A price index with variable mark-ups and changing variety

by Thomas Demuynck and Mathieu Parenti

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Abstract

This paper proposes an estimation of an augmented Tornqvist price-index - featuring demand shifters - which is exact for homothetic translog preferences. Contrary to previous work based on a constant elasticity of substitution across varieties, this demand system allows for changes in markups even when the number of products is large. We then propose a structural decomposition of this index in terms of changes in markups, productivity, variety and demand shocks. We illustrate our approach using sample data from ACNielsens Homescan Panel. For instance, our results are consistent with competition effects where a decrease in per-product demand translates into lower markups.

1 Introduction

The construction of price indices are central to both policy makers and academics. They aim at allowing meaningful comparisons of standards of living across individuals, countries and time periods. In this paper, we propose a new theoretically grounded price-index - an augmented Tornqvist index - whose variations can be estimated and decomposed into four channels: demand shifters (product appeal), supply shocks, mark-ups and variety.

When theoretically derived, a price-index is determined by the functional form of consumer demand. Constructing a price-index relies therefore on the estimation of demand parameters that are assumed to be fixed. This, in turn, relies implicitly on the existence taste/demand shocks that shift the demand curve so as to map the observed price and quantity consumed of each good. In other words, these shocks are the micro-foundation of the error terms in the econometric model used in the estimation. Without these demand shifters, demand parameters would be over-identified with a couple of observations. As emphasized by Redding and Weinstein (2018) this

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leads to a paradoxical situation in which price-indices - whose fundamental aim is to make meaningful welfare comparisons across time - rely implicitly on the assumption that preferences change over time. In this paper, we build on the insight of Redding and Weinstein (2018) who show that this tension can be solved by mapping taste shocks into price variations entering a fixed utility function. This allows then for meaningful welfare comparisons across time through the construction of an augmented price index.

Redding and Weinstein (2018) introduce a unified price index derived from CES preferences augmented with multiplicative demand shifters. While the tractability and the potential applications arising from CES preferences is clear, it remains ill-equipped to capture variations in markups. When working with a large number of products, each good consumed exhibits, mechanically, a low market share so that the elasticity of demand remains in the neighbourhood of the assumed constant elasticity of substitution. As a result, markups do not vary across goods in a given CES nest. In this paper instead, we want to shed light on the importance of markups for the fluctuations of the price-index. We thus consider the translog expenditure system of Feenstra (2003). Like the CES, cross and own price elasticity of demands are entirely determined by a single parameter. Unlike the CES however, when the market share of a product is small, demand and cost shocks can still lead to fluctuations in the markup associated with that product. Furthermore, while the CES does not require any assumptions on unavailable goods, translog demand is defined for a finite number (or mass) of potential varieties. The prices of those varieties that are not consumed are then implicitly defined at consumer’s reservation price which is itself a function of the average log price of consumed varieties. Consumers demand for a good available is therefore a direct function of the number of goods that are not consumed.

In order to estimate our price index, we build on the insight of Feenstra (1994) by modelling the supply-side of the economy and then assuming that (residual) demand and supply shocks are uncorrelated. This defines a moment condition that determines the demand semi-elasticity, our variable of interest. We then apply our methodology to estimate a price-index using data from ACNielsens Homescan Panel from the Denver area, a large homescan dataset. We show how the variations of our price-index can be decomposed into four different channels: markups, cost shocks, demand shocks and product variety. Estimating price indices for 30 product categories, we are then able to see how these different channels correlate. For instance, our results are consistent with competition effects where a decrease in per-product demand translates into lower markups.

Related literature.

Our paper is related to at least two strands of the literature. The first one is the vast literature on theoretically grounded price indices with the pioneering contribution of Diewert (1976). When assuming CES preferences on the demand side, one gets the so-called Sato-Vartia index (Sato, 1976 and Vartia, 1976). Feenstra (1994) shows how to incorporate new/exiting varieties in the Sato-Vartia index while Redding and Weinstein (2018) propose a structural estimation of this price-index where demand
shocks rationalize exactly the data while preserving a money-metric utility. Our contribution is to allow for the variation in the markups associated with each product. Contrary to the CES case, assuming a translog expenditure function (Feenstra, 2003) retains markup variability even when the number of goods is large. In that spirit, our paper is also related to Feenstra and Weinstein (2017) but our approach gives a direct mapping between our augmented price-index and the data along the lines of Redding and Weinstein (2018). In our application, we also show how to structurally decompose the price-index along the markup, demand, supply and variety channels. Our empirical estimation is in the spirit of Feenstra (1994) and Imbs and Mejean (2015). We specify the supply side of this economy by assuming that all products are horizontally differentiated. Each of these products are supplied by a firm which competes oligopolistically la Bertrand-Nash. We obtain a closed-form expression for the mark-ups as a function of firms’ marginal cost and the product’s market share. As will be clear however, since we are working with a large number of products, the monopolistically competitive limit can be assumed without loss of generality. Nevertheless and contrary to the CES demand system, the monopolistically competitive limit under a translog expenditure function leads to variable markups. Equipped with a supply-side linking variety marginal costs of production and prices, our identification relies on a moment condition linking demand and supply shocks. Specifically, we assume that their variations, defined as a double difference i.e. across different varieties within a product group and between the two periods, are orthogonal. This leads to a consistent estimator or our parameter of interest.

The remainder paper is organized as follows. Section 2 describes the translog expenditure model augmented with demand shocks under imperfect competition. Section 3 presents the identification strategy. Section 4 presents an application to supermarket scanner data. Section 5 concludes.

2 Model

We consider a setting with a single representative consumer who buys $N$ goods. The prices of the various goods are represented by a price vectors, $p \in \mathbb{R}_+^N$. Consumption bundles are represented by vectors $q \in \mathbb{R}_+^N$. Let $u : \mathbb{R}_+^N \rightarrow \mathbb{R}$ be the utility function of the consumer. We assume that the consumer is expenditure minimizing in the sense that for a given desired level of utility $u$, and prices $p$ the consumer chooses the bundle $q$ that solves the following expenditure minimization problem,

$$e(p, u) = \arg\min_q \; p'q \text{ s.t. } u(q) \geq u.$$  

Following Redding and Weinstein (2018), we augment this model by allowing for preference shifters that increases or decreases the attractiveness of certain goods for the consumer. We do this by introducing a vector $\varphi \in \mathbb{R}_+^N$ and setting, $u(q; \varphi) = u(\varphi q)$, where $\varphi q$ is shorthand for the vector in $\mathbb{R}_+^N$ with $i$th element $\varphi_i q_i$. Changes in $\varphi$ can be seen as subjective valuations for quality of the various goods. For example if in period 0, $\varphi_i = 1$ and in period 1, $\varphi_i = 2$ then consuming 1 unit of good $i$ in period 1 feels like consuming two units of the same good in period 2, i.e. the subjective valuation.
quality doubled. Using this, we get,
\[ e(p, u; \varphi) = \arg\min_q p'q \text{ s.t. } u(\varphi q) \geq u, \]
\[ = \arg\min_q \sum_{i=1}^N (p_i/\varphi_i) \tilde{q}_i \text{ s.t. } u(\tilde{q}) \geq u, \]
\[ = e(p/\varphi, u). \]

where the second line follows from the change of variables \( \tilde{q}_i = \varphi_i q_i \) and where we abuse notation by denoting \( p/\varphi \) as the vector where at element \( i \), we have the value \( p_i/\varphi_i \). If the utility function is homothetic then the expenditure function is also homogeneous of degree one in \( u \),
\[ e(p/\varphi, u) = e(p/\varphi)u. \]

Here \( e(p/\varphi) \) is called the unit expenditure function. Applying Shephard's lemma to this expenditure function gives,
\[ \frac{\partial e(p/\varphi, u)}{\partial (p_i/\varphi_i)} = \varphi_i q_i, \text{ and } \]
\[ \frac{\partial \ln(e(p/\varphi, u))}{\partial \ln(p_i/\varphi_i)} = \frac{p_i q_i}{e(p/\varphi, u)} = s(p/\varphi, u), \]

where \( s(p/\varphi, u) \) is the vector of demand shares at prices \( p/\varphi \) and utility level \( u \). In cases where the expenditure function is homogeneous of degree one, this function is independent of \( u \).

One of the most popular unit expenditure functions is the translog expenditure function. This function satisfies the following specification,
\[ \ln(e(p/\varphi)) = \alpha_0 + \sum_{i=1}^N \alpha_i \ln(p_i/\varphi_i) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \gamma_{i,j} \ln(p_i/\varphi_i) \ln(p_j/\varphi_j), \]
\[ = \alpha_0 + \alpha' \ln(p/\varphi) + \frac{1}{2} \ln(p/\varphi)'\Gamma \ln(p/\varphi). \]

Here \( \sum_{i=1}^N \alpha_i = 1, \gamma_{i,j} = \gamma_{j,i} \) and \( \sum_{i=1}^N \gamma_{i,j} = 0 \). From Shephard’s lemma, we obtain the following expression for the demand shares,
\[ s_i = \alpha_i + \sum_{j=1}^N \gamma_{i,j} \ln(p_j/\varphi_j). \]

or in matrix form,
\[ s = \alpha + \Gamma \ln(p/\varphi). \]

To make the problem tractable, we consider in this paper the Feenstra and Weinstein (2017) specification where for all goods \( i \in N \), \( \Gamma_{i,i} = -\gamma(1 - 1/N) \) and for
all distinct goods $i, j$ and $k$, $\Gamma_{i,j} = \Gamma_{i,k} = \gamma/N$. This gives the following substitution matrix,

$$\Gamma = -\gamma \left( I_N - \frac{1}{N} 1_N \right),$$

where $1_N$ is the $N \times N$ matrix of ones and $I_N$ is the $N \times N$ identity matrix. Substituting this into the translog specification gives,

$$\ln(e(p/\varphi)) = \alpha_0 + \alpha \ln(p/\varphi) - \gamma/2 \ln(p/\varphi)\ln(p/\varphi) + \gamma/2N \left( \sum_{i=1}^{N} \ln(p_i/\varphi) \right)^2,$$

$$s = \alpha - \gamma \ln(p/\varphi) + \gamma \ln(p/\varphi)\Gamma \ln(p/\varphi),$$

where $\ln(p/\varphi) = \frac{1}{N} \sum_{i=1}^{N} \ln(p_i/\varphi_i)$.

**The price index**  Consider a setting with two periods $t = 0, 1$ where in each period not necessarily all goods are available. We will use superscript to denote the relevant period under consideration. In order to model such situation, we first fix the total number of potential goods to some (large) number $N$. If $i$ is a good that is not available at period $t$, we have that the share of good $i$, $s_{t,i} = 0$ while the price, $p_{t,i}/\varphi_{t,i}$ of this good is set equal to its shadow prices, i.e. the choke price that sets $s_{t,i} = 0$. Notice that these shadow prices are endogenous and depend on the prices and demand shifters of the other available goods. For ease of notation, we do not make this explicit.

Let $N_t$ be the set of goods available in period $t$. The expenditure function is still takes the form,

$$e(p_t/\varphi_t) = \alpha_0 + \alpha \ln(p_t/\varphi_t) + \ln(p_t/\varphi_t)\Gamma \ln(p_t/\varphi_t),$$

except that now, for goods not in in $N_t$, the prices are set equal to the shadow prices.

Our aim is to get an expression for the cost of living,

$$\frac{e(p_1/\varphi_1)}{e(p_0/\varphi_0)}.$$

This fraction measures the minimal expenditure needed at prices $p_1/\varphi_1$ in order to be as well off as under the prices $p_0/\varphi_0$.

Let $I = N^0 \cap N^1$ be the set of goods that are available in both periods and let $S_t^I = \sum_{i \in I} s_{t,i}^I$ be the total share of those goods in period $t$. In Appendix A we show that the log cost of living can be written in the following form,

$$\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0)) = \frac{1}{\gamma} \sum_{i=1}^{N} \frac{(s_{0,i}^I)^2 - (s_{1,i}^I)^2}{2} - \frac{1}{\gamma} \sum_{i \in I} \frac{(s_{0,i}^I)^2 - (s_{1,i}^I)^2}{S_{t}^0 + S_{t}^I},$$

$$+ \sum_{i \in I} \frac{s_{0,i}^I + s_{1,i}^I}{S_{t}^0 + S_{t}^I} \ln(p_{t,i}^1/p_{t,i}^0),$$

$$- \sum_{i \in I} \frac{s_{0,i}^I + s_{1,i}^I}{S_{t}^0 + S_{t}^I} \ln(\varphi_{t,i}^1/\varphi_{t,i}^0).$$

(1)
The left hand side is the log price index which measures the log of the cost of living. The right hand side contains four terms. In case where the set of available goods common in both periods (i.e. \( I = N^1 = N^0 \)), then the first two terms cancel out (observe that in this case, \( S^0_I + S^1_I = 2 \)). If there are also no change in demand shifters, i.e. \( \varphi^1_i / \varphi^0_i = 1 \) for all \( i \), then also the last term cancels out. If so, we are left with \( \sum_{i=1}^{N^t} s^0_i I + s^1_i I \ln(p^1_i / p^0_i) \) which equals the Törnqvist price index, which is an exact price index for the translog expenditure function.

Observe that none of the terms on the right hand side contains expressions involving shadow prices for the unobserved goods.

**Supply side** In order to derive the pricing rule of the firms, we assume a setting with oligopolistic competition. Let \( x_i(p) \) be the demand function for good \( i \). Remember that,

\[
 s^t_i(p/\varphi) = \frac{p^t_i x^t_i(p/\varphi)}{E^t} = \alpha_i - \gamma \ln(p^t_i / \varphi^t_i) + \gamma \ln(p^t / \varphi^t).
\]

Notice that the last term on the right hand side contains shadow prices for the non-available goods. We can get rid of this term by first summing over all goods in \( N^t \), giving,

\[
 \gamma \ln(p^t / \varphi^t) = \frac{1}{N^t} \left( 1 - \sum_{i \in N^t} \alpha_i \right) + \gamma \frac{1}{N^t} \sum_{i \in N^t} \ln(p^t_i / \varphi^t_i),
\]

And then substituting back to obtain,

\[
 s^t_i(p/\varphi) = \frac{p^t_i x^t_i(p/\varphi)}{E^t} = \alpha_i - \gamma \ln(p^t_i / \varphi^t_i) + \frac{1}{N^t} \left( 1 - \sum_{i \in N^t} \alpha_i \right) + \gamma \frac{1}{N^t} \sum_{i \in N^t} \ln(p^t_i / \varphi^t_i).
\]

As such,

\[
 \frac{\partial \ln(s^t_i(p^t / \varphi^t))}{\partial \ln(p^t_i)} = 1 + \frac{\partial \ln(x^t_i(p^t / \varphi^t))}{\partial \ln(p^t_i)} = -\frac{\gamma}{s^t_i(p^t / \varphi^t)} \frac{N^t - 1}{N^t}.
\]

Here we made the assumption that firms treat total expenditure \( E \) as exogenous. The firm that produces good \( i \) sets the price \( p^t_i \) in order to maximize profits \( \pi_i(p^t_i) \)

\[
 \pi_i(p^t_i) = x_i(p^t / \varphi^t)(p^t_i - c_i^t),
\]

where \( c_i^t \) is the marginal cost of producing good \( i \) at period \( t \). The first order condition gives,

\[
 \frac{\partial x_i(p^t / \varphi^t)}{\partial p^t_i} (p^t_i - c_i^t) + x_i(p^t / \varphi^t) = 0.
\]

Then,

\[
 \frac{p^t_i - c_i^t}{p^t_i} = -\frac{x_i(p^t / \varphi^t)}{p^t_i \frac{\partial x_i(p^t / \varphi^t)}{\partial p^t_i}} = -\frac{1}{\frac{\partial \ln(x_i(p^t / \varphi^t))}{\partial \ln(p^t_i)}}
\]

\[
 = \frac{1}{1 + \frac{\gamma}{s^t_i(p^t / \varphi^t)} \frac{N^t - 1}{N^t}}.
\]
Equivalently,

\[ p_t^i = c_t^i \left( 1 + \frac{s_t(p_t^i / \phi_t^i) N_t^i}{\gamma (N_t^i - 1)} \right) \]

This shows that the markup is identified if \( \gamma \) is known and the shares are observed.

If the number of firms \( N_t \) is large then the mark up approximates,

\[ \frac{p_t^i - c_t^i}{c_t^i} \approx \frac{s_t^i (p_t^i / \phi_t^i)}{\gamma} \]

The mark-up is increasing in the market share of the good and decreasing in \( \gamma \) which measures the responsiveness of the demand with respect to an increase in prices.

3 Identification and estimation of \( \gamma \)

Consider a random sample existing of shares and prices for a variety of goods for two periods \( \{(p_t^0, s_t^0)_{i \in N^0}, (p_t^1, s_t^1)_{i \in N^1}\} \). We assume that this random sample originates from random demand and supply shocks, denoted by, \( \{((\phi_t^i)^1, c_t^1)_{i \in N^0}; (\phi_t^0, c_t^0)_{i \in N^1}\} \). Let \( I = N^1 \cap N^0 \) be the collection of goods that are available in both periods. In our setting \( |I| = m \) is the sample size. We assume that the random 2 dimensional vectors \( [\phi_t^1 / \phi_t^0, c_t^1 / c_t^0]_{i \in I} \) are i.i.d. drawn from some common distribution.

In order to obtain our identification, we impose that the log of demand and supply shocks have zero covariance,

\[ \text{cov}[\ln(\phi_t^1 / \phi_t^0) ; \ln(c_t^1 / c_t^0)] = 0. \]

A sufficient condition for this to hold is that demand and supply shocks are independent,

\[ \phi_t^1 / \phi_t^0 \perp c_t^1 / c_t^0 \quad (i, j \in I). \]

The approach that we use to estimate \( \gamma \) is based on the fact that we can use the sample of shares and prices to identify the various demand and supply shocks. Once we have identified these, we can use the covariance moment condition above to identify \( \gamma \).

We start from the share demand equations: for \( t \in \{0, 1\} \),

\[ s_t^i = \alpha_i - \gamma \ln(p_t^i / \phi_t^i) + \gamma \ln(p_t^i / \phi_t^i). \]

Taking differences of this equation over the two time periods allows us to get rid of the common factor \( \alpha_i \),

\[ s_t^1 - s_t^0 = -\gamma \ln(p_t^1 / p_t^0) + \gamma \ln(p_t^1 / \phi_t^1) + \gamma \ln(p_t^1 / \phi_t^1) - \gamma \ln(p_t^0 / \phi_t^0). \]

In order to get rid of the last two terms, we can take the difference over two goods \( i, j \in I \),

\[ (s_t^1 - s_t^0) - (s_t^1 - s_t^0) = -\gamma \ln(p_t^1 / p_t^0) + \gamma \ln(p_t^1 / p_t^0) + \gamma \ln(p_t^1 / \phi_t^0) - \gamma \ln(\phi_t^1 / \phi_t^0). \]
Observe that for \( \delta \)

\[
\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0) = \frac{1}{\gamma} \left( (s_i^1 - s_i^0) - (s_j^1 - s_j^0) \right) + \ln(p_i^1/p_i^0) - \ln(p_j^1/p_j^0).
\]

This shows that the left hand side, which is a double difference over time and goods of the demand shocks, is identified given the value of \( \gamma \). From the previous section, we have that the marginal costs \( c_i^t \) satisfies the following identity \( \langle t \in \{0, 1\} \rangle \),

\[
\ln(c_i^t) = \ln(p_i^t) - \ln \left( 1 + \frac{s_i^t N_t}{\gamma(N_t^2 - 1)} \right).
\]

Let us summarize the random variables corresponding to good \( i \) as \( d_i = [s_i^1, s_i^0, p_i^1, p_i^0] \) and define the following functions,

\[
f(d_i, d_j; \delta) = \delta \left( (s_i^1 - s_i^0) - (s_j^1 - s_j^0) \right) + \ln(p_i^1/p_i^0) - \ln(p_j^1/p_j^0).
\]

\[
g(d_i, d_j; \delta) = \ln(p_i^1/p_i^0) - \ln(p_j^1/p_j^0) - \ln \left( 1 + \delta \frac{s_i^1 N^1}{N^1 - 1} \right) + \ln \left( 1 + \delta \frac{s_j^0 N^0}{N^0 - 1} \right) + \ln \left( 1 + \delta \frac{s_j^0 N^0}{N^0 - 1} \right)
\]

Observe that for \( \delta = 1/\gamma \),

\[
f(d_i, d_j; 1/\gamma) = \ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0),
\]

\[
g(d_i, d_j; 1/\gamma) = \ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0).
\]

As such,

\[
e(1/\gamma) \equiv \mathbb{E}[f(d_i, d_j; 1/\gamma)g(d_i, d_j; 1/\gamma)],
\]

\[
= \mathbb{E} \left[ (\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0))(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0)) \right],
\]

\[
= \sum_{k \in \{i, j\}} \text{cov}(\ln(\varphi_k^1/\varphi_k^0); \ln(c_k^1/c_k^0)) = 0.
\]

We estimate \( \gamma \) by the value of \( \delta \) that sets the corresponding finite moment condition equal to zero.

\[
e_m(1/\gamma) \equiv \frac{1}{m(m-1)} \sum_{i,j \in I} f(d_i, d_j; 1/\gamma)g(d_i, d_j; 1/\gamma) = 0.
\]

Assume, for a moment that \( e_m(1/\gamma) \) has an asymptotic normal distribution,

\[
\sqrt{m} \ e_m(1/\gamma) \sim^a N(0, \sigma^2)
\]

Given that the functions \( g \) and \( f \) are differentiable in \( \delta \), we can use the Delta method to obtain the asymptotic distribution of \( 1/\gamma \),

\[
\sqrt{m} \left( \frac{1}{\gamma} - \frac{1}{\gamma} \right) \sim^a N \left( 0, \sigma^2 \left( \frac{\partial e(1/\gamma)}{\partial \delta} \right)^{-2} \right).
\]
The term $\left( \frac{\partial e(1/\gamma)}{\partial \delta} \right)^{-2}$ can be consistently estimated using the finite sample analogue, 

$$\frac{\partial e_m(1/\hat{\gamma})}{\partial \delta}.$$ 

In order to obtain an expression for the variance $\sigma^2$. Notice that 

$$e(1/\gamma) = \mathbb{E} \left[ (\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0))(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0)) \right],$$ 

This is a U-statistic of order 2 (see, for example, van der Vaart (1998)). As such, 

$$\sigma^2 = 4 \text{var} \left[ F(\varphi_i^1/\varphi_i^0; c_i^1/c_i^0) \right], \text{ where}$$ 

$$F(\varphi_i^1/\varphi_i^0; c_i^1/c_i^0) = \mathbb{E} \left[ (\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0))(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0)) | \varphi_i^1/\varphi_j^1; c_i^1/c_j^1 \right]$$ 

The value of $\sigma^2$, can, again, be estimated consistently using finite sample analogues and the consistent estimator $\hat{\gamma}$. 

We summarize the procedure in the following steps, 

step 1: Collect a finite sample $(d_i)_{i \in I}$.
step 2: Minimize the function, 

$$e_m(\delta) = \left( \frac{1}{m(m-1)} \sum_{i,j} f(d_i, d_j; \delta)g(d_i, d_j; \delta) \right)^2$$ 

with respect to $\delta$ and set $\hat{\gamma} = 1/\delta$. Check that the minimization problem attains the value of zero.
step 3: construct the values of, 

$$h(d_j) = \sum_{i \in I} \frac{1}{(m-1)} f(d_i, d_j; 1/\hat{\gamma})g(d_i, d_j; 1/\hat{\gamma})$$ 

Set $\hat{\sigma}^2$ to be 4 times the sample variance of $h(d_j)$.
step 4: compute $w = \frac{\partial e_m(1/\hat{\gamma})}{\partial \delta}$.
step 5: The variance is estimated as $\frac{\hat{\sigma}^2}{(w)^{-2}}$.

The table below shows simulation results for the type one error for different sizes $|I| = m$ where we set the total number of potential goods equal to $N = 900$. We see that for $m \geq 50$ the nominal level is close to the real rejection rate.

| rejection rate | $|I|$ (nominal level = 5%) |
|----------------|--------------------------|
|                | 10                        | 0.142                     |
|                | 30                        | 0.084                     |
|                | 50                        | 0.072                     |
|                | 100                       | 0.056                     |
|                | 300                       | 0.054                     |
Table 1: Summary statistics of estimates

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<th>mean</th>
<th>std</th>
<th>min</th>
<th>max</th>
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<td>0.0707</td>
<td>0.0127</td>
<td>0.3166</td>
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<tr>
<td>mean mark-up</td>
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<td>0.0983</td>
<td>0.0987</td>
<td>0.5418</td>
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<td>$</td>
<td>I</td>
<td>= m$</td>
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4 Application

We illustrate our results using data from ACNielsens Homescan Panel from the Denver area, a large homescan dataset. The dataset captures a large variety of grocery packed goods purchased by a large number of households at a large number of retail shops. We define a good by the UPC (Unique product code). For each transaction, we have the amount of money paid and the total quantity bought. The data set covers transactions from the period from January 1993 through March 1995. As we compute price indices we aggregate over all consumers on a yearly level for two years, 1993 and 1994. From this we compute total expenditures per good and total amount bought per good. Prices are computed by dividing total expenditures by total amounts.

We conduct our analysis separately for various product subcategories (orange juice, tomato sauce, instant food, etc). Towards this end, we restrict ourselves to product categories for which the number of goods common to both periods (i.e. $|I| = m$) is at least 30. This gives a total number of 30 product categories.

Table 1 provides summary statistics of our estimates over these 30 groups. Appendix B contains results for each group separately. The first row of table 1 gives information on the estimates $\hat{\gamma}$. Appendix A gives summary statistics of this estimate for all 30 product groups separately. The second line gives the mean mark-up which is computed as,

$$
\frac{1}{N_t} \sum_{i \in N_t} \frac{p_i - c_i}{c_i} = \frac{1}{N_t} \sum_{i \in N_t} \frac{s_i}{\gamma} \frac{N_t}{N_t - 1} = \frac{1}{\gamma (N_t - 1)}.
$$

The last line gives the number of common goods in the two periods.
It is possible to decompose our expression for the price index in the following way,

$$\ln(\tilde{e}(p^1/\varphi^1)) - \ln(\tilde{e}(p^0/\varphi^0)) = \frac{1}{\gamma} \sum_{i=1}^{N} \left( \frac{(s_i^0)^2 - (s_i^1)^2}{2} - \frac{1}{\gamma} \sum_{i \in I} \frac{(s_i^0)^2 - (s_i^1)^2}{S_i^0 + S_i^1} \right),$$

$$+ \sum_{i \in I} \left( \frac{s_i^0 + s_i^1}{S_i^0 + S_i^1} \left( \ln(p_i^1/c_i^1) - \ln(p_i^0/c_i^0) \right) \right),$$

$$+ \sum_{i \in I} \left( \frac{s_i^0 + s_i^1}{S_i^0 + S_i^1} \left( \ln(c_i^1) - \ln(c_i^0) \right) \right),$$

$$- \sum_{i \in I} \left( \frac{s_i^0 + s_i^1}{S_i^0 + S_i^1} \left( \ln(\varphi_i^1) - \ln(\varphi_i^0) \right) \right).$$

The first line gives the part of the price index that is due to the expanding or shrinking number of varieties available. The second line is a weighted average of the change in mark-ups of the producers. The third line is a weighted average of the change in supply shifters. Finally, the fourth line is the weighted average of the change in demand shifters. This last part is not entirely identified from the model. For this, we impose the additional condition that the average of the change of the demand shifters over all common goods equals zero.

$$\sum_{i \in I} \ln(\varphi_i^1/\varphi_i^0) = 0.$$

Table 2 provides summary statistics of this decomposition for the 30 groups. Appendix B shows the decomposition in pictures for the 30 groups separately. All numbers are in %-points. Table 3 computes the correlation between the %-points of the various parts of the price index. There are three types of exogenous shocks in our model: the number of varieties $N^t$, the demand shocks $\varphi^t_i$ and the supply shocks $c^t_i$. All of these have an influence on the mark-up of the firms. From Table 3, we see that the %-points change in the price index due to a change in varieties is highly correlated with the %-point change in the price index due to the change in
Table 3: Correlation matrix of decomposition in % points

<table>
<thead>
<tr>
<th></th>
<th>varieties</th>
<th>mark-up</th>
<th>supply</th>
<th>demand</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>varieties</td>
<td>1.000</td>
<td>0.932</td>
<td>0.056</td>
<td>-0.460</td>
<td>0.235</td>
</tr>
<tr>
<td>mark-up</td>
<td>1.000</td>
<td>1.000</td>
<td>-0.005</td>
<td>-0.507</td>
<td>0.125</td>
</tr>
<tr>
<td>supply</td>
<td>1.000</td>
<td>1.000</td>
<td>-0.581</td>
<td>0.782</td>
<td></td>
</tr>
<tr>
<td>demand</td>
<td>1.000</td>
<td>1.000</td>
<td>-0.140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>

mark-up. If more varieties enter the market this leads to a decrease of the price index as the prices drop from their shadow (choke) price to the actual price. Also, more goods in the market lowers (on average) the share of all other products and increases competition, which leads to a decrease of the mark-ups. Next, we see that there is a negative correlation between the price index component for the demand shocks and the component attributed to changes in the mark-up. This can be explained by the fact that if a good receives a positive demand shock, the consumer needs less of the good for a given level of utility. As such, the price index should decrease. On the other hand, the share for the corresponding good increases, which has a positive effect on the mark-up for the corresponding good and, hence, raises the component of the price index for this part. Finally, we see that the part of the price index due to the supply shocks has a small but negative correlation with the part for the mark-up. Higher marginal costs lead to higher prices which increases the price index. Higher prices, in turn, lead to lower demand shares, which lowers the mark-up for the corresponding goods. As such, the component of the price index due to changing mark-ups decreases.

5 Conclusion

In this paper we have proposed an estimation of an augmented Tornqvist price-index derived from a translog expenditure system augmented with demand shocks. Following Redding and Weinstein (2018) our framework allows for the estimation of meaningful price index variations in a framework where all the observed prices and quantities are the outcome of a well-defined utility maximizing consumer. Contrary to previous work based on a constant elasticity of substitution across varieties, the translog expenditure system allows us to uncover a new channel through which demand and supply shocks can affect price indices: mark-ups.

Applying our methodology to a sample dataset from ACNielsens Homescan Panel in the Denver area, we have shown how the price index can be decomposed along four microeconomic channels: markups, demand shocks, cost shocks and variety. By computing the correlation between these channels across different product groups, our estimates are consistent with competition effects where a decrease in per-product demand translates into lower markups.
There are many directions in which our framework could be extended. First, we have assumed that a single parameter on the demand side was enough to capture own and cross-price demand elasticities. Differentiating varieties that are supplied by the same firm - and hence considering multi-product firms with cannibalization effects - through a nested translog expenditure system is a promising avenue for future research. Second, our framework could be used to quantify the gains from trade through the estimation of the variations of the price-index over time. Along the lines of Feenstra and Weinstein (2017), our methodology could be used to decompose these gains and quantify the importance of the mark-up channel. Last, the literature has so far assumed a demand side parameterized either by CES preferences or a translog expenditure system. Allowing for a less parametric approach that would nest these two models seems inevitable to understand the true impact of markups which are ruled out by assumption under CES preferences in the presence of a large number of goods.

References


A Derivation of equation (1)

The expenditure function is homogeneous of degree one in prices, so we can divide all prices by the price of, say, the \(i\)-th good \((i \in \mathbb{N})\). Doing this gives us an \(N - 1\) dimensional price vector \(\ln(q^t)\) where \(\ln(q^t) = \ln(p^t_j / \varphi^t_j) - \ln(p^t_i / \varphi^t_i)\), i.e. \(q^t_j = \frac{p^t_j \varphi^t_i}{p^t_i \varphi^t_j}\).

Let \(\alpha_{-i}\) be the sub-vector of \(\alpha\) consisting of all elements except the \(i\)th and let \(\Gamma_{-i}\) be the \((N - 1) \times (N - 1)\) submatrix of \(\Gamma\) with the \(i\)th row and column deleted,

\[
\Gamma_{-i} = -\gamma \left( I_{N-1} - \frac{1}{N} \mathbf{1}_{N-1} \right).
\]

Then, we can write

\[
\ln(e(p^t / \varphi^t)) = \alpha_0 + \ln(p^t_i / \varphi^t_i) + \alpha_{-i} \ln(q^t) + \frac{1}{2} \ln(q^t)' \Gamma_{-i} \ln(q^t).
\]

Shephard’s lemma gives,

\[
s^t_{-i} = \alpha_{-i} + \Gamma_{-i} \ln(q^t).
\]

where now \(s^t_{-i}\) is the vector of shares with the \(i\)th entry removed.

Consider the matrix \(\Delta = -\frac{1}{\gamma}(I_{N-1} + \mathbf{1}_{N-1})\) which is the inverse of \(\Gamma_{-i}\). Indeed,

\[
\Delta \Gamma_{-i} = -\frac{1}{\gamma}(I_{N-1} + \mathbf{1}_{N-1})(-\gamma(I_{N-1} - \mathbf{1}_{N-1}/N)),
\]

\[
= I_{N-1} - \frac{1}{N} \mathbf{1}_{N-1} + \mathbf{1}_{N-1} - \frac{1}{N} \mathbf{1}_{N-1} \mathbf{1}_{N-1},
\]

\[
= I_{N-1} - \frac{1}{N} \mathbf{1}_{N-1} + \mathbf{1}_{N-1} - \frac{N-1}{N} \mathbf{1}_{N-1},
\]

\[
= I_{N-1}.
\]

If we pre and post multiplying the matrix \(\Delta\) by \(s^t_{-i}\), we obtain,

\[
(s^t_{-i})' \Delta (s^t_{-i}) = (\alpha_{-i} + \Gamma_{-i} \ln(q^t))' \Delta (\alpha_{-i} + \Gamma_{-i} \ln(q^t)),
\]

\[
= \alpha_{-i} \Delta \alpha_{-i} + 2 \alpha_{-i} \ln(q^t) + \ln(q^t)' \Gamma_{-i} \ln(q^t).
\]

If we take the difference of above expression for \(t = 1\) and \(t = 0\), then,

\[
\frac{(s^1_{-i})' \Delta (s^1_{-i})}{2} - \frac{(s^0_{-i})' \Delta (s^0_{-i})}{2} = \alpha'_{-i} \ln(q^1) - \alpha'_{-i} \ln(q^0) + \frac{1}{2} \ln(q^1)' \Gamma_{-i} \ln(q^1) - \frac{1}{2} \ln(q^0)' \Gamma_{-i} \ln(q^0),
\]

\[
= \ln(e(p^1 / \varphi^1)) - \ln(e(p^0 / \varphi^0)) - \ln(p^1_i / \varphi^1_i) + \ln(p^0_i / \varphi^0_i).
\]

Rewriting gives,

\[
\ln(\tilde{e}(p^1 / \varphi^1)) - \ln(\tilde{e}(p^0 / \varphi^0)) = \frac{(s^1_{-i})' \Delta (s^1_{-i})}{2} - \frac{(s^0_{-i})' \Delta (s^0_{-i})}{2} + \ln(p^1_i / p^0_i) - \ln(\varphi^1_i / \varphi^0_i).
\]

Now, \(\Delta = -\frac{1}{\gamma}(I_{N-1} + \mathbf{1}_{N-1})\), so the first two terms on the right hand side simplify to,

\[
(s^t_{-i})' \Delta (s^t_{-i}) = -\frac{(s^t_{-i})' (s^t_{-i})}{\gamma} - \frac{1}{\gamma} (1 - s^t_i)^2,
\]

\[
= -\frac{(s^t ' (s^t)}{\gamma} - \frac{1 - 2s^t_i}{\gamma}.
\]
As such,

$$\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0)) = -\frac{(s^1)'(s^1)}{2\gamma} + \frac{(s^0)'(s^0)}{2\gamma} + \frac{s^1_i - s^0_i}{\gamma} + \ln(p^1_i/p^0_i) - \ln(\varphi^1_i/\varphi^0_i).$$

If we multiply above expression by \(\frac{s^0_i + s^1_i}{2}\), then we obtain,

$$\frac{s^0_i + s^1_i}{2} (\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0))) = \frac{s^0_i + s^1_i}{2} \left( -\frac{(s^1)'(s^1)}{2\gamma} + \frac{(s^0)'(s^0)}{2\gamma} \right),$$

$$+ \frac{(s^1_i)^2 - (s^0_i)^2}{2\gamma} + \frac{s^0_i + s^1_i}{2} \left( \ln(p^1_i/p^0_i) \right),$$

$$- \frac{s^0_i + s^1_i}{2} \ln(\varphi^1_i/\varphi^0_i).$$

Let \(I = N^0 \cap N^1\) be the set of goods that are common in both periods and define \(S^I = \sum_{i \in I} s^I_i\) as the sum of shares of the goods in \(I\). Then, summing above expression over all goods in \(I\) gives,

$$\frac{S^0_I + S^1_I}{2} (\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0))) = \frac{S^0_I + S^1_I}{2} \left( -\frac{(s^1)'(s^1)}{2\gamma} + \frac{(s^0)'(s^0)}{2\gamma} \right),$$

$$+ \sum_{i \in I} \frac{(s^1_i)^2 - (s^0_i)^2}{2\gamma} + \sum_{i \in I} \frac{s^0_i + s^1_i}{2} \left( \ln(p^1_i/p^0_i) \right),$$

$$- \sum_{i \in I} \frac{s^0_i + s^1_i}{2} \ln(\varphi^1_i/\varphi^0_i).$$

**B  Estimation results**

The following table gives the estimate of \(\gamma\), the 95% asymptotic confidence interval together with the sample size for all thirty groups of goods.
The following graphs provide for each group of goods the decomposition of the price index into the different parts.
<table>
<thead>
<tr>
<th>Varieties</th>
<th>Mark-up</th>
<th>Supply Shifters</th>
<th>Demand Shifters</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.01</td>
<td>0.99</td>
<td>1.02</td>
<td>1.02</td>
</tr>
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<td>1.02</td>
<td>1.02</td>
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</tr>
<tr>
<td>1</td>
<td>1.02</td>
<td>0.99</td>
<td>1.02</td>
<td>1.04</td>
</tr>
</tbody>
</table>

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305. “Forward guidance, quantitative easing, or both?”, by F. De Graeve and K. Theodoridis, Research series, October 2016.


