A macroeconomic model with heterogeneous and financially-constrained intermediaries

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Abstract

This paper analyses the risk amplification inherent in a macroeconomic model with a heterogeneous financial sector. It extends a model with an equity-constrained intermediary by adding a shadow banking intermediary with pro-cyclical leverage. It is shown that the inclusion of this intermediary significantly amplifies financial frictions and adds to financial instability. Quantitative effects on asset prices are magnified, and the amplification propagates to the real side of the macroeconomy. Reducing the size of the shadow banking sector involves a trade-off between stabilizing the economy and the expected growth of economic activity. Ignoring the heterogeneity of the financial sector may lead to an underestimation of the excess risk-taking due to the anticipation of expansionary policies and of financial and macroeconomic responses to shocks.

JEL classification: G2, G12, E44.

Keywords: Financial frictions, Financial constraints, Endogenous risk, Shadow banking.

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1 Introduction

Since the global financial crisis of 2007-2008, academics and policymakers have underlined the importance of a good understanding of the risk inherent in the financial system and its propagation to the real side of the economy. A recent literature has elaborated on this endogenous financial risk with well-known examples including Mendoza (2010), He and Krishnamurthy (2013), Adrian and Shin (2010, 2014), Brunnermeier and Sannikov (2014) and Nuno and Thomas (2014). These papers share the central role of financial constraints in the risk channel. In particular, He and Krishnamurthy (2013, 2017) consider a financial intermediary sector subject to an occasionally-binding equity constraint. Negative shocks may constrain the issuance of equity, thereby depleting the capital base of the financial intermediary and increasing the riskiness of its balance sheet. The intermediary reacts by adjusting its asset and credit evaluation standards which in turn adversely affect asset prices. Negative corrections in asset prices feed back to real investment and output, disrupting further financial intermediation and raising the probability of a vicious cycle.

The He and Krishnamurthy (hereafter HK) model generates interesting asymmetries in the equilibrium dynamics of financial variables, with higher volatility during the distress periods than in the non-distress periods. The model considers a single intermediary, which is intended to represent the financial sector as a whole. This intermediary is associated with a counter-cyclical aggregate leverage that emanates from the fact that households do not have direct access to the risky asset coupled with the equity constraint. Therefore, the HK model does not shed any light on the role played by the deleveraging of important segments of the financial intermediation sector in the propagation of the recent financial disruption, a role emphasized by a major strand of the literature (see for instance Brunnermeier and Pedersen, 2009; Geanakoplos, 2010; Adrian and Shin, 2010, 2014). This deleveraging phenomenon adds to financial instability, and is likely to amplify the endogenous mechanism of the HK model. Dewachter and Wouters (2014) document that the endogenous mechanism remains confined to financial variables, and generates small quantitative effects on real variables. As a result, the HK model needs rather important exogenous shocks to explain the propagation of recent financial turmoil to the real economy. A comprehensive modeling of the financial sector may help in generating more important responses of macroeconomic variables to the health of the sector.

The objective of this paper is the investigation of endogenous risk amplification mechanisms in a macroeconomic model with heterogeneous and financially-constrained intermediaries. We propose an extension of the HK model by explicitly modeling two types of financial intermediaries: traditional banks and shadow banking institutions. The separation between traditional and shadow banking activities is motivated by the important heterogeneity that can be observed in the financial sector. Financial intermediaries differ in many aspects such as the main orientation of their activities (commercial-investment), their liabilities (depository versus non-depository institutions), the degree of wholesale funding (e.g. repo and other collateral financing), and their access to central bank facilities. Throughout the paper, we use the terms "traditional" banks and "shadow banking" institutions to distinguish between the two types of intermediaries. Traditional banks refer to depository institutions with mainly commercial activities and access to central bank liquidity. Consistent with Pozsar et al. (2010), the shadow banking intermediaries refer to non-depository institutions without any access to central bank liquidity or public sector credit guarantees. This sector is characterized by a relatively high degree of wholesale funding, which involves securitization and collateral funding as described for instance in Claessens et al. (2012).

These different aspects of financial intermediaries have different implications for their behavior along the business cycle. In economic upturns, shadow banking activities expand as a result of easy access to wholesale funding and regulatory arbitrage incentives. The leverage of shadow bankers is therefore high in good times. During crisis times, the drop in value of assets that served as collateral and the tightening of collateral requirements (e.g. rise in haircuts) leads to a sharp contraction of wholesale funding. Moreover, due to the uncertainty surrounding the reliability of counterparties and individual exposures to those collateral assets, some investors decide to stop rolling over their lending, which could cause a run-like event as described in Gorton and Metrick (2012). The worsening of funding conditions could eventually force shadow banking intermediaries to deleverage, applying downward pressure on asset prices. As a result, the leverage of shadow bankers is low in times of crises. This mechanism is consistent with the ones described in, among others, Adrian and Ashcraft (2012), Claessens et al. (2012), Bakk-Simon et al. (2012), Luck and Schempp (2014), and with the haircut theories developed in Geanakoplos and Fostel (2008), Adrian and Shin (2009) and Brunnermeier and Pedersen (2009), where intermediaries choose the maximum leverage given a haircut. In our model, traditional banks are the acquirer of assets shed by shadow banks.[1] However, due to asset quality deterioration, or for financial regulation reasons (Huang, 2018), traditional banks require prices to decline significantly for them to be willing to purchase those assets. This phenomenon leads to fire-sale prices for certain assets. As a result, the price of assets declines more than it would if there were no shadow banking sector in the economy.

The modeling of traditional and shadow banking intermediation is analyzed in a continuous-time macrofinance framework as in He and Krishnamurthy (2017) and Brunnermeier and Sannikov (2014). Such a set-up characterizes the full dynamics of the economy, including both normal and crisis states, and not only the dynamics around a steady state. The study of full equilibrium dynamics is important in order to capture relevant asymmetries in variables and the effects of the shadow banking sector, which are likely to be more pronounced in extreme states. The leverage behavior of the shadow banking intermediary is determined by a leverage rule similar to the one proposed by Danielsson et al. (2011) and Adrian and

[1] This buyer role of traditional banks during crisis times is consistent with the empirical results by He et al. (2010), who document that US commercial banks have increased asset holdings during the recent financial crisis. Moreover it is also supported by the reintermediation phenomenon observed during the global financial crisis of 2007-2008 (Pozsar, 2008). In many cases, traditional banks provided sponsoring to their off-balance-sheet special purpose vehicles (SPVs). This sponsoring carried implicit guarantees in the case of poor performance by their SPVs' loan portfolios. These implicit guarantees were extensively realized in the recent crisis (Acharya et al., 2013), while reputation risks also lead banks to bring assets of its off-balance-sheet vehicles onto its balance sheet. He et al. (2010) also report that commercial banks have absorbed such assets.

Boyarchenko (2015b,a). The leverage rule emanates from a Value-at-Risk (VaR) type of constraint, which implies a negative correlation between shadow bankers' leverage and the uncertainty in the economy. It appears as a mirror image of increased collateral requirements and contraction in rollover lending during downturns. Hence, our model uncovers important features of shadow banking observed during the 2007-2009 period, such as the deleveraging of the sector and the fire-sale phenomenon. As in the original HK model, the traditional sector is the marginal investor in capital, and its leverage decisions affect risk-pricing in the capital market.

The inclusion of the shadow banking intermediary is shown to add to financial instability by generating an extra acceleration in the development of endogenous risk. In crisis times, the equity constraint is binding, and the mechanism of He and Krishnamurthy applies. However, its leverage constraint forces the shadow banking intermediary to deleverage, and exerts an additional pressure on the traditional intermediary who has to absorb the assets sold. This in turn implies a deeper correction in asset prices, which become more volatile. As a result, the economy appears to be less stable in and around crisis states. Responses of financial and macroeconomic variables to exogenous shocks are amplified due to the shadow banking extra loop. Policies that aim at tightening wholesale funding conditions in order to limit the size of the shadow banking face a trade-off between stabilizing the economy and reducing the expected growth of economic activity. Moreover, neglecting the differences in the cyclical properties of intermediaries' leverage may lead to an under-assessment of the excess risk-taking due to the anticipation of expansionary policies. The loss in efficiency of anticipated policy compared to unanticipated policy is also more important in a model with shadow banking.

The model proposed in this paper combines two of the most popular types of funding constraints for intermediaries found in the literature: an equity-based constraint à la He and Krishnamurthy (2013, 2017) and a leverage rule inspired by Adrian and Shin (2014). In this respect, the papers most closely related to ours are Adrian and Boyarchenko (2015a) and Huang (2018). As in our paper, these authors propose the reconciliation of equity-constrained and leverage-constrained theories inside a continuous-time framework. Adrian and Boyarchenko (2015a) identify the two intermediaries according to their behavior in terms of book leverage, while we separate traditional banking activities from shadow banking activities. We are therefore more informative on the spillover effect of the shadow banking system on traditional intermediation. In this aspect, our research also relates to papers that attempt to model shadow banking in a macroeconomic context (Verona et al., 2011; Goodhart et al., 2012; Gennaioli et al., 2013; Meeks et al., 2013; Aoki and Nikolov, 2013; Luck and Schempp, 2014; Moreira and Savov, 2014). In addition, Adrian and Boyarchenko (2015a) feature acyclical leverage for the equity-constrained intermediary and do not shed any light on the acquirers of assets shed in times of crisis. Huang (2018) uses Brunnermeier and Sannikov's (2014) framework to include shadow bankers and focuses his study on the impact of financial regulation on the size and instability effects of shadow banking activities. Pro-cyclical shadow banking leverage emanates from an endogenous enforcement problem, while we use an exogenous VaR constraint. Huang's (2018) approach is qualitative, and suggests a U-shaped relationship between financial regulation and financial instability. In contrast, our contribution is mainly quantitative. We focus on the quantitative assessment of amplification effects due to shadow banking activities. For this purpose, we rely on the HK structure which helps us to study the amplification effects in an environment that generates quantitatively significant non-linearities (see He and Krishnamurthy, 2017). Furthermore, we examine the influence of the heterogeneity in the financial sector on the response of real variables to shocks. Our model underlines the role played by the relative leverage of the two intermediaries on the volatility of financial variables and their propagation to real variables. Finally, we also examine the importance of agents' anticipation of the disruption caused by a binding financial constraint, and document its implication for policy implementation.

The paper is organized as follows. Section 2 presents the two-intermediary sector model in a AK framework. This model features constant equity allocation between the two intermediaries. In Section 3, differences in global dynamics and model simulations with respect to a model without shadow banking are stressed. Section 4 discusses policy experiments within the two-intermediary model. Robustness checks with endogenous equity allocation are presented in Section 5, while Section 6 concludes.

2 A two-intermediary model

The theoretical framework is similar to He and Krishnamurthy (2017) and to the continuous-time model presented in Dewachter and Wouters (2014). We consider a simple AK type of economy in a continuous time setting, where firms produce output with technology

$$Y_t = AK_t \tag{1}$$

and pay wage bills as a fixed proportion of output, $l_t w_t = lY_t$. The remaining portion of output is paid out to shareholders in the form of dividend, $div_t = (1-l)Y_t$. Capital K_t evolves according to

$$dK_t = (i_t - \delta)K_t dt + \sigma K_t dZ_t \tag{2}$$

where σ is the instantaneous standard deviation parameter that governs the exogenous shock on K_t. The term σdZ_t can be interpreted as a capital quality shock, with K_t being the degree of efficiency or the effective quality of capital. The capital quality shock is a simple device to introduce an exogenous source of variation in the value of capital, and is standard in the macrofinance literature (Gertler and Kiyotaki, 2010; Brunnermeier and Sannikov, 2014; Nuno and Thomas, 2014). As in He and Krishnamurthy (2017), it is the only exogenous shock in the model.[2] New capital can be created through endogenous investment i_t. We assume a

[2]An alternative way to introduce such an exogenous shock into the economy would be to use a stochastic productivity A_t. However, this implies an additional state variable and complicates the solution of the model. As underlined in He and Krishnamurthy (2017), the main difference lies on a direct impact on capital prices q. Following a negative productivity shock, capital prices are expected to directly fall before any equilibrium adjustment effects, while with a capital quality shock q is only affected by general equilibrium effects.

standard investment technology with adjustment costs: the total cost of installing $i_t K_t$ units of new capital requires $\Psi(K_t, i_t)$ units of consumption goods,

$$\Psi(K_t, i_t) = i_t K_t + \frac{\kappa}{2}(i_t - \delta)^2 K_t \tag{3}$$

Risk-neutral capital good producers[3] sell one unit of capital at a price q_t. Their profit maximization leads to the optimal investment rule

$$i_t = \delta + \frac{q_t - 1}{\kappa} \tag{4}$$

Figure 1 below shows the balance sheet representation of the three main sectors in the model: household, traditional intermediary and shadow banking.[4] As in He and Krishnamurthy (2017), the household sector is composed of a constant fraction λ of debt household and a fraction $(1-\lambda)$ of equity household. While the former invests all attributed wealth λW_t in the risk-free debt, the equity household allocates between risky equity of the intermediary sector α^H and risk-free debt $(1 - \alpha^H)$. The portion invested in equity is shared between the traditional sector $(1 - \phi)$ and the shadow banking sector (ϕ). Both intermediaries leverage equity (respectively E_t^{TI} and E_t^{SB}) with risk-free debt to invest in the risky capital market. We assume that intermediaries do not consume and invest maximally if the opportunity arises. Capital market equilibrium is therefore

$$q_t K_t = q_t K_t^{TI} + q_t K_t^{SB} = \alpha_t^{TI} E_t^{TI} + \alpha_t^{SB} E_t^{SB} \tag{5}$$

where TI stands for traditional intermediary and SB for shadow banking. The debt market equilibrium is

$$\lambda W_t + (1 - \alpha^H)(1 - \lambda) W_t = (\alpha_t^{TI} - 1) E_t^{TI} + (\alpha_t^{SB} - 1) E_t^{SB} \tag{6}$$

From capital and debt market equilibria we obtain $q_t K_t = W_t$, that is, total household financial wealth is equal to the total market value of capital assets. The next sections provide more details on the behavior of the three agents.

[3] We follow He and Krishnamurthy (2017) and consider the risk-neutrality of capital good producers. It is a simplifying assumption that enables us to derive a simple link between investment decisions and the price of capital, and hence to relate the real side and the financial side of the economy. Under risk aversion, however, the drift of the pricing kernel of agents is affected by uncertainty (see for instance Lioui and Poncet, 2008, for the impact of uncertainty in a macro-money model), and firms might prefer to maximize value instead of profits.

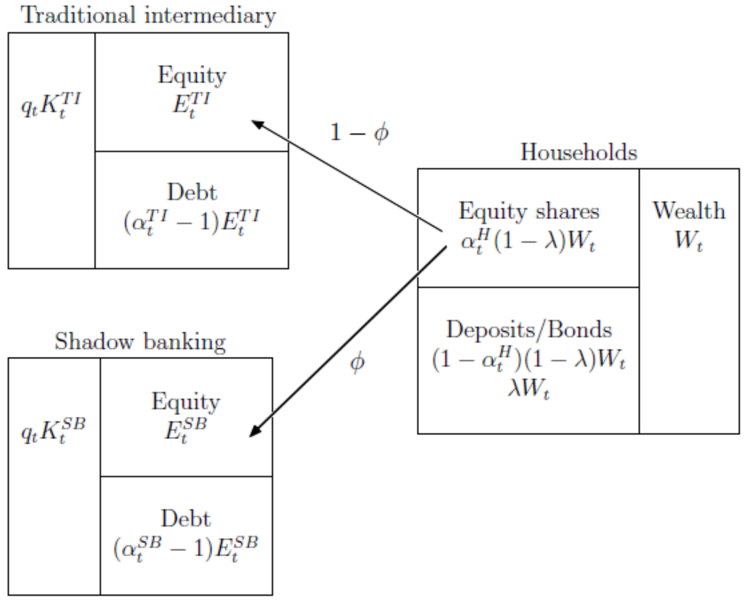
[4] We follow He and Krishnamurthy (2017), and most of the related macroeconomic literature, by not giving an explicit role for money in our model. Recent studies have nevertheless shown that the quantitative effects of money may be non negligible. In Sidrauski's (1967) framework, Reis (2007) shows that, under certain conditions, money is not super-neutral both on and off the steady state. Lioui and Poncet (2008, 2012) underline that, when uncertainty is introduced in this model, money is not super-neutral even if the utility function is separable in consumption and money. As our objective is to document the extension of the existing HK model, we follow its related literature and use a similar "no-money" framework, keeping in mind that an analysis with an explicit role for money might be interesting as well.

The single balance-sheet representation of the shadow banking sector in Figure 1 should be seen as a simplified picture of the complex interconnections between financial institutions involved in shadow banking activities. In particular, many different institutions such as money market funds (MMFs), special purpose vehicles (SPVs), broker-dealer banks (especially important for the US) and hedge funds are involved in the shadow banking system. The shadow banking mechanisms depicted in the literature (Adrian and Ashcraft, 2012; Claessens et al., 2012; Gorton and Metrick, 2012; Bakk-Simon et al., 2012) generally involve the securitization of banks' assets (loans) into asset backed-securities (ABS). This process is undertaken by SPVs created by banks or broker-dealers. These SPVs help to remove items from banks' balance sheets, the so-called off-balance-sheet items (Duffie, 2010). The ABS are then sold to other special vehicles or to financial intermediaries such as broker-dealers. The purchases of ABS are funded by money market instruments, such as asset-backed commercial paper (ABCP) or repos. Maturity transformation of loans into short term instruments is therefore involved in the process. Money market funds (MMFs) form the major buyer side of the money market. They finance their purchase by the issuance of stable net asset value (NAV) claims, which are in turn sold to ultimate savers, that is, short-term households and corporate savings. The MMFs' NAV claims correspond to safe and liquid assets, which were perceived as alternatives to traditional deposits by financial actors in the pre-2007 period (McCabe, 2010; Sunderam, 2015). Another shadow banking channel involves the intensive re-use of collateral within the system. The broker-dealer banks have a central role in this channel. They receive collateral from leveraged counterparties that require funding such as hedge funds. The collateral is then re-used to obtain funding or support other contracts. Funding using repurchase agreements (repo) is very common in this collateral channel.

The simplified representation in Figure 1 is consistent with these shadow banking mechanisms. Part of the household debt is dedicated to the shadow banking system, and can be seen as holdings under the form of NAV claims issued by MMFs, while the remaining part is held under traditional deposits in commercial banks.[5] The equity of the shadow banking sector includes equity shares of broker-dealers and hedge funds. Commercial banks also use off-balance-sheet operations which, through the creation of SPVs, help to extend businesses outside their balance-sheet in order to circumvent financial regulation. Beccalli et al. (2015) point out that the degree of off-balance-sheet securitization has important implications for the effective leverage behavior of US commercial banks. In many case, commercial banks provide implicit guarantees (or sponsoring) to support their off-balance-sheet financing, and their wealth (or their reputation, a concept described later in this paper) can influence SPVs investors (Gorton and Souleles, 2007; Huang, 2018). Our definition of the equity of the shadow bankers can also be seen as to cover the equity shares of commercial banks with a high degree of off-balance-sheet securitization activity.

[5]This representation is similar to the one used in Luck and Schempp (2014) and Huang (2018).

Figure 1: Balance sheet representation of the economy



2.1 Financial intermediation

Households are assumed to have no direct access to capital. This constitutes a first source of financial friction, and establishes the need for financial intermediation to channel household funds to the capital market.[6] As in He and Krishnamurthy (2017), there is a continuum of competitive intermediaries owned by households but controlled by specialists independently of household preferences. Therefore, there is a separation between ownership and control of the intermediary. Specialists have the know-how to invest in capital markets, while the intermediaries' equity capital is provided by households.

There are two types of intermediary: a traditional intermediary (denoted by TI) and a shadow banking intermediary (SB). They both raise equity and issue debt to fund their investment in capital assets. Suppose the law of motion of the price per unit of capital q_t is $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$, which is endogenously determined in the model. By Ito's lemma, the

[6] This assumption implies that the aggregate financial sector has to absorb all the capital assets with an equity base that is only a fraction of total household wealth. This set-up creates the need for leveraged positions for intermediaries, and generates counter-cyclical leverage.

returns on capital evolve according to the following process[7]

$$\begin{aligned} dR_t &= \frac{dq_t + A(1-l)dt}{q_t} - \delta dt + \sigma dZ_t + \frac{< dq_t dK_t >}{q_t K_t} \\ &= \left(\mu_t^q + \frac{A(1-l)}{q_t} - \delta + \sigma_t^q \sigma\right) dt + (\sigma_t^q + \sigma) dZ_t \\ &= \mu_t^R dt + \sigma_t^R dZ_t \end{aligned} \tag{7}$$

It follows that the return on equity of financial intermediaries is given by

$$\begin{aligned} dR_t^j &= \alpha_t^j dR_t + (1-\alpha_t^j) r_t dt \\ &= \left[\alpha_t^j(\mu_t^R - r_t) + r_t\right] dt + \alpha_t^j \sigma_t^R dZ_t \end{aligned} \tag{8}$$

for $j = TI$ (traditional intermediary) or $j = SB$ (shadow banking), and where r_t is the risk-free interest rate.

2.1.1 Shadow banking

The shadow banking intermediary leverages its equity E_t^{SB} in order to maximize expected returns.[8] We assume that this intermediary is restricted in its portfolio choice by the respect of a risk constraint that ensures that equity E_t^{SB} is large enough to cover its Value-at-Risk (VaR). This type of constraint is inspired by the Value-at-Risk constraint proposed in Danielsson et al. (2011) and Adrian and Boyarchenko (2015b,a).[9] As exogenous shocks are modeled with Brownian motions, their Gaussian nature enables us to define VaR as a constant z multiplied by the standard deviation of the return on equity ($\alpha_t^{SB}\sigma_t^R E_t^{SB}$). Shadow bankers are assumed to maximize their expected return on equity subject to their Value-at-Risk constraint

$$\max_{\alpha_t^{SB}} \left[\alpha_t^{SB}\left(\mu_t^R - r_t\right) + r_t\right] E_t^{SB} \quad s.t. \quad z\alpha_t^{SB}\sigma_t^R E_t^{SB} \leq E_t^{SB} \tag{9}$$

where z is a parameter that governs the tightness of the risk-based constraint. We assume that shadow bankers always leverage at the maximum, i.e. the VaR constraint always binds. We therefore obtain the following simple leverage rule

$$\alpha_t^{SB} = \frac{1}{z\sigma_t^R} \tag{10}$$

[7] We compute here the returns on existing capital, meaning that new investments are not taken into account as they are not yet part of the investment portfolio. This explains why investments i_t does not appear in the process describing the evolution of capital returns.

[8] Our objective is the documentation of amplification effects due to the presence of the shadow banking system. For this reason, we do not explicitly model the motivation behind the existence of the shadow banking sector. However, three main raisons d'être for shadow banking activities can usually be identified: regulatory arbitrage, the demand for (alternative) safe and liquid assets from institutional investors, and the demand from financial institutions for securitized debts in order to use them as collateral to attract repo funding, and hence boost leverage and returns (Claessens et al., 2012).

[9] Adrian and Shin (2014) provide a microfoundation for the VaR constraint as a consequence of constraints imposed by lenders in a moral-hazard setting. Nuno and Thomas (2014) combine bank default risk, limited liability and moral hazard to obtain a set-up that links volatility of asset returns and leverage.

The VaR constraint is used as a tool to generate the observed pro-cyclical leverage movements of shadow bankers and enables us to replicate important stylized behavior of the shadow banking sector. It is consistent with the findings of Nuno and Thomas (2014) that volatility shocks produce empirically relevant fluctuations in leverage. Their discrete-time set-up underlines the role played by exogenous volatility, while in our model, the leverage of shadow bankers directly depends on the level of overall volatility σ^R, that is, the levels of both endogenous (σ^q) and exogenous (σ) volatilities (see equation 7). In normal or boom time, endogenous volatility is low and the leverage of shadow bankers is relatively important. The leverage rule therefore mimics easy wholesale funding conditions and incentives to extend off-balance-sheet operations, which generate expansions in shadow banking activities. In crisis periods, endogenous volatility is high, and the leverage rule implies that shadow bankers enter into a deleveraging process. Through this mechanism, the leverage rule drives the demand of shadow banking intermediaries for risky capital asset. It is consistent with empirical evidence that wholesale funding dependent intermediaries substantially deleveraged and shed assets during the recent financial crisis (He et al., 2010; Ang et al., 2011; Damar et al., 2013). It can also be related to recent theories on market liquidity and haircuts developed in Geanakoplos and Fostel (2008), Brunnermeier and Pedersen (2009), Gorton and Metrick (2012) and Moreira and Savov (2014). In times of high uncertainty, concerns about market liquidity make collateralized funding tighter, raise haircuts and incite money market lenders to stop the rollover of a part of their lending. As a consequence, institutions that rely heavily on wholesale funding are forced to reduce their positions.

2.1.2 Traditional banking

Traditional intermediaries hold the remaining market value of the capital asset. They are not constrained on the debt side of their balance sheet and hence are the marginal investors, as in the original HK framework. This is consistent with He et al. (2016), who find that the marginal value of wealth of financial intermediaries with counter-cyclical leverage is a relevant pricing kernel for several asset classes, suggesting that they are marginal investors in many asset markets. We follow He and Krishnamurthy (2013, 2017) and assume that traditional intermediaries make investment decisions to maximize a mean-variance type of objective function:

$$\max_{\alpha_t^{TI}} E_t\left[dR_t^{TI}\right] - \frac{m}{2} Var_t\left[dR_t^{TI}\right] \tag{11}$$

where m represents the risk-aversion coefficient of the traditional intermediary. In A.3, we show that the optimization problem (11) comes from the assumption that traditional intermediaries maximize the log of their performance (also called reputation in He and Krishnamurthy, 2017). The solution to this problem leads to the following optimal investment strategy

$$m\alpha_t^{TI}\sigma_t^R = \frac{\mu_t^R - r_t}{\sigma_t^R} \tag{12}$$

where the term on the right of (12) corresponds to the Sharpe ratio, i.e. the excess return per unit of volatility. This optimal condition intuitively implies that the Sharpe ratio increases if the traditional intermediary bears more risk in its portfolio and/or becomes more risk-averse (i.e. m becomes larger). This relation is the key link between leverage and financial developments in the HK framework: the evolution of required returns are determined by movements in traditional leverage.

2.2 Households and occasionally-binding constraint

Households are assumed to maximize the expected CRRA utility of total consumption

$$\max_{C_t} E \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} dt$$

subject to the budget constraint

$$\begin{aligned} dW_t &= (lY_t - C_t)dt + (1-\phi)\alpha_t^H (1-\lambda) W_t \left[\alpha_t^{TI}(dR_t - r_t dt) + r_t dt\right] \\ &+ \phi \alpha_t^H (1-\lambda) W_t \left[\alpha_t^{SB}(dR_t - r_t dt) + r_t dt\right] + \left(\lambda W_t + (1-\alpha_t^H)(1-\lambda) W_t\right) r_t dt \end{aligned}$$

Aggregate wealth evolves according to income from labor services, consumption and allocation to the equity and debt of financial intermediaries.[10] Optimal conditions leads to a standard Euler equation

$$r_t = \rho + \theta E_t \left[\frac{dC_t}{C_t}\right] - \frac{\theta(\theta+1)}{2} Var_t \left[\frac{dC_t}{C_t}\right] \tag{13}$$

Total equity allocation is expressed as a fraction α_t^H of equity household wealth $(1-\lambda)W_t$. In a first analysis, we consider constant equity allocation ϕ. In the results reported below, this simple assumption is found to be sufficient in order to underline interesting amplification effects. A constant ϕ is therefore retained as our baseline configuration. As an alternative, we discuss in Section 5 a framework with varying ϕ determined by portfolio choices that nests the constant shares as a special case.

Following the HK mechanism, equity households' investment decisions are based on the reputation (or equivalently performance) of the aggregate financial sector, ε_t. It is defined as

$$\begin{aligned} d\varepsilon_t &= (1-\phi)\varepsilon_t m \left(\alpha_t^{TI}(dR_t - r_t dt) + r_t dt\right) \\ &+ \phi \varepsilon_t \left(\alpha_t^{SB}(dR_t - r_t dt) + r_t dt\right) - \eta dt + d\psi_t \end{aligned} \tag{14}$$

The total financial wealth that equity households are willing to invest into the equity of intermediaries is equal to $(1-\lambda)W_t$. However, given the aggregate reputation of the financial sector, the actual amount that equity households invest must not exceed ε_t. If the reputation is sufficiently high (i.e., $\varepsilon_t \geq (1-\lambda)W_t$), equity household invest all their allocated wealth in the equity of intermediaries.[11] In contrast, if intermediaries reputation is not sufficient

[10] Investment adjustment costs do not enter the budget equation, as new investments (and associated costs) are not yet part of the investment portfolio.

[11] Though He and Krishnamurthy (2017) do not provide an explicit proof of equity dominance if the constraint is not binding, it can actually be shown that equity is dominant as long as the risk aversion of households is lower than or equal to the risk aversion of intermediaries.

to absorb the maximum allocation (i.e, $\varepsilon_t < (1-\lambda)W_t$), equity households restrict their investment and invest the remaining wealth in the debt market. This occasionally-binding constraint is motivated by a skin-in-the-game argument in He and Krishnamurthy (2013, 2017),[12] and can be expressed as follows

$$\alpha_t^H = \min\left(1, \frac{\varepsilon_t}{(1-\lambda)W_t}\right) \tag{15}$$

It is worth noting that the equity capital constraint applies to both intermediaries. If the constraint is binding, equity households reduce their allocation to both intermediaries in proportion of the sector weight ϕ. This comes in contrast with Adrian and Boyarchenko (2015a) who make a distinction between equity-constrained and leverage-constrained intermediaries, and apply equity constraints only to the former. In our model, we assume that equity household's decisions are based on an aggregate measure of reputation. In distressed times, the performance of the shadow banking sector also deteriorates, and negatively contributes to the reputation of the aggregate financial sector. The extension of the equity constraint to shadow bankers is consistent with falls in assets under management of certain types of shadow banking institutions, such as hedge funds, reported in He et al. (2010) and Ang et al. (2011).

The occasionally-binding constraint is a key instrument in the HK mechanism. It sets up the channel through which reputation of financial intermediaries affect required premium and asset prices. Low performance and hence low reputation of financial intermediaries are associated with lower equity allocation from households. With a constant sector allocation ϕ, a lower funding from households translates into a higher leverage for traditional intermediaries. This in turn raises required returns per unit of volatility (i.e. the Sharpe ratios) as entailed in the optimal portfolio strategy (12). As a result, asset prices fall in order to deliver these higher required returns. As reputation is a multiple of asset returns, this fall in capital prices further tightens the equity constraint and leads to an amplification mechanism. The inclusion of a shadow banking sector has the potential to amplify the endogenous mechanism. In distressed periods, the HK mechanism generates important endogenous volatility, reflected in the volatility of capital returns. Due to the leverage rule, an environment with high endogenous volatility forces shadow bankers to reduce importantly their demand for capital assets. Traditional intermediaries have to absorb a more substantial part of the risky asset, with a lower level of equity due to the binding equity constraint. Therefore they need to rely on a higher level of leverage. The higher riskiness of their balance-sheet further raises the equilibrium Sharpe ratios and applies an additional downward pressure on asset prices.

Anticipation effects also emerge from the global solution of the model in regions where the constraint is not binding. To understand how these anticipation effects work, let us consider the one-intermediary model and note that without any equity constraint, $q(e)$ would be a flat line. This comes from the fact that traditional leverage would always be constant as households always allocate a constant share $(1-\lambda)$ of their wealth to intermediary equity. As

[12] See also Phelan (2014) for an alternative formulation of an equity constraint in a continuous-time macro-finance framework.

a result of constant leverage, there is no change in the risk premium. This implies that prices are not affected by the evolution of intermediaries' reputation, i.e. $q'(e) = 0$, and there is no endogenous volatility in the economy ($\sigma^R = \sigma$). When an equity constraint is introduced, the economy becomes dependent on the evolution of financial reputation. As intermediaries' equity becomes constrained, traditional leverage increases and risk premia are raised. Asset prices are now sensitive to the evolution of reputation. In the unconstrained area, far away from the constrained states, the economy behaves as if there were no constraint: prices are not much sensitive to reputation, endogenous volatility is close to 0, and risk premia are relatively flat. However, as the economy moves closer to the constrained area, it becomes more and more sensitive to the evolution of intermediaries' reputation. In these states, there is a non-negligible probability that a decrease in reputation leads the economy to fall in the constrained region, where prices are highly sensitive to reputation and endogenous volatility surges. As asset prices are present values of future cash flows, the anticipation of future constrained states is reflected in higher discount rates, and causes risk premia to increase and asset prices to fall when the economy moves closer to the constrained area.[13] Therefore, anticipation effects implies that financial frictions effects are not only active in the constraint area, but also affect significantly some unconstrained states.

The introduction of shadow banking activities implies anticipation effects that are more important than in the HK model, as it implies more severe developments in the constrained area, and hence deeper corrections in expectations. In the unconstrained area, the HK model is characterized by a constant traditional leverage (this can be seen by replacing ϕ_t with 0 and α_t^H with 1 in equation 18). As the economy moves closer to constrained states, endogenous volatility increases (i.e. σ_t^R increases) and reflects the higher likelihood of entering the constrained area. According to their optimal portfolio rule (equation 12), higher uncertainty leads intermediaries to raise risk premia (i.e. discount rates) for the same level of leverage. In the two-intermediary model, traditional leverage is not constant in the unconstrained area. An increase in uncertainty (i.e. volatility) leads shadow bankers to reduce their leverage, and their decision applies an upward pressure on traditional leverage. This additional effect is absent in the HK model without shadow banking. The increase in traditional leverage in the two-intermediary model leads to a more important effect on risk premia than the initial increase in endogenous uncertainty alone, and anticipation effects are more important in the two-intermediary model.

2.3 Equilibrium and model solution

The equilibrium is a map from histories of macro shocks to a set of prices (q_t, r_t) and decisions (C_t, α_t^{TI}, α_t^{SB}, i_t) such that, given the prices, optimal decisions are made (equations 4, 10, 12, 13, 21), and given decisions, markets clear at those prices. Clearing conditions on the

[13] These anticipation effects are similar to the adjustments in dynamics found in the simple model of Krugman (1991) about the exchange rate behavior under a target zone regime. In his model, it is shown that the expectation that monetary policy will be adjusted to prevent the exchange rate from leaving the zone affects exchange rate behavior even when it lies inside the zone, and is thus not being defended actively.

goods market requires

$$Y_t = C_t + \Psi(K_t, i_t) \tag{16}$$

Financial intermediaries do not consume, and hence do not enter the goods market clearing condition. Therefore, the aggregate identity implies the following rule for households' consumption:

$$C_t = \left(A - \delta + \frac{1 - q_t^2}{2\kappa} \right) K_t \tag{17}$$

and dynamics for dC_t can be inferred from this relation (see A.1).

The capital market equilibrium has already been expressed above, in equation (5). Using the shadow banking leverage rule (10) and the equilibrium relation $q_t K_t = W_t$, we are able to derive the traditional leverage required for capital market clearing

$$\alpha_t^{TI} = \frac{1}{(1 - \phi_t)\alpha_t^H (1 - \lambda)} - \frac{\phi_t}{1 - \phi_t} \alpha_t^{SB} \tag{18}$$

Traditional leverage depends on the evolution of shadow banking leverage and the importance of this second sector. If $\phi = 0$, we are back to the HK model with one type of financial intermediary. The HK one-intermediary model is thus a special case of our two-intermediary model, and will be used as a benchmark case in model simulations. A positive ϕ means that some equity is diverted from the traditional sector to shadow bankers, and this has an increasing effect on traditional leverage through the term $(1 - \phi)$ in the first fraction on the right of equation (18). On the other hand, the demand for capital asset from the shadow banking sector is now positive, and traditional intermediaries are required to absorb less of the total capital assets. This in turn tends to decrease traditional leverage. The second term on the right-hand side of equation (18) reflects this lower residual demand. In distress times, the deleveraging process of shadow bankers sharply decreases α_t^{SB} and has an upward effect on traditional leverage. The overall effect of the inclusion of a shadow banking sector should be assessed by the analysis of the overall dynamics, and depends on the effects of a lower share of total equity allocated to traditional intermediaries and the remaining capital market shares they are required to hold. Movements in traditional leverage implied by capital market clearing are propagated to asset prices through the optimal investment decisions of traditional intermediaries (equation 12).

To find an equilibrium, we follow He and Krishnamurthy (2017) and express the relationships between variables and the realizations of shocks dZ_t in terms of a single state variable: the scaled reputation of the aggregate financial sector, $e_t = \frac{\varepsilon_t}{K_t}$, which evolves according to the following stochastic process $de_t = \mu_e dt + \sigma_e dZ_t$. Appendix A reports the different steps involved in order to express all prices and decisions variables in terms of this state process. The equilibrium is characterized by a second-order differential equation for $q(e_t)$ and is solved using numerical techniques for systems of ODEs. The numerical solution of the second-order ODE requires two boundary conditions. In order to obtain a stable solution, we follow He and Krishnamurthy (2017) and express boundary constraints in terms of price derivatives at the lowest and the highest state values, i.e. $q'(\underline{e}) = 0$ and $q'(\bar{e}) = 0$. He and Krishnamurthy (2017) justify the lower bound by a no-arbitrage argument. They interpret $\underline{e}$ as a reflecting boundary: entry occurs as the Sharpe ratio is high[14] and instantaneously moves $\underline{e}$ to $\underline{e} + \epsilon$ for any positive and small ϵ. This move is sustained by the re-allocation of β units of capital. In this case imposing $q'(\underline{e}) = 0$ prevents actors in the capital market to bet on a certain increase in q.[15] At upper end, the economy with $\bar{e} \to \infty$ behaves as if the equity constraint never binds. An economy without equity constraint is characterized by variables that are constant with respect to reputation, hence we impose $q'(\bar{e}) = 0$. The results are not sensitive to $\bar{e}$ provided it is sufficiently important to ensure that all meaningful regions are covered in the simulations of reputation states.

3 Results

3.1 Global dynamics

Table 1 reports the calibration of the two-intermediary model with constant equity share, which we refer to as our baseline model. In the simulations, the effects of introducing shadow banking activities into the HK structure are assessed by comparing this baseline model to a benchmark model with traditional banking only (i.e., a model with $\phi = 0$). Most of the baseline calibration is similar to the one in Dewachter and Wouters (2014). A depreciation rate δ of 10% is in line with the literature. In the AK production technology, an aggregate productivity constant of 0.35 corresponds to a GDP to capital share of about 1/3. Adjustment costs ($\kappa = 20$) are chosen to strike a balance between investment sensitivity to capital prices and volatility in the capital price. In line with average consumption (70%) and investment share (30%) in GDP, we fix the wage bill share in GDP to 60% ($l = 0.6$). Capital efficiency shock σ governs the exogenous uncertainty in the model. As in the one-intermediary model, it impacts volatilities of quantities and prices. In the two-intermediary version, it also influences the overall level of shadow bankers' leverage through the volatility of capital returns. If σ is too low, the leverage and hence the demand for capital assets from the shadow banking system is so important that traditional leverage achieves unreasonably low levels. A too high value for σ reduces substantially the demand for risky assets from shadow bankers. A σ of 5% delivers a good trade-off with reasonable levels of leverage and plausible volatilities in other variables. Appendix B reports robustness checks with different values for σ.

[14]This reflects the fact that when the Sharpe ratio is high, the value of entry is high. As in Dewachter and Wouters (2014), we calibrate $\underline{e}$ such that the entry Sharpe ratio is equal to 2 at the lower bound in the one-intermediary solution that is used as a benchmark (i.e. in a model where $\phi = 0$). It does not impose that the Sharpe ratio in the two-intermediary model is equal to 2 at the lower bound. A too low value for the entry Sharpe ratio leads to a more important value of $\underline{e}$ and might thus prevent the solution from covering meaningful reputation states of the constrained area. Higher values for the entry Sharpe ratio reduces the entry point $\underline{e}$ and increases the constrained area. This leads to more important amplification effects in global dynamics.

[15]A similar boundary condition can be found in the exchange-rate model of Krugman (1991).

The intermediation part is close to He and Krishnamurthy (2017). A risk aversion parameter of traditional intermediaries (m) equal to 2.5 implies Sharpe ratios between 0.24 and 2 in the benchmark model (with traditional banking only), with an average equal to 0.29. In the original HK framework, parameter λ entirely drives the level of traditional leverage when the equity constraint does not bind. In the two-intermediary model, λ is no longer the only determinant of traditional leverage in the unconstrained area. For comparison purposes we use the same value for the parameter (i.e. $\lambda = 0.5$) in both versions, keeping in mind that its implications have slightly changed compared to the original HK model. Exit rate η is fixed at 0.15 to ensure good model dynamics. This parameter has important impact on the drift of the reputation process and is adjusted to ensure reasonable simulated distribution.[16] The tightness of the leverage constraint z is fixed at 5 in order to ensure reasonable levels of shadow bankers' leverage. In order to select an appropriate value of the shadow banking share ϕ, we consider the following trade-off. Low values are associated with small shadow banking sector and small amplifications effects with respect to a model with traditional banking only. However, too important values lead to unrealistically small traditional leverage at high reputation states when the equity constraint does not bind.[17] Concerning households, a discount rate ρ of 3% is a standard value in the literature.

Figure 2 presents the global dynamics for the baseline case with a fixed equity share in the two-intermediary AK economy. The x-axis represents different values for the state variable e (scaled reputation of the financial sector as a whole). Areas where the equity constraint binds are the crisis states and lie on the left of the vertical black line. Blue dashed lines represent the benchmark model without shadow banking. Dynamics of the baseline model with shadow banking are plotted in red straight lines, while leverage and equity of shadow bankers are in red dashed-dotted lines. As in the benchmark model, the funding constraint generates non-linear dynamics in the two-intermediary model, with an important asymmetry between constrained and unconstrained areas. However, variables appear to be more volatile, and the asymmetry is amplified. Dynamics of the two-intermediary model are characterized by a more important endogenous volatility than in the benchmark, as reflected in higher volatility in capital returns σ_R, around and in the constrained area. The risk premium de-

[16]Typically, high values of the exit rate shift the simulated distribution of reputation states to the left, towards lower reputation states. A too important value would lead to unreasonably high frequency of constrained states (i.e. crisis states). In the two-intermediary model, we assume that the same exit rate applies to both traditional and shadow bankers.

[17]It is hard to rely on data to motivate the choice of ϕ. Few studies provide precise estimates of the importance of the shadow banking sector in the total of financial intermediation. A recent paper of Adrian et al. (2013) underscores the difficulties to collect and use information about the shadow banking system. Their monitoring exercise covers 25 countries and the euro area as a whole. They estimate the size of the shadow banking sector to be around 25% of total assets of the financial system, with a peak at 27% in 2007. In the U.S. this share achieved 44% in 2005, but decreased after the financial turmoil to about 35% in 2011. In the euro area, the share went from 31% in 2005 to 33% in 2011. Focusing on the U.S., Adrian and Ashcraft (2012) estimate traditional liabilities to be around 45% of the total liabilities of the financial system in the 2000 decade. In the euro area, Bakk-Simon et al. (2012) report non-banks intermediaries to have a share of about 28% of total assets of the financial sector. Though most of these estimates are based on the asset side of the balance sheet, they are broadly consistent with a share ϕ of 0.3 used in our baseline configuration. We also report some robustness analysis to the impact of changes in ϕ on global dynamics in Appendix B.

Table 1: Calibrated values for parameters (in per annum terms)

Parameters	Description	Baseline
Production		
σ	Capital efficiency shock	0.05
δ	Depreciation rate	0.1
κ	Inv. adjust. cost	20
A	Productivity constant	0.35
l	Wage share	0.6
Intermediation		
m	traditional int. risk aversion	2.5
λ	share of debt households	0.5
η	bankers exit rate	0.15
z	tightness of shadow bankers' funding constraint	5
ϕ	Shadow B, equity share	0.3
$\bar{SR}$	Entry Sharpe ratio	2
Households		
ρ	discount rate	0.03
θ	households risk aversion	2

pends on the leverage of traditional intermediaries. It is lower than the benchmark in most of the unconstrained area, as traditional leverage is now reduced due to the presence of a shadow banking sector that holds a part of the assets. However, when the capital constraint binds, volatility achieves higher levels than in a model without shadow banking. Therefore, higher risk premium are required by traditional intermediaries for a same level of leverage. Moreover, traditional leverage achieves higher levels than in the benchmark, and this adds an extra pressure on the risk premium. This evolution leads to more sensitive asset prices, with higher values relative to the benchmark at high reputation states and lower levels in the constrained area. Since investment policy is driven by the asset price, the dynamics of the investment rate also display more important non-linearities. The resource constraint of this simple AK economy implies a counter-cyclical behavior for the consumption-to-capital variable. The interest rate is highly sensitive to the reputation state. However the effects on interest rates are likely to be over-stated in both the baseline and the benchmark model. The reason of this overstatement comes from the fact that the models ascribe all movements in interest rates to shocks to intermediaries' reputation. In practice, interest rates are also affected by changes in the demand for bonds from sectors that are not influenced by the intermediation constraint.

The introduction of a shadow banking sector in the HK model has two separate effects on global dynamics. A first effect appears as funds are diverted from traditional intermediaries and allocated to the shadow banking sector. We call this effect the *equity effect*. For a same amount of equity provided by households to the financial sector, the traditional intermediary receives less equity. Ceteris paribus, this reduction in equity implies that the traditional intermediary has to rely on more leverage to fund its assets than in a model without shadow banking. This in turn generates higher risk premium and puts a downward pressure on asset prices. However, this picture is incomplete and a second effect should also be considered. This second effect emanates from the positive demand of the shadow banking sector for the risky asset. For a same level of asset market value, the traditional intermediary is required to hold a smaller portion of the risky asset. This suggests a downward effect on traditional leverage. This effect is denoted as the *asset effect*.

The total outcome of these two effects depends on the evolution of the relative leverage of the two sectors (i.e. $\alpha_t^{SB}/\alpha_t^{TI}$), which plays an important role in our model. The outcomes are illustrated in Figure 2. The dynamics in the two-intermediary model cover both situations where the equity effect prevails, and situations where the asset effect is more important. In economic upturns, shadow banking activities expand as a result of easy access to wholesale funding and regulatory arbitrage incentives to extend off-balance-sheet operations. Consequently, the leverage of the shadow banking sector is high. In our model, this situation corresponds to regions with low volatility in the unconstrained area, associated with a shadow banking leverage that exceeds the one of traditional banks (see reputation states higher than 2 in Figure 2). In this case, shadow banks' holding of capital assets is so important that the asset effect prevails over the equity effect. The expansion of shadow banking activities helps traditional intermediaries to reduce risky asset exposures. Traditional leverage in the two-intermediary model is thus lower than its equivalent in a model without shadow banking. Risk premia are then lower and asset prices are higher than in the benchmark case. In contrast, crisis periods, on the left of the constraint boundary, are characterized by higher uncertainty and a contraction in wholesale funding that leads shadow banking institutions to reduce their leverage. At the beginning of the constrained area, volatility jumps and conduces shadow bankers to implementing important deleveraging. The resulting re-intermediation of assets increases traditional intermediaries' exposures. Assets shed by the shadow banking sector have to be absorbed by the traditional sector and this leads to an increase in traditional leverage which becomes more important than in the benchmark model. Hence, relative leverage inverts (i.e. $\alpha_t^{SB}/\alpha_t^{TI}$ becomes < 1) and continues his downward spiral in the constrained area. The equity effect now prevails over the asset effect. As a consequence, risk premia are raised and apply a downward pressure on asset prices.

Changes in the relative importance of the two shadow banking effects caused by changes in relative leverage makes traditional leverage more volatile than it would be in a model without shadow banking. Higher volatility in risk premia and asset prices emanates from the relative leverage movements. More sensitive asset prices in turn increase endogenous volatility and apply an additional downward pressure on relative leverage and an upward pressure on the risk premium, with a loop effect on asset prices. Therefore the two-intermediary model generates higher endogenous risk than the benchmark model.[18] As capital price affects the

[18]In Figure 2, a counter-intuitive effect appears at the left extreme of the constrained area. From right-to-left, volatility in capital returns suddenly declines and translates into corresponding jumps in the risk premium and shadow banking leverage. This effect is also present in dynamics of the different models reported in He and Krishnamurthy (2017). This is a consequence of the boundary condition $q' = 0$ used at

equilibrium levels of consumption and investment, the amplification propagates to the real side of the economy.

The introduction of shadow banking in the constrained area has significant consequences on the anticipation of crisis states. Financial frictions effects appear earlier and are more significant in the model with shadow banking. As can be seen from Figure 2, endogenous volatility and risk premia starts to increase earlier when we move from the right to the left on the x-axis, and achieve higher levels in unconstrained reputation states close to the constraint boundary. The expectation of a more severe disruption than in a model without shadow banking is reflected in asset valuation by higher risk premia in the two-intermediary model. As explained above, traditional leverage is not longer constant in the unconstrained area compared to the HK model. Indeed, the reduction in shadow banking leverage due to higher volatility applies an upward pressure on traditional leverage, which in turn raises risk premia. Consequently, risk premia increase more importantly than they would due to movements in endogenous volatility alone, and the effects of an anticipation of crisis states are amplified.

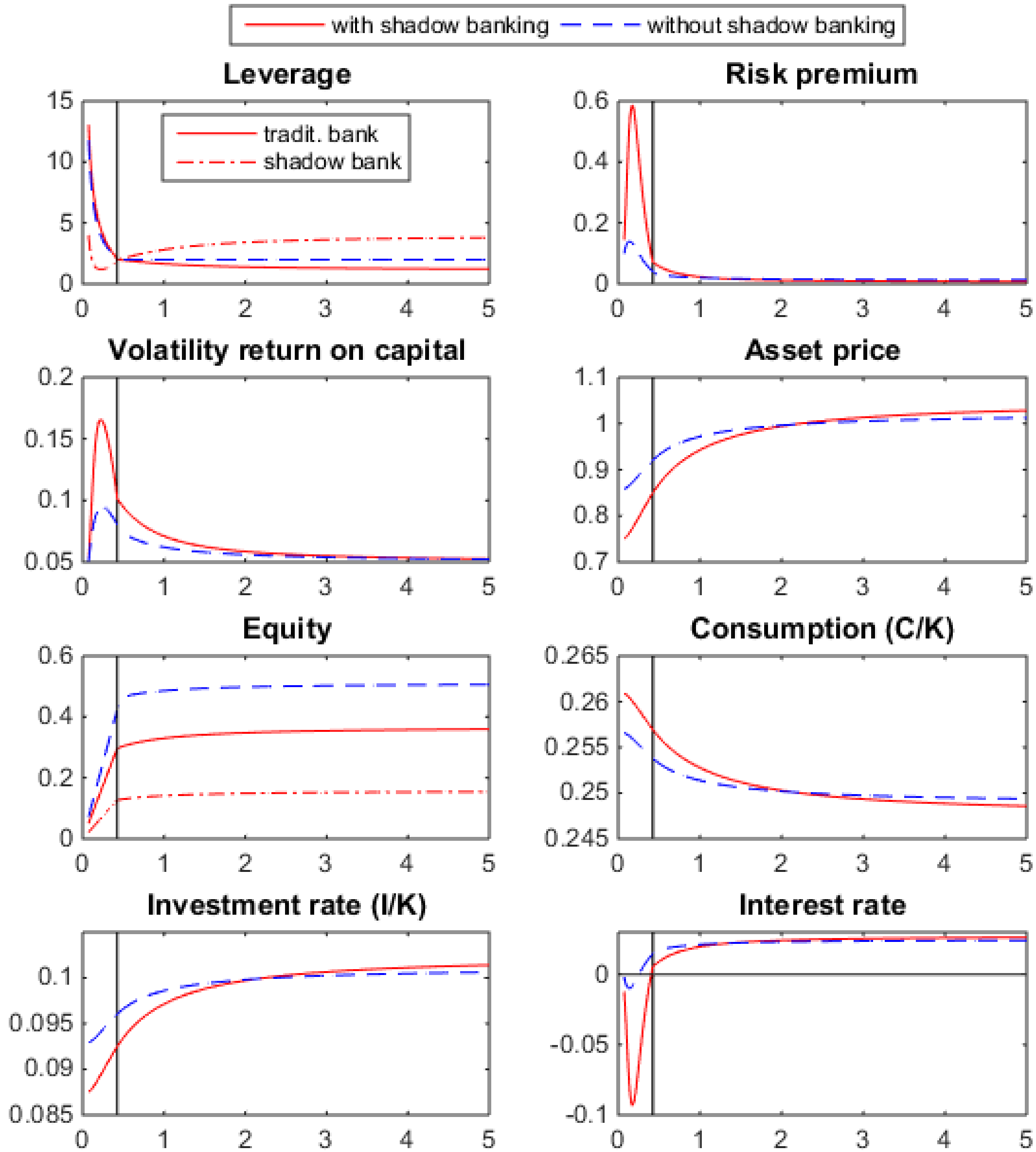
Through its effect on shadow banking leverage, exogenous volatility σ has an influence on the relative importance of the amplification of dynamics and anticipation effects. According to the leverage rule, low values for exogenous volatility shifts the levels of shadow bankers' leverage upward. As a result, shadow bankers' leverage crosses later (i.e. at lower reputation states) with traditional leverage and relative leverage inverts later. Hence, the equity effect also becomes prevalent later, and the amplification of dynamics and anticipation effects are less important. In Appendix B, Figures 15 and 16 show global dynamics in models with low values for the exogenous volatility (respectively $\sigma = 0.04$ and $\sigma = 0.03$). Though anticipation effects are reduced, we continue to observe important amplifications in global dynamics. For our calibration, a too low value for the exogenous volatility leads to a demand for risky asset from shadow bankers so important that traditional leverage achieves unrealistically low levels of leverage. This effect is illustrated in Figure 16 where α^{TI} is close to 0 in the unconstrained area.

In Appendix B, we also present robustness analyses to different values of the equity share ϕ. The global dynamics for a small and a large equity share of the shadow banking sector, i.e. for respectively $\phi = 0.2$ and $\phi = 0.5$, are shown in Figures 17 and 18. We obtain similar amplified dynamics than in the results of the baseline case with $\phi = 0.3$. The larger the

the lower bound. This condition cancels endogenous volatility at the lower bound and forces the convergence of volatility in capital returns to exogenous volatility σ. The difference with the original HK model is the transmission of this convergence to the shadow bankers' leverage through their leverage rule. The higher shadow banking leverage at lower bound cancels some of the amplification effect in global dynamics (i.e. it reduces somewhat the magnitude of the equity effect). A solution would be to deviate from the leverage rule at some point in the constrained area, for instance by imposing a constant minimum leverage for the shadow banking sector. This helps to avoid the counter-intuitive increase in shadow bankers' leverage and generates a higher amplification of dynamics. It however introduces some arbitrary choices concerning the shadow banking behavior that we prefer to avoid. Moreover this issue remains marginal as it does not prevent the model from generating interesting amplification effects in global dynamics.

equity share ϕ, the more important the amplification effects. Indeed, higher ϕ increases the sensitivity of traditional leverage to the leverage of shadow bankers. It therefore increases the magnitude of equity and asset effects, and leads to more volatile variables and more important endogenous risk. Consequently, more important anticipation effects are also observed. Similar to the effect of a too low value for exogenous volatility, a too important equity share of the shadow banking intermediaries raises their asset demand at levels that are so high that the residual demand absorbed by the traditional sector becomes unreasonably low and traditional leverage approaches 0, as shown in Figure 18.

Figure 2: Global dynamics, framework with aggregate financial reputation and constant equity allocation. Red lines correspond to the baseline calibration for the two-intermediary model. Dashed blue lines are for the model without shadow banking. Left-hand side of the vertical line indicates constrained area.



3.2 Simulation and impulse-response analysis

The amplification effects due to the introduction of a shadow banking sector can be further illustrated by means of simulations. We simulate the economy, yearly, for 5,000 years. The first 500 data points are dropped to randomize the initial point, while the next 4,500 yearly data are stored in order to compute sample statistics. We repeat the simulation 1,000 times and report the sample averages. Figure 3 shows the simulated distributions of reputation for the baseline model with constant equity shares and the benchmark model. The distribution of financial reputation states simulated for the baseline model concentrates in a smaller range of values than for the model without shadow banking. The 25^{th}, 50^{th} and 95^{th} percentiles of the distribution of reputation states equal respectively [0.65, 0.88, 1.66] against [0.57, 0.85, 3.02] in the benchmark model. This indicates that extreme reputation states are less frequent in the two-intermediary model. Moreover, the probability to visit states higher than 3, which corresponds to highly favorable economic activity, is close to 0, while it has a non-negligible probability in the one-intermediary model. On the other hand, crisis states are also less frequent in the baseline model: the unconditional probability of constrained states is around 5.7% in the two-intermediary model, against about 14.6% in the model without shadow banking. This significant difference is explained by the evolution of the risk premium. As observed in the dynamics presented above, the crisis states of the baseline model are associated with higher risk premia than in the benchmark model. As intermediaries hold leveraged position in the risky asset, the capital base (and hence reputation) of the financial sector is expected to re-build much faster and the economy is expected to spend less time in crisis states.[19] At the other extreme, much lower risk premia are associated with high reputation states in the baseline economy (i.e., states that exceed a value of 2), and this tends to push reputation towards lower levels at a higher force than in the model without shadow banking.[20]

Though crisis states are less frequent in the baseline model, the simulated distribution concentrates on reputation states that correspond to more severe developments in financial and macro variables. This result is illustrated in simulated cumulative distribution functions reported in Figure 4. Higher values for the risk premium and the volatility in capital returns, as well as more depressed asset prices are more likely in the baseline model. The volatility in variables is also more pronounced in the baseline case. Table 2 reports sample average of simulated moments for financial and real variables. Following He and Krishnamurthy (2017), distressed periods are defined as the 33% highest realizations of the Sharpe Ratio. Therefore, the distressed states are not limited to the states associated with the constrained region, and also cover the distressed area that emanates from anticipation effects, as underlined in the global dynamics analysis presented earlier. For comparison purposes, U.S. data reported in He and Krishnamurthy (2017) are presented in the first column of the table. It should be emphasized that the objective of this analysis is not to provide the best fit of the data, but to assess if the two-intermediary model is able to generate quantitatively more important volatility during distressed and non-distressed periods. Therefore, the goodness-of-fit is not the primary criterion in the calibration of the two models. Though the benchmark and the baseline model do not perform optimally in replicating the empirical moments, they are able to generate important asymmetries in the simulated series. Simulations underline the important amplification effects in variables due to the introduction of the shadow banking intermediary. Variables are more volatile in both distressed and non-distressed periods in

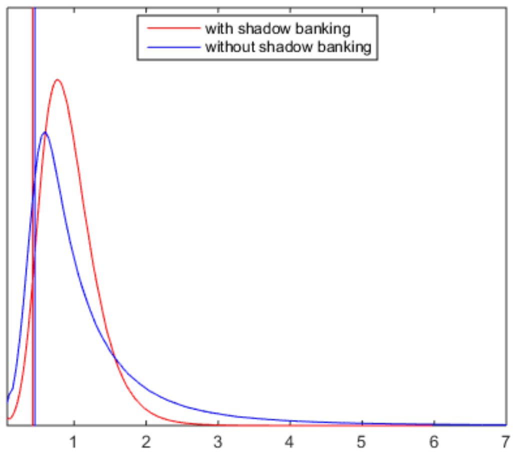
[19] In stochastic terms, the drift of the reputation process is higher for the baseline model in the crisis region.

[20] For extremely high reputation states, the drift in the reputation process is lower in the baseline model than its equivalent in the benchmark model.

the two-intermediary model. More volatile asset prices are particularly interesting, as the endogenous propagation to the real variables goes through q in the model. Dewachter and Wouters (2014) report small quantitative effects of the propagation of shocks from financial asset prices to real variables in an AK version of the HK model. The results reported in Table 2 suggest that movements in the financial variables simulated from the two-intermediary model have the potential to generate significant non-linearities in real variables. However, in this simple AK set-up, the modeling of the real side is relatively simple, and the propagation of amplification effects to real variables remains limited. Investment volatility is more important than in the model without shadow banking, even though the non-linearities are less marked than in asset prices.

The effects of the additional intermediary with a pro-cyclical leverage on the amplification of exogenous shock is further illustrated in the impulse-response functions reported in Figure 5. We consider the introduction of an identical shock on capital in the two models. We simulate monthly data and study the effect of a -5% shock in σdZ_t, introduced in the first month of the simulation (i.e. shocks are then set to 0 after the initial shock). The construction and interpretation of impulse-response functions is slightly different from the usual impulse response computed for traditional linear models with a non-stochastic steady state. Inspired by the generalized approach in Koop et al. (1996), we compute impulse responses as the (log) difference between a simulated path of variables subject to the initial shock and the path without any shock (i.e. only driven by the endogenous drift of the model, which is different between the baseline case and the benchmark model). Using this method to compute impulse responses, we obtain the marginal effect of the shock on the mean path of the variables (i.e. the path where all shocks are set to their mean value of 0). The impulse-responses of reputation, investment and Sharpe ratio in the 5 years that follow the initial shock are reported in Figure 5. We consider the introduction of the capital shock at three different initial states: one at the constrained boundary (distress state), one at the median value of constrained reputation states (crisis state), and the other at the median value of the simulated distribution (normal state). In both models, the negative shock has more important consequences when it is introduced in a crisis state. However, the effect of the shock on impact is always more important in the two-intermediary model than in the model without shadow banking. For instance, in the baseline model, the -5% shock in capital is amplified to an initial reduction of about -8% in investment in distressed times and about -9% in crisis times, against respectively -6.8% and -7% in the benchmark model. In both models, the importance of the initial impact in a crisis state dies out through time. Indeed, intermediaries re-build their capital and pave the way to recovery with high risk premia in crisis times. In the two-intermediary model, risk premia are more important than in the benchmark for the same exit rate. As a result, higher risk premia accrue to the intermediaries and they recover faster. Impulse responses that die out faster in the two-intermediary model illustrate this result. In normal states, a same initial shock propagates more importantly in the model with shadow banking. The outcomes of the impulse-response analysis have important policy implications. Shadow banking activities worsen financial frictions and reinforce the endogenous mechanism of the equity constraint. Ignoring the heterogeneity in the financial sector therefore leads to an underestimation of financial and macroeconomic responses to exogenous shocks. The

Figure 3: Simulated distribution of reputation states in the baseline model (red) and in the model without shadow banking (blue). Constrained states are located on the left of the respective vertical lines.



consideration of shadow banking features thus offers a more comprehensive view on financial frictions at stake and helps to reduce biases in the policy analysis of the responses of financial and macroeconomic variables to shocks.

Figure 4: Simulated cumulative distribution functions for risk premium, volatility and asset price in the baseline model (solid red) and in the model without shadow banking (dashed blue).

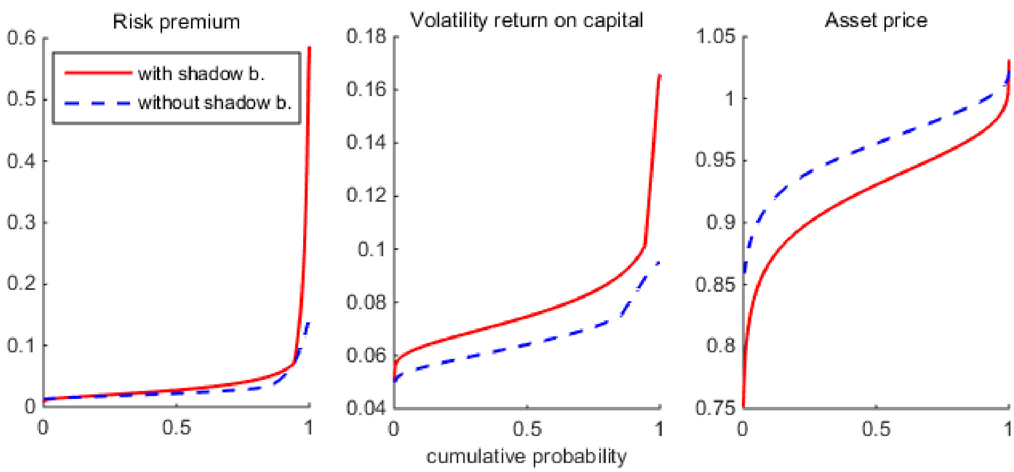
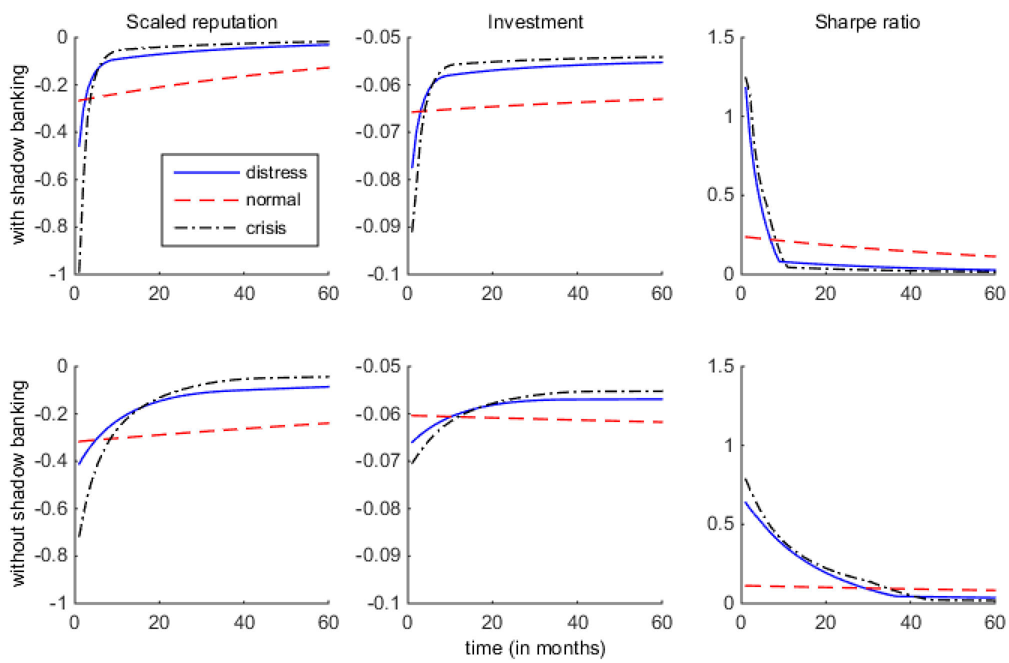


Table 2: Summary statistics of simulated series. Statistics reported under the column. U.S. data reported in the first column are from He and Krishnamurthy (2017).

	Data		**One intermediary**			**Two intermediaries (baseline)**		
	Distressed	Non distressed	Distressed	Non distressed	Overall	Distressed	Non distressed	Overall
Vol(Eq)	31.48	17.54	54.17	11.66	32.81	27.32	18.11	21.83
Vol(SR)	60.14	12.72	33.91	3.40	23.72	56.11	6.12	37.93
Vol(ER)			2.89	0.35	2.15	9.19	0.61	5.95
Vol(q)			2.98	1.39	2.11	4.33	3.59	4.04
Vol(I/K)			1.43	0.67	1.02	2.02	1.63	1.86
Vol(I)	8.05	6.61	5.48	5.24	5.81	5.71	5.58	6.42
Vol(C/K)			0.50	0.24	0.36	0.65	0.52	0.60
Vol(C)	1.71	1.28	3.81	4.46	4.72	3.52	4.14	4.58
Vol(K)			4.23	4.67	5.00	4.03	4.46	5.00
cov(Eq,I)	1.31	0.07	1.75	0.18	0.65	0.69	0.32	0.51
cov(Eq,C)	0.25	0.03	1.01	0.11	0.36	0.28	0.01	0.15
cov(Eq,SR)	-6.86	-0.14	-5.05	0.06	-1.35	-5.98	0.01	-2.43
cov(Eq,Q)			1.20	0.11	0.46	0.74	0.53	0.63

Figure 5: Impulse-response functions for the effect of a -5% capital shock on scaled reputation, investment and the Sharpe ratio.



4 Policies

4.1 Changes in the tightness of shadow bankers' leverage constraint

The results of previous sections underline the amplification effects that emanate from the introduction of a shadow banking sector. Figure 6 shows the overall dynamics for different values of the tightness coefficient z of the leverage constraint of shadow bankers. An increase in z corresponds to tighter wholesale funding conditions and leads to a contraction in shadow bankers' leverage. Consequently, traditional leverage becomes higher and raises the risk premium everywhere. This in turn corresponds to lower asset prices, and hence an equity constraint that binds "later", i.e. at lower reputation states. The crisis boundary is therefore lower the tighter the coefficient z. Higher risk premia raise the profitability of the financial sector, thereby increasing reputation states: a tighter coefficient shifts the mass of the simulated reputation distribution to the right, as shown in Figure 7.

These findings are consistent with the risk-return trade-off in Adrian and Boyarchenko (2015b): tighter intermediary funding requirements tend to shift systemic risks downward, at the cost of increased risk-pricing at every reputation states. Increased risk premium are associated with lower asset prices and lower investment rates. In the stochastic AK economy, the level of capital is described by $dK_t = K_t(i_t - \delta)dt + \sigma K_t dZ_t$. The ex-ante expected growth rate is $E\left(\frac{dK}{K}\right) = \left(E(i_t) - \delta\right) dt = \left(\frac{E(q_t)-1}{\kappa}\right) dt$. Consequently, the expected growth rate of output is $E\left(\frac{dY}{Y}\right) = A\left(\frac{E(q_t)-1}{\kappa}\right) dt$. Lower expected asset prices, as the ones obtained in the economy with high z, reduce the expected investment rate, and lead to a decline in the expected growth rate. In order to deepen the investigation of the impact of a change in the tightness coefficient of shadow bankers' leverage on capital and output, we generate 5,000 simulated paths of the level of the capital stock (with $K_0 = 1$) over 100 years. The simulation is implemented for the baseline economy (with $z = 5$) and for an economy with a tight coefficient $z = 7$. The same 5,000 x 100 series of random shocks are applied to both economies. For each of the 100 simulated years, Figure 8 shows the medians and quantiles of differences between the simulated levels of the capital stock of the baseline economy and the simulated capital stocks of an economy with a tight coefficient $z = 7$. For the first 28 years, differences in simulated capital levels are positive for every of the 5,000 simulated paths, indicating that a tighter z has a negative impact on economic growth. For longer periods, medians are always positive, and the distribution of differences is concentrated on positive values. After 28 years, a minority of paths lead to negative differences between the simulated levels of the capital stock of the baseline economy and the simulated capital stocks of an economy under a tight policy regime ($z = 7$). This is illustrated by negative quantiles in Figure 8. The reason for these negative differences comes from the fact that a high level of capital at t amplifies the volatility component in $dK_t = K_t(i_t - \delta)dt + \sigma K_t dZ_t$. Therefore, the simulated capital stock in the baseline economy might become so important compared to the one simulated for the economy with a tight z, that this amplification effect in the volatility component might on a few occasions more than offset the drift effect described above. These negative areas reflect a reduction in the probabilities of extremely large future costs (tail risk) under the tighter policy regime.[21] This is however balanced by lower expected growth as underlined by the distribution that mainly spans positive areas. This outcome attests to a negative impact of a tighter coefficient in the shadow bankers' leverage constraint on

[21] Note that in a simulation over an extended period of 1,000 years, the lowest quantile of differences in capital levels does not continue to decrease as suggested in Figure 8. It stabilizes between 200 and 300 years and then returns slowly to positive areas. The reduction in tail risk from a tighter policy regime slowly fades away.

expected economic growth. The results of this simulation underline that the increase of the tightness coefficient on shadow bankers' leverage involves a trade-off between stabilizing the economy and the growth of economic activity.

Figure 6: Changes in the tightness coefficient z of the leverage constraint of shadow bankers. Overall dynamics. The constrained area, represented by a vertical line, corresponds to the baseline configuration.

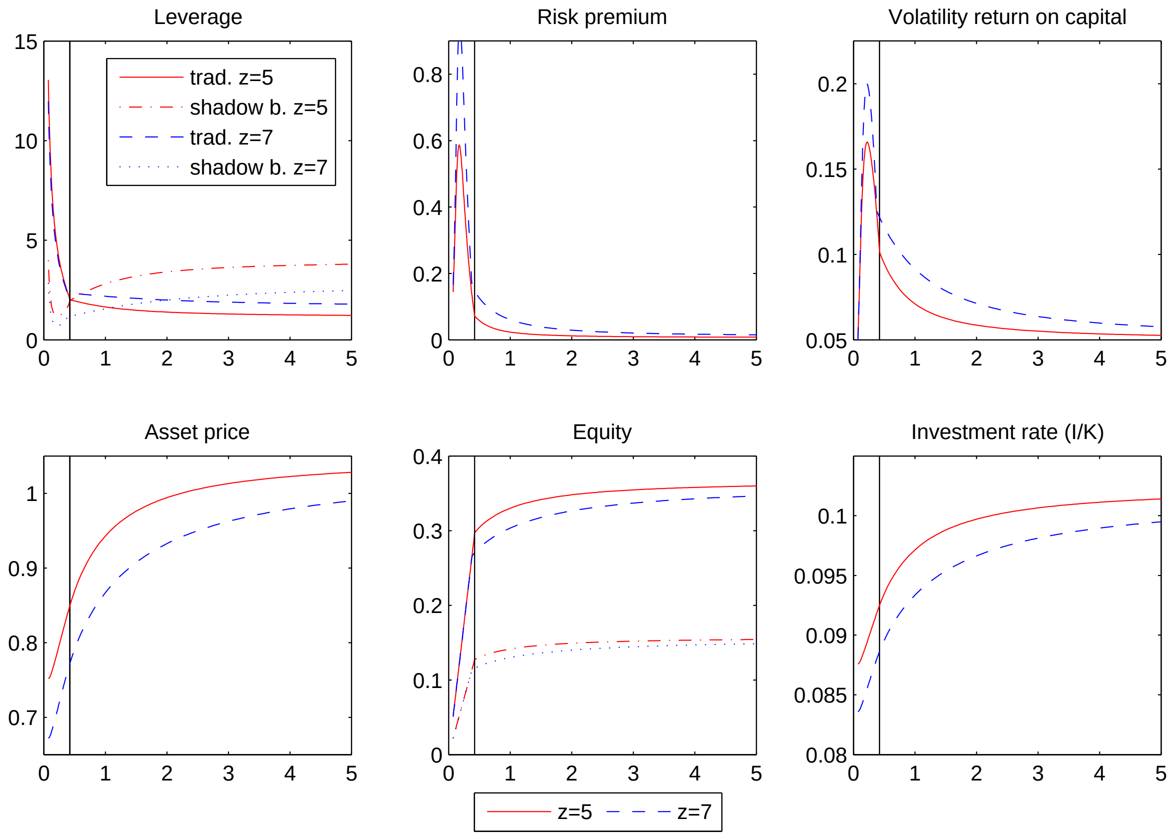


Figure 7: Changes in the tightness coefficient z of the leverage constraint of shadow bankers. Simulated reputation distributions.

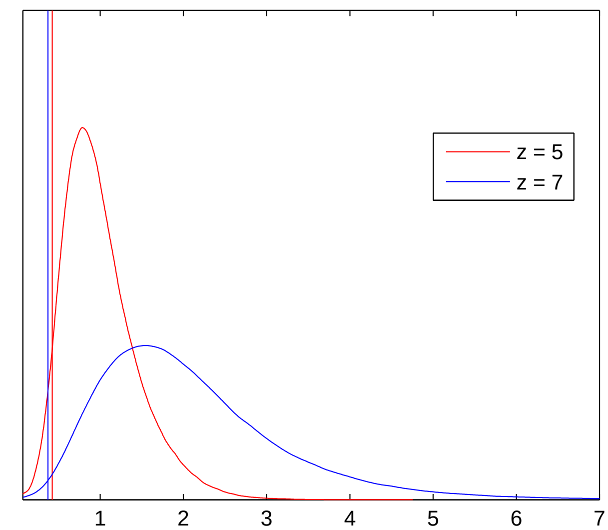
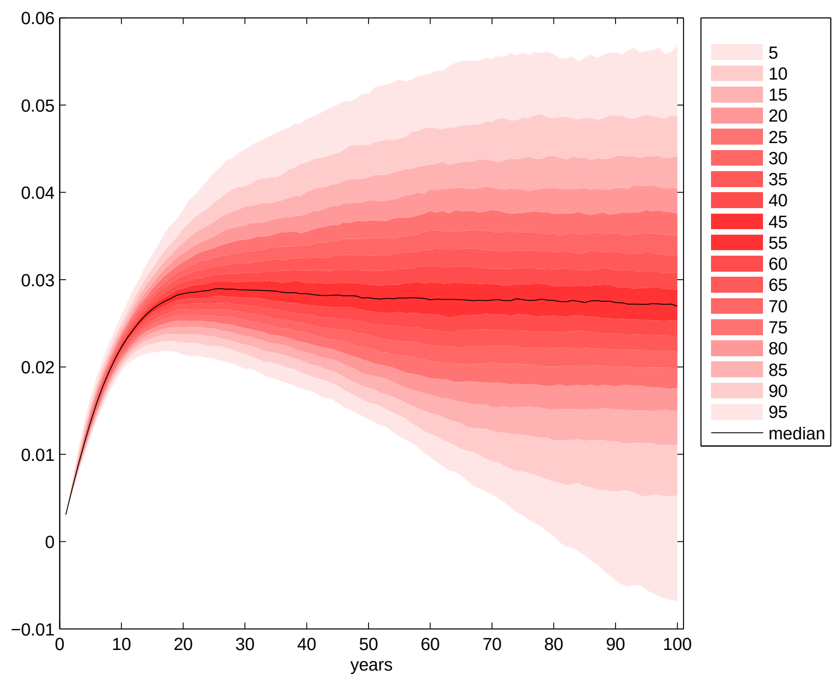


Figure 8: Medians and quantiles of differences between the simulated levels of the capital stock of the baseline economy and the simulated capital stocks of an economy with a tight coefficient (i.e., $K_t^{z=5} - K_t^{z=7}$), for each t of the 100 simulated years.



4.2 Unanticipated versus anticipated policies

In a second experiment, we analyze the differences between unanticipated and anticipated policies. The anticipation of expansionary policies in times of crisis might lead to an increase in risk-taking (or lower valuation of risks) due to a lower uncertainty in normal times. This is likely to destabilize the economy and to affect the efficiency of policies. In the two-intermediary economy, these effects are significant. In the unconstrained states of the economy, the anticipation of expansionary policies activated in the constrained area leads to lower uncertainty (i.e. it corresponds to an anticipation of less severe developments in crisis times). As the risk-taking behavior of shadow bankers is directly affected by the uncertainty in the economy, they raise leverage and add an extra downward pressure on traditional leverage and the risk premium. The excess risk-taking effects of anticipated policy compared to a model without any policy are then likely to be more important in a model with shadow banking.

We implement a simple policy experiment in order to illustrate these effects. The experiment consists in introducing direct asset purchases from the government. As suggested by He and Krishnamurthy (2013), such an experiment relates to the asset purchase program conducted by the Federal Reserve and government-sponsored enterprises during the 2007 subprime crisis.[22] In our framework, the government intervention in the capital market should partly compensate the decrease in demand from shadow bankers. As a result, the traditional sector has to absorb less assets, and traditional leverage (and risk premium) do not increase as much as without any government intervention. The policy is implemented as follows: we assume that the government buys a proportion s of the risky asset if intermediaries become constrained, and that this purchase is financed by issuing sq_tK_t of risk-free debt. The cash-flow from this transaction is $sq_tK_t(dR_t - r_tdt)$, and the government raises lump-sum taxes from (or rebates to) the households to balance this cash-flow. This last assumption enables us to introduce the policy in a very tractable way: it is an exogenous diversion of households' wealth to the risky capital market, and does not modify the household optimal decisions.[23] In the model, the expression of traditional leverage is affected by a term in $(1-s)$, as intermediaries have to hold less capital than before the policy implementation:

$$\alpha_t^{TI} = \frac{[(1-\phi)\alpha_t^H(1-\lambda)]^{-1}\mathbf{(1\text{-}s)}\left(z(1-\frac{q_t'}{q_t}e_t)\sigma + \frac{q_t'}{q_t}e_t\phi\right) - \frac{\phi}{1-\phi}}{z(1-\frac{q_t'}{q_t}e_t)\sigma + (1-m)\phi\frac{q_t'}{q_t}e_t} \tag{19}$$

In the simulation of anticipated policies, equation (19) is only used in the constrained area

[22]See He and Krishnamurthy (2013): The Federal Reserve and GSEs intervened for an amount of respectively \$ 1.25tn and \$550bn in the purchase of distressed assets such as mortgage-backed securities from mid-2007 to mid-2009.

[23]The tractability of this simple experiment, without explicit modeling of the government, motivated its use in order to show the effects of anticipated and unanticipated policy. To the extent that we do not try to determine an optimal government policy, we find that this simple treatment of government intervention is sufficient. However, a more rigorous welfare analysis (involving a more rigorous modeling of the government or monetary authorities) is required in order to assess the welfare cost and benefits of reducing the potential instability (and additional risk-taking) due to the introduction of the shadow banking sector. Our objective is to present evidence of this instability, and not to measure its consequences in terms of welfare.

(or equivalently, $s = 0$ in unconstrained states). Similar to the anticipation of constrained states, anticipation effects of the government policy results from the global dynamics solution. Figure 9 shows the differences in the dynamics of a two-intermediary model without any policy (i.e. our baseline case) and a model with anticipated government asset purchase of a portion $s = 0.10$ of the capital assets in the constrained area.[24] Dynamics in the risk premium and the asset price are represented in respectively the left plot and the right plot. Anticipation effects can be observed in the unconstrained area: risk premia are lower than in the baseline model. As market participants anticipate that the government would step in to alleviate tensions, the perception of risks is affected. A lower risk valuation corresponds to higher asset prices, and the resulting excess risk-taking makes the constraint binding at higher reputation states. This implies that the government should intervene earlier (i.e. at a higher reputation state) than in the baseline model, due to more important risk-taking generated by policy anticipations. Moreover, lower required returns on assets make it harder for intermediaries to rebuild reputation. Consequently, the economy spend more time in sensitive states. The simulated distributions with and without policies for the baseline model, reported in the left subplot of Figure 10, support this argument. Distressed states (i.e., states around and on the right of the binding state) are more frequent, and the probability of constrained states is more important in a model with an anticipated policy. Moreover, these negative outcomes due to the anticipation of the policy are more important in the model with heterogeneous intermediaries than in the model with one representative sector. The right subplot of Figure 10 underlines that the difference in distribution is less significant in the model without shadow banking. As an example, the probability of constrained states increases by about 59% in the benchmark model, from 14.6% in a model without policy to 23.2% in the model with anticipated policy. This increase in the probability of constrained states is more pronounced in the two-intermediary model, and amounts to 212% (from 5.7% to 17.8%), which attests to the extra loop due to the presence of shadow banking explained earlier.

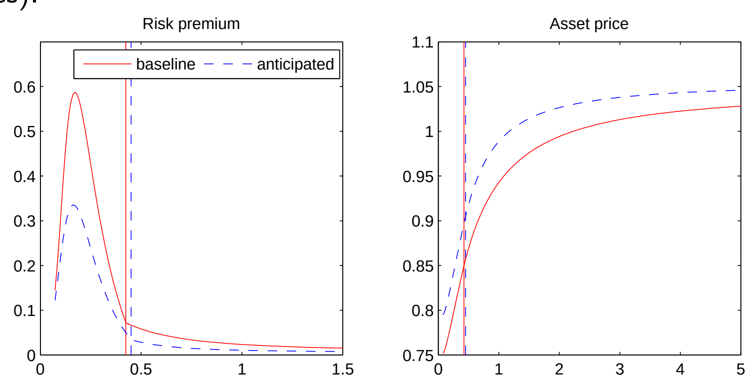
We now examine differences and loss in efficiency of an anticipated policy compared to an unanticipated policy in the baseline and the benchmark models. The unanticipated policy is simulated as follows. We compute two equilibria: one without the policy (i.e. the equilibrium described in the previous sections), and one with the policy (i.e. where equation 19 is applied both in and out the constrained area). As long as the economy remains in the unconstrained area, no policy is activated nor anticipated, and the dynamics are described by the model without any policy. Once the economy achieves a constrained state, the government announces the direct asset purchase policy and this action causes asset prices to jump instantaneously. The policy is activated until the unconstrained states are recovered. Impulse-responses when the policy is activated are computed as the (log) difference between a simulated path subject to a shock in the equilibrium with policy, and the path without any shocks in the no-policy economy.

Differences in impulse-response functions of investment and the Sharpe ratio are shown

[24]Results are robust to different specifications of the portion of government purchases.

in Figure 11. They are computed at the constrained boundary. Both anticipated and unanticipated policies bring improvement in the amplified reaction to an exogenous shock. As expected, the unanticipated policy is the most efficient government intervention.[25] Bottom plots in Figure 11 show a comparison with the impulse-response differences between anticipated and unanticipated policies in the one-intermediary (dashed lines) and the two-intermediary model (straight lines). The loss in efficiency of the anticipated policy is quantitatively more important in the model with a shadow banking sector. This reflects the additional effect from the risk-taking behavior of shadow bankers. The lower uncertainty generated by policy anticipations leads shadow bankers to rely on a higher leverage. This in turn adds an extra downward pressure on traditional leverage and the risk premium is further reduced in normal times.

Figure 9: Government policy in the baseline model (two intermediaries): direct asset purchase of a portion of $s = 0.10$ of capital assets. Dynamics of the risk premium and the asset price q in the without-policy model (i.e. the baseline model in solid red) and in the anticipated policy model (in dashed blue lines).



[25]Note that the impulse-responses of unanticipated policy should be ignored beyond period 30 as the economy goes back to the unconstrained area.

Figure 10: Government policy in the baseline model: direct asset purchase. Simulated distributions in the baseline model (with shadow banking) and the benchmark model (without shadow banking) when anticipated policy are activated.

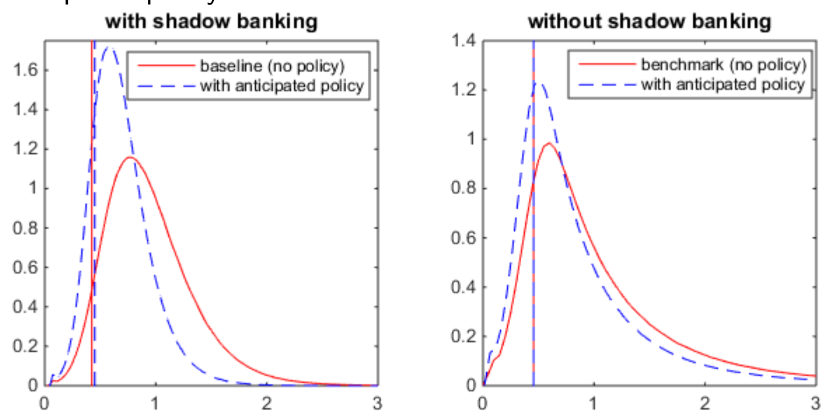
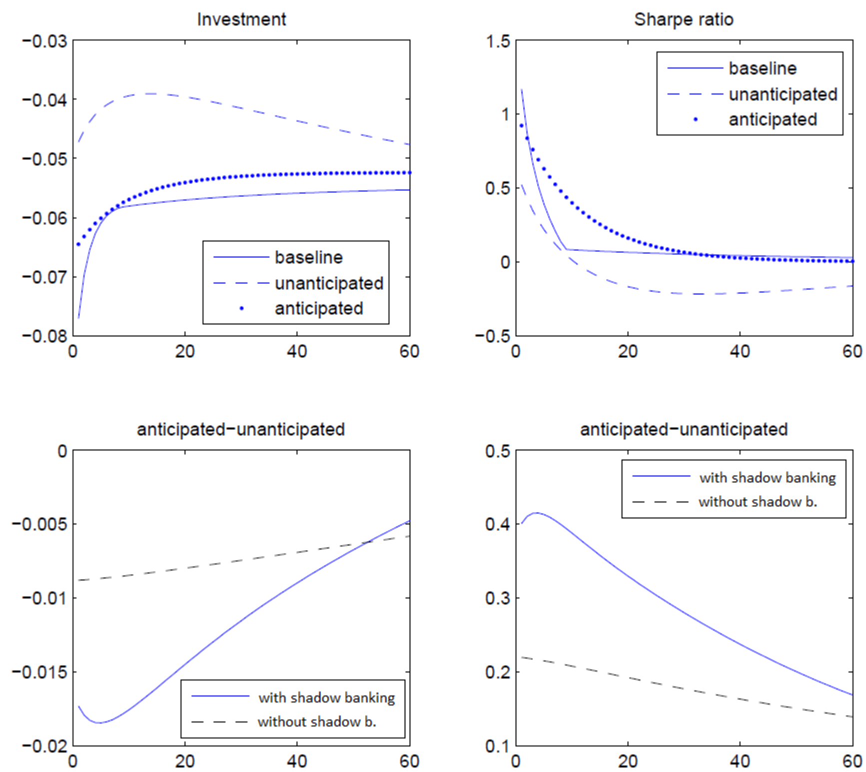


Figure 11: Government policy: direct asset purchase. Impulse-response functions for the effect of a -5% capital shock on investment and the Sharpe ratio.



5 Endogenous equity portfolio choices

In the baseline model with shadow banking, we assumed a constant equity allocation between the two intermediaries. In this section, we consider two simple ways to introduce endogenous (i.e. state-dependent) equity allocation. As a first alternative, we propose a general framework that nests the constant solution, and where equity allocation is the result of optimal portfolio choices. A second alternative considers varying allocation according to the relative (expected) performance of the two intermediaries. The objective is to add more flexibility in the modeling of the relative size of traditional and banking activities and to test the potential propagation that could emanate from different equity allocation processes. It also enables us to verify the robustness of the results under different assumptions.

5.1 Optimal equity portfolio

We assume that equity households save in a fund the amount they are willing to invest in equity. This fund is a financial vehicle designed to aggregate equity funds from household and allocate them to financial intermediaries. It is assumed to follow a mean-variance type of strategy:

$$\max_{\phi_t} E_t\left(\frac{d\tilde{W}_t}{\tilde{W}_t}\right) - \frac{\gamma}{2}Var_t\left(\frac{d\tilde{W}_t}{\tilde{W}_t}\right) - \frac{\psi}{2}\left(\phi_t - \phi^*\right)^2 \tag{20}$$

where $\tilde{W}_t$ designates households' wealth allocated to equity (i.e., $\tilde{W}_t = \alpha_t^H(1-\lambda)W_t$), γ is the risk aversion parameter of the funds, and reflects risk aversion of equity households. The last term $\frac{\psi}{2}\left(\phi_t - \phi^*\right)^2$ introduces some adjustment costs with respect to a target weight ϕ^*. Intuitively, the adjustment costs can be interpreted as a way to reflect preferences for a specific share ϕ^*. We assume that these costs are non-pecuniary in order to keep the balanced relationship $q_tK_t = W_t$ and ease the solution of the model. The optimal condition of portfolio problem (20) is

$$\phi_t = \frac{(\alpha_t^{SB} - \alpha_t^{TI})\alpha_t^{TI}(\sigma_t^R)^2(m-\gamma) + \psi\phi^*}{\gamma(\alpha_t^{SB} - \alpha_t^{TI})^2(\sigma_t^R)^2 + \psi} \tag{21}$$

A constant equity weight can be replicated by picking an extremely high value for the adjustment cost parameter ψ. It generates an optimal allocation that does not significantly deviate from the constant target ϕ^*. Low values for the adjustment costs allow for varying equity allocations. As both intermediaries are exposed to the same risky asset, it is intuitive to find that the optimal allocation depends on the relative degree of exposure, i.e. the relative difference in leverage $(\alpha_t^{SB} - \alpha_t^{TI})$. The solution is also strongly related to differences in risk aversion coefficients $m - \gamma$. Intuitively, if equity households are less risk averse than traditional intermediaries ($m > \gamma$), they are willing to allocate above target to the intermediary who is more exposed to the risky asset (i.e. uses more leverage). In contrast, more risk-averse households would decrease their allocation with respect to target in the intermediary with higher exposure. Moreover, when differences in exposures are too important, equity households tend to privilege allocation to the traditional sector. The coexistence of the two intermediaries is ensured by imposing the following boundary constraints

$$\phi_t > 0 \Leftrightarrow (\alpha_t^{SB} - \alpha_t^{TI})\alpha_t^{TI}(\sigma_t^R)^2(m-\gamma) > -\psi\phi^*$$

It is worth noting that without the adjustment costs mechanism (i.e., $\psi = 0$), $(\alpha_t^{SB} - \alpha_t^{TI})$ would be required to have the same sign as $m - \gamma$. Introducing adjustment costs therefore allows some difference in sign between relative risk-aversion and relative leverage depending on the term $(-\psi\phi^*)$. The complement boundary for coexistence is

$$\phi_t < 1 \Leftrightarrow (\alpha_t^{SB} - \alpha_t^{TI})\alpha_t^{TI}(\sigma_t^R)^2(m - \gamma) < \gamma(\alpha_t^{SB} - \alpha_t^{TI})^2(\sigma_t^R)^2 + \psi(1 - \phi^*)$$

The right-hand side of the last boundary is always positive, and this constraint also allows for a difference in the signs of $(\alpha_t^{SB} - \alpha_t^{TI})$ and $(m - \gamma)$. We are thus able to introduce more flexibility in the modeling of portfolio choice than would have been the case without any adjustment costs, thereby obtaining more realistic dynamics in portfolio shares.

The evolution in allocations and associated dynamics can be observed in Figure 12. We consider the two different cases: $m < \gamma$ and $m > \gamma$. For each case we present the results for two representative targets, i.e. $\phi^* = 0.3$ and $\phi^* = 0.5$. When equity households are more risk averse than traditional bankers (i.e. $m < \gamma$), they allocate below the target to the shadow banking sector if this sector has a higher exposure to the risky asset than the traditional sector. At the upper end of reputation states in Figure 12, the leverage of shadow bankers is higher than traditional leverage, and equity households want to reduce allocation to the shadow banking sector compared to the target. As the economy approaches the constrained area, shadow bankers' leverage decreases, and the allocation to this sector increases. At reputation states close to the constrained area (states around 0.5), the leverage of shadow bankers becomes lower than traditional leverage and equity households now allocate to these intermediaries above the target. However, as the difference between the two leverages becomes relatively important, equity households start to re-balance their portfolio towards traditional intermediaries. In the constrained area, this difference explodes due to the deleveraging of the shadow banking system, and their allocated equity sharply declines. We obtain similar dynamics than in the baseline case with constant equity shares. A lower target allocation ϕ^* is associated with lower equity levels for the shadow banking sector, and this decreases the magnitude of amplification effects. As the equity share of shadow bankers decreases in the constrained area, traditional intermediaries' equity is sustained and this reduces the importance of the equity effect relative to the baseline case with constant equity shares. A target allocation of 0.5 generates similar amplification effects to the baseline with a constant equity share ϕ of 0.3.

We now consider a situation where equity households are less risk averse than traditional intermediaries (i.e. $m > \gamma$). In this case, equity households are willing to allocate above the target to the shadow banking sector if this sector is more exposed to the risky asset (i.e. uses more leverage) than the traditional sector. Figure 13 illustrates the global dynamics. The major difference with the baseline case occurs around and in the constrained area (i.e., states around and on the left of the value of 0.5). Funds allocated to the shadow banking sector decline sharply as this sector proceeds to important deleveraging and the leverage gap expands. This in turn partly supports traditional equity, reduces the importance of the equity effect, and attenuates the amplification effects.

Figure 12: Global dynamics under the alternative with endogenous equity shares derived from optimal portfolio choices. Configuration with $m < \gamma$. Red lines correspond to a target allocation $\phi^* = 0.3$. Dash-dot black lines correspond to $\phi^* = 0.5$. Dashed blue lines are for the model without shadow banking. Lines with larger width are associated with shadow bankers' variables.

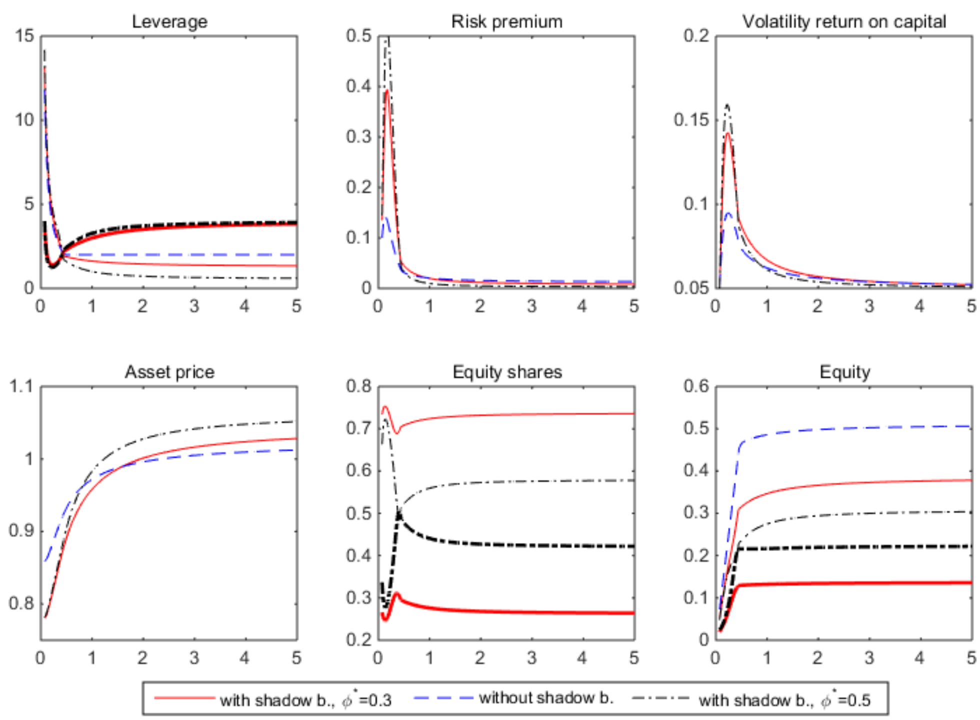
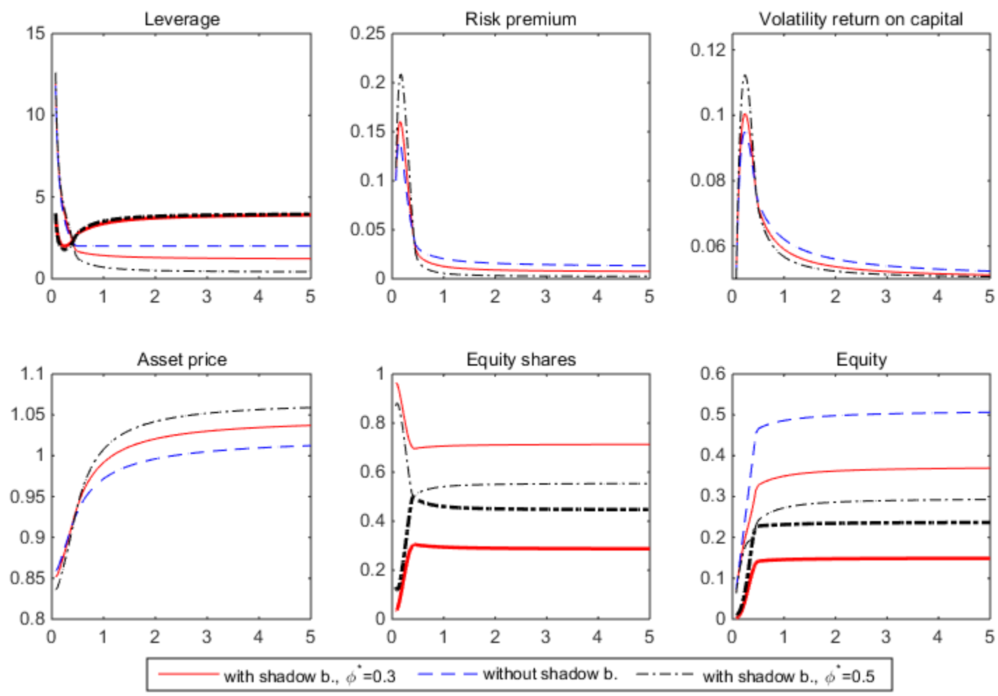


Figure 13: Global dynamics under the alternative with endogenous equity shares derived from optimal portfolio choices. Configuration with $m > \gamma$. Red lines correspond to a target allocation $\phi^* = 0.3$. Dash-dot black lines correspond to $\phi^* = 0.5$. Dashed blue lines are for the model without shadow banking. Lines with larger width are associated with shadow bankers' variables.



5.2 Relative performance

In this section we investigate a second alternative to constant equity allocation. A simple allocation rule is proposed according to the realistic idea that equity households allocate funds to intermediaries based on their relative expected performance. Both intermediaries invest in the same asset and only differ with respect to their exposure to this asset, which is determined by their leverage. Therefore, their relative performance directly depends on their relative leverage. This observation motivates the use of the following simple allocation rule:

$$\phi_t = d \frac{\alpha_t^{SB}}{\alpha_t^{TI} + \alpha_t^{SB}} \tag{22}$$

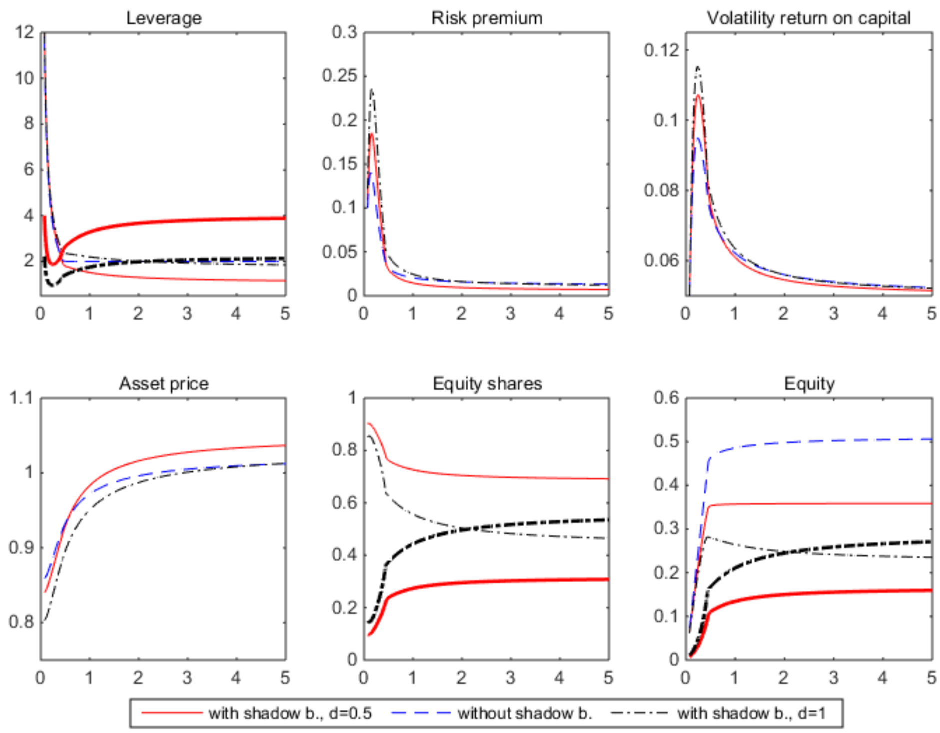
where d is a strictly positive scale parameter to ensure a stable solution.[26] The intuition behind (22) is as follows: any increase in the shadow banking leverage relative to the traditional sector leverage increases the exposure of shadow bankers to positive expected excess returns, measured by $\mu_t^R - r_t$, and hence raises its relative equity allocation. It is important to notice that the portfolio rule (22) focuses on relative expected performance only, and does not account for any risk consideration in allocations. This simple rule generates an intuitive pattern in equity allocation: funds allocated to the shadow banking sector increase in normal times when volatility is low and risk-taking is important, while they are sharply reduced during periods of high uncertainty.

Figure 14 illustrates the global dynamics in a framework where equity allocation depends on the relative expected performance of the two financial sectors. Results are reported for two different values of the scale parameter d, which drives the relative importance of equity shares. In both cases, the equity shares allocated to shadow bankers decrease as the economy moves from unconstrained to constrained areas, due to a decrease in their leverage. In the first case, represented in red solid lines, these shares are relatively lower compared to equity shares allocated to traditional intermediaries. In the second case (dash-dot lines), the shadow banking sector receives relatively more importance in the unconstrained area, before its equity share drops below the one of the traditional financial sector.[27] Dynamics associated with these two patterns continue to display more asymmetries than in the one-intermediary model. We nevertheless observe smaller amplification effects compared to the baseline two-intermediary model, due to a sharp reduction in the importance of the shadow banking sector in the constrained area.

[26]Parameter d is strictly positive and lower or equal to 1. It is introduced to ensure that the equity share of the shadow banking sector does not become too important in such a way that the sector overwhelms the demand for capital assets.

[27]In order to generate this second pattern, we need to consider a trade-off between the importance of shadow bankers' leverage and their equity share. As can be seen from equation (18), a too important shadow banking leverage coupled with high shadow banking equity share might lead to unreasonably low (or even negative) levels for the traditional banking leverage. Negative levels for α^{TI} involves convergence problem in the solution algorithm. In the simulation of the second pattern of allocations based on (22), we therefore use a different calibration for the tightness coefficient z of the leverage rule. It is set at $z = 9$, a value that reduces the average level of shadow banking leverage, and avoids any convergence problems.

Figure 14: Global dynamics under the alternative with endogenous equity shares derived from relative performance. Solid red lines correspond to a scale parameter $d = 0.5$ and the tightness of the risk-based rule set at $z = 5$. Dash-dot black lines correspond to a scale parameter $d = 1$ and the tightness of the risk-based rule set at $z = 9$. Dashed blue lines are for the model without shadow banking. Lines with larger width corresponds to shadow bankers.



6 Conclusion

The paper documents the effects of the introduction of a shadow banking sector into the macroeconomic framework developed by He and Krishnamurthy (2017). A leverage rule inspired by Danielsson et al. (2011) and Adrian and Boyarchenko (2015b) is used as a tool to obtain leverage behavior consistent with the empirical evidence on financial institutions with a high degree of wholesale funding (shadow bankers). The two-intermediary model is shown to generate more pronounced endogenous risk in global dynamics. This result is related to the evolution of the relative leverage of the two intermediaries. Stable times are characterized by a low level of volatility in the economy, important demand for capital assets from shadow banking institutions, and low leverage for the traditional financial sector. Crisis times are associated with high endogenous volatility, the deleveraging of shadow bankers and reintermediation of assets back onto traditional banks' balance sheets. These financial disturbances come in addition to the equity constraint of the He and Krishnamurthy model, and lead to a bigger increase in traditional leverage and a deeper asset price correction than in a model without shadow banking. The policy analyses implemented in the two-intermediary model suggest a trade-off between stabilizing the economy and the growth of economic activity for policy that aims at limiting the size of the shadow banking sector. Moreover, ignoring the heterogeneity of the financial sector may lead to an underestimation of the excess risk-taking due to the anticipation of expansionary policies and of financial and macroeconomic responses to shocks. The loss in efficiency of anticipated policy compared to unanticipated policy is also more important in a model with shadow banking.

Compared to the original model, the introduction of a shadow banking sector generates more volatile variables and more important responses of financial and real variables to economic shocks. These amplification effects show the potential of the two-intermediary model for the propagation of the performance and leverage of financial institutions to the real economy. The simple AK framework used in the paper illustrates the different mechanisms without adding any superfluous complexity. However, the modeling of a more sophisticated real block, with for instance habits in consumption and endogenous labor demand/supply and wage decisions, would contribute to a better investigation as to how the transmission mechanism to the real economy works and whether potential feedback from real to financial risk also matters.

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A Solution

A.1 Households consumption and interest rate

The marginal utility of consumption defines the pricing kernel of households: $\Lambda_t = e^{-\rho t}u'(C_t)$. Assuming $dC_t = \mu_t^C C_t dt + \sigma_t^C C_t dZ_t$ and CRRA utility, we have by Ito's lemma

$$\frac{d\Lambda}{\Lambda} = \left(-\rho - \theta\mu_t^C + 0.5\theta(\theta+1)(\sigma_t^C)^2\right)dt - \theta\sigma_t^C dZ_t \tag{23}$$

The process Λ_t can be used to price the risk-free asset with process $\frac{dp_t}{p_t} = r_t dt$ without any coupon payment

$$\begin{aligned} E_t\left[\frac{dp_t}{p_t} + \frac{d\Lambda_t}{\Lambda_t} + \frac{dp_t d\Lambda_t}{p_t\Lambda_t}\right] &= 0 \\ \Leftrightarrow r &= \rho + \theta\mu_t^C - 0.5\theta(\theta+1)(\sigma_t^C)^2 \end{aligned}$$

The aggregate identity of the economy leads to equilibrium (17), which we can use to solve for the stochastic process of C. To save space and improve clarity, we omit the time subscripts in the following equations. Impose $f = A - \delta + \frac{1-q^2}{2\kappa}$,

$$
\begin{aligned}
dC &= d(fK) = Kdf + fdK + dfdK \\
&= K\left(-\frac{q}{\kappa}\frac{q'}{q}\mu_e q - \frac{q}{2\kappa}\frac{q''}{q}\sigma_e^2 q - \frac{1}{2\kappa}(\frac{q'}{q}\sigma_e)^2 q^2\right)dt + K\left(-\frac{q}{\kappa}\frac{q'}{q}\sigma_e q\right)dZ + f(i-\delta)Kdt \\
&\quad + f\sigma K dZ + K\sigma\left(-\frac{q}{\kappa}\frac{q'}{q}\sigma_e q\right)dt \\
\Leftrightarrow \frac{dC}{C} &= \left(-\frac{qq'}{\kappa f}\mu_e - \frac{qq''}{2\kappa f}\sigma_e^2 - \frac{q'q'}{2\kappa f}\sigma_e^2\right)dt + (i-\delta)dt + \sigma\left(-\frac{qq'}{\kappa f}\sigma_e\right)dt + \left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)dZ \\
\Leftrightarrow \frac{dC}{C} &= \left(i - \delta - \frac{qq'}{\kappa f}\mu_e - \frac{qq''}{2\kappa f}\sigma_e^2 - \frac{q'q'}{2\kappa f}\sigma_e^2 - \sigma\frac{qq'}{\kappa f}\sigma_e\right)dt + \left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)dZ \qquad (24)
\end{aligned}
$$

Using (24) we can further proceed in the characterization of the risk-free interest rate.

$$
r = \rho + \theta\left(i - \delta - \frac{qq'}{\kappa f}\mu_e - \frac{qq''}{2\kappa f}\sigma_e^2 - \frac{q'q'}{2\kappa f}\sigma_e^2 - \sigma\frac{qq'}{\kappa f}\sigma_e\right) - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2 \qquad (25)
$$

A.2 State process

The aggregate reputation of the intermediary sector can be defined as an average of each intermediary's reputation:

$$
d\varepsilon = (1-\phi)\varepsilon m\left(\alpha^{TI}(dR - rdt) + rdt\right) + \phi\varepsilon\left(\alpha^{SB}(dR - rdt) + rdt\right) - \eta dt + d\psi \qquad (26)
$$

The state variable is the aggregate reputation normalized by the capital stock $e = \frac{\varepsilon}{K}$. We further have

$$
\begin{aligned}
\frac{de}{e} &= \frac{d\varepsilon}{\varepsilon} - (i-\delta)dt - \sigma dZ - \sigma\frac{\sigma_e}{e}dt \\
&= \left[\delta - i + \left((1-\phi)m\alpha^{TI} + \frac{\phi}{z\sigma_R}\right)(\mu_R - r) + (m - \phi m + \phi)r - \eta + \mu_\psi - \sigma\frac{\sigma_e}{e}\right]dt \\
&\quad + \left[(1-\phi)m\alpha^{TI}\sigma_R + \phi z^{-1} - \sigma\right]dZ \qquad (27)
\end{aligned}
$$

Hence

$$
\sigma_e = \frac{e\left[((1-\phi)m\alpha^{TI} - 1)\sigma + \phi z^{-1}\right]}{1 - (1-\phi)m\alpha^{TI}\frac{q'}{q}e} \qquad (28)
$$

Let's pose $X_1 = (1-\phi)m\alpha^{TI} + \frac{\phi}{z\sigma_R}$. The drift in the state process is

$$
\begin{aligned}
\mu_e q &= eq(m - \phi m + \phi)r - eq\eta + eX_1(-rq - \delta q + 0.5q''\sigma_e^2 + A(1-l) + q'\sigma_e\sigma + q'\mu_e) \\
&\quad - (ieq - \delta eq + q\sigma\sigma_e) \\
\Leftrightarrow \mu_e &= (q - eX_1 q')^{-1}\left(eq(m - \phi m + \phi)r - eq\eta + eX_1(-rq - \delta q + 0.5q''\sigma_e^2 + A(1-l) + q'\sigma_e\sigma)\right. \\
&\quad \left.- (ieq - \delta eq + q\sigma\sigma_e)\right) \qquad (29)
\end{aligned}
$$

Terms on $\sigma\sigma_e$ are moved to the left-hand side to ease further derivations. We obtain

$$\begin{aligned}
(q - eX_1 q')(\mu_e + \sigma\sigma_e) &= eq(m - \phi m + \phi - X_1)\left[\rho + \theta\left(i - \delta - \frac{qq'}{\kappa f}\mu_e - \frac{qq''}{2\kappa f}\sigma_e^2\right.\right. \\
&- \left.\frac{q'q'}{2\kappa f}\sigma_e^2 - \sigma\frac{qq'}{\kappa f}\sigma_e\right) \\
&\left.-\frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\
&+eX_1(-\delta q + A(1-l)) + eX_1 0.5q"\sigma_e^2 - eq(\eta + i - \delta) \\
\Leftrightarrow (q - eX_1 q' &+ eq(m - \phi m + \phi - X_1)\theta\frac{qq'}{\kappa f})(\mu_e + \sigma\sigma_e) \\
&= -eq(\eta + i - \delta) + eX_1(-\delta q + A(1-l)) + eX_1 0.5q"\sigma_e^2 \\
&+eq(m - \phi m + \phi - X_1)\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2\right. \\
&- \left.\frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\
&+eq(m - \phi m + \phi - X_1)\left(-\theta\frac{qq''}{2\kappa f}\sigma_e^2\right)
\end{aligned}$$

As in He and Krishnamurthy (2017), we pose $F = q/e - X_1 q'$ and $G = \kappa f q/e - \kappa f X_1 q' + q(m - \phi m + \phi - X_1)\theta qq'$, hence $G = \kappa f F + q(m - \phi m + \phi - X_1)\theta qq'$. We obtain

$$\begin{aligned}
\frac{G}{\kappa f}(\mu_e + \sigma\sigma_e) &= -q(\eta + i - \delta) + X_1(-\delta q + A(1-l)) + X_1 0.5q"\sigma_e^2 \\
&+q(m - \phi m + \phi - X_1)\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2 - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\
&+q(m - \phi m + \phi - X_1)\left(-\theta\frac{qq''}{2\kappa f}\sigma_e^2\right)
\end{aligned}$$

A.3 Traditional intermediaries

As in He and Krishnamurthy (2017), traditional intermediaries maximize the log of their reputation, denoted ε_t^{TI}. Therefore, their portfolio choice problem belongs to the CRRA log-normal case. As shown below, due to the log shape of the utility function, this maximization problem becomes equivalent to the "mean-variance"-type of objective function (11) presented in the theoretical set-up.

$$\max_{\alpha_t^{TI}} E\int_0^\infty e^{-\eta t}\ln(\varepsilon_t^{TI})dt$$

where $\frac{d\varepsilon_t^{TI}}{\varepsilon_t^{TI}} = mdR_t^{TI}$ and R_t^{TI} is the return of the traditional intermediaries' portfolio. Let's write $d\varepsilon_t^{TI} = A_t\varepsilon_t^{TI}dt + B_t\varepsilon_t^{TI}dZ_t$, with $A_t = m[\alpha_t^{TI}(\mu_t^R - r_t) + r_t]$ and $B_t = m\alpha_t^{TI}\sigma_t^R$. The unique solution of the Ito process is $\varepsilon_t^{TI} = exp\left[\int_0^t (A_s - 0.5B_s^2)\,ds + \int_0^t B_s dZ_s\right]$.[28] Consequently,

$$\begin{aligned} E\int_0^\infty e^{-\eta t}\ln(\varepsilon_t^{TI})dt &= E\int_0^\infty e^{-\eta t}\left[\int_0^t \left(A_s - 0.5B_s^2\right)ds + \int_0^t B_s dZ_s\right]dt \\ &= E_0\int_0^\infty e^{-\eta t}\left[\int_0^t \left(A_s - 0.5B_s^2\right)ds\right] + E_0\int_0^\infty e^{-\eta t}\left[\int_0^t B_s dZ_s\right]dt \end{aligned}$$

where the second term vanishes as $E\left[\int_0^t B_s dZ_s\right] = 0$. In this case, the portfolio choice problem reduces to finding the optimal leverage α_t^{TI} such that

$$\begin{aligned} &\max_{\alpha_t^{TI}} E\left[\left(A_t - 0.5B_t^2\right)dt\right] \\ =\ &\max_{\alpha_t^{TI}} E\left[A_t dt\right] - 0.5E\left[B_t^2 dt\right] \\ =\ &\max_{\alpha_t^{TI}} E\left(\frac{d\varepsilon_t^{TI}}{\varepsilon_t^{TI}}\right) - \frac{1}{2}Var\left(\frac{d\varepsilon_t^{TI}}{\varepsilon_t^{TI}}\right) \\ =\ &\max_{\alpha_t^{TI}} mE\left(dR_t^{TI}\right) - \frac{m^2}{2}Var\left(dR_t^{TI}\right) \end{aligned} \tag{30}$$

A.4 Second-order ODE

Using capital market equilibrium (5), the VaR leverage rule (10) and the volatility of the state process (28), we are able to solve for traditional leverage as a function of the state process e and prices variables q and q'.

$$\alpha^{TI} = \frac{[(1-\phi)\alpha^H(1-\lambda)]^{-1}\left(z(1-\frac{q'}{q}e)\sigma + \frac{q'}{q}e\phi\right) - \frac{\phi}{1-\phi}}{z(1-\frac{q'}{q}e)\sigma + (1-m)\phi\frac{q'}{q}e} \tag{31}$$

In order to express the second-order term q" in terms of lower order terms, we start with intermediary portfolio rule $\alpha^{TI}m(\sigma^R)^2 = \mu^R - r$ and isolate terms in q" in μ_e and r.

[28] To verify this assume $Y_t = \int_0^t \left(A_s - 0.5B_s^2\right)ds + \int_0^t B_s dZ_s$ which satisfies $dY_t = \left(A_t - 0.5B_t^2\right)dt + B_t dZ_t$. Posing $X = e^Y$ and applying Ito's lemma, we obtain:

$$\begin{aligned} dX &= e^Y(A - 0.5B^2)dt + 0.5e^Y B^2 dt + e^Y B dZ_t \\ dX &= AXdt + BXdZ_t \end{aligned}$$

$$
\begin{aligned}
0.5q"\sigma_e^2 &= \alpha^{TI} m\sigma_R^2 \frac{1}{q} - A(1-l) + \delta q \\
&- q'\frac{\kappa f}{G}\left[\begin{array}{c} -q(\eta + i - \delta) + X_1(-\delta q + A(1-l)) + X_1 0.5q"\sigma_e^2 \\ +q(m - \phi m + \phi - X_1)\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2 - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\ +q(m - \phi m + \phi - X_1)\left(-\theta\frac{qq''}{2\kappa f}\sigma_e^2\right) \end{array}\right] \\
&+ q\left[\begin{array}{c} \frac{\kappa f F}{G}\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2 - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\ -\theta 0.5\frac{qq"}{\kappa f}\sigma_e^2 \\ -\theta\frac{qqq'}{G}(-\eta - i + \delta) - \theta\frac{qq'}{G}X_1(-\delta q + A(1-l)) \\ -\theta\frac{qq'}{G}\left[X_1 0.5q"\sigma_e^2 + q(m - \phi m + \phi - X_1)\left(-\theta\frac{qq''}{2\kappa f}\sigma_e^2\right)\right] \end{array}\right]
\end{aligned}
$$

Isolating terms in $q"$ we find

$$
\begin{aligned}
& q"\left[\begin{array}{c} 0.5\sigma_e^2 + q'\frac{\kappa f}{G}\left(0.5X_1\sigma_e^2 - \theta 0.5\frac{qq}{\kappa f}\sigma^2(m - \phi m + \phi - X_1)\right) \\ \frac{q}{G}\left(\theta qq' X_1 0.5\sigma_e^2 + \theta 0.5 qF\sigma_e^2\right) \end{array}\right] \\
&= \alpha^{TI} m\sigma_R^2 \frac{1}{q} - A(1-l) + \delta q \\
&- q'\frac{\kappa f}{G}\left[\begin{array}{c} -q(\eta + i - \delta) + X_1(-\delta q + A(1-l)) \\ +q(m - \phi m + \phi - X_1)\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2 - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \end{array}\right] \\
&+ \frac{q}{G}\left[\begin{array}{c} \kappa f F\left[\rho + \theta(i-\delta) - \theta\frac{q'q'}{2\kappa f}\sigma_e^2 - \frac{\theta(\theta+1)}{2}\left(\sigma - \frac{qq'}{\kappa f}\sigma_e\right)^2\right] \\ -\theta qqq'(-\eta - i + \delta) - \theta qq' X_1(-\delta q + A(1-l)) \end{array}\right]
\end{aligned} \tag{32}
$$

B Figures

Figure 15: Global dynamics, robustness check with exogenous volatility of $\sigma = 0.04$. Red lines correspond to the baseline calibration for the two-intermediary model. Dashed blue lines are for the model without shadow banking. Left-hand side of the vertical black line indicates constrained area.

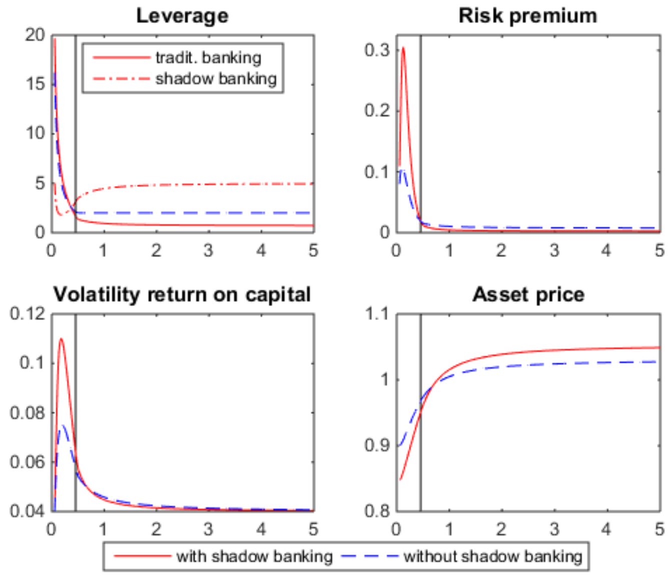


Figure 16: Global dynamics, robustness check with exogenous volatility of $\sigma = 0.03$. Red lines correspond to the baseline calibration for the two-intermediary model. Dashed blue lines are for the model without shadow banking. Left-hand side of the vertical black line indicates constrained area.

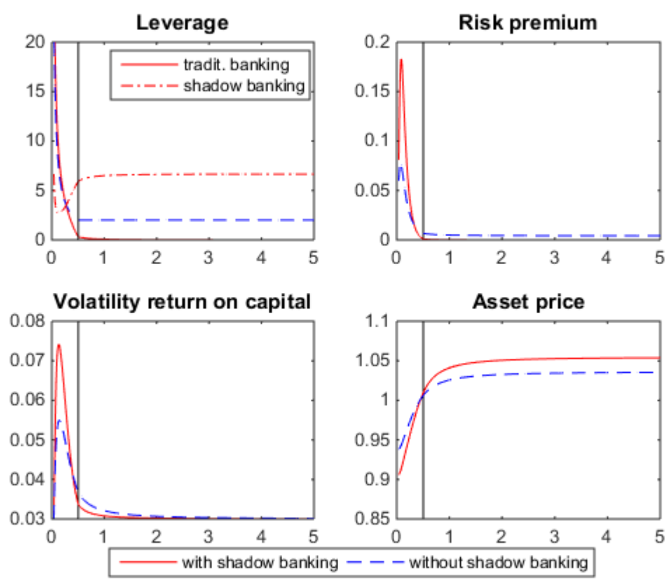


Figure 17: Global dynamics, robustness check with a households share allocated to shadow bankers of $\phi = 0.2$. Red lines correspond to the baseline calibration for the two-intermediary model. Dashed blue lines are for the model without shadow banking. Left-hand side of the vertical line indicates constrained area.

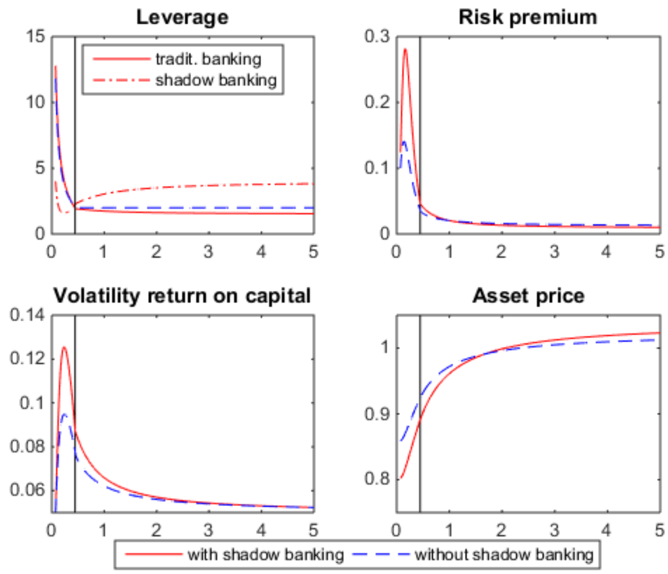
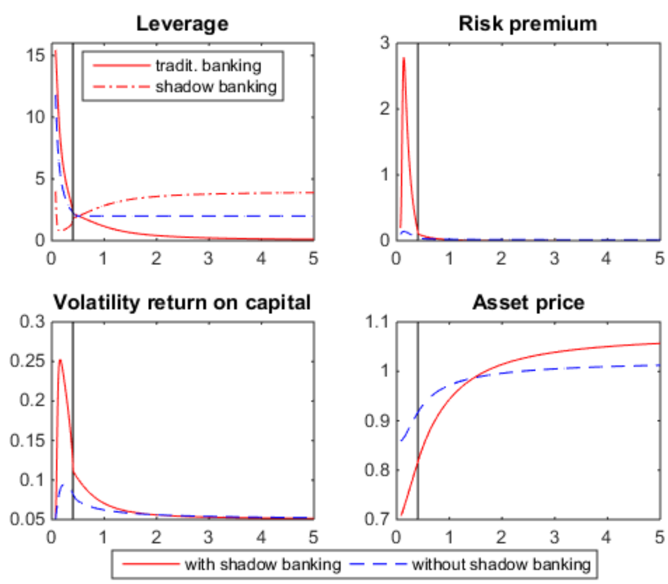


Figure 18: Global dynamics, robustness check with a households share allocated to shadow bankers of $\phi = 0.5$. Red lines correspond to the baseline calibration for the two-intermediary model. Dashed blue lines are for the model without shadow banking. Left-hand side of the vertical line indicates constrained area.



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