of consumer price (HICP energy) for the BE and EA blocs, for which the energy component of the private consumption deflator is not directly available. In contrast, this energy component is available for the US private consumption deflator (on the website of the Bureau of Economic Analysis) and can thus be used as an observable in the model.

countries helps the model to generate a co-movement between Belgian and euro area internal demands in line with the data. Though this also improves business cycle synchronisation, the model has more difficulties to replicate the large unconditional correlation between BE and EA GDP found in the data. This echoes the well-known challenge of the new open-economy models in the literature to generate (unconditional) international synchronisation,[3] and highlights that this challenge is also present in the case of a small open economy inside a monetary union. On the nominal side, the mechanisms limiting the ERPT (and the energy pass-through) to final prices help the model to generate a high correlation between exchange rates and import and export price inflations, but a small and limited correlation between exchange rates (respectively crude oil prices) and final price inflation.

However, and interestingly, conditional to the main international shocks in the model, BEMGIE is able to produce a significant synchronisation between the growth rates of BE and EA output. The fully-fledged EA-US structure makes it possible to endogenise the evolution of international variables, such as the euro-dollar exchange rate, the euro area short-term interest rate and main trade flows. These variables play a key role in the transmission of international shocks to the Belgian economy. The paper illustrates these transmission mechanisms by presenting the reaction of BE macroeconomic variables to several relevant external shocks. We first analyse the fluctuations in BE prices and quantities generated by responses to shocks in the rest of the euro area. The model generates strong endogenous spillovers from EA monetary and productivity shocks to BE macro variables, but negative co-movements after EA demand shocks. The euro area Taylor rule, common to the two blocs, has important implications for explaining these results. A positive TFP shock in the rest of the euro area triggers a monetary expansion that stimulates Belgian domestic demand. This goes together with an increase in Belgian exports to intra-zone partners and makes BE and EA outputs move in the same direction. Concerning the demand shock, different scenarios are considered: (i) no correlation between EA and BE shock innovations, (ii) estimated correlation, (iii) full correlation. In the first case, the model produces a de-synchronisation between BE and EA business cycles. In response to an adverse demand shock in the EA bloc, the central bank conducts an expansionary monetary policy that leads to a stimulus in BE internal demand. This effect compensates the negative influence of a drop in BE net exports on output. This finding complements the issue raised in estimated SOE (Justiniano and Preston, 2010) and two-country models (de Walque et al., 2017), by underlining the lack of endogenous transmission of demand shocks in the rest of the zone to a SOE inside a monetary union. A co-movement between EA and BE variables can be obtained by making the BE demand shock correlated to the EA ones, as in scenarios (ii) and (iii). Such correlated disturbances can help to account for absent, but potentially relevant, cross-country transmission mechanisms. For instance, they can represent a (admittedly rough) short-cut for the representation of an internationally integrated financial system as developed by Dedola et al. (2012) or Kollman (2013).

Shocks to the euro exchange rate, to oil prices and to extra-euro area demand are other common shocks to intra-zone countries. As illustrated in the paper, these shocks also produce significant co-movement between BE and EA variables. In response to an adverse movement in extra-euro area demand, monetary policy becomes expansionary in the euro area, and has supportive effects on BE and EA internal demands. This supportive effect cancels out the impact of a contraction in net exports on euro area production. Introducing correlation between the extra-euro area demand and EA and BE demand shocks helps the model to generate a decline in eurozone internal demand and an international synchronization of business cycles.

The simulations presented in this paper also underlines two structural characteristics of the Belgian economy after an energy price shock. After a sudden change in wholesale energy prices, consumer price inflation

[3] see for instance the influential paper by Justiniano and Preston (2010), the EA-US model of de Walque et al. (2017; 2023b) and the recent two-country setup, MAJA, developed at the Riksbank by Corbo and Strid (2020).

is found to increase more in BE than in EA. This leads to a larger drop in real purchasing power for Belgian households, and to a slightly more persistent decline in private consumption and hence in output. The Belgian automatic and complete wage indexation adds to the gap between BE output responses relative to the euro area as it deteriorates the competitiveness of Belgian firms on both domestic and foreign markets.

The paper is organised as follows. Section 2 describes the model structure and the optimisation problem of agents. In Section 3, we present key log-linearised equilibrium equations and provide intuition on their economic mechanisms. Section 4 gives details on the estimation procedure, the data and shocks used, and the estimated parameters of the model. The ability of the estimated model to replicate relevant unconditional moments of the data is assessed in Section 5. In Section 6, we analyse the dynamics generated by BEMGIE and report the responses of key BE and EA macro variables after the main eurozone and extra-euro area shocks. The estimated DSGE model has the potential to provide a plausible account of the evolution of macroeconomic times series in terms of the underlying exogenous forces (shocks). This analysis is presented for Belgian GDP and inflation in Section 7. Finally, Section 8 concludes and discusses future extensions of the model.

2 The model

BEMGIE consists of four blocs: Belgium (BE), the euro area (EA), the United States (US), and the Rest-of-the-World (RoW). The first three economies are fully-fledged, while the RoW appears under the form of exogenous shock processes. The core structure of modelled economies is as in Smets and Wouters (2007) and features standard bells and whistles aimed to fit the data: habit preferences, investment adjustment costs, variable utilisation rate of capital and price and wage stickiness augmented with indexation schemes. Households consume domestic and foreign goods and energy, invest in physical capital, supply differentiated labour services and set wages. They have access to saving instruments that takes the form of one-period risk-free bonds. The US bond is traded internationally, and gives rise to an uncovered interest rate parity condition for the euro-dollar exchange rate.

There are three layers of firms. First, intermediate good firms act under monopolistic competition and use domestic and foreign value added (imported inputs and oil) to produce goods sold to homogeneous-good assemblers. This second layer of firms act in perfect competition, combine differentiated intermediate goods and use Kimball's (1995) technology to produce homogeneous domestic and export goods. Homogeneous domestic and foreign goods are then combined in a CES production function by retailers to produce a final good sold to end users in a perfectly competitive environment.

BE is modelled as a small open economy inside a monetary union. We assume that Belgian economic developments are too small to generate meaningful feedbacks on the other blocs of the model. BE shares the nominal interest rate and exchange rate with the euro area. The EA and US economies form the endogenous international environment of BE and are modelled according to an updated version of the model presented in de Walque et al. (2017). The main differences between the original and updated versions of the EA-US setup, as well as with the BE bloc, are detailed in Appendix 9.1. In the following lines we describe the model from a BE perspective, and refer to this appendix and to de Walque et al. (2017) for the design of EA and US areas. Compared to these two regions, the modelling of the BE economy is more sophisticated in several dimensions. More detailed attention has been given to the modelling of BE GDP components. The import content across consumption, investment and public goods is allowed to differ. Housing investment is modeled separately from business investment. Public investment and consumption are regrouped into an exogenous process that matches public expenditure data, used in the estimation of the model. On the government income side, labour income taxation, social contributions, rental capital income taxation and value-added taxation are introduced. Compared to the EA and the US, wage setting in BE differs in an automatic indexation mechanism where wages gradually adjust to the inflation of a 'health' consumption price index. Moreover, and as discussed in Section 4.1, the Belgian block is estimated on a larger set of time series compared to EA and US blocks with, in particular, measures of intra-euro area & extra-euro area import prices and measures of intra- and extra-euro area demands for BE goods.

In the following sections, we present the equations from the perspective of the BE bloc. To simplify notation and for clarity reasons, we do not use any specific subscript for a BE variable or parameter when it represents a domestic element. For the other blocs, we use the notation X_j to represent a variable or parameter X of region j. We use $X_{j,k}$ for goods coming from country k at the destination of country j. This double subscript is also used for parameters.

2.1 Households

There is a continuum of households indexed by $h \in [0,1]$. Each household consumes a final good $\mathbb{C}$, supplies a differentiated labour service L and maximizes the following non-separable inter-temporal utility:

$$U_t(h) \equiv E_t \sum_{j=0}^{\infty} \beta^j \varepsilon_t^{bb} \left(\frac{1}{1-\sigma_c} (\mathbb{C}_{t+j}(h) - \tilde{C}_{t+j})^{1-\sigma_c} \right) \exp \left(\frac{\sigma_c - 1}{1+\sigma_\ell} L_{t+j}(h)^{1+\sigma_\ell} \right) \tag{1}$$

where β is a constant discount factor, σ_c measures the degree of relative risk aversion (i.e. the inverse of the inter-temporal elasticity of substitution for constant labour), σ_ℓ is the inverse of the Frisch elasticity (i.e. the elasticity of work effort with respect to real wage), and $\tilde{C}_t = \lambda \mathbb{C}_{t-1}$ is the external habit variable, which is proportional to aggregate past consumption. Exogenous variable ε_t^{bb} is an AR(1) preference shock that affects the discount rate of households, and hence their intertemporal substitution between current and future consumption. Ceteris paribus, an increase in the preference shock enlarges the gap between current and future marginal utilities of consumption, and stimulates the contemporaneous demand for goods. Note that for the limiting case of $\sigma_c \to 1$, (1) converges to the simpler case of log-separable preferences.[4]

Each household maximizes its utility subject to a budget constraint in period t which states that its expenditure on consumption $P_t \mathbb{C}_t(h)$, housing and capital investment goods (respectively $P_{h,t} I_{h,t}(h)$ and $P_{I,t} I_t(h)$) and the net accumulation of financial assets must equal its disposable income:

$$\begin{aligned}
&\left(1+\tau_t^{vat}\right)\left[P_t \mathbb{C}_t(h) + P_{h,t} I_{h,t}(h)\right] + P_{I,t} I_t(h) \\
&+ \frac{B_t(h)}{\exp\left(\varepsilon_t^b\right) R_t} - B_{t-1}(h) + \frac{B_t^{EA}(h)}{\exp\left(\varepsilon_t^b\right) R_{EA,t} \Gamma_t^{EA}} - S_{EA,BE,t} B_{t-1}^{EA}(h) \\
\leq &\left(1-\tau_t^{w23}\right) W_t^h(h) L_t(h) \\
&+ \left(1-\tau_t^k\right) R_t^k u_t^k(h) K_{t-1}(h) - a[u_t^k(h)] K_{t-1}(h) \\
&+ P_t \tau_t^k \delta^k K_{t-1}(h) + R_t^h u_t^h(h) H_{t-1}(h) - a[u_t^h(h)] H_{t-1}(h) \\
&+ Div_{f,t} + Div_{u,t} + P_t TR_t - P_t T_t \quad .
\end{aligned} \tag{2}$$

[4]That means, per period utility would be given by $ln\left(\mathbb{C}_{t+j}(h) - \tilde{C}_{t+j}\right) + \frac{L_{t+j}(h)^{1+\sigma_\ell}}{1+\sigma_\ell}$.

Household's income comes from different sources. The first source is after-tax labour revenues, $\left(1-\tau_t^{w23}\right) W_t^h\left(h\right) L_t\left(h\right)$, where $W_t^h(h)$ is the gross hourly wage and $\tau_{w23,t}$ represents employee's social security contributions and withholding tax on earned income. The second source represents income for the effective capital services rented to intermediate good firms $R_t^k u_t^k\left(h\right) K_{t-1}\left(h\right)$ taken after taxation τ_t^k and net of utilisation costs $a[u_t^k(h)]K_{t-1}(h)$. Capital utilisation costs are modelled as a function a of the utilisation rate u^k which respects the following properties: $a\left(\bar{u}\right)=0$, a' and $a" > 0$. The aggregate revenues collected from utilisation costs, $Div_{u,t}$, are assumed to be rebated lump-sum to the household sector. The term $P_t\tau_t^k\delta_K K_{t-1}\left(h\right)$ takes into account that depreciated capital is not taxed. In our model, households are the owners of the housing stock and receive another source of income from renting their housing services, $R_t^h u_t^h(h)H_{t-1}(h)$.[5] Finally, households also receive dividends $Div_{f,t}$ from owning domestic firms, dividends paid by labour unions to households, receive lump-sum transfer TR_t from the government and pay a lump-sum tax T_t to the public authorities.

Capital investment

Households are the owners of physical capital assets and rent them to firms who combine these assets with labour input for production. They decide on the utilisation intensity $u_t^k\left(h\right)$ of the stock of physical capital, K_{t-1}, and on capital asset investment, I_t, taking into account the following standard capital accumulation equation:

$$K_t\left(h\right)=\left(1-\delta^k\right) K_{t-1}\left(h\right)+\varepsilon_t^I\left(1-S\left(I_t\left(h\right)/I_{t-1}\left(h\right)\right)\right) I_t\left(h\right) \quad . \tag{3}$$

Adjustment costs to investment are a function S of the ratio of current investment relative to past investment and ε_t^I is a shock to the marginal efficiency of the transformation of investment into physical capital assets. This function respects the following properties: $S\left(\centerdot\right)=0$ in steady state where the growth rate of investment corresponds to the balance growth path and $S'\left(\centerdot\right)=0$, $S"\left(\centerdot\right)>0$.

Housing investment

We follow a financial frictionless version of Aoki et al. (2004) and treat housing investment in a similar way to the case of physical capital investment. This modelling choice enables us to reproduce the dynamics of the national account concepts for housing investment and its deflator while paving the way for a future extension of the model to financial frictions à la Bernanke et al. (1999) similar to the one implemented in Aoki et al. (2004).[6] Households are both owners of the housing stock and consumers of housing services. They decide on housing investment, taking into account depreciation δ^h, investment adjustment costs S and the housing stock accumulation, which is as follows:

$$H_t\left(h\right)=\left(1-\delta^h\right) H_{t-1}\left(h\right)+\varepsilon_t^h\left(1-S\left(I_{h,t}\left(h\right)/I_{h,t-1}\left(h\right)\right)\right) I_{h,t}\left(h\right) \quad . \tag{4}$$

Bond markets

In BE, households have access to one-period riskless domestic bonds (B, traded only in BE), and to EA

[5]The rental price of housing R_t^h can be seen as a counterpart for actual and imputed rents in the data. As for physical capital the response of the rental rate to shocks can be attenuated with a variable utilisation capacity. This device helps to make the model concept more in line with rigid rents observed in Belgian data.

[6]In this future extension to financial frictions, we find this mechanism more appropriate for the BE housing market compared to an alternative à la Iacoviello (2005) where borrowing households use real estate as a collateral to finance consumption expenditures. This feature suits the US market where home equity loans are widely used, and enable households to obtain additional credit for consumption against the value of their home. BE households, however, rarely rely on these types of loans and the estimated effect of house prices on aggregate consumption is small in BE compared to countries where these products are frequently used (Reusens and Warisse, 2018).

bonds (B^{EA}, traded only in the euro area). Nominal gross interest rates received on bond holdings are subject to an exogenous domestic risk premium, ε_t^b, which follows an AR(1) process. Fisher (2015) interprets this risk premium shock as a structural shock to the demand for safe and liquid assets, which picks up 'flight-to-safety' types of behaviours. Moreover, as explained later, a larger ε_t^b leads to a reduction of current consumption and to an increase in the required return on capital and real estate assets. This in turn leads to a decline in capital and housing asset prices which discourages investment. The risk premium shock thus moves consumption and investment in the same direction, which makes it particularly adequate to simulate demand shocks in the model. As discussed in Smets and Wouters (2007), the risk premium shock shares similarities with the effects of an external finance premium in a model with financial frictions (as for instance in Bernanke et al., 1999).[7]

In the model, stationarity is induced by assuming that BE bondholders pay a country specific interest rate premium Γ_t^{EA} on EA bonds. As in Schmitt-Grohe and Uribe (2003), this premium depends on the net foreign asset position – here of BE vis-à-vis the EA – and has the following form:

$$\Gamma_t^{EA} = \exp\left(-\Gamma^{EA}\frac{B_t^{EA}}{P_t\gamma^t}\right) \quad . \tag{5}$$

The net foreign assets evolve according to previous positions and the trade balance of Belgium:

$$\frac{B_t^{EA}}{R_{EA,t}\Gamma_t^{EA}} = B_{t-1}^{EA} + P_{X,t}X_t - P_{M,t}M_t \tag{6}$$

where $P_{X,t}X_t$ and $P_{M,t}M_t$ are total export and total imports in nominal terms ,and $R_{EA,t}$ is the nominal interest rate on EA bonds. The optimization of the constrained problem of households with respect to each bond holding leads to the following Euler equations:

$$R_t\beta E_t\left[\frac{\Xi_{t+1}}{\Xi_t}\left(\Pi_{t+1}\frac{1+\tau_{t+1}^{vat}}{1+\tau_t^{vat}}\right)^{-1}\right]\exp\left(\varepsilon_t^b\right) = 1 \quad , \tag{7}$$

$$R_{EA,t}\Gamma_t^{EA}\beta E_t\left[\frac{\Xi_{t+1}}{\Xi_t}\left(\Pi_{t+1}\frac{1+\tau_{t+1}^{vat}}{1+\tau_t^{vat}}\right)^{-1}\right]\exp\left(\varepsilon_t^b\right) = 1 \quad , \tag{8}$$

where $\beta E_t\left[\frac{\Xi_{t+1}}{\Xi_t}\right]$ is the stochastic discount factor of households, with the marginal utility of consumption, $\Xi_t = U'(C_t)$, $\Pi_t = P_t/P_{t-1}$ is the – before value-added tax – BE consumption inflation rate. Combining (7) and (8), one obtains that $R_{BE,t} = R_{EA,t}\Gamma_t^{EA}$. The transaction cost coefficient, Γ^{EA}, which is typically calibrated to a small value in the literature, is imposed to 10^{-7} which ensures that the nominal interest rates of BE and EA are numerically identical while preserving the stationarity of the model.

Labour supply and wage setting

The modelling of the labour market follows Erceg et al. (2000) and Smets and Wouters (2007). Labour supply is the same for all households and is given by the following first order condition to their utility maximisation problem with respect to labour services:

$$\frac{W_t^h}{P_t}\frac{\left(1-\tau_t^{w23}\right)}{\left(1+\tau_t^{vat}\right)} = \frac{\left[\frac{1}{1-\sigma_c}\left(\mathbb{C}_t-\tilde{C}_t\right)^{1-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_\ell}L_t^{1+\sigma_\ell}\right)(\sigma_c-1)L_t^{\sigma_\ell}\right]}{\Xi_t} \quad . \tag{9}$$

[7] At the level of the euro area and the US, de Walque et al. (2017) find that it is a major driver of the fluctuations in EA and US real variables during the global financial crisis of 2007-2008 and the euro sovereign crisis periods.

This optimal condition reflects that the after tax real wage desired by households depends on the marginal rate of substitution between leisure and consumption. Intermediate labour unions bring together the homogeneous labour supplied by households and differentiate labour services. Unions thus have market power on their variety and set wages, taking the marginal rate of substitution represented in Equation (9) as their 'marginal costs'. The mark-up above the marginal dis-utility of labour is redistributed to households. Staggered wage contracts à la Calvo (1983), where a fraction $(1-\xi_w)$ of unions are allowed to set optimal wages at each period, help the model to generate sluggish wage adjustments. The non-optimised wages are indexed to the deterministic growth rate of the economy, γ, and to the gross rate of the past inflation rate derived from a "health" consumption price index (π_{health}, detailed here below in Section 2.2). We use a lag structure close to the one of the NONAME model (Burggraeve and Jeanfils, 2008), a semi-structural model developed for the Belgian economy, to account for the fact that wages are indexed with some lags to the health index.[8] Labour unions who can adjust wages set an optimal wage, $\tilde{W}_t$, taking into account all the future states where the union is stuck with that wage. Indexed by ℓ, a representative union faces the following inter-temporal optimisation problem:

$$\max_{\tilde{W}_t(\ell)} E_t \sum_{j=0}^{\infty} (\beta\xi^w)^j \frac{\Xi_{t+j} P_{,t}}{\Xi_t P_{t+j}} \left[\tilde{W_t(\ell)} \chi^w_{t,j} - W^h_{t+j} \right] L_{t+j}(\ell) \tag{10}$$

where

$$\chi^w_{t,j} = \begin{cases} 1 & \text{if } j=0 \\ \Pi^j_{k=1} \gamma \pi^{3/4}_{health,t+k-1} \pi^{1/4}_{health,t+k-2} & \text{if } j=1,...,\infty \end{cases} . \tag{11}$$

A complete set of securities markets and full consumption risk sharing across households is assumed in order to make budget constraints independent from the optimal wage set. Consequently, the optimisation problem is identical across unions and they select the same optimal wage. The aggregate wage expression is then as follows:

$$W_t = \left[(1-\xi^w) \left(\tilde{W}_t \right)^{1/\lambda^w_t} + \xi^w \left(W_{t-1} \gamma \pi^{3/4}_{health,t-1} \pi^{1/4}_{health,t-2} \right)^{1/\lambda^w_t} \right]^{\lambda^w_t} . \tag{12}$$

Intermediate good firms use an index of differentiated labour services supplied by labour unions given by the following Dixit-Stiglitz-type aggregator function:

$$L_t(f) = \left(\int_0^1 \left(L_t(\ell, f) \right)^{\frac{1}{1+\lambda^w_t}} d\ell \right)^{1+\lambda^w_t} . \tag{13}$$

The elasticity of substitution between differentiated labour services, $(1+\lambda^w_t)/\lambda^w_t$, can be interpreted as the degree of union market power in the wage setting process, and reflects the deviation from free competition in the labour market. The lower the elasticity, the more market power the unions have in negotiating wages (i.e. the higher is the wage mark-up). We allow $(1+\lambda^w_t)$ to be time-varying and interpret it as a wage mark-up shock whose logarithm, later denominated ε^w_t, follows an ARMA(1,1) process with a moving average term to capture some of the high-frequency fluctuations in wages.

[8]This delayed indexation aims to mimic as closely as possible some features of the Belgian wage indexation scheme. The indexation mechanism differs across sectorial collective agreement. In most sectors the indexation is based on an average of the health index over the last four months. For some of them, the process is purely time dependent (with different frequencies, the lowest being once a year) while in others (as for example the public sector) it is level dependent, i.e. the indexation occurs whenever the smoothed health index has increased by 2%. The lag structure imposed in BEMGIE (and NONAME) attempts to approximate this features and reflect that indexation occurs with delays compared to the evolution of the health consumption price index.

2.2 Production of domestic goods

Intermediate good firms

Intermediate good firm f combines domestic inputs, J_t, with imported homogeneous inputs, $Y^p_{BE,j,t}$ coming from $j \in \{EA, US, RoW\}$, and oil, O_t,[9] to produce its differentiated gross output, $Y_{H,t}$, according to the following production system:

$$J_t(f) = \exp(\varepsilon^a_t)\left(u^k_t K_{t-1}(f)\right)^{\alpha}\left(\gamma^t L_t(f)\right)^{1-\alpha} \quad , \tag{14}$$

$$U_t(f) = \left[\left(1-\sum_j \phi_{BE,j}\right)^{\frac{1}{\lambda_m}} J_t(f)^{\frac{\lambda_m-1}{\lambda_m}} + \sum_j \phi_{BE,j}^{\frac{1}{\lambda_m}}\left(\left(1-\Omega^p_{BE,j,t}\right)Y^p_{BE,j,t}(f)\right)^{\frac{\lambda_m-1}{\lambda_m}}\right]^{\frac{\lambda_m}{\lambda_m-1}} , \tag{15}$$

$$Y_{H,t}(f) = \left[(1-\phi_{oil})^{\frac{1}{\lambda_{oil,p}}} U_t(f)^{\frac{\lambda_{oil,p}-1}{\lambda_{oil,p}}} + \phi_{oil}^{\frac{1}{\lambda_{oil,p}}}\left(O^p_t(f)\right)^{\frac{\lambda_{oil,p}-1}{\lambda_{oil,p}}}\right]^{\frac{\lambda_{oil,p}}{\lambda_{oil,p}-1}} - \Phi\gamma^t \tag{16}$$

where α measures the share of capital in domestic value added , λ_m and λ_{oil} are elasticities of substitution between respectively domestic value added and imported inputs and oil, while $\phi_{BE,j}$ and ϕ_{oil} are bias towards respectively imported inputs and oil products, and Φ denotes a fixed cost of production. Variable ε^a_t is a transitory productivity shock. Adjustments in the proportion of foreign value added relative to domestic inputs is costly, according to the following cost function:

$$\Omega^p_{BE,j,t} = \frac{\Omega^p_{BE,j}}{2}\left(\frac{Y^p_{BE,j,t}/J_t}{Y^p_{BE,j,t-1}/J_{t-1}} - 1\right)^2 \tag{17}$$

with $j \in \{EA, US, RoW\}$.

To generate price rigidities, price setting is subject to a Calvo's (1983) friction according to which only a proportion of firms receives permission to optimally reset prices in a given period t, while the remaining firms index their last price to a combination of observed past inflation and constant trend inflation (where the indexation weight is ι_p). The optimal price is set such as to maximize the firm's profit:

$$\max_{\tilde{P}_{H,t}(f)} E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \Lambda_{t,t+j}\left[\tilde{P}_{H,t}(f)\chi_{t,j} Y_{H,t+j}(f) - MC_{t+j} Y_{H,t+j}(f)\right] \tag{18}$$

where

$$\chi_{t,j} = \begin{cases} 1 & \text{if } j = 0 \\ \Pi^j_{i=1}\pi^{\iota_p}_{H,t+i-1}\bar{\pi}_H^{1-\iota_p} & \text{if } j = 1,...,\infty \end{cases} . \tag{19}$$

When evaluating future cash flows, firms use the nominal stochastic discount factor of households, $\Lambda_{t,t+j} = \frac{\Xi_{t+j}P_t(1+\tau^{vat}_t)}{\Xi_t P_{t+j}(1+\tau^{vat}_{t+1})}$, who are the final owners of firms. Marginal costs are denoted by MC_t and are a function of the prices of domestic inputs (i.e., wages and the rental rate of capital assets) and foreign inputs (i.e., imported price and oil price at the border):

[9] The absence of an observed energy price component of the private value added deflator, as there can be at the level of consumer prices, makes it more complicated to include an energy production input (oil and non-oil) in the model. To the extent that non-oil wholesale prices correlate with the price of crude oil, the present formulation can be used to capture some of their effects on firms' marginal costs. We plan to investigate this issue more in future research, for instance by looking at the dependence of available producer price time series and wholesale gas and electricity prices.

$$MC_t = \left[(1-\phi_{oil})MC_{Y,t}^{1-\lambda_{oil,p}} + \phi_{oil}P_{oil,t}^{1-\lambda_{oil,p}}\right]^{\frac{1}{1-\lambda_{oil,p}}}, \tag{20}$$

$$MC_{Y,t} = \left[\left(1-\sum_j \phi_{BE,j}\right) MC_{J,t}^{1-\lambda_m} + \sum_j \phi_{BE,j}\left(P_{BE,j,t}^{p}\right)^{1-\lambda_m}\right]^{\frac{1}{1-\lambda_m}}, \tag{21}$$

$$MC_{J,t} = \frac{\left[(1+\tau_t^{w1})W_t\right]^{1-\alpha}\left(R_t^k\right)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\exp\left(\varepsilon_t^a\right)} \tag{22}$$

where τ^{w1} are social contributions of employers, $P_{BE,j}^{p}$ is the price of foreign non-oil inputs coming from $j \in \{EA, US, RoW\}$ and P_{oil} is the price of crude oil in euro.[10]

Homogeneous assemblers and final good firms

Homogeneous assemblers purchase differentiated outputs from domestic good firms, indexed by f. They use a generalized aggregator to assemble them into a final, homogeneous, product. Such an aggregator accounts for a demand elasticity of differentiated varieties that depends on their relative prices. As intermediate good firms consider the final demand for their (aggregated) goods, this real rigidity helps to estimate a more reasonable degree of price stickiness, as shown in Eichenbaum and Fisher (2007). Their production functions are as follows:

$$1 = \int_0^1 G\left(\frac{Y_{H,t}(f)}{Y_{H,t}}\right) df \tag{23}$$

with $G(1) = 1, G'(x) > 0$ and $G"(x) < 0$ for all $x \geq 0$. By the zero-profit condition of homogeneous assemblers, and accounting for optimal or indexed prices set by intermediate good firms, the aggregate price index is given by:

$$\begin{aligned} P_{H,t} = {} & \xi_p \pi_{H,t-1}^{\iota_p}\bar{\pi}_H^{1-\iota_p} P_{H,t-1} G'^{-1}\left(\frac{\pi_{H,t-1}^{\iota_p}\bar{\pi}_H^{1-\iota_p} P_{H,t-1}}{P_{H,t}}\mathcal{I}_{H,t}\right) \\ & + (1-\xi_p)\tilde{P}_{H,t} G'^{-1}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\mathcal{I}_{H,t}\right) \end{aligned} \tag{24}$$

where $\mathcal{I}_{H,t} = \int_0^1 G'\left(\frac{Y_{H,t}(f)}{Y_{H,t}}\right)\frac{Y_{H,t}(f)}{Y_{H,t}} df$ and $\tilde{P}_{H,t}(f) = \tilde{P}_{H,t}$ as intermediate goods face the same optimisation problem, and hence, all firms that receive permission to reset prices choose the same optimal price.

Homogeneous goods producers sell their goods to final good firms, where the products proceed towards final uses, that is consumption (private C and public G) and investment (capital I and housing I_h):

$$Y_{H,t} = C_{H,t} + I_{H,t} + I_{h,H,t} + G_{H,t}\ . \tag{25}$$

For each use, domestic homogeneous goods are combined with foreign homogeneous products in a CES aggregator to produce a final consumption or investment good:

[10] Note that we ignored adjustment costs in (21), as these terms vanish up to a first-order approximation of the model.

$$Z_t = \left[\left(1 - \sum_j \phi_{BE,j}\right)^{1/\lambda_z} Z_{H,t}^{\frac{\lambda_z - 1}{\lambda_z}} + \sum_j \phi_{BE,j}^{\frac{1}{\lambda_z}} \left\{(1 - \Omega_{BE,j,t}^Z) Z_{BE,j,t}\right\}^{\frac{\lambda_z - 1}{\lambda_z}} \right]^{\frac{\lambda_z}{\lambda_z - 1}} \tag{26}$$

where $Z \in \{C, I, I_h, G\}$, indexing respectively private consumption, capital investment, housing investment and public expenditures, and $j \in \{EA, US, RoW\}$. Adjusting the relative proportion of foreign goods is costly, with a cost function given as follows:

$$\Omega_{BE,j,t}^Z = \frac{\Omega}{2} \left(\left(\varepsilon_t^M\right)^{-1/\Omega} \frac{Z_{BE,j,t}/Z_{H,t}}{Z_{BE,j,t-1}/Z_{H,t-1}} - 1 \right)^2 . \tag{27}$$

As in Christoffel et al. (2008), we refer to ε_t^M as an import demand shock. We assume the same trade elasticity λ for consumption (public and private) and investment bundles (capital and housing). The final good for private consumption is subject to an additional layer of modelling where housing services (H) and energy (N) are added to the basket of consumers. Two other layers of the CES aggregator are used in order to model these additions:

$$C_{nh,t} = \left[(1 - \phi_{n,c})^{1/\lambda_{n,c}} C_t^{\frac{\lambda_{n,c} - 1}{\lambda_{n,c}}} + \phi_{n,c}^{1/\lambda_{n,c}} \left(N_t^d\right)^{\frac{\lambda_{n,c} - 1}{\lambda_{n,c}}} \right]^{\frac{\lambda_{n,c}}{\lambda_{n,c} - 1}} \tag{28}$$

$$\mathbb{C}_t = \left[(1 - \phi_h)^{1/\lambda_h} C_{nh,t}^{\frac{\lambda_h - 1}{\lambda_h}} + \phi_h^{1/\lambda_h} \left(u_t^h H_{t-1}\right)^{\frac{\lambda_h - 1}{\lambda_h}} \right]^{\frac{\lambda_h}{\lambda_h - 1}} \tag{29}$$

where $C_{nh,t}$ denotes the consumption basket of non-housing good C_t and energy products N_t^d, while $\mathbb{C}_t$ stands for the total private consumption composite, including housing. Energy products N_t^d are produced by a distribution sector which combines wholesale energy N_t with domestic refining and distribution services. This distribution mechanism helps to attenuate the pass-through of highly volatile wholesale energy price to the smoother energy component of the price of consumption expenditures. Accordingly, the following distribution function for energy is used:

$$N_t^d = \min \left[(1 + \delta_n) N_t; \frac{1 + \delta_n}{\delta_n} Y_{H,t} \right] \tag{30}$$

where $\delta_n/(1 + \delta_n)$ is the distribution margin and the price of distributed energy, expressed in home currency, is:

$$P_{n,t}^d = \frac{1}{1 + \delta_n} P_{n,t} + \frac{\delta_n}{1 + \delta_n} P_{H,t} , \tag{31}$$

$$P_{n,t} = S_{US,EA,t} \left(\nu_{oil,1} P_{oil,t} + \nu_{oil,2} P_{oil,t-1} + \nu_{oil,3} P_{oil,t-2} + \nu_{oil,4} P_{oil,t-3}\right) + \varepsilon_t^n . \tag{32}$$

The last equation helps to account for a gradual dependence of distributed energy price (and hence consumption price) to oil price. An AR(1) shock process ε_t^n is introduced to better account for non-oil energy price shocks in the model. The weights ν_{oil} are estimated based on an unrestricted regression of the quarterly growth rates of the energy price on the growth rates of raw oil price. These coefficients thus capture dependencies of energy prices to raw oil prices, as well as to non-oil wholesale energy prices to the extent that they correlate with oil prices and have explanatory power for HICP energy components.[11] The dynamics

[11] Using monthly data on wholesale gas and electricity price changes (source: ECB macroeconomic projection exercise), we

produced by the model after an oil price shock, which affects energy prices through oil prices via Equation (32), may thus also reflect sensitivities to the co-movement of non-oil energy commodities, and thus reflect the responses to an energy shock beyond that of a pure oil price shock (i.e. in a scenario where the wholesale price of non-oil energy commodities remain the same).[12]

The price indices associated with final consumption and investment goods in (26), with the consumption of non-housing goods represented in (28) and the model concept for the private consumption deflator are as follows:[13]

$$P_{Z,t} = \left[\left(1 - \sum_j \phi_{BE,j}\right) P_{H,t}^{1-\lambda_z} + \sum_j \phi_{BE,j} \left(P^f_{BE,j,t}\right)^{1-\lambda_z} \right]^{\frac{1}{1-\lambda_z}} \tag{33}$$

$$P_{nh,t} = \left[(1-\phi_{n,c}) P_{C,t}^{1-\lambda_{n,c}} + \phi_{n,c} \left(P^d_{n,t}\right)^{1-\lambda_{n,c}}\right]^{\frac{1}{1-\lambda_{n,c}}} \tag{34}$$

$$P_t = \left[(1-\phi_h) P_{nh,t}^{1-\lambda_h} + \phi_h \left(R^h_t\right)^{1-\lambda_h}\right]^{\frac{1}{1-\lambda_h}} \tag{35}$$

with $Z \in \{C, I, I_h, G\}$ and where $P^f_{BE,j}$ represents the price of finished imported goods from area $j \in \{EA, US, RoW\}$.

Health consumption price index In Belgium, wages (as well as housing rents, pension and social security benefits) are automatically and fully indexed based on a so-called 'health' consumption price index. This health price index is derived from the consumption price index from which tobacco, alcoholic beverages and motor fuels (except LPG) are excluded.[14] Its composition also differs from the one of the private consumption price by the absence of the rental rate of housing services. The energy component of this health index also diverges from the one of other concepts of consumption prices. It is characterised by a smaller weight attached to energy products to account for the exclusion of (most) motor fuels from its calculation. This is reflected in the calibration of specific coefficients in front of contemporaneous and lagged crude oil prices:

$$P_{health,t} = \left[(1-\phi_{n,c}) P_{C,t}^{1-\lambda_{n,c}} + \phi_{n,c} \left(P_{n,health,t}\right)^{1-\lambda_{n,c}}\right]^{\frac{1}{1-\lambda_{n,c}}}, \tag{36}$$

$$\begin{aligned} P_{n,health,t} = {} & S_{US,EA,t} \left(\nu_{oil,health,1} P_{oil,t} + \nu_{oil,health,2} P_{oil,t-1} \right. \\ & \left. + \nu_{oil,health,3} P_{oil,t-2} + \nu_{oil,health,4} P_{oil,t-3}\right) + \varepsilon^n_{health,t}, \end{aligned} \tag{37}$$

obtain an unconditional correlation around 0.3 between crude oil price changes and gas and electricity price changes in euro for the period 2004-2021. A time-varying correlation computed using moving windows of 2 years however indicates that co-movements can be more substantial during specific periods, as for instance for 2014-2015 where the correlation figures between oil and electricity prices go above 0.6, and 2016-2018 where the correlation between oil and gas price changes fluctuated around 0.4-0.5.

[12] Thanks to the inclusion of an energy price concept in BEMGIE, and its link to the data on HICP-energy prices, it is however straightforward to simulate such a pure oil price shock. Outside the model, it is possible to compute how oil-related categories of the HICP energy price (that is, motor fuels and heating fuels) adjust after a movement in crude oil price, leaving unchanged the price of non-oil energy categories. Using the weights of the oil-related categories, one can calculate the change in the HICP energy prices, and calibrate the shock ε^n_t such as to replicate the pass-through of the pure oil price shock to energy prices in the model.
Similarly it is also straightforward to simulate a broader energy shock, by computing the changes in all HICP-energy categories (oil and non-oil related) after sudden changes in oil and non-oil wholesale prices (gas, electricity) using bridge equations outside the model. The simulated path for HICP-energy prices can then be replicated in BEMGIE using an appropriate calibration of the energy price shock ε^n_t. We are grateful to Koen Burggraeve for his invaluable work in developing and estimating these energy bridge equations for both NONAME and BEMGIE models.

[13] Note that we ignored adjustment costs in (33), as these terms vanish up to a first-order approximation.

[14] Hence the name 'health' of the index refers to a consumption basket where some unhealthy products have been excluded.

where weights $\nu_{oil,health}$ are estimated based on a regression of the quarterly growth rate of the energy component of the health index on the oil price inflation rates. A shock $\varepsilon^n_{health,t}$ is introduced to better accommodate for the non-oil items of the energy component of the health index. When computing the inflation rate of the health index, we account for the effects of value-added taxation, that is $\pi_{health,t} = \frac{(1+\tau^{vat}_t)P_{health,t}}{(1+\tau^{vat}_{t-1})P_{health,t-1}}$.

2.3 Production of exported goods

There are two types of monopolistic firms in the export sector: final good exporters that trade a product, Y^f, aimed to be sold abroad for final use (consumption and investment), and input good exporters that trade an intermediate good, Y^p, used in the production process in other countries. Final good firms buy the homogeneous domestic good Y_H from domestic assemblers, and transform it into a specialized export good through a differentiating technology. Their marginal costs thus equals the price of the homogeneous domestic good, $MC^f_t = P_{H,t}$. As domestic intermediate good producers, the exported input firms combine domestic labour and capital inputs with foreign and energy components to produce their differentiated output (see Equations 14-16). We thus have that $MC^p_t = MC_t$. When setting prices, both type of exporters apply local currency pricing and face local price rigidities, ξ^{pF}_k, modelled as a Calvo's (1983) friction. The main difference between the two firms thus lies in their marginal cost, and their price setting problem can be generalized as follows:

$$
\max_{\substack{\tilde{P}^z_{US,BE,t}(f)\\ \tilde{P}^z_{EA,BE,t}(f)\\ \tilde{P}^z_{RoW,BE,t}(f)}} \sum_k E_t \sum_{j=0}^{\infty} (\beta\xi^{pF}_k)^j \Lambda_{t,t+j} \Big[S_{k,EA,t+j}\tilde{P}^z_{k,BE,t}(f)\chi_{k,t,j}Y^z_{k,BE,t+j}(f) - MC^z_{t+j}Y^z_{k,BE,t+j}(f) \Big]
$$

where

$$
\chi_{k,t,j} = \begin{cases} 1 & \text{if } j=0 \\ \Pi^j_{i=1}\left(\pi^z_{k,BE,t+i-1}\right)^{\iota_{px}} \bar{\pi}_k^{1-\iota_{px}} & \text{if } j = 1,...,\infty \end{cases}
$$

with $k \in \{EA, US, RoW\}$ and $z \in \{f, p\}$ for respectively final and input exports.[15] The differentiated goods are then converted into homogeneous exported goods by export good assemblers, according to Kimball's (1995) technology:

$$
1 = \int_0^1 G\left(\frac{Y^z_{k,BE,t}(f)}{Y^z_{k,BE,t}}\right) df
$$

with $G(1) = 1, G'(x) > 0$ and $G"(x) < 0$ for all $x \geq 0$. By the zero-profit condition of homogeneous assemblers, and accounting for optimal or indexed prices set by intermediate firms, a general formula for the aggregate export price index is expressed as follows:

[15] Note that in the absence of a fully-fledged RoW bloc, we use $\bar{\pi}_{US}$ as a proxy for $\bar{\pi}_{RoW}$.

$$P^z_{k,BE,t} = \xi^{pF}_{BE,k} \left(\pi^z_{k,BE,t-1}\right)^{\iota_{px}} \bar{\pi}_k^{1-\iota_{px}} P^z_{k,BE,t-1} G^{'-1}\left(\frac{\left(\pi^z_{k,BE,t-1}\right)^{\iota_{px}} \bar{\pi}_k^{1-\iota_{px}} P^z_{k,BE,t-1}}{P_{H,t}} \mathcal{I}^z_{k,BE,t}\right)$$
$$+ (1-\xi^{pF}_{BE,k})\tilde{P}^z_{k,BE,t} G^{'-1}\left(\frac{\tilde{P}^z_{k,BE,t}}{P^z_{k,BE,t}} \mathcal{I}^z_{k,BE,t}\right) \tag{38}$$

with $k \in \{EA, US, RoW\}$ and $z \in \{f, p\}$. As a result of this setup with two types of exports, we obtain two New Keynesian Phillips curves for exported products: one for finished exports that will compete with local final goods used in the total consumption or investment baskets of foreign consumers, and one for exported inputs, that are substitutable abroad with local value added in production processes, as depicted in the foreign equivalent of Equation (16).

2.4 International environment

An updated version of the two-country NK-DSGE model described in de Walque et al. (2017) is used for the dynamics of EA and US variables. Compared to de Walque et al. (2017), intermediate good firms are allowed to make substitution between domestic and foreign inputs according to a CES production function that replaces the Leontieff equation, which assumes complementarity, used in the original EA-US model. We use the same structure as the one described in Section 2.2 for BE firms. Finally, we remove distribution costs from the EA-US bloc as this device generates effects similar to other mechanisms (in terms of limiting exchange rate pass-through and international spillovers, see de Walque et al. (2023b) on this topic). Consequently, its parameter appears to be significantly collinear with other parameters in the model, and generates identification issues in the estimation.[16] More detail on the updated EA-US setup can be found in Appendix 9.1. We describe here the linkages between the international setup and BE external variables.

Total exports

BE total exports is an aggregation of homogeneous exported good (of both the finished f and input p types) with a foreign good, $X_{F,t}$. Total exports thus write as follows:

$$X_t = \left[\phi_X^{1/\lambda_x} \left(X_{H,t}\right)^{\frac{\lambda_x-1}{\lambda_x}} + (1-\phi_X)^{1/\lambda_x} \left(X_{F,t}\right)^{\frac{\lambda_x-1}{\lambda_x}}\right]^{\frac{\lambda_x}{\lambda_x-1}}, \tag{39}$$
$$X_{H,t} = Y^f_{EA,BE,t} + Y^f_{US,BE,t} + Y^f_{ROW,BE,t} + Y^p_{EA,BE,t} + Y^p_{US,BE,t} + Y^p_{ROW,BE,t}, \tag{40}$$
$$X_{F,t} = X_{BE,EA,t} + X_{BE,US,t} + X_{BE,ROW,t}. \tag{41}$$

The foreign good component can be seen as a proxy for transit goods. Also known as re-exports, these transit goods are defined as products that are directly re-exported, usually with no or negligible domestic value added. As pointed out in Duprez (2014), they come partly from the development of international trade routes and can be particularly important for small open economies with important international hubs (e.g. the port of Antwerp for Belgium and of Rotterdam for the Netherlands). Therefore, we assume that the domestic and foreign components of total exports are complements (i.e., $\lambda_x = 0$). Transit goods and the foreign value added contained in the production of domestic exported goods together form the total import

[16] Moreover, as explained in Section 4.1, we also use real import and export times series as observables instead of net trade data used in de Walque et al. (2017). For the EA, these series account for extra-euro area trade only.

content of exports.

The demand for BE products from other fully-fledged regions is derived from a CES aggregator used by foreigners to produce EA or US imported goods:

$$Z_{j,t} = \left[\left(1 - \sum_{k \neq j} \phi_{j,k} \right)^{\frac{1}{\lambda_j}} (Z_{j,H,t})^{\frac{\lambda_j - 1}{\lambda_j}} + \sum_{k \neq j} \phi_{j,k}^{\frac{1}{\lambda_j}} \left\{ \left(1 - \Omega^Z_{j,k,t}\right) Z_{j,k,t} \right\}^{\frac{\lambda_j - 1}{\lambda_j}} \right]^{\frac{\lambda_j}{\lambda_j - 1}} \tag{42}$$

with $Z \in \{C, I, G, Y^p\}$, $j \in \{EA, US\}, k \in \{BE, EA, US, RoW\}$, and with $Y^f_{j,k,t} = C_{j,k,t} + I_{j,k,t} + G_{j,k,t}$. Changes in the good basket is subject to adjustment costs represented by the following functions:

$$\Omega^Z_{j,k,t} = \frac{\Omega^Z_{j,k}}{2} \left(\left(\varepsilon^M_{j,k,t}\right)^{-1/\Omega^Z_{j,k}} \frac{Z_{j,k,t}/Z_{j,H,t}}{Z_{j,k,t-1}/Z_{j,H,t-1}} - 1 \right)^2 \tag{43}$$

Similar to the case of BE imports – see Equation (27) – we refer to $\varepsilon^M_{j,k,t}$ as a shock to the demand for region j imports of BE products. Consistent with our assumption of a negligible feedback from BE to the EA-US bloc, we calibrate $\phi_{j,BE}$ to an insignificant value. We introduce variables $M_{j,BE,t}$ as the total imports of region j of products coming from BE, and relate it to imports for consumption, investment,[17] production and export purposes as follows:

$$M_{j,BE,t} = C_{j,BE,t} + I_{j,BE,t} + Y^p_{j,BE,t} + X_{j,BE,t} \tag{44}$$

with $j \in \{EA, US\}$.

Total demand for BE exports equals the sum of the demand for BE goods from the EA, the US and the RoW:

$$X_t = M_{EA,BE,t} + M_{US,BE,t} + M_{RoW,BE,t} \tag{45}$$

As the RoW is not fully-fledged, the demand of RoW for BE products, $M_{ROW,BE,t}$, is modelled as an AR(1) shock process. This shock process is pinned down by the observation of a measure of the extra-euro area demand for Belgian exporters coming from the Trade Consistency Exercise (see Hubrich and Karlsson, 2010).

Export prices associated with the Cobb-Douglas formulation are as follows:

$$P_{X,t} = \left[\phi_X \left(P_{XH,t}\right)^{1-\lambda_x} + (1 - \phi_X) \left(P_{F,t}\right)^{1-\lambda_x} \right] \tag{46}$$

$$P_{XH,t} = \sum_z \sum_k \frac{\overline{Y}^z_{k,BE}}{\overline{X}_H} P^z_{k,BE,t} \tag{47}$$

with $z \in \{f, p\}$ for finished or input exports and $k \in \{EA, US, ROW\}$.

[17] Note that in the EA and US bloc, and as e.g. in the NAWM model (Christoffel et al., 2008), investment in capital assets and housing are not modelled separately, in contrast with our modelling for BE. There is therefore only one 'investment' variable in each of the EA and US blocs.

Total imports

Optimal demand for bilateral imports are obtained from the first-order conditions of BE firms who minimize the total costs of the baskets described in (26). From those bilateral imports, we construct a non-energy import aggregate as follows:

$$Y_{F,t} = M_{BE,EA,t} + M_{BE,US,t} + M_{BE,RoW,t} \tag{48}$$

where

$$M_{BE,j,t} = C_{BE,j,t} + I_{BE,j,t} + I_{h,BE,j,t} + G_{BE,j,t} + Y^p_{BE,j,t} + X_{BE,j,t} \tag{49}$$

with $j \in \{EA, US, RoW\}$. Total BE imports M_t consists in non-energy imports, $Y_{F,t}$, and oil and energy products used for either consumption (N_t) or production (O^p_t) purposes:

$$M_t = Y_{F,t} + N_t + O^p_t \tag{50}$$

Import prices associated with the different layers of the aggregation of imported quantities are as follows:

$$P_{M,t} = \frac{\bar{Y}_F}{\bar{M}} P_{F,t} + \frac{\bar{N}}{\bar{M}} P_{n,t} + \frac{\bar{O}^p}{\bar{M}} P_{oil,t} \tag{51}$$

$$P_{F,t} = \phi_F \sum_j \phi_{BE,j} P^f_{BE,j,t} + (1 - \phi_F) \sum_j \phi_{BE,j} P^p_{BE,j,t} \tag{52}$$

where ϕ_F is the share of imported finished goods,[18] and where for $j \in \{EA, US, RoW\}$. The following aggregate bilateral import prices is obtained using the same structure as for BE exporters (i.e. Calvo price setting, local currency pricing and generalized Kimball aggregation) applied abroad:

$$\begin{aligned} P_{BE,j,t} = {} & \xi_{BE,j} \pi^{\iota_{BE,j}}_{BE,j,t-1} \bar{\pi}_H^{1-\iota_{BE,j}} P_{BE,j,t-1} G^{'-1} \left(\frac{\pi^{\iota_{BE,j}}_{BE,j,t-1} \bar{\pi}_H^{1-\iota_{BE,j}} P_{BE,j,t-1}}{P_{BE,j,t}} \mathcal{I}_{BE,j,t} \right) \\ & + (1 - \xi_{BE,j}) \tilde{P}_{BE,j,t} G^{'-1} \left(\frac{\tilde{P}_{BE,j,t}}{P_{BE,j,t}} \mathcal{I}_{BE,j,t} \right) . \end{aligned} \tag{53}$$

2.5 Government

The government collects taxes and issues debt to finance its public expenditures. Its budget constraint is as follows:

$$\begin{aligned} P_{G,t} G_t + P_t TR_t = {} & \left(\tau^{w23}_t + \tau^{w1}_t \right) W_t L_t + \tau^k_t R^k_t u^k_t K_{t-1} - \tau^k_t \delta^k P i_t K_{t-1} \\ & + \tau^{vat}_t \left(P_t \mathbb{C}_t + P_{Ih,t} I_{h,t} \right) + P_t T_t + \frac{B_t}{R_t} - B_{t-1} \end{aligned} \tag{54}$$

where G_t represents government purchases of private goods and services, health care and public investment. On the government receipt side, one finds taxes on labour income (including direct household income taxes, social contributions of firms and employees), taxes on capital income, value-added taxes, lump-sum taxes,

[18]This share is inferred from other parameters and steady-state ratios.

T_t, and public debt net of interest payments, $\frac{B_t}{R_t} - B_{t-1}$. Transfers TR_t are a generic variable that collects unmodelled public expenditures (public wages, transfers to households, net capital transfers, depreciation of government capital, subsidies) net of unmodelled public receipts (other indirect taxes, companies direct taxes, and other net government revenues), and is used to obtain an adequate path for the government primary balance and public debt in simulations. Transfers are rebated lump-sum to households. Lump-sum taxes are used to prevent government debt from following an unstable path and ensure the stationarity of the model. They are defined as a fraction of steady state nominal output and are adjusted according to the following fiscal rule:

$$\frac{T_t}{\bar{P}_{\mathcal{Y}}\bar{\mathcal{Y}}} = \varkappa_{B_{H/y}} \left(\frac{B_t}{\bar{P}_{\mathcal{Y}}\bar{\mathcal{Y}}} - \overline{B} \right) \tag{55}$$

where $\overline{B}$ is the fiscal authority's target for the government debt-to-GDP ratio. In the literature on estimated NK-DSGE, it is standard to use non-distortionary taxes or transfers to close the fiscal authority's budget in the estimation of DSGE models.[19] In simulation, however, the impact of distortionary fiscal rules, using adjustments in the modelled tax rates, can be evaluated with the model.

2.6 Closing the model

Goods market clearing and private value added

Total gross output consists in the production of goods sold at home and abroad. Integrating over the continuum of firms f, the following resource constraint is obtained:

$$\mathbb{Y}_t = s_{H,t} Y_{H,t} + \sum_j s^f_{j,BE,t} Y^f_{j,BE,t} + \sum_j s^p_{j,BE,t} Y^p_{j,BE,t} \tag{56}$$

where the terms

$$s_{H,t} = \int_0^1 G'^{-1} \left(\frac{P_{H,t}(i)}{P_{H,t}} \mathcal{I}_{H,t} \right) di$$

and

$$s^z_{j,BE,t} = \int_0^1 G'^{-1} \left(\frac{P^z_{j,BE,t}(i)}{P^z_{j,BE,t}} \mathcal{I}^z_{j,BE,t} \right) di$$

with $z \in \{f, p\}$, reflect the degree of price dispersion across the different varieties of intermediate goods i, and vanish up to a first-order approximation of the model.

Market clearing condition for the final goods market is obtained by integrating the household budget constraint across households, combining it with the government budget constraint, firms and unions profits, and by using the net foreign asset accumulation equation. This derivation leads to the following equilibrium equations that reconciles nominal private value added, $P_{\mathcal{Y},t}\mathcal{Y}_t$, computed using the value-added approach (57) and private value added calculated with the expenditure approach (58):

$$P_{\mathcal{Y},t}\mathcal{Y}_t = P_{\mathbb{Y},t}\mathbb{Y}_t - P^p_{F,t} Y^p_{F,t} - P_{oil,t} O^p_t \tag{57}$$

$$= P_t \mathbb{C}_t + P_{Ih,t} I_{h,t} + P_{I,t} I_t + P_{G,t} G_t + P_{X,t} X_t - P_{M,t} M_t + SCR_t \tag{58}$$

[19]See for instance the estimation methodology followed by Christoffel et al. (2008) for the New Area-Wide Model (NAWM).

where $P^p_{F,t}Y^p_{F,t} = \sum_j P^p_{BE,j,t}Y^p_{BE,j,t}$ with $j \in \{EA, US, RoW\}$. Variable SCR_t denotes an exogenous expenditure, which captures the discrepancies between the value-added and the expenditure approaches, e.g. inventories. We denote the deviation of detrended SCR_t from its steady state, expressed as a percentage of detrended steady-state output as ε^{SCR}_t, i.e. $\varepsilon^{SCR}_t = \frac{scr_t - s\bar{c}r}{\bar{\mathcal{Y}}}$, with $scr_t \equiv \frac{SCR_t}{\gamma^t}$ and assume that it follows an AR(1) process.[20]

Monetary policy

In the EA and US blocs of the model, a standard Taylor rule is used to represent the ECB and Fed monetary policies and to close the model by pinning down the evolution of short-term interest rates. We follow Smets and Wouters (2007) and use the following monetary policy rule (which is identical to the one used in de Walque et al., 2017):

$$\frac{R_{j,t}}{\bar{R}_j} = \left(\frac{R_{j,t-1}}{\bar{R}_j}\right)^{\varphi_r} \left[\left(\frac{\Pi_{j,t}}{\bar{\Pi}_j}\right)^{\varphi_\pi} \left(\frac{\mathcal{Y}_{j,t}}{\mathcal{Y}^*_{j,t}}\right)^{\varphi_y}\right]^{1-\varphi_r} \left(\frac{\mathcal{Y}_{j,t}/\mathcal{Y}_{j,t-1}}{\mathcal{Y}^*_{j,t}/\mathcal{Y}^*_{j,t-1}}\right)^{\varphi_{\Delta y}} \varepsilon^r_{j,t} \tag{59}$$

with $j \in \{EA, US\}$ and where variables with a bar sign indicate steady-state values. Monetary policy implements smooth interest rate changes and reacts to inflation, output and changes in output in deviation from potential output, $\mathcal{Y}^*_{j,t}$, and changes in potential output. The variable $\mathcal{Y}^*_{j,t}$ corresponds to the hypothetical output level that would be observed in the absence of nominal wage and price rigidities and wage and price markup shocks. The exogenous variables $\varepsilon^r_{j,t}$ are AR(1) monetary policy shock processes.

3 Linearisation and mechanics

Trended variables are made stationary prior to the estimation of the model. They return to their steady-state after the economy has been hit by temporary shocks. Real variables are de-trended with the level of technology of the economy, γ^t, which is common to the different regions of the model. The price of the consumption good is used as the numeraire. Other prices are expressed relative to the consumption price, and relative prices are stationary. In the notation of the linear equations, small capital letters indicate de-trended real variables, e.g. $y_t = Y_t\gamma^{-t}$, or relative prices, e.g. $p_{H,t} = P_{H,t}/P_t$. Hat variables stand for variables in percentage deviation from their steady-state, e.g. $\hat{y} = (y_t - \bar{y})/\bar{y}$. The optimisation problems of agents, presented in Section 2 are solved with the Lagrange multiplier method. The model equilibrium equations consist in the resulting first order conditions which define agents' decision rules and market clearing equilibria. This set of non-linear equilibrium equations are then linearised around the non-stochastic steady-state using a first-order Taylor approximation. The resulting system of linear equations is used for the estimation of the model. To gain intuition into the mechanics of the model, and the dynamics of its key variables, we present here a selection of key linearised equilibrium equations.

3.1 Consumption

The linearisation of the first order condition with respect to domestic bonds, Equation (7), combined with the first order condition with respect to consumption generates the following Euler consumption equation:

[20] In national accounts, a variable representing changes in inventories explains the differences between value-added and expenditure approaches. This variable is constructed based on inventories reported in the required full-format accounting reports of large-size BE firms as well as on statistical arbitrage used to match the two approaches. After estimation, the model-generated smoothed shock process ε^{SCR}_t is highly correlated (i.e., around 90%) with the changes in inventories divided by lagged GDP, though it is found to be less volatile than the national account corresponding concept.

$$
\begin{aligned}
\hat{\mathbf{c}}_t = c_1 \hat{\mathbf{c}}_{t-1} + (1-c_1)\mathrm{E}_t \hat{\mathbf{c}}_{t+1} + c_2 \left(\hat{\ell}_t - \mathrm{E}_t \hat{\ell}_{t+1} \right) \\
- c_3 \left(\hat{r}_t - \mathrm{E}_t \hat{\pi}_{t+1} - \frac{E_t \Delta \varepsilon^{vat}_{t+1}}{1+\bar{\tau}^{vat}} + \varepsilon^b_t + E_t \Delta \varepsilon^{bb}_{t+1} \right)
\end{aligned} \tag{60}
$$

where $c_1 = \frac{\lambda_{hab}/\gamma}{1+\lambda_{hab}/\gamma}$, $c_2 = \frac{\bar{W}^h \bar{L}}{\bar{\mathbb{C}}} \frac{(\sigma_c - 1)}{[\sigma_c(1+\lambda_{hab}/\gamma)]}$,$c_3 = \frac{(1-\lambda_{hab}/\gamma)}{\sigma_c(1+\lambda_{hab}/\gamma)}$, and $\hat{\mathbf{c}}_t$ is the aggregate private consumption, $\mathbb{C}_t$, detrended and expressed in percentage deviation from its steady-state. External habit formation in consumption (i.e. λ_{hab}>0 and thus $c_1 > 0$) creates a role for lagged consumption, at the expense of the impact of future consumption $(1 - c_1)$ and the total benefit of postponing consumption c_3, with $c_3 \approx 0$ with full habit formation ($\lambda_{hab} = 1$). Hence the response of consumption to its drivers becomes more gradual.

While we assume a representative household with unrestricted access to credit markets and consumption smoothing, it is worthwhile noting that for $\sigma_c > 1$ and thus $c_2 > 0$, there is a positive effect of current hours worked on current consumption. Linnemann (2006) shows that as a result, in the absence of habit formation and with $\sigma_c > 1$ and when wages and labour demand allocation is independent of the household type, Equation (60) becomes observationally equivalent with an aggregate Euler equation derived by Galí et al. (2007), which adds 'rule of thumb' households who always consume their current income to the standard new Keynesian model.[21] In fact, this mechanism helps to attenuate or avoid a crowding-out effect of public expenditures on private consumption.

3.2 Investment

The dynamics of investment are obtained from the first order conditions of households' optimisation problem with respect to the real value of investment, capital and the capital utilisation rate. In linearised terms, they can be written as follows:

$$
\hat{i}_t = i_1 \hat{i}_{t-1} + (1-i_1)\mathrm{E}_t \hat{i}_{t+1} + i_1 i_2 \left(\hat{q}_t - \hat{p}_{I,t} \right) + \varepsilon^I_t \tag{61}
$$

$$
\hat{q}_t = q_1 \mathrm{E}_t \hat{q}_{t+1} + q_2 \mathrm{E}_t \hat{r}^k_{t+1} - q_3 \mathrm{E}_t \hat{\tau}^k_{t+1} + q_4 \mathrm{E}_t \hat{p}_{I,t+1} - \left(\hat{r}_t - \mathrm{E}_t \hat{\pi}_{t+1} + \varepsilon^b_t \right) \tag{62}
$$

$$
\hat{u}^k_t = u_1 \hat{r}^k_t \tag{63}
$$

where $i_1 = \frac{1}{1+\bar{\beta}\gamma}$, $i_2 = \frac{1}{\varphi\gamma^2}$, $q_1 = \bar{\beta}(1-\delta^k)$, $q_2 = [1-\bar{\beta}(1-(1-\bar{\tau}^k)\delta^k)]$, $q_3 = \frac{1-\bar{\beta}}{1-\tau^k}$, $q_4 = \bar{\beta}\tau^k\delta^k$ and $u_1 = \frac{\upsilon}{1-\upsilon}$. Parameter φ is the steady-state elasticity of the capital adjustment cost function, and υ is a capital utilisation adjustment parameter. If υ takes a small value, close to 0, it is extremely costly to change the intensity of the use of physical capital assets, and the utilisation of capital remains constant, at its steady-state value. In contrast, when υ takes a large value, the utilisation rate becomes volatile while the evolution of the rental rate of capital, $\hat{r}^k$, is smooth. Real investment evolves according to Tobin's Q theory of investment (Tobin, 1969). Business investment expands when the market value of capital, $\hat{q}$, exceeds the replacement cost of

[21] The non-additively separable utility used in the model and the estimation of σ_c thus provide some insight on the importance of hand-to-mouth households for BE aggregate consumption dynamics. It is however more limited than a two-household framework (hand-to-mouth and Ricardian households) with different wages and labour demand allocation. In an alternative version of the model, BE hand-to-mouth consumers are introduced along these lines and their share in total private consumption expenditures is estimated. This share of hand-to-mouth households seems however weakly identified by our dataset of macroeconomic variables, so that we present here a version without an explicit modelling of BE hand-to-mouth households. In future research, we plan to calibrate this share based on BE microeconomic estimates, and evaluate the effect of a presence of hand-to-mouth households for the dynamics of the model, in particular after fiscal policy shocks.

physical assets, $\hat{p}_{I,t}$. The market value of capital depends on expected capital gains, (after-tax) expected rental income, and the stochastic discount factor of investors (i.e. households). The larger the interest rate or the exogenous risk premium, the higher the required return on capital assets, and hence the smaller its market price. The decline in the market value of capital assets in turn discourages investment.

The dynamics of housing investment are governed by similar equilibrium equations compared to business investment.

$$\hat{i}_{h,t} = i_1 \hat{i}_{h,t-1} + (1-i_1)\mathrm{E}_t \hat{i}_{h,t+1} + i_1 i_{h,2}\left(\hat{q}_{h,t} - \hat{p}_{Ih,t}\frac{\varepsilon_t^{vat}}{1+\bar{\tau}^{vat}}\right) + \varepsilon_t^{Ih} \tag{64}$$

$$\hat{q}_{h,t} = q_{h,1}\mathrm{E}_t \hat{q}_{h,t+1} + q_{h,2}\mathrm{E}_t \hat{r}_{t+1}^h - \left(\hat{r}_t - \mathrm{E}_t \hat{\pi}_{t+1} + \varepsilon_t^b\right) \tag{65}$$

$$\hat{u}_t^h = u_{h,1}\hat{r}_t^h \tag{66}$$

where $i_{h,2} = \frac{1}{\varphi_h \gamma^2}$, $q_{h,1} = \bar{\beta}(1-\delta^h)$, $q_{h,2} = [1-\bar{\beta}(1-\delta^h)]$, $u_{h,1} = \frac{\upsilon^h}{1-\upsilon^h}$, and where a VAT shock ε_t^{vat} appears since housing investment goods are subject to indirect taxes. A major difference between business and housing investment comes from the demand side for the rental price of assets. While the rental rate of capital assets is sensitive to firms' demand for capital inputs in production, the rental price of housing assets depends on households' consumption of housing services. It is thus more directly linked to the aggregate consumption dynamics described in the previous paragraph. The linearised optimal demand for housing services is derived from (29) and reads as follows:

$$\hat{h}_t = \hat{\mathbf{c}}_t - \lambda_h \hat{r}_t^h \tag{67}$$

where the rental price of housing services, $\hat{r}_t^h$, is expressed relative to the consumption price, and $\hat{h}_t = \hat{u}_t^h + \hat{h}_{t-1}$.

3.3 Real wage dynamics

Profit maximization by imperfectly competitive labour unions gives rise to the following dynamics for the real wage:

$$\begin{aligned}\hat{w}_t = {} & w_1\left(\hat{w}_{t-1} + \hat{\pi}_{t-1}^{index} - \hat{\pi}_t\right) \\ & + (1-w_1)\left(E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1} - \hat{\pi}_t^{index}\right) - w_2 \hat{\mu}_t^w + \varepsilon_t^w\end{aligned} \tag{68}$$

where

$$\begin{aligned} w_1 &= \left(1+\beta\gamma^{1-\sigma_c}\right)^{-1} \\ w_2 &= \left[\left(1-\beta\gamma^{1-\sigma_c}\xi_w\right)(1-\xi_w)/\left(\xi_w\left\{1+\sigma_\ell\left(1+\lambda_w\right)/\lambda_w\right\}\right)\right]\left(1+\beta\gamma^{1-\sigma_c}\right)^{-1}. \end{aligned}$$

The main difference with the real wage equation detailed in Smets and Wouters (2007) is the full indexation to a moving average of inflation in the BE health consumption index: $\hat{\pi}_t^{index} = \frac{3}{4}\hat{\pi}_t^{health} + \frac{1}{4}\hat{\pi}_{t-1}^{health}$.[22] Due to nominal wage stickiness, real wages adjust gradually to the desired wage-markup, $\hat{\mu}_t^w$, which equals the

[22] The weights imposed to current and lagged value of the inflation of the health index are close to the one used in NONAME. They help to account for a gradual implementation of the wage indexation at the aggregate level.

difference between the real wage and the marginal rate of substitution between working and consuming, that is:

$$\hat{\mu}_t^w = w_t - \sigma_\ell \hat{\ell}_t - \frac{1}{1-\lambda_{hab}} \left(\hat{\mathbf{c}}_t - \frac{\lambda_{hab}}{\gamma} \hat{\mathbf{c}}_{t-1} \right) - \frac{\varepsilon_t^{w23}}{1+\bar{\tau}^{w23}} - \frac{\varepsilon_t^{vat}}{1+\bar{\tau}^{vat}} \tag{69}$$

Ceteris paribus, an increase in the (contemporaneous or expected) costs of supplying an additional unit of labour (i.e. an increase in the disutility of labour) and/or a decline in the benefits of doing so, measured by the marginal utility of consumption, push real wages upwards. An increase in the taxation on labour income or on VAT lowers the later and make unions willing to adjust real wages upwards.

The first term on the right-hand side of (68) highlights the dependence of current wages to past levels of wages indexed on past inflation, taking into account the effect of realized inflation on the purchasing power. Furthermore, note that the real wage is increasing in expected inflation (second term) as unions care about the evolution of their expected purchasing power. The foreseen indexation to the evolution of the contemporaneous health index helps to reduce the effects of shocks that erode the future purchasing power and thus appears with a negative sign. The efficiency of this buffer is however reduced by the gradual implementation of wage indexation. For example, since some oil products are excluded from the wage indexation mechanism, an increase in the price of oil reduces households' purchasing power, and gives an incentive for unions to renegotiate higher real wages.[23]

3.4 Price Phillips curves

The price setting for the intermediate goods of imperfectly competitive domestic firms leads to the following New Keynesian Phillips curve for domestic prices:

$$\hat{\pi}_{H,t} = \pi_1 \hat{\pi}_{H,t-1} + \pi_2 E_t \hat{\pi}_{H,t+1} - \pi_3 \hat{\mu}_t^{pH} + \varepsilon_t^{pH} \tag{70}$$

with $\pi_1 = \iota_p \left(1 + \beta\gamma^{1-\sigma_c}\iota_p\right)^{-1}$, $\pi_2 = \beta\gamma^{1-\sigma_c}\left(1 + \beta\gamma^{1-\sigma_c}\iota_p\right)^{-1}$ and $\pi_3 = \left[(1-\xi_p)(1-\xi_p\beta\gamma)(\epsilon_p - 1) / (\xi_p \{\eta_p + \epsilon_p - 1\})\right]\left(1 + \beta\gamma^{1-\sigma_c}\iota_p\right)^{-1}$. The exogenous variable ε_t^{pH} is a domestic price mark-up shock that follows and ARMA(1,1) process. Due to Calvo's (1983) mechanism of price stickiness, domestic prices only adjust gradually to the evolution in the price mark-up, $\hat{\mu}_t^{pH}$, defined as the difference between selling price and marginal costs:

$$\hat{\mu}_t^{pH} = \hat{p}_{H,t} - \hat{mc}_t \tag{71}$$

where

$$\begin{aligned} \hat{mc}_t &= mc_1 \left[\alpha \hat{r}_t^k + (1-\alpha)\left(\hat{w}_t + \frac{\hat{\tau}^{w1}}{1+\bar{\tau}^{w1}} \right) - \varepsilon_t^a \right] \\ &\quad + mc_2\, \hat{p}_{F,t}^p + (1 - mc_1 - mc_2)\, \hat{p}_{oil,t}, \\ mc_1 &= 1 - \psi\left(\rho_m + \rho_{oil}\right), \\ mc_2 &= \psi \rho_m \, . \end{aligned} \tag{72}$$

Parameter ψ represents the ratio of output gross of fixed costs to output net of fixed costs, while ρ_m and ρ_{oil} denote the steady-state shares of imported intermediate inputs and oil used in production in total output

[23]This is however notwithstanding general equilibrium effects of the shock on the real activity which will impact the disutility of labour and push in the other direction.

gross of fixed costs. Current domestic price inflation depends on its own lag due to indexation to past inflation for firms who are not allowed to reset prices. Price rigidities lead firms who are allowed to re-optimize to consider the probability that they are stuck with the optimal price set today for some period. They thus consider both contemporaneous and expected future changes in their mark-up. This forward-looking behaviour is reflected in the term $E_t\hat{\pi}_{H,t+1}$ in (70).

Similar New Keynesian Phillips curves hold for bilateral import prices (for finished products f and inputs p):

$$\hat{\pi}^z_{BE,j,t} = \pi_{BE,j,1}\,\hat{\pi}^z_{BE,j,t-1} + \pi_{BE,j,2}\,E_t\hat{\pi}^z_{BE,j,t+1} - \pi_{BE,j,3}\,\hat{\mu}^z_{j,t} + \varepsilon^{pF}_{BE,j,t} + \varepsilon_{MTD,t} \tag{73}$$

with

$$\begin{aligned}
\pi_{BE,j,1} &= \iota^{pF}_j\left(1+\beta_j\gamma_j^{1-\sigma^c_j}\iota^{pF}_j\right)^{-1},\\
\pi_{BE,j,2} &= \beta_j\gamma_j^{1-\sigma^c_j}\left(1+\beta_j\gamma_j^{1-\sigma^c_j}\iota^{pF}_j\right)^{-1},\\
\pi_{BE,j,3} &= \left[(1-\xi_{BE,j})\left(1-\xi_{BE,j}\beta_j\gamma_j\iota^{pF}_j\right)(\epsilon_p-1)\,/\,(\xi_{BE,j}\left\{\eta_p+\epsilon_p-1\right\})\right]\left(1+\beta_j\gamma_j^{1-\sigma^c_j}\right)^{-1}
\end{aligned}$$

with $j \in \{EA, US, RoW\}$ and $z \in \{f, p\}$. The exogenous variables $\varepsilon^{pF}_{BE,j,t}$ are bloc-specific price mark-up disturbances, and $\varepsilon_{MTD,t}$ is a common foreign price mark-up shock.[24] The mark-up shocks follow and ARMA(1,1) process. Under local currency pricing, foreign firms set prices in the currency of the destination market. They thus take strategic complementarities at the local level into account (see the home demand curvature parameter, η_p, in coefficient $\pi_{BE,j,3}$) and consider their marginal costs expressed in the local currency. This is reflected by the presence of the real exchange rate in the price mark-up of foreign firms:

$$\hat{\mu}^z_{j,t} = \hat{p}^z_{BE,j,t} - \left(\hat{mc}^z_{j,t} + \hat{rs}_{j,BE,t}\right) \tag{74}$$

As the RoW region is not fully-fledged, we assume that RoW exporters face the same marginal costs as their US counterparts. The exchange rate pass-through to domestic prices is limited twice: first via the nominal and real rigidities that apply at the border via the price setting of foreign exporters (Equation 73), and second via the nominal and real rigidities that apply later in the production chain at the level of domestic producers (Equation 70). We refer to de Walque et al. (2023b) for more details on the implications of foreign inputs in production in terms of exchange rate pass-through and international spillovers.

3.5 International trade

The euro-dollar exchange rate is pinned down by an uncovered interest rate parity condition (UIRP), obtained from EA household's first-order conditions with respect to EA and US bond holdings. Expressed in real terms, we obtain the following equation:

$$\begin{aligned}
\hat{rs}_{EA,US,t} = {} & (1-\theta_s)\,E_t\hat{rs}_{EA,US,t+1} + \theta_s\,\hat{rs}_{EA,US,t-1} + \hat{r}_{US,t} - E_t\hat{\pi}_{US,t+1}\\
& - (\hat{r}_{EA,t} - E_t\hat{\pi}_{EA,t+1}) - \rho_{nfa}nfa_{EA,US,t} + \varepsilon^s_t
\end{aligned} \tag{75}$$

[24] We use specific mark-up shocks and a common mark-up shock to accommodate the multiple import price series observed for the BE bloc: total import prices, a proxy for intra-euro area import prices and for extra-euro area import prices.

where $nfa_{EA,US,t}$ is the net foreign asset position of EA and comes from the presence of transaction costs, modelled as a function of EA real holdings of foreign assets, necessary to ensure the stationarity of the model. In the calibration of the model, we set $\rho_{nfa} = 10^{-7}$ which means that the net foreign asset position does not play a significant role in euro-dollar relative currency dynamics. The coefficient θ_s measures the degree of persistence in euro-dollar exchange rate data. The AR(1) exogenous variable ε_t^s is an international risk premium shock that captures changes in international investors sentiment towards the euro relative to the dollar that are orthogonal to changes in relative economic outlooks or monetary policies. An increase in $\hat{rs}_{EA,US,t}$ indicates a depreciation of the euro vis-à-vis the dollar.

BE and EA economies share the same nominal exchange rate relative to the US dollar. This gives rise to the following relationships for BE real exchange rates:

$$\hat{rs}_{EA,BE,t} = \hat{rs}_{EA,BE,t-1} + \hat{\pi}_{EA,t} - \hat{\pi}_t \tag{76}$$

$$\hat{rs}_{US,BE,t} = \hat{rs}_{EA,BE,t-1} + \hat{rs}_{EA,US,t} \tag{77}$$

$$\hat{eer}_t = \phi_{BE,EA}\hat{rs}_{EA,BE,t} + (1 - \phi_{BE,EA})\left(\phi^*_{BE,US}\hat{rs}_{US,BE,t} + \varepsilon_t^{eer}\right) \tag{78}$$

where the last equation describes the evolution of BE effective real exchange rate, which is a weighted average of bilateral rates relative to modelled areas and a shock process ε_t^{eer}. Coefficient $\phi^*_{BE,US}$ measures the sensitivity of the bilateral real exchange rate between BE and extra-euro area countries to the BE-US real exchange rate.

The linearised optimal demands of region $j \in \{BE, EA, US\}$ for foreign goods coming from other regions can be written as follows:

$$\begin{aligned}\hat{z}_{j,k,t} = {} & \hat{z}_{j,t} - \lambda_j\left(\hat{p}^i_{j,k,t} - \hat{p}_{Z,j,t}\right) + \varepsilon^M_{j,t} \\ & - \lambda_j\Omega_j\left(\Delta\hat{z}_{j,k,t} - \Delta\hat{z}_{H,j,t}\right) \\ & + \beta_j\gamma_j^{-\sigma_j^c}\lambda_j\Omega_j\left(E_t\Delta\hat{z}_{j,k,t+1} - E_t\Delta\hat{z}_{H,j,t+1}\right)\end{aligned} \tag{79}$$

with $z \in \{c, i, ih, g, y^p\}$, $i \in \{f, p\}$ applies alternatively for finished (in the case of the first four elements of z) or input products, and $k \in \{BE, EA, US, RoW\}$ and $j \neq k$. The first term on the right-hand side, $\hat{z}_{j,t}$ reflects the aggregate demand channel (hereafter AD) of the transmission of international shocks while the second term contains price differentials driving the terms-of-trade channel (hereafter ToT). Terms in the last two lines of (79) emanate from adjustment costs in the changes in relative quantities, and generate a gradual evolution in the demand for foreign goods. It is interesting to note that the two trade channels go in the opposite (same) directions for foreign shocks that drive foreign output and interest rate in the opposite (respectively same) direction. For instance, a positive productivity shock outside the euro area leads to an increase in the demand for BE goods via an increase in foreign aggregate demand for all products, giving rise to a positive AD externality. However, as foreign marginal costs decline and as the foreign currency depreciates with respect to the euro, foreign goods become cheaper relative to BE products and the ToT channel oppose to this AD externality. A similar reasoning applies to an expansionary monetary policy shock in extra-euro area countries. In contrast, a positive demand shock outside the eurozone (e.g. generated by a decline in the foreign risk premium) generates an increase in foreign quantities and prices, and an appreciation of the foreign currency vis-à-vis the euro. Consequently, the AD and ToT move in the same direction, increasing the foreign demand for BE products.

4 Estimation

We use a Bayesian full-information maximum likelihood approach (see for instance Smets and Wouters, 2007) to estimate the model, using the Matlab code package Dynare (Adjemian et al., 2021). For computational reasons related to the scale of our three-region model, we estimate the model in two steps. In the first step, we estimate the updated two-region EA-US DSGE model of de Walque et al. (2017), facilitated by our assumption of no feedback from Belgian variables to EA and US variables. Taking the parameters estimated in the first step as given, we estimate the parameters of the Belgian bloc in a second step.

In this section, we start with a discussion on the data used to match model concepts and inform the estimation, as well as on the shock processes. We then present the prior distributions and comment the posterior mode estimates.

4.1 Data and shocks

The sample period considered for the estimation of the DSGE ranges from 1995Q1 to 2019Q4, using the period 1990Q1 to 1994Q4 as a training sample to initialise the Kalman filter iterations. For the EA and the US, we follow de Walque et al. (2017) and use real gross domestic product (YER), real private consumption (PCR), real investment (ITR), hours worked (LAB), real wages (WAGE), as well as the GDP deflator (YED), the private consumption deflator (PCD), the import deflator (MTD) and the short-term interest rate (STI).[25] Different from de Walque et al. (2017), we add the energy component (PEN) of the euro area HICP and the one of the US private consumption deflator to the set of observables. Moreover, we separately observe real export and import growth series (XTR and MTR respectively), instead of the export surplus. For the EA, these concepts are proxies for the trade flows between EA members and countries outside the monetary union. The linearisation implies constant weights of trade variables (and other GDP components) in the GDP equation in the model. However the export- and import-to-GDP ratios significantly trend upwards over the sample. The use of traditional measurement equations with the growth rates of these trade variables and a deterministic constant would thus generate a too high (or too low) impact on GDP in the model at the beginning (respectively end) of the sample period. Alternatively, we choose to observe the contributions of the deviation of exports and imports growths from their historical average multiplied by the share of their levels in GDP lagged. For instance, for exports, we have:

$$(dXTR_t - const_{XTR})\frac{XTR_{t-1}}{YER_{t-1}} = \alpha_x(\hat{x}_t - \hat{x}_{t-1}) \tag{80}$$

where $const_{XTR}$ is the sample average of export growth, α_x is the calibrated steady-state share of exports in GDP. The larger the (lagged) share of export to GDP in the data, the bigger becomes the impact of a deviation of export growth from its trend growth on the model concept $\hat{x}$, and thus on GDP in the model, even though the weight α_x remains constant.

The EA import deflator reflects extra-euro area foreign prices only and does not include intra-zone import prices. Finally, we also include observables for the nominal euro–dollar exchange rate (EXCRN) and the price of crude oil in dollar (POIL).[26] For the US dataset, time series come from the Bureau of Economic

[25] The 3-month Federal Fund rate and the 3-month EURIBOR interest rate are used for the US and EA respectively.
[26] UK Brent, US dollars per barrel.

Analysis. EA data, the exchange rate and oil price series up to 2017Q4 are extracted from the updated Area Wide Model database, described in Fagan et al. (2001). Those time series are updated until 2019Q4 using data from the ECB Macroeconomic Projection Exercise (MPE). Prior to estimation, we express real variables in per capita quarter-on-quarter growth rates and all price deflators as quarter-on-quarter inflation rates.

For the estimation of BE parameters and shocks, we use Belgian counterparts of the real and nominal variables considered for the EA and US described above. For output, we include the growth rates of the real value added by the private sector (YVAR) in the dataset. This measure corresponds to real GDP growth where public wages and pensions as well as indirect taxes and subsidies have been removed.[27] Correspondingly, its deflator (YVAD) measures the modelled domestic output price. The health consumption price index (HEALTH), as well as its energy component (PEN_{health}), used for the automatic wage indexation in BE are also used as observables. In addition, we include real public expenditures (i.e., public consumption and investment, GCIR), the implicit value-added tax (VAT) rate and the implicit employers and employees social contribution rate (TW1 and TW23 respectively). Moreover, we observe business (IOR) and housing investment (IHR) and their respective deflators (IOD and IHD) separately, to match the more disaggregate modelling of investment in the BE bloc. We also include contributions of total exports (XTR) and imports (MTR) to GDP and their respective deflators (XTD and MTD) among the observable variables. BE time series come from NBB stat. Finally, we include measures from the ESCB Trade Consistency Exercise (hereafter TCE) (Hubrich and Karlsson, 2010) for intra- and extra-euro area demand for BE products (WDR_{IN} and WDR_{EX}) as well as measures of the prices of BE imports coming from exporters inside and outside the eurozone (CMD_{IN} and CMD_{EX} respectively), and the BE nominal effective exchange rate (EEN).[28] These external data are extracted from the dataset of the Broad Macroeconomic Projection Exercise (BMPE) in which the NBB participates. Note that the TCE extra-eurozone concepts include US trade data, and are thus related to the US and RoW blocs of the model. The following measurement equations link the BE observed macroeconomic time series to their model counterparts:

[27]BE GDP can thus easily be retrieved from the model estimates for BE private value added in simulation/forecasting.

[28]These TCE measures come particularly in handy during the Broad Macroeconomic Projection Exercise (BMPE) where model projections are undertaken conditional on a path for these variables.

$$
\begin{bmatrix}
dYVAR_t \\ dPCR_t \\ dIOR_t \\ dIHR_t \\ dGCIR_t \\ (dXTR_t-\bar{\gamma}_x)\frac{XTR_{t-1}}{YER_{t-1}} \\ (dMTR_t-\bar{\gamma}_x)\frac{MTR_{t-1}}{YER_{t-1}} \\ dWDR_{IN,t} \\ dWDR_{EX,t} \\ dWAGE_t \\ dLAB_t \\ dYVAD_t \\ dPCD_t \\ dIOD_t \\ dIHD_t \\ dMTD_t \\ dXTD_t \\ dCMD_{IN,t} \\ dCMD_{EX,t} \\ \\ dEEN_t \\ \\ VAT_t \\ TW1_t \\ TW23_t \\ dPEN_t \\ dHEALTH_t \\ dPEN_{health,t}
\end{bmatrix}
=
\begin{bmatrix}
\bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ 0 \\ 0 \\ \bar{\gamma}_{IN} \\ \bar{\gamma}_{EX} \\ \bar{\gamma}_w \\ \bar{\ell} \\ \bar{\pi} \\ \bar{\pi} \\ \bar{\pi}_I \\ \bar{\pi}_{Ih} \\ \bar{\pi}_M \\ \bar{\pi}_X \\ \bar{\pi}_{IN} \\ \bar{\pi}_{EX} \\ \\ \bar{\gamma}_s \\ \\ \bar{\tau}^{vat} \\ \bar{\tau}^{w1} \\ \bar{\tau}^{w23} \\ \bar{\pi}_n \\ \bar{\pi} \\ \bar{\pi}_{n,health}
\end{bmatrix}
+
\begin{bmatrix}
\hat{\boldsymbol{y}}_t-\hat{\boldsymbol{y}}_{t-1} \\
\hat{\mathbf{c}}_t-\hat{\mathbf{c}}_{t-1} \\
\hat{\imath}_t-\hat{\imath}_{t-1} \\
\hat{ih}_t-\hat{ih}_{t-1} \\
\hat{g}_t-\hat{g}_{t-1} \\
\alpha_x(\hat{x}_t-\hat{x}_{t-1}) \\
\alpha_m(\hat{m}_t-\hat{m}_{t-1}) \\
\hat{m}_{EA,BE,t}-\hat{m}_{EA,BE,t-1} \\
\nu_{US}\left(\hat{m}_{US,BE,t}-\hat{m}_{US,BE,t-1}\right)+(1-\nu_{US})\left(\hat{m}_{RoW,BE,t}-\hat{m}_{RoW,BE,t-1}\right) \\
\hat{w}_t-\hat{w}_{t-1} \\
\hat{\ell}_t-\hat{\ell}_{t-1} \\
\hat{\pi}_{H,t} \\
\hat{\pi}_t \\
\hat{p}_{I,t}-\hat{p}_{I,t-1}+\hat{\pi}_t \\
\hat{p}_{Ih,t}-\hat{p}_{Ih,t-1}+\hat{\pi}_t \\
\hat{p}_{M,t}-\hat{p}_{M,t-1}+\hat{\pi}_t \\
\hat{p}_{X,t}-\hat{p}_{X,t-1}+\hat{\pi}_t \\
\phi_F\hat{\pi}^f_{BE,EA,t}+(1-\phi_F)\hat{\pi}^p_{BE,EA,t} \\
\frac{\phi_{BE,US}}{\phi_{BE,US}+\phi_{BE,RoW}}\left(\phi_F\hat{\pi}^f_{BE,US,t}+(1-\phi_F)\hat{\pi}^p_{BE,US,t}\right) \\
+\frac{\phi_{BE,RoW}}{\phi_{BE,US}+\phi_{BE,RoW}}\left(\phi_F\hat{\pi}^f_{BE,RoW,t}+(1-\phi_F)\hat{\pi}^p_{BE,RoW,t}\right) \\
\hat{eer}_t-\hat{eer}_{t-1}-\phi_{BE,US}\hat{\pi}_{US,t} \\
-\phi_{BE,EA}\hat{\pi}_{EA,t}+\hat{\pi}_t \\
\varepsilon^{\text{vat}}_t \\
\varepsilon^{w1}_t \\
\varepsilon^{w23}_t \\
\hat{p}^d_{n,t}-\hat{p}^d_{n,t-1}+\hat{\pi}_t \\
\hat{p}_{health,t}-\hat{p}_{health,t-1}+\hat{\pi}_t \\
\hat{p}_{n,health,t}-\hat{p}_{n,health,t-1}+\hat{\pi}_t
\end{bmatrix}
\tag{81}
$$

where $dX_t = \ln X_t - \ln X_{t-1}$, $\bar{\gamma} = 100(\gamma - 1)$ denotes the deterministic trend GDP growth rate, common to all three blocs. We calibrate the trend growth rate of exports and imports ($\bar{\gamma}_x$) to its sample average, and $\bar{\ell}, \bar{\gamma}_{IN}, \bar{\gamma}_{EX}, \bar{\gamma}_w, \bar{\gamma}_s, \bar{\pi}_I, \bar{\pi}_{Ih}, \bar{\pi}_M, \bar{\pi}_{IN}, \bar{\pi}_{EX}, \bar{\tau}_{vat}, \bar{\tau}_{w1}, \bar{\tau}_{w23}$ are constants that are estimated along with the structural parameters of the model. The observable on private value-added growth is related to variable $\hat{\boldsymbol{y}}_t$, which corresponds to the log-linearised output of Equation (57), $\mathcal{Y}_t$. Parameter $\bar{r}_{EA} = 100(\bar{\pi}_{EA}/\bar{\beta}_{EA} - 1)$ is the steady-state nominal interest rate of the euro area. We assume that BE consumption price inflation and household discount factor equals the EA ones at steady-state: $\bar{\pi} = \bar{\pi}_{EA}$ and $\bar{\beta} = \bar{\beta}_{EA}$. Hence, the BE short-term interest rate has the same steady-state as the eurozone rate.

Bayesian estimation of DSGE models requires at least as many stochastic shocks as observable variables in order to avoid stochastic singularity. Our model has 47 observables and 48 structural shocks. Firstly, in each bloc we have the six shocks familiar from the Smets and Wouters (2007) closed economy setup, namely an exogenous expenditure shock ε_t^{SCR} (see Equation 58), a domestic risk premium shock ε_t^b, which increases the demand for safe assets at the expense of consumption and capital goods (see Equations 60 and 62), an investment specific technology shock ε_t^I (see 61), a price markup ε_t^{pH} and wage markup shock ε_t^w in the domestic price and wage Phillips Curves (see Equations 68 and 70), an energy price shock ε_t^n (Equation 32) and a total factor productivity shock ε_t^a (Equation 14). Moreover, we have two monetary policy shocks associated with the EA and US Taylor rules, $\varepsilon_{j,t}^r$ (see Equation 59). Although in general equilibrium, a shock can and in practice typically does affect many or all model variables, these shocks can be thought of accommodating (or being identified by) the following observable variables in particular: Respectively, GDP,[29] consumption, investment, the GDP deflator, the real wage, energy prices, hours worked, and the Euro Area and US policy rates. Furthermore, to accommodate the inclusion of the consumption expenditure deflator, the health consumption price and its energy component, the business investment price deflator and the housing investment price deflators among the observable variables, we add a specific shock to each of these prices.

Furthermore, a number of shocks are present in the BE bloc only to accommodate the additional number of variables observed for Belgium. Firstly, there are two investment specific technology shocks in the BE bloc instead of just one as in the other regions, as the estimation observes BE business and housing investment separately. Secondly, there are four fiscal shocks to account for the observation of government expenditures, the implicit VAT rate, the implicit social contribution rate and the implicit labour tax rate (τ_t^{w1} and τ_t^{w23}, see equations 54 and 69). Thirdly, we add a discount factor shock ε_t^{bb} (see equation 60) specific to BE consumption, which comes in addition to the number of shocks required to avoid stochastic singularity. Unlike the risk-premium shock, the discount factor shock has a direct impact on consumption only, to allow for the model to account for the fact that BE real consumption and investment growth co-move significantly less than in the EA and the US.[30]

The observation of variables related to the open-economy dimension of the estimation motivates the inclusion of a number of additional shocks. Each of the three fully-fledged blocs (i.e. BE, EA, US) has an import demand shock. Moreover, we associate a shock with the demand of the RoW for exports of the three modelled regions. For BE, as we observe the demand for Belgian goods coming from intra-euro area and extra-euro area partners, two additional shocks are used to accommodate these variables. We include price mark-up shocks into import price Phillips curves. As we observe proxies for the price of BE intra- and extra-euro area imports, as well as for the total import deflator, the BE bloc has two specific foreign price mark-up shocks ($\varepsilon_{BE,EA,t}^{pF}$ and $\varepsilon_{BE,RoW,t}^{pF}$ in Equation 73) and a common foreign price mark-up shock ($\varepsilon_{MTD,t}^{pF}$).[31] A price mark-up shock is also added to the export price Phillips curve of BE, as we observe the Belgian export deflator. Furthermore, the estimation features a UIP shock affecting the euro-dollar exchange rate (see Equation 75) and a separate shock to the effective exchange rate of Belgium. Finally, an oil price shock accommodates the inclusion of the crude oil price observable.

All shocks follow AR(1) processes, except for price and wage mark-up shocks, which also feature a moving average term to better fit high frequency fluctuations typically found in inflation observables. To enhance the ability of the model to account for international business cycle correlation we allow for cross-country correlation of the innovations of the total factor productivity shocks, the risk premium shocks, the import price mark-up shocks, the import demand and the monetary policy shocks. Though crude oil price shock are

[29]Specifically, the component of GDP which the estimation does not observe, i.e. in BE inventories and in the EA and the US the sum of inventories and government expenditure on goods and services.

[30]This flexibility is especially welcome in projection exercises where modellers would have a consumption-specific shock available to fine-tune projected consumption paths. After estimation, it appears that the model picks a combination of a persistent risk premium shock and a fast-adjusting preference shock.

[31]The US specific foreign mark-up shock $\varepsilon_{BE,US,t}^{pF}$is not separately identified from $\varepsilon_{BE,RoW,t}^{pF}$ as we do not observe a time series for import price for US separated from RoW. We thus dropped it from the estimation.

common to all blocs in the model, we also introduce correlation between energy price shocks to account for global changes in non-oil wholesale energy prices.

In the BEMGIE setup, shocks to the EA bloc can be used to simulate shocks specific to the rest of the euro area, i.e. excluding Belgium. For instance they could be calibrated to reflect a shock to one or several eurozone partners. However, their estimation is also influenced by more outspread euro-area effects, or euro-area wide disturbances, that are not necessarily well captured endogenously in our model. This is the reason why we allow some BE shock processes –TFP, risk premium and consumer preference and investment technology– to react to the innovations of their EA counterparts. For instance, the integration of the Belgian financial sector in the euro area one can lead to an important missing transmission channel. Ueda (2012) and Alpanda and Aysun (2014) show in multi-country DSGE the importance of banking globalization and financial frictions/shocks for generating international spillovers. Correlations between risk premium shocks and possibly investment-specific shocks are good candidates to exogenously account for the financial channel, not explicitly modelled in the current version of BEMGIE. Smets and Wouters (2007) mention that this shock has effects comparable to the one of an external finance premium (as explicitly modelled in (Bernanke et al., 1999)). Justiniano et al. (2010) relate the shock to the marginal efficiency of investment to disruptions in the functioning of the financial sector and de Walque et al. (2017) find that the risk premium shock is an important contributor to US and EA real GDP fluctuations during the Global Financial Crisis episode. The correlation in these shocks can thus be seen as a (admittedly rough) short-cut for the representation of an internationally integrated financial system as developed by Dedola et al. (2012) or Kollman (2013). Moreover, as explained in Corbo and Strid (2020), small open economy agents are to a large extent influenced by the same news as foreigners. This is even more true for a SOE inside a monetary union as BE. As a result, agents may display the same waves of optimism and pessimism, as reflected by correlated sentiment/confidence indicators.[32] Correlated demand shocks can also help to capture these dependencies. Finally, as Belgium is well integrated in the euro area and world goods markets, it makes the supply side of its economy quite open to foreign technology developments. The introduction of a correlated TFP shock helps to capture the fact that, to some extent, the Belgian productivity should follow the one observed in the euro area. This co-movement is consistent with the empirical measures of BE and EA TFP growth, as reported in De Mulder and Godefroid (2018).

4.2 Calibration and priors

We calibrate a number of parameters in advance of the estimation. Firstly, we restrict some parameters in order to set steady-state ratios equal to empirical targets calculated as averages over the period over which we estimate the model or obtained from other sources. In the Belgian bloc, we target a share of private consumption, business investment, housing investment, public expenditures (excluding government wage bill) and imports in private value added equal to, respectively, 0.61, 0.18, 0.06, 0.15 and 0.75. Given the values of the otherwise calibrated or estimated model parameters, these targets pin down the capital income share in domestic value added α and the home bias in the non-oil-non-housing final consumption goods basket ϕ_C (see Equation 26).[33]

Furthermore, we follow Duprez (2014) and assume an import content of exports (i.e. foreign value added in production and transit goods) of 60% of total exports. We set the share of finished imported goods in

[32] For instance, OECD consumer confidence indicators for Belgium and the euro area are correlated around 60% for the 1995-2019 period.

[33] We estimate the home bias of the investment good baskets.

public expenditure ϕ_H^g to 0.07, which after estimation helps to obtain an import content of public consumption in line with the share of government consumption expenditures of foreign origin reported in the World Input–Output Database (WIOD, 2014 vintage, see Timmer et al., 2015). The share of energy in the private consumption basket and in the health index $\phi_{n,c}$ is calibrated to 0.08. The national accounts deliver an annual detail of private consumption expenditures per categories. One category is related to 'electricity, gas and other fuels' used for housing, and has an average share of about 0.05 of total expenditures (period 1995-2019). This constitutes a lower bound for our parameter, as energy products are also significant in transport expenditures. The transport item in the national account data is however not sufficiently granular to compute an accurate share of energy used in transportation. In the HICP (Eurostat) the weight of the transport fuel category averages to about 3/5 of the one for 'Electricity, gas and other fuels' for the period 1996-2019. Applying this proportion to the private consumption deflator, a share of energy in total expenditures is found to be 0.08. This number also corresponds to the average weight of the energy component in the health consumption index for the period 2004-2019. The weights ν_{oil} and $\nu_{oil,health}$ in Equations (32) and (37) are calibrated based on a regression of the QoQ inflation rates of the HICP energy price and respectively the health index energy price on the QoQ inflation rates of crude oil price converted in euro over the period 1995-2019. For Equation (32) (and Equation 37), we obtain values of 0.16, 0.06, 0.06 and 0.03 (respectively 0.04, 0.07, 0.06 and 0.03) for the coefficients in front of respectively the contemporaneous and the first three lags of crude oil price. The price elasticity of oil demand for consumption purposes is calibrated at 0.3, a value consistent with estimates found in Natal (2012) and Kilian and Murphy (2014). As in de Walque et al. (2017), the elasticity of substitution between oil and non-oil inputs in production is set to 0, assuming complementarity.

In the euro area and US regions, we target the GDP shares of private consumption (EA: 0.56/ US: 0.67), investment (0.22/0.17), government expenditure (0.20/0.15) and imports (0.22/0.15), which analogously to the Belgian bloc pin down α, and the home bias in the non-oil final goods basket. For further details on the empirical targets used to pin down the EA and US blocs we refer to de Walque et al. (2017).

In all three blocks, we adopt standard values for the labour market related parameters. We set the steady-state wage mark-up, $1 + \lambda_w$, to 1.25, and the inverse elasticity of labour supply to wages, σ_ℓ to 2. Following Smets and Wouters (2007), the curvature of the demand for labour inputs from unions or for goods from homogeneous assemblers are set to 10 (Kimball curvature, η_w and η_p). In line with the literature, the depreciation rates on capital assets, δ^k, equals 2.5% on a quarterly basis. Based on Clerc et al. (2015), the depreciation rate δ^h in the real estate asset accumulation equation is calibrated at 0.75% on a quarterly basis. Given this value for the depreciation rate and the consumption- and housing investment-to-GDP ratios, the share of housing services in total consumption is obtained to be 0.12. This is in line with, though a bit smaller than, the average share of actual, imputed rents and other housing services in private consumption expenditures in national accounts, obtained to be around 0.16 for 1995-2019. The coefficient linking the adjustment of lump-sum taxes to the deviation of public debt from its target, $\varkappa_{B_{H/y}}$, equals 0.10.

The prior distributions used in the Bayesian estimation of the model are in line with the literature. Standard deviations of shock processes are assumed to follow an inverted Gamma distribution with mean 0.2 and 2 degrees of freedom. The prior set on their autoregressive coefficient is a Beta distribution with a mean of 0.5 and a standard deviation of 0.2. Moving average coefficients of price and wage mark-up shocks are assumed to follow a Beta(0.4,0.15). When we allow for correlation between shock innovations, the Gaussian law with mean 0 and a standard deviation of 0.5 is used as a prior distribution.

In the three blocs of the model, we use the same prior distributions and associated parameters as Smets and Wouters (2007) for fixed costs, habit coefficient, investment adjustment costs, the degree of relative risk aversion, Taylor rule coefficients, the discount rate of households, price and wage indexation and the deterministic growth rate of the economy. Our prior for the capital/housing utilisation rates are somewhat laxer, set as a Beta(0.5,0.2). We also use a Beta distribution, with mean 0.5 and standard deviation 0.15, for Calvo price rigidities. The QoQ inflation rates at steady-state follows a Gamma prior around 0.5 with a standard deviation of 0.1. The elasticity of substitution between housing expenditures and other consumption items is also a Gamma prior with mean 1 and standard deviation of 0.5.

For open economy parameters, we follow de Walque et al. (2017) and use a Gamma(3,1) for trade elasticities and a Normal(4,1) for adjustment costs in foreign quantities (both for intermediate and final goods). The prior set on the share of foreign inputs in production is similar to this paper for EA and US areas: a Normal distribution with mean 0.06 and standard deviation of 0.02. We use a larger prior mean, 0.12, for BE as, as explained earlier, the model relies on foreign value added in production for the simultaneous replication of a high Belgian imports-to-GDP ratio and a limited ERPT to consumer prices, via a large home bias in the consumption basket (i.e. the share of domestically produced goods in the basket of final goods). The home biases in the two investment baskets (business and housing investment) follow a Beta prior with mean 0.5 and dispersion 0.25. Beta priors are also used for the sensitivity of total imports to intra-euro area products and the sensitivity of total exports to intra-zone demand, with a prior mean of 0.5, respectively 0.6, and standard deviations of 0.2 and 0.05. These prior means are close to the historical averages over (2018-2021) available in the monthly bulletins for NBB Foreign Trade statistics since 2019 (see for instance Institut des comptes nationaux 2021). A Beta(0.5,0.2) is set for the sensitivity of extra-euro area demand for BE goods to the US demand. Normal distribution with large dispersion are used for remaining constant parameters, as those parameters are generally well pinned down by the data.

4.3 Posterior estimates

Table 1 reports the posterior estimates of EA-US shock processes. Regarding the exogenous driving processes, we find only few differences in the estimated AR(1) coefficients across regions. As commonly found in estimated DSGEs on these two areas with data that covers the Global Financial Crisis, we obtain a persistent domestic risk premium shock, with a significant correlation between the innovations of this shock for the two countries. The finding of highly persistent TFP and exogenous expenditure processes is also standard. In contrast with Christoffel et al. (2008), we allow for autocorrelation in the monetary policy shocks, and estimate non-negligible AR(1) coefficients.

Turning to the posterior estimates of main EA-US structural parameters, reported in Table 2, we find that the parameters driving the response of consumption to interest rate and risk premium shock changes, i.e. the degrees of relative risk aversion σ_c (i.e. the inverse of the inter-temporal substitution elasticity of households) and habit formation λ_{hab}, are very close. By contrast, investment responds somewhat more smoothly to changes in Tobin's Q in the US than in the EA. The marginal cost coefficient in the domestic price Phillips Curve π_3 (see Equation 70) implied by the estimated degree of nominal price rigidity and the fixed cost is close across the two regions.[34] Furthermore, import prices are relatively more flexible, with Calvo (1983) parameters of 0.27 and 0.49, implying a rapid exchange rate pass-through at the border. The

[34]The fixed costs affects the Phillips Curve slope by directly determining the price markup of firms and thus the elasticity of substitution between varieties of the intermediate good ϵ_p ,which affects π_3.

Taylor rule reaction coefficient to inflation is estimated to be somewhat smaller in the US, while steady-state inflation rates are both around 0.5% on quarterly basis. The QoQ deterministic growth rate of the modelled economies is estimated to be 0.34%, in line with the historical averages of the three real GDP growths for the period 1995-2019 (i.e. 0.32% and 0.33% for EA and US real growth, and 0.39% for the growth rate of BE private value added). The trade elasticity between domestic and foreign final goods is found to be close to 1 in the US, and close to 2 in the EA. The order of magnitude is found to be opposite for estimated elasticities for foreign inputs in production. The share of those imported intermediate goods in total output (gross of fixed costs) is around 7% for the EA and 5% in the US. While the share of oil in total output (gross of fixed costs) is estimated to be about the same in the two countries (around 1%), refining and distribution costs are larger in the US. Despite a more important calibrated share of oil in the US consumption basket, these estimates imply a smaller pass-through of oil price to US consumption prices compared to the one to EA consumption prices.

Posterior estimates of BE shock processes can be found in Table 3 for the standard deviation of shocks, and Table 4 for their estimated persistence and moving average coefficients. On the demand shock side, we obtain a combination of a persistent BE risk premium shock, common to consumption and investment expenditures, with volatile preference, capital investment and housing investment shocks. The BE risk premium shock is significantly correlated with the EA one, as indicated by a large estimated value for the reaction of BE innovations to EA ones (0.79). The BE TFP shock is estimated to be as persistent as the EA and US counterparts. The three external demand shock are significantly used by the model to explain fluctuations in total BE exports. Intra- and extra-euro area demands are found to be long-lasting processes. Like in the EA and the US, the exogenous expenditure shock is highly persistent. Note that in the BE bloc, this shock represents changes in inventories, as the estimation observes all components of expenditure-approach based GDP, and is highly correlated with changes in inventories in the data.

Table 5 shows the posterior mode estimates of BE structural parameters. While a similar value is obtained in BE and EA for the posterior mode of the habit coefficients, the coefficient of relative risk aversion of Belgian households, σ_c, is found to be larger. As detailed in Section 3.1, this value implies a complementarity between hours worked and consumption expenditures, and a smaller sensitivity of BE consumption to changes in the real rate compared to EA consumption. Real rigidities on investment (capital and housing investment adjustment costs) display larger values than EA counterparts. The adjustment costs on housing investment are significantly higher than for capital investment. The utilisation adjustment parameters are high, meaning that the model relies on the variable utilisation intensity of capital and real estate assets to fit the data, in contrast with EA and US where this parameter is close to 0. The elasticity of substitution between housing and other consumption expenditures is relatively low, with an estimated mode at 0.72. The marginal cost coefficient of the domestic price NKPC is very close to its EA counterpart, due to a very similar Calvo (1983) probability and degree of fixed costs. The slope of the wage Phillips curve is however found to be steeper in BE, due to a smaller estimated value for the wage Calvo (1983) probability (0.79 vs 0.88 in EA). The import price Phillips curve are quite steep. The value of the Calvo (1983) parameter on extra-euro area import price is very small, 0.12, which implies more flexible prices than what is obtained for the EA. The flexible import prices generate a large exchange rate pass-through at the border, in line with the findings of de Walque et al. (2017). The pass-through to final prices is substantially limited by a high estimated share of foreign inputs in production (0.15), meaning that an important part of imports is rerouted away from the final goods basket and towards domestic production. The impact of exchange rate fluctuations is thus attenuated at two stages: slightly at the border, where the NKPC slope is relatively steep, but much more via the price setting of

Table 1: Posterior estimates of EA and US shock processes.

		Prior distribution				Posterior mode	
Standard deviations		Distr.	Mean	St. Dev.	Third param.	US	EA
Euro-dollar risk premium	ε^s	Inv. Gamma	0.20	2.00		0.47	
Oil price	ε^{oil}	Inv. Gamma	0.20	2.00		0.12	
Energy price	ε^n	Inv. Gamma	0.20	2.00		5.16	1.58
TFP	ε^a	Inv. Gamma	0.20	2.00		0.45	0.24
Domestic risk premium	ε^b	Inv. Gamma	0.20	2.00		0.24	0.18
Government spending	ε^g	Inv. Gamma	0.20	2.00		0.30	0.35
Investment technology	ε^i	Inv. Gamma	0.20	2.00		0.24	0.63
Monetary policy	ε^r	Inv. Gamma	0.20	2.00		0.10	0.07
Domestic wage mark-up	ε^w	Inv. Gamma	0.20	2.00		0.43	0.13
Domestic price mark-up	ε^{pH}	Inv. Gamma	0.20	2.00		0.12	0.12
Foreign price mark-up	ε^{pF}	Inv. Gamma	0.20	2.00		1.45	1.40
Consumption price	ε^{pC}	Inv. Gamma	0.20	2.00		0.15	0.17
Export demand	ε^x	Inv. Gamma	0.20	2.00		1.08	1.29
Import demand	ε^m	Inv. Gamma	0.20	2.00		4.89	7.67
Autoregressive coefficients							
Euro-dollar risk premium	ρ^s	Beta	0.50	0.20		0.83	
Oil price	ρ^{oil}	Beta	0.50	0.20		0.91	
Energy price	ρ^n	Beta	0.50	0.20		0.67	0.93
TFP	ρ^a	Beta	0.50	0.20		0.98	0.97
Domestic risk premium	ρ^b	Beta	0.50	0.20		0.89	0.93
Government spending	ρ^g	Beta	0.50	0.20		0.97	0.98
Investment technology	ρ^i	Beta	0.50	0.20		0.84	0.39
Monetary policy	ρ^r	Beta	0.50	0.20		0.60	0.40
Domestic wage mark-up	ρ^w	Beta	0.50	0.20		0.35	0.74
Domestic price mark-up	ρ^{pH}	Beta	0.50	0.20		0.89	0.80
Foreign price mark-up	ρ^{pF}	Beta	0.50	0.15		0.95	0.80
Consumption price	ρ^{pC}	Beta	0.50	0.20		0.98	0.94
Export demand	ρ^x	Beta	0.50	0.20		0.95	0.72
Import demand	ρ^m	Beta	0.50	0.20		0.74	0.92
Moving average coefficients							
Oil price	μ^{oil}	Beta	0.40	0.15		0.27	
Domestic wage mark-up	μ^w	Beta	0.40	0.15		0.30	0.65
Domestic price mark-up	μ^{pH}	Beta	0.40	0.15		0.82	0.73
Foreign price mark-up	μ^{pF}	Beta	0.40	0.15		0.55	0.15
Correlations							
TFP	ν^a	Beta	0.00	0.35	-1.00	0.26	
Domestic risk premia	ν^b	Beta	0.00	0.35	-1.00	0.26	
Interest rates	ν^r	Beta	0.00	0.35	-1.00	0.40	
Foreign prices	ν^{pF}	Beta	0.00	0.35	-1.00	0.30	
Energy prices	ν^n	Beta	0.00	0.35	-1.00	0.45	
Import demands	ν^m	Beta	0.00	0.35	-1.00	0.38	

Table 2: Posterior estimates of selected EA and US structural parameters.

		Prior distribution			Posterior mode	
Parameter		Distr.	Mean	St. Dev.	US	EA
Fixed cost in prod.	Ψ	Normal	1.25	0.125	1.18	1.64
Habit coefficient	λ_{hab}	Beta	0.50	0.20	0.70	0.74
Investm. Adj. Cost	φ	Normal	4.00	1.50	6.25	4.72
Relative risk aversion	σ_c	Gamma	1.00	0.50	1.09	1.04
Utilisation cost	ν	Beta	0.50	0.20	0.05	0.24
Domestic wage rigidity	ξ_w	Beta	0.50	0.15	0.77	0.88
Domestic wage indexation	ι_w	Beta	0.50	0.15	0.44	0.20
Domestic price rigidity	ξ_p	Beta	0.50	0.15	0.84	0.79
Domestic price indexation	ι_p	Beta	0.50	0.15	0.23	0.17
Foreign price rigidity	ξ_{pF}	Beta	0.50	0.15	0.27	0.49
Foreign price indexation	ι_{pF}	Beta	0.50	0.15	0.17	0.22
Taylor rule, inflation	φ_π	Normal	1.50	0.25	1.63	1.69
Taylor rule, smoothing	φ_r	Beta	0.75	0.10	0.90	0.90
Taylor rule, output gap	φ_y	Normal	0.13	0.05	0.14	0.10
Taylor rule, Δ output gap	$\varphi_{\Delta y}$	Normal	0.13	0.05	0.12	0.14
Trade elasticity (final goods)	λ	Gamma	3.00	1.00	1.07	2.27
Adj. Costs imports (final goods)	Ω	Normal	4.00	1.00	3.94	4.37
Trade elasticity (interm. goods)	λ_m	Gamma	3.00	1.00	2.85	0.65
Adj. Costs imports (interm. goods)	Ω_m	Normal	4.00	1.00	4.85	4.66
Sensitivity to bilateral exports	$\phi_{i,j}$	Beta	0.50	0.10	0.08	0.15
Sensitivity to exports to ROW	$\phi_{i,RoW}$	Beta	0.50	0.10	0.67	0.58
Energy distribution cost	δ_{oil}	Uniform	0.00	3.00	1.32	0.49
Share of foreign inputs in prod.	ρ_m	Normal	0.06	0.02	0.050	0.071
Share of oil in prod.	ρ_{oil}	Gamma	0.01	0.0075	0.008	0.012
Inflation at st.-st.	$\bar{\pi}$	Gamma	0.50	0.10	0.46	0.48
Nominal rate at st.-st.	$100(1/\beta - 1)$	Gamma	0.25	0.10	3.74	3.81
Growth at st.-st.	γ	Normal	0.40	0.10	0.34	

Table 3: Posterior estimates of the standard deviation and correlation of BE shock processes.

		Prior distribution				Posterior mode
Standard deviations		Distr.	Mean	St. Dev.	Third param.	
TFP	ε^{a}	Inv. Gamma	0.20	2.00		0.46
Domestic risk premium	ε^{b}	Inv. Gamma	0.20	2.00		0.08
Preference	ε^{bb}	Inv. Gamma	0.20	2.00		1.61
Government spending	ε^{g}	Inv. Gamma	0.20	2.00		1.96
Business investm. Technology	ε^{i}	Inv. Gamma	0.20	2.00		0.99
Housing investm. Technology	ε^{ih}	Inv. Gamma	0.20	2.00		0.30
Domestic wage mark-up	ε^{w}	Inv. Gamma	0.20	2.00		0.32
Domestic price mark-up	ε^{pH}	Inv. Gamma	0.20	2.00		0.15
Intra-euro area import price mark-up	$\varepsilon^{pF}_{BE,EA}$	Inv. Gamma	0.20	2.00		0.61
Extra-euro area import price mark-up	$\varepsilon^{pF}_{BE,RoW}$	Inv. Gamma	0.20	2.00		2.94
Consumption price	ε^{pC}	Inv. Gamma	0.20	2.00		0.26
Health consumption price	ε^{n}	Inv. Gamma	0.20	2.00		0.25
Energy price	ε^{n}	Inv. Gamma	0.20	2.00		2.45
Energy price (health index)	ε^{n}	Inv. Gamma	0.20	2.00		2.38
Business investm. Price	ε^{pI}	Inv. Gamma	0.20	2.00		0.37
Housing investm. Price	ε^{pIh}	Inv. Gamma	0.20	2.00		0.64
Total import price	ε^{pM}	Inv. Gamma	0.20	2.00		1.07
Total export price	ε^{pX}	Inv. Gamma	0.20	2.00		1.38
Import demand	ε^{m}	Inv. Gamma	0.20	2.00		1.68
RoW demand	$\varepsilon^{M}_{ROW,BE}$	Inv. Gamma	0.20	2.00		1.48
Intra-euro area demand	$\varepsilon^{M}_{EA,BE}$	Inv. Gamma	0.20	2.00		1.06
Extra-euro area demand	$\varepsilon^{M}_{WDR_{EX}}$	Inv. Gamma	0.20	2.00		1.20
Effective exchange rate	ε^{eer}	Inv. Gamma	0.20	2.00		0.57
Changes in inventories	ε^{SCR}	Inv. Gamma	0.20	2.00		0.26
Social contribution (firms)	ε^{tw1}	Inv. Gamma	0.10	2.00		0.009
Social contr.(employees) and lab. inc. tax	ε^{tw23}	Inv. Gamma	0.10	2.00		0.012
VAT tax	ε^{vat}	Inv. Gamma	0.10	2.00		0.008
Correlations (between innovations)						
Energy price (BE-EA)	ν^{n}	Beta	0.00	0.35	-1.00	0.15
Energy price (CPI - health index)	$\nu^{n,health}$	Beta	0.00	0.35	-1.00	0.69
Correlations (reaction to EA innovations)						
TFP	ν^{a}	Normal	0.00	0.50		0.60
Domestic risk premium	ν^{b}	Normal	0.00	0.50		0.79
Preference	ν^{r}	Normal	0.00	0.50		0.30
Business investm. Technology	ν^{pF}	Normal	0.00	0.50		-0.02

Table 4: Posterior estimates of the autoregressive and moving average coefficients of BE shock processes.

		Prior distribution			Posterior mode
Autoregressive coefficients		Distr.	Mean	St. Dev.	
TFP	ρ^{a}	Beta	0.50	0.20	0.98
Domestic risk premium	ρ^{b}	Beta	0.50	0.20	0.93
Preference	ρ^{bb}	Beta	0.50	0.20	0.38
Government spending	ρ^{g}	Beta	0.50	0.20	0.83
Business investm. Technology	ρ^{i}	Beta	0.50	0.20	0.08
Housing investm. Technology	ρ^{ih}	Beta	0.40	0.10	0.70
Domestic wage mark-up	ρ^{w}	Beta	0.50	0.20	0.60
Domestic price mark-up	ρ^{pH}	Beta	0.50	0.20	0.62
Intra-euro area import price mark-up	$\rho^{pF}_{BE,EA}$	Beta	0.50	0.15	0.97
Extra-euro area import price mark-up	$\rho^{pF}_{BE,ROW}$	Beta	0.50	0.15	0.85
Consumption price	ρ^{pC}	Beta	0.50	0.20	0.98
Energy price	ρ^{n}	Beta	0.50	0.20	0.87
Energy price	ρ^{n}	Beta	0.50	0.20	0.98
Energy price	ρ^{n}	Beta	0.50	0.20	0.93
Business investm. Price	ρ^{pI}	Beta	0.50	0.20	0.94
Housing investm. Price	ρ^{pIh}	Beta	0.50	0.20	0.99
Total import price	ρ^{pM}	Beta	0.50	0.20	0.82
Total export price	ρ^{pX}	Beta	0.50	0.20	0.09
Import demand	ρ^{m}	Beta	0.50	0.20	0.88
ROW demand	$\rho^{M}_{ROW*,BE}$	Beta	0.50	0.20	0.63
Intra-euro area demand	$\rho^{M}_{EA,BE}$	Beta	0.50	0.20	0.94
Extra-euro area demand	$\rho^{M}_{WDR_{EX}}$	Beta	0.50	0.20	0.93
Effective exchange rate	ρ^{eer}	Beta	0.50	0.20	0.90
Changes in inventories	ρ^{SCR}	Beta	0.50	0.20	0.97
Social contribution (firms)	ρ^{tw1}	Beta	0.50	0.20	0.88
Social contr.(employees) and lab. inc. tax	ρ^{tw23}	Beta	0.50	0.20	0.90
VAT tax	ρ^{vat}	Beta	0.50	0.20	0.82
Moving average coefficients					
Domestic wage mark-up	μ^{w}	Beta	0.10	0.075	0.03
Domestic price mark-up	μ^{pH}	Beta	0.10	0.075	0.12
Intra-euro area import price mark-up	μ^{pF}	Beta	0.40	0.15	0.50
Extra-euro area import price mark-up	μ^{pX}	Beta	0.40	0.15	0.22
Social contribution (firms)	μ^{tw1}	Beta	0.50	0.20	0.16
Social contr.(employees) and lab. inc. tax	μ^{tw23}	Beta	0.50	0.20	0.54
VAT tax	μ^{vat}	Beta	0.50	0.20	0.39

domestic producers, which is subject to more nominal rigidity. Therefore the model simultaneously matches the high ERPT at the border and a low ERPT at the final good level, a property labelled the 'ERPT disconnect' in de Walque et al. (2023b). Given the aforementioned calibration of the import-content of exports, this estimated share of foreign value added in production implies a share of transit goods in total BE exports around 0.5, which is in line with Duprez (2014). The implied or estimated home biases in the final consumption, business investment and housing investment baskets equal respectively 0.96, 0.88 and 0.86 (both estimated). The home bias in consumption expenditures is similar to the EA one, obtained to be around 0.98.

The pass-through of oil price in BE is attenuated by (i) a small estimated share of the oil price in the marginal costs of domestic intermediate good producers, and (ii) a distribution margin of $0.35\times(1+0.35)^{-1}$ =26% for energy processed towards the consumption good. Finally, the bias of total BE imports towards intra-euro area imports is estimated to be around 0.74, which is significantly above the prior mean and the 2018-2021 average weight of 0.55 that can be computed from Institut des comptes nationaux (2021). It is however close to the share of imports coming from European countries calculated on this same dataset (0.72). The sensitivity of BE total exports to intra-zone demand for BE products is found to be around 0.67, a bit above the 2018-2021 average weight (0.58) computed from Institut des comptes nationaux (2021). Finally, the sensitivity of the Belgian effective exchange rate to the euro-dollar is significant, with a mode value at 0.53.

Table 5: Posterior estimates of selected BE structural parameters.

		Prior distribution			Posterior mode
Parameter		Distr.	Mean	St. Dev.	
Fixed cost	Ψ	Normal	1.25	0.125	1.42
Habit coefficient	λ_{hab}	Beta	0.70	0.10	0.73
Business investm. Adj. Cost	φ	Gamma	5.00	2.50	6.32
Housing investm. Adj. Cost	φ_h	Gamma	5.00	2.50	13.37
Business investm. Utilisation	ν	Beta	0.50	0.20	0.72
Housing investm. Utilisation	ν_h	Beta	0.50	0.20	0.46
Substitution housing	λ_h	Gamma	1.00	0.50	0.76
Relative risk aversion	σ_c	Normal	1.50	0.375	1.45
Domestic wage rigidity	ξ_w	Beta	0.50	0.15	0.79
Domestic price rigidity	ξ_p	Beta	0.50	0.15	0.82
Domestic price indexation	ι_p	Beta	0.50	0.15	0.50
Intra-euro area import price rigid.	$\xi^{pF}_{EA,BE}$	Beta	0.50	0.15	0.26
Extra-euro area import price rigid.	$\xi^{pF}_{ROW,BE}$	Beta	0.50	0.15	0.12
Export price indexation	ι_{px}	Beta	0.50	0.15	0.18
Energy distribution cost	δ_{oil}	Uniform	0.00	3.00	0.35
Share of foreign inputs in prod.	ρ_m	Normal	0.12	0.02	0.15
Share of oil in prod.	ρ_{oil}	Gamma	0.02	0.01	0.014
Home bias, busin. Investm.	ϕ_i	Beta	0.50	0.25	0.88
Home bias, housing investm.	ϕ_h	Beta	0.50	0.25	0.86
Trade elasticity (final goods)	λ	Gamma	3.00	1.00	1.51
Adj. Costs imports (final goods)	Ω	Normal	4.00	1.00	3.38
Trade elasticity (interm. goods)	λ_m	Gamma	3.00	1.00	2.06
Adj. Costs imports (interm. goods)	Ω_m	Normal	4.00	1.00	4.72
Share of intra-euro area imports	$\phi_{EA,BE}$	Beta	0.50	0.20	0.74
Share of intra-euro area exports	$\phi_{BE,EA}$	Beta	0.60	0.05	0.67
Sensitivity to euro-dollar exch. Rate	$\phi^*_{BE,US}$	Beta	0.50	0.20	0.53

5 Moment analysis

Table 6 reports model-implied unconditional mean and standard deviations estimated at the posterior mode, along with the data moments estimated over the period 1995Q1 to 2019Q4 for a subset of key BE variables. Model-based moments are computed using the linear state-space representation of the model to analytically derive moments of endogenous variables (see Hamilton, 1994). In general, the model fits the data well. Model-based averages are broadly in line with the data. Business and housing investment display respectively significantly larger (0.71) and smaller (0.08) sample means compared to the common deterministic trend in real variables estimated by the model (0.34).[35] The model tends to overestimate the volatility of the growth rates of private value added,[36] private consumption, housing investment, hours worked and their deflators.[37] However, their relative magnitude, reported in the third column by the ratio of a variable's standard deviation over GDP growth volatility for real variables and over consumption inflation for nominal ones, are well captured by the model. The model also replicates the sample co-movements between GDP growth and its components and between consumption price inflation and the growth rate of other deflators well. The correlation between imports (hours worked) and GDP growth is somewhat smaller (respectively larger) than in the data. The model-based co-movement between the inflation rate of the import deflator and the consumption price inflation seems to be significantly underestimated compared to the sample correlation. However, when we use data on import deflators filtered from oil price effects, their sample correlation with consumption price inflation drops to 0.32 and are closer to the ones generated by the model for the correlation between consumption inflation and non-oil import price inflation (0.18).

[35]Our results are however robust to the estimation of separate deterministic trend for investment growth rates or to their calibration to their respective sample mean.

[36]It is interesting to note that model-based volatility for private value added growth is slightly more in line with sample standard deviation of private value added computed according to the expenditure approach once the statistical variable 'changes in inventories' has been removed from the calculation. In this case, the standard deviation computed for private value added growth is 0.69 for the period 1995-2019. An endogenous modelling of inventories, for instance along the lines of Iacoviello et al. (2011), might help to improve the model fit in this dimension. We leave this modelling and further investigation of the statistical variable for future research.

[37]Note that sample variance is a statistic that is not explicitly targeted in the estimation. The over-prediction of sample variance is frequent in the estimated DSGE literature, see for instance the NAWM (Christoffel et al., 2008) and NAWM II (Coenen et al., 2018).

Table 6: Data- and model-based unconditional mean and standard deviations of the growth rate of selected Belgian variables. Data-based moments are computed using data over the period 1995Q1 to 2019Q4. Model-implied moments are generated using the posterior mode estimates.

	mean(x)		std(x)		std(x)/std(YVAR)		corr(x,YVAR)	
	data	model	data	model	data	model	data	model
real variables								
private value added (YVAR)	0.43	0.34	0.60	0.78	1.00	1.00	1.00	1.00
private consumption	0.31	0.34	0.42	0.63	0.70	0.81	0.49	0.59
business investment	0.71	0.34	2.06	2.84	3.43	3.64	0.36	0.54
housing investment	0.08	0.34	2.00	2.74	3.33	3.51	0.34	0.39
exports (contrib. to GDP)	-0.02	0.00	1.68	1.60	2.80	2.05	0.69	0.58
imports (contrib. to GDP)	0.01	0.00	1.52	1.29	2.53	1.65	0.65	0.28
hours worked	0.17	0.17	0.59	0.82	0.98	1.05	0.49	0.70
real wages	0.09	0.18	0.82	0.89	1.37	1.14	-0.23	0.06
deflators/price indices					std(x)/std(PCD)		corr(x,PCD)	
value added	0.34	0.48	0.25	0.40	0.60	0.68	0.59	0.68
private consumption (PCD)	0.42	0.48	0.42	0.59	1.00	1.00	1.00	1.00
business investment	0.30	0.43	0.40	0.54	0.95	0.92	0.45	0.48
housing investment	0.63	0.71	0.68	0.75	1.62	1.27	0.41	0.34
imports	0.31	0.35	1.41	1.41	3.36	2.39	0.67	0.32
exports	0.26	0.33	1.11	1.14	2.64	1.93	0.60	0.51
health consumption (HEALTH)	0.43	0.48	0.33	0.61	0.79	1.03	0.70	0.85
energy (HICP)	0.75	0.76	3.41	3.25	8.12	5.51	0.73	0.62
energy (HEALTH)	0.61	0.60	3.04	3.13	7.24	5.31	0.62	0.52

Table 7 reports selected unconditional cross-correlation between EA or international variables and BE variables. On the real side, without the estimated dependence of BE demand shocks on EA innovations, the model generates opposite unconditional co-movements in cross-country internal demands, consistent with what is illustrated later in Section 6 under the ‘no correlation’ scenario. The estimated correlation between EA and BE demand shocks helps the model to produce an unconditional synchronisation of internal demand components in line with the data. This co-movement in internal demand is however not sufficient to generate a large business cycle synchronisation as reflected by a significantly smaller model-based correlation between EA and BE output growth (0.14) compared to the sample counterpart (0.74).[38] Correlation in EA-BE TFP shock processes is not more successful, though it does help to reverse the sign of the unconditional co-movement between hours worked. This seems related to a much lower cross-country correlation between exports in the model than in the data and to a smaller model-generated co-movement between EA GDP and BE exports.[39] The latter weakness is obtained even though the unconditional correlation of Belgian exports with the TCE proxy for intra-zone demand for BE products is matched (0.58), and the one with EA imports is pretty fair (0.40). A closer look at cross-correlations between EA GDP and its components underlines a lack of model-generated co-movement of EA imports and output, despite substantial correlation with internal demand components.[40] Moreover, the over-prediction of BE real growth volatility observed in Table 6 also

[38]The statistical variable ‘changes in inventories’ seems to inflate the data correlation figure. Once this variable is removed from the computation of private value added growth, the standard deviation of the latter significantly increases. The covariance between private value added growth and EA GDP growth does not significantly change after this transformation, which drives the sample correlation to 0.53 instead of 0.74.

[39]Though not reported here, a similar picture emerges between BE and US aggregates. The pattern in correlation obtained within US GDP components is also similar to the one of the EA bloc.

[40]The calibration of a larger share of foreign inputs in gross output helps to significantly increase this correlation between EA GDP and imports, and generates a slight improvement in the synchronization between EA GDP and BE GDP and exports. It comes however at the expense of a smaller unconditional co-movement between final expenditures and imports in the EA bloc. Moreover, the current estimation implies a share of imported inputs in total imports of 0.61 in the euro area, quite in line with the ratio that can be computed from OECD TiVA (0.64 on average over 2005-2015) and somewhat larger than what is reported

contributes to the gap between data and model-based unconditional correlation in output growth. Assuming EA-wide shocks (i.e. full correlation between the innovations of TFP, risk premium, and investment-specific shocks in EA and BE areas) does not lead to a significant improvement in output synchronization, and comes at the expense of too large model-based co-movement in internal demands. Similarly, while the model is able to generate substantial co-movement in consumer price inflation, it still falls short of the value in the data. That being said, the model succeeds to reproduce the low correlation between the euro exchange rate and consumer price inflation in Belgium and the somewhat higher correlation between the exchange rate and import prices, as well as the correlation of consumer price and oil price inflation. This success of the model is due to features limiting exchange rate pass-through to the consumer price, namely imported oil and non-oil inputs in production and transit goods.

The fact that matching international business cycle poses a challenge for open-economy macro models has been widely discussed in the literature. For instance, using the workhorse small open-economy model of Galí and Monacelli (2005) fitted to Canada, Justiniano and Preston (2010) show that US shocks are not significant contributors to the variability of Canadian time series, in contrast with what can be found in empirical studies. In two-country setups, de Walque et al. (2017) and Corbo and Strid (2020) experience similar issues when modelling respectively EA-US and Sweden-ROW linkages.[41]

in Kose (2002) (0.47 for G7 countries). A deeper investigation of this issue stands high in our research agenda.

[41] Several mechanisms have been recently explored in the literature to address these shortcomings. On the trade channel, Engel and Wang (2011) consider international trade in durable goods and find that a two-country setup is able to endogenously reproduce the pro-cyclicality of imports and exports and their co-movement relatively well. In a small open-economy DSGE, Houssa et al. (2019) find that trade in commodities plays an important role in the transmission of foreign shocks for emerging economies. Financial spillovers have also been found to be relevant for international shock propagations in the studies of Alpanda and Aysun (2014); Ueda (2012). Further investigation of these mechanisms is listed in our research agenda.

Table 7: Data- and model-based cross-correlation for selected Belgian and euro area variables. Data-based moments are computed using data over the period 1995Q1 to 2019Q4. Model-based moments are generated using the posterior mode estimates.

		data	baseline	no shock corr.	EA wide shocks
EA	BE	real variables			
gdp	private value added	0.74	0.14	-0.07	0.26
private cons.	private cons.	0.38	0.43	-0.17	0.55
business investm.	business investm.	0.23	0.16	-0.15	0.49
exports	exports	0.60	0.21	0.21	0.22
imports	imports	0.57	0.37	0.32	0.39
hours worked	hours worked	0.57	0.11	-0.08	0.22
gdp	exports	0.64	0.15	0.16	0.15
imports	exports	0.55	0.40	0.41	0.40
intrazone demand	exports	0.58	0.58	0.59	0.58
EA	EA				
gdp	private cons.	0.67	0.45	0.45	0.45
gdp	business investm.	0.58	0.66	0.66	0.66
gdp	exports	0.78	0.30	0.30	0.30
gdp	imports	0.74	0.08	0.08	0.08
EA	BE	nominal var.			
gdp deflator	priv. value added defl.	0.03	0.21	0.16	0.22
private cons. defl.	private cons. defl.	0.73	0.31	0.28	0.32
extra-ea imp. price	extra-ea imp. price	0.70	0.26	0.25	0.25
International	BE				
oil price	private cons. defl.	0.48	0.33	0.31	0.34
oil price	HICP energy	0.65	0.64	0.64	0.64
oil price	health index	0.15	0.12	0.11	0.12
oil price	health index - energy	0.23	0.26	0.26	0.26
euro-dollar exch. rate	private cons. defl.	0.03	0.003	0.03	-0.01
euro-dollar exch. rate	health index	0.06	0.03	0.03	0.02
euro-dollar exch. rate	intra-ea imp. price	0.18	-0.001	0.00	0.00
euro-dollar exch. rate	extra-ea imp. price	0.74	0.61	0.62	0.61
effective exch. rate	total import price	0.27	0.33	0.34	0.33
effective exch. rate	total export price	0.31	0.17	0.18	0.17

6 Dynamics

An attractive aspect of having a fully-fledged EA-US environment when modelling the Belgian economy, is the possibility to make the evolution of relevant international variables endogenous. The analysis of the effects of international shocks, and in particular euro area shocks, on interest rates, exchange rates, international prices and the demand for BE products is key to understand the dynamics of BE macro variables. In this section, we illustrate the mechanics of BEMGIE by simulating the responses of the BE price and quantities to intra- and extra-euro area shocks. We start the analysis with a euro area monetary policy, exchange rate, demand and productivity shocks. For the last two, we first consider exogenous disturbances in the rest of the euro area. Next, we allow for EA and BE innovations to be correlated as in the case of euro-area wide shocks. Finally, we analyse the responses to shocks coming from outside the euro area: a sudden movement in the oil price and a shock to the extra-zone demand for euro area products. For each shock, we compare dynamics in the BE variables with the evolution of EA counterparts.

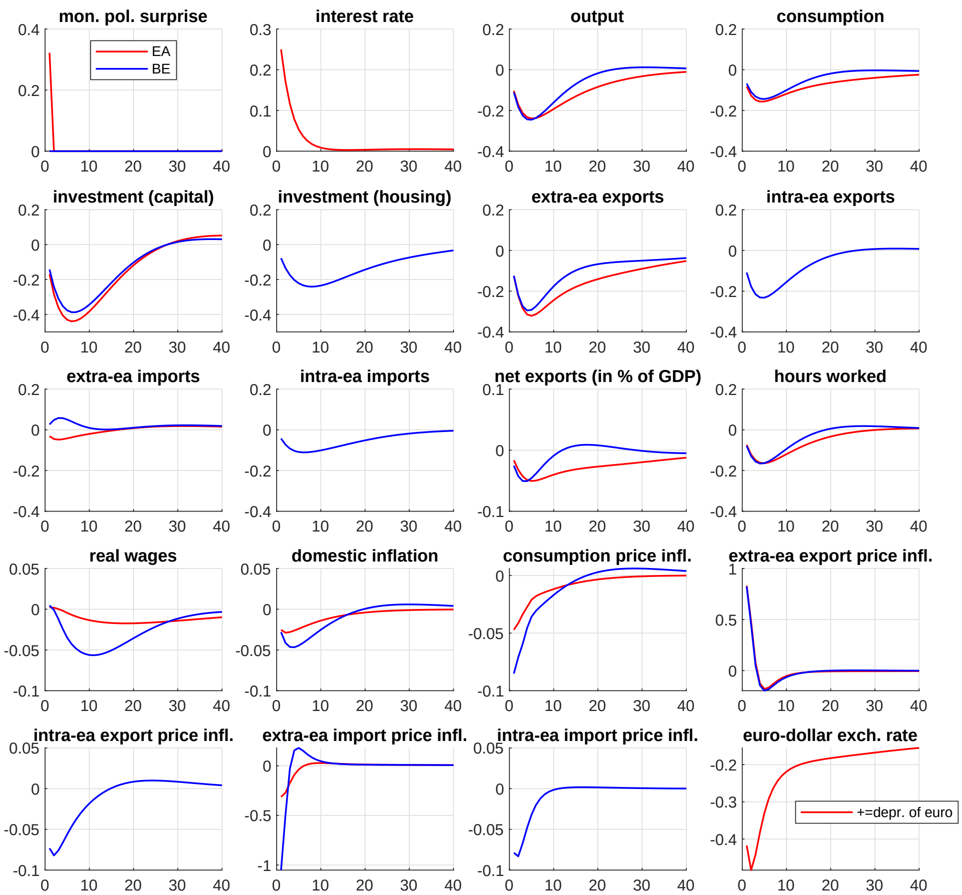
6.1 A conventional monetary policy shock

The impulse responses associated with an unexpected monetary policy tightening are reported in Figure 1. The shock is such that there is a 25 basis points increase in the annualized short-term interest rate of the euro area. We assume no persistence in the policy surprise, but the lagged interest rate term in the monetary policy rule (see Equation 59) causes the interest rate to be persistently above its steady-state value. Both in Belgium and at the euro area level, the resulting increase in the real interest rate lowers private domestic demand, with investment declining more strongly than consumption. Furthermore, extra-euro area exports decline as the higher short term interest rate triggers an appreciation of the euro exchange rate via the UIP condition (see Equation 75), and the associated loss in competitiveness. The decline in aggregate demand lowers employment and thus the real wage, as well as increasing the marginal product of capital, and the exchange rate appreciation causes an import price drop. Consequently, marginal costs of production and thus the price of domestic output fall. Note that the consumption price reacts more strongly on impact, but then closely tracks the domestic output price. Indeed the share of foreign goods entering directly into the consumption basket is small, as most imports go either into production or directly into exports, in the form of transit goods. This mitigates seriously the exchange rate pass-through to consumer price, as explained in Section 3.4. The responses of EA variables to the shock are in line with other models estimated on euro area data.[42]

The declines of consumption price inflation, domestic price inflation and real hourly wage are larger in Belgium than in the EA. This result is due to a somewhat higher wage Phillips curve slope and the automatic wage indexation –which makes the decline in real wages stronger and more persistent in BE– and a larger share of foreign inputs in production in BE than in the EA. The latter increases the impact of the –very flexible– import prices on domestic marginal costs. Consumption contracts slightly less in BE due to different consumer inflation responses, and a smaller inter-temporal elasticity of substitution which makes consumption less sensitive to the real rate increase, and more to changes in employment (hours worked). With larger investment adjustment costs and a more flexible utilisation rate of capital assets, BE capital investment responds slightly less in the short run. Finally, BE extra-euro area import prices respond more strongly to the exchange rate appreciation due to lower price rigidities at the border in BE compared to EA. Consequently, the substitution effect is strong enough to generate a small increase in Belgian imports of goods coming from outside the eurozone.

[42]For instance, Christoffel et al. (2008) report a decline of private consumption, investment and GDP of respectively about 35, 60 and 30 basis points for a contractionary monetary policy shock that increases the nominal interest rate by about 32 basis points. We obtain a fall of respectively 20, 56 and 31 basis points for EA private consumption, investment and GDP for the same scaling of the shock. Concerning nominal variables, their paper indicates a maximum drop of around 15 basis points for annual consumption inflation in EA, while our model generates a smaller peak decline of around 6 basis points in annual inflation.

Figure 1: Selected impulse responses to a monetary policy tightening shock that increases the nominal interest rate of the euro area by 25 basis points (in annual terms) on impact. There is no persistence in the monetary policy surprise. Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively).



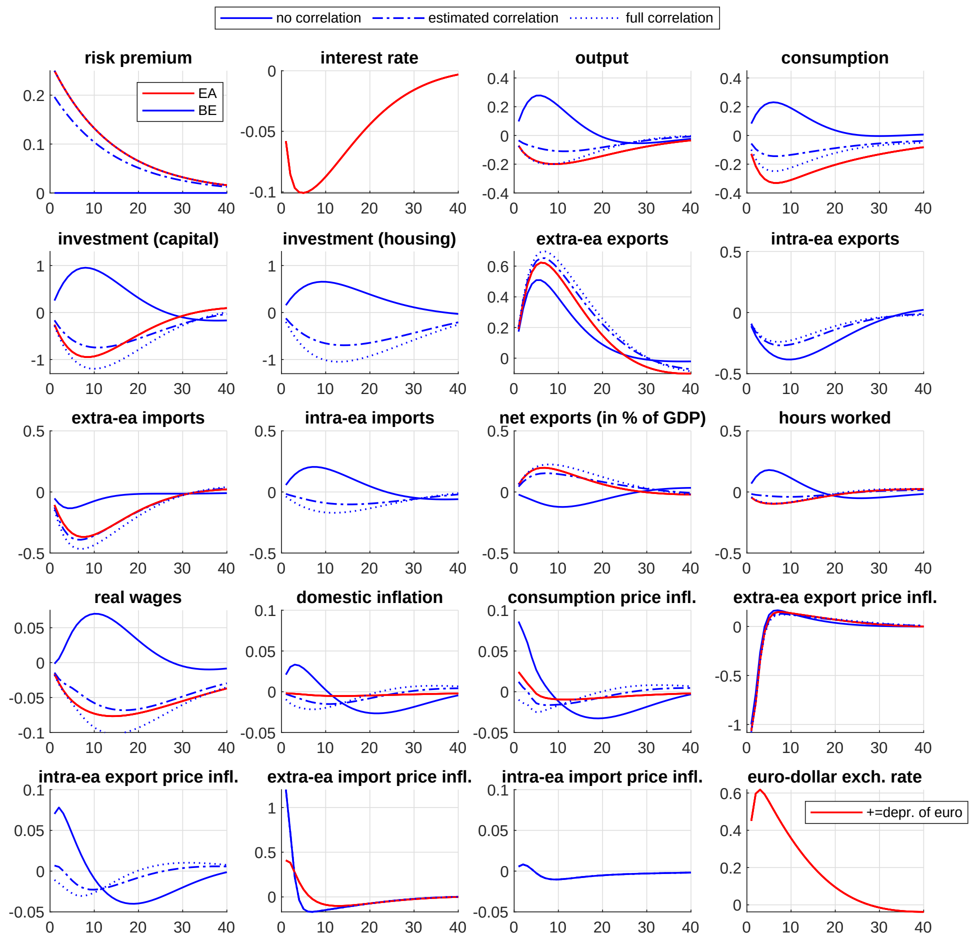
6.2 A demand shock in the euro area

We simulate a decline in EA internal demand (private consumption and investment) using a positive innovation in the EA risk premium shock, ε_t^b. This shock increases the household's demand for safe assets at the expense of consumption and physical capital (see Equations 60 and 62). The shock is calibrated such as to produce an increase of 25 basis points in the annualized EA risk premium. Figure 2 shows the impulse responses to the contractionary EA demand shock. In a first simulation (blue straight lines for BE variables), we assume no change in the BE risk premium. This helps to assess the model transmission of a demand shock that only appears in the rest of the euro area via endogenous channels, with no effect on the BE risk premium. In a second step (blue dashed-dotted lines for BE variables), the estimated correlation between BE and EA risk premium innovations is activated. Finally, in a third simulation (blue dotted lines for BE variables), a euro area wide shock is considered, assuming the same value for the BE and EA risk premium innovations (keeping their persistence coefficients at their estimated levels). Correspondingly, the dynamics of EA variables are identical across the three simulations: consumption and investment are temporarily curtailed, the consumption price gradually declines. In reaction to the economic slowdown, monetary authorities provide an accommodative stance through interest rate cuts.

In the first simulation, when BE risk premium does not react to the shock, the expansionary effects of this endogenous monetary policy stimulates BE consumption and investment. The impact of this channel on BE value added compensates the negative effects of a smaller EA demand for BE products and an increase in BE imports, reflected in negative net exports. The resulting de-synchronisation of BE GDP and the output of the rest of the euro area highlights the lack of endogenous international synchronization in the model in the face of demand shocks. This finding complements the issue raised in estimated SOE (Justiniano and Preston, 2010) and two-country models (de Walque et al., 2017), by underlining the lack of business cycle synchronization after demand shocks in the rest of the zone to a SOE inside a monetary union.

When the correlation of BE and EA shock innovations are set to their estimated value, the BE domestic risk premium rises on impact, though less than in the EA bloc. The contractionary effects of the increased premium dominate the effects of the supportive monetary policy, and BE domestic demand, GDP and imports decline. The decline in imports is more important than the slowdown in BE intra-euro area exports and the response of net exports turns positive. These effects, and the resulting synchronisation of BE and EA value added, are amplified when a euro area wide demand shock is assumed, i.e. when BE and EA shock innovations are fully correlated, as in the third simulation.

Figure 2: Selected impulse responses to a contractionary euro area demand, simulated by a 25 basis points increase in the annualised EA risk premium. Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively). Solid lines correspond to a simulation where there is no correlation between shock innovations (i). Estimated correlations are used in the simulation (ii) represented by dashed-dotted lines. Full correlation is assumed in the simulation reported in dotted lines (iii).

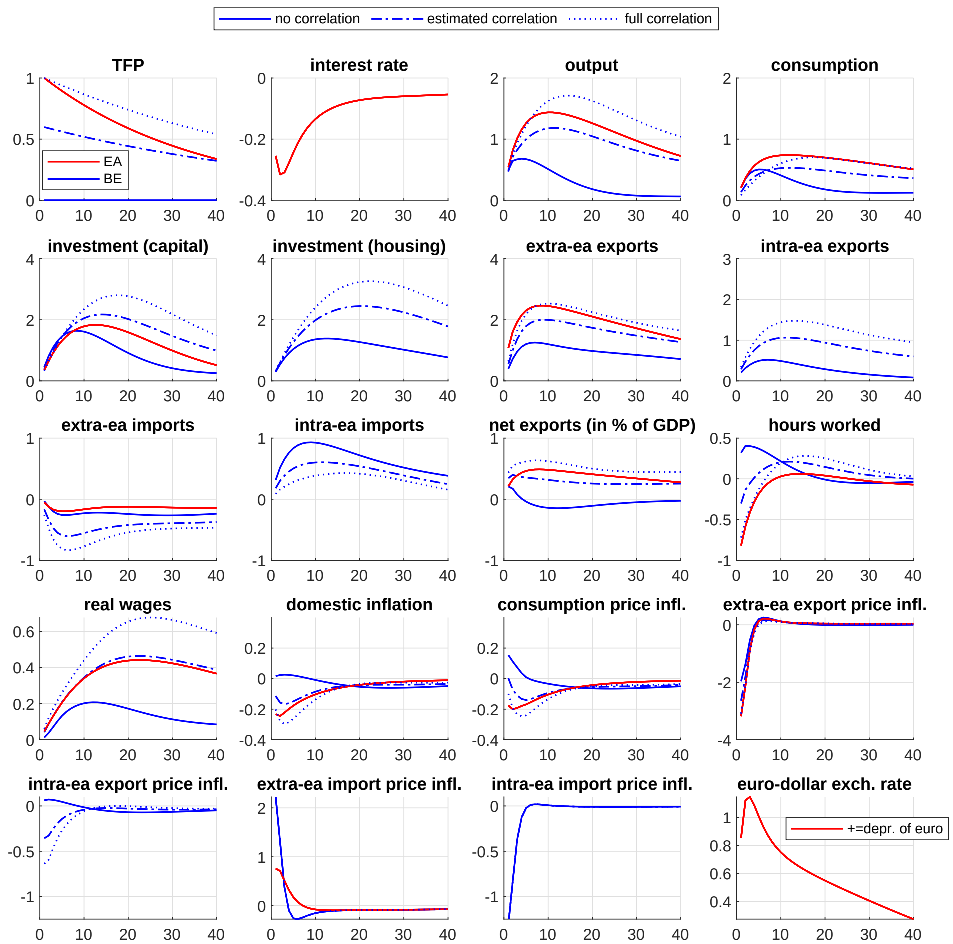


6.3 A productivity shock in the euro area

In Figure 3, we simulate an increase in EA total factor productivity, ε_t^a, of 1%. As for the EA risk premium shock, we consider three scenarios regarding the response of Belgian total factor productivity: i) no correlation between EA and BE productivity shock innovations (blue straight lines for BE variables), (ii) estimated correlation (blue dashed-dotted lines for BE), (iii) full correlation (blue dotted lines for BE). In the EA, the shock lowers marginal costs of production of firms and thus inflation (see 70), which triggers a decline in the policy interest rate via the policy rule and thus an increase in consumption and investment. The decline in the policy rate causes an exchange rate depreciation on impact, though in the long run there is an appreciation due to the permanently lower price level.

Even under the assumption of no correlation between Belgian and EA TFP, due to the monetary expansion, the improvement in EA TFP has positive spillovers on Belgian consumption, investment, real wages, and thus domestic prices and imports. Exports to the EA increase as well due to a higher demand from eurozone partners, but less than imports due to the increased competitiveness of EA production. As a result, net exports turn persistently negative. Once there is some impact of EA TFP on Belgian TFP, as in simulations (ii) and (iii), the Belgian price level also declines, implying that exports to the euro area increase substantially more than in a simulation without any BE productivity reaction. Moreover, exports to the extra-euro area countries also respond by more. Thus, in contrast to scenario (i), there is an improvement in the net foreign asset position, which via the endogenous risk premium stimulates Belgian consumption and investment on top of the stimulus provided by the EA monetary expansion. Note that the persistence of the BE TFP shock process is estimated to be slightly more important than the EA one, which leads to larger output effects in BE in the case of the full correlation scenario (iii).

Figure 3: Selected impulse responses to an expansionary productivity shock in the euro area, simulated by increasing EA TFP by 1%. Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively). Solid lines correspond to a simulation where there is no correlation between shock innovations (i). Estimated correlations are used in the simulation (ii) represented by dashed-dotted lines. Full correlation is assumed in the simulation reported in dotted lines (iii).



6.4 A euro depreciation shock

We simulate a depreciation of the euro via the UIP condition shock, ε_t^s (see Equation 75). This shock is caused by non-fundamental exchange rate movements. It is calibrated such as to produce a 10% depreciation of the euro on impact. As can be observed in Figure 4, the exchange rate depreciation lowers the marginal costs of exporters, expressed in foreign currency, and thus the prices of exports to extra-euro area countries. This decline is less than the amount of the currency depreciation as prices are nominally rigid in terms of the foreign currency (local currency pricing).[43] Similarly, EA import prices increase, but to a lower extent

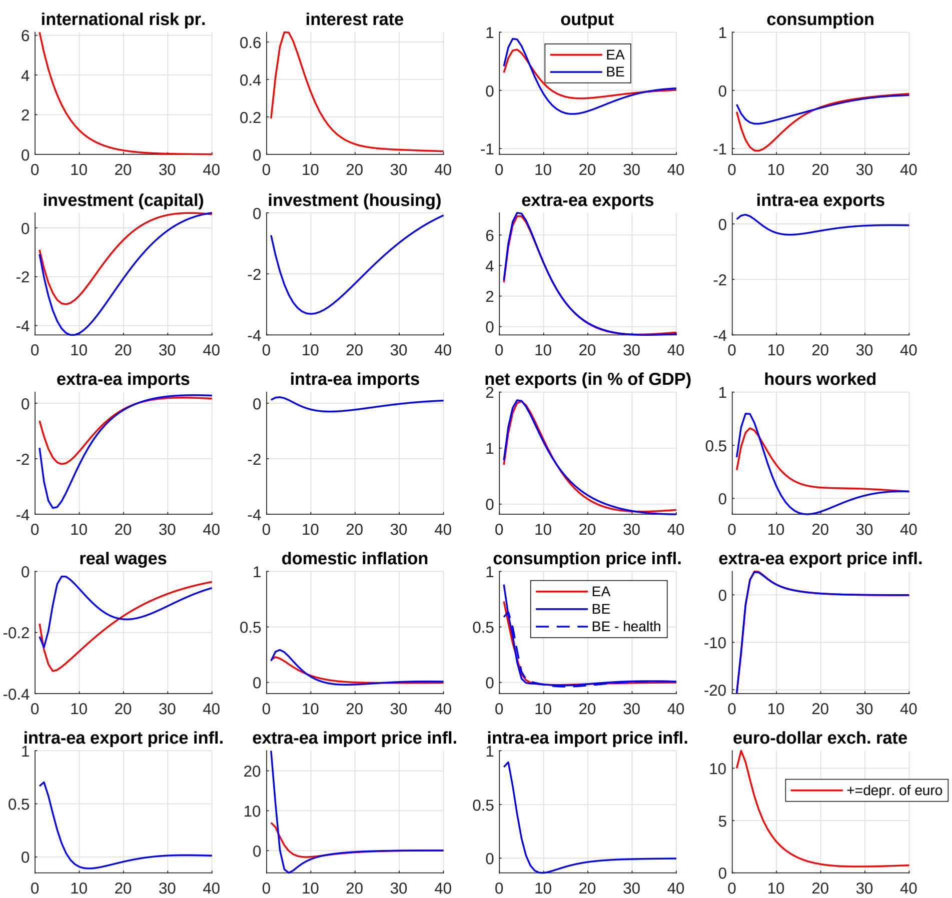
[43]Note that the inflation figures are presented in annual terms (that is the quarterly log deviation is multiplied by a factor 4), while the euro-dollar exchange rate is presented in levels. The short-run ERPT to extra-euro area export and import prices is

compared to the amount of the depreciation (40% in annual terms). The resulting higher competitiveness of EA production boosts extra-euro area exports and lowers imports. There is thus an expenditure switching away from foreign products and towards EA goods. Furthermore, by the higher import price, it results in higher inflation, which triggers an increase in the nominal and real interest rate via the monetary policy rule. As a result, private consumption and investment both contract. This effect, which originates from the terms-of-trade deterioration, partially compensates the positive expenditure switching effect on EA and BE productions.

In the BE bloc, output is raised by more than in EA in the short run. This outcome mainly results from a smaller decline of private consumption. This is due to (i) the automatic wage indexation (to the health consumption price) in BE which produces a significant hump in real wage response, and (ii) a smaller inter-temporal elasticity of substitution which makes consumption less sensitive to the real rate increase, and more to current labour income via the – here positive – changes in hours worked. BE net exports expressed in percentage of GDP increase similarly compared to the euro area as a whole. The rise in BE total exports is attenuated due to a less responsive and fluctuating intra-euro area demand for goods – and thus also for Belgian products – compared to the demand coming from outside the eurozone. While BE imports from extra-euro area countries decline more than their EA counterparts as import prices at the border are less rigid, they account for less than a third of total imports. Total BE imports decline less relative to EA imports due to the much smaller reduction in imports from inside than from outside the EA and to a larger share of transit goods (not shown). Finally, Belgian investment reacts more to the shock compared to its EA counterpart, which contributes to drive BE output to more negative areas in the medium run. This difference in sensitivity mainly comes from a capital utilisation that is estimated to be more variable in BE (i.e. we found a larger estimate for the utilisation coefficient in Equation 63), which makes the adjustment in the rental rate of capital smoother in response to changes in output. Consequently, the medium and long run responses of the rental rate are raised by a more important amount in EA, and forward-looking asset prices do not drop as much as in BE. Consumption prices increase slightly more in BE compared to EA, due to a combination of stronger wage indexation, larger share of imported content in final and intermediate goods and sensitive import prices.

thus smaller than 1, as explained earlier in the text.

Figure 4: Selected impulse responses to UIP shock that depreciates the euro by 10% relative to foreign currencies. Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively).



6.5 An oil price shock

Figure 5 displays the macroeconomic effects of a 10% transitory increase in the world market price of crude oil (in dollar) on BE and EA economies. The rise in oil prices generates both direct and indirect inflationary pressures on consumption prices. The direct effect materialises through the presence of energy products in the consumption basket, and is somewhat diluted by the presence of refining and distribution margins (see Equation 31). It can be observed in the second subplot that reports the responses of the inflation rates of energy components of the consumer prices. Note that these responses capture sensitivities of energy prices to crude oil price, as well as potentially to non-oil wholesale energy prices (e.g. gas) to the extent that they correlate with crude oil prices. Therefore, as explained in Section 2.2, the dynamics after an oil price shock may to some degree reflect the responses to an energy shock beyond that of a pure oil price shock (i.e. in a scenario where the wholesale price of non-oil energy commodities remain the same).[44] The indirect effect of the shock is obtained through an increase in domestic prices due to an augmentation in the price of oil inputs used in the production of domestic consumption goods. The price of the investment good (not shown), which is not affected by the direct effect of oil price, closely follows the domestic price evolution. As a result of the increase in oil prices, total energy consumption (for both consumption and production purposes) decline, as shown in the 6th subplot. The ratio of these impulse responses to the responses of the real crude oil price enables us to compute a short-run – i.e. after one year – 'empirical' energy demand elasticity of around -0.05 and -0.07 for EA and BE respectively (and -0.07 for the US bloc). This elasticity is in line with the empirical literature on oil demand elasticity. Cooper (2003) estimates a short-run elasticity demand for crude oil in 22 developed countries and China to be around -0.05 (and -0.06 for the US). Using an alternative identification strategy based on IV regressions,[45] Caldara et al. (2019) find a demand elasticity after an increase in the real price of crude oil between -0.055 and -0.08.

As reported on the second subplot of Figure 5, the BE energy component of consumption prices is more sensitive to the oil price shock compared to its euro area counterpart. This finding is consistent with the analysis found in Baugnet and Dury (2010). This article underlines that factors such as the low level of excise duty on diesel and on heating oil and more concentration in the market of petrol stations can imply a faster and stronger transmission of the crude oil price to Belgian consumer prices, compared to the euro area as a whole and to the three main neighbouring partners (Germany, France and the Netherlands). They also document a reaction of the consumer price of gas to crude oil price movements considerably faster in Belgium compared to neighbouring countries. Quicker adjustment in tariff setting and lower levels of gross margin[46] and excise duty are cited as contributors to the higher elasticity of Belgian consumer prices of gas.[47]

The more sensitive energy component implies a larger direct pass-through to final consumer price in Belgium. The indirect effect, passing through oil input in production is similar in BE and EA, as the share of oil in marginal cost is roughly the same (0.02). However, Belgian domestic firms are found to index their prices more importantly than in the euro area, which contributes to the gap in headline inflation. In addition, automatic wage indexation in Belgium generates significant second-round effects as it pushes up nominal wages, and hence domestic firms' marginal costs, following the rise in the health consumption index (reported in dashed lines on the 15th subplot). The latter jumps less than the consumer price on impact, and effect of the shock are delayed, due to the exclusion of some oil products from the basket used in its calculation and a more gradual adaptation to raw oil price changes.[48] The ratio between BE and EA consumer price impulse responses - that is, cumulated inflation responses - over three years is around 1.5. It is in line with the ratio of 1.6 obtained from the differences in BE and EA total HICP responses to a 10% oil price shock in Baugnet and Dury (2010).[49]

[44] As detailed in the footnotes of Section 2.2, it is possible to be more accurate, for instance by using bridge equations to simulate the effects of such a pure oil price shock, or a broader energy shock that involves sudden changes in other energy commodities, on HICP and health energy components, and to feed BEMGIE with these simulated paths. We chose here to follow the literature and present a traditional oil price shock, simulated fully with the endogenous mechanisms estimated in the model.

[45] Oil consumption of the following OECD countries is used in their study: Canada, France, Germany, Italy, Japan, South Korea, United Kingdom, and the United States.

[46] Defined as the difference between the consumer price of energy excluding tax and the import price of the energy commodity.

[47] In a separate study on wholesale gas, electricity and oil prices for the period 2021-2022, we also found that the subcomponents of the HICP energy index related to gas and electricity react stronger and faster in Belgium relatively to the euro area after the historical sequence of wholesale energy price surge.

[48] Moreover, as excluding motor fuels from the health index magnifies the weights of gas and electricity prices, the response of the price used as reference for wage indexation can also be stronger in the case of a shock affecting all energy commodity prices (as for instance the sequence of shocks occurred in 2021-2022), compared to our reported traditional oil price shock.

[49] This comparison should however been taken with a grain of salt as these responses are computed based on backward-looking semi-structural models (e.g. NONAME for Belgium) and for a permanent increase of the crude price of oil.

The increase in the consumption price generates a decline in internal demand that is reinforced by an endogenous monetary policy tightening (see the last subplot that reports the nominal interest rate). Difference in consumption price and employment responses leads to a more important response of Belgian consumption. Higher estimated investment and utilisation adjustment costs in BE makes the decline in investment smoother but more persistent than in EA. Moreover, higher domestic inflation reduces the competitiveness of BE firms on foreign markets, which has a more negative impact on BE exports. Extra-euro area imports slightly goes up for BE due to less sensitive extra-euro area import prices compared to BE nominal marginal costs, and hence an increase in imports of US and RoW production inputs.[50] As a result of these internal and external demand reactions, BE GDP declines by 0.35% and is more persistently affected than EA output (-0.19%) after a transitory oil price shock.[51]

The stronger GDP response obtained for BE compared to the euro area is mainly due to the quicker pass-through to consumer price and the impact of price and wage indexation depicted above. To better gauge the role of automatic wage indexation after an increase in oil prices, we simulate a counterfactual in BEMGIE where wage indexation and rigidity parameters are set in BE to the same values as estimated for the EA. Figure 9 in the appendix shows a comparison of the dynamics under this counterfactual (dashed-dotted lines) and baseline simulation (solid lines). Belgian real wages are not protected anymore by the strong wage indexation. The magnitude of the reaction of the Belgian firms' marginal cost gets much closer to this of their European competitors, together with the prices they set on domestic and foreign markets. The competitiveness handicap highlighted in the baseline simulation is significantly reduced. The implied smaller reaction of domestic prices also impacts consumption prices. As they increase somewhat less in this counterfactual economy, and that hours worked are less impaired, the drop in consumption is attenuated. Overall, about one third of the gap between the troughs of the reaction of GDP in Belgium and euro area has disappeared compared to the baseline simulation. In the dotted lines of Figure 9, we add to the counterfactual a correction for the consumer energy price formation process. We impose that the energy component reacts in the same way to wholesale oil prices in Belgium as in the rest of the euro area. Not surprisingly, this assumption has the strongest effect on private consumption: The gaps in consumer prices inflation and consumption expenditure responses almost disappear. There is however no significant change in the supply-side developments, as reflected in domestic and export price inflation (as indicated by the extra-euro area export prices), and hence no further improvement in the competitiveness of BE firms, nor in investment responses. Consequently, though reduced, the GDP gap persists. In a third counterfactual, we also harmonise investment and capital utilization adjustment parameters, as well as households risk aversion coefficient and the degree of price indexation

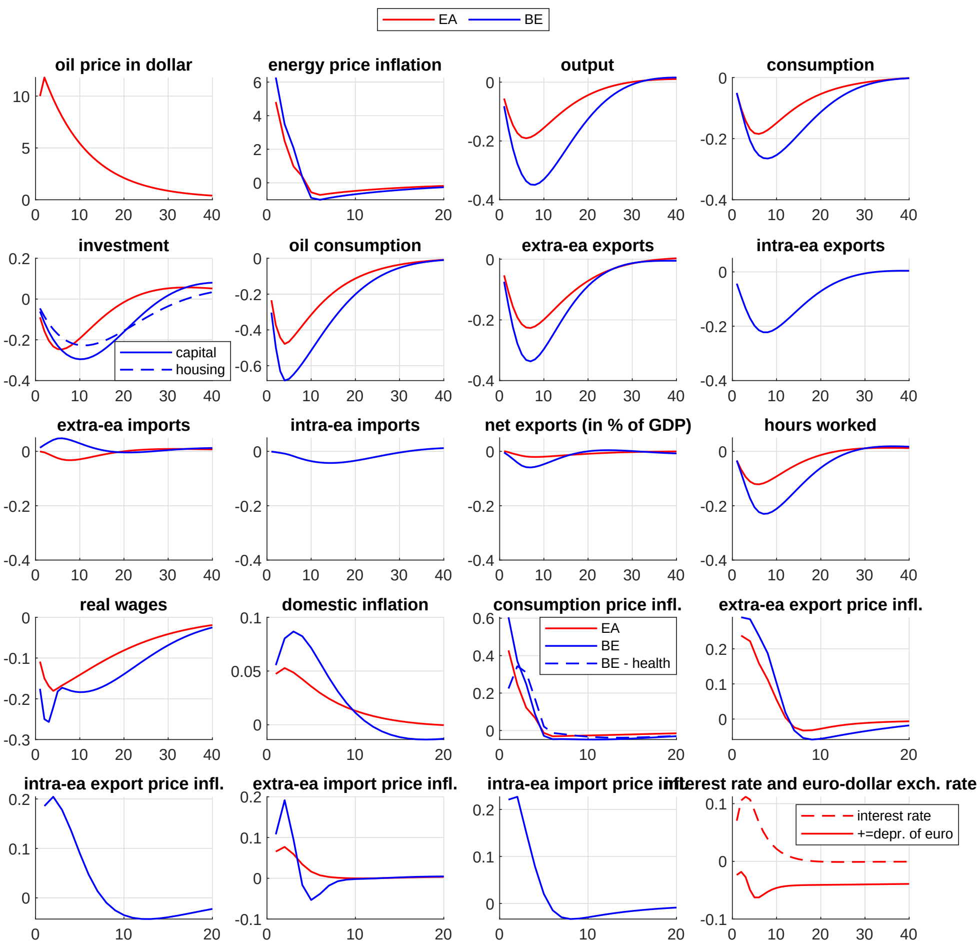
[50] This comes in part from a smaller calibrated share of oil in US marginal costs compared to BE and the small appreciation of the euro. This appreciation emanates from a slightly smaller reaction of the US economy to the oil price shock compared to the euro area.

[51] The magnitude of the responses generated by BEMGIE falls in the ballpark of those obtained in the literature. Forni et al. (2015) estimate a two-country DSGE model for the euro area and the rest of the world where oil price endogenously emanates from oil demand and supply shocks in the global oil market. After a 10% transitory increase in oil prices (whether it is due to an oil supply shock or an oil demand shock), they find an increase in euro area annual CPI inflation rate up to around 0.10% and a persistent reduction in GDP around 0.10%. When the same scaling and persistence of the shock (i.e. an AR(1) process with a coefficient of 0.9) is applied in BEMGIE, the annual euro area consumption inflation rate displays a peak increase around 0.17% in deviation from its steady-state and output declines by about 0.14%. Caldara et al. (2019) find a decline of industrial production in advanced economies to be around -0.40% two and a half years after a shock that increases monthly crude oil prices by 6% (i.e. 18% in quarterly terms). Scaling their results for a 10% oil price increase in the first quarter, one obtains a decrease in industrial production of 0.22% in advanced countries. The recent study of Bjornland (2022) is however more pessimistic than our results, and reports a reduction in euro area GDP by 0.5% after 2 years when the world economy is hit by a sudden jump in oil prices of 10% amid geopolitical tensions or supply constraints. Finally, Jeanfils and Burggraeve (2005) simulate the effect of a 10% permanent increase in crude oil price on the Belgian economy with NONAME. They find a peak increase in consumption price inflation of about 0.50% in annual terms, and a reduction in GDP by 0.35%. The simulation of a near-unit root shock in BEMGIE gives +0.59% and -0.45% for BE annualized consumption price inflation and GDP respectively.

of domestic firms to the parameters estimated for the EA. As domestic inflation and investment dynamics in Belgium now closely matches the euro area ones, the remaining gaps between BE and EA are mostly closed.

In summary, the simulation underlines two structural characteristics of the Belgian economy in the presence of oil price shocks.[52] First, the automatic wage indexation mechanism amplifies the wage-price spiral, and significantly deteriorates the competitiveness of Belgian firms on both domestic and foreign markets. Second, the quicker and stronger transmission of wholesale oil prices to consumers harms the households purchasing power and domestic demand more than what is simulated for the euro area.

Figure 5: Selected impulse responses to a 10% increase in the crude price of oil (in dollar). Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively).



[52]These two elements also appear to play a major role in simulations with BEMGIE that combine increases in oil, gas and electricity wholesale prices (see NBB, 2023, Chapter 4, Box 3).

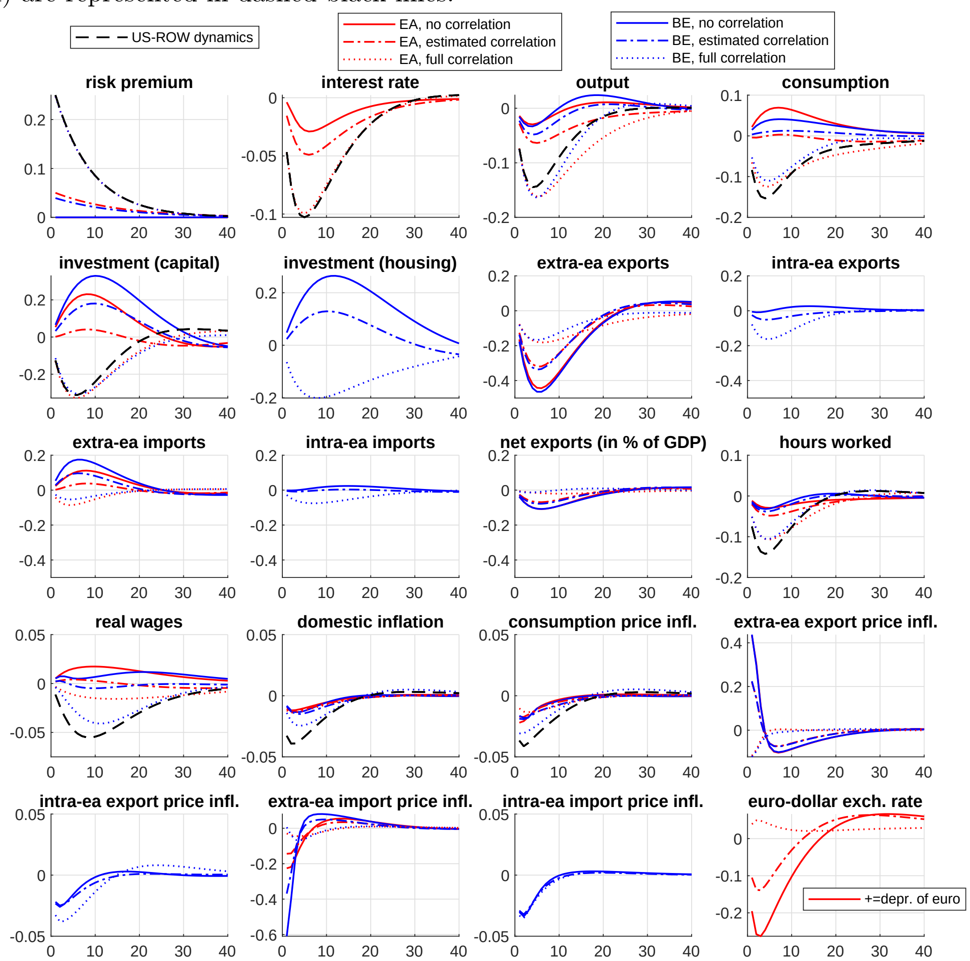
6.6 Extra-euro area (and global) demand shocks

We now discuss the effect of a contractionary demand shock coming from outside the euro area and a global shock. The estimated US risk premium shock process is used for the simulation, and applied to both US and RoW blocs. To make RoW dynamics endogenous, we assume that the RoW economy behaves as in the US. This shock has the capacity to drive both consumption and investment in the same direction, and makes it thus an interesting and significant demand shock in the model. Moreover, it can be related to financial frictions and in particular the demand for safe and/or liquid assets (Fisher, 2015), and is found to be an important contributor during the 2007-2008 Global Financial crisis (see for instance Christiano et al., 2015; de Walque et al., 2017). We distinguish between (i) a scenario where the effect of the extra-euro area risk premium on the EA and BE risk premia is set to zero, (ii) a scenario where the correlation between the US, EA and BE shock innovations equals their estimated value, and (iii) full correlation between the three innovations as well as same persistence in the shock process (the estimated US one). The first scenario without correlated shocks is interesting to underline the effects of a demand shock limited to extra-euro area countries and its endogenous propagation to the eurozone economies. The second scenario, with estimated correlations, can capture additional propagation mechanisms that may be missing in the DSGE. For instance, risk premium shock correlations can be seen as a short-cut for propagations coming from of an international financial system as developed by Dedola et al. (2012) or Kollman (2013). The last scenario with fully correlated innovations and same persistence represents a global demand shock, and raises the risk premia the same way in all blocs of the model. In the discussion, we focus on the effect of the shock on EA and BE variables, as its effect on US and RoW variables is analogous to the effect of an EA risk premium shock on EA variables (see Section 6.2).

As shown in Figure 6, the extra-euro area risk premium shock lowers the extra-euro area demand for eurozone exports by lowering foreign aggregate demand, and, by triggering a decline in the foreign policy rate, an appreciation of the euro. However, it also stimulates euro area domestic demand by lowering inflation and thus –via the monetary policy rule– the nominal and real interest rate. Due to the compensating effect of the domestic demand expansion, the effect of the shock on eurozone economic activity remains small and actually turns positive after about two years. To illustrate the degree of spillover, selected US/RoW responses are reported in black dashed lines.[53] The degree of synchronization between extra-euro area and EA and BE responses is limited under this scenario, while consumption and investment respond positively in eurozone blocs. Setting the correlation between the US and the EA innovations to its estimated value helps to mitigate the increase in investment and consumption and improves the GDP synchronization. Assuming a global shock as in simulation (iii) further strengthens the downturn in the eurozone and produces a strong synchronization in real variables across regions, as well as a significant improvement in nominal spillovers. The responses of the BE and EA variables go in the same direction across the three scenarios. Similar to other common shocks, differences in their magnitude can be mainly explained by different estimates for real rigidities, inter-temporal elasticities, capital utilisation variabilities, the slope of respective Phillips curves, the share of foreign value added in production and the importance of net exports in GDP.

[53] In order to avoid the addition of lines in an already well loaded figure, we only report the US-RoW dynamics associated with scenario (iii) in Figure 6. Even if they slightly differ across scenarios, they remain representative and helpful to gauge the degree of international spillover.

Figure 6: Selected impulse responses to extra-euro area demand shocks. Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively). Solid lines correspond to a simulation where there is no correlation between shock innovations (i). Estimated correlations are used in the simulation (ii) represented by dashed-dotted lines. Full correlation and same persistence in the shock processes is assumed in the simulation reported in dotted lines (iii). RoW is endogenised by assuming the responses to behave as in the US. The US/RoW responses associated with simulation (iii) are represented in dashed black lines.



7 Shock decomposition

An important added-value of a fully articulated and estimated DSGE model such as BEMGIE is to be a suitable tool to decompose observable (as well as projected) time series in terms of the underlying exogenous forces driving them. This decomposition can prove useful for story-telling, see for instance Boeckx et al. (2018) for an example in practice. In this section, we report the unconditional and historical shock decomposition of Belgian GDP growth (as measured by the real growth rate of private value added) and consumption inflation (as measured by the inflation rates of the private consumption deflator). Unconditional variance decomposition (or forecast error variance decomposition) delivers insight on which shocks are the most important contributors 'on average' over the sample period. Historical decomposition is useful to assess the importance of shocks during specific historical episodes, as for instance the Global Financial Crisis (2008-2009, hereafter GFC), or the euro area sovereign debt crisis (2010-2014). Due to the large number of shocks in BEMGIE, we regroup them into 8 categories. The category 'EA demand' regroups the EA risk premium shock, investment-specific shock, exogenous spending, and EA export and import shocks. 'EA supply' stands for EA TFP shock, price and wage mark-up shocks and the EA consumption price shock. 'EA interest rate' is the EA monetary policy shock. 'RoW shocks' collect US disturbances, extra-euro area demand and price mark-up shocks as well as exchange rate shocks. 'Energy prices' refers to the oil price innovation and shocks associated with energy prices observed in the model. 'BE demand' corresponds to BE preference and risk premium shocks, import demand, and capital and housing investment shocks. 'BE supply' stands for the BE TFP shock, BE prices and markup shocks. 'BE others' captures the discrepancies between the value-added and the expenditure approaches, e.g. inventories, and shocks associated with the observed tax rates.

Table 8 reports the contributors to the unconditional variance of BE real GDP growth and consumption inflation. Internal demand forces contribute for about one third in the business cycle fluctuations. Among the most important contributors of this category, we find the risk-premium shock, the investment-specific shock and the import preference disturbance. Domestic price markup and TFP shocks are the main contributors from the internal supply classification. Foreign forces play an important role in Belgian GDP volatility, led by both intra- and extra-euro demand drivers, and the EA risk-premium shock. EA supply and energy price innovations explain a relatively smaller share of the unconditional variability of BE growth rates. Finally, BE exogenous expenditure, associated with the shock introduced in the resource constraint (see Equation 58) appears to be a major contributor. This shock captures among others changes in inventories as well as statistical discrepancies between the value-added and expenditure approach for the calculation of GDP.

Fluctuations in the inflation rate of the Belgian private consumption deflator are mostly driven by internal supply forces and energy price shocks. In the former category, consumer price shocks (used at the level of the CES aggregator, see Equation 35, to accommodate the observable) and domestic price markup shocks are dominant. Oil price shocks accounts for around 16% of the unconditional variance of the consumption price inflation rates. Other foreign disturbances, emanating both from intra-zone or extra-euro area trade, are also significant, their shares summing up to 10% of the unconditional volatility of headline inflation. Among those foreign forces, exchange rate shocks have a relatively small contribution (less than 1%), reflecting the limited and diffuse ERPT to final prices in the model. It does not, however, prevent those shocks to appear significant at several occasion during specific episodes in the sample period, as discussed here below with the historical decomposition.

The historical decomposition of the year-on-year growth rate of BE real GDP, in deviation from its steady-state (the trend growth rate), is displayed in Figure 7. Foreign shocks are important contributors in periods of slowdown or recession. The 2002-2004 period of lower growth is characterized by large negative contributions from EA demand forces. They are the major driving force behind the GFC and the euro area debt crisis. BE internal demand shocks sometimes bring additional pressures on the top of EA ones (e.g. 2002-2004), while they contribute in an opposite direction in 2008-2009 and 2010-2012. Foreign shocks coming from outside the eurozone also apply significant downward pressures in these two crisis periods. They then become important positive contributors in the aftermath. EA monetary policy shocks contribute negatively to BE growth in 2008-2014, while these negative contributions decay and become expansionary after 2015. Energy

Table 8: Forecast error variance decomposition of Belgian real GDP growth and private consumption deflator. The decomposition has been computed based on the posterior mode estimates. Contributions are expressed in percentage of total variance.

	BE real GDP growth	**BE private consumption inflation**
EA demand	11.7	2.8
EA supply	1.6	2.4
EA mon. pol.	2.0	1.0
Extra-euro area shocks	22.6	5.1
Energy prices	1.5	25.6
BE demand	32.3	2.5
BE supply	11.7	60.4
BE others	16.6	0.1

price shocks are found to be supportive in the period 2014-2016, characterized by one of the largest drop in oil prices in the sample.

Volatile energy prices are among of the most important drivers of historical fluctuations in the year-on-year inflation rate of the private consumption deflator, as reported in Figure 8. Among other foreign forces, intra-euro area factors have a persistently negative contribution on headline inflation after the GFC period. As in the case of BE growth, EA monetary policy shocks contribute negatively from 2009 until the aftermath of the sovereign debt crisis. Their contribution then quickly reduces and turns positive at the end of the sample period. Extra-euro area shocks play an important role in the beginning of the euro era and during the GFC recession episodes. The early 2000 period is characterized by an important depreciation followed by an important appreciation of the euro versus the dollar, which makes the exchange rate shocks especially significant in this period. As reflected by the forecast error variance decomposition, internal drivers to consumption price inflation are mostly on the supply side. Nonetheless, BE demand innovations bring significant deflationary pressures after 2016, reflecting diffuse effects coming from previous episodes of slowdown in private demand.

Figure 7: Historical shock decomposition of the year-on-year growth rates of the real GDP of Belgium. The solid black line indicates real GDP growth in deviation from its steady-state trend of 1.4% per annum. The decomposition has been computed based on the posterior mode estimates.

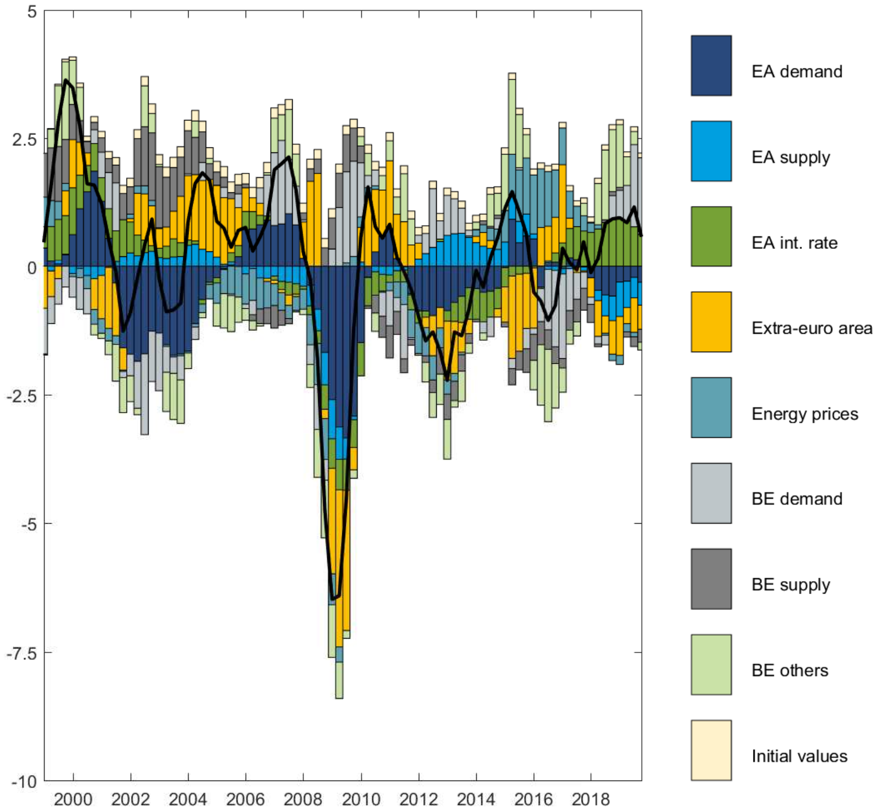
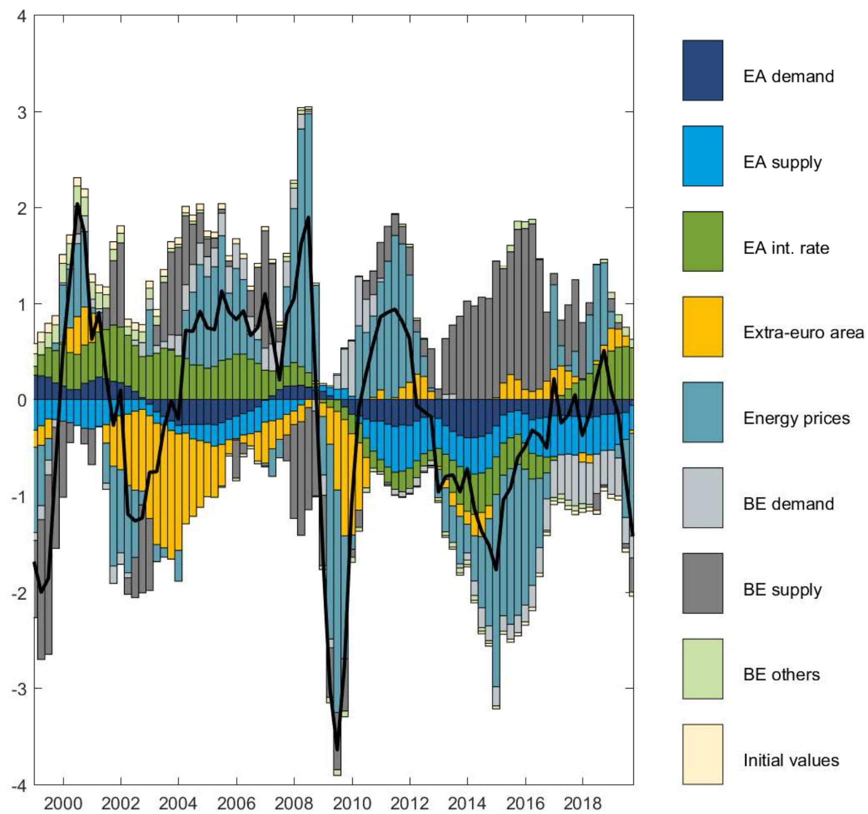


Figure 8: Historical shock decomposition of the year-on-year inflation rates of the Belgian private consumption deflator. The solid black line indicates consumption price inflation in deviation from its steady-state trend of 1.9% per annum. The decomposition has been computed based on the posterior mode estimates.



8 Conclusion

This paper provides a detailed description of BEMGIE, the estimated multi-country NK-DSGE of the National Bank of Belgium. The model is added to the NBB toolkit for macroeconomic simulation and projections. It is log-linearised and estimated using Bayesian techniques on BE, EA and US data covering the 1995-2019 period. The usefulness of the structure imposed on the international environment of the Belgian bloc is illustrated with an analysis of responses of BE macro variables to external shocks. The model endogenously generates significant shock-dependent spillovers after exogenous changes in oil prices, the euro exchange rate, EA productivity and monetary policy. For external demand shocks, coming from intra- or extra-euro area partners, correlation between BE and foreign shock innovations are needed to obtain a synchronisation between Belgian and foreign business cycles.

The version of BEMGIE presented in this paper can be extended in many directions. The lessons learned for macro models after the Global Financial crisis and the tasks assigned to the NBB in terms of macro-prudential supervision naturally put financial frictions on top of our agenda. We plan to equip the Belgian bloc of the model with corporate and mortgage credit and a financial accelerator mechanism à la Bernanke et al. (1999). Monopolistic banks with sector-specific capital requirements can be introduced in this framework to obtain rigidities in the setting of commercial bank rates and to evaluate macroprudential policies.

In an investigation on how to best generate model forecasts conditional on a fixed interest rate path, we consider the inclusion of yield curve data to guide interest rate expectations in BEMGIE and the introduction of agents' preference over safe assets (POSA). The latter strongly attenuates the forward guidance puzzle (Del Negro et al., 2012) by reducing household's responsiveness to future interest rates and by creating a consumption wealth effect from government bonds (Rannenberg, 2019). Moreover, in a DSGE estimated on euro area data, de Walque et al. (2023a) show that the inclusion of interest rate expectations in the dataset improves the model forecast of GDP and its components. They obtain the best forecasting performance with a model that combines POSA and interest rate expectation measures. This extension may thus turn to be particularly useful in the context of the use of BEMGIE in projection exercises where forecasts are typically conditioned on the market expectations of the short-term interest rate.

References

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., Mutschler, W., Perendia, G., Pfeifer, J., Ratto, M., and Villemot, S. (2021). Dynare: Reference Manual Version 4. Dynare Working Papers 1, CEPREMAP.

Adolfson, M., Laséen, S., Lindé, J., and Villani, M. (2008). Evaluating an estimated new keynesian small open economy model. *Journal of Economic Dynamics and Control*, 32(8):2690–2721.

Alpanda, S. and Aysun, U. (2014). International transmission of financial shocks in an estimated DSGE model. *Journal of International Money and Finance*, 47(C):21–55.

Aoki, K., Proudman, J., and Vlieghe, G. (2004). House prices, consumption, and monetary policy: a financial accelerator approach. *Journal of Financial Intermediation*, 13(4):414–435.

Baugnet, V. and Dury, D. (2010). Energy markets and the macroeconomy. *Economic Review*, (ii):61–82.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). Chapter 21 the financial accelerator in a quantitative business cycle framework. volume 1 of *Handbook of Macroeconomics*, pages 1341–1393. Elsevier.

Bjornland, H. (2022). The effect of rising energy prices amid geopolitical developments and supply disruptions . Mimeo, ECB.

Boeckx, J., de Sola Perea, M., Deroose, M., de Walque, G., Lejeune, T., and Nieuwenhuyse, C. V. (2018). What will happen when interest rates go up ? *Economic Review*, (iii):35–56.

Burggraeve, K. and Jeanfils, P. (2008). "NONAME": A new quarterly model for Belgium. *Economic Modelling*, 25(1):118–127.

Caldara, D., Cavallo, M., and Iacoviello, M. (2019). Oil price elasticities and oil price fluctuations. *Journal of Monetary Economics*, 103:1–20.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the great recession. *American Economic Journal: Macroeconomics*, 7(1):110–67.

Christoffel, K., Coenen, G., and Warne, A. (2008). The New Area-Wide Model of the euro area: a micro-founded open-economy model for forecasting and policy analysis. Working Paper Series 944, European Central Bank.

Clerc, L., Derviz, A., Mendicino, C., Moyen, S., Nikolov, K., Stracca, L., Suarez, J., and Vardoulakis, A. P. (2015). Capital Regulation in a Macroeconomic Model with Three Layers of Default. *International Journal of Central Banking*, 11(3):9–63.

Coenen, G., Karadi, P., Schmidt, S., and Warne, A. (2018). The New Area-Wide Model II: an extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector. Working Paper Series 2200, European Central Bank.

Cooper, J. C. (2003). Price elasticity of demand for crude oil: estimates for 23 countries. *OPEC Review*, 27(1):1–8.

Corbo, V. and Strid, I. (2020). MAJA: A two-region DSGE model for Sweden and its main trading partners. Working Paper Series 391, Sveriges Riksbank (Central Bank of Sweden).

De Mulder, J. and Godefroid, H. (2018). Slowdown in productivity. *Economic Review*, (iv):51–66.

de Walque, G., Lejeune, T., and Rannenberg, A. (2023a). Empirical DSGE model evaluation with interest rate expectations measures and preferences over safe assets. Working Paper Research 433, National Bank of Belgium.

de Walque, G., Lejeune, T., Rannenberg, A., and Wouters, R. (2023b). Low pass-through and international synchronization in general equilibrium: Reassessing vertical integration. *Journal of International Economics*, 140(C).

de Walque, G., Lejeune, T., Rychalovska, Y., and Wouters, R. (2017). An estimated two-country EA-US model with limited exchange rate pass-through. Working Paper Research 317, National Bank of Belgium.

Dedola, L., Lombardo, G., and Broner, F. (2012). Financial frictions, financial integration and the international propagation of shocks. *Economic Policy*, 27(70):319–359.

Del Negro, M., Giannoni, M., and Patterson, C. (2012). The forward guidance puzzle. Staff Reports 574, Federal Reserve Bank of New York.

Duprez, C. (2014). Creating export value. an analysis of belgium. *Economic Review*, (ii):23–38.

Eichenbaum, M. and Fisher, J. D. (2007). Estimating the frequency of price re-optimization in Calvo-style models. *Journal of Monetary Economics*, 54(7):2032–2047.

Engel, C. and Wang, J. (2011). International trade in durable goods: Understanding volatility, cyclicality, and elasticities. *Journal of International Economics*, 83(1):37–52.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281–313.

Fagan, G., Henry, J., and Mestre, R. (2001). An area-wide model (AWM) for the euro area. Working Paper Series 42, European Central Bank.

Fisher, J. D. (2015). On the Structural Interpretation of the Smets-Wouters "Risk Premium" Shock. *Journal of Money, Credit and Banking*, 47(2-3):511–516.

Forni, L., Gerali, A., Notarpietro, A., and Pisani, M. (2015). Euro area, oil and global shocks: An empirical model-based analysis. *Journal of Macroeconomics*, 46:295–314.

Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.

Galí, J. and Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, 72(3):707–734.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, 1 edition.

Houssa, R., Mohimont, J., and Otrok, C. (2019). A model for international spillovers to emerging markets. Working Paper Research 370, National Bank of Belgium.

Hubrich, K. and Karlsson, T. (2010). Trade consistency in the context of the Eurosystem projection exercises - An overview. Working Paper Series 108, European Central Bank.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3):739–764.

Iacoviello, M., Schiantarelli, F., and Schuh, S. (2011). Input and output inventories in general equilibrium. *International Economic Review*, 52(4):1179–1213.

Institut des comptes nationaux (2021). Statistique du commerce extérieur. Bulletin mensuel 6, National Bank of Belgium.

Jeanfils, P. and Burggraeve, K. (2005). Noname - A new quarterly model for Belgium. Working Paper Research 68, National Bank of Belgium.

Justiniano, A. and Preston, B. (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics*, 81(1):61–74.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2):132–145.

Kilian, L. and Murphy, D. P. (2014). The role of inventories and speculative trading in the global market for crude oil. *Journal of Applied Econometrics*, 29(3):454–478.

Kimball, M. S. (1995). The Quantitative Analytics of the Basic Neomonetarist Model. *Journal of Money, Credit and Banking*, 27(4):1241–1277.

Kollman, R. (2013). Global banks, financial shocks, and international business cycles: Evidence from an estimated model. *Journal of Money, Credit and Banking*, 45(s2):159–195.

Kose, M. (2002). Explaining business cycles in small open economies: "how much do world prices matter?". *Journal of International Economics*, 56(2):299–327.

Linnemann, L. (2006). The effect of government spending on private consumption: A puzzle? *Journal of Money, Credit and Banking*, 38(7):1715–1735.

Natal, J.-M. (2012). Monetary Policy Response to Oil Price Shocks. *Journal of Money, Credit and Banking*, 44(1):53–101.

National Bank of Belgium (2023). Annual Report 2022 - Economic and financial developments . Report, NBB.

Rannenberg, A. (2019). Forward guidance with preferences over safe assets. Working Paper Research 364, National Bank of Belgium.

Reusens, P. and Warisse, C. (2018). House prices and economic growth in Belgium. *Economic Review*, (iv):81–106.

Schmitt-Grohe, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61(1):163–185.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., and de Vries, G. J. (2015). An illustrated user guide to the world input-output database: the case of global automotive production. *Review of International Economics*, 23(3):575–605.

Tobin, J. (1969). A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking*, 1(1):15–29.

Ueda, K. (2012). Banking globalization and international business cycles: Cross-border chained credit contracts and financial accelerators. *Journal of International Economics*, 86(1):1–16.

9 Appendix

9.1 Endogenous international environment

The EA and US economies form the endogenous international environment of BEMGIE and are modelled according to an updated version of the model presented in de Walque et al. (2017). We compare here their structure with the one described in the main text for the BE bloc, and underline the main differences between the updated setup and its original version detailed in de Walque et al. (2017). This two-country environment is an integration of the closed economy models of Smets and Wouters (2003) and Smets and Wouters (2007) through international trade in goods and assets.

9.1.1 Households

Each household h in the EA bloc maximizes Utility (1) subject to a budget constraint very similar to the one outlined in Equation (2):

$$\begin{aligned} & P_t \left[\mathbb{C}_t (h) + I_t (h)\right] + \frac{B_{H,t} (h)}{\exp \left(\varepsilon_t^b\right) R_t} - B_{t-1} (h) + \frac{B_t^{US} (h)}{\exp \left(\varepsilon_t^b\right) R_{US,t} \Gamma_t^{US}} - S_{US,EA,t} B_{t-1}^{US} (h) \\ & \leq W_t^h (h) L_t (h) + R_t^k u_t^k (h) K_{t-1} (h) - a[u_t^k(h)] K_{t-1}(h) + Div_{f,t} + Div_{u,t} \end{aligned} \tag{82}$$

Different from the BE bloc, and similar to Smets and Wouters (2007), consumption and investment goods have the same price, while $I_t(h)$ includes both business and housing investment, and we do not model taxation. It is assumed that EA households have access to both a EA bond and an internationally traded bond is issued by the US, which is expressed in dollar currency. They receive a return on foreign bonds equal to the interest rate bearing on US bonds and subject to real costs following Adolfson et al. (2008):

$$\Gamma_t^{US} = exp \left(-\theta_a \frac{S_{US,EA,t} B_t^{US}}{P_t \gamma^t} - \theta_s \left(\frac{E_t S_{US,EA,t+1}}{S_{US,EA_t}} \frac{S_{US,EA,t}}{S_{US,EA,t-1}} - 1 \right) + \varepsilon_t^{UIP} \right)$$

US households face the same maximization problem, with the exception that they invest all their financial wealth in the US bond. Consequently the fourth and fifth terms of (82) drops from the US household's problem. Combining the Euler equations of the EA household's optimization with respect to domestic and foreign bond leads to the uncovered interest-rate parity that pins down the nominal exchange rate:

$$E_t \left[\frac{S_{US,EA,t+1}}{S_{US,EA,t}} \right] = \frac{R_{EA,t}}{R_{US,t} \Gamma_t^{US}} \tag{83}$$

Households' optimal rules for consumption, investment, capital and capital utilization rates are standard and not reported here for brevity reasons (see Smets and Wouters, 2007; de Walque et al., 2017). Labour supply and wage setting follows closely the description in Section 2.1 for the BE bloc. In contrast with Belgium, however, there is no health consumption price index. Instead, at each quarter wages are indexed on the evolution of the private consumption price inflation rate in the previous quarter, as is standard in the estimated DSGE literature.

9.1.2 Firms

The EA-US firm sectors display an updated structure compared to the original version of the two-country NK-DSGE model described in de Walque et al. (2017), which closely follows the one described in Section

2.2 for the Belgian bloc. Monopolistic intermediate good firms are allowed to make substitution between domestic and foreign inputs according to a CES production function, which replaces the Leontieff equation used in the original EA-US model. We also remove distribution costs from the EA-US bloc as this device generates effects similar to other mechanisms (in terms of limiting exchange rate pass-through and international spillovers). Consequently, its parameter appears to be significantly collinear with other parameters in the model, and generates identification issues in the estimation.

The intermediate goods are aggregated using Kimball's (1995) assembler into a final homogeneous good, leading to the same Equations (23)-(25) as for BE (except that housing investment is not separated from business investment, and is thus included in $I_t(h)$). On the demand side, CES aggregators combine this domestically produced good with foreign homogeneous goods to generate final consumption and investment baskets as depicted in Equation (26). For consumption, this aggregate is then combined with an energy component, while there is no separate housing consumption in contrast with the BE bloc. Hence we obtain the following relationships for real private consumption and its deflator in EA and US regions:

$$\mathbb{C}_t = \left[(1-\phi_{n,c})^{1/\lambda_{n,c}} C_t^{\frac{\lambda_{n,c}-1}{\lambda_{n,c}}} + \phi_{n,c}^{1/\lambda_{n,c}} \left(N_t^d\right)^{\frac{\lambda_{n,c}-1}{\lambda_{n,c}}}\right]^{\frac{\lambda_{n,c}}{\lambda_{n,c}-1}} \tag{84}$$

$$P_t = \left[(1-\phi_{n,c}) P_{C,t}^{1-\lambda_{n,c}} + \phi_{n,c}\left(P_{n,t}^d\right)^{1-\lambda_{n,c}}\right]^{1-\lambda_{n,c}} \tag{85}$$

where the distributed price of energy $P_{n,t}^d$ depends on the price of domestic distributed services and contemporaneous and lagged prices of crude oil, as for the BE economy.

9.1.3 Imports and exports

We assume that BE flows from/to the EA and the US are too small to be relevant for the dynamics of the total exports and imports of the two regions. We therefore only keep track of transatlantic trade flows as well as imports from and exports to the RoW. As a result, total exports (including transit goods) of country $k \in \{EA, US\}$ meet demands from the country $j \in \{US, EA\}$with $j \neq k$, and the RoW:

$$X_t = M_{j,k,t} + M_{RoW,k,t} \tag{86}$$

Note that compared to the original version of the EA-US blocs, we use the TCE variable for the world demand from extra-euro area countries (WDR_{EX}, controlled for US trade flows) as a proxy to guide the demand coming from the RoW ($M_{RoW,k,t}$).

On the import side, as in the BE bloc, total non-energy imports are combined with imports of energy. Total non-energy imports of country $j \in \{EA, US\}$writes as follows:

$$Y_{F,t} = M_{j,k,t} + M_{j,RoW,t}$$

with $k \in \{US, EA\}$for $k \neq j$.

9.1.4 Closing conditions

A real resource constraint ensures that domestic value-added production equals demand at equilibrium. It is similar to the BE bloc and writes as follows:

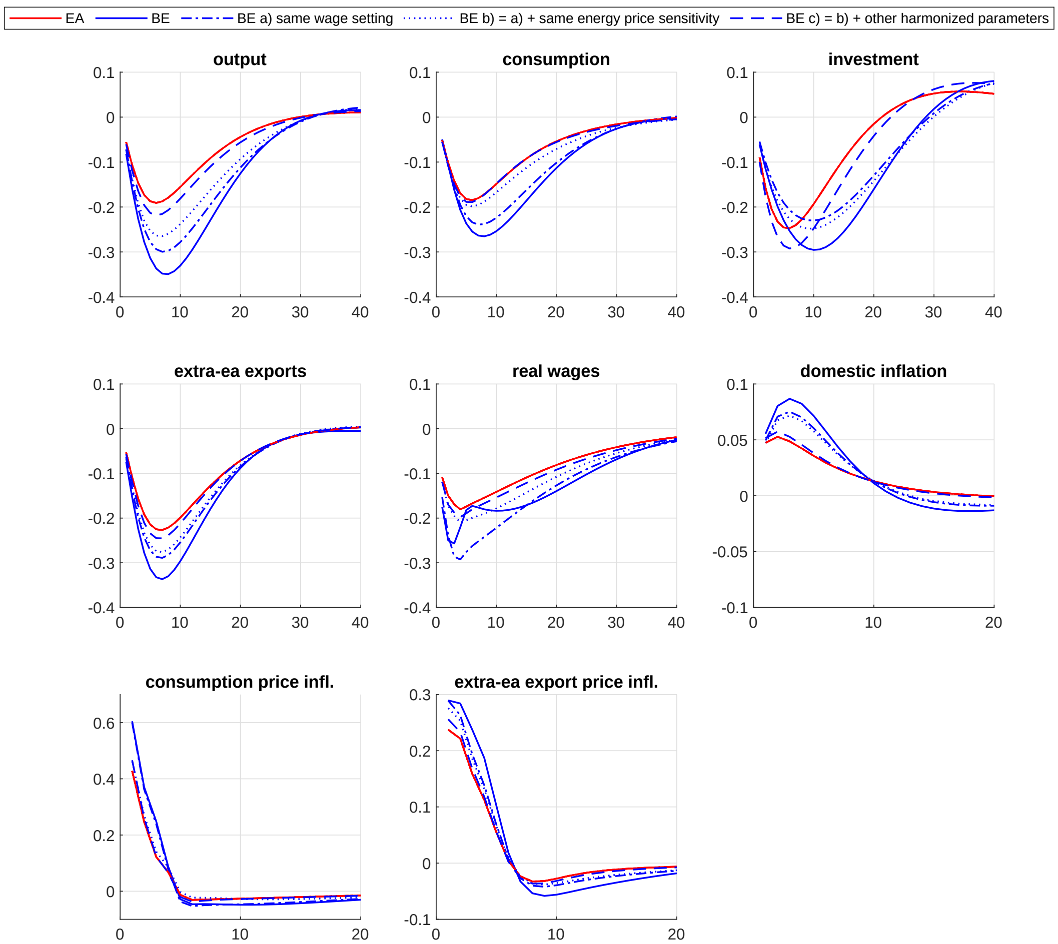
$$P_{\mathcal{Y},t}\mathcal{Y}_t = P_t\mathbb{C}_t + P_{,t}I_t + P_{H,t}G_t + P_{X,t}X_t - P_{M,t}M_t \tag{87}$$

It is assumed that EA and US government expenditures consist of home produced goods only, and are treated as an exogenous expenditure AR(1) process. The modelling of the government sector is simpler than in the BE bloc, as we do not keep track of the evolution of public revenues and public debt.

As explained in Section 2.6, a standard Taylor rule is used to represent central bank policies in EA and US economies. Monetary policy reacts to inflation and the output gap by gradually adjusting the nominal interest rate of the economy. To generate a measure for the output gap, we solve for counterfactual EA and US economies in a frictionless equilibrium where prices and wages are fully-flexible and the economies are not hit by price and mark-up shocks.

9.2 Additional figures

Figure 9: Selected impulse responses to a 10% increase in the crude price of oil (in dollar). Variables are expressed in percentage deviations from steady-state. The impulse responses for the interest rate and inflation rates are in nominal terms and annualised. An increase in the euro-dollar exchange rate indicates a nominal depreciation of the euro vs the dollar. The effects of transit goods (larger for BE than EA) are removed from import and export impulse responses to make figures more comparable across blocs. Import price inflation rates are in euro. Intra-euro area (extra-euro area) export price inflation rates are expressed in euro (in US dollar, respectively).



Solid lines indicate baseline simulation. In dashed-dotted lines, BE wage indexation and rigidity is set as in the EA. The simulation in dotted lines adds to this counterfactual by harmonizing the response of BE energy price to the EA ones. Finally, in dashed lines, we set the BE parameters driving adjustment in investment and capital utilisation, the degree of price indexation and the coefficient of households risk aversion to the EA estimated parameters.

NATIONAL BANK OF BELGIUM - WORKING PAPERS SERIES

The Working Papers are available on the website of the Bank: http://www.nbb.be.

396. “Daily news sentiment and monthly surveys: A mixed–frequency dynamic factor model for nowcasting consumer confidence”, by A. Algaba, S. Borms, K. Boudt and B. Verbeken, *Research series*, February 2021.
397. “A bigger house at the cost of an empty fridge? The effect of households' indebtedness on their consumption: Micro-evidence using Belgian HFCS data”, by Ph. Du Caju, G. Périleux, F. Rycx and I. Tojerow, *Research series*, March 2021.
398. “Low interest rates and the distribution of household debt”, by M. Emiris and F. Koulischer, *Research series*, March 2021.
399. “The interplay between green policy, electricity prices, financial constraints and jobs. Firm-level evidence”, by G. Bijnens, J. Hutchinson, J. Konings and A. Saint Guilhem, *Research series*, April 2021.
400. Economic importance of the Belgian maritime and inland ports – Report 2019”, by I. Rubbrecht, E. Dhyne and C. Duprez, *Research series*, May 2021.
401 “The return on human (STEM) capital in Belgium”, by G. Bijnens and E. Dhyne, *Research series*, July 2021.
402. “Unraveling industry, firm and host-region effects on export behaviors of international new ventures and established exporters”, by I. Paeleman, S. A. Zahra and J. W. B. Lang, *Research series*, July 2021.
403 “When trust is not enough: Bank resolution, SPE, Ring-fencing and group support” by M. Dewatripont, M. Montigny and G. Nguyen, *Research series*, August 2021.
404 “Bank specialization and zombie lending”, by O. De Jonghe, K. Mulier and I. Samarin, *Research series*, November 2021.
405. “Robert Triffin, Japan and the quest for Asian Monetary Union”, I. Maes and I. Pasotti, *Research series*, February 2022.
406. “The impact of changes in dwelling characteristics and housing preferences on house price indices”, by P. Reusens, F. Vastmans and S. Damen, *Research series*, May 2022.
407. “Economic importance of the Belgian maritime and inland ports – Report 2020”, by I. Rubbrecht, *Research series*, May 2022.
408. “New facts on consumer price rigidity in the euro area”, by E. Gautier, C. Conflitti, R. P. Faber, B. Fabo, L. Fadejeva, V. Jouvanceau, J. O. Menz, T. Messner, P. Petroulas, P. Roldan-Blanco, F. Rumler, S. Santoro, E. Wieland and H. Zimmer, *Research series*, June 2022.
409. “Optimal deficit-spending in a liquidity trap with long-term government debt”, by Charles de Beauffort, *Research series*, July 2022.
410. “Losing prospective entitlement to unemployment benefits. Impact on educational attainment”, by B. Cockx, K. Declercq and M. Dejemeppe, *Research series*, July 2022.
411. “Integration policies and their effects on labour market outcomes and immigrant inflows”, by C. Piton and I. Ruyssen, *Research series*, September 2022.
412. “Foreign demand shocks to production networks: Firm responses and worker impacts”, by E. Dhyne, A. K. Kikkawa, T. Komatsu, M. Mogstad and F. Tintelnot, *Research series*, September 2022.
413. “Economic research at central banks: Are central banks interested in the history of economic thought?”, by I. Maes, *Research series*, September 2022.
414. “Softening the blow: Job retention schemes in the pandemic”, by J. Mohimont, M. de Sola Perea and M.-D. Zachary, *Research series*, September 2022.
415. “The consumption response to labour income changes, by K. Boudt, K. Schoors, M. van den Heuvel and J. Weytjens, *Research series*, October 2022.
416. “Heterogeneous household responses to energy price shocks, by G. Peersman and J. Wauters, *Research series*, October 2022.
417. “Income inequality in general equilibrium”, by B. Bernon, J. Konings and G. Magerman, *Research series*, October 2022.
418. “The long and short of financing government spending”, by J. Mankart, R. Priftis and R. Oikonomou, *Research series*, October 2022.
419. “Labour supply of households facing a risk of job loss”, by W. Gelade, M. Nautet and C. Piton, *Research series*, October 2022.
420. “Over-indebtedness and poverty: Patterns across household types and policy effects”, by S. Kuypers and G. Verbist, *Research series*, October 2022.
421. “Evaluating heterogeneous effects of housing-sector-specific macroprudential policy tools on Belgian house price growth”, by L. Coulier and S. De Schryder, *Research series*, October 2022.
422. “Bank competition and bargaining over refinancing”, by M. Emiris, F. Koulischer and Ch. Spaenjers, *Research series*, October 2022.

423. “Housing inequality and how fiscal policy shapes it: Evidence from Belgian real estate”, by G. Domènech-Arumì, P. E. Gobbi and G. Magerman, *Research series*, October 2022.
424. “Income inequality and the German export surplus”, by A. Rannenberg and Th. Theobald, *Research series*, October 2022.
425. “Does offshoring shape labor market imperfections? A comparative analysis of Belgian and Dutch firms”, by S. Dobbelaere, C. Fuss and M. Vancauteren, *Research series*, November 2022.
426. “Sourcing of services and total factor productivity”, E. Dhyne and C. Duprez, *Research series*, December 2022.
427. " Employment effect of citizenship acquisition: Evidence from the Belgian labour market", S. Bignandi and C. Piton, *Research series*, December 2022.
428. "Identifying Latent Heterogeneity in Productivity", R. Dewitte, C. Fuss and A. Theodorakopoulos, *Research series*, December 2022.
429. “Export Entry and Network Interactions - Evidence from the Belgian Production Network”, E. Dhyne, Ph. Ludwig and H. Vandenbussche, *Research series*, January 2023.
430. "Measuring the share of imports in final consumption", E. Dhyne, A.K. Kikkawa, M. Mogstad and F. Tintelnot, *Research series*, January 2023.
431. "From the 1931 sterling devaluation to the breakdown of Bretton Woods: Robert Triffin’s analysis of international monetary crises", I. Maes and I. Pasotti, *Research series*, January 2023.
432. “Poor and wealthy hand-to-mouth households in Belgium”, L. Cherchye, T. Demuynck, B. De Rock, M. Kovaleva, G. Minne, M. De Sola Perea and F. Vermeulen, *Research series*, February 2023.
433. “Empirical DSGE model evaluation with interest rate expectations measures and preferences over safe assets”, G. de Walque, Th. Lejeune and A. Rannenberg, *Research series*, February 2023.
434. “Endogenous Production Networks with Fixed Costs”, E. Dhyne, A. K. Kikkawa, X. Kong, M. Mogstad and F. Tintelnot, *Research series*, March 2023.
435. “BEMGIE: Belgian Economy in a Macro General and International Equilibrium model”, G. de Walque, Th. Lejeune, A. Rannenberg and R. Wouters, *Research series*, March 2023.

National Bank of Belgium
Limited liability company
RLP Brussels – Company's number: 0203.201.340
Registered office: boulevard de Berlaimont 14 – BE-1000 Brussels
www.nbb.be

Editor

Pierre Wunsch
Governor of the National Bank of Belgium

Layout: Analysis and Research Group
Cover: NBB CM – Prepress & Image

Published in March 2023