Monetary and macroprudential policy games in a monetary union

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Abstract

We use the two-country model of the euro area developed by Quint and Rabanal (2014) to study policymaking in the European Monetary Union (EMU). In particular, we focus on strategic interactions: 1) between monetary policy and a common macroprudential authority, and; 2) between an EMU-level monetary authority and regional macroprudential authorities. In the first case, price stability and financial stability are pursued at the EMU level, while in the second case each macroprudential authority adopts region-specific objectives. We compare cooperative equilibria in the simultaneous-move and leadership solutions, each obtained assuming policy discretion. Further, we assess the effects on policy performance of assigning shared objectives across policymakers and of altering the level of importance attached to various policy objectives.

Keywords: Monetary policy, macroprudential policy, policy coordination

JEL Codes: E42, E44, E52, E58, E61

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1 Introduction

This paper studies the strategic interactions between monetary and macroprudential policies in the context of the two-country DSGE model of Quint and Rabanal (2014) estimated for the euro area. Their framework allows us to analyze the current policy landscape in the EMU, where the ECB is charged as the single monetary authority with the task to maintain price stability, while at the same time its macroprudential responsibilities at area-wide level are shared with national prudential authorities. This multilayer-feature of policy setting makes cooperation across the several authorities a challenging task. We investigate the consequences on macroeconomic stability when this cooperation fails, and identify the cases in which it might not even be desirable for authorities to cooperate when it concerns the pursuit of regional (or country-specific) objectives. We perform our analysis in a setup where each authority is assumed to minimize an assigned loss function, and behave as a player in an interaction game that is either cooperative or non-cooperative. We compute nash and leadership equilibria in order to distinguish between alternative timings of the players' moves. We compare the solutions in the following two alternative settings. First, we distinguish between a common EMU-wide monetary and macroprudential authority, both pursuing area-wide objectives, and therefore ignoring regional imbalances and heterogeneity across the euro-area. This is a case of two players similar in spirit to the approaches undertaken particularly in closed economy setups, such as the analysis performed by De Paoli and Paustian (2013), Angelini et al. (2012), Bean et al. (2010), Beau et al. (2012), Darracq Paries et al. (2011), Gelain and Ilbas (2016), and others. In the second setting, we assume that macroprudential duties are performed at the regional level[1], where one authority is in charge of prudential policy in the core and another one in the periphery, both pursuing regional financial stability objectives, while the ECB continues being in charge of area-wide price stability. Therefore, in this second setting, there are three players whose cooperative and non-cooperative interactions are analyzed. Allowing for three players to interact in the policy game yields a broader, and more realistic policy analysis within the context of a monetary union. In this respect, the current paper differs most from the existing literature. In particular, to our knowledge, other papers that consider a similar, two-country setup to analyze interactions between policy makers, such as Brzoza-Brzezina et al. (2013) and Quint and Rabanal (2014), do not take into account the effects of strategic interactions between more than two players.

We find that the gains from leadership of either the ECB or the prudential regulator in the two-player setting, giving the leader the first mover advantage, are limited. In this case where the ECB and macroprudential policy are conducted at the union-wide level, the most favorable outcome is achieved under cooperative and nash equilibria. However, successful stabilization of union-wide objectives comes at the cost of highly volatile credit-to-GDP ratios in both the core and the periphery (while being very stable in the union as a whole). Focusing on non-cooperative policies but assigning real union-wide GDP growth as a common objective leads

[1] While in reality the ECB and national authorities have shared competences, the role of the former is limited to imposing stricter capital requirements as foreseen in the EU legislations. Allowing for only national prudential authorities however simplifies the analysis without too much loss of realism.

to non-cooperation outperforming cooperation, but it does not remove the excessive regional-level volatility in the credit-to-GDP ratio. Introducing regional prudential regulators focusing on their own, region-specific objectives leads to a situation in which non-cooperation performs better that cooperation, provided policymakers choose their policies simultaneously. Assigning to the ECB the leadership role can lead to worse outcomes in the three-player setting since it implies more focus on union-wide objectives. However, assigning macroprudential policy to regional authorities contains the volatility in the credit-to-GDP ratios in the core and the periphery, at the cost of slightly higher union-wide inflation and output growth volatility.

This paper is organized as follows. Section 2 outlines the modeling framework based on Quint and Rabanal (2014). Section 3 explains the monetary and macroprudential policy framework adopted throughout the paper. Section 4 presents the results of the strategic interactions between monetary and macroprudential policy at the union-wide level, while section 5 shows the results of interactions between monetary policy and regional macro-prudential authorities. Finally, section 6 concludes.

2 The modeling framework

Our analysis is conducted using the model developed and estimated in Quint and Rabanal (2014). Briefly, the Quint and Rabanal (2014) model is one of Europe and contains two regions, which are labeled the "core" and the "periphery". The core consists of France and Germany while the periphery contains Greece, Ireland, Italy, Portugal, and Spain. Each region has sectors that produce non-durables, which can be consumed or traded, and durables, which cannot be traded and are accumulated (subject to an adjustment cost) to augment the housing stock. The durables and non-durables sectors are monopolistically competitive, consisting of intermediate-good producers who set their price subject to a Calvo-based (Calvo, 1983) price rigidity and inflation indexation (Christiano, Eichenbaum and Evans, 2005).

On the demand side, each region is populated by two types of households: savers and borrowers, where borrowers are more impatient than savers and are characterized by lower discount factors. Both types of household obtain utility by consuming non-durable goods and housing services, and obtain disutility by supplying labor. The nominal wage is perfectly flexible and both types of households work to produce durables and non-durables. However, there is a labor reallocation cost associated with labor moving from one sector to the other. For both patient and impatient households, the consumption of non-durable goods exhibits external habit formation.

Each region contains financial intermediaries. These intermediaries receive the savings of the patient households and lend to the impatient households. Excess savings in one region can be transferred to the other through the purchase of one-period nominal bonds from an international intermediary. Impatient households, or borrowers, obtain one-period nominal loans from their regional financial intermediary, using the value of their housing stock as collateral. However, the quality of housing as collateral is subject to an idiosyncratic shock and this creates some risk that a borrower may not repay their loan. When the value of the housing stock is less than the loan repayment, then the borrower will default. When default occurs, the borrower pays the house's collateral value, which is split between the financial intermediary and a debt collection agency, and keeps their house. The revenues generated by the debt collectors are returned (lump-sum) to savers. Where the regional financial intermediaries are risk neutral, the international financial intermediary charges a risk premium (related to the region's debt-to-GDP ratio) on regional borrowing.

While the resulting model is large, with multiple regions, sectors, and agents, we use this model, not only because it is estimated, but because it provides roles for both monetary policy and macroprudential policy and it allows the effects of these policies on the economies of the core and the periphery to be considered.

2.1 Key shocks and policy channels

Risk shocks are an important feature of the model. Borrowers face an idiosyncratic shock to the value of their collateral (their housing stock) and are more likely to default on their loan when the idiosyncratic shock is small and their collateral has less value. The standard deviation of each idiosyncratic shock is stochastic and assumed to follow an autoregressive process. An increase in the standard deviation affects the volatility, but not the mean of the shock to collateral quality, and is termed a risk shock. When a risk shock occurs, because the variance of collateral quality rises, it becomes more likely that realized collateral quality will be below the default threshold, increasing the default rate, which raises the credit spread and adversely effects the balance sheet of financial intermediaries with ongoing adverse effects on bank lending. One way to think about these risk shocks is that they raise the loan-to-value ratio for borrowers. Other shocks that also raise the loan-to-value ratio will have similar effects on the default rate, the credit spread, and bank lending.

The policy instrument for the monetary authority is assumed to be the bank deposit rate, which, because monetary policy is conducted at the union-wide level, means financial intermediaries in both regions pay the same rate on deposits. A tightening of monetary policy has the effect of raising the deposit rate, which encourages saving by the patient households and discourages borrowing by the impatient households. As such, tighter monetary policy has the usual damping effect on consumption. However, the higher deposit rate also raises the cost of providing loans which gets passed on to borrowers through a higher lending rate and can lower the value of housing collateral. All else constant, a higher lending rate raises the loan-to-value ratio and increases the default rate.

Macroprudential policy is assumed to operate through a variable lending fraction, with tighter macroprudential policy associated with a lower lending fraction. In this model, when the macroprudential authority lowers the lending fraction, perhaps by requiring financial intermediaries to increase their reserves or raise their loan-loss provisions, financial intermediaries must restrict their lending, which leads to an increase in the lending rate and hence in credit spreads and has an adverse effect on the demand for goods by borrowers, particularly.

3 Monetary and macroprudential policy framework

In addition to private agents, the model is inhabited by a monetary authority and a macroprudential authority and is one in which the policies of each authority can have important economic effects. Recognizing the model's size and complicated structure, we approach the decision problems faced by the monetary authority and the macroprudential authority in terms of constrained optimization problems in which each authority chooses its policy to optimize an objective function, subject to constraints, some of which are forward-looking. We take the "primal" approach and express policy objectives in terms of loss functions that view unfavorably volatility in key macroeconomic aggregates, such as inflation, output growth and the credit-to-GDP ratio. The obvious alternative would be to use welfare-based policy objectives. We take the approach of specifiying policy objectives in terms of loss functions for a couple of reasons. First, policy goals involving targets for inflation and credit-to-GDP ratios appear in legislation and/or policy mandates, unlike goals expressed (directly) in terms of welfare, and can be readily captured through loss functions. Second, exercises such as Schmitt-Grohe and Uribe (2007) showed that welfare-based objectives can often lead to extreme policies, ones in which policy coefficients need to be artificially constrained.[2]

3.1 Objectives and instruments of the monetary authority

The central bank operates at the union-wide level and its objectives are assumed to be captured by the following intertemporal loss function:

$$L_t^{CB} = (1-\delta)\, E_t \sum_{i=0}^{\infty} \delta^i \left[\pi_{t+i}^2 + \lambda_{\Delta y}^{CB} (y_{t+i} - y_{t-1+i})^2 + \lambda_{\Delta r}^{CB} (r_{t+i} - r_{t-1+i})^2\right], \qquad (1)$$

where $|\delta| < 1$ denotes the central bank's discount factor and E_t denotes the expectation operator conditional on information available at time t. With π_t, y_t, and r_t representing (union-wide) inflation, output, and the deposit rate (the monetary policy instrument), respectively, the central bank is assumed to set the deposit rate in order to stabilize union-wide annual inflation and union-wide real output growth, without creating large changes in the deposit rate. The weight on the inflation target is normalized to one, hence the weights on the output gap, $\lambda_{\Delta y}^{CB} > 0$, and on the change in the interest rate, $\lambda_{\Delta r}^{CB}$, indicate the importance of stabilizing these variables relative to stabilizing inflation.

As is well-known, in the limit as $\delta \uparrow 1$, equation (1) converges to:

$$L_t^{CB} = Var\left(\pi_t\right) + \lambda_{\Delta y}^{CB} Var\left(\Delta y_t\right) + \lambda_{\Delta r}^{CB} Var\left(\Delta r_t\right), \qquad (2)$$

where $Var\left(\pi_t\right)$, for example, denotes the unconditional variance of inflation. The central bank chooses policy to minimize equation (2), subject to restrictions that come from the structural model, under discretion.

[2] Further, although it is possible to construct second-order accurate measures of welfare for each household-type in each region of the model, exactly how to perform the aggregation is unclear. The method of Negishi (1960) is often invoked, but cannot be used here as it requires the welfare theorems to hold.

3.2 Objectives and instruments of the macroprudential authority

To define the macroprudential loss function we consider two distinct cases. In the first case, macroprudential policy is conducted at the union-wide level and the macroprudential authority (or regulator) has the union-wide loss function:

$$L_t^{MP} = (1-\delta)\, E_t \sum_{i=0}^{\infty} \delta^i \left[\lambda_{cr/y}^{MP} (cr_{t+i}/y_{t+i})^2 + \lambda_{\Delta\eta}^{MP} (\eta_{t+i} - \eta_{t-1+i})^2 \right], \tag{3}$$

where cr_t represents credit. Following Quint and Rabanal (2014), the regulator sets the macroprudential instrument, η_t, which influences credit spreads by affecting the fraction of funds that financial intermediaries are able to lend. We assume that the union-wide macroprudential authority has a double mandate. Specifically, it is tasked with stabilizing both the union-wide credit-to-GDP ratio, cr_t/y_t, without making large changes in the lending fraction, so the weights on these two objectives, $\lambda_{cr/y}^{MP}$ and $\lambda_{\Delta\eta}^{MP}$, respectively, are positive. As earlier, we consider the limiting case where $\delta \uparrow 1$ so that the macroprudential authority conducts policy by choosing η_t to minimize:

$$L_t^{MP} = \lambda_{cr/y}^{MP} Var\left(cr_t/y_t\right) + \lambda_{\Delta\eta}^{MP} Var\left(\Delta\eta_t\right), \tag{4}$$

subject to restrictions reflected in the structural model, under discretion.

In the second case, we assume that each region—the core and the periphery—has its own macroprudential authority or regulator and that each regulator seeks to stabilize its own region's credit-to-GDP ratio and to smooth its own lending fraction. For this case, the macroprudential loss functions for the core and periphery are, respectively:

$$L_t^{MP,c} = (1-\delta)\, E_t \sum_{i=0}^{\infty} \delta^i \left[\lambda_{cr/y}^{MP,c} (cr_{t+i}^c/y_{t+i}^c)^2 + \lambda_{\Delta\eta}^{MP,c} (\eta_{t+i}^c - \eta_{t-1+i}^c)^2 \right], \tag{5}$$

$$L_t^{MP,p} = (1-\delta)\, E_t \sum_{i=0}^{\infty} \delta^i \left[\lambda_{cr/y}^{MP,p} (cr_{t+i}^p/y_{t+i}^p)^2 + \lambda_{\Delta\eta}^{MP,p} (\eta_{t+i}^p - \eta_{t-1+i}^p)^2 \right], \tag{6}$$

where we distinguish between the macroprudential instruments for core and the periphery, η_t^c and η_t^p, respectively. Again, in the limit as $\delta \uparrow 1$, these loss functions converge to:

$$\begin{aligned} L_t^{MP,c} &= \lambda_{cr/y}^{MP,c} Var\left(cr_t^c/y_t^c\right) + \lambda_{\Delta\eta}^{MP,c} Var\left(\Delta\eta_t^c\right), && (7) \\ L_t^{MP,p} &= \lambda_{cr/y}^{MP,p} Var\left(cr_t^p/y_t^p\right) + \lambda_{\Delta\eta}^{MP,p} Var\left(\Delta\eta_t^p\right). && (8) \end{aligned}$$

3.3 Strategic interaction between monetary and macroprudential policy

We consider optimal discretionary policies under both cooperation and non-cooperation. Cooperative policies are the outcome of shared policy objectives while non-cooperative policies allow the central bank and the macroprudential regulator to have distinct policy objectives. For the cooperative case, we consider three timing environments. The first of these timing environments is where both policymakers move simultaneously whereas the second and third environments correspond to those where either the monetary authority or the macroprudential authority has a first-mover advantage, i.e., moves first within the period. These three discretionary policies are compared to an optimal benchmark, which is described by the optimal commitment policy under cooperation.

Clearly, the interaction between monetary policy and macroprudential policy will depend on whether macroprudential policy operates at the regional level or the union-wide level, and we consider each in turn below.

3.3.1 Cooperation

In the case of a union-wide macroprudential authority, the policy problem involves two "players", a union-wide monetary authority and a union-wide macroprudential authority, whose policies can potentially interact. When these two policymakers cooperate, their respective loss functions, equations (2) and (4), are combined into a single joint loss function, which is given by:

$$L_t^{coop} = Var\left(\pi_t\right) + \lambda_{\Delta y}^{CB} Var\left(\Delta y_t\right) + \lambda_{\Delta r}^{CB} Var\left(\Delta r_t\right) + \lambda_{cr/y}^{MP} Var\left(cr_t/y_t\right) + \lambda_{\Delta \eta}^{MP} Var\left(\Delta \eta_t\right). \quad (9)$$

Where the two policymakers make their decisions simultaneously, we can treat this problem as one in which there is a single policymaker whose task is to minimize the joint loss function, L_t^{coop}, having two instruments at its disposal, i.e. the interest rate r_t and η_t. Although there are two policy instruments, this decision problem boils down to a standard discretionary problem and can be solved using standard methods (Dennis, 2007).

As mentioned above, we also consider the cooperative cases where one policymaker (the leader) has a first-mover advantage with respect to other policymaker (the follower). In these cases, the instrument of the leader is chosen first and the follower sets its instrument taking the leader's policy into account. With this timing structure, the leader can predict the follower's reaction and exploit this reaction when setting its own policy. We consider both monetary leadership and macroprudential leadership.

3.3.2 Non-cooperation

Non-cooperation differs from cooperation in as much as the two policymakers do not share a common objective function. The non-cooperative environments we consider are those where the monetary authority and the macroprudential authority move separately, but simultaneously, each formulating their policy to minimize their respective objective functions. Accordingly, r_t and η_t are chosen simultaneously, with r_t chosen to minimize equation (2) and η_t chosen to minimize equation (4), subject to constraints imposed by the model equations and with each policymaker taking as given the decision of the other policymaker.

3.3.3 Regional macroprudential policies

When macroprudential policy is formulated at the regional level the model has two macroprudential authorities, one conducting macroprudential policy in the core and the other in the periphery. Because there are two macroprudential authorities and a union-wide central bank, the model contains three policymakers with each choosing its policy optimally under discretion.

With three policymakers all optimizing, the range of strategic environments that could be considered proliferate. For the cooperative case, the (cooperative) loss function is given as the sum of their respective loss functions, i.e., the sum of equations (2), (7) and (8):

$$\begin{aligned} L_t^{CB} + L_t^{MP,c} + L_t^{MP,p} &= Var(\pi_t) + \lambda_{\Delta y}^{CB} Var(\Delta y_t) + \lambda_{\Delta r}^{CB} Var(\Delta r_t) \\ &\quad + \lambda_{cr/y}^{MP,c} Var(cr_t^c / y_t^c) + \lambda_{\Delta\eta}^{MP,c} Var(\Delta\eta_t^c) \\ &\quad + \lambda_{cr/y}^{MP,p} Var(cr_t^p / y_t^p) + \lambda_{\Delta\eta}^{MP,p} Var(\Delta\eta_t^p). \end{aligned} \tag{10}$$

As above, this cooperative loss function is minimized as a standard discretionary problem with the three instruments, r_t, η_t^c, and η_t^p chosen simultaneously.

In addition to the simultaneous-move cooperative case, we also consider the leadership case where the union-wide central bank has a first-mover advantage with respect to the two macroprudential authorities, who are assumed to choose their policies simultaneously with each other, but following the central bank. Finally, we also consider the non-cooperative case where the central bank and the core and periphery macroprudential authorities have differing policy objectives, governed by equations (2), (7) and (8), respectively.

4 Monetary-macroprudential interactions at union-wide level

In this section we focus on union-wide policymaking and explore the interactions between monetary policy and macroprudential policy for a range of cooperative and non-cooperative environments. We begin by comparing optimal discretionary policy to the optimal commitment policy and to an estimated Taylor-rule policy.

4.1 Taylor rule vs. optimal cooperative policies

The first three columns of Table 1 compare the model's solution with monetary policy conducted according to an estimated Taylor rule (and with macroprudential policy described by a constant lending fraction) to the optimal cooperative policies obtained under commitment and discretion when policymakers move simultaneously. With policymakers cooperating, we assume that the loss function shared by the monetary authority and the macroprudential authoity is given by equation (9) with $\lambda_{\Delta y}^{CB} = 1$, $\lambda_{\Delta r}^{CB} = 0.5$, $\lambda_{cr/y}^{MP} = 1$, and $\lambda_{\Delta\eta}^{MP} = 0.5$. It is clear from the table that the biggest gain when going from the estimated Taylor rule policy to optimal cooperative policy (whether under commitment or discretion) is achieved through annual inflation and credit-to-GDP ratio becoming more stable at the EMU-wide level. Not surprisingly, the decline in the volatility of the credit-to-GDP ratio is very large as macroprudential policy, in addition to monetary policy, is used actively to stabilize it. Comparing the commitment and discretionary policies, the main difference between them that stands out is the higher inflation volatility under commitment. This result might be a bit surprising from the viewpoint of models without financial frictions and in which monetary policy only is used for macro-stabilization. However, in the presence of an additional macroprudential policymaker, and with time-inconsistency affecting more than just the trade-off between inflation and output volatility, higher inflation volatility under commitment can arise when policy promises, or forward guidance, is directed more toward stabilizing the credit-to-GDP ratio than towards stabilizing inflation.[3]

The final column of Table 1 considers the case where only macroprudential policy is optimal while monetary policy is conducted according to the estimated Taylor rule. Compared to the cases where both policy instruments are directed at minimizing the joint loss function, when only macroprudential policy is optimizing the result is higher volatility in both annual inflation and the credit-to-GDP ratio. Interestingly, no stable discretionary solution could be found when monetary policy only was used to minimize equation (9) (with macroprudential policy set to deliver a constant lending fraction). This result is due primarily to the high volatility in the credit-to-GDP ratio, which cannot be effectively addressed by monetary policy only.

Table 1: **Some benchmark cooperative solutions**

	Estimated Taylor rule	Optimal cooperative policy		Optimal macroprudential policy (monetary policy = estimated Taylor rule)
		$\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$		
		Commitment	Discretion	Discretion
σ_{π}^2	1.123	0.159	0.081	1.276
$\sigma_{\Delta y}^2$	1.973	1.545	1.757	1.744
$\sigma_{\Delta r}^2$	0.305	0.162	0.287	0.280
$\sigma_{cr/y}^2$	82.647	0.027	0.080	0.430
$\sigma_{\Delta\eta}^2$	0.000	0.163	0.1780	0.188
L^{coop}	85.896	1.892	2.150	3.683

Note: The table reports the variances and the unconditional losses under the estimated Taylor rule (and absent macroprudential policy) and under optimal cooperative policies (commitment and discretion) where the EMU prudential regulator and the central bank jointly minimize equation (9), and the case where macroprudential policy only acts optimally to minimize equation (9).

The following two figures compare the responses to a risk shock in the periphery and the core, respectively, under the estimated Taylor rule and the optimal cooperative policies (commitment and discretion) for the parameterization of the joint loss function (9) considered above. First of all, the spillover of the shock from the periphery to the core is qualitatively similar to the spillover of the shock from the core to the periphery, but is quantitatively smaller as the periphery risk shock has a standard deviation that is twice that of the core. The figures also reveal only small differences in the responses under commitment and discretion, suggesting that time-inconsistency considerations do not seem to cause discretionary policy to respond with great inefficiency to risk shocks. Regarding the differences between the optimal cooperative policies and the Taylor rule policy, Figure 1 shows that the absence of macroprudential policy results in a more accommodative interest rate than the optimal polices, which is needed to address the larger fall in the EMU-wide credit-to-GDP ratio and the deeper recession at the EMU level. In contrast to the optimal cooperative policies—where the shock creates a boom in the core—under the Taylor rule policy the core is also hit by a recession. With both the core and the periphery experiencing a recession, the Taylor rule's policy response is the appropriate one for both regions.

[3] With a lower weight assigned to stabilizing the credit-to-GDP ratio in the shared loss function, the usual stabilization bias associated with discretionary policymaking reasserts itself.

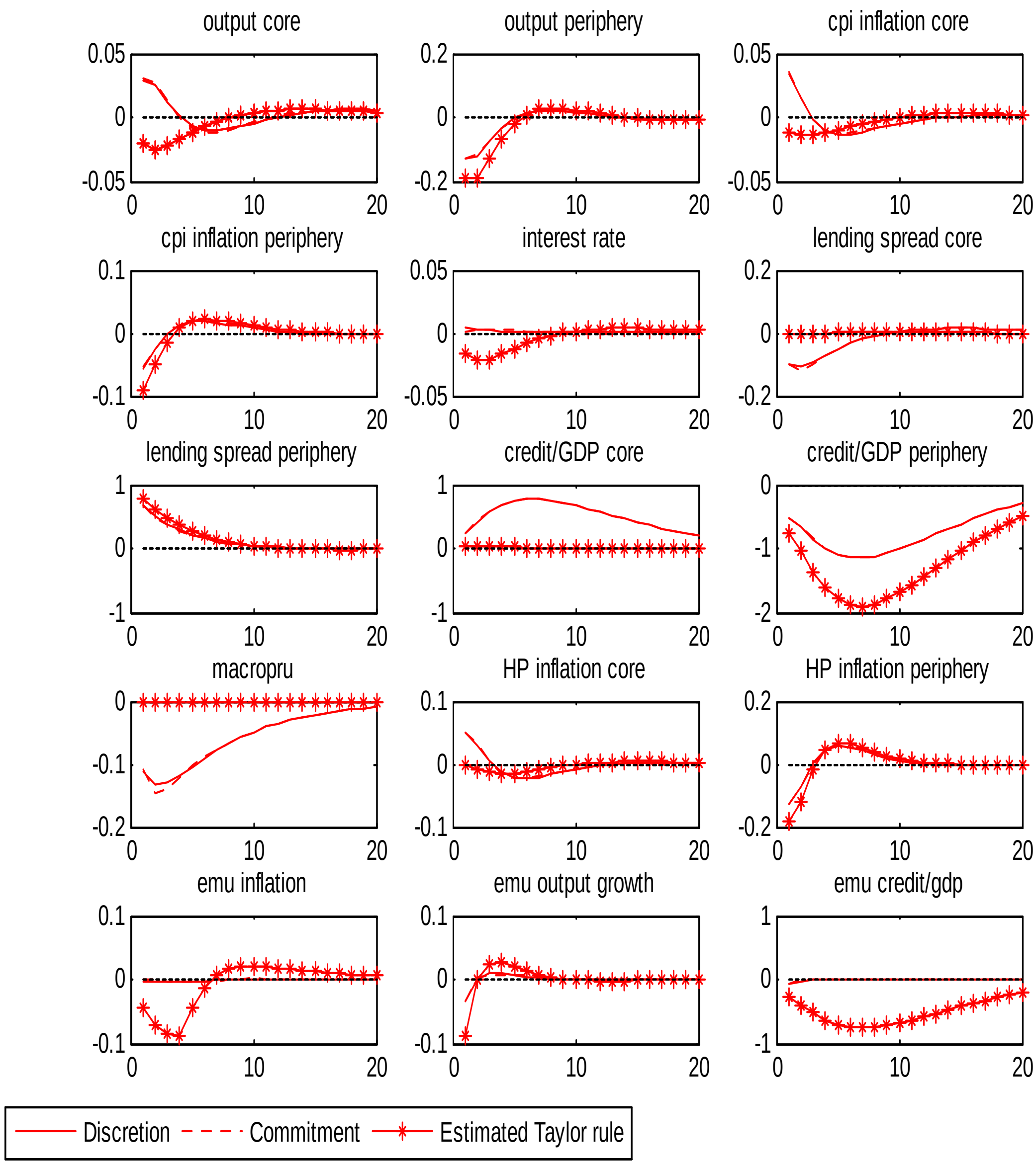

Figure 1. Impulse responses risk shock in the periphery: estimated TR vs. cooperative policies (commitment vs. discretion)

Note: The figure plots the responses to a risk shock in the periphery under the estimated Taylor rule and the optimal cooperative policies (commitment and discretion) for the loss function, equation (9), with the following weights: $\lambda^{CB}_{\Delta y} = 1$, $\lambda^{CB}_{\Delta r} = 0.5$, $\lambda^{MP}_{cr/y} = 1$, $\lambda^{MP}_{\Delta \eta} = 0.5$.

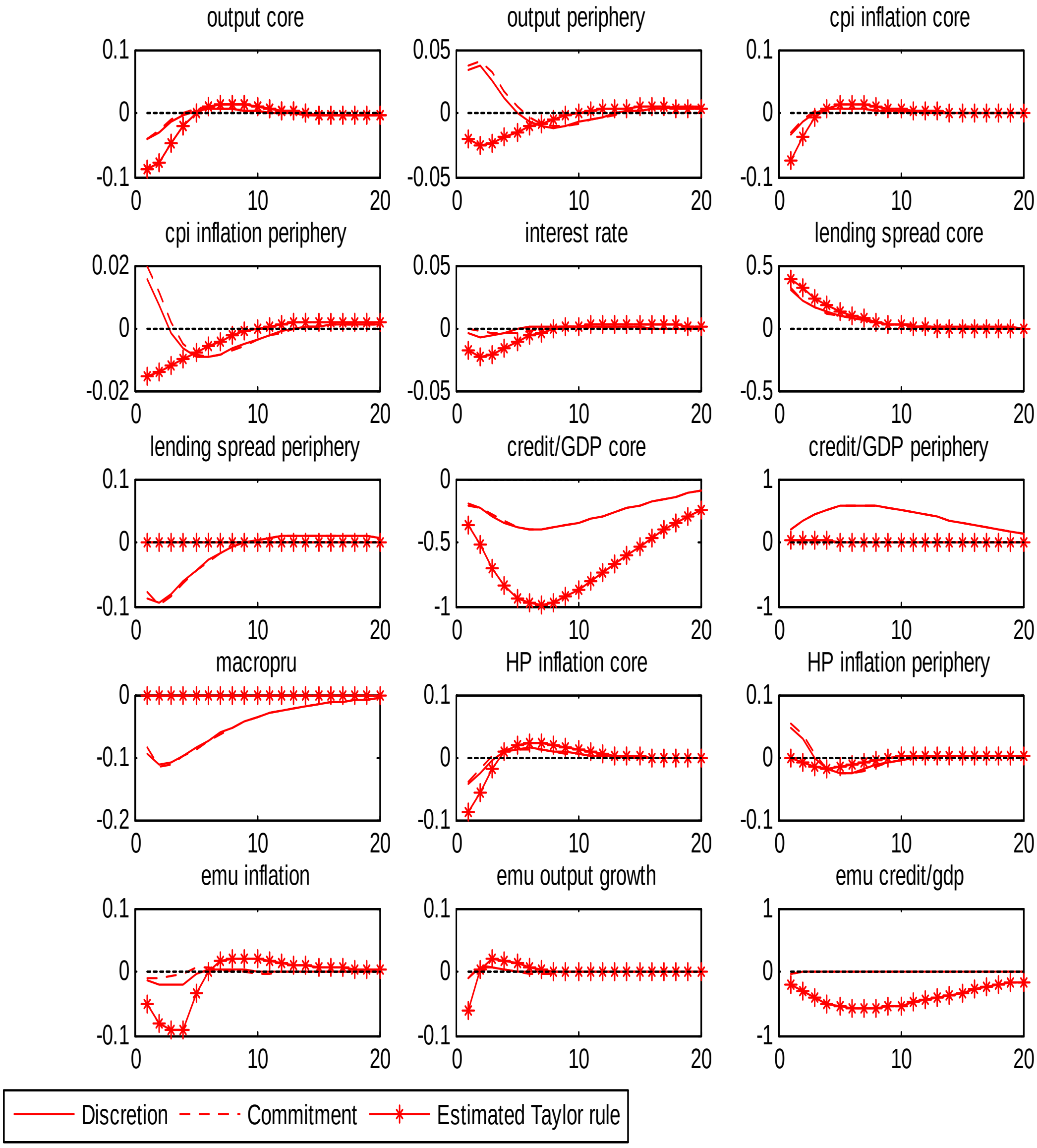

Figure 2. Impulse responses for a risk shock in the core: estimated TR vs. cooperative policies (commitment vs. discretion)

Note: The figure plots the responses to a risk shock in the core under the estimated Taylor rule and the optimal cooperative policies (commitment and discretion) for the loss function, equation (9), with the following weights: $\lambda_{\Delta y}^{CB} = 1$, $\lambda_{\Delta r}^{CB} = 0.5$, $\lambda_{cr/y}^{MP} = 1$, $\lambda_{\Delta \eta}^{MP} = 0.5$.

4.2 Leadership equilibria

The results above assumed that the central bank and the prudential regular make their policy decision simultaneously. We now extend upon that analysis by considering the alternative environments in which either the central bank or the prudential regular moves first—serving as the leader—within the period. The results are shown in Table 2, which compares the solution under simultaneous move (Nash) to the two leadership cases, where the maintained assumption is that the policymakers cooperate and therefore share the same loss function, equation (9).

Table 2: **Losses under cooperative policy (discretion)**

	Nash	Monetary leadership	Prudential leadership
	$\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$		
σ_{π}^2	0.081	0.078	0.083
$\sigma_{\Delta y}^2$	1.757	1.758	1.758
$\sigma_{\Delta r}^2$	0.287	0.288	0.303
$\sigma_{cr/y}^2$	0.080	0.084	0.082
$\sigma_{\Delta\eta}^2$	0.178	0.188	0.178
L^{coop}	2.150*	2.158	2.164

Note: The table reports the variances and the unconditional losses in the model under the Nash and Leadership solutions where the EMU regulator and the central bank both minimize the common loss function, equation (9), either at the same time (Nash), or in the leader-follower setup.

Although the differences among the losses is small, Table 2 shows that the preferred environment is the one in which the policymakers choose simultaneously (Nash), for while the policymaker that moves first gains from that advantage it does so at the expense of the policymaker that moves second, with the cost to the follower outweighing the gain to the leader. However, although the simultaneous-move environment is preferred, the differences in outcomes across the three decision problems are generally quite small, suggesting that qualitatively and quantitatively similar results are obtained from this two-player policy problem regardless of the particular timing assumption made.

4.3 Non-cooperation

Where the previous results related to the case where policymakers cooperated, sharing the same policy objectives, we now turn to the non-cooperative case. We assign distinct loss functions to the monetary authority and the prudential regulator in order to assess the extent to which the conclusions from the previous section, where policymakers cooperate, are impacted.

Table 3 reports the unconditional variances and the losses under cooperative and non-cooperative equilibria. As before the cooperative environment assumes that the two policymakers share equation (9) as the loss function. For the non-cooperative policies we assume that the central bank minimizes the loss function (2) with $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, and that the prudential regulator policy minimizers equation (4) with $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$. A key feature of this assignment is that when added together the loss functions for the central bank and the prudential regulator has the same structure as the cooperative loss function. With distinct policy objectives, we consider timing environments in which both policymakers move simultaneously (Nash) and in which either the central bank or the prudential authority has a first-mover advantage.

Table 3: **Comparing cooperation and non-cooperation**

	Cooperation	Non-cooperation		
	Nash	Nash	Monetary leadership	Prudential leadership
	$\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$			
σ_{π}^2	0.081	0.100	0.100	0.117
$\sigma_{\Delta y}^2$	1.757	1.834	1.819	1.835
$\sigma_{\Delta r}^2$	0.287	0.349	0.3850	0.404
$\sigma_{cr/y}^2$	0.080	0.033	0.041	0.031
$\sigma_{\Delta\eta}^2$	0.178	0.076	0.115	0.080
L^{CB}	1.981	2.108	2.112	2.154
L^{MP}	0.168	0.071	0.098	0.071
L^{coop}	2.150*	2.179	2.210	2.224
L^c	37.990	37.882	37.920	37.884
L^p	85.271	37.844	85.213	85.209

Note: The table reports the variances and the unconditional losses under the alternative coordination schemes, where the prudential regulator minimizes loss function (4) with $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$ and the central bank minimizes loss function (2) with $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$.

Overall, the cooperative solution outperforms the non-cooperative solutions, driven primarily by greater stability of the monetary policy objectives. Considered from the individual policymakers' perspective, column two shows that the central bank clearly gains from cooperation. In contrast, macroprudential policy would fare better with less cooperation, particularly under the prudential leadership scheme, as non-cooperation allows the prudential regulator to better stabilize the credit-to-GDP ratio with smaller changes in the lending fraction. These conclusions remain valid when a lower weight on output growth is assigned in the central bank's loss function ($\lambda_{\Delta y}^{CB}=0.5$), and when the weight on both smoothing coefficients is increased ($\lambda_{\Delta r}^{CB}=\lambda_{\Delta\eta}^{MP}=1$).

While the policy objectives all relate to union-wide outcomes, we also report in Table 3 the corresponding losses for the core and the periphery. What we see is that although union-side policy objectives delivers stability at the union-wide level, this stability masks considerable volatility at the regional level. In particular, where the credit-to-GDP ratio is very stable at the union-wide level the underlying credit-to-GDP ratios in the core and the periphery are very volatile. This result suggests that, while some union-wide stability might be sacrificed, prudential policies directed at stability regional variables might drastically reduce regional-level volatility.

Figure 3 plots the impulse responses for a positive risk shock in the periphery for the same parameterization of the loss function weights (2) and (4) as in Table 3, i.e., $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr/y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$. The figure compares the best of the cooperative and non-cooperative environments (Nash, in both cases) and shows the consequences of non-cooperation among the monetary and the prudential authorities when the economy is hit by a positive risk shock in the periphery. The risk shock increases the lending-deposit spread in the periphery, which lowers credit and house prices in the periphery, leading to a recession in the periphery. Because the union-wide credit-to-GDP ratio decreases, the union-wide prudential regulator loosens, raising the lending fraction, in order to stimulate lending. The lower credit standards adopted at the union-wide level, however, leads to a boom in economic activity and to higher inflation in the core. Under cooperation, the ECB slightly increases the interest rate in order to contain union-wide inflation. Under non-cooperation, the ECB is less restrictive than under cooperation, and, as a result, union-wide inflation is slightly higher, while the regional-level variables are largely unaffected.

Figure 4 shows the impulse responses to a union-wide technology shock. Both the core and the periphery react in a similar way to the shock, with higher output and lower inflation. The ECB responds to the decrease in union-wide inflation by lowering the interest rate, while the prudential regulator reacts to a higher credit-to-GDP ratio by increasing the lending spreads in both regions. Under cooperation, however, macroprudential policy loosens, in line with monetary policy's accommodative move, which leads to lower lending spreads and higher credit-to-GDP ratios in both regions under cooperation than under non-cooperation. As a result, the union-wide credit-to-GDP ratio increases and takes longer to return to steady state.

Our analysis of non-cooperative policy thus far has assumed that the monetary policy objective and the macroprudential objectives are distinct, having no overlap. In Table 4, we now assume that union-wide output growth is assigned as a common goal to the two policymakers. Hence, the individual loss functions have one goal in common, and with equal weights, i.e., $\lambda_{\Delta y}^{CB} = \lambda_{\Delta y}^{MP} = 0.5$. These weights imply that the central bank cares relatively less than previously about output growth, but, at the same time, macroprudential policy is now charged with stabilizing output growth, such that the total weight assigned to growth across the two policymakers remains equal to 1.0. Of central interest resides in whether the shared objective closes the gap between cooperation and non-cooperation and even allows the non-cooperative policy environment to be preferred. The results in Table 4 reveal that assigning a common objective (with the current parameterization of the model) leaves the central bank best off under the cooperative scheme. However, this is not the case for macroprudential policy, who gains most from macroprudential leadership by trading off slightly higher output volatility with more stable macroprudential objectives. Indeed, the gain to the macroprudential policymaker is such that the combined loss is now smaller under non-cooperation than under cooperation, suggesting that there can be advantages to allowing the two policymakers to act independently of each other provided their incentives are guided by a common objective.

In Table 5, we alternatively assign the union-wide credit-to-GDP ratio as a common objective to the two policymakers, where $\lambda_{cr/y}^{CB} = \lambda_{cr/y}^{MP} = 0.5$. In this case the central bank has an additional objective, while the prudential regulator assigns relatively less importance (with the weight going from 1.0 to 0.5) to its main objective. Importantly, although now assigned as an objective to both policymakers, the overall importance of the credit-to-GDP ratio in the joint loss function remains unchanged, with its weight still equally 1.0. The preferred cooperation scheme for the individual policymakers is in line with the previous case where output growth was assigned as a common objective, i.e., macroprudential leadership yields the lowest loss for the regulator, while the central bank fares best under cooperation. However, the gains from

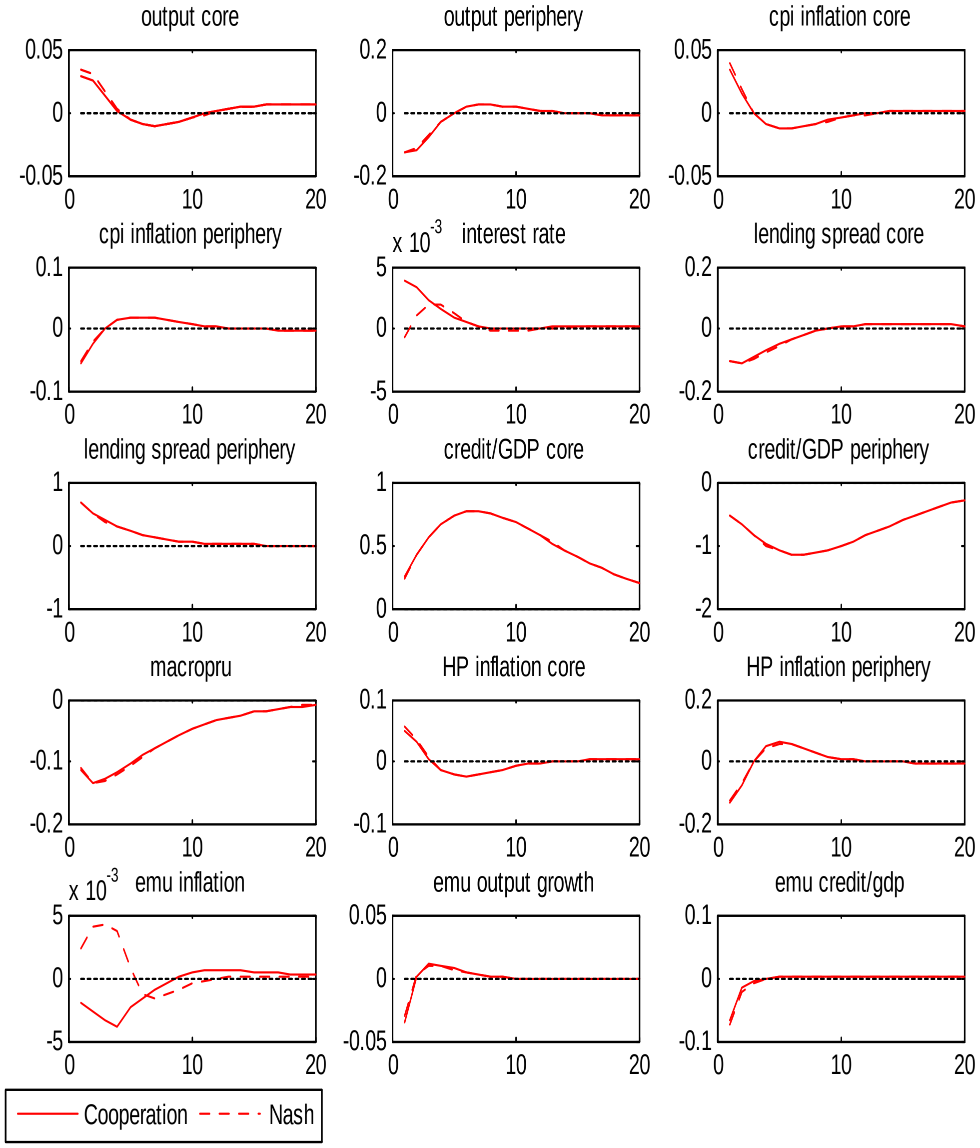

Figure 3. Impulse responses for a risk shock in the periphery: cooperation vs. non-cooperation

Note: The figure plots the impulse responses to a risk shock in the periphery under cooperation and non-cooperation for the following parameterizations of the loss function weights: $\lambda^{CB}_{\Delta y} = 1$ and $\lambda^{CB}_{\Delta r} = 0.5$ in equation (2) and $\lambda^{MP}_{cr/y} = 1$, $\lambda^{MP}_{\Delta\eta} = 0.5$ in equation (4)

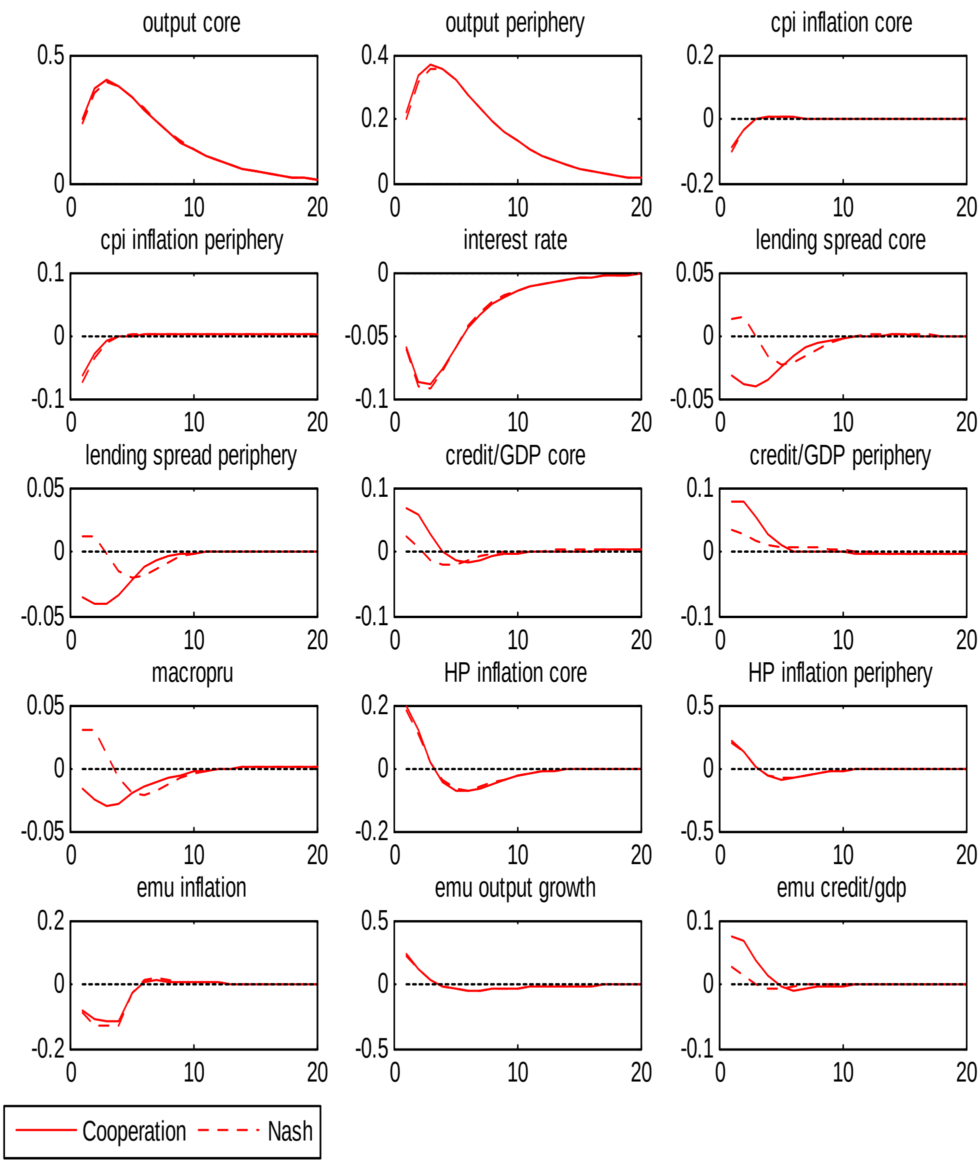

Figure 4. Impulse responses for an EMU-wide technology shock: cooperation vs. non-cooperation

Note: The figure plots the impulse responses to a positive union-wide technology shock under cooperation and non-cooperation for the following parameterizations of the loss function weights: $\lambda^{CB}_{\Delta y} = 1$ and $\lambda^{CB}_{\Delta r} = 0.5$ in equation (2) and ${}^{MP}_{cr/y} = 1$, $\lambda^{MP}_{\Delta \eta} = 0.5$ in equation (4)

Table 4: **Comparing cooperation and non-cooperation: output as common goal**

	Cooperation	Non-cooperation		
	Nash	Nash	Monetary leadership	Prudential leadership
	$\lambda^{CB}_{\Delta y} = 0.5$, $\lambda^{CB}_{\Delta r} = 0.5$, $\lambda^{MP}_{cr/y} = 1$, $\lambda^{MP}_{\Delta\eta} = 0.5$, $\lambda^{MP}_{\Delta y} = 0.5$			
σ^2_{π}	0.081	0.081	0.081	0.094
$\sigma^2_{\Delta y}$	1.757	1.846	1.841	1.866
$\sigma^2_{\Delta r}$	0.287	0.233	0.244	0.276
$\sigma^2_{cr/y}$	0.080	0.040	0.044	0.034
$\sigma^2_{\Delta\eta}$	0.1780	0.116	0.138	0.104
L^{CB}	1.103	1.121	1.123	1.165
L^{MP}	1.047	1.021	1.033	1.019
L^{coop}	2.150	2.142*	2.156	2.185
L^{c}	37.990	37.921	37.945	37.908
L^{p}	85.271	85.204	85.207	85.205

Note: The table reports the variances and the unconditional losses under the alternative coordination schemes, where union-wide output growth is assigned as an objective to both policymakers. Hence, the union-wide regulator minimizes loss function (4) augmented with union-wide output growth with a weight of $\lambda^{MP}_{\Delta y} = 0.5$. The central bank minimizes loss function (2) with $\lambda^{CB}_{\Delta y} = 0.5$.

non-cooperation received by the prudential regulator are now not sufficient to compensate for the central bank's higher loss, with cooperation remaining the preferred scheme.

Table 5: **Comparing cooperation and non-cooperation: EMU-wide credit-to-gdp as common goal**

	Cooperation	Non-cooperation		
	Nash	Nash	Monetary leadership	Prudential leadership
	$\lambda^{CB}_{\Delta y} = 1$, $\lambda^{CB}_{\Delta r} = 0.5$, $\lambda^{CB}_{cr/y} = 0.5$, $\lambda^{MP}_{cr/y} = 0.5$, $\lambda^{MP}_{\Delta\eta} = 0.5$			
σ^2_{π}	0.081	0.097	0.100	0.109
$\sigma^2_{\Delta y}$	1.757	1.862	1.843	1.861
$\sigma^2_{\Delta r}$	0.287	0.336	0.367	0.379
$\sigma^2_{cr/y}$	0.080	0.063	0.075	0.059
$\sigma^2_{\Delta\eta}$	0.1780	0.046	0.073	0.050
L^{CB}	2.021	2.159	2.164	2.188
L^{MP}	0.129	0.054	0.074	0.054
L^{coop}	2.150*	2.213	2.238	2.242
L^{c}	37.990	37.877	37.918	37.878
L^{p}	85.271	85.233	85.243	85.235

Note: The table reports the variances and the unconditional losses under the alternative cooperative schemes, where the union-wide credit-to-GDP ratio is assigned as a common objective to both policymakers. Hence, the central bank minimizes loss function (2) augmented with the union-wide credit-to-GDP ratio with a weight of $\lambda^{CB}_{c/y} = 0.5$. The union-wide prudential regulator minimizes loss function (4) with $\lambda^{MP}_{cr/y} = 0.5$.

4.4 A robustness exercise

In this section we briefly consider an alternative loss function for the prudential regulator, one that depends on the credit spread rather than on the credit-to-GDP ratio. Table 6 shows the simulation results for the case where the union-wide credit-to-GDP ratio is replaced by the union-wide average spread between the lending and deposit rate in the macroprudential policymaker's loss function.

Table 6: **Comparing cooperation and non-cooperation: alternative measures of financial stability**

	Cooperation	Non-cooperation		
	Nash	Nash	Monetary leadership	Prudential leadership
	$\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{spread}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$			
σ_{π}^2	2.028	2.200	2.199	2.197
$\sigma_{\Delta y}^2$	2.281	2.388	2.389	2.388
$\sigma_{\Delta r}^2$	0.915	1.301	1.302	1.324
σ_{spread}^2	0.043	0.012	0.012	0.012
$\sigma_{\Delta\eta}^2$	0.067	0.051	0.053	0.051
L^{CB}	4.766	5.238	5.239	5.247
L^{MP}	0.080	0.037	0.038	0.037
L^{coop}	4.843*	5.275	5.277	5.285

Note: The table reports the variances and the unconditional losses under the alternative cooperative schemes, where the central bank minimizes loss function (2) and the EMU regulator minimizes loss function (4), and where the spread instead of the credit-to-GDP ratio is used as financial objective in the latter's loss function.

Looking at Table 6, the results and conclusions are qualitatively in line with those reported in Table 3. Specifically, the cooperative scheme delivers outcomes that are preferred to the three non-cooperative schemes, the central bank benefits from cooperative while the prudential regulator prefers non-cooperation, and the environments with leadership generate outcomes that are very similar to the simultaneous move (Nash) case.

4.5 Optimal assignment of objectives and weights under non-cooperation

In this section we assume that society's loss function is represented by the benchmark cooperative loss function adopted in the previous simulations:

$$L_t^{society}=Var\left(\pi_t\right)+\lambda_{\Delta y}^{CB}Var\left(\Delta y_t\right)+\lambda_{\Delta r}^{CB}Var\left(\Delta r_t\right)+\lambda_{cr/y}^{MP}Var\left(cr_t/y_t\right)+\lambda_{\Delta\eta}^{MP}Var\left(\Delta\eta_t\right) \quad (11)$$

where $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr/y}^{MP}=1$ and $\lambda_{\Delta\eta}^{MP}=0.5$. Society minimizes the above loss function with respect to the policy instruments r_t and η_t. The outcome of this optimization exercise will serve as a benchmark and used to help assign the individual loss functions under the non-cooperative solution. Following Debortoli et al. (2015) and Gelain and Ilbas (2016), the objectives and the weights in the individual loss functions under non-cooperation are chosen to match or improve upon the benchmark cooperative policy. We find that the following assignment of loss functions do a reasonably good job of approximating the outcome achieved under the benchmark cooperative optimization:

$$L_t^{CB}=Var\left(\pi_t\right)+\lambda_{\Delta y}^{CB}Var\left(\Delta y_t\right)+\lambda_{\Delta r}^{CB}Var\left(\Delta r_t\right)$$

for monetary policy, where $\lambda_{\pi}^{CB}=1.5$, $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$ and

$$L_t^{MP}=\lambda_{cr/y}^{MP}Var\left(cr_t/y_t\right)+\lambda_{\Delta y}^{MP}Var\left(\Delta y_t\right)+\lambda_{\Delta\eta}^{MP}Var\left(\Delta\eta_t\right)$$

for macroprudential policy, where $\lambda_{cr/y}^{MP}=1.5$, $\lambda_{\Delta y}^{MP}=1$, $\lambda_{\Delta\eta}^{MP}=0.5$. The non-cooperative solution with these individual loss functions yields a total loss of 2.127, when evaluated at the benchmark loss, equation (11), compared to 2.150 obtained under the cooperative solution. Concerning the loss function assigned to the central bank, we find that a higher weight on inflation than society is required in order to improve on the cooperative outcome. Greater weight on inflation stabilization ensures better control over inflation stabilization under non-cooperative, with some of the (relative) importance of stabilizing output growth being passed to the prudential regulator. As a result, both policymakers have a common objective, i.e., output growth, that helps to bring the non-cooperative outcome closer to the cooperative outcome (see also Gelain and Ilbas, 2016). Introducing the output growth component in the prudential regulator's loss function leads to a higher weight than society on the credit-to-GDP ratio in order to ensure its stabilization under non-cooperation.

5 Interactions between monetary policy and regional macroprudential authorities

We now assume that macroprudential policy is conducted at the regional level, i.e., that there is a prudential regulator in the core and another in the periphery and that each one of them is charged with safeguarding financial stability in its own region. Their focus on their own region sees these prudential authorities ignoring outcomes in the other region and at the EMU-wide level. As before, the central bank is in charge of conducting monetary policy at EMU-wide level. Because there are three players, a much wider range of strategic interaction schemes can be considered than in the two-player case. However, we focus on what we believe are the three most interesting cases: the cooperative solution and the two non-cooperative solutions where all policymakers move simultaneously (Nash) and where the ECB has a first-mover advantage with respect to the two prudential regulators (monetary leadership).

Table 7 reports the results for the case where the ECB minimizes its usual loss function, equation (2), and the regulators in the core and the periphery minimize their region-specific loss functions, equations (7) and (8), respectively. The loss functions for the prudential authorities include the regional credit-to-GDP ratio and smoothing of the regional lending fraction. In the cooperative solution the shared loss function is given by the sum of the loss functions for the three policymakers. The results indicate that, although the central bank prefers the cooperative environment, both regional prudential authorities prefer outcomes under the simultaneous move non-cooperative solution. Because the two prudential authorities desire to focus on their own regions, they can be more effective at stabilizing these variables in the non-cooperative solution than if they cooperate with the ECB, or follow its actions in the leader-follower setup. Interestingly, in the absence of commitment, society's loss, $L^{CB+MP,c+MP,p}$, is lower in the simultaneous move non-cooperative solution than it is in the cooperative solution, suggesting that some level of regional-based macroprudential policymaking can be desirable.

Figure 5 plots the responses to a risk shock in the periphery under the policy schemes considered in Table 7. Unlike the case where there was a common prudential regulator at the EU level—where this risk shock causes a boom in the core—, when we allow for separate regulators to respond to regional variables this is no longer the case. The reason is that

Table 7: **CComparing coperation and non-cooperation: three policymakers**

	Cooperation	Non-cooperation	
	Nash	Nash	Monetary leadership
	$\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, $\lambda_{cr^c/y^c}^{MP,c}=1$, $\lambda_{\Delta\eta^c}^{MP,c}=0.5$, $\lambda_{cr^p/y^p}^{MP,p}=1$, $\lambda_{\Delta\eta^p}^{MP,p}=0.5$		
σ_{π}^2	0.082	0.100	0.101
$\sigma_{\Delta y}^2$	1.817	1.830	1.815
$\sigma_{\Delta r}^2$	0.328	0.353	0.391
σ_{cr^c/y^c}^2	0.081	0.048	0.055
$\sigma_{\Delta\eta^c}^2$	0.142	0.118	0.161
σ_{c^p/y^p}^2	0.064	0.061	0.068
$\sigma_{\Delta\eta^p}^2$	0.199	0.180	0.221
L^{CB}	2.062	2.107	2.111
$L^{MP,c}$	0.153	0.107	0.136
$L^{MP,p}$	0.163	0.152	0.179
$L^{CB+MP,c+MP,p}$	2.378	2.366*	2.426

Note: The table reports variances and the unconditional losses under the alternative non-cooperative schemes, where the central bank the minimizes loss function (2) with $\lambda_{\Delta y}^{CB}=1$, $\lambda_{\Delta r}^{CB}=0.5$, the core's prudential regulator minimizes equation (7) with $\lambda_{cr^c/y^c}^{MP,c}=1$, $\lambda_{\Delta\eta^c}^{MP,c}=0.5$, and the periphery's regulator minimizes equation (8) with $\lambda_{cr^p/y^p}^{MP,p}=1$, $\lambda_{\Delta\eta^p}^{MP,p}=0.5$.

macroprudential policy in the periphery loosens in response to the recession created by the shock, while the regulator in the core counteracts the immediate spillover effect of the shock that hits the periphery by increasing the lending-deposit spread, which has a restraining effect on the credit-to-GDP ratio and output in the core. As a result, inflation slightly decreases in the core as well, which also leads to lower EMU-wide inflation, to which monetary policy reacts by lowering the nominal interest rate. The net result of the opposing regional prudential policies is that the shock has a more synchronized effect across the core and the periphery.

The scope for conducting independent macroprudential policy leads to diverging policy responses when the risk shock hits only the periphery. The picture however changes somewhat when we consider a common, area-wide technology shock. Because both economies are affected similarly by the shock, the regional prudential authorities respond in a similar way. Therefore, the effects of the shock and the policy responses resemble closely those obtained when macroprudential policy is operated at the common EMU-level. The added value of allowing for independent prudential policies hence mainly arises from the possibility of responding more efficiently to the spillover effects arising from region-specific shocks.

In Table 8, the regional prudential authorities are assumed to receive, in addition to their existing objectives described in equations (7) and (8), the goal of stabilizing output growth in their respective region. Moreover, the ECB is assumed to place weight on stabilizing these region-specific output growth variables, instead of EMU-wide output growth. Introducing this common objective between the ECB and each regional authorities does not alter the earlier finding that society prefers the non-cooperative Nash solution to the cooperative solution.

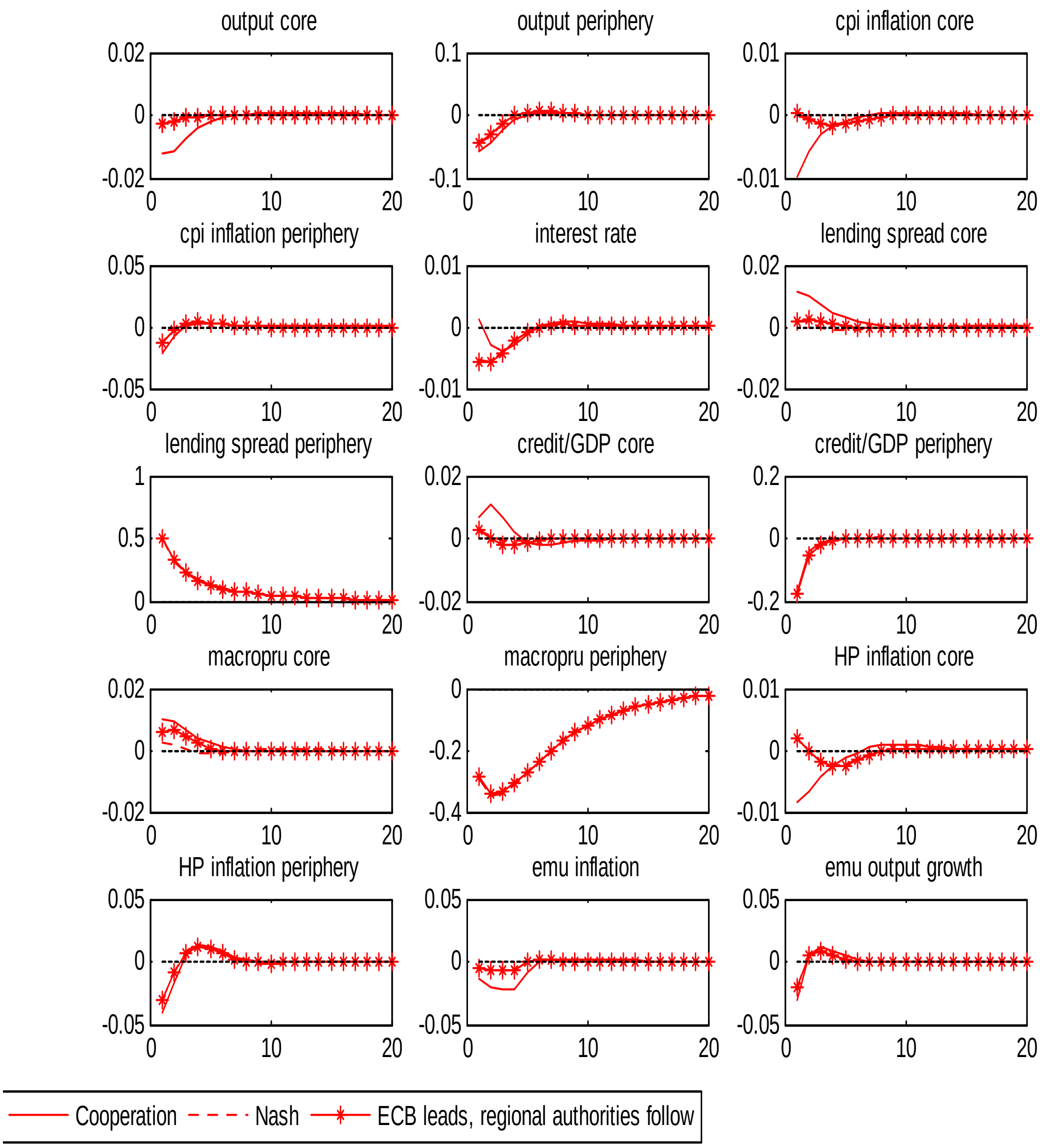

Figure 5. Impulse responses for a risk shock in the periphery

Note: The figure plots the impulse responses to a risk shock in the periphery under alternative interaction schemes (cooperation, non-cooperation, and monetary leadership) for the following parameterizations of loss function weights: $\lambda_{\Delta y}^{CB} = 1$, $\lambda_{\Delta r}^{CB} = 0.5$ in equation (2), $\lambda_{cr/y}^{MP,c} = 1$, $\lambda_{\Delta \eta}^{MP,c} = 0.5$ in equation (7), and $\lambda_{cr/y}^{MP,p} = 1$, $\lambda_{\Delta \eta}^{MP,p} = 0.5$ in equation (8).

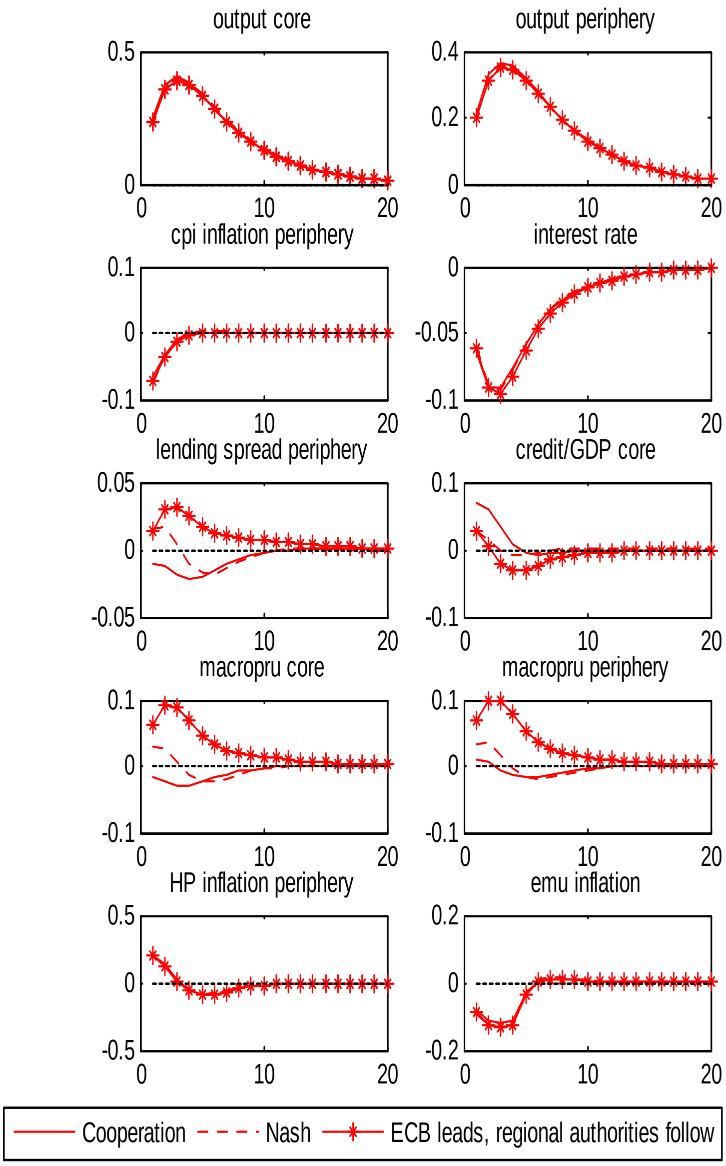

Figure 6. Impulse responses for an area-wide technology shock

Note: The figure plots the impulse responses to a common area-wide technology shock under alternative interaction schemes (cooperation, non-cooperation, and monetary leadership) for the following parameterizations of loss function weights: $\lambda_{\Delta y}^{CB} = 1$, $\lambda_{\Delta r}^{CB} = 0.5$ in equation (2), $\lambda_{cr/y}^{MP,c} = 1$, $\lambda_{\Delta\eta}^{MP,c} = 0.5$ in equation (7), and $\lambda_{cr/y}^{MP,p} = 1$, $\lambda_{\Delta\eta}^{MP,p} = 0.5$ in equation (8).

Table 8: **Comparing cooperation and non-cooperation: three policymakers with additional objectives**

	Cooperation	Non-cooperation	
	Nash	Nash	Monetary leadership
	$\lambda^{CB}_{\Delta y^c} = 0.3$, $\lambda^{CB}_{\Delta y^p} = 0.2$, $\lambda^{CB}_{\Delta r} = 0.5$, $\lambda^{MP,c}_{cr^c/y^c} = 1$, $\lambda^{MP,c}_{\Delta\eta^c} = 0.5$, $\lambda^{MP,c}_{\Delta y^c} = 0.3$, $\lambda^{MP,p}_{cr^p/y^p} = 1$, $\lambda^{MP,p}_{\Delta y^p} = 0.2$,		
σ^2_π	0.081	0.079	0.078
$\sigma^2_{\Delta y^c}$	2.035	2.109	2.102
$\sigma^2_{\Delta y^p}$	1.915	1.979	1.972
$\sigma^2_{\Delta r}$	0.326	0.239	0.252
$\sigma^2_{cr^c/y^c}$	0.087	0.054	0.058
$\sigma^2_{\Delta\eta^c}$	0.138	0.124	0.157
$\sigma^2_{cr^p/y^p}$	0.067	0.061	0.066
$\sigma^2_{\Delta\eta^p}$	0.194	0.183	0.214
L^{CB}	1.238	1.227	1.229
$L^{MP,c}$	0.766	0.748	0.767
$L^{MP,p}$	0.547	0.548	0.567
$L^{CB+MP,c+MP,p}$	2.551	2.523*	2.563

Note: The table reports the variances and the unconditional losses under the alternative non-cooperative schemes, where the central bank the minimizes loss function (2) with $\lambda^{CB}_{\Delta y^c} = 0.3$, $\lambda^{CB}_{\Delta y^p} = 0.2, \lambda^{CB}_{\Delta r} = 0.5$, the prudential regulator in the core minimizes equation (7) with $\lambda^{MP,c}_{cr^c/y^c} = 1$, $\lambda^{MP,c}_{\Delta\eta} = 0.5$, $\lambda^{MP,c}_{\Delta y^c} = 0.3$, and the periphery's regulator minimizes equation (8) with $\lambda^{MP,p}_{cr^*/y^*} = 1$, $\lambda^{MP,p}_{\Delta y^p} = 0.2, \lambda^{MP,p}_{\Delta\eta} = 0.5$.

5.1 Common objective for the central bank and the prudential policies across the core and periphery

Table 9 reports the simulation results when EMU-wide output growth is assigned as a common objective in the loss functions of the three policymakers. It becomes clear from the table that introducing a common objective does not changes significantly the story that emerged earlier based on separated objectives (Table 7). In order to better stabilize the objectives of the three policymakers, separating the loss functions and allowing each policymaker to act independently from the others therefore seems to be a better strategy because it compensates to a certain degree for the lack of independent monetary policy in the regions.

Table 9: **Losses under alternative coordination schemes: cooperation vs. non-cooperation: three policymakers with a common EMU-wide objective**

	Cooperation	Non-cooperation	
	Nash	Nash	Monetary leadership
	$\lambda_{\Delta y}^{CB} = 0.5, \lambda_{\Delta r}^{CB} = 0.5, \lambda_{cr^c/y^c}^{MP,c} = 1,\ \lambda_{\Delta \eta^c}^{MP,c} = 0.5,\ \lambda_{\Delta y}^{MP,c} = 0.3, \lambda_{cr^*/y^*}^{MP,p} = 1,\ \lambda_{\Delta y}^{MP,p} = 0.2, \lambda_{\Delta \eta^p}^{MP,p} = 0.5$		
σ_π^2	0.082	0.078	0.078
$\sigma_{\Delta y}^2$	1.817	1.900	1.892
$\sigma_{\Delta r}^2$	0.328	0.240	0.254
σ_{cr^c/y^c}^2	0.081	0.049	0.053
$\sigma_{\Delta \eta^c}^2$	0.142	0.124	0.158
σ_{cr^p/y^p}^2	0.064	0.060	0.065
$\sigma_{\Delta \eta^p}^2$	0.199	0.180	0.214
L^{CB}	1.154	1.148	1.150
$L^{MP,c}$	0.698	0.680	0.700
$L^{MP,p}$	0.527	0.529	0.550
$L^{CB+MP,c+MP,p}$	2.378	2.358*	2.400

Note: The table reports the variances and the unconditional losses under the alternative non-cooperative schemes with EMU-wide output growth as a common objective assigned to the regional prudential policy makers. Therefore, the core minimizes loss function (7), augmented with EMU-wide output growth, $\lambda_{\Delta y}^{MP,c} = 0.5$, and the periphery minimizes loss function (8), augmented with EMU-wide output growth $\lambda_{\Delta y}^{MP,p} = 0.5$.

6 Conclusion

We use an estimated DSGE model of the EURO area to study the interaction between monetary policy and macroprudential policy in a monetary union. The model is one in which monetary policy and macroprudential policy interact through the behavior of savers and borrowers and through the balance sheets of financial intermediaries, with the economic effects of tighter monetary policy potentially offset through looser macroprudential policy (a higher lending fraction). A useful feature of the model is that it contains two regions—the core and the periphery—which allows the effects of union-wide and region-specific policies and shocks on both the core and the periphery to be explored. We assume that policymakers behave purposefully, but do not have access to a commitment technology, so that monetary policy and macroprudential policy are each formulated to be optimal under discretion. With policymakers optimizing, we consider a range of cooperative and non-cooperative decision problems, examine the effects of leadership, and compare the implications of having two region-focused rather than a single union-wide-focused prudential regulator.

Using the optimal commitment policy as one benchmark we find that the discretionary stabilization bias leads to inefficient responses to shocks, but in a way that is different from DSGE models that focus only on monetary policy. With time-inconsistency also affecting the prudential regulator, inflation volatility is actually lower under discretion than commitment, a result that arises as policymakers use policy promises to instead secure greater stability in the credit-to-GDP ratio, the nominal interest rate, and in the lending fraction. In both cooperative and non-cooperative environments, we found little benefit to policy leadership, regardless of whether it was the monetary authority or the prudential regulator that had the first mover advantage. Indeed, when both monetary policy and macroprudential policy are both conducted at the union-wide level, the preferred decisionmaking environment had policymakers cooperating and choosing their policies simultaneously. However, although union-wide policies were able to successfully stabilize union-wide variables, we found that this stability masked considerable underlying volatility at the regional level. In particular, the credit-to-GDP ratio remained highly volatile in both the core and the periphery while being very stable in the union as a whole. Focusing on non-cooperative policies but assigning real union-wide GDP growth as a common objective led to non-cooperation outperforming cooperation, but it did not remove the excessive regional-level volatility in the credit-to-GDP ratio.

Introducing regional-level prudential regulators we found that non-cooperation performed better that cooperation, provided policymakers choose their policies simultaneously. With the regional prudential regulators focusing on regional-level outcomes and the central bank focusing on union-wide outcomes, giving the monetary authority a first-mover advantage can be detrimental because it leads to a greater focus on union-wide variables. With macroprudential policy conducted at the regional level the excessive regional-level volatility in the credit-to-GDP ratio is eliminated, but at the expense of slightly higher volatility in union-wide inflation and output growth.

Acknowledgements

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7 Appendix I - Solution Procedure

With two regions, two productive sectors, and two types of households, along with regional and international financial intermediaries and a host of real and nominal rigidities, the model is large, containing over 100 equations. To solve for the optimal discretionary policies we draw on the solution methods developed by Dennis (2007) which can be applied to larger models because they do not require the model to be put in a state space form. De Paoli and Paustain (2013) extended the methods in Dennis (2007) to two-player settings, and the solution methods we employ in this paper are straightforward extensions of that work.

In what follows we describe the solution for both the simultaneous move and the leader-follower cases, both of which work with the equations summarizing the model expressed in the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2 E_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_4 E_t\mathbf{x}_{t+1} + \widetilde{\mathbf{A}}_4 E_t\widetilde{\mathbf{x}}_{t+1} + \mathbf{A}_5\mathbf{v}_t, \tag{12}$$

where $\mathbf{y}_t$ is a vector of endogenous variables, $\mathbf{x}_t$ is the vector of policy instruments for one policymaker, $\widetilde{\mathbf{x}}_t$ is the vector of policy instruments of the other policymaker, $\mathbf{v}_t \sim i.i.d.[\mathbf{0}, \mathbf{\Omega}]$ is a vector of stochastic disturbances, and the matrices $\mathbf{A}_0$—$\mathbf{A}_5$ contain the model's parameters.

For convenience we will label the policymakers 1 and 2, then the loss functions for the two policymakers are assumed to be given by

$$\begin{aligned} Loss_1 &= E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{x}_t'\mathbf{Q}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\mathbf{Q}_2\widetilde{\mathbf{x}}_t\right)\right], &(13)\\ Loss_2 &= E_0\left[\sum_{t=0}^{\infty}\widetilde{\beta}^t\left(\mathbf{y}_t'\widetilde{\mathbf{W}}\mathbf{y}_t + \mathbf{x}_t'\widetilde{\mathbf{Q}}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\widetilde{\mathbf{Q}}_2\widetilde{\mathbf{x}}_t\right)\right], &(14)\end{aligned}$$

where the $\mathbf{W}$ and $\mathbf{Q}$ matrices (symmetric and positive semi-definite) contain the policy preferences of the two policymakers. In the case where the two policymakers cooperate the loss function parameters must satisfy $\beta = \widetilde{\beta}$, $\mathbf{W} = \widetilde{\mathbf{W}}$, $\mathbf{Q}_1 = \widetilde{\mathbf{Q}}_1$, and $\mathbf{Q}_2 = \widetilde{\mathbf{Q}}_2$. The two discount factors, β and $\widetilde{\beta}$, lie between 0 and 1. Note that equations (24) and (25) differ from De Paoli and Paustain (2013) because they allow each policymaker's loss to depend on both policymaker's policy instruments, not just their own. Although the solution methods described below are closely related to those presented in De Paoli and Paustain (2013), we present them below, partly for completeness and partly because our use of different loss functions leads to different updating equations.

7.1 Simultaneous move

Employing results from Dennis (2007), the conjectured solution is

$$\begin{aligned} \mathbf{y}_t &= \mathbf{H}_1\mathbf{y}_{t-1} + \mathbf{H}_2\mathbf{v}_t, &(15)\\ \mathbf{x}_t &= \mathbf{F}_1\mathbf{y}_{t-1} + \mathbf{F}_2\mathbf{v}_t, &(16)\\ \widetilde{\mathbf{x}}_t &= \widetilde{\mathbf{F}}_1\mathbf{y}_{t-1} + \widetilde{\mathbf{F}}_2\mathbf{v}_t, &(17)\end{aligned}$$

allowing the constraints (equation 12) to we written as

$$\mathbf{D}\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t, \tag{18}$$

where

$$\mathbf{D} = \mathbf{A}_0 - \mathbf{A}_2\mathbf{H}_1 - \mathbf{A}_4\mathbf{F}_1 - \widetilde{\mathbf{A}}_4\widetilde{\mathbf{F}}_1. \tag{19}$$

After a few substitutions and exploiting the properties of convergent geometric series we obtain

$$\begin{aligned} Loss_1 &= \left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right)' \mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right) \\ &\quad + \mathbf{x}_t'\mathbf{Q}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\mathbf{Q}_2\widetilde{\mathbf{x}}_t + \frac{\beta}{1-\beta} tr\left[\left(\mathbf{F}_2'\mathbf{Q}_1\mathbf{F}_2 + \widetilde{\mathbf{F}}_2'\mathbf{Q}_2\widetilde{\mathbf{F}}_2 + \mathbf{H}_2\mathbf{P}\mathbf{H}_2\right)\mathbf{\Omega}\right], \quad (20) \\ Loss_2 &= \left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right)' \mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right) \\ &\quad + \mathbf{x}_t'\widetilde{\mathbf{Q}}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\widetilde{\mathbf{Q}}_2\widetilde{\mathbf{x}}_t + \frac{\widetilde{\beta}}{1-\widetilde{\beta}} tr\left[\left(\mathbf{F}_2'\widetilde{\mathbf{Q}}_1\mathbf{F}_2 + \widetilde{\mathbf{F}}_2'\widetilde{\mathbf{Q}}_2\widetilde{\mathbf{F}}_2 + \mathbf{H}_2\widetilde{\mathbf{P}}\mathbf{H}_2\right)\mathbf{\Omega}\right], \quad (21) \end{aligned}$$

where

$$\begin{aligned} \mathbf{P} &= \mathbf{W} + \beta\left(\mathbf{F}_1'\mathbf{Q}_1\mathbf{F}_1 + \widetilde{\mathbf{F}}_1'\mathbf{Q}_2\widetilde{\mathbf{F}}_1 + \mathbf{H}_1'\mathbf{P}\mathbf{H}_1\right), \quad (22) \\ \widetilde{\mathbf{P}} &= \widetilde{\mathbf{W}} + \beta\left(\mathbf{F}_1'\widetilde{\mathbf{Q}}_1\mathbf{F}_1 + \widetilde{\mathbf{F}}_1'\widetilde{\mathbf{Q}}_2\widetilde{\mathbf{F}}_1 + \mathbf{H}_1'\widetilde{\mathbf{P}}\mathbf{H}_1\right). \quad (23) \end{aligned}$$

Differentiating $Loss_1$ with respect to $\mathbf{x}_t$ and $Loss_2$ with respect to $\widetilde{\mathbf{x}}_t$ gives the first order conditions

$$\begin{aligned} \frac{\partial Loss_1}{\partial \mathbf{x}_t} &= \mathbf{A}_3'\mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right) + \mathbf{Q}_1\mathbf{x}_t = 0, \quad (24) \\ \frac{\partial Loss_2}{\partial \widetilde{\mathbf{x}}_t} &= \widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right) + \widetilde{\mathbf{Q}}_2\mathbf{x}_t = 0. \quad (25) \end{aligned}$$

The simultaneous move solution can now be obtained using the following iterative scheme

1. Initialize $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\widetilde{\mathbf{F}}_1$, and $\widetilde{\mathbf{F}}_2$.

2. Compute $\mathbf{D}$ using equation (18), $\mathbf{P}$ using equation (22) and $\widetilde{\mathbf{P}}$ using equation (23).

3. Update $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\widetilde{\mathbf{F}}_1$, and $\widetilde{\mathbf{F}}_2$ according to

$$\begin{aligned} \mathbf{F}_1 &= -\left(\mathbf{Q}_1 + \mathbf{A}_3'\mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\mathbf{A}_3\right)^{-1}\mathbf{A}_3'\mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}(\mathbf{A}_1 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_1), \\ \mathbf{F}_2 &= -\left(\mathbf{Q}_1 + \mathbf{A}_3'\mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\mathbf{A}_3\right)^{-1}\mathbf{A}_3'\mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_5 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_2\right), \\ \widetilde{\mathbf{F}}_1 &= -\left(\widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\widetilde{\mathbf{A}}_3\right)^{-1}\widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}(\mathbf{A}_1 + \mathbf{A}_3\mathbf{F}_1), \\ \widetilde{\mathbf{F}}_2 &= -\left(\widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\widetilde{\mathbf{A}}_3\right)^{-1}\widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_3\mathbf{F}_2\right), \\ \mathbf{H}_1 &= \mathbf{D}^{-1}\left(\mathbf{A}_1 + \mathbf{A}_3\mathbf{F}_1 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_1\right), \\ \mathbf{H}_2 &= \mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_3\mathbf{F}_2 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_2\right). \end{aligned}$$

4. Iterate over steps 2—4 until convergence.

7.2 Leader-follower

For the leader-follower case, to ease exposition let us designate policymaker 1 as the leader and policymaker 2 as the follower. Then the conjectured reaction function for the follower takes the form

$$\widetilde{\mathbf{x}}_t = \widetilde{\mathbf{F}}_1 \mathbf{y}_{t-1} + \widetilde{\mathbf{F}}_2 \mathbf{v}_t + \mathbf{L}\mathbf{x}_t, \tag{26}$$

with the remainder of the conjectured solution continuing to be given by equations (15) and (16). With this reaction function, policymaker 1 can take the behavior (reaction) of policymaker 2 into account when formulating policy.

The solution procedure now precedes as before. Substituting the conjectured solution into the constraints gives equation (18), but where now $\mathbf{D}$ is given by

$$\mathbf{D} = \mathbf{A}_0 - \mathbf{A}_2\mathbf{H}_1 - \mathbf{A}_4\mathbf{F}_1 - \widetilde{\mathbf{A}}_4\widetilde{\mathbf{F}}_1 - \widetilde{\mathbf{A}}_4\mathbf{L}\mathbf{F}_1. \tag{27}$$

Then the loss functions for the two policymakers are given by

$$Loss_1 = \mathbf{y}_t'\mathbf{P}\mathbf{y}_t + \mathbf{x}_t'\mathbf{Q}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\mathbf{Q}_2\widetilde{\mathbf{x}}_t + \frac{\beta}{1-\beta} tr\left[\left(\mathbf{F}_2'\mathbf{Q}_1\mathbf{F}_2 + \left(\widetilde{\mathbf{F}}_2 + \mathbf{L}\mathbf{F}_2\right)' \mathbf{Q}_2 \left(\widetilde{\mathbf{F}}_2 + \mathbf{L}\mathbf{F}_2\right) + \mathbf{H}_2\mathbf{P}\mathbf{H}_2\right)\boldsymbol{\Omega}\right], \tag{28}$$

$$Loss_2 = \mathbf{y}_t'\widetilde{\mathbf{P}}\mathbf{y}_t + \mathbf{x}_t'\widetilde{\mathbf{Q}}_1\mathbf{x}_t + \widetilde{\mathbf{x}}_t'\widetilde{\mathbf{Q}}_2\widetilde{\mathbf{x}}_t + \frac{\widetilde{\beta}}{1-\widetilde{\beta}} tr\left[\left(\mathbf{F}_2'\widetilde{\mathbf{Q}}_1\mathbf{F}_2 + \left(\widetilde{\mathbf{F}}_2 + \mathbf{L}\mathbf{F}_2\right)' \widetilde{\mathbf{Q}}_2 \left(\widetilde{\mathbf{F}}_2 + \mathbf{L}\mathbf{F}_2\right) + \mathbf{H}_2\widetilde{\mathbf{P}}\mathbf{H}_2\right)\boldsymbol{\Omega}\right], \tag{29}$$

where

$$\mathbf{P} = \mathbf{W} + \beta\left(\mathbf{F}_1'\mathbf{Q}_1\mathbf{F}_1 + \widetilde{\mathbf{F}}_1'\mathbf{Q}_2\widetilde{\mathbf{F}}_1 + \mathbf{H}_1'\mathbf{P}\mathbf{H}_1\right), \tag{30}$$

$$\widetilde{\mathbf{P}} = \widetilde{\mathbf{W}} + \beta\left(\mathbf{F}_1'\widetilde{\mathbf{Q}}_1\mathbf{F}_1 + \left(\widetilde{\mathbf{F}}_1 + \mathbf{L}\mathbf{F}_1\right)' \widetilde{\mathbf{Q}}_2 \left(\widetilde{\mathbf{F}}_1 + \mathbf{L}\mathbf{F}_1\right) + \mathbf{H}_1'\widetilde{\mathbf{P}}\mathbf{H}_1\right). \tag{31}$$

After substituting equations (18) and (26) into the two loss functions, differentiating them with respect to $\mathbf{x}_t$ and $\widetilde{\mathbf{x}}_t$, respectively, gives

$$\begin{aligned}\frac{\partial Loss_1}{\partial \mathbf{x}_t} &: \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)' \mathbf{D}'^{-1}\mathbf{P}\mathbf{D}^{-1}\left(\left(\mathbf{A}_1 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_1\right)\mathbf{y}_{t-1} + \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)\mathbf{x}_t + \left(\mathbf{A}_5 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_2\right)\mathbf{v}_t\right) \\ &: +\mathbf{Q}_1\mathbf{x}_t + \mathbf{L}'\mathbf{Q}_2\left(\widetilde{\mathbf{F}}_1\mathbf{y}_{t-1} + \widetilde{\mathbf{F}}_2\mathbf{v}_t + \mathbf{L}\mathbf{x}_t\right) = 0,\end{aligned} \tag{32}$$

$$\frac{\partial Loss_2}{\partial \widetilde{\mathbf{x}}_t} = \widetilde{\mathbf{A}}_3'\mathbf{D}'^{-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\left(\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_3\mathbf{x}_t + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{x}}_t + \mathbf{A}_5\mathbf{v}_t\right) + \widetilde{\mathbf{Q}}_2\mathbf{x}_t = 0. \tag{33}$$

The leader-follower solution can now be obtained using the following iterative scheme

1. Initialize $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\widetilde{\mathbf{F}}_1$, $\widetilde{\mathbf{F}}_2$ and $\mathbf{L}$.

2. Compute $\mathbf{D}$ using equation (27), $\mathbf{P}$ using equation (30) and $\widetilde{\mathbf{P}}$ using equation (31).

3. Update $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$, $\widetilde{\mathbf{F}}_1$, $\widetilde{\mathbf{F}}_2$ and $\mathbf{L}$ according to

$$\begin{aligned}
\mathbf{F}_1 &= -\left(\mathbf{Q}_1 + \mathbf{L}^{'}\mathbf{Q}_2\mathbf{L} + \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)^{'}\mathbf{D}^{'-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)\right)^{-1} \\
&\quad \times \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)^{'}\mathbf{D}^{'-1}\mathbf{P}\mathbf{D}^{-1}(\mathbf{A}_1 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_1 + \mathbf{L}^{'}\mathbf{Q}_2\widetilde{\mathbf{F}}_1), \\
\mathbf{F}_2 &= -\left(\mathbf{Q}_1 + \mathbf{L}^{'}\mathbf{Q}_2\mathbf{L} + \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)^{'}\mathbf{D}^{'-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)\right)^{-1} \\
&\quad \times \left(\mathbf{A}_3 + \widetilde{\mathbf{A}}_3\mathbf{L}\right)^{'}\mathbf{D}^{'-1}\mathbf{P}\mathbf{D}^{-1}\left(\mathbf{A}_5 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_2 + \mathbf{L}^{'}\mathbf{Q}_2\widetilde{\mathbf{F}}_2\right), \\
\widetilde{\mathbf{F}}_1 &= -\left(\widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\widetilde{\mathbf{A}}_3\right)^{-1}\widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}(\mathbf{A}_1 + \mathbf{A}_3\mathbf{F}_1), \\
\widetilde{\mathbf{F}}_2 &= -\left(\widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\widetilde{\mathbf{A}}_3\right)^{-1}\widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_3\mathbf{F}_2\right), \\
\mathbf{H}_1 &= \mathbf{D}^{-1}\left(\mathbf{A}_1 + \mathbf{A}_3\mathbf{F}_1 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_1 + \widetilde{\mathbf{A}}_3\mathbf{L}\mathbf{F}_1\right), \\
\mathbf{H}_2 &= \mathbf{D}^{-1}\left(\mathbf{A}_5 + \mathbf{A}_3\mathbf{F}_2 + \widetilde{\mathbf{A}}_3\widetilde{\mathbf{F}}_2 + \widetilde{\mathbf{A}}_3\mathbf{L}\mathbf{F}_2\right), \\
\mathbf{L} &= -(\widetilde{\mathbf{Q}}_2 + \widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\widetilde{\mathbf{A}}_3)^{-1}\widetilde{\mathbf{A}}_3^{'}\mathbf{D}^{'-1}\widetilde{\mathbf{P}}\mathbf{D}^{-1}\mathbf{A}_3.
\end{aligned}$$

4. Iterate over steps 2—4 until convergence.

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