

Endogenous forward guidance



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# Endogenous Forward Guidance\*

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## Abstract

We propose a novel framework where forward guidance (FG) is endogenously determined. Our model assumes that a monetary authority solves an optimal policy problem under commitment at the zero-lower bound. FG derives from two sources: 1. from committing to keep interest rates low at the exit of the liquidity trap, to stabilize inflation today. 2. From debt sustainability concerns, when the planner takes into account the consolidated budget constraint in optimization. Our model is tractable and admits an analytical solution for interest rates in which 1 and 2 show up as separate arguments that enter additively to the standard Taylor rule.

In the case where optimal policy reflects debt sustainability concerns (satisfies the consolidated budget) monetary policy becomes subservient to fiscal policy, giving rise to more volatile inflation, output and interest rates. Liquidity trap (LT) episodes are longer, however, the impact of interest rate policy commitments on inflation and output are moderate. 'Keeping interest rates low' for a long period, does not result in positive inflation rates during the LT, in contrast our model consistently predicts negative inflation at the onset of a LT episode.

In contrast, in the absence of debt concerns, LT episodes are shorter, but the impact of commitments to keep interest rates low at the exit from the LT, on inflation and output is substantial. In this case monetary policy accomplishes to turn inflation positive at the onset of the episode, through promising higher inflation rates in future periods.

We embed our theory into a DSGE model and estimate it with US data. Our findings suggest that FG during the Great Recession may have partly reflected debt sustainability concerns, but more likely policy reflected a strong commitment to stabilize inflation and the output gap. Our quantitative findings are thus broadly consistent with the view that the evolution of debt aggregates may have had an impact on monetary policy in the Great Recession, but this impact is likely to be small.

**Keywords:** Bayesian estimation, DSGE model, Fiscal policy, Forward Guidance, Inflation, Liquidity trap, Monetary policy.

**JEL:** E31, E52, E58, E62, C11

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# 1 Introduction

What forces lie behind the Federal Reserve’s policy to keep interest rates close to the zero lower bound (ZLB) for several years following the burst of the Great Recession in 2008-9? A standard view in monetary theory is that a central bank which focuses on stabilizing inflation and the output gap, should commit to keep interest rates at the ZLB for a long period of time, as this can mitigate the negative effects of shocks on output and inflation (Eggertsson and Woodford (2003)). However, the post-2009 developments in monetary policy, most notably the purchases of substantial amounts of long term government bonds by the Federal Reserve, coupled with the mounted debt and deficit levels in the US economy, have also led many economists to think that ‘keeping interest rates low’ was (at least partially) driven by the (implicit) objective to contain debt servicing costs.

Which of these alternative views is more plausible? A large body of recent literature has used medium-scale DSGE models to structurally identify key forces behind the behavior of macroeconomic aggregates in the Great Recession. In some of these studies (e.g., Del Negro et al. (2015); Kollman et al. (2016); Fratto and Uhlig (2014)) government debt aggregates are of little relevance for monetary policy. Instead, other work (e.g., Bianchi and Melosi (2017)) emphasizes the importance of the interactions between monetary and fiscal policies in explaining the data. These medium-scale models, however, summarize monetary policy in simple (ad hoc) Taylor rules which are estimated from the data. To match interest rates in the Great Recession they augment these rules with exogenous shocks which make interest rates deviate (persistently) from the value implied by the Taylor rule. Typically, these shocks are interpreted as a model-consistent form of forward guidance (FG).

As such, the models cannot inform us about whether keeping the interest rate low is an optimal policy response of a central bank which focuses on inflation stabilization, and beyond this, of one, which may have debt sustainability concerns, the latter being motivated by a desire to keep the costs of debt refinancing low. Our contribution in this paper is to provide a framework which can be used to answer this question. We propose a model of endogenous optimal interest rates at the ZLB, which nests both the case where the Fed’s policies reflect debt sustainability concerns and the case where they do not. Our model is tractable and it enables us to derive interest rates in closed form which in turn allows us to develop several new analytical insights about the efficacy of forward guidance in liquidity trap (LT) episodes. Moreover, the model can be easily embedded in a medium scale DSGE; to demonstrate this we perform a quantitative evaluation of optimal policies in the Great Recession.

Our theoretical framework is developed in Section 2 and assumes that a Ramsey planner under commitment sets allocations to minimize the deviations of inflation, output and interest rates from their respective target levels, subject to the standard set of dynamic equations which define the competitive equilibrium. In the baseline version of the Ramsey policy equilibrium, in which we assume that the Fed’s policies are also driven by debt sustainability concerns, we assume that this set also includes the consolidated budget constraint which determines the value of net debt in the hands of the private sector. Taxes are distortionary and moreover, tax policy is assumed to be exogenous to the planner’s problem; it follows a simple rule, which determines the tax rate as a function of lagged debt, a standard assumption in the DSGE literature (e.g., Leeper, (1991)).

Our first analytical result derives the interest rate rule that emerges from optimal policy in this model. We show that the policy function is similar to the interest rate rule assumed in the recent DSGE literature: the short-term rate is expressed as the sum of a Taylor rule component (a function of inflation and output growth) and “forward guidance shocks”. In our model these shocks are endogenous and expressed as the sum of two components. The first is a component which represents commitment to keep interest rates low at the exit from a LT episode (as in e.g., Eggertsson and Woodford (2003, 2006)) and is captured by the lags of the Lagrange multiplier attached to the occasionally binding ZLB constraint. The second component measures the impact of past shocks to the consolidated budget constraint, capturing the planner’s commitment to “twist interest rates” in order to satisfy the intertemporal budget (e.g., Faraglia, Marcet, Oikonomou and Scott (2016), henceforth FMOS). Since debt is distortionary, ours is a model of optimal policy under incomplete markets (as in e.g., Aiyagari et al. (2002), FMOS) and the Lagrange multiplier on the consolidated budget constraint is a state variable. Interest rate twisting is captured by the lags of this multiplier.

As an alternative to this benchmark model, we consider a Ramsey policy equilibrium where the consolidated budget does not enter into the constraint set. In this case, endogenous forward guidance emerges only from the impact of the occasionally-binding ZLB and the model does not feature any interest rate twisting effects from shocks to the intertemporal budget. By switching on and off the consolidated budget from the Ramsey program in this way, we are able to contrast the properties of equilibria where monetary policy has concerns over debt sustainability with equilibria where monetary policy has none of such concerns.

We first focus on the properties of the two versions of the model in the neighborhood of the steady state, that is away from the ZLB. We show analytically that in order for the rational expectations to be determinate in the baseline case, where optimization is subject to the consolidated budget, fiscal policy needs to not respond strongly to the deviations of government debt from its steady state level. Debt becomes an explosive process and monetary policy is charged with satisfying intertemporal solvency. In contrast, in the case where the consolidated budget does not enter in optimization, taxes strongly adjust to high debt levels and debt becomes a mean reverting process.

We interpret this finding in light of the analysis of Leeper (1991) on the joint monetary/fiscal policy mix. As in Leeper (1991) determinacy in our model requires an active/passive or passive/active mix for the equilibrium to be unique. Monetary policy is active when the Fed has no debt concerns and passive when it does. However, in contrast to Leeper (1991), the monetary policies which emerge from our model are optimal and derived from a Ramsey program where the Fed is assumed to take as given the tax adjustment rule followed by the fiscal authority.

After studying the properties of the model in the neighborhood of the steady state we turn to the case of the binding ZLB. As is standard, we assume that the economy falls into a LT following negative demand shocks which make households willing to postpone consumption. Our key finding is that, in the case where the Fed has debt concerns, LTs last longer; optimal interest rates remain at the ZLB for a longer period and display considerable persistence. However, although private agents expect interest rates to be kept low, output and inflation drop significantly at the beginning of the episode. This is in contrast to the no debt concerns case where monetary policy under commitment is very effective in stabilizing both output and inflation around the steady state. Our finding is thus, that when a central bank cares both about stabilizing inflation and government debt, monetary policy at the ZLB is overridden by debt sustainability concerns and this reduces the effectiveness of forward guidance on inflation.

To understand this finding, note that a negative preference shock which drives the economy to the LT, has two effects on the intertemporal consolidated budget. First, it increases the market value of debt-to GDP ratio. This is because real bond prices increase sharply in the LT. And second, it increases the real present discounted value of government surpluses, again since real interest rates drop in response to the shock. Which of these two effects dominates over the other, determines the optimal response of monetary policy to the shock. In the case where the market value rises faster than the surpluses, the optimal policy is to promise high inflation rates in the future. In the case where the surpluses rise more, letting inflation become negative to balance the intertemporal budget is optimal. We establish that in our model the second effect always dominates. Hence, under debt concerns the planner gives up on the goal of stabilizing inflation and output in favor of avoiding to destabilize debt, which is explosive.

In Section 3 we embed our optimal policy framework into a medium-scale DSGE model. In order to keep the analysis as rooted as possible to the analytical benchmark developed in Section 2, our quantitative model extends the baseline with preferences exhibiting habit formation, shocks to TFP, markup shocks etc. The model however has a rich enough structure to match the US data; it is broadly similar to the DSGE models without capital used in Bianchi and Ilut (2017), Bianchi and Melosi (2017) among others. In Section 4 we estimate our quantitative model with standard Bayesian techniques. To do so we follow the trail of the recent DSGE literature which uses observations prior to the Great Recession to estimate the posterior distributions of structural parameters (e.g. Del Negro et al. (2015), De Graeve and Theodoridis (2017), Chen et al (2012) and others). We thus focus on a sample which starts in 1980:Q1 and ends in 2008:Q4, the last quarter before interest rates

hit the ZLB in the United States.<sup>1</sup>

In Section 5 we use the model to investigate the implications of optimal monetary policies under 'debt concerns' and 'no debt concerns' in the Great Recession. We first consider a simple "forecasting" exercise in which we recover the realized vector of disturbances which drove the US economy to the LT trap in the first quarter of 2009, and simulate the paths of macroeconomic variables for several quarters thereafter.<sup>2</sup> This experiment enables us to evaluate the ability of the models to predict interest rates that remain persistently at the ZLB, in line with the US data, through their endogenous propagation mechanisms. It also allows us to investigate the implications of optimal policies for inflation and output growth.

We find that under both 'debt concerns' and 'no debt concerns', our framework is able to capture the persistence of interest rates at the ZLB, due to endogenous forward guidance. However, the effects on inflation are strikingly different across the two models. The 'no debt concerns' model predicts positive inflation, whereas the debt concerns model gives rise to the opposite outcome; inflation becomes very negative, and even falls below -10 percent, at the onset of the recession.

This is at odds with the data; as it is well known, inflation was not negative during the Great recession (except in the second quarter of 2009).<sup>3</sup> Our benchmark quantitative experiment thus seems to provide strong evidence against debt concerns. But we find that the actual data lie somewhere in between the two models, since the no debt concerns model overpredicts positive inflation. This motivates us to consider a simple scenario under which monetary policy begins to have debt sustainability concerns in 2009, but later on reverts back to the Ramsey equilibrium without debt concerns. A 'temporary switch' of this sort can match the data patterns reasonably well.

Towards the end of Section 5 we use our models to perform shock decompositions and recover the underlying shocks which make both models fit the US observations. Our findings suggest that in the no debt concerns model the lack of deflation in the Great Recession is explained by policy commitments to keep interest rates at the ZLB. In turn, in the debt concerns model, positive inflation rates are explained by shocks which lower the surplus of the government (e.g., negative tax/ positive spending shocks). Under both scenarios we find that markup shocks are not relevant to explain missing deflation; if anything they contributed negatively to the inflation rate observed in our sample. We link these findings to the recent DSGE literature.

## Related literature

This paper is related to several strands of the literature. First, as discussed previously, our analysis is motivated by the wideheld view that monetary policy in the Great Recession begun to take into account the evolution of debt aggregates. This view is analogous to the view that unbacked fiscal deficits during the late 1970s made difficult for monetary policy to disinflate the US economy under the "fiscally led" regime which prevailed at that time.<sup>4</sup> As in the late 1970s, at the onset of the Great Recession, we have seen a large run-up in US government debt and sizable deficits. As a result, concerns over the ability of the fiscal authority to generate sufficient surpluses have resurfaced.<sup>5</sup> When

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<sup>1</sup>Our choice of sample is based on the following considerations: first, there is a wideheld view that towards the end of the 1970s there was a shift in the monetary/fiscal policy mix in the US (see e.g. Bianchi and Ilut (2017), Bianchi and Melosi (2017) for empirical evidence). Our framework could be designed to capture transitions across regimes, however, in the interest of emphasizing the theoretical analysis, we chose not to do so in this paper. More crucially, estimating models with transitions and a binding ZLB turns out to be a formidable task even for state of the art Bayesian estimation methods. We leave this to future work.

Over the sample period 1980:Q1 to 2008:Q4 we assume that optimal policy had no debt concerns. This assumption is consistent with the findings of (for instance) Bianchi and Ilut (2017).

<sup>2</sup>Similar experiments are considered in Bianchi and Melosi (2017), Del Negro et al (2015) among others.

<sup>3</sup>See for example Hall (2001) and the considerable literature on the missing deflation puzzle.

<sup>4</sup>See for example Bianchi and Ilut (2017) and also Sims's (2011) well known "stepping on a rake" analysis.

<sup>5</sup>Public debt over GDP basically doubled in the US since 2009, following a series of tax cuts and hikes in spending levels which led to some of the largest deficit levels ever recorded. For example, in 2009 the government's deficit to GDP ratio was the largest since WWII. In 2010 it was the second largest. In 2010 the CBO, in its budget outlook report, projected that the ratio of debt held by the public to GDP would stabilize at around 70 percent (already the highest level since the 1950s). Today public debt is expected to continue to grow unless significant increases in tax

fiscal policy cannot sufficiently adjust taxes to debt, monetary policy must ensure debt solvency. This is essentially our debt concerns model.

Moreover, several papers have explored the impact of the buybacks of long term government bonds by the Fed, on the design of US monetary policy in the Great Recession. Del Negro and Sims (2015) use a microfounded model to study the issue of the solvency of a central bank when it accumulates long term assets on its balance sheet and to characterize the implications for monetary policy. Cochrane (2018), building on Cochrane (2001) and Sims (2011), presents a fiscal theory of the price level microfoundation of quantitative easing and forward guidance, and characterizes their effects on inflation. Bhattarai et al. (2015) use an optimal Ramsey policy program to propose a microfoundation of the so called signalling theory of quantitative easing, that buybacks of long-term bonds made the Fed more willing to keep interest rates low during the recession.

Our focus in this paper is not on quantitative easing per se.<sup>6</sup> However, implicitly we adopt the view that quantitative easing is a likely turning point for monetary policy in the US. In our quantitative experiment in Section 5 we take the 1st quarter of 2009 (which marks the start of QE) to be the date where policy begins to take into account the consolidated budget. Moreover, in our debt concerns model we let the planner choose the quantity of (net) long term debt held by private agents. This can be seen as being broadly consistent with the Fed's buybacks of long bonds from the secondary market. Cochrane (2018) models quantitative easing in a similar manner.

Related to our paper is also the work of Bianchi and Melosi (2017) who provide evidence that policy uncertainty increased during the recession, when the passive monetary /active fiscal regime resurfaced. In their model, LTs lead to a rise in the market value of debt-to-GDP and a rise in inflation, when monetary policy becomes subservient to fiscal policy. Their model is thus able to resolve the so called *missing deflation puzzle*. As discussed previously, in our debt concerns model, LTs lead to deflation because preference shocks increase the present value of fiscal surpluses more than the market value of debt. Therefore, the properties of inflation are opposite to the ones Bianchi and Melosi (2017) obtain from their model. We argue that this difference probably derives from commitment. When we assume that the planner cannot commit to future policies, debt and inflation comove positively when the economy falls in the LT. Under no commitment, however, the model does not give rise to any (endogenous) FG.

Our model also draws heavily from the literature, which studies optimal monetary policy at the ZLB and in a variety of economic environments (e.g., Eggertsson and Woodford (2003, 2006), Jung et al. (2005), Adam and Billi (2006, 2008), Bhattarai et al. (2015), Bouakez et al. (2018)). These models also give rise to endogenous FG and some of them consider the interactions between monetary and fiscal policies as we do. However, the focus of these papers is mainly theoretical and generally disconnected from the DSGE literature. Crucially, because these models are highly non-linear they characterize optimal policies through first order conditions, which are not straightforward to map into simple interest rate rules comparable to the ones assumed in the DSGE literature. As discussed previously, a contribution of our paper is to provide an optimal policy microfoundation for the ad hoc "forward guidance shocks" assumed in recent DSGE models. To accomplish this, we cast our optimal policy problem in the linear quadratic (LQ) framework of Giannoni and Woodford (2003), Benigno and Woodford (2003) and others, this enables to derive microfounded Taylor rules. We augment the LQ framework with the ZLB constraint and with a fiscal block assuming that taxes are distortionary and government debt is of long-term maturity.

Finally, our paper is more broadly related to numerous studies on optimal policies under the Ramsey equilibrium. SGU, Faraglia et al (2013), Lustig et al. (2008), and Siu (2004) consider the case of coordinated monetary and fiscal policies with distortionary taxation and incomplete

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rates and decreases in spending are introduced (see the 2018-2028 outlook).

<sup>6</sup>Also there is no explicit modelling of the Fed's balance sheet here, like for example in Del Negro and Sims (2015). We follow the trail of numerous macro papers which use the consolidated budget in models of optimal policy (e.g. Schmitt-Grohe and Uribe (2004), henceforth SGU, Faraglia et al (2013), Lustig et al (2009) and others).

Bhattarai et al (2015) show that the optimal policies which emerge from the Ramsey program when optimization is subject to the consolidated budget, are similar to policies when optimization is subject to the Fed's balance sheet, when the planner targets the net worth of the central bank.

markets. FMOS extend the optimal fiscal policy model of Aiyagari et al. (2002) to the case of long-term government debt. Unlike in these papers our tax policies are not optimal; we assume that taxes follow an exogenous rule, which relates the level of taxes to the lagged value of debt. This assumption makes our framework fit better the DSGE literature. However, the findings of these papers continue to be relevant here. For example, our microfoundation of FG in the debt concerns model draws from the interest rate twisting concept developed in FMOS. In that paper, interest rate twisting emerges when the fiscal authority distorts the tax schedule to influence long bond prices. In our model it emerges because the monetary authority distorts the path of inflation to satisfy the consolidated budget.

## 2 Theoretical Framework

We begin with a simple setup of the Ramsey policy equilibrium which allows us to illuminate key forces driving interest rates in our model. In the baseline version of the Ramsey policy we let the planner choose interest rates, inflation and output, subject to the consolidated budget constraint (of the Fed and the government). We refer to this model as the 'debt concerns' model. The aim is to capture a scenario under which the planner (Fed) has concerns over the sustainability of debt. We show analytically that in this case the optimal interest rate rule has three components: First, a standard Taylor rule component, which links interest rates to realized inflation, output growth and lagged values of interest rates. Second, a component which represents a commitment to keep interest rates low at the exit from a LT episode (as in e.g., Eggertsson and Woodford (2003, 2006)). Third, a component which measures the impact of past promises made by the planner to alter interest rates in the face of shocks to the consolidated intertemporal budget constraint. The third component is an "interest twisting effect" which emerges because debt is distortionary and long-term as in FMOS.

As an alternative to the baseline, we consider the case where the planner has "no debt concerns" and does not account for the consolidated budget constraint in optimization. The optimal policy rule in this case is identical to the baseline in terms of the first two components, but features no interest rate twisting impacts.

Endogenous FG emerges in our model because of these extra components which make interest rates deviate from the Taylor rule. Our definition of FG is consistent with recent papers in the DSGE literature in which interest rate rules are of the following form: <sup>7</sup>

$$\hat{i}_t^{DSGE} = \max\{\mathcal{T}_t^{DSGE} + \sum_{l=0}^M \epsilon_{t-l}^{l,DSGE}, -\hat{i}^*\} \quad (1)$$

where  $M > 0$ .  $\mathcal{T}_t^{DSGE}$  represents the Taylor rule component in these papers a function of inflation, output growth and lags of the interest rate.  $\epsilon_t^0$  is a standard monetary policy shock announced in period  $t$  and applying to policy in that period, whereas the vector  $\left(\epsilon_{t-1}^1, \dots, \epsilon_{t-M}^M\right)$  summarizes shocks which have been revealed in past periods and apply to the policy rule in  $t$ . These shocks capture *forward guidance* in monetary policy.

We show below that the optimal policy rules in our model are of the form (1). The shocks  $\left(\epsilon_{t-1}^1, \dots, \epsilon_{t-M}^M\right)$  are endogenous and, moreover, they are functions of economic fundamentals and of the structure of the Fed's optimal policy program. In the case of 'no debt concerns' these shocks reflect the Fed's commitment to keep interest rates low after a liquidity trap. In the case of 'debt concerns' they also reflect interest rate twisting in response to shocks to the consolidated budget.

Our analysis then focuses on comparing the properties of the two alternative setups focusing on the interactions between monetary and fiscal policies. We show that in the case where the Fed has concerns over debt sustainability, monetary policy becomes subservient to fiscal policy, partially

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<sup>7</sup>See for example Laseen and Swensson (2011), Campbell et al (2012, 2016), Del Negro et al (2015), De Graeve and Theodoridis (2017) and De Graeve, Ilbas and Wouters (2017) and others.

losing its control over inflation and the output gap. Fiscal policy then needs to be 'active' in the sense of Leeper (1991) for the equilibrium to be uniquely defined. In contrast, under 'no debt concerns' monetary policy is active and fiscal policy is passive. These differences lead to substantially different effects of economic shocks on macroeconomic variables, both at and away from the ZLB. To highlight these differences we use a mixture of analytical results and numerical simulations.

## 2.1 The baseline model

The building blocks of our model are a standard NK Phillips curve, an IS (Euler) equation which prices a short nominal bond, and the consolidated budget constraint which determines the value of (net) debt held by the private sector. The model is a simplified version of the NK model that we will later employ in estimation and which we formally describe in Section 3. For brevity, we summarize here the competitive equilibrium in the linearized version of the model; for detailed derivations we refer the reader to the online appendix.

Let  $\hat{x}$  denote the log deviation of variable  $x$  from its steady state value,  $\bar{x}$ . The competitive equilibrium equations are the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \quad (2)$$

where  $\kappa_1 \equiv -\frac{(1+\eta)\bar{Y}}{\theta}(\gamma_h + \sigma\frac{\bar{Y}}{C}) > 0$ ,  $\kappa_2 \equiv -\frac{(1+\eta)\bar{Y}}{\theta}\frac{\bar{\tau}}{(1-\bar{\tau})} > 0$ ,  $\kappa_3 \equiv -\frac{(1+\eta)\bar{Y}}{\theta}\sigma\frac{\bar{G}}{C} > 0$ ,

$$\hat{i}_t = E_t \left( \hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma \left[ \frac{\bar{Y}}{C} (\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\bar{G}}{C} (\hat{G}_t - \hat{G}_{t+1}) \right] \right) \quad (3)$$

$$\hat{i}_t \geq -\frac{1}{\beta} + 1 \equiv -i^* \quad (4)$$

$$\begin{aligned} & \frac{\beta \bar{b}_\delta}{1 - \beta \delta} \hat{b}_{t,\delta} + \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[ E_t \left( -\sigma \left( \frac{\bar{Y}}{C} \hat{Y}_{t+j} - \frac{\bar{G}}{C} \hat{G}_{t+j} \right) - \sum_{l=1}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \right] \\ & + \frac{\bar{\tau}(1+\eta)\bar{Y}}{\eta} \left( (\gamma_h + 1) \hat{Y}_t + \hat{\xi}_t + \frac{\hat{\tau}_t}{1 - \bar{\tau}} \right) - \bar{G} \left( \hat{G}_t - \sigma \frac{\bar{Y}}{C} \hat{Y}_t + \sigma \frac{\bar{G}}{C} \hat{G}_t + \hat{\xi}_t \right) + \bar{b}_\delta \sigma \left( \frac{\bar{Y}}{C} \hat{Y}_t - \frac{\bar{G}}{C} \hat{G}_t \right) - \bar{b}_\delta \hat{\xi}_t \\ & = \frac{\bar{b}_\delta}{1 - \beta \delta} (\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t \left( -\sigma \left( \frac{\bar{Y}}{C} \hat{Y}_{t+j} - \frac{\bar{G}}{C} \hat{G}_{t+j} \right) - \sum_{l=1}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \end{aligned} \quad (5)$$

(2) is the Phillips curve at the heart of our model.  $\hat{Y}_t$  is the output gap and  $\hat{\tau}$  denotes a distortionary tax levied on the labor income of households.  $\hat{G}_t$  denotes government spending in  $t$ . Parameters  $\eta$  and  $\theta$  denote the elasticity of substitution across differentiated products and the degree of price stickiness respectively.<sup>8</sup>

(3) is the log-linear Euler equation which prices a short term nominal asset.  $\hat{\xi}$  is a standard preference shock which affects the relative valuation of current vs. future utility by the household. A drop in  $\hat{\xi}$  makes the household relatively patient, willing to substitute current for future consumption.

(4) is the ZLB constraint on the short-term nominal interest rate.

(5) is the consolidated budget constraint. As Bianchi and Melosi (2017) we assume that debt is issued in a perpetuity bond with decaying coupons where  $\delta$  denotes the decay factor. Short debt is in zero net supply. Finally, parameter  $\sigma$  denotes the inverse of the intertemporal elasticity of substitution and  $\gamma_h$  is the inverse of the Frisch elasticity of labor supply.

Tax policies in our model follow a simple rule of the form:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \phi_{\tau,b} \hat{b}_{t-1,\delta} + u_{\tau,t} \quad (6)$$

where  $u_{\tau,t}$  is a shock to the tax rate following a first order autoregressive process and  $\phi_{\tau,b}$  measures the feedback effect of debt issued in  $t - 1$  on the tax rate in  $t$ .

<sup>8</sup>We assume price adjustment costs as in Rotemberg (1989).  $\theta$  governs the magnitude of these costs. When  $\theta$  equals zero, prices are fully flexible.

## 2.2 Ramsey Policy

The Ramsey policy chooses inflation, output and interest rate sequences to maximize the following objective

$$-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \hat{\pi}_t^2 + \lambda_Y \hat{Y}_t^2 + \lambda_i \hat{i}_t^2 \right\} \quad (7)$$

subject to equations (2) to (5) and given (6).  $\lambda_Y$  and  $\lambda_i$  govern the relative weights attached to output gap and interest rate stabilization by the planner. For brevity, we leave it to the appendix to derive the Lagrangian function that we solve to find the optimal policies. Letting  $\psi_{\pi,t}$  be the multiplier attached to the Phillips curve constraint,  $\psi_{i,t}$ ,  $\psi_{ZLB,t}$  and  $\psi_{gov,t}$  the analogous multipliers for the Euler equation, the ZLB constraint and the consolidated budget respectively, the first order conditions for the optimum are given by:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} + \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} = 0 \quad (8)$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t} \kappa_1 + \sigma \frac{\bar{Y}}{C} \left( \psi_{i,t} - \frac{\psi_{i,t-1}}{\beta} \right) + \sigma \frac{\bar{Y}}{C} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} + \omega_Y \psi_{gov,t} = 0 \quad (9)$$

$$-\lambda_i \hat{i}_t + \psi_{i,t} + \psi_{ZLB,t} = 0 \quad (10)$$

$$\frac{\beta \bar{b}_\delta}{1-\beta\delta} \left( \psi_{gov,t} - E_t \psi_{gov,t+1} \right) - \mathcal{I}_P \beta \phi_{\tau,b} (1-\rho_\tau) E_t \left( \sum_{j=0}^{\infty} (\rho_\tau \beta)^j (\kappa_2 \psi_{\pi,t+1+j} - \frac{\bar{\tau} \bar{Y} (1+\eta)}{\eta(1-\bar{\tau})} \psi_{gov,t+1+j}) \right) = 0 \quad (11)$$

$$\psi_{ZLB,t} \geq 0 \quad \text{and} \quad (\hat{i}_t + i^*) \psi_{ZLB,t} = 0 \quad (12)$$

where  $\omega_Y \equiv \bar{G} \sigma \frac{\bar{Y}}{C} + \frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} (1+\gamma_h)$ . (8) is the FONC with respect to  $\hat{\pi}_t$ ; (9), (10), (11) are first order conditions with respect to  $\hat{Y}_t$ ,  $\hat{i}_t$  and  $\hat{b}_{t,\delta}$  respectively. (12) is the complementary slackness condition for the ZLB constraint. Finally,  $\mathcal{I}_P$  is an indicator function that takes the value 1 when the planner internalizes the feedback effect of debt on taxes in equation (6) and is zero otherwise. For the remainder of the paper we will set  $\mathcal{I}_P = 0$ .<sup>9</sup>

### 2.2.1 Optimal interest rate rules

Combining the first order conditions (8), (9) and (10) and following the argument of Giannoni and Woodford (2001) we can derive analytically the optimal interest rate rule which emerges from the Ramsey program. The following proposition summarizes the result:

**Proposition 1.** *The optimal interest rate rule which derives from the Ramsey program is:*

$$\hat{i}_t = \max\{\mathcal{T}_t + \mathcal{D}_t + \mathcal{Z}_t, -i^*\} \quad (13)$$

$$\mathcal{T}_t \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta} \Delta \hat{i}_{t-1}$$

with  $\phi_\pi = \frac{\kappa_1 \bar{C}}{\lambda_i \sigma \bar{Y}}$ ,  $\phi_i = (1 + \frac{\kappa_1 \bar{C}}{\beta \sigma \bar{Y}})$  and  $\phi_Y = \frac{\lambda_Y \bar{C}}{\sigma \lambda_i \bar{Y}}$ ,

$$\mathcal{D}_t = -\frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\lambda_i \sigma} \frac{\bar{b}_\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} - \frac{\bar{b}_\delta}{\lambda_i} \sum_{l=0}^{\infty} \delta^l \left( \Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \right) - \frac{\bar{C} \omega_Y}{\bar{Y} \sigma \lambda_i} \Delta\psi_{gov,t} \quad (14)$$

<sup>9</sup>This assumption is made to keep with the rest of the literature on fiscal/monetary policy interactions which assumes independent policies.

and

$$\mathcal{Z}_t = -\frac{1}{\lambda_i} \left(1 + \frac{1}{\beta} + \frac{\kappa_1 \bar{C}}{\sigma \beta \bar{Y}}\right) \psi_{ZLB,t-1} + \frac{1}{\beta \lambda_i} \psi_{ZLB,t-2}. \quad (15)$$

**Proof** See appendix.

As equation (13) shows there are three distinct components in the interest rate rule. The first,  $\mathcal{T}_t$ , is a standard Taylor rule component which links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on the interest rate policy rule depends on the weights  $\lambda_i$ ,  $\lambda_Y$  which capture the output and interest rate stabilization objectives of the planner. It also depends on the structural parameters  $\sigma$ ,  $\kappa_1$  and  $\beta$ .

The second component,  $\mathcal{Z}_t$ , is a pull factor which relates interest rates to the “bindness” of the ZLB constraint. If for example  $\hat{i}_{t-1} = i^*$  then (from complementary slackness) we have  $\psi_{ZLB,t-1} > 0$ . In this case  $\mathcal{Z}_t$  becomes negative and so the planner keeps the interest rate in  $t$  lower than the value implied by the Taylor rule component (and partially reverses the effect in  $t + 1$ ). The impact of this channel on the level of interest rates depends on  $\lambda_i$ , with higher values of  $\lambda_i$  leading to a lower impact, since the planner’s objective to minimize the deviation of the interest rate from its target becomes stronger.

Finally, the term  $\mathcal{D}_t$  measures the impact of changes in the value of the multiplier  $\psi_{gov,t}$  on the optimal interest rate path. To analyze this term use equation (11) setting  $\mathcal{I}_P = 0$ . We have

$$\psi_{gov,t+1} = \psi_{gov,t} + \epsilon_{t+1,G} \quad (16)$$

In other words, the multiplier  $\psi_{gov,t}$  is a random walk, and  $\epsilon_{t+1,G}$  denotes a mean zero, i.i.d shock to the value of the multiplier. We can now write (14) as:

$$\mathcal{D}_t = -\frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\lambda_i \sigma} \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \epsilon_{t-l,G} - \frac{\bar{b}_\delta}{\lambda_i} \sum_{l=0}^{\infty} \delta^l \left( \epsilon_{t-l,G} - \epsilon_{t-l-1,G} \right) - \frac{\bar{C} \omega_Y}{\bar{Y} \sigma \lambda_i} \epsilon_{t,G} \quad (17)$$

which relates  $\mathcal{D}_t$  to current and past shocks to the value of the multiplier.

What do these shocks capture? Notice that since (real) debt can either be financed through distortionary taxes or through distortionary inflation, ours is a standard model of optimal policy under incomplete markets (e.g. Aiyagari et al (2002), SGU (2004), Faraglia et al (2013), FMOS among others). As is well known, in these models shocks to the economy translate to changes in the excess burden of distortions, the multiplier  $\psi_{gov}$  that measures the magnitude of these distortions behaves like a random walk, since the planner wants to spread evenly the costs across periods.<sup>10</sup> In the presence of long term debt ( $\delta > 0$ ) the sequence of shocks  $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$  influences interest rates because all the lags of these variables enter into the state vector, as the FONC reveal.

To clarify further the role played by the sequence  $\{\epsilon_{t-l,G}\}_{l=0}^{\infty}$  we iterate forward on constraint (5) to get:

$$E_t \sum_{j=0}^{\infty} \beta^j \bar{S} \hat{S}_{t+j} = \frac{\bar{b}_\delta}{1 - \beta \delta} \left( \hat{b}_{t-1,\delta} + \hat{\xi}_t \right) + \bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j E_t \left[ -\sigma \left( \frac{\bar{Y}}{\bar{C}} \hat{Y}_{t+j} - \frac{\bar{G}}{\bar{C}} \hat{G}_{t+j} \right) - \sum_{l=0}^j \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right] \quad (18)$$

<sup>10</sup> To be accurate, even in the case of lump sum taxation  $\psi_{gov}$  evolves as a random walk when  $\mathcal{I}_P = 0$ , since the planner does not control taxes. However, in the case where  $\mathcal{I}_P = 1$ , where the planner influences taxes through debt, assuming non distortionary taxation would imply  $\psi_{gov,t} = 0$  for all  $t$ . To see this note that in this case (11) can be written as:

$$\frac{\beta \bar{b}_\delta}{1 - \beta \delta} \left( \psi_{gov,t} - E_t \psi_{gov,t+1} \right) + \beta \phi_{\tau,b} (1 - \rho_\tau) E_t \left( \sum_{j=0}^{\infty} (\rho_\tau \beta)^j \left( \frac{\bar{\tau} \bar{Y} (1 + \eta)}{\eta (1 - \bar{\tau})} \psi_{gov,t+1+j} \right) \right) = 0,$$

since lump sum taxes will not show up in the Phillips curve we have  $\kappa_2 = 0$ . The above difference equation gives  $\psi_{gov,t} = 0$  when  $\phi_{\tau,b} > 0$ .

In the case where inflation is not distortionary (i.e. the model does not admit a Phillips curve and  $\lambda_Y = 0$ ) it is easy to show that  $\psi_{gov,t} = 0$ .

where  $\bar{S}\hat{S}_t \equiv \left[ -\bar{G} \left( \hat{G}_t(1 + \sigma \frac{\bar{G}}{\bar{C}}) - \sigma \frac{\bar{Y}}{\bar{C}} \hat{Y}_t + \hat{\xi}_t \right) + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left( (1 + \gamma_h)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\tau} + \hat{\xi}_t \right) \right]$  is the surplus of the government multiplied by marginal utility.

(18) is the *intertemporal consolidated budget constraint* linking the present discounted value of the fiscal surplus to the real value of debt outstanding in  $t$ . Notice also that (18) is equivalent to (5) in terms of the Ramsey policy.<sup>11</sup> Consider the impact of a shock which lowers the LHS of (18) relative to the RHS. This may, for example, occur following a shock which lowers taxes. In response to this shock the constraint tightens, the value of the multiplier  $\psi_{gov}$  increases. Therefore,  $\epsilon_{t,G} > 0$ . To satisfy the constraint, the monetary authority needs to engineer a drop in the real payout of debt (the RHS of (18)) either through increasing future inflation and /or increasing future output when  $\sigma > 0$ . Note that under commitment it is feasible to make such promises about the future course of economic variables. The terms that enter in  $\mathcal{D}_t$  in equation (17) are essentially the promises made by the planner to manipulate inflation and output, and hence also the interest rate, in response to shocks to the consolidated budget which have occurred in the past. Following FMOS, we label this impact an 'interest rate twisting' effect of optimal policy.

## 2.3 Assuming No Debt Concerns

We now present an alternative setup in which the planner maximizes (7) subject to (2), (3) and (4) and leaves the consolidated budget (5) outside the optimal policy program. (5) continues to hold but is satisfied in equilibrium given optimal policies and the tax rule (6). In this version of the model the multiplier  $\psi_{gov,t}$  is obviously dropped from the list of model variables.

Letting superscript *NDC* denote the equilibrium under "no debt concerns", the first order conditions<sup>12</sup> become:

$$\begin{aligned} -\hat{\pi}_t^{NDC} + \Delta\psi_{\pi,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta} &= 0 \\ -\lambda_Y \hat{Y}_t^{NDC} - \psi_{\pi,t}^{NDC} \kappa_1 + \sigma \frac{\bar{Y}}{\bar{C}} (\psi_{i,t}^{NDC} - \frac{\psi_{i,t-1}^{NDC}}{\beta}) &= 0 \\ -\lambda_i \hat{i}_t^{NDC} + \psi_{i,t}^{NDC} + \psi_{ZLB,t}^{NDC} &= 0 \end{aligned}$$

The above equations give rise to the following interest rate rule:

**Proposition 2.** *Assume that the planner does not account for the consolidated budget in optimization. The optimal interest rate rule is given by*

$$\hat{i}_t^{NDC} = \max\{\mathcal{T}_t^{NDC} + \mathcal{Z}_t^{NDC}, -i^*\} \quad (19)$$

$$\mathcal{T}_t^{NDC} \equiv \phi_\pi \hat{\pi}_t^{NDC} + \phi_Y \Delta \hat{Y}_t^{NDC} + \phi_i \hat{i}_{t-1}^{NDC} + \frac{1}{\beta} \Delta \hat{i}_{t-1}^{NDC}$$

and

$$\mathcal{Z}_t^{NDC} = -\frac{1}{\lambda_i} \left( 1 + \frac{1}{\beta} + \frac{\kappa_1 \bar{C}}{\sigma \beta \bar{Y}} \right) \psi_{ZLB,t-1}^{NDC} + \frac{1}{\beta \lambda_i} \psi_{ZLB,t-2}^{NDC}.$$

Notice that the expressions for components  $\mathcal{T}_t^{NDC}$  and  $\mathcal{Z}_t^{NDC}$  in (19) are essentially the same as in the optimal rule (13) of the benchmark model. The difference between the two policy rules emerges because (19) omits the term  $\mathcal{D}_t$  which appears in the baseline 'debt concerns' version of optimal policy.

The two policy rules are also similar to equation (1), the interest rate rule employed by the recent DSGE literature. The terms  $\mathcal{Z}_t$  and  $\mathcal{D}_t$  (the latter in the case  $\delta > 0$ ) in the debt concerns model and

<sup>11</sup>See for example Aiyagari et al (2002).

<sup>12</sup>See online appendix for a detailed description of the planner's program.

the term  $Z_t^{NDC}$  in the no debt concerns model are functions of lagged state variables, and summarize changes in interest rates that have been revealed in the past and apply to policy in  $t$ . These terms capture *endogenous forward guidance*.

We next focus on the properties of the two models and show that they give rise to substantially different policy outcomes.

## 2.4 Model Properties Away from the ZLB

### 2.4.1 Determinacy under optimal policies

Let us first consider the case where the ZLB constraint is not binding. Since ours is a linear model approximated around a non-stochastic steady state with a positive nominal interest rate, equilibria with non-binding ZLB constraints can occur if shocks are not too big to drive the economy far away from steady state.<sup>13</sup> We begin by discussing the determinacy of the equilibrium under the two model versions.

In the case where the ZLB is not binding we have  $\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t$  and  $\hat{i}_t^{NDC} = \mathcal{T}_t^{NDC}$ . To derive analytical results we first consider a simplistic setup setting  $\lambda_Y = \sigma = \delta = \rho_\tau = \bar{G} = 0$ . We further assume that tax shocks are the only source of uncertainty in the economy.<sup>14</sup> Under these assumptions and using equations (9) and (10) to substitute out  $\psi_{\pi,t}$  and  $\psi_{i,t}$  in the debt concerns model we get:

$$-\hat{\pi}_t - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}_\delta \tilde{\eta} \Delta \psi_{gov,t} = 0 \quad (20)$$

where  $\tilde{\eta} = \left(1 + \frac{(1-\beta)(1+\gamma_h)}{\kappa_1}\right)$ .<sup>15</sup>  $\psi_{gov,t}$  is again a martingale and so  $E_t \Delta \psi_{gov,t+1} = 0$ . From the Euler equation we have  $\hat{i}_t = E_t \hat{\pi}_{t+1}$ .

Using the Phillips curve to substitute  $\hat{Y}_t = \frac{\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \kappa_2 \hat{\tau}_t}{\kappa_1}$  the consolidated budget constraint can be written as follows:

$$\beta \bar{b}_\delta \hat{b}_{t,\delta} - \beta \bar{b}_\delta \tilde{\eta} E_t \hat{\pi}_{t+1} + (1-\beta) \bar{b}_\delta \left( \frac{1}{1-\bar{\tau}} - \frac{\kappa_2}{\kappa_1} (1+\gamma_h) \right) \hat{\tau}_t = \bar{b}_\delta \hat{b}_{t-1,\delta} - \bar{b}_\delta \tilde{\eta} \hat{\pi}_t \quad (21)$$

Finally, using  $\frac{\kappa_2}{\kappa_1} = \frac{\bar{\tau}}{(1-\bar{\tau})\gamma_h}$  and the tax rule (6) we have:

$$\hat{b}_{t,\delta} - \tilde{\eta} E_t \hat{\pi}_{t+1} = \underbrace{\frac{1}{\beta} \left[ 1 - \frac{(1-\beta)\phi_{\tau,b}}{(1-\bar{\tau})\gamma_h} \left( \gamma_h - \bar{\tau}(1+\gamma_h) \right) \right]}_{\tilde{\epsilon}} \hat{b}_{t-1,\delta} - \underbrace{\frac{\tilde{\eta}}{\beta} \hat{\pi}_t - \frac{1}{\beta} \frac{(1-\beta)}{(1-\bar{\tau})\gamma_h} \left( \gamma_h - \bar{\tau}(1+\gamma_h) \right)}_A \epsilon_{\tau,t} \quad (22)$$

Coefficient  $\tilde{\epsilon}$  is a key object in determining the dynamics of government debt in our model. In the case where  $\tilde{\epsilon} > 1$  government debt becomes an explosive process, whereas if  $\tilde{\epsilon} < 1$  debt is a stationary process. Notice that the magnitude of  $\tilde{\epsilon}$  hinges crucially on the size of the feedback effect of lagged debt on taxes,  $\phi_{\tau,b}$ , and also on the steady state level of taxes and the Frisch elasticity  $\frac{1}{\gamma_h}$  since these quantities influence the responsiveness of aggregate hours and tax revenues to shocks when taxes are distortionary.

To identify which values for  $\tilde{\epsilon}$  give rise to a unique rational expectations equilibrium use the Euler equation together with (20) to write

$$\frac{\lambda_i \hat{i}_t}{\beta} = -E_t \hat{\pi}_{t+1} + \bar{b}_\delta \tilde{\eta} E_t \Delta \psi_{gov,t+1} = -\hat{i}_t \quad (23)$$

<sup>13</sup>In Section 4 we will estimate our model setting  $\psi_{ZLB,t} = 0$  for all  $t$  with data from the US economy and over the period 1980 to 2008. We will then apply the estimates to the Great Recession allowing for occasionally binding ZLB constraints. The analysis of this subsection is therefore important to understand the working of the model.

<sup>14</sup>Our findings in this section do not hinge on the nature of shocks that hit the economy. We assume only tax shocks and set  $\bar{G} = 0$  to simplify the algebra; otherwise we could assume more than one disturbance in the model and all results described below would carry through.

<sup>15</sup>Under  $\sigma = \bar{G} = 0$  we have  $\omega_Y = \frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} (1+\gamma_h)$ . From the budget constraint we get  $\frac{\bar{\tau}(1+\eta)}{\eta} \bar{Y} = (1-\beta) \bar{b}_\delta$ .

where the last equality makes use of the martingale property of  $\psi_{gov,t+1}$ . From (23) we get  $\hat{i}_t = 0$  when  $\lambda_i \geq 0$ . Therefore, we have  $\hat{\pi}_t = \bar{b}_\delta \tilde{\eta} \Delta \psi_{gov,t}$  and with this we can write (22) as

$$\hat{b}_{t,\delta} = \tilde{\epsilon} \hat{b}_{t-1,\delta} - \frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t} - A \epsilon_{\tau,t} \quad (24)$$

Equation (24) together with the martingale property  $E_t \Delta \psi_{gov,t+1} = 0$  pin down the values of  $\hat{b}_{t,\delta}$  and  $\Delta \psi_{gov,t}$ .

It is now easy to show that the solution to this system of equations is unique only when  $\tilde{\epsilon} > 1$ . To see this assume that  $\tilde{\epsilon}$  is less than one and solve equation (24) backwards to obtain:

$$\hat{b}_{t,\delta} = - \sum_{j=0}^{\infty} \tilde{\epsilon}^j \left( \frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t-j} + A \epsilon_{\tau,t-j} \right) \quad (25)$$

which determines the debt level at  $t$  as function of the lagged shocks  $\Delta \psi_{gov,t-j}$  and  $\epsilon_{\tau,t-j}$ . Notice that the value of  $\Delta \psi_{gov,t-j}$  is not pinned down in this model; the martingale property  $E_{t-j-1} \Delta \psi_{gov,t-j} = 0$  does not determine a unique value for this object.

In the unique rational expectations equilibrium with  $\tilde{\epsilon} > 1$ , (24) is solved forward giving

$$\hat{b}_{t-1,\delta} = E_t \sum_{j=0}^{\infty} \frac{1}{\tilde{\epsilon}^{j+1}} \left( \frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t+j} + A \epsilon_{\tau,t+j} \right) = \frac{1}{\tilde{\epsilon}} \left( \frac{\bar{b}_\delta \tilde{\eta}^2}{\beta} \Delta \psi_{gov,t} + A \epsilon_{\tau,t} \right) \quad (26)$$

From (26) the equilibrium satisfies  $\Delta \psi_{gov,t} = -\frac{\beta}{\bar{b}_\delta \tilde{\eta}^2} A \epsilon_{\tau,t}$  and therefore  $\hat{b}_{t,\delta} = 0$  for all  $t$ <sup>16</sup>.

Now consider the case of no debt concerns. We have  $\psi_{gov,t} = 0$  for all  $t$  and the consolidated budget constraint can be written as:

$$\hat{b}_{t,\delta}^{NDC} = \tilde{\epsilon} \hat{b}_{t-1,\delta}^{NDC} - A \epsilon_{\tau,t} \quad (27)$$

A unique equilibrium can be found here when  $\tilde{\epsilon} < 1$  so that (27) is solved backwards. We summarize the above findings in the following Proposition.

**Proposition 3.** *Assume that parameters  $\lambda_Y$ ,  $\bar{G}$ ,  $\delta$  and  $\sigma$  equal zero. Assume further that the planner takes into account the consolidated budget constraint. Determinacy of the equilibrium requires:*

$$\tilde{\epsilon} \equiv \frac{1}{\beta} \left[ 1 - \frac{(1-\beta)\phi_{\tau,b}}{(1-\bar{\tau})\gamma_h} \left( \gamma_h - \bar{\tau}(1+\gamma_h) \right) \right] > 1$$

*In contrast in the “no debt concerns case”, where the planner does not account for the consolidated budget, determinacy requires  $\tilde{\epsilon} < 1$ .*

Several comments are in order. Notice first that the above condition extends the analysis of Leeper (1991) to the case of distortionary taxes and in a model where monetary policy is optimal. In the case where  $\tilde{\epsilon} < 1$  (the no debt concerns model) fiscal policy is sufficiently responsive to debt levels, and so taxes adjust to guarantee the sustainability of debt. In the sense of Leeper (1991), fiscal policy is “passive”. In contrast, in the baseline model with debt concerns, fiscal policy needs to be “active” for the rational expectations equilibrium to be unique or, to put it differently, taxes should not respond aggressively to deviations of debt from its steady state value.

Second, note that the above finding, that determinacy requires an explosive debt process in the baseline model, does not hinge on the assumptions made in Proposition 3. It holds more generally, for positive values of  $\lambda_Y$ ,  $\sigma$ ,  $\delta$ , and  $\bar{G}$ , and therefore, it holds when the optimal policies are as described in Propositions 1 and 2. What do these optimal policies tell us about whether monetary policy is active/passive?

<sup>16</sup>In other words debt will not explode in this example if it equals zero for all  $t$ .

Generally, in the no debt concerns model monetary policy is active since by construction the Fed does not take into account the debt constraint, and equilibrium inflation in this model will not be impacted by tax shocks, beyond the extent that these shocks influence the inflation output tradeoff through the Phillips curve. In contrast, in the debt concerns model the opposite holds; monetary policy is passive because the Fed adjusts inflation in response to changes in the level of deb; shocks to distortionary taxes influence inflation also through their impacts on the budget constraint (see the simulation results of the next subsection).

Admittedly, Leeper’s classification of monetary policy into active/passive which hinges on the response of interest rates on inflation (and the output gap) is not easy to map into our optimal policy rule analytically. We can however verify numerically that the rule defined in Proposition 2 is an active monetary policy, in the sense of Leeper, when  $\lambda_Y, \lambda_i, \sigma$  are positive. Indeed, we found this is the case. And since  $\mathcal{T}^{NDC}$  defines an active monetary policy, then this must mean that  $\hat{i}_t = \mathcal{T} + \mathcal{D}$  defines a passive monetary policy, when  $\mathcal{T}^{NDC} = \mathcal{T}$  as we previously showed.

This may seem odd since  $\mathcal{D}$  is a moving average of mean zero shocks to the Lagrange multiplier. However, note that these shocks ought to be correlated with inflation, output growth etc. If they are not correlated, and interest rates are on average equal to  $\mathcal{T}$  in the model, then the equilibrium is indeterminate and so inflation becomes an explosive process (see for example Leeper and Leith (2016)). Another way of saying this, is the following: Suppose we use simulated data from the model to regress interest rates on inflation, output growth and lagged values of interest rates, but assume that  $\mathcal{D}$  is unobservable. Then, if  $\mathcal{D}$  is orthogonal to the right hand side variables, the estimates of  $\mathcal{T}$  should be close to the values corresponding to the analytical solution of Proposition 1. In contrast, if  $\mathcal{D}$  is not orthogonal, then the coefficients on the variables in  $\mathcal{T}$  will differ from the analytical expression.

In Table 1 we show the output of this regression. The model is calibrated assuming the parameter values reported in Table 2 (see below). The first column of Table 1 reports the values of the coefficients of the independent variables. The second column reports the estimates obtained from the ‘no debt concerns’ model, and columns 3 -5 report the debt concerns model’s estimates when we vary the sources of uncertainty in the model. Column 3 assumes only preference shocks, column 4 spending shocks and column 5 tax shocks. In all cases we assume a first order autocorrelation of 0.9. Clearly, under all scenarios the debt concerns regressions deliver a weaker reponse of interest rates to inflation and output growth than in the analytical solution, and obviously this implies that  $\mathcal{D}$  is not orthogonal to these variables.<sup>17</sup>

[ Table 1 About Here]

## 2.4.2 The effects of shocks

We now investigate how economic shocks affect the behavior of interest rates and inflation in the two versions of the model. We first derive the impulse responses analytically, in a simplified setup assuming, as previously,  $\lambda_Y = \lambda_i = \sigma = 0$  but now we let  $\bar{G}, \delta, \rho_\tau > 0$ . We consider the impact of shocks at date 0 which change the values of parameters  $\{G_0, \xi_0, \tau_0\}$  assuming that after  $t = 0$  there are no further shocks to the economy. Under these assumptions we derive in the appendix analytically the optimal paths of inflation and interest rates and the path of the multiplier  $\psi_{gov,t}$ .

The appendix shows that equilibrium inflation and interest rates under debt concerns are given

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<sup>17</sup>Do we also get passive monetary policy in all cases in Columns 3-5? No we do not! When we use the estimates reported in Column 3 (for instance) as values for an ad hoc policy rule, we find that fiscal policy needs to be passive for the equilibrium to be determinate. This may seem odd but note that it is consistent with previous findings in the literature. For instance, SGU reach a similar conclusion when they estimate model implied Taylor rules from their joint monetary/fiscal policy program.

by

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta \delta^t}{1 - \beta\delta} \Delta\psi_{gov,0} \quad (28)$$

$$\hat{i}_t = -\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0 + \frac{\bar{b}_\delta \delta^{t+1}}{1 - \beta\delta} \Delta\psi_{gov,0} \quad (29)$$

for  $t = 0, 1, 2, \dots$  and where  $\mathcal{I}_{t=0}$  takes the value 1 at  $t = 0$  and 0 otherwise. Moreover,  $\Delta\psi_{gov,0}$  is a linear function of  $\{\hat{G}_0, \hat{\xi}_0, \hat{\tau}_0\}$ , the partial derivatives satisfy  $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{G}_0} > 0$ ,  $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\tau}_0} < 0$  and finally  $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\xi}_0} > 0$ .

In the no debt concerns model we have  $\hat{i}_t = -\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$  and  $\hat{\pi}_t = 0$ .

Consider first the impact of a preference shock which lowers the value  $\hat{\xi}_0$ , focusing on the no debt concerns equilibrium. The predictions of this model are standard: Since the planner does not want to smooth the nominal interest rate, i.e.  $\lambda_i = 0$ ,  $\hat{i}_t$  drops one for one with the real rate thus fully stabilizing inflation.

Now consider the case of the 'debt concerns' model. Since  $\frac{\partial \Delta\psi_{gov,0}}{\partial \hat{\xi}_0} > 0$ , a drop in  $\hat{\xi}_0$  lowers  $\Delta\psi_{gov,0}$ . From (28) and (29) we see that inflation turns negative after the shock and the nominal interest rate drops below real rate,  $-\rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$ .

Notice that a preference shock has two impacts on the consolidated budget: First, it increases real bond prices and hence increases the real payout of government debt; second, it increases the present value of surpluses which finance the debt. Since the planner has to satisfy the intertemporal constraint she will either use inflation or use deflation to adjust the real payout of debt so that the constraint is satisfied with equality. If the first effect is stronger, and real debt increases faster than the surpluses, then inflation is optimal. In contrast, if the second effect dominates, then it is optimal to make inflation negative. We find that the second effect is stronger and so inflation becomes negative in response to the preference shock.<sup>18</sup>

A similar argument applies to the case where spending levels increase in period 0 and in the case where taxes drop. In both cases the consolidated budget constraint tightens and so  $\Delta\psi_{gov,0} > 0$ . From (28) and (29), interest rates rise in response to the shocks in the debt concerns model and inflation rises above its steady state value. Positive inflation here ensures the solvency of the consolidated budget because the shocks lower the surplus. In contrast, under no debt concerns inflation does not respond to these shocks.

Figure 1 shows the above responses over 40 periods. The top two rows show the responses of inflation and interest rates, whereas the third and fourth rows plot the analogous responses of output and the (market value of) debt to GDP ratio. The first column shows the case of the preference shock, the middle column spending, and the right column considers the case of a negative shock to taxes. The values we assign to the model's parameters are presented in Table 2. The top panel of the table reports values for parameters which are common across the two versions of the model, the bottom panel reports the value that we assign to  $\phi_{\tau,b}$  in each version separately. In the debt concerns model we set  $\phi_{\tau,b} = 0.0$  and in the case where the Fed has "no debt concerns" we set  $\phi_{\tau,b} = 0.07$ .<sup>19</sup>

[ Table 2 About Here ]

[Figure 1 About Here]

The equilibrium output in the debt concerns model is given by:

$$\hat{Y}_t = \frac{1}{\kappa_1} \left( \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0 \right)$$

<sup>18</sup>This property holds for all values  $\delta < 1$ . In the case where the long bonds are consols preference shocks have zero impact on the consolidated budget.

<sup>19</sup> $\phi_{\tau,b} = 0.07$  is chosen in the no debt concerns model so that debt has a near unit root, consistent with the empirical evidence on US government debt (see e.g. Marcat and Scott (2009) among others).

The value of  $\phi_{b,\tau}$  which gives  $\tilde{\epsilon} = 1$ , the cutoff for determinacy, is equal to 0.065.

(see appendix), and so output drops in response to the preference shock and rises in response to the spending and tax shocks. Under no debt concerns, output changes in response to changes in taxes.<sup>20</sup> This implies a slight increase in output in response to the preference shock in that model, a slight drop in response to the  $\hat{G}$  shock and a rise in response to the negative shock in taxes.

Finally, notice that because of the differences in the responses of interest rates and output in the two models, the market value of debt to GDP responds differently to the shocks. For instance, in the case of the spending shock, the market value increases in the no debt concerns model, because the surplus drops, and, at the same time, interest rates do not respond strongly to the shock. In the debt concerns model, however, since taxes do not respond to debt, the market value of debt needs to drop in period 0 so that the intertemporal budget is satisfied. This is accomplished through a sharp rise in interest rates, which lowers bond prices.

In Figure 2 we assume  $\lambda_i = \lambda_Y = 0.5$  and  $\sigma = 1$ . Notice that now inflation and output change also in the no debt concerns model in response to the shocks, but qualitatively the comparison between the two models remains. In the debt concerns model inflation becomes negative in response to the preference shock, and positive in response to the spending and tax shocks. The (numerical) partial derivatives  $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{G}_0}$ ,  $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\tau}_0}$ ,  $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\xi}_0}$  have the same sign as before. This implies that interest rates drop more in the debt concerns model, following a negative shock to preferences and they rise (relative to the no debt concerns model) in response to shocks to the fiscal variables.<sup>21</sup>

[Figure 2 About Here]

## 2.5 Optimal Policies at the ZLB

We now turn to the case where interest rates hit the ZLB and analyse the properties of the two versions of our model. We first consider the impact of a shock, which lowers the value of  $\hat{\xi}_0$  sufficiently so that the ZLB binds. Before resorting to numerical simulations to compute impulse responses with the calibrated parameter values reported in Table 2, we present analytical results in this subsection assuming  $\sigma = \lambda_Y = 0$  but now letting wlog  $\lambda_i \geq 0$ . In the case where the preference shock is i.i.d we can derive in both models analytically the path of inflation and the nominal interest rates. The results are summarized in the following Proposition.

**Proposition 4.** *Assume  $\sigma = \lambda_Y = 0$  and consider the impact of an i.i.d shock which lowers the value of  $\hat{\xi}_0$  so that the ZLB binds in period 0 ( $\hat{\xi}_0 < -\hat{i}^*(1 + \frac{\lambda_i}{\beta})$ ). Thereafter assume that no further shock occurs. The optimal paths of inflation and the nominal interest rate are given by*

$$\hat{\pi}_t = \begin{cases} \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} + \frac{\bar{b}_\delta}{1-\beta\delta} \Delta \psi_{gov,0} & t = 0 \\ -\hat{i}^* - \hat{\xi}_0 & t = 1 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^t}{1-\beta\delta} \frac{\Delta \psi_{gov,0}}{1+\frac{\lambda_i}{\beta}}\} & t \geq 2 \end{cases} \quad \hat{i}_t = \begin{cases} -\hat{i}^* & t = 0 \\ \max\{-\hat{i}^*, \frac{\bar{b}_\delta \delta^t}{1-\beta\delta} \frac{\Delta \psi_{gov,0}}{1+\frac{\lambda_i}{\beta}}\} & t \geq 1 \end{cases} \quad (31)$$

<sup>20</sup>From the Phillips curve we have

$$\hat{Y}_t^{NDC} = -\frac{\kappa_2}{\kappa_1} \hat{\tau}_t^{NDC} \quad (30)$$

where now  $\kappa_1 \equiv -\frac{(1+\eta)\bar{Y}}{\theta} \gamma_h$ ,  $\kappa_2 \equiv -\frac{(1+\eta)\bar{Y}}{\theta} \frac{\bar{\tau}}{(1-\bar{\tau})}$ .

In the case of the negative shock in preferences taxes drop. Even though the market value to GDP increases, due to the rise in bond prices, fewer bonds need to be issued and so  $\hat{b}_{t,\delta}$  is lower along the optimal path. Analogously, in the case of the positive spending shock, the face value of debt is higher and this implies higher taxes.

<sup>21</sup>At first sight this may appear inconsistent with (13) since (for example)  $\frac{\partial \Delta \psi_{gov,0}}{\partial \hat{\xi}_0} < 0$  implies that  $\mathcal{D}$  turns positive in response to the  $\hat{x}_i$  shock. However, the fact that  $\mathcal{D}$  is greater than zero, does not imply that interest rates should be higher in the debt concerns model. Recall that  $\mathcal{D}$  is correlated with the variables in  $\mathcal{T}$ .

in the case of the debt concerns model and

$$\hat{\pi}_t^{NDC} = \begin{cases} 0 & t = 0 \text{ and } t \geq 2 \\ -\hat{i}^* - \hat{\xi}_0 & t = 1 \end{cases} \quad \hat{i}_t^{NDC} = \begin{cases} -\hat{i}^* & t = 0 \\ 0 & t \geq 1 \end{cases} \quad (32)$$

in the no debt concerns model.

**Proof.** See appendix.

There are several noteworthy features. First, as is evident from (32), in the no debt concerns model, the planner does not commit to keep interest rates low at the exit from the LT episode. Instead,  $\hat{i}_t^{NDC}$  is at the ZLB in  $t = 0$  only and when the shock dies out, the optimal interest rate returns to the steady state. This is clearly a simplistic example, in which to satisfy the ZLB the planner promises to keep inflation high in period 1 only.<sup>22</sup> As a result we have  $\mathcal{Z}_t^{NDC} = 0$ .

In the case of the debt concerns model interest rates remain at the ZLB even after the shock dies out, due to the impact of  $\mathcal{D}$  on the optimal interest rate path. Notice that following a shock to preferences the present value of the surplus will again rise and  $\Delta\psi_{gov0} < 0$ . From (31)  $\hat{i}_t$  is negative for  $t > 0$  and gradually converges towards zero. Inflation is negative at  $t = 0$ , then switches sign at  $t = 1$ , in which case we have  $\hat{\pi}_1 = -\hat{i}^* - \hat{\xi}_0 > 0$ , and for  $t > 1$  inflation turns again negative.

What is the planner trying to do? Since debt is an explosive process, to satisfy the consolidated budget, inflation must become negative in response to the shock. However, with negative inflation, the ZLB would be violated in period zero and so the planner sets  $\hat{\pi}_1 > 0$  to satisfy the constraint. Inflation becomes again negative in periods  $t = 2, 3, \dots$  to ensure debt sustainability. This requires to keep the nominal interest rate below the steady state value.<sup>23</sup>

The above properties continue to hold under the more plausible calibration reported in Table 2, letting  $\sigma = 1$  and  $\lambda_Y = \lambda_i = 0.5$ . Figure 3 shows the responses of inflation, output, interest rates debt and taxes under the two versions of the model. The solid (blue) line shows responses in the baseline debt concerns model. The dashed line shows responses under the “no debt concerns” model. Qualitatively, the pattern of adjustment of inflation, output and interest rates resembles the pattern described in Proposition 4. Under “no debt concerns” inflation becomes only slightly negative when the shock hits, and then turns positive during the period when the interest rate is at the ZLB. Inflation gradually goes back to zero after the economy escapes from the LT, as interest rates gradually return to their steady state value. Notice that since we now assume  $\sigma > 0$ , component  $\mathcal{Z}$  exerts an influence on the response of the nominal interest rate to the shock. When the shock hits aggregate output drops, however, the planner promises to gradually increase output and ultimately engineer a boom as the economy escapes from the LT. These are standard properties of the NK model’s response to the LT shock (see Eggertsson and Woodford (2003)). Under optimal policy with commitment the planner can ward off the LT shock very effectively.

[Figure 3 About Here]

In the case of the debt concerns model, we see that following the preference shock in period 0, inflation drops sharply. Inflation becomes positive for few periods, but subsequently turns negative again. As discussed previously, there are two opposing forces in this model that influence the path of inflation: The first, is that a positive inflation rate at the ZLB is needed to mitigate the impact of the ZLB. The second, is that since debt is explosive, negative inflation rates are necessary to ensure debt sustainability. The planner responds to this tradeoff through raising inflation in short term and committing to a negative inflation rate in the longer term.

<sup>22</sup>Higher inflation in period 1 is achieved by increasing output  $\hat{Y}_1$  sufficiently.  $\hat{Y}_2, \hat{Y}_3, \dots$  equal zero.

An alternative policy would be to raise  $\hat{\pi}_1$  to satisfy the ZLB through promising a boom beyond period 1. This would imply a smaller increase in  $\hat{Y}_1$  and  $\hat{Y}_2, \hat{Y}_3, \dots > 0$ . This policy is not optimal here, since the planner does not want to smooth output and higher output in periods 2,3,... implies higher inflation in these periods.

<sup>23</sup> Notice that these properties hinge on the presence of long term debt in the model. In the case where  $\delta = 0$  optimal policy would engineer a strong deflation in period 0 and positive inflation in period 1 to satisfy both constraints. Thereafter,  $\hat{\pi}_t$  and  $\hat{i}_t$  would equal 0.

## 2.6 The role of commitment

The key result of this section is that the presence of debt concerns can destabilize inflation during LTs. Inflation can drop considerably and over many periods and at the same time, interest rates remain at the ZLB for many quarters. Therefore, when monetary policy is overridden by debt sustainability concerns, keeping interest rates low does not imply that the negative effects of the LT on inflation are warded off. In contrast, in the no debt concerns model, monetary policy is very effective in stabilizing output and inflation, the binding ZLB does not lead to negative inflation rates or substantial output losses.

In the next sections we will put our theory to work, estimating the model with data from the US, and applying the estimates to the Great Recession. As described in the introduction, our aim is to outline the strengths and the weaknesses of each model in matching the data observations, most notably the observed persistence of interest rates at the ZLB and the lack of deflation in the US economy. Since the debt concerns model implies that inflation turns negative in response to shocks to preferences, a pure debt concerns equilibrium policy will not be able to explain the data.

Before delving in the quantitative evaluation of the model, it is worth exploring in this final paragraph of the theory section, how our assumption of full commitment affects the responses of economic variables to the LT shock. In the online appendix we setup an optimal policy problem without commitment, essentially assuming that the planner reoptimizes in every period, and sets the lagged values of the multipliers  $\psi$  to zero.<sup>24</sup> In this model  $\mathcal{D}$  and  $\mathcal{Z}$  both equal 0 and so there is no FG of interest rates. The optimal interest rate rules become:

$$\hat{i}_t = \max\left\{\frac{\kappa_1 \bar{C}}{\sigma \lambda_i \bar{Y}} \hat{\pi}_t + \frac{\lambda_Y \bar{C}}{\sigma \lambda_i \bar{Y}} \hat{Y}_t - \omega_{gov} \psi_{gov,t}, -\hat{i}^*\right\} \quad (33)$$

in the case the planner takes into account the consolidated budget<sup>25</sup> and

$$\hat{i}_t = \max\left\{\frac{\kappa_1 \bar{C}}{\sigma \lambda_i \bar{Y}} \hat{\pi}_t + \frac{\lambda_Y \bar{C}}{\sigma \lambda_i \bar{Y}} \hat{Y}_t, -\hat{i}^*\right\} \quad (34)$$

under no debt concerns.

Figure 4 shows the impulse responses to the LT shock in this no commitment model. As before, the debt concerns model is represented with solid lines and no debt concerns with dashed lines. Notice first that in both models, removing commitment, leads to a large drop in inflation at the start of the episode and to substantial output losses, consistent with previous findings in the literature.<sup>26</sup> Second, note that inflation now becomes more negative in the no debt concerns model and also interest rates remain longer at the ZLB in this model. Therefore, lack of commitment reverses the conclusions of the previous section, that LT last longer in the debt concerns model but policy fails to stabilize inflation.

[Figure 4 About Here]

What is going on? There are two important effects. Firstly, since output now drops more in response to the preference shock, government deficits are larger in the short run and this contains the rise of the present value of surpluses which is due to the drop in interest rates. After the shock, the market value increases more than the surpluses and we find that  $\psi_{gov}$  turns positive in response to the shock. This contributes towards making inflation higher. Second, under no commitment, inflation responds to high debt levels and hence in every period where debt is above its steady state value, inflation is higher. This explains why the inflation rate in the debt concerns model is higher than in the no debt concerns case even when the shocks have died out.

<sup>24</sup>This follows, for instance, Debortoli and Lakdawala (2016).

<sup>25</sup> $\omega_{gov}$  equals  $\frac{\bar{C}}{\bar{Y} \sigma \lambda_i} (\sigma \frac{\bar{Y}}{\bar{C}} \bar{b}_\delta + \omega_Y + \frac{\kappa_1 \bar{b}_\delta}{(1-\beta\delta)})$ . Moreover,  $\psi_{gov}$  continues to evolve as a random walk.

See online appendix for these derivations.

<sup>26</sup>See e.g. Eggertsson and Woodford (2003, 2006).

Our focus in this paper is on FG and thus on policies under commitment, however, it is important to note that commitment exerts a crucial influence. The recent work of Bianchi and Melosi (2017) finds that inflation in the US did not turn very negative in 2009-10, because rational agents anticipated that the Fed would monetize part of government debt. In their model, in response to a LT shock, there is a rise in the debt to GDP ratio, and a rise in inflation under the fiscally led regime. A likely source of the differences between our model's predictions and theirs, is commitment.

### 3 Quantitative Model

In this section we lay out the quantitative model that we estimate with US data. The model augments the baseline NK model we developed in the previous section with habit formation, a trend in TFP, and shocks to markups, TFP and to the consolidated budget.

#### 3.1 Households, Firms and Government

##### 3.1.1 Households

We assume that household preferences are of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left( \log(C_t - \Omega C_{t-1}^A) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

where  $C_t$  denote the consumption of the household,  $\Omega C_{t-1}^A$  is an external habit stock, where  $0 < \Omega < 1$  and  $C_{t-1}^A$  denotes the average level of consumption in  $t-1$ . The household derives disutility from exerting labor effort  $h_t$ . Parameters  $\chi$  and  $\gamma_h$  govern the household's preferences over leisure. As previously,  $\xi_t$  is the preference shifter which impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_{t,L} B_{t,\delta} + P_t C_t + B_{t,S} P_{t,S} = P_t W_t h_t (1 - \tau_t) + B_{t-1,S} + (1 + \delta P_{t,L}) B_{t,\delta} + P_t D_t$$

$B_{t,\delta}$  is a long government bond which pays decaying coupons to the household.  $P_{t,L}$  is the price of the asset.  $B_{t,S}$  denotes the quantity of short term (one-period) government debt purchased by the household. We assume that short debt is in zero net supply.  $W_t$  denotes the nominal wage and  $P_t$  is the price level. Finally,  $D_t$  is real dividends paid by monopolistically competitive firms to the household.

The first order conditions from the household's program are:

$$\begin{aligned} P_{t,L} &= \beta E_t \frac{u_c(C_{t+1} - \Omega C_t^A)}{u_c(C_t - \Omega C_{t-1}^A)} \frac{\xi_{t+1}}{\xi_t} \frac{P_t}{P_{t+1}} (1 + \delta P_{L,t+1}) \\ \chi \frac{h_t^{\gamma_h}}{u_c(C_t - \Omega C_{t-1}^A)} &= w_t (1 - \tau_t) \quad \text{and} \\ P_{t,S} &= \beta E_t \frac{u_c(C_{t+1} - \Omega C_t^A)}{u_c(C_t - \Omega C_{t-1}^A)} \frac{P_t}{P_{t+1}} \frac{\xi_{t+1}}{\xi_t} \end{aligned}$$

where  $u_c$  denotes the marginal utility of the consumption.

##### 3.1.2 Firms

We assume that output is produced by a continuum of monopolistically competitive firms which operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer, that aggregates the intermediate products of firms using a Dixit-Stiglitz aggregator.

The production function of the generic intermediate good firm  $k$  is  $y_{k,t} = A_t h_{k,t}^{1-\alpha}$ , where  $A_t$  denotes the level of TFP in the economy. The demand for product  $k$  is given by  $Y_t d(P_{k,t}/P_t, \eta_t)$ , where  $P_{k,t}$  is the price of  $k$ , and  $Y_t$  is output in the final-good sector.  $\eta_t$  is a (time varying) parameter that governs the elasticity of substitution across differentiated products. The demand function,  $d$ , satisfies standard assumptions that guarantee the existence of a symmetric equilibrium.

We further assume that intermediate goods firms face price adjustment costs as in Rotemberg (1982). The cost function of firm  $i$  is the following:  $\frac{\theta}{2} \left( \frac{P_{k,t}}{P_{k,t-1}} - \bar{\pi}^{1-\zeta} \pi_{t-1}^\zeta \right)^2$ .  $\theta \geq 0$  governs the degree of price stickiness.  $\bar{\pi}$  is the steady state level of gross inflation.

Intermediate-good producers seek to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \frac{u_c(C_{t+i} - \Omega C_{t+i-1}^A) \xi_{t+i}}{u_c(C_t - \Omega C_{t-1}^A) \xi_t} \left\{ \frac{P_{k,t+i}}{P_{t+i}} Y_{t+i} d\left(\frac{P_{k,t+i}}{P_{t+i}}, \eta_{t+i}\right) - w_{t+i} h_{k,t+i} - \frac{\theta}{2} \left( \frac{P_{k,t+i}}{P_{k,t+i-1}} - \bar{\pi}^{1-\zeta} \pi_{t+i-1}^\zeta \right)^2 \right\}, \quad (35)$$

subject to the constraint  $A_{t+i} h_{k,t+i}^{1-\alpha} = Y_{t+i} d(P_{k,t+i}/P_{t+i}, \eta_{t+i})$ . The first-order condition with respect to  $P_{k,t}$  is given by

$$\begin{aligned} \frac{1}{P_t} Y_t d\left(\frac{P_{k,t}}{P_t}\right) + \frac{P_{k,t}}{P_t^2} Y_t d'\left(\frac{P_{k,t}}{P_t}, \eta_t\right) - w_t Y_t d'\left(\frac{P_{k,t}}{P_t}, \eta_t\right) \frac{1}{P_t} - \theta \left( \frac{P_{k,t}}{P_{k,t-1}} - \bar{\pi}^{1-\zeta} \pi_{t-1}^\zeta \right) \frac{1}{P_{k,t-1}} \\ + \beta E_t \frac{u_{c,t+1} \xi_{t+1}}{u_{c,t} \xi_t} \theta \left( \frac{P_{k,t+1}}{P_{k,t}} - \bar{\pi} \right) \frac{P_{k,t+1}}{P_{k,t}^2} = 0. \end{aligned}$$

Finally, we assume that TFP evolves according to the following stochastic process

$$\ln(A_t/A_{t-1}) = \gamma + \hat{a}_t$$

where  $\hat{a}_t$  is a first order autoregressive process (see below).

### 3.1.3 Government

The government levies distortionary taxes to finance exogenous spending  $G_t$ . Imposing that short debt is in zero net supply we write the flow budget constraint as:

$$P_{t,L} B_{t,\delta} = (1 + \delta P_{t,L}) B_{t-1,L} + P_t (G_t + T_t - w_t h_t \tau_t) + \Lambda_t$$

$G_t - w_t h_t \tau_t$  is the real primary surplus. As Bianchi and Melosi (2017), we augment the flow budget with an exogenous shock variable  $\Lambda_t$  capturing features that we have left outside the model. These could derive from changes in the maturity of debt or the term premium, but also (more crucially) by variation in revenues and spending from sources that we do not model explicitly here (e.g. tax revenues from capital income / consumption, public investment, transfers etc.).

As in Section 2 we assume that taxes follow an exogenous rule which relates the current tax rate to the lagged value of debt. In the next subsection we define this rule in the log-linear version of the model.

## 3.2 Log-Linear Model

Since productivity grows over time in our model we rescale model variables and linearize the model equations around the deterministic steady state. For brevity we delegate all derivations to the online

appendix. The linear equations which define the constraint set of the planner are the following:

$$\hat{C}_t = -\tilde{h}e^{-\gamma}\Delta\hat{C}_t^A + E_t\hat{C}_{t+1} + \tilde{h}e^{-\gamma}E_t\Delta\hat{a}_{t+1} + \left(1 - \tilde{h}e^{-\gamma}\right)\left(-\Delta\hat{\xi}_{t+1} - \hat{i}_t + \hat{\pi}_{t+1} + E_t\hat{a}_{t+1}\right) \quad (36)$$

$$\hat{p}_{L,t} = -\hat{C}_t - \tilde{h}e^{-\gamma}\Delta\hat{C}_t^A + E_t\hat{C}_{t+1} + \tilde{h}e^{-\gamma}E_t\Delta\hat{a}_{t+1} + \left(1 - \tilde{h}e^{-\gamma}\right)\left(-\Delta\hat{\xi}_{t+1} + \frac{\beta\delta e^{-\gamma}}{\bar{\pi}}\hat{p}_{L,t+1} + \hat{\pi}_{t+1} + E_t\hat{a}_{t+1}\right) \quad (37)$$

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t \quad (38)$$

$$\hat{\mu}_t = \frac{1}{1 - \tilde{h}e^{-\gamma}}\left(\hat{C}_t - \tilde{h}e^{-\gamma}\hat{C}_{t-1} - \hat{a}_t\right) + \frac{\hat{\tau}_t}{1 - \bar{\tau}} + \frac{\alpha + \gamma h}{1 - \alpha}\hat{Y}_t \quad (39)$$

$$\bar{p}_L\bar{b}_\delta\left(\hat{p}_{L,t} + \hat{b}_{t,\delta}\right) + (1 - \nu)\bar{\tau}\bar{Y}\left(\hat{Y}_t + \hat{\mu}_t\right) + (1 - \nu)\bar{Y}\hat{\tau}_t = \bar{G}\hat{G}_t + \frac{\bar{p}_L\bar{b}_\delta}{\beta}\left(\hat{b}_{t-1,\delta} - \hat{\pi}_t - \hat{a}_t\right) + \delta e^{-\gamma}\frac{\bar{p}_L\bar{b}_\delta}{\bar{\pi}}\hat{p}_{L,t} + \hat{\Lambda}_t \quad (40)$$

$$\hat{\pi}_t = \kappa\hat{\mu}_t + \beta E_t\hat{\pi}_{t+1} + \hat{u}_{\mu,t} \quad (41)$$

$$\hat{i}_t \geq -i^* \quad (42)$$

where  $\mu$  denotes marginal costs of production. In equilibrium it holds that  $\hat{C}_t = \hat{C}_t^A$ .

We assume the following feedback rule for taxes:

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1 - \rho_\tau)\left(\phi_{\tau,b}\hat{b}_{t-1,\delta} + \phi_{\tau,Y}\hat{Y}_t + \phi_{\tau,G}\hat{G}_t\right) + u_{\tau,t}. \quad (43)$$

Notice that we now allow aggregate output and the level of spending to impact directly the tax rate in period  $t$ . (43) is a standard specification of the tax rule which follows, for instance, Bianchi and Ilut (2017).  $u_{\tau,t}$  is an exogenous tax shock process.

We specify the shock processes in the model as follows:

$$\hat{x}_t = \rho_x\hat{x}_{t-1} + u_{x,t}$$

for  $\hat{x} = \{\hat{G}, \hat{a}, \hat{\Lambda}, \hat{\xi}\}$ . In other words, exogenous shocks to spending, TFP,  $\hat{\Lambda}$  and preferences follow first order autoregressive processes. In the case of markup shocks we assume

$$u_{\mu,t} = \rho_\mu u_{\mu,t-1} + \epsilon_{\mu,t} - \nu_\mu \epsilon_{\mu,t-1}$$

thus allowing for both an AR(1) and a moving average component, as Del Negro et al (2015) do. Finally, we assume  $u_{\tau,t}$  is an i.i.d process.

### 3.3 The planner's objective

For the quantitative model of this section we adopt the following objective function:

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\left(\hat{\pi}_t^2 + \lambda_Y\left(\hat{Y}_t - \hat{Y}_t^n\right)^2 + \lambda_i\left(\hat{i}_t - \hat{i}_{t-1}\right)^2\right) \quad (44)$$

assuming that the planner seeks to minimize the deviation of inflation in  $t$  from the steady state level  $\bar{\pi}$ , the deviation of output from its natural level  $\hat{Y}_t^n$ ,<sup>27</sup> and the change in value of the nominal interest rate relative to the value in the previous period.

Note that (44) is commonly used in estimated DSGE models of optimal monetary policy (see for example Debortoli and Lakdawala (2016), among others). We thus follow the recent literature in our choice of the objective. Moreover, with (44) our optimal policy model continues to admit a closed form expression for the interest rate rule. For brevity, we leave it to the online appendix to derive this expression.

<sup>27</sup>We define the natural level of output as the flexible price level. When the planner chooses optimal policies we assume she does not account for their impact on  $\hat{Y}_t^n$ . For brevity, we discuss the details in the online appendix.

## 4 Estimation

Our quantitative analysis will be carried out in two steps. First, we will estimate the model using data until the fourth quarter of 2008, ignoring the ZLB constraint. Second, we will use the estimates of model parameters to investigate the performance of the model in the Great Recession accounting for the ZLB. In this section we describe our approach in model estimation and report the estimates of the structural parameters.

### 4.1 Selecting the “Correct” Model to Estimate

The task of estimating the model is complicated because we have two model versions we can choose from: the version where monetary policy takes into account the consolidated budget and the version where monetary policy has no debt concerns. Which of these models do we want to estimate? Should we estimate both?

If we estimate both versions, the estimates of the structural parameters will vary across model versions. This would imply that potential differences in terms of the performance of the models in the Great Recession will be partly driven by the different values of the structural parameters, not by the models’ endogenous propagation mechanisms which we would like to analyse. We thus choose to estimate one model only. To choose which one, we have to decide whether assuming no debt concerns describes more accurately US monetary policy since the 1980s when our sample begins, or over this period, monetary policy was subservient to fiscal policy as in the debt concerns model. Bianchi and Ilut (2017) estimate a DSGE model allowing for the monetary/fiscal policy mix to vary through time. They find that from the 1980s and until 2008 monetary policy was ‘active’ and fiscal policy passive, or, in other words, monetary policy was not affected by the debt aggregate. Since this corresponds closely to the no debt concerns model we estimate our structural optimal policy framework assuming that the Fed chooses allocations without taking into account the consolidated budget.

### 4.2 Sample and Variables

We fit the model to US observations using data from the period 1980 Q1 -2008 Q4. The macroeconomic aggregates that we employ in estimation are: consumption of nondurable goods and services (CONS), inflation (INFL), the federal funds rate (FFR), tax revenues (TAX), government debt (MVDEBT) and government spending (GOV). For the debt series we use the market value of debt that is held by the public, the measure which corresponds closely to our definition of debt in the model. Inflation is calculated as the annualized growth rate of the GDP deflator. Spending is consumption of goods and services by the public sector in the US. Finally, tax revenues correspond to tax receipts on personal income. The details on the sources of these variables are spelled out in the online appendix.

The measurement equations we employ in estimation are the following:

$$\mathbf{Y}^{\text{OBS}}_t = \begin{pmatrix} \Delta \text{CONS}_t \\ \text{FFR}_t \\ \text{INFL}_t \\ \Delta \text{GOV}_t \\ \Delta \text{TAX}_t \\ \Delta \text{MVDEBT}_t \end{pmatrix} = \begin{pmatrix} 100\gamma + \Delta \hat{C}_t + u_{a,t} \\ 100 \log R + \hat{i}_t \\ 100 \log \bar{\pi} + \hat{\pi}_t \\ 100\gamma + \Delta \hat{G}_t + u_{a,t} \\ 100\gamma + \Delta \hat{\mu}_t + \Delta \hat{Y}_t + \frac{1}{\lambda} \hat{A} \Delta \hat{\tau}_t + u_{a,t} \\ 100\gamma + \Delta \hat{p}_{t,L} + \Delta \hat{b}_{t,\delta} + u_{a,t} \end{pmatrix} + \begin{pmatrix} 0 \\ u_{i,t} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (45)$$

As is standard, our model will fit the US data through finding parameters and shock processes such that these measurement equations are satisfied.  $u_{i,t}$  is a mean zero measurement error that we use to match the interest rate data with our model.

## 4.3 Estimates

### 4.3.1 Priors and Calibrated Parameters

As is typical in the literature, we proceed with the estimation of the model by first selecting prior distributions for the parameters we wish to estimate and picking values for parameters that we want to fix in estimation. Table 3 summarizes the calibrated values of the parameters that we fix and the right side of Table 4 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g. Bianchi and Ilut (2017)) and are relatively loose.

[ Tables 3 and 4 About Here ]

We fix the values of the labor share,  $\alpha$ , the elasticity of labor supply,  $\frac{1}{\gamma_h}$ , the demand elasticity parameter,  $\eta$ , the decay factor of the long term bond,  $\delta$ , the debt to GDP and the spending to GDP ratios in steady state. We assume  $\alpha = 0.66$  and  $\gamma_h = 1$ . The decay factor  $\delta$  is set to 0.95 which gives us an average maturity of 5 years, consistent with US data. The steady state value of  $\eta$  is such that markups are 17 percent. Finally, the debt to GDP ratio is equal to 34 percent and the steady state level of the spending to output ratio is 10 percent. These values are chosen to be in range with the average ratios in our data.

### 4.3.2 Posterior Distributions

The left side of table 4 reports the posterior estimates of the model parameter distributions. According to the values reported in the table, the degree of price indexation  $\zeta$  is modest (0.2 at the mean), the Phillips curve is relatively flat (the mean estimate of  $\kappa$  is 0.0086) and the distribution of the habit parameter  $\Omega$  is centered around roughly 0.5. Moreover, the estimated response of taxes to the lagged value of debt is low ( $\phi_{\tau,b} = 0.066$  at the mean). These values are close to the analogous objects reported in Bianchi and Ilut (2017).

Notice also that our model's estimates of the coefficients  $\lambda_i$  and  $\lambda_Y$  are in range of recent estimates of optimal Ramsey models with US data (see Debortoli and Lakdawala (2016) and references therein). To show how optimal policy in our model maps into simple interest rate rules commonly used in the DSGE literature, we use simulated data from the posterior distribution, and run the following specification:

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i)(\phi_1 \hat{\pi}_t + \phi_2 (\hat{Y}_t - \hat{Y}_t^n) + \phi_3 \Delta \hat{Y}_t) + v_t \quad (46)$$

where  $v_t$  is the residual from the regression. Least squares give us the following fitted values

$$\hat{i}_t = 0.977 \hat{i}_{t-1} + (1 - 0.977) \left[ 2.70 \hat{\pi}_t - 0.001 (\hat{Y}_t - \hat{Y}_t^n) + 0.763 \Delta \hat{Y}_t \right] \quad (47)$$

In other words, interest rates display substantial persistence, respond strongly to inflation and to output growth and the response to the output level is insignificant. These properties are consistent with the findings of several papers that estimate DSGE models with rules of the form (46).<sup>28 29</sup>

[ Figures 5 and 6 About Here ]

<sup>28</sup>For instance Bianchi and Ilut (2017) obtain a coefficient  $\rho_i \approx 0.91$  at the mean of the posterior distribution, and a coefficient  $\phi_1 \approx 2.73$  in the active monetary regime. These values are close to our estimates reported in (47).

The property that output growth is a significant variable in the interest rate rule, but not the level of the output gap, is consistent with the findings of (for example) Smets and Wouters (2007) and Fratto and Uhlig (2014).

<sup>29</sup>Using simulated model data from the DC model and assuming  $\phi_{\tau,b} = 0$  we obtain:

$$\hat{i}_t = 0.811 \hat{i}_{t-1} + (1 - 0.811)(0.833 \hat{\pi}_t + 0.116 (\hat{Y}_t - \hat{Y}_t^n) + 0.053 \Delta \hat{Y}_t)$$

Interest rates are thus characterised by lower persistence and a much more moderate response to inflation. We verified that these values are consistent with 'passive' monetary policy.

Figures 5 and 6 show the responses of inflation, interest rates, output and debt to each of the shock processes considered in the model, when parameter values are set at the means of posteriors. We assume a one standard deviation innovation to each process, based on the estimates reported in Table 4. Figure 5 considers the case of a preference shock (left panel), a spending shock (middle panel) and a tax shock (right panel). Figure 6 shows the responses to a markup shock, a shock to the government budget and a shock to TFP (from left to right). The dashed (red) line is the no debt concerns model estimated in this section. For the sake of comparison we have added the responses under debt concerns with the solid (blue) line. Notice that the impact effects of each of the shocks on the model variables are now in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase etc.

From the Figures we see that under no debt concerns, inflation is basically unaffected by shocks to preferences, spending, taxes, and shocks to the budget. Shocks to TFP exert a small influence on inflation, but mainly markup shocks are the key driving force behind inflation variability. This model property can be explained as follows: First, preference and spending shocks exert only a minor influence because these shocks are persistent and also because monetary policy is optimal. Due to high persistence these shocks do not provoke large movements to the real rate. Given welfare losses derive from interest rate growth in (44), the planner can adjust the nominal interest rate by a few basis points, without impinging substantial welfare losses. Tax shocks on the other hand, under no debt concerns, affect the inflation output tradeoff through their influence on the Phillips curve, but our estimates in Table 4 suggest that this effect is not large. This also applies to shocks to the consolidated budget which lead to changes in taxes.

Notice that the finding that markup shocks are a key driving force behind inflation dynamics is not out of line with the rest of the literature. Several studies have reached a similar conclusion (e.g. Fratto and Uhlig (2014), Hall (2011), Michallat and Saez (2014)). Some authors have suggested that this property hints at a failure of the New Keynesian model in (endogenously) explaining inflation. It is important, however, to stress that our optimal policy model offers a different perspective: Inflation is driven by shocks to markups only, because monetary policy is very effective in stabilizing inflation against other types of shocks.

This however, does not hold for the equilibrium Ramsey policy under debt concerns. As can be seen from Figures 5 and 6, in this case, inflation responds strongly to all shocks, including to shocks in fiscal variables. In the debt concerns model the planner gives up on the goal of stabilizing interest rates, output and inflation for the sake of stabilizing the government debt.

## 5 Quantitative Analysis: Optimal Policies in the Great Recession

In this section we evaluate our model's properties in the Great Recession. Our main goal is to identify the forces behind the Fed's forward guidance policies, through evaluating the ability of each of the two versions of the model to match the behavior of macroeconomic variables during the recession. To this end we perform a 'forecasting' exercise, whereby we recover the shocks which, in the first quarter of 2009, brought the US economy to the LT, and trace their impacts on the behavior of macroeconomic variables throughout the Great Recession. We thus trace our model's ability to capture: i) the persistence of interest rates we observe in the data, ii) the lack of deflation observed, and iii) the dynamics of output growth and government debt throughout the sample.<sup>30</sup> A similar exercise is considered in Bianchi and Melosi (2017), Del Negro et al (2015) and others.

We do this (essentially impulse response analysis) separately for the debt concerns and no debt concerns models in order to assess their relative fit. Since we assumed that up to the fourth quarter of 2008 monetary policy did not take into account the evolution of debt aggregates, when we consider

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<sup>30</sup>Recall that in estimation we asked the model to match the consumption growth series. Aggregate output in the model is thus the sum of consumption and spending. However, in our analysis below we will use the GDP data from the US to evaluate the performance of the models.

the debt concerns equilibrium post 2009, we essentially assume that there was a shift in policy at the onset of the recession. For our first experiment we make the shift permanent assuming an unexpected permanent shock which changed the structure of policy. We contrast the properties of the model when the shift occurs, with the properties when it doesn't occur and monetary policy adheres to its objective to stabilize inflation and the output gap.

## 5.1 Constructing Paths of Macroeconomic Variables

We first describe formally our approach in constructing the paths of macroeconomic variables. Essentially, our methodology consists of recovering the initial values of the state variables and the shocks which drove the US economy to the LT in the first quarter of 2009, then we solve the model forward to trace the effects of the shocks on key variables and in each of the two versions of the model.

Formally, let  $X_t^{NDC}$  denote the vector of endogenous variables in the NDC model and  $\epsilon_t^{NDC}$  the vector of exogenous disturbances in that model. Denote by  $X_{1,t}^{NDC}$  the set of variables that are chosen by the planner and by  $X_{2,t}^{NDC}$  the remaining model variables. In the case where the ZLB is not binding (i.e. until the 4th quarter of 2008) the competitive equilibrium in the NDC model can be summarized in the following linear system:

$$\tilde{A}_1 E_t X_{t+1}^{NDC} + \tilde{A}_0 X_t^{NDC} + \tilde{A}_{-1} X_{t-1}^{NDC} + \tilde{B} \epsilon_t^{NDC} = 0 \quad (48)$$

and the subsystem of equations which represent the planner's constraints can be written as:

$$\tilde{A}_{1,1} E_t X_{1,t+1}^{NDC} + \tilde{A}_{1,2} E_t X_{2,t+1}^{NDC} + \tilde{A}_{0,1} X_{1,t}^{NDC} + \tilde{A}_{0,2} X_{2,t}^{NDC} + \tilde{A}_{-1,1} X_{1,t-1}^{NDC} + \tilde{A}_{-1,2} X_{2,t-1}^{NDC} + \tilde{B}_1 \epsilon_t = 0 \quad (49)$$

Letting  $\psi_t^{NDC}$  denote the multipliers attached to constraints (49) the first order conditions (see online appendix) of the planner's program are

$$2W X_{1,t}^{NDC} + \tilde{A}'_{1,1} \frac{\psi_{t-1}^{NDC}}{\beta} + \tilde{A}'_{0,1} \psi_t^{NDC} + \tilde{A}'_{-1,1} E_t \psi_{t+1}^{NDC} = 0 \quad (50)$$

Moreover, denoting  $Y_t^{NDC'} = [X_t^{NDC'}, \psi_t^{NDC'}]$ , the system of dynamic equations which needs to be resolved to characterize the equilibrium under optimal Ramsey NDC policy is

$$\tilde{C}_1 E_t Y_{t+1}^{NDC} + \tilde{C}_0 Y_t^{NDC} + \tilde{C}_{-1} Y_{t-1}^{NDC} + \tilde{D} \epsilon_t = 0 \quad (51)$$

The solution is the following transition equation

$$Y_t^{NDC} = \tilde{E} Y_{t-1}^{NDC} + \tilde{F} \epsilon_t^{NDC} \quad (52)$$

Using (52) and the sample 1980Q1- 2008Q4, employed in estimation, we can recover initial conditions  $Y_{\bar{t}-1}^{NDC}$  through a standard Kalman filter and smoother, where  $\bar{t}$  denotes the first quarter of 2009.

From period  $\bar{t}$  onwards the solution of the DSGE model needs to take into account the possibility that the ZLB may bind. We follow the methodology developed in Guerrieri and Iacoviello (2015); suppose that in  $\bar{t} + j$ ,  $j > 0$  the ZLB is expected to bind for  $T_{\bar{t}+j}$  periods. The model's equilibrium conditions become

$$\tilde{C}_1^* Y_{k+1}^{NDC} + \tilde{C}_0^* Y_k^{NDC} + \tilde{C}_{-1}^* Y_{k-1}^{NDC} + \mathcal{R}^* = 0 \quad (53)$$

for  $k = \bar{t} + j, \bar{t} + j + 1, \dots, \bar{t} + j + T$ , and the solution to (53) takes the following time varying form:

$$Y_k^{NDC} = \tilde{E}_k^{T_{\bar{t}+j},*} Y_{k-1}^{NDC} + \tilde{R}^* \quad (54)$$

We use the Occbin algorithm to determine  $T_{\bar{t}+j}$  endogenously. In period  $\bar{t}$  we let the economy be hit by exogenous shocks  $\epsilon_{\bar{t}}^{NDC}$ . Given a duration of the LT episode  $T_{\bar{t}}$  we have

$$Y_{\bar{t}}^{NDC} = \tilde{E}_{\bar{t}}^{T_{\bar{t}},*} Y_{\bar{t}-1}^{NDC} + \tilde{R}^* + \tilde{F}_{\bar{t}}^{T_{\bar{t}},*} \epsilon_{\bar{t}}^{NDC} \quad (55)$$

We recover the entries of the vector  $\epsilon_{\bar{t}}^{NDC}$  which match the US data through a non-linear filter. Applying the above model solution we generate paths of endogenous variables during the Great Recession period.

### 5.1.1 Switching to the DC equilibrium

In the case where policy switches to the debt concerns equilibrium and in periods where the ZLB binds the dynamic equations of the model can be written as

$$C_1^* Y_{k+1} + C_0^* Y_k + C_{-1} Y_{k-1} + \mathcal{R} = 0 \quad (56)$$

where  $C_1, C_0, C_{-1}$  can be derived similarly to  $\tilde{C}_1, \tilde{C}_0, \tilde{C}_{-1}$  under the Ramsey optimal policy, and the vector of endogenous variables  $Y$  now includes the multiplier on the consolidated budget constraint. We initiate  $Y_{\bar{t}-1}$  using the entries of  $Y_{\bar{t}-1}^{NDC}$  and letting  $\psi_{gov, \bar{t}-1}$  be equal to zero. Finally, we use the same shocks  $\epsilon_{\bar{t}}^{NDC}$  for the DC model to generate paths of macroeconomic variables.

## 5.2 Model Evaluation: Optimal Policies in the Great Recession

Figure 7 plots the paths of inflation (top left), interest rate (top right), debt and output growth (bottom left and right respectively). The solid lines show again the case of the baseline model with debt concerns, the dashed lines show the no debt concerns model. The dotted (black) line is the filtered data observations.

[Figure 7 About Here]

### 5.2.1 Forecasts of Macro Variables: No Debt Concerns

Consider first the performance of the no debt concerns model and its predicted paths for the macroeconomic variables after 2009:Q1. There are several noteworthy features: First, note that the model generates a positive inflation rate during the Great Recession; the inflation rate drops to around 1 per cent in the beginning of 2009, and very fast –after a couple of quarters– reaches a level, which exceeds the steady state level of inflation (roughly 2 per cent). Inflation in the model is thus higher than in the data (where we observe slightly negative inflation for a couple of quarters in 2009) and displays high persistence. Second, the model predicts that interest rates remain at the ZLB for several quarters after 2009. In particular, interest rates are at the ZLB until 2011:Q3 and then gradually return to steady state. Third, the model performs very well in matching the behavior of the growth rate of output. As can be seen from the bottom right panel, the model predicts that output growth recovers rapidly in 2009 and subsequently is stabilized close to the steady state rate of TFP growth. Finally, the model underperforms in matching the dynamics of government debt: Public debt increases only slightly in the model whereas the rise in the data is considerable.

As it is well known, both the lack of deflation during the Great Recession and the high persistence of interest rates at the ZLB are facts which are difficult to explain through a NK model’s endogenous propagation mechanisms. NK models typically imply negative inflation rates at the ZLB and also imply that interest rates recover fast and move away from the ZLB (see for example Del Negro et al (2015), Fratto and Uhlig (2014) among numerous others). The no debt concerns model has little difficulty in generating persistently low rates and also generates positive inflation at the ZLB, though inflation is higher than in the data.

To understand these findings note that the model impinges two key channels through which keeping interest rates at the ZLB for a long period of time is an optimal policy outcome: persistent preference shocks and endogenous FG. Both of these matter for the persistence of interest rates. However, whereas preference shocks tend to make inflation negative (see below) the model’s prediction that inflation rates are positive even at the onset of the LT can be fully accounted by the Fed’s endogenous FG policies. This is consistent with the findings of Eggertsson and Woodford (2003): Under full commitment, Ramsey policy can nearly fully offset the negative effects of a LT shock on inflation through promising to keep the policy rate low when the economy escapes from the LT. Higher future (expected) inflation translates into high current inflation because of the forward looking Phillips curve. This also explains why inflation remains higher than the steady state value throughout the entire simulated path, thus overshooting the data.

Is it crucial that the model cannot match the behavior of debt? Not at all. Recall that under no debt concerns, optimal inflation is not impacted by the evolution of the debt level. Therefore, the fact that the debt to GDP ratio does not rise sufficiently in response to the shocks which hit the economy in 2009:Q1 is inconsequential for the path of inflation. As we shall later see, shocks which explain the rise in government debt in the no debt concerns models (tax shocks and shocks to the government budget) have only small effects on the rate of inflation.

### 5.2.2 Forecasts of Macro Variables: Debt Concerns

Consider now the responses in the debt concerns model shown in the Figure. Notice that inflation drops a lot in 2009, interest rates stay even longer at the ZLB, output growth drops sharply in 2009 before it rises again above the steady state level in 2010-11. Moreover, the model tracks well the behavior of debt.

These properties are in line with our previous theoretical findings. As we demonstrated, following a shock which drives the economy to the LT, in the debt concerns model the planner finds optimal to let inflation become negative and at the same time promise to keep the interest rate at the ZLB for many periods. Since the preference shock increases the present value of future surpluses more than it increases the real payout of government bonds, the price level must drop so that the intertemporal budget is satisfied. Because the maturity of debt is long, the drop in prices is gradual and so inflation remains negative until the second quarter of 2011.

Notice, also, that differently from the model of Section 2, where inflation first falls in response to the LT shock, then rises for few periods before turning negative again (see e.g. Figure 3), in Figure 7 inflation remains below the steady state level throughout the entire sample. To satisfy the ZLB constraint the planner now promises more rapid output growth, leading the economy into a boom in 2010 which persists for several quarters.

Finally, note that the behavior of debt is influenced by two factors: First, negative inflation contributes to the increase in the growth rate of the real debt, plotted in Figure 7. Second, when policy switches to the debt concerns equilibrium in the beginning of 2009, the tax rate drops, since  $\phi_{\tau,b}$  becomes equal to 0. Since debt is above its steady state level before the beginning of the LT episode, tax revenues automatically drop when the regime switches. This contributes towards a lower fiscal surplus in the short term.

## 5.3 Transitory Switch

The previous paragraph highlighted the relative successes and failures of the two models when we impose at the onset of the LT episode a permanent shock that determines the regime. The NDC model (over)predicts positive inflation rates and generates interest rates which remain at the ZLB for several quarters after 2009. In the case where policy switches to the debt concerns equilibrium, inflation becomes severely negative however interest rates stay even longer at the ZLB.

The failure of the debt concerns model to capture the dynamics of inflation may have left the reader with the impression that the results imply there is no trace of debt concerns in monetary policy during the Great Recession. However, since the inflation data clearly lie between the two models, debt aggregates may still have exerted an influence on monetary policy, albeit a small one.

In this paragraph we attempt to bring our simulations closer to the data through combining the two models. We ask what would happen if the switch from the NDC equilibrium to the debt concerns model in 2009 is not permanent and the private sector anticipates with positive probability that optimal monetary policy will return to its previous pre Great Recession structure. We do this in a simple fashion, assuming that at the end of the LT episode another shock is realized which may (or may not) return optimal policy to the NDC equilibrium. If this 'linear combination' of the two models can match the data better then we conclude that, at least in terms of the simple forecasting exercise we present here, the Fed's policy is somewhere between the two cases.

[Figure 8 About Here]

The results are shown in Figure 8 where we assume that, at the end of our sample, monetary policy returns to NDC with probability a half.<sup>31</sup> We see clearly that a temporary shift towards debt concerns can capture the data patterns better. For example, the solid (blue) line which now shows the policy under 'temporary debt concerns', in the top left panel lies close to the dashed line which represents the data for inflation; the model matches both the mild drop in prices in early 2009, and the recovery of price growth in subsequent quarters. The model can also match better the data pattern of the growth of output. Its performance in terms of predicting interest rates that remain persistently at the ZLB is not affected.

These results are clearly tentative. Ideally, we would combine both models in estimation and let the data tell us which of the two regimes is more likely, presumably through estimating a Markov switching model across the two regimes. As discussed in the introduction, this task is beyond the scope of the paper. We leave this to future work.

## 5.4 Shock Decompositions

Our main quantitative exercise is an impulse response analysis, in the spirit of the analogous exercises shown in Del Negro et al (2015), Bianchi and Melosi (2017) and others. It enables us to evaluate the models' internal propagation mechanisms and their ability to predict paths of macroeconomic variables close to the data counterparts. Both models, however, can obviously match the data perfectly if we find for every period and for each model separately, the appropriate shock vector through standard shock decompositions. We perform this exercise in this section. Our goal is to see which are the key driving forces behind macroeconomic fluctuations in each model and compare the output with the recent literature.

Note that the literature has offered alternative explanations for why inflation did not turn negative during the Great Recession. Many authors, including Fratto and Uhlig (2014), Kollmann et al (2015) and Linde et al (2016), advocate that missing deflation is due to positive price and wage markup shocks. Others, assign a less crucial role to exogenous variations in inflation through markups and instead seek to improve the fit of the NK model to the inflation data through extending the model. For example Del Negro et al (2015) add financial market frictions and show that the rapid growth in marginal costs of production coupled with the steep Phillips curve which characterizes the US data, can rationalize the inflation pattern. Bianchi and Melosi (2017) present a model where the monetary/fiscal policy mix can vary through time. They show that a shift towards the fiscally led regime in the Great Recession explains missing deflation.

Are markup shocks a key ingredient of our models? Recall that in the case of no debt concerns, inflation is indeed driven by markups, other sources of shocks did not matter much for this variable (see Figures 5 and 6). Hence, it would seem that at least for this model, markups may prove significant also in the Great Recession just like in Fratto and Uhlig (2014). In the debt concerns model, all shocks exert a strong impact on inflation and so it is also possible that markup shocks are important.

[Figure 9 About Here]

Our decompositions are shown in Figure 9 where we focus on matching the inflation observa-

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<sup>31</sup> To understand why we need to assume that the regime reverts back to NDC with probability less than one, consider a scenario in which, following the LT trap shock, monetary policy shifts to the DC model, but this shift lasts deterministically for  $T$  periods. In this case we can argue that  $\mathcal{D}_t = 0$ , for  $t = \bar{t}, \bar{t} + 1, \dots, \bar{t} + T$ . Why? Since the multiplier is a forward looking process (a random walk) and agents anticipate that in the NDC equilibrium it will be  $\psi_{gov, \bar{t}+T} = 0$ , the unique value of the multiplier for  $t = \bar{t}, \bar{t} + 1, \dots, \bar{t} + T - 1$  is zero.

Under this scenario we have (temporarily) a policy mix in which monetary policy is 'active' because  $\mathcal{D}_t = 0$ , and fiscal policy is also 'active' because taxes do not adjust to the debt level. Our simulations (not shown here) suggest that this brings us very close to the NDC equilibrium.

In the case where the DC regime survives with positive probability at  $\bar{t} + T$ , the expectation  $E_t \psi_{gov, \bar{t}+T}$  is non-zero and hence  $\mathcal{D}$  exerts an influence on interest rates.

tions.<sup>32</sup> The Figure shows the following: First, in the no debt concerns model (top panel) markups do not contribute positively to inflation. Rather inflation is pushed downwards by negative markup shocks and by negative preference shocks. Second, in the debt concerns model shocks which push inflation upwards are 'fiscal' shocks (i.e. spending, tax and  $\hat{\Lambda}$  shocks); again markups and preference shocks contribute negatively.

What explains these findings? First, focusing on the no debt concerns model, note that it is not surprising that preference shocks impact inflation in the Great Recession. This is due to the binding ZLB. Second, as we have seen, monetary policy in this model is very effective in stabilizing output and inflation in the LT. The planner is able to avoid deflation by committing to keep interest rates low and thus generate positive inflation rates at the exit from the episode. This results in too high inflation; to match the data, the model requires negative markup shocks. At the same time, monetary policy is very effective in shielding inflation from other types of shocks (for example fiscal shocks which drive debt upwards).

In the debt concerns model positive markup shocks push inflation up but lower government debt (see Figure 6). Since debt increases over the sample, our decomposition favors fiscal shocks in response to which inflation and debt are positively correlated.

## 5.5 Debt Concerns at QE 2

In the previous paragraphs we evaluated the performance of the models during the recession when a large and negative demand shock drives interest rates to the ZLB. In this response to this shock the debt concerns model predicts negative inflation rates. Prices dropped because the shock raised the present discounted value of surpluses more than the market value of debt.

As is well known, after 2009 the US economy experienced dramatic shifts in fiscal variables. The debt to GDP ratio doubled relative to its prerecession level, spending increased and taxes dropped significantly, following the enactment of the American Investment and Recovery Act by the US Congress in early 2009. Whereas demand shocks slacken the intertemporal budget, fiscal shocks put the government budget in strain, and a monetary authority which has to ensure debt sustainability finds optimal to let inflation rise. This property was shown in Section 2.

[Figure 10 About Here]

We now perform the same exercise as in Section 5.2, however, instead of assuming that policy switches towards the debt concerns equilibrium in 2009, we assume a switch in the fourth quarter of 2010, when the second round of quantitative easing was announced. The results are displayed in Figure 10. As the Figure shows, the debt concerns model now predicts inflation rates above 4 percent and interest rates that rapidly increase above their ZLB in early 2011. These predictions are at odds with the data. In contrast, the no debt concerns model predicts paths of interest rates and inflation which match closely their data counterparts. We conclude that a switch towards debt concerns after 2009 is unlikely.

## 6 Conclusion

We offer a tractable framework of endogenous forward guidance which allows us to investigate the key forces that determined the behavior of interest rates in the Great Recession. Our model nests both the case where the Fed's policies reflect debt sustainability concerns and the case where they do not. A Ramsey planner (the Fed) sets allocations under commitment to minimize the deviations of inflation, output and interest rates from their respective target levels. If the planner has debt sustainability concerns then optimization is subject to the consolidated budget constraint; otherwise it is not.

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<sup>32</sup>We leave the decompositions of the remaining variables outside the figures for brevity. The results are shown in the online appendix.

The optimal policy rule we obtain from our model, endogenizes the 'forward guidance' shocks assumed in the recent DSGE literature. The short-term rate in our model is expressed as the sum of a Taylor rule component (a function of inflation and output growth), a component, which represents commitment to keep interest rates low at the exit from a LT episode, and, in the case the planner has debt concerns, an additional component which captures the impact of past shocks to the consolidated budget constraint. These additional components which make interest rates deviate from the Taylor rule capture endogenous forward guidance.

We show that in the presence of debt concerns, monetary policy becomes subservient to fiscal policy. Taxes do not increase in response to a high debt level and inflation adjusts to make the budget solvent. As a result, inflation, output and interest rates become more volatile responding to fiscal shocks. LT episodes are longer, however 'keeping interest rates low' for a long period, does not result in positive inflation rates at the onset of a LT episode. In contrast, in the absence of debt concerns, LT episodes are shorter, but the impact of commitments to keep interest rates low at the exit from the LT, on inflation and output is substantial. In this case monetary policy accomplishes to turn inflation positive at the onset of the episode, through promising higher inflation rates in future periods.

We embed our optimal policy framework in a medium-scale DGSE model and estimate it with US data. Our quantitative findings suggest that interest rate policies in the Great recession reflected mainly the Fed's commitment to stabilize inflation and the output gap.

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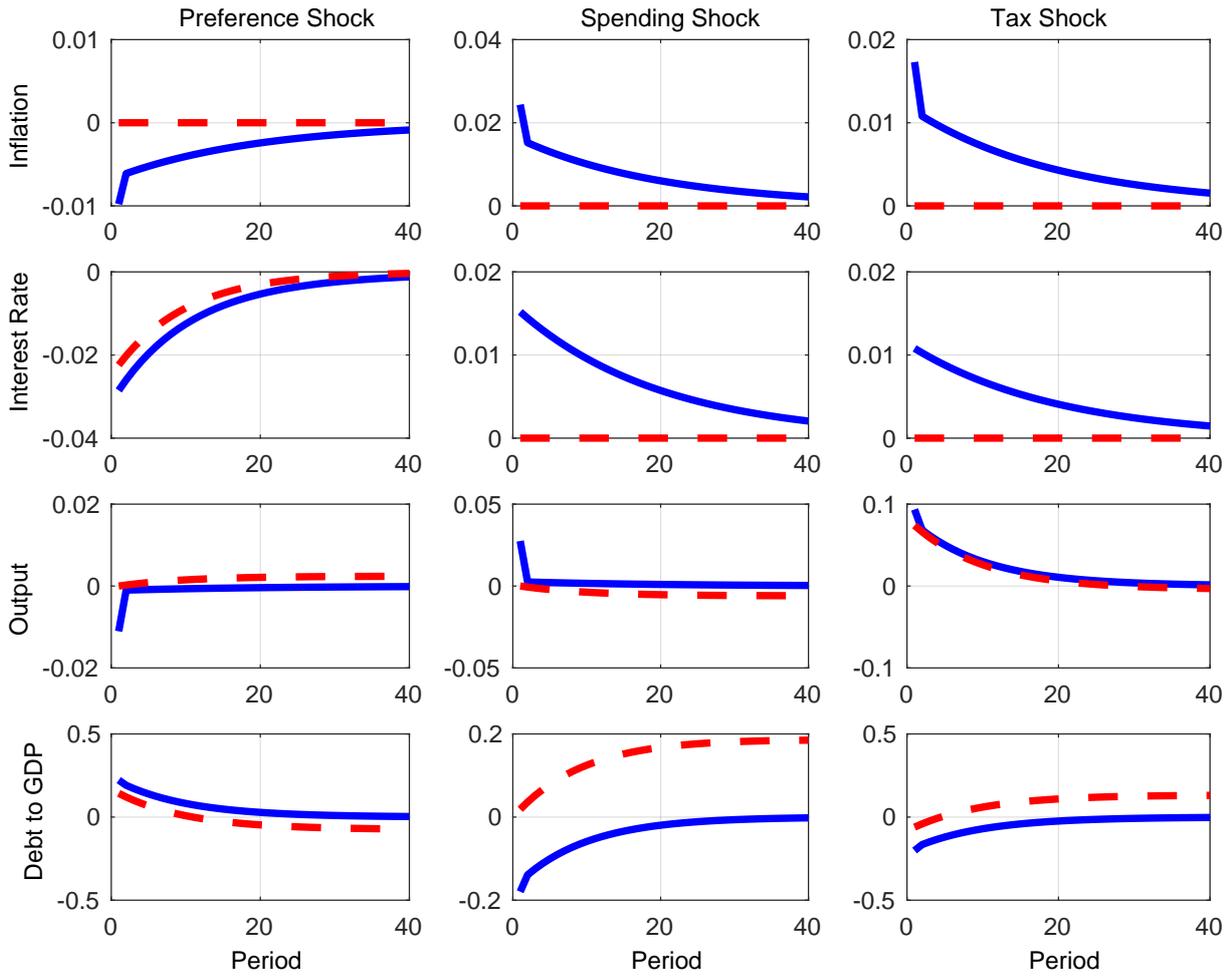
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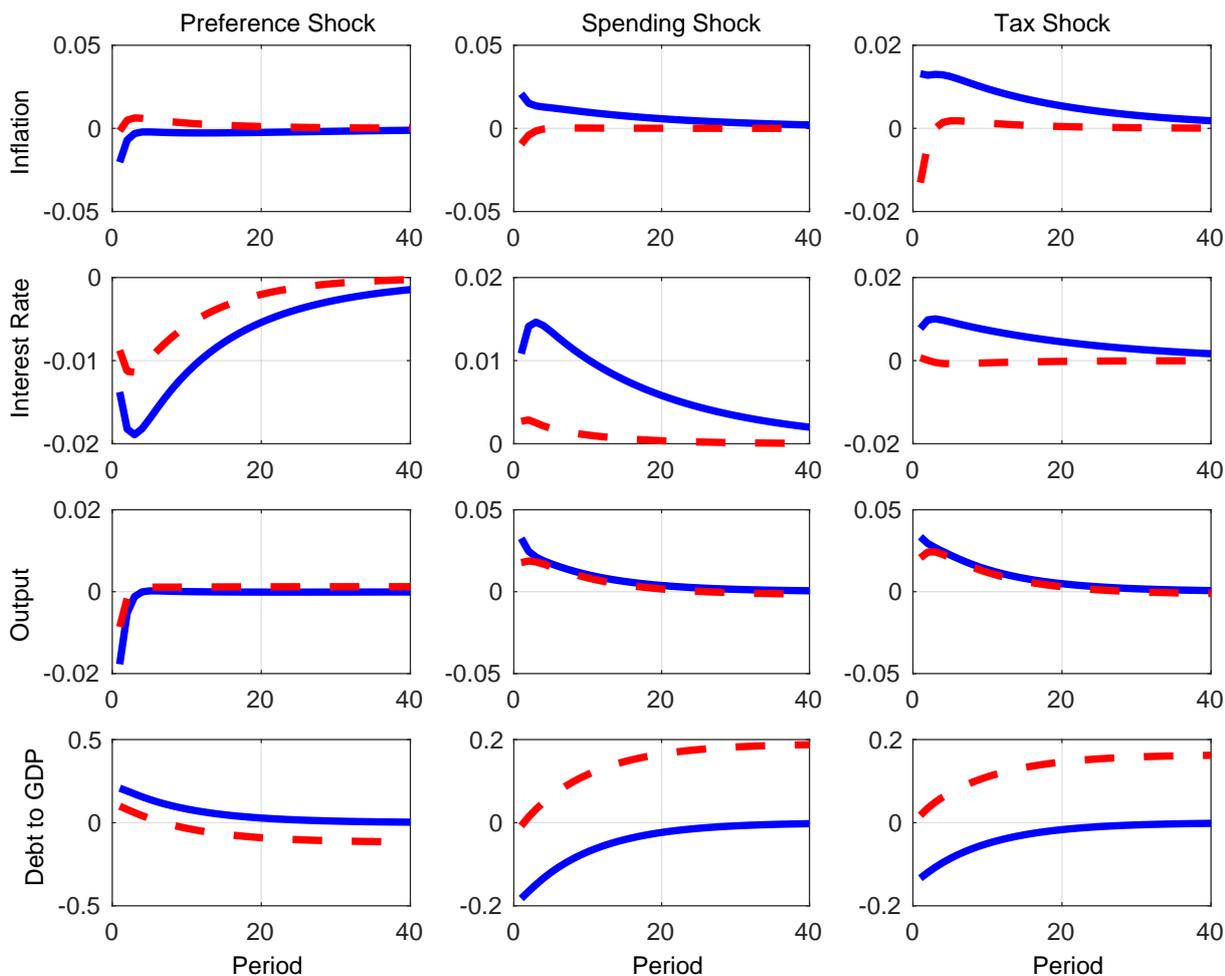
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Figure 1: **Optimal Policies away from the ZLB:  $\lambda_Y = \lambda_i = 0$ .**



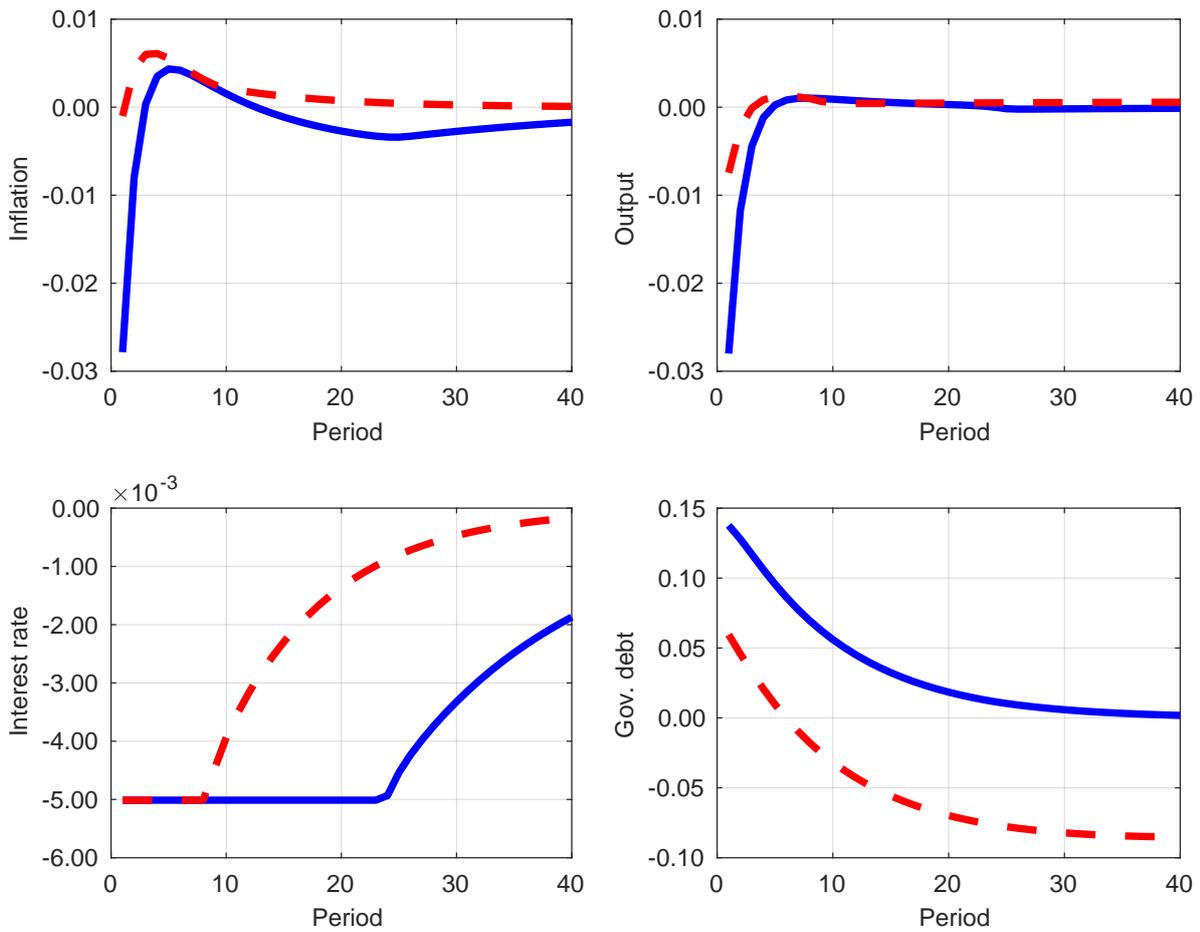
Notes: The figure plots the response of model variables to a shock in preferences (left panel), spending (middle panels) and interest rates (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set  $\lambda_Y = \lambda_i = 0$  in the planner's objective. See Section 2.4.2 for details.

Figure 2: **Optimal Policies away from the ZLB:**  $\lambda_Y = \lambda_i = 0.5$ ,  $\sigma = 1$ .



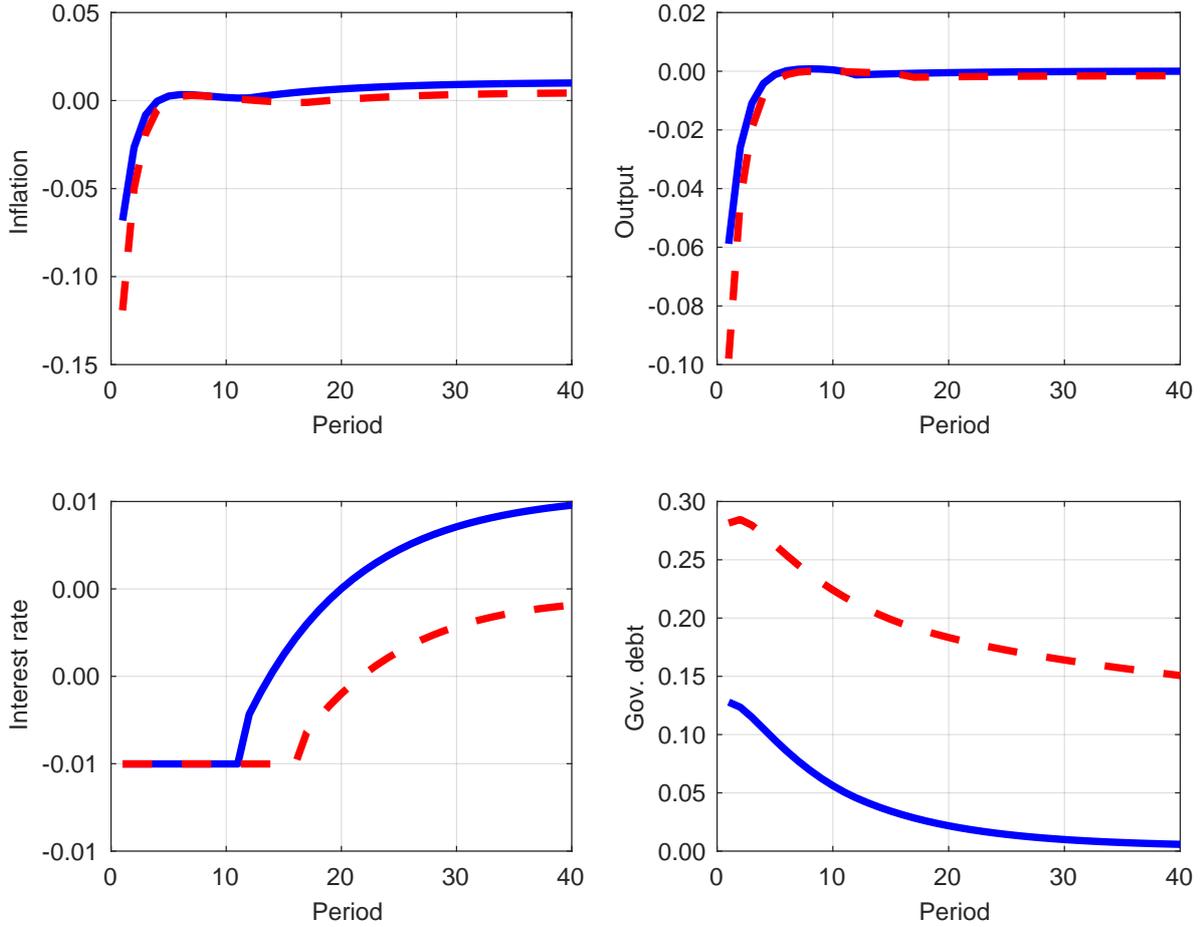
Notes: The figure plots the response of model variables to a shock in preferences (left panel), spending (middle panels) and interest rates (right panels). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set  $\lambda_Y = \lambda_i = 0.5$  in the planner's objective. See Section 2.4.2 for details.

Figure 3: Optimal Policies at the ZLB.



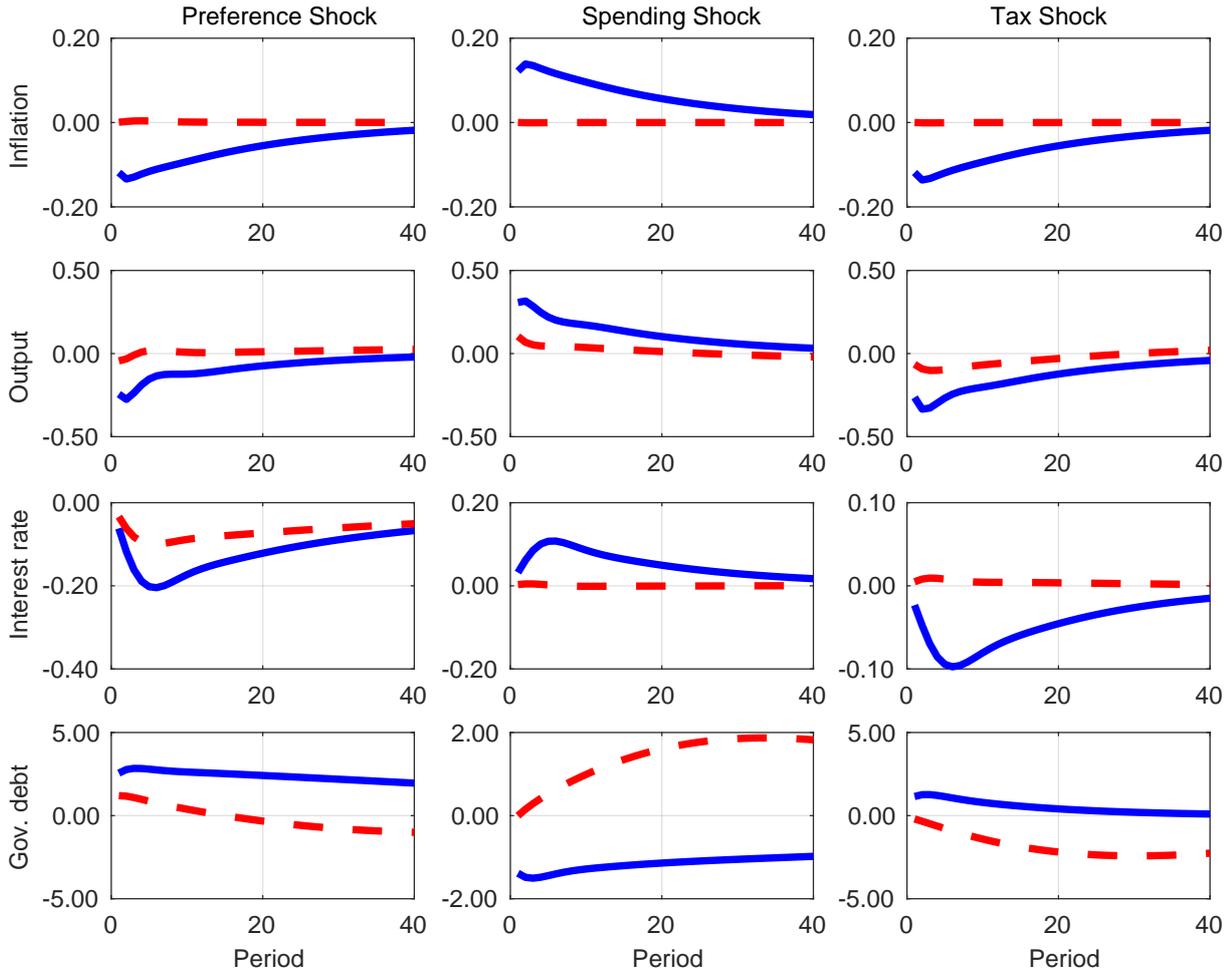
Notes: The figure plots the response of model variables to a preference shock which drives the economy to the LT. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set  $\lambda_Y = \lambda_i = 0.5$  in the planner's objective.

Figure 4: No commitment at the ZLB.



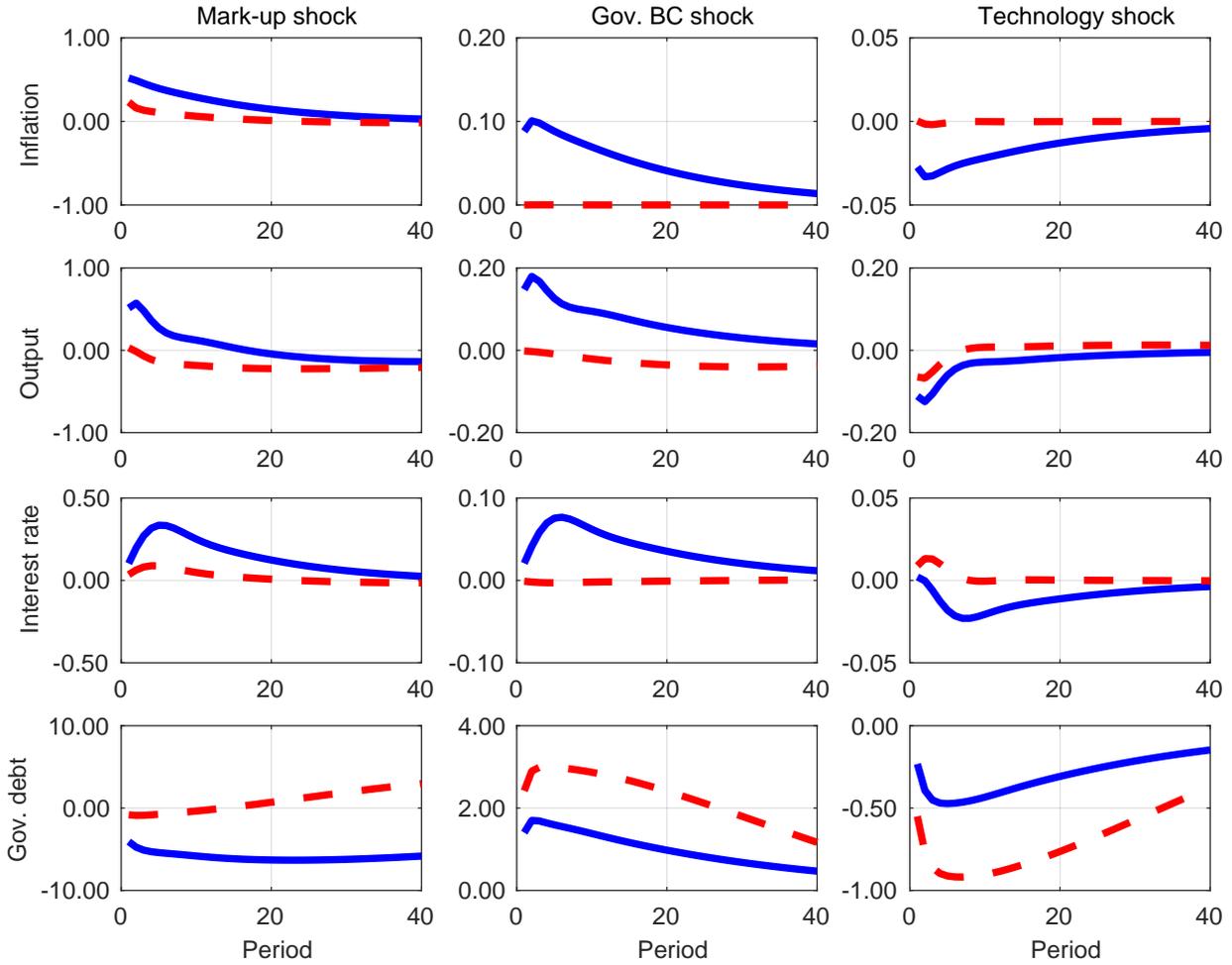
Notes: The figure plots the response of model variables to a LT shock in the case where the planner cannot commit to future policies (see Section 2.6 and the online appendix for details). The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model. We set  $\lambda_Y = \lambda_i = 0.5$  in the planner's objective.

Figure 5: Responses to Shocks: Quantitative Model



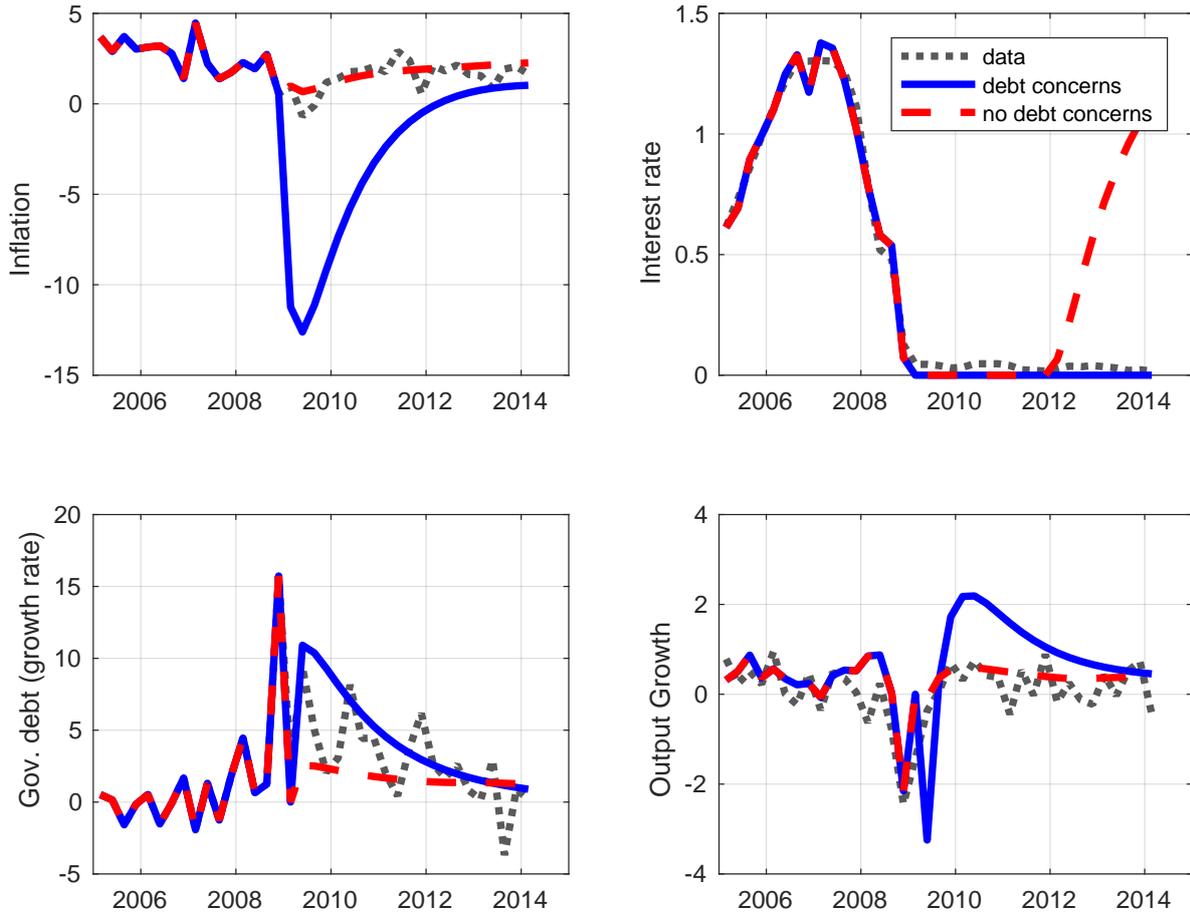
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 3. To construct the impulse response functions we apply the estimates of the model reported in Section 4. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a preference shock, the middle panels the responses to a spending shock and the right panels the responses to a tax shock. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 6: Responses to Shocks: Quantitative Model



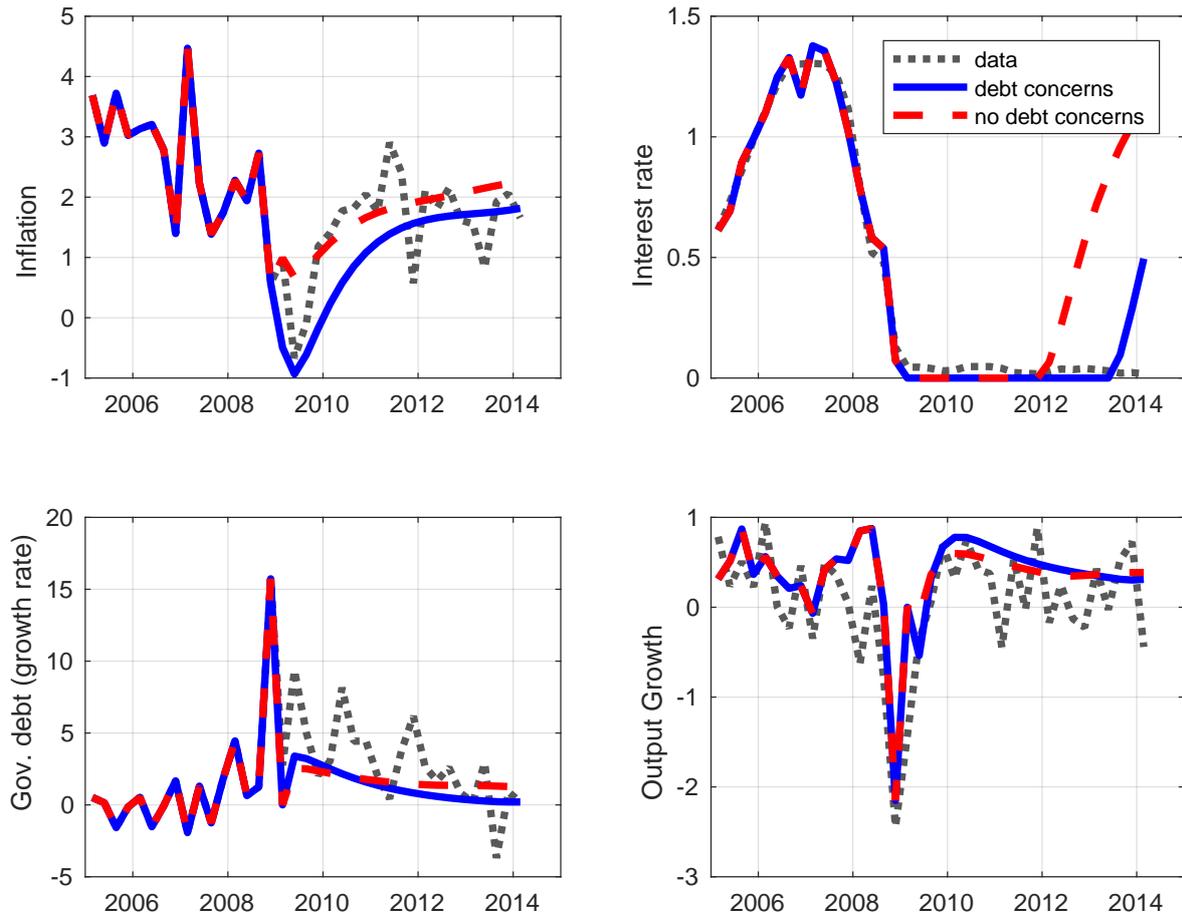
Notes: The figure plots the responses of model variables to economic shocks in the quantitative model of Section 3. To construct the impulse response functions we apply the estimates of the model reported in Section 4. The size of each shock is 1 standard deviation from its posterior distribution estimate. The left panels show the responses to a markup shock, the middle panels the responses to a shock to the government budget constraint and the right panels the responses to a TFP shock. The solid (blue) line represents the debt concerns model and the dashed (red) line the no debt concerns model.

Figure 7: **Baseline Forecasts in the Great Recession**



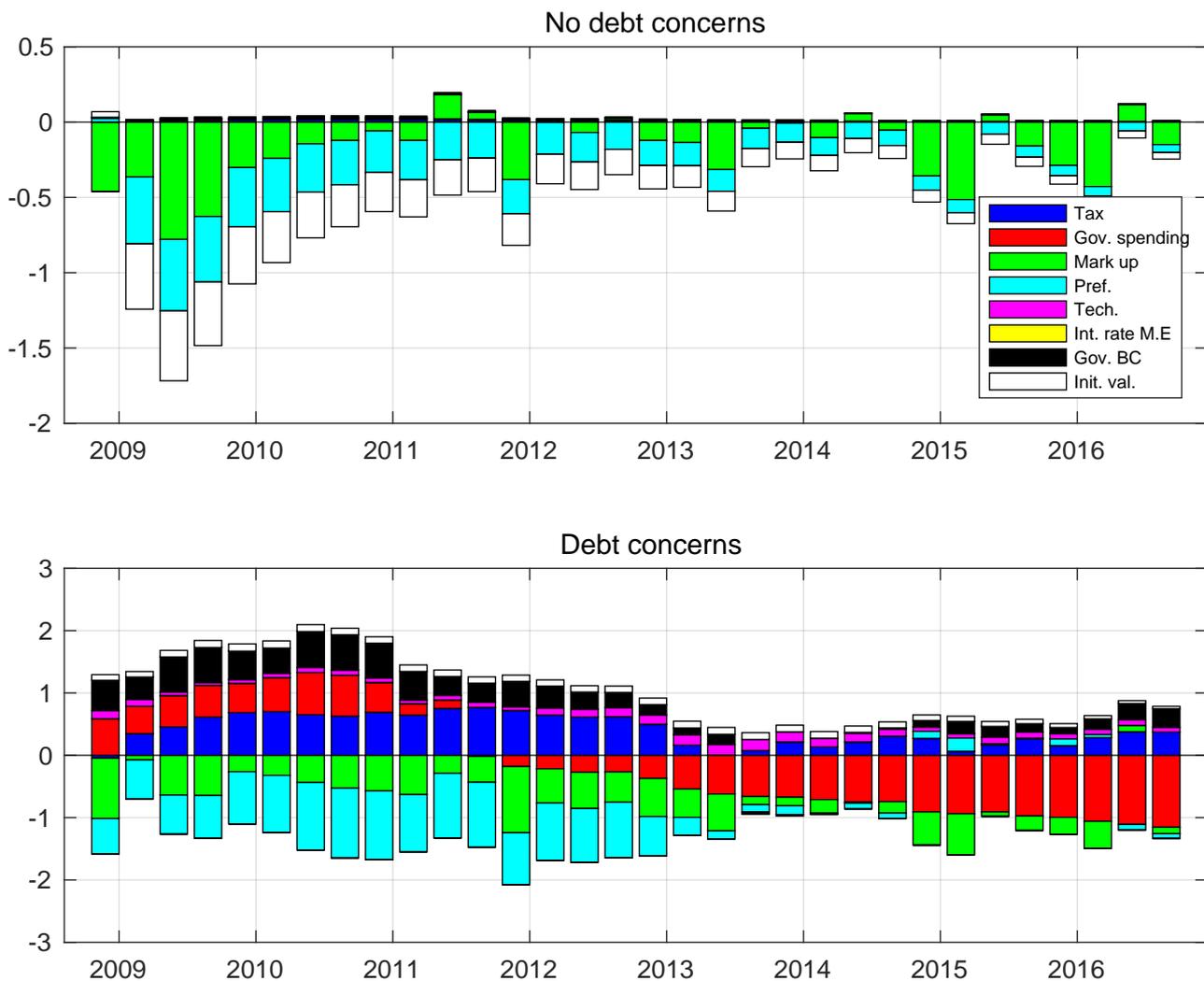
Notes: The figure plots forecasts of inflation (top left), interest rates (top right), government debt (bottom left) and output growth (bottom right) in the models. The solid (blue) line represents the model with debt concerns. The dashed (red) line is the no debt concerns model. The model series are obtained using the Kalman filter and accounting for the piecewise linear solution of the model under the ZLB. The data for inflation, debt growth and interest rates (represented with dashed, grey lines) correspond to the time series used in estimation (see Section 4). Output growth is the growth rate of GDP.

Figure 8: **Baseline Forecasts in the Great Recession**



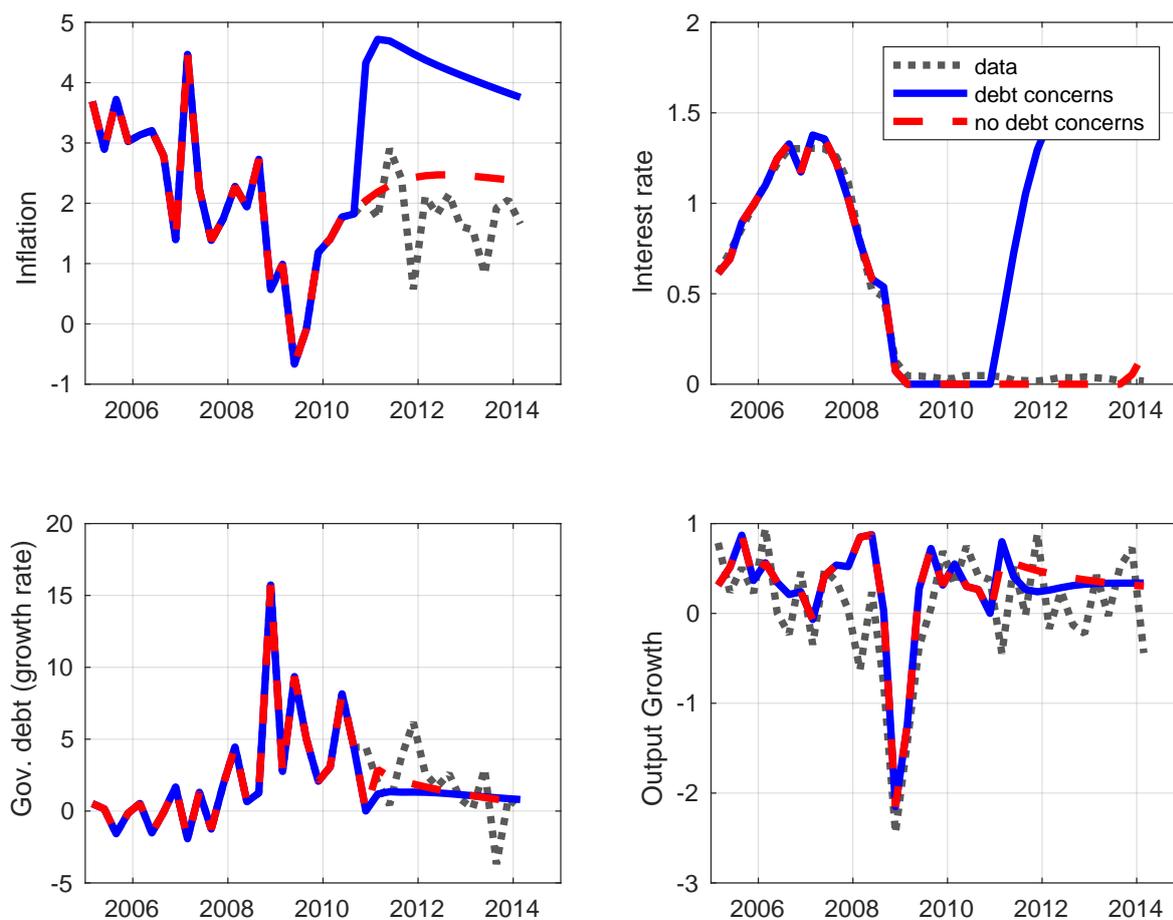
Notes: The figure plots forecasts of inflation (top left), interest rates (top right), government debt (bottom left) and output growth (bottom right) in the models. The solid (blue) line represents the 'temporary debt concerns model', that is when we assume that monetary policy reverts back to no debt concerns with positive probability at the end of the LT episode. The dashed (red) line is the no debt concerns model. The data is the same as in Figure 7.

Figure 9: Shock Decomposition: Inflation in the two models



Notes: The Figures shows the shock decomposition of inflation, in the no debt concerns model (top panel) and the debt concerns model (bottom panel). In both cases we assume that the monetary policy regime is permanent.

Figure 10: **Switching at Quantitative Easing 2**



Notes: The figure plots forecasts of inflation (top left), interest rates (top right), government debt (bottom left) and output growth (bottom right) in the models. We assume that monetary policy switches to debt concerns in the 4th quarter of 2010. The data is the same as in Figure 7.

Table 1: Model Implied Taylor Rules

Parameter	Calibrated values	No debt concerns	Debt concerns		
			Preference shock	G shock	Tax shock
$\phi_\pi$	0.6048	0.6048	0.0302	-0.1298	0.1384
$\phi_Y$	0.8000	0.8000	0.8790	0.3041	0.1664
$\phi_i$	1.6078	1.6078	0.9932	1.1251	0.8468
$\phi_{\Delta i}$	1.0050	1.0050	1.0330	0.3702	0.2143

Notes: The table reports the OLS estimates of the model implied Taylor rule  $\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \Delta \hat{i}_{t-1} + \phi_{\Delta i} \Delta \hat{i}_{t-1}$  (See subsection 2.4.1). The first column shows the values of parameters ( $\phi_\pi, \phi_Y, \phi_i, \phi_{\Delta i}$ ) we obtain from the analytical solution of the model using the calibration reported in Table 2. The second column shows the estimates we obtain from the no debt concerns model. Columns 3-5 show the debt concerns model estimates when we vary the source of shocks that hit the economy. In all cases we assume a first order autocorrelation of 0.9 for the shock processes.

Table 2: **Calibration**  
Parameters Common Across Models

Parameter	Value	Label
$\beta$	0.995	Discount factor
$\lambda_Y$	{0, 0.5}	Loss function - weight on output
$\lambda_i$	{0, 0.5}	Loss function - weight on interest rate
$\theta$	17.5	Price Stickiness
$\eta$	-6.88	Elasticity of Demand
$\sigma$	{0, 1}	Inverse of IES
$\gamma_h$	1	Inverse of Frisch Elasticity
$\rho_\tau$	0.9	Persistence of Taxes
$\bar{b}_\delta$	0.1321	Debt Level
$\bar{\tau}$	0.2545	Tax Rate
$\bar{Y}$	1	Output
$\bar{G}$	0.1	Spending

Parameters Not Common Across Models

Parameter	Value	Label
$\phi_{\tau,b}$	0.00 0.07	Tax rule coefficients

Notes: The table reports the values of model parameters assumed in the numerical experiments in Section 2.  $\beta$  notes the discount factor chosen to target a steady state real interest rate of 2 percent.  $\lambda_Y$  and  $\lambda_i$  are the weights on output and interest rates in the objective of the planner. Parameter  $\eta$  is calibrated to target markups of 17 percent in steady state.  $\theta$  is calibrated as in SGU. Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output which is normalized to unity in steady state.

Finally, the values  $\{\lambda_Y, \lambda_i, \sigma\} = \{0, 0, 0\}$  correspond to the impulse responses shown in Figure 1. The values  $\{\lambda_Y, \lambda_i, \sigma\} = \{0.5, 0.5, 1\}$  correspond to Figure 2.

The bottom panel of the Table reports the value of the coefficient  $\phi_{\tau,b}$  in the tax policy rule (6). As discussed in text we set  $\phi_{\tau,b} = 0.07$  in the no debt concerns model to have a determinate equilibrium. In the debt concerns case we set  $\phi_{\tau,b} = 0.00$  to find a unique equilibrium. See text for further details.

Table 3: **Calibrated parameters**

Parameter	Value
$1 - \alpha$ labor share	0.66
$\delta$ decaying rate of coupon bonds	0.95
$\eta$ demand Elasticity	-6.88
$\gamma_h$ inverse of Frisch elasticity	1
$\frac{\bar{G}}{\bar{Y}}$ steady-state g-to-GDP ratio	0.099
$\frac{\bar{P}_L \bar{b}_\delta}{\bar{Y}}$ steady-state m.v of debt-to-GDP ratio	0.344

**Notes:** The table reports model parameters whose values we fix in estimation. See text for details.

Table 4: **Prior and posteriors**

Parameter	Posterior		distrib	Prior	
	mean	90 % interval		mean	std
<b>Quarterly trends</b>					
$100\gamma$ growth rate	0.4152	[ 0.337 ; 0.489 ]	G	0.4	0.05
$100(\beta^{-1} - 1)$ discount rate	0.1767	[ 0.065 ; 0.288 ]	G	0.25	0.1
$100 \ln \bar{\Pi}$ inflation	0.5706	[ 0.478 ; 0.658 ]	G	0.5	0.05
<b>HH and firms</b>					
$\Omega$ habit formation	0.4757	[ 0.378 ; 0.574 ]	B	0.7	0.1
$\kappa$ slope Phillips curve	0.0086	[ 0.003 ; 0.014 ]	G	0.3	0.15
$\zeta$ price indexation	0.2031	[ 0.076 ; 0.328 ]	B	0.5	0.15
<b>CB preferences</b>					
$\lambda_i$ interest rate smoothing	1.1302	[ 0.768 ; 1.446 ]	G	0.25	0.1
$\lambda_Y$ output smoothing	0.2053	[ 0.097 ; 0.310 ]	G	1	0.2
<b>Fiscal policy</b>					
$\rho_\tau$ tax smoothing	0.958	[ 0.924 ; 0.993 ]	G	0.5	0.25
$\phi_{\tau,b}$ tax response to debt	0.066	[ 0.036 ; 0.092 ]	G	0.07	0.02
$\phi_{\tau,Y}$ tax response to output	0.400	[ 0.099 ; 0.718 ]	N	0.4	0.2
$\phi_{\tau,G}$ tax response to G	0.071	[ -0.025 ; 0.187 ]	N	0.5	0.25
<b>Shock processes</b>					
$\rho_a$ technology	0.428	[ 0.327 ; 0.535 ]	B	0.5	0.2
$\rho_g$ gov. spending	0.989	[ 0.980 ; 0.998 ]	B	0.5	0.2
$\rho_\xi$ preference	0.982	[ 0.972 ; 0.994 ]	B	0.5	0.2
$\rho_\lambda$ gov b.c	0.199	[ 0.062 ; 0.344 ]	B	0.5	0.2
$\rho_p$ price mark-up	0.980	[ 0.964 ; 0.997 ]	B	0.5	0.2
$\mu_p$ price moving average	0.870	[ 0.779 ; 0.958 ]	B	0.5	0.2
<b>Shocks, Std</b>					
$\sigma_a$ technology	0.553	[ 0.467 ; 0.642 ]	IG	0.5	2
$\sigma_g$ gov. spending	1.780	[ 1.581 ; 1.968 ]	IG	1	2
$\sigma_\xi$ preference	5.78	[ 2.686 ; 9.131 ]	IG	1	2
$\sigma_\lambda$ gov b.c	3.259	[ 2.841 ; 3.665 ]	IG	1	2
$\sigma_p$ price mark-up	0.117	[ 0.084 ; 0.149 ]	IG	0.1	2
$\sigma_\tau$ labor tax	0.303	[ 0.266 ; 0.339 ]	IG	0.5	2
$\sigma_r$ interest rate, m.e.	0.036	[ 0.030 ; 0.042 ]	IG	0.1	2

**Notes:** The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90% HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.

### Proof of Proposition 1.

From the first order conditions of the planner's program (equations (8) to (12)) we have:

$$\begin{aligned} \kappa_1 \Delta \psi_{\pi,t} &= -\lambda_Y \Delta \hat{Y}_t + \sigma \frac{\bar{Y}}{\bar{C}} \left( \lambda_i \Delta \hat{i}_t + \Delta \psi_{ZLB,t} - \frac{\lambda_i \Delta \hat{i}_{t-1} + \Delta \psi_{ZLB,t-1}}{\beta} \right) \\ &\quad + \sigma \frac{\bar{Y}}{\bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \omega_Y \Delta \psi_{gov,t} \end{aligned}$$

and so (8) becomes

$$-\hat{\pi}_t + \frac{1}{\kappa_1} \left[ -\lambda_Y \Delta \hat{Y}_t + \sigma \frac{\bar{Y}}{\bar{C}} \left( \lambda_i \Delta \hat{i}_t + \Delta \psi_{ZLB,t} - \frac{\lambda_i \Delta \hat{i}_{t-1} + \Delta \psi_{ZLB,t-1}}{\beta} \right) \right] \quad (57)$$

$$+ \sigma \frac{\bar{Y}}{\bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \omega_Y \Delta \psi_{gov,t} \left] - \frac{\lambda_i \hat{i}_{t-1} + \psi_{ZLB,t-1}}{\beta} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} = 0 \quad (58)$$

$$(59)$$

Rearranging (58) we get:

$$\frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \bar{C}} \hat{i}_t = \hat{\pi}_t + \frac{\lambda_Y \Delta \hat{Y}_t}{\kappa_1} + \left( \frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \bar{C}} + \frac{\lambda_i}{\beta} \right) \hat{i}_{t-1} + \frac{\sigma \lambda_i \bar{Y}}{\kappa_1 \beta} \Delta \hat{i}_{t-1} - \frac{\bar{Y}}{\bar{C}} \frac{\sigma}{\kappa_1} \psi_{ZLB,t} + \frac{\sigma \bar{Y}}{\kappa_1 \bar{C}} \left( 1 + \frac{1}{\beta} + \frac{\bar{C}}{\bar{Y}} \frac{\kappa_1}{\beta \sigma} \right) \psi_{ZLB,t-1} \quad (60)$$

$$- \frac{\sigma \bar{Y}}{\kappa_1 \bar{C}} \bar{b}_\delta \sum_{l=0}^{\infty} \delta^l (\Delta \psi_{gov,t-l} - \Delta \psi_{gov,t-l-1}) + \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,t} + \frac{\bar{b}_\delta}{(1 - \beta \delta)} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (61)$$

We have two cases. 1.  $\psi_{ZLB,t} > 0$  so that the ZLB binds. In this case it follows trivially that  $\hat{i}_t = -i^*$ . 2.  $\psi_{ZLB,t} = 0$ . In this case rearranging (61) we can easily get (13) in Proposition 1. ■

### Proof of Proposition 2.

The proof is trivial: Following the steps in the proof of Proposition 1 and setting  $\psi_{gov,t} = 0$  for all  $t$  we can arrive to the optimal policy in the no debt concerns model. ■

### Derivations for the analytical model of subsections 2.5.2 and 2.6

Assuming  $\lambda_Y = \lambda_i = \sigma = 0$  and focusing on the case where the ZLB does not bind, the first order conditions become:

$$\begin{aligned} -\hat{\pi}_t + \Delta \psi_{\pi,t} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} &= 0 \\ -\psi_{\pi,t} \kappa_1 + \omega_Y \psi_{gov,t} &= 0 \end{aligned}$$

Rearranging we get

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,t} + \frac{\bar{b}_\delta}{1 - \beta \delta} \sum_{l=0}^{\infty} \delta^l \Delta \psi_{gov,t-l} \quad (62)$$

the optimal inflation path in this model.

Under perfect foresight (from period 0 onwards) we can drop conditional expectations and the multiplier  $\psi_{gov}$  satisfies  $\psi_{gov,t} = \psi_{gov,t+1}$ . Therefore,  $\Delta\psi_{gov,t} = 0$  for  $t \geq 1$  and  $\Delta\psi_{gov,0} \neq 0$  since a shock hits in period 0. Therefore, (62) becomes:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta\psi_{gov,0} \quad (63)$$

as is claimed in text. Moreover, from the Euler equation we have

$$\hat{i}_t = \hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t = \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^{t+1} \Delta\psi_{gov,0} - \rho_\xi^t (\rho_\xi - 1) \hat{\xi}_0$$

From the Phillips curve we obtain the following expression for  $\hat{Y}_t$ :

$$\hat{Y}_t = \frac{1}{\kappa_1} \left[ \hat{\pi}_t - \beta \hat{\pi}_{t+1} - \kappa_2 \hat{\tau}_t + \kappa_3 \hat{G}_t \right] \quad (64)$$

which, after noting that taxes dont respond to lagged debt in this version of the model and  $\kappa_3 = 0$ , becomes:

$$\hat{Y}_t = \frac{1}{\kappa_1} \left[ \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} - \kappa_2 \rho_\tau^t \hat{\tau}_0 \right] \quad (65)$$

Now lets turn to the intertemporal budget constraint to find an analytical expression for  $\Delta\psi_{gov,0}$  as a function of the shocks. Notice that since there is perfect certainty after period 0, the date 0 intertemporal constraint (18) is sufficient for the equilibrium (see FMOS). Given the parameterization of the model and assuming for simplicity  $\hat{b}_{-1,\delta} = 0$  we have

$$\sum_{t=0}^{\infty} \beta^t \left[ -\bar{G} \left( \hat{G}_t + \hat{\xi}_t \right) + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left( (1 + \gamma_h) \hat{Y}_t + \frac{\hat{\tau}_t}{1 - \bar{\tau}} + \hat{\xi}_t \right) \right] = \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t E_t \left[ \hat{\xi}_t - \sum_{l=0}^t \hat{\pi}_l \right] \quad (66)$$

Substituting out the expressions for output and inflation, the LHS of the above equation becomes:

$$\begin{aligned} & -\hat{G}_0 \bar{G} \sum_{t=0}^{\infty} \beta^t \rho_G^t + \hat{\tau}_0 \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left( \frac{1}{1 - \bar{\tau}} - (1 + \gamma_h) \frac{\kappa_2}{\kappa_1} \right) \sum_{t=0}^{\infty} \beta^t \rho_\tau^t \\ & + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \frac{1}{\kappa_1} \sum_{t=0}^{\infty} \beta^t \left[ \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \bar{b}_\delta \delta^t \Delta\psi_{gov,0} \right] + \left( -\bar{G} + \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \right) \sum_{t=0}^{\infty} \beta^t \rho_\xi^t \hat{\xi}_0 = \\ & \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \rho_\xi^t \hat{\xi}_0 - \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \left( \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,0} \mathcal{I}_{t=0} + \sum_{l=0}^t \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^l \Delta\psi_{gov,0} \right) \end{aligned}$$

Rearranging all  $\Delta\psi_{gov,0}$  terms to the LHS of the above equation we get:

$$\underbrace{\left[ \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \frac{1}{\kappa_1} \left( \frac{\omega_Y}{\kappa_1} \sum_{t=0}^{\infty} \beta^t + \bar{b}_\delta \delta^t \right) + \bar{b}_\delta \sum_{t=0}^{\infty} \beta^t \delta^t \left( \frac{\omega_Y}{\kappa_1} + \sum_{l=0}^t \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^l \right) \right]}_{v_{gov}} \Delta\psi_{gov,0}$$

on the LHS where  $v_{gov} > 0$ . On the RHS we have:

$$\underbrace{\frac{\bar{G}}{1 - \beta\rho_G}}_{v_G} \hat{G}_0 - \underbrace{\frac{1}{1 - \beta\rho_\tau} \frac{\bar{\tau}\bar{Y}(1+\eta)}{\eta} \left( \frac{1}{1 - \bar{\tau}} - (1 + \gamma_h) \frac{\kappa_2}{\kappa_1} \right)}_{v_\tau} \hat{\tau}_0 + \underbrace{\bar{b}_\delta \frac{\beta(1 - \rho_\xi)(1 - \delta)}{(1 - \beta\rho_\xi)(1 - \beta\delta)(1 - \beta\rho_\xi\delta)}}_{v_\xi} \hat{\xi}_0$$

The coefficient  $v_\xi$  is positive and becomes zero when  $\delta = 1$ . Coefficient  $v_G$  is positive.

#### Proof of Proposition 4.

It is straightforward to show that the FONC give:

$$-\hat{\pi}_t + \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta \psi_{gov,0} - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \frac{\psi_{ZLB,t-1}}{\beta} = 0 \quad (67)$$

in the debt concerns model and

$$-\hat{\pi}_t^{NDC} - \frac{\lambda_i \hat{i}_{t-1}^{NDC}}{\beta} + \frac{\psi_{ZLB,t-1}^{NDC}}{\beta} = 0 \quad (68)$$

in the no debt concerns model.

Focus first on the NDC equilibrium. According to (68)  $\hat{\pi}_0^{NDC} = 0$  clearly. Moreover, since the shock is large we have:  $\hat{i}_0^{NDC} = \hat{\pi}_1^{NDC} - \hat{\xi}_0 = -\hat{i}^*$ . This gives the value of  $\hat{\pi}_1^{NDC}$ . From  $t = 1$  onwards, let's guess (and then verify) that  $\hat{i}_t^{NDC} = \hat{\pi}_{t+1}^{NDC} > -\hat{i}^*$ . From (68) we have  $-\hat{\pi}_{t+1}^{NDC} - \frac{\lambda_i \hat{\pi}_{t+1}^{NDC}}{\beta} = 0$  and so inflation equals zero from period 2 onwards. Clearly this gives  $\hat{i}_t^{NDC} = 0$  for  $t \geq 0$  consistent with the guess.

Now consider the debt concerns model. From (67) we have:  $\hat{\pi}_0 = \frac{\omega_Y}{\kappa_1} \Delta \psi_{gov,0} \mathcal{I}_{t=0} + \frac{\bar{b}_\delta}{1 - \beta\delta} \Delta \psi_{gov,0}$ . Moreover, from the binding ZLB  $\hat{i}_0 = -\hat{i}^*$ , we have  $\hat{\pi}_1 = -\hat{i}^* - \hat{\xi}_0$ . For  $t > 1$  it must be  $\hat{\pi}_t = -\hat{i}^*$  if  $\hat{i}_{t-1} = -\hat{i}^*$ , however,

$$-\hat{\pi}_t + \frac{\bar{b}_\delta}{1 - \beta\delta} \delta^t \Delta \psi_{gov,0} - \frac{\lambda_i \hat{\pi}_t}{\beta} = 0$$

if  $\hat{i}_{t-1} > -\hat{i}^*$ . ■

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