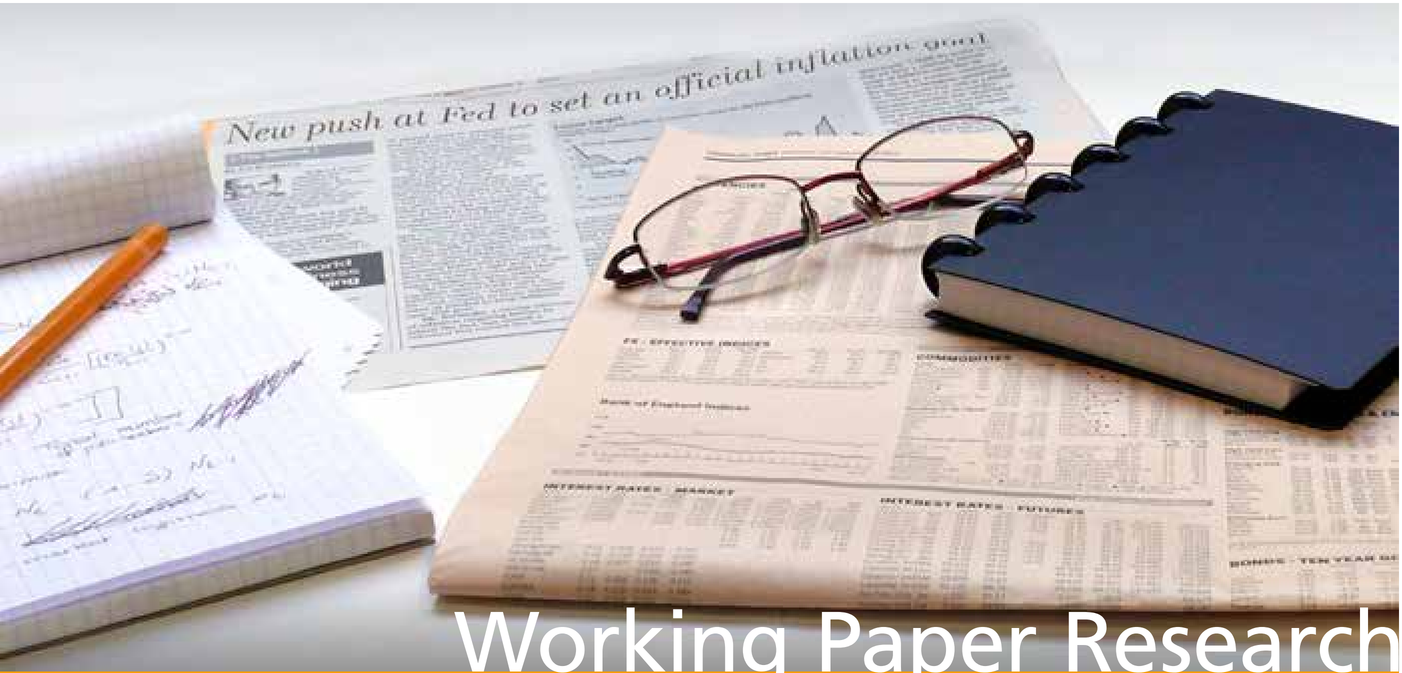
Portfolio choice and investor preferences: A semi-parametric approach based on risk horizon



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by Georges Hübner and Thomas Lejeune

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Abstract

The paper proposes an innovative framework for characterizing investors' behavior in portfolio selection. The approach is based on the realistic perspective of unknown investors' utility and incomplete information on returns distribution. Using a four-moment generalization of the Chebyshev inequality, an intuitive risk measure, risk horizon, is introduced with reference to the speed of convergence of a portfolio's mean return to its expectation. Empirical implementation provides evidence on the consistency of the approach with standard portfolio criteria such as, among others, the Sharpe ratio, a shortfall probability decay-rate optimization and a general class of flexible three-parameter utility functions.

JEL Classification: G11, G12, C14.

Keywords: Portfolio choice, risk-return trade-off, horizon.

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1 Introduction

The characterization of investors' behavior in portfolio decision problems is a central research question in finance. Markowitz's (1952) seminal work is one of the early attempts to address this issue, through the mean-variance paradigm. If investors are characterized by quadratic preferences and/or if asset returns display spherically generated distributions (Chamberlain, 1983), portfolios can be ranked according to the Sharpe ratio (Sharpe, 1994), computed ex-post on historical data. However, the first condition leads to a decreasing marginal utility, an implausible characterization of the preferences of most investors. The second one contradicts substantial empirical evidence on the unconditional distribution of asset returns (see, for instance, Badrinath and Chatterjee, 1988; Corrado and Sun, 1997; Fong, 1997; Harvey and Siddique, 2000; Aparicio and Estrada, 2001). Even when one only fits the unconditional distribution of stock returns, departures from normality due to fat tails (high kurtosis) and to left or right asymmetry (skewness) are pervasive, precluding the straight application of the mean-variance framework.

Another approach to portfolio decision problems relies on expected utility maximization. Restrictions on higher-order derivatives of utility functions allow for the consideration of asymmetric risks (Kraus and Litzenberger, 1976) or fat-tail risks (Fang and Lai, 1997). However, no utility-based framework has managed to circumvent the fundamental argument put forward by Brockett and Kahane (1992): for any well-behaved utility function, i.e. one whose derivatives alternate in signs, it is possible to find returns distributions such that expected utility maximization leads to a solution with a low expectation, a high variance and a low skewness. A comprehensive theoretical framework with limited assumptions over returns distributions should therefore not posit moment preference, although parametric models of the expected utility approach necessarily do so. For this reason, the fundamental question: "which utility function of the rational investor, universal enough to encompass each profile, but sufficiently well-behaved to get a tractable treatment, should be used in practice?" remains unanswered.

Pure parametric approaches to utility maximization are doomed to expose themselves to a similar criticism. Furthermore, imposing a parametric utility function results in a "one-size-fits-it-all" characterization of investors' preferences, which is incompatible with the mental accounting behavior of individuals. In this paper, we move away from this traditional utility theory stream, and develop an alternative framework that diverges in the following aspects: (i) it starts from an inductive and positive treatment of investors' portfolio choices rather than a deductive and normative set of predictions derived from a pre-specified utility function; (ii) it acknowledges that individual investors are likely to agree on a loose information set, like some characteristics of the distribution of financial returns, rather than imposing a homogenous and sophisticated probabilistic knowledge across all potential states of the world; and – as the proof of the pudding is in the eating – (iii) it ensures that portfolio choices obtained with reasonably parameterized utility functions are attainable with a reasonable calibration in our framework as well, even though the opposite may not be true. We believe that the first two differentiators represent potential improvements over utility theory. Our purpose is not to *impose* them as criteria of dominance over traditional utility functions; rather, we *propose* an alternative, hopefully intuitive way of framing the investor's behavior. The last aspect, which is output-oriented, imposes itself as an effectiveness criterion. Finally, we will show that the implementation of our framework does not impose the estimation of a large set of parameters: the individual investor profile is characterized with three distinct parameters. This number is equal to the number of parameters that is necessary to implement the general class of flexible utility functions (encompassing all HARA functions), but with a larger set of resulting optimal portfolio allocations. In that sense, by improving the set of outputs with a comparable set of inputs, we also show that our framework achieves a superior efficiency over the flexible utility class.

Our approach is based on the following intuitive observation: in practice, investors do not know their utility function and have to rely on incomplete but readily usable returns characteristics, such as some unconditional moments of the distribution. Their associated measure of risk should reflect this incomplete information set. We propose a semi-parametric framework, compatible with this observation, where neither utility function nor full distribution structures are specified. This is achieved through the introduction of an innovative notion of risk: investor's perception of the riskiness of an asset or a portfolio is tied to its "risk horizon". This notion corresponds to the amount of time necessary to obtain a sufficient level of convergence of the mean asset return around its expectation. In more intuitive terms, risk horizon is the assessment of how long investors need to hold the asset in order to ensure that the probability of deviating from expected returns becomes negligible. We associate the notion of negligibility with a target level of value-at-risk (VaR) at a pre-determined confidence level adjusted downwards with a determined proportion of the value-at-potential (VaP, i.e. the opposite of the value-at-risk). Risk horizon can be viewed as a fixed date, leading to portfolio reallocations over time, or a fixed time interval from the date of the portfolio composition decision, leading to fixed allocation portfolios. Because it is not meant to encompass the full investor's risk profile, the concept can be used for multiple simultaneous portfolio choices by a single person (mental accounting), thus making it usable in the context of goal-based investing.

The notion of risk horizon is intuitive and presumably accessible to any investor. It means that the longer the required period before average returns are secured, the higher the risk horizon and the riskier the asset. To illustrate this intuition, let us temporarily restore some assumptions on the form of returns distribution and consider a simple example of a portfolio whose continuous excess returns are i.i.d. and normally distributed with an expectation of 0.5% and a monthly standard deviation of 2%. It takes 62 months to reach a confidence interval containing 95% of the probability mass with strictly positive bounds (namely [0.002%, 0.998%]), i.e. to obtain a VaR of 0.498% at the 97.5% confidence level. If the representative investor is of the "safety first" kind (i.e. not concerned about value-at-potential), his or her assessment of this portfolio's risk horizon will thus be 5.17 years. If the investor now prices the 97.5% value-at-potential (VaP) at half of the value-at-risk, he or she obtains an adjusted confidence level of $97.5\% - 0.5 \times \mathbb{P}\left[X \geq \text{VaP}_{97.5\%}\right]$ equal to 96.25%, and a VaR level equivalent to 0.498% after only 51 months. For the same investor, another portfolio with the same expectation but a volatility of 2.5% per month is riskier because one needs to wait for 97 months to obtain the same VaR, or a little longer than 80 months under the confidence level adjusted for VaP. Naturally, the tolerance criterion may be stricter. For instance, if the safety-first investor wishes to ensure that the VaR at a 97.5% confidence level is equal to 0.25%, it takes much longer (246 months for the first portfolio, 317 months for the second) but the order remains unchanged. An asset that takes more time to converge to its expectation, after adjusting for its upside potential, is perceived as riskier by an investor who adopts this criterion.

The previous example does not rely on any specification of the investor's utility function but requires the knowledge of the unconditional distribution of returns, namely the Gaussian distribution in that case. In reality, investors usually realize that asset returns are not normally distributed, but they do not reach a wide consensus on which distribution should apply instead. An agreement is easier to reach on point estimates, such as the centered sample moments of the returns distributions. The risk horizon measure we propose reflects this common denominator. Its construction only requires the knowledge of moments of order 2, 3 and 4 of the unconditional distribution. A generalized four-moment Chebyshev inequality borrowed from Mallows (1956) is used as a tool to characterize the risk dependence on the first four moments. This semi-parametric approach has the advantage of making use of the information about the skewness and kurtosis in a tractable way, without the need to impose any specific functional form on the returns distribution.

The use of the risk horizon in portfolio decision problems also obviates the need to specify any functional form for investors' utility. Though this aspect of our risk measure is appealing from a practical point of view, denying the availability of any information about the parametric specification of investors' preferences could ultimately cast doubt on the validity of this setup. In particular, if this framework appears to be inconsistent with any reasonable general characterization of utility functions, the adoption of the risk horizon as a market-wide measure of risk would not be justifiable. In the empirical part of the article, we propose an implementation of the risk horizon criterion for portfolio selection using Fama and French's 10 domestic industry indices. This implementation provides an explicit connection to prominent portfolio approaches. By varying the parameters that define investors' attitude towards risk, we show that asset allocations implied from the risk horizon framework are able to accurately replicate the allocation from standard portfolio criteria, i.e. the Sharpe ratio, Kelly's (1956) growth-optimal strategy, an objective function related to Bell's (1995) utility and Stutzer's (2003) decay rate optimization which is a generalization of the expected power utility criterion. Moreover, it is shown that the risk horizon approach is able to replicate allocations derived from a flexible and general utility framework (i.e. the flexible three-parameter model, or FTP, described in Conniffe, 2007; Meyer, 2010), while the reverse is not always true. The framework therefore appears as a generalization of several standard approaches to portfolio decision problems.

The paper refers extensively to the literature on shortfall-based portfolio selection. The risk measure we define requires the respect of a VaR type of constraint. In this regard, it shows some analogies with the "safety-first" literature, formalized by Roy (1952) and further analyzed by Arzac and Bawa (1977) and more recently by Moore et al. (2013), and with the family of lower-partial-moments (LPM) asset pricing models (Bawa and Lindenberg, 1977; Harlow and Rao, 1989). In these approaches, investors associate risk with the downside potential of securities: every realization that is not within the disaster area of the distribution is neglected in the measurement of risk. This approach might characterize some, but presumably not all investors. Our framework provides a more general perspective as it encompasses downside risk but also upside potential (VaP). The risk definition explicitly treats the possible trade-off between the negative impact of the left tail and the desirable impact of the right tail of portfolio returns. This adds flexibility to the characterization of investors, and enables us to obtain wide heterogeneity in risk profiles. This flexibility is shown to be of high relevance in the replication of traditional allocations. Typically, risk horizon investors with a negligible consideration for VaP and a tight confidence level tend to select portfolios that correspond closely to mean-variance optimal allocations. In contrast, risk horizon investors who behave similarly to those who maximize Stutzer's (2003) decay rate are characterized by their significant level of consideration for VaP.

The framework is also consistent with the observation outlined in Fishburn (1977), Browne (1999), Stutzer (2003) and Puhalskii (2011) among others, that investors frequently associate risk with the failure to achieve a certain target return. In particular, the convergence property embodied in the risk horizon criterion is close to the decay rate formulation proposed by Stutzer (2003). In Stutzer's approach, the optimal portfolio is the one with the fastest decay rate for the probability of not realizing a target return. We adopt an alternative view: investors choose their portfolio for a given risk horizon so as to maximize its expected return. The formulation of this problem corresponds closely to the common practice emphasized by Marshall (1994) of selecting portfolio compositions that primarily fit the investor's need for liquidity at his or her personal horizon. Moreover, the fact that investors select a portfolio and hold it until its risk horizon means that they are indifferent to the behavior of its mean return before this moment. In this context, it is natural to adopt the risk horizon in the definition of a commonly shared risk measure as it explicitly accounts for the timing of convergence of the returns distribution towards the expectation. In addition, Stutzer's approach relates to one particular type of utility, i.e. a power utility with constant risk aversion. We further show that our semi-parametric approach encompasses allocations implied by decay rate optimization, among other reasonable characterizations of investors' preferences.

The risk horizon framework also relates to the literature on the introduction of asymmetric risks in a risk-return setup, as it accounts for (incomplete) information in higher order moments in investors' perception of the riskiness of securities. Well-known examples in the asset-pricing literature are Kraus and Litzenberger (1976), Simaan (1993), Fang and Lai (1997), Harvey and Siddique (2000), Jurczenko and Maillet (2001), Dittmar (2002) and Chabi-Yo (2012). To derive their equilibrium relationships, these approaches generally rely on restrictive but necessary assumptions on either returns distributions or utility functions. As we adopt a semi-parametric approach, we contribute to this literature by avoiding the specification of any utility or distribution functional form. Furthermore, risk horizon is not necessarily a monotonic function of any of the centered moments. The model is therefore consistent with the critique in Brockett and Kahane (1992) as it does not imply any preferences for particular moments.

The paper is organized as follows. Section 2 presents the theoretical setup, the characterization of the risk measure, risk horizon, and the portfolio choice problem. The third section presents a representative set of competing standard portfolio selection programs against which we challenge our approach. Section 4 is dedicated to the empirical implementation, with optimal portfolio allocations derived within the risk horizon framework. A comparison with traditional portfolio criteria is made in this section, and we present the results of a horse race with the general FTP utility framework. Section 5 concludes.

2 Theoretical framework

This section describes the theoretical framework. We start with the set of assumptions required to design the portfolio problem and investors' perception of the riskiness of assets. The risk horizon criterion is then described with reference to standard results in the semi-parametric statistical literature. The portfolio selection program is then introduced and discussed.

2.1 Key assumptions

The first set of assumptions that surrounds the model is basic and similar to the modern portfolio theory: there is no transaction cost or tax, all investors are price-takers, all securities are infinitely divisible and the investor has full use of the proceeds from any short sales. The design of our framework departs from the assumptions that lead to the standard CAPM results, namely the structure of investors' preferences and the distribution of returns.

2.1.1 Structure of preferences

Following Hakansson and Ziemba (1995), we assume that the value of an initial investment of W_0 in a portfolio p over a horizon T is $W_T = W_0 \prod_{t=1}^{T} \tilde{R}_{p,t}$, where $\tilde{R}_p$ represents gross returns (i.e. 1 + net returns). The investment can be equivalently re-expressed as

$$W_T = W_0 e^{\left(T^{-1} \sum_{t=1}^{T} R_{p,t}\right) T}$$

where $R_p = \log \tilde{R}_p$ are continuously compounded portfolio returns (or more commonly "log returns"), and $T^{-1} \sum_{t=1}^{T} R_{p,t}$ is the compound growth rate of capital or what we simply denote as the "mean returns". The use of continuously compounded rates of return is important because these are the rates investors care about when looking at their realized returns over any given horizon (see, for instance, the discussion in Hakansson and Ziemba, 1995, page 69).

If the sequence $R_{p,1}, R_{p,2}, ...$ is mean ergodic, the mean returns $\overline{R}_p^T = T^{-1} \sum_{t=1}^{T} R_{p,t}$ converge to $E\left[R_p\right]$ as horizon $T \to \infty$. In other words, investors know that the mean returns can be "secured" over an infinite time horizon (i.e. they are indistinct from their expectation). Over a finite horizon, though, this convergence might be sufficient for investors to consider the sample mean as being not significantly different from the expectation. The shorter the required horizon for such a sufficient convergence, the less risky the asset or the portfolio.[1] This proposition is formalized in the following assumption.

Assumption 1: *All agents are rational with unknown utility functions. They consider that the risk horizon H_i of a security or portfolio is the shortest number of periods such that*

$$\mathbb{P}\left[\overline{R}_i^{H_i} \leq E_i - \lambda\right] \leq \Omega + \gamma \mathbb{P}\left[\overline{R}_i^{H_i} \geq E_i + \lambda\right] \tag{1}$$

for constants $\lambda \geq 0$, $0 \leq \gamma \leq 1$, $0 \leq \Omega \leq 1$ and where E_i is the expected return on the security or the portfolio.[2]

This assumption imposes no parametric structure on investors' preferences. Instead, they are inferred from their assessment of the riskiness of an asset or a portfolio. Such an assessment is characterized by risk horizon parameters λ, Ω and γ.

Parameter λ determines the boundaries of the interval around the expectation. It defines the VaR and VaP thresholds, and therefore reflects the risk tolerance of investors: low values for λ imply a narrower interval, and a higher risk horizon H_i is needed to achieve a sufficient level of density concentration. The interpretation of λ can be further developed within the context of portfolio selection. In Section 2.3, the portfolio selection problem is formulated such that investors maximize the expected return of their portfolio for a given level of risk horizon (matched with their investment horizon). Consider two investors with the same investment horizon and whose attitudes towards risk differ solely in terms of λ. The investor with the smaller λ reacts more adversely to the uncertainty around the expectation and is therefore more risk-averse than the investor with a larger λ. In the

[1] Our argument should not be confused with the idea of *time diversification* which suggests that risk decreases with time horizon. The convergence of mean returns does not imply time diversification as it allows for a potential increase in wealth at risk (Samuelson, 1966) and for within-horizon negative returns (Kritzman et al., 2001).

[2] It is interesting to note that paradigm (1) suggests the existence of a "least risky asset", that is the asset whose horizon is the lowest of all existing assets. Therefore the use of the risk horizon as a risk measure provides an interesting definition of what is traditionally meant as the risk-free rate, and allows it to be a stochastic variable.

empirical part of the paper, results from optimal portfolio allocation confirm this interpretation.

Parameter Ω represents the probability mass of mean returns that the investor tolerates outside the interval determined by λ. Hence, $(1-\Omega)$ corresponds to a pre-determined confidence level assigned by investors, similar to a traditional VaR confidence level. For a given interval around the expectation, lower values for Ω ensure that a higher mass of the distribution lies inside the interval. Consequently, if two investors differ only by their tolerance probability Ω, the investor characterized by a lower Ω associates a higher risk horizon with the same portfolio, and can be considered as more risk averse. Altogether, the pair (λ, Ω) play the same role as the quantile value and confidence level, respectively, in a system in which the VaR would be the measure of risk.[3]

Finally, a proportionality coefficient γ is introduced to count as a reward for the upside potential of assets. Probabilities of being outside the interval on the right-hand side of the VaP threshold are generally beneficial to investors as they usually correspond to probabilities of extreme gains. To take these into account, we introduce an adjustment in expression (1), which increases the tolerance probability by a probability of extreme gains proportional to γ. In other words, the pre-determined confidence level $(1-\Omega)$ is adjusted downward by the second term in the right-hand side of (1). Parameter γ refers to the trade-off coefficient between the downside and upside potential of the asset.[4] If γ is equal to 0, investors are only concerned about the downside risk: the risk horizon of a security is simply the number of periods necessary to respect a VaR constraint. A γ equal to 0 typically characterizes the risk horizon version of "safety-first" investors, who may differ in their degree of risk

[3] According to Assumption 1, the risk horizon measure implies that the risk of being outside a certain interval around expectation is considered as negligible beyond a certain horizon (i.e. the risk horizon). More risk-averse investors are likely to pick a very small λ in order to obtain very low probabilities of extreme losses. However, at the individual level, it is reasonable to consider that investors might still suffer from those unlikely extreme events, depending on their risk aversion with respect to extreme loss. In the current setup, our risk horizon measure does not consider the magnitude of the losses outside the interval, nor the preferences of investors related to this extreme risk. This matter is similar to the one with the VaR measure and its complement, the conditional VaR (a.k.a. expected shortfall). While the former measure does not reveal anything about what is beyond the VaR threshold, the second one takes into account the magnitude of extreme losses. Similarly to the VaR measure, our risk horizon measure does not consider the magnitude nor the average of losses outside the interval. In this case, it might be interesting to define a complementary measure to risk horizon which, for instance, takes into account the horizon required for the average of "outside losses" to become sufficiently negligible. It is nevertheless not straightforward to characterize such a measure in a semi-parametric framework as the one presented in this paper. In particular, this complementary measure would require a non-parametric approximation of the expectation of returns beyond $E_i - \lambda$. We therefore leave the implementation of such a measure to future research.

[4] This distinction between considerations for downside and the upside potential is consistent with one of the features underlined by prospect theory (see, for instance, Kahneman and Tversky, 1979; He and Zhou, 2011): investors are not uniformly risk averse as they behave differently when considering losses or gains. They tend to be more sensitive to losses than to gains. The trade-off parameter introduced in the risk horizon framework reflects this asymmetry by overweighting losses relative to gains for $\gamma < 1$.

aversion by parameters λ and Ω. On the other hand, a large γ value implies that investors assign a high importance to the upside return potential of the security. The limiting case $\gamma = 1$ leads investors to assign the same weight to returns above $E_i + \lambda$ as to returns below $E_i - \lambda$. In this particular case, for an asset with a symmetric distribution of returns, $\mathbb{P}\left[\overline{R}_i^{H_i} \geq E_i + \lambda\right] = \mathbb{P}\left[\overline{R}_i^{H_i} \leq E_i - \lambda\right]$ and the constraint given by (1) is always respected, meaning that the risk horizon of this security is null.

At this point, it is important to emphasize the intuitive nature of the risk horizon parameters λ, γ and Ω. These parameters characterize investors' preferences and their knowledge is required in the determination of the risk horizon of a portfolio, in the same way as a risk aversion parameter in the utility approach. However, in contrast with the utility approach where the practical determination of an appropriate value for the risk aversion parameter is not obvious, the selection of suitable parameters for the risk horizon approach is intuitive. Parameter λ can be determined as the percentage deviation from expected returns tolerated by an investor. It can therefore be expressed in return percentage and its interpretation is meaningful to the investor. As illustrated in the introduction, $(1 - \Omega)$ can be directly related to a VaR-type confidence level, which is typically selected among the values of 90%, 95%, 97.5% or 99%. Finally, values of γ could be selected with reference to a gradual scaling from 0 to 1, i.e. from extreme loss aversion (0) to equal consideration for downside and upside potential of the portfolio (1). From a portfolio adviser point of view, it is therefore practicable to solicit information from the investor in order to select appropriate risk preference parameters, and more intuitive than the assessment of the risk aversion coefficient in the utility approach. Moreover, in the utility approach, it is unclear how an adviser should determine the risk aversion coefficient for an investor with a particular investment horizon length of H periods. By separating the risk aversion dimension from the investment horizon dimension, the risk horizon approach has the advantage of avoiding this issue.

2.1.2 Information on returns

Even though the paradigm underlying equation (1) obviates the need to specify any utility function, it still relies on knowledge of the probability distribution $\mathbb{P}$. Instead of assuming any specific functional form for $\mathbb{P}$, we adopt an intuitive perspective. In practice, the entire probability distribution of asset returns is barely observable, and investors have to rely instead on less complete but readily usable returns characteristics, such as some unconditional moments of the distribution representing volatility, asymmetry and fat-tailness. The investors' associated measure of risk has to make use of all such available information. As a result, we assume the existence[5] and knowledge of unconditional moments up to order 4.

Assumption 2: *Continuously compounded returns of each asset i for period t, denoted R_{it}, are i.i.d. with an unknown distribution. Only centered moments of orders 2 (μ_2), 3 (μ_3) and 4 (μ_4) of the unconditional distribution of returns of any asset or portfolio i, which all exist, are known and are denoted V_i, S_i and K_i respectively.*

For i.i.d. period returns, it is easy to derive the moments of the mean of a sequence of n drawings of these returns, $\overline{R}_i^n = \frac{1}{n}\sum_{t=1}^{n} R_{it}$. These moments are equal to: $V_i^n = \frac{V_i}{n}$, $S_i^n = \frac{S_i}{n^2}$, $K_i^n = \frac{K_i}{n^3} + \frac{3(n-1)V_i^2}{n^3}$.

2.2 Characterization of risk horizon

As we have assumed an unknown returns distribution, we need to link the risk horizon constraint (1) with the set of available information, that is, the moments of orders 2, 3 and 4. Chebyshev's inequality already provides such an estimate[6] in the form of an upper bound for the cumulative distribution, but such bounds are "much too wide" and do not exploit the information on moments higher than 2. Mallows (1956) generalizes a Chebyshev-type of bound to moments of orders 3 and 4, and provides an attractive approach for estimating the probability of mean convergence while considering information on asymmetric and fat-tail risks. Information given in Assumption 2 enables us to adapt this classical result and derive an expression for the risk horizon as defined in Assumption 1:

[5] Although affirming the existence of these moments represents a strong assumption regarding some evidence from many series of returns (see Jondeau and Rockinger, 1999), the assumption is commonly used for the derivation of asset-pricing models.

[6] See Roy (1952) for the use of Chebyshev's inequality to characterize the safety-first investor's portfolio problem.

If S_i and K_i exist and are known, the risk horizon of a security or portfolio i is given by:

$$\begin{aligned}
H_i &= \arg\min\{H : \Omega \geq \pi_i(-\lambda, H) - \gamma\pi_i(\lambda, H)\} && (2)\\
\text{where}\ \ \pi_i(x,H) &= \frac{\Delta_i}{Q_i^2(x) + \Delta_i(1+\frac{Hx^2}{V_i})} && (3)\\
Q_i(x) &= -\frac{Hx^2}{V_i} + \frac{S_i x}{V_i^2} + 1 && (4)\\
\Delta_i &= \frac{1}{H}\left(\frac{K_i}{V_i^2} - \frac{S_i^2}{V_i^3} - 3\right) + 2 && (5)\\
\text{under the constraint}\ \ \Delta_i &> 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi_i(x,H)}{\partial \mid x \mid} &< 0\ \forall x && (6)\\
\frac{\partial \pi_i(x,H)}{\partial H} &< 0\ \forall \varphi_i \leq H \leq \Phi_i && (7)\\
\text{where}\ \ \varphi_i &= \min\left[\frac{(-x^2S_i^2 + S_i^2 - V_iK_i - 3V_i^3x^2 + 3V_i^3 + V_iK_ix^2)^2}{4V_i^3x^2S_i^2}, \frac{x^2S_i^2}{V_i^3(1-x^2)^2}\right] && (8)\\
\Phi_i &= \max\left[\frac{(-x^2S_i^2 + S_i^2 - V_iK_i - 3V_i^3x^2 + 3V_i^3 + V_iK_ix^2)^2}{4V_i^3x^2S_i^2}, \frac{x^2S_i^2}{V_i^3(1-x^2)^2}\right] && (9)
\end{aligned}$$

The proof of this proposition follows directly from Mallows (1956) by using the moments of $\overline{R}_i^{H_i}$. The proof for the sign of the derivative with respect to $\mid x \mid$ is straightforward. The sign of the second derivative can be obtained by using the variable transformation $y = \frac{\sqrt{H}x}{\sqrt{V_i}}$ and noting that $\frac{d\pi_i(y,H)}{dH} = \frac{\partial\pi_i(y,H)}{\partial y}\frac{dy}{dH} + \frac{\partial\pi_i(y,H)}{\partial H}$. The first term is always strictly negative and the second term is non-positive for $\varphi_i \leq H \leq \Phi_i$, which completes the proof. □

The probability function $\pi_i(x, H)$ represents the upper bound of the corresponding probabilities defined in equation (1) and is expressed as a function of the unconditional moments. The function $\pi_i(x, H)$ approximates $\mathbb{P}\left(\overline{R}_i^H - E_i > x\right)$ for $x > 0$ and $\mathbb{P}\left(\overline{R}_i^H - E_i < x\right)$ for $x < 0$.[7] Note that the

[7]We assume that the inequality is binding in the framework, i.e. the approximation for $\mathbb{P}$ is exactly equal to the upper bound π. We acknowledge that the true value might actually be lower than this upper bound. However, it is not straightforward to assess how much lower it could be, and this assessment is likely to vary from portfolio to portfolio. Therefore, by using π as an approximation for $\mathbb{P}$, we think that investors make an efficient use of the available information (i.e. the unconditional moments of distributions). Moreover, for the probabilities of downside risk, using π implies a conservative choice as it considers the worst case for this probability. A parallel can be drawn

behavior of π_i with respect to x and H is desirable. The higher the interval x around expectations, the greater the probability mass inside it, and the lower the π. Moreover, the higher the risk horizon H, the more concentrated the distribution around the expectation and the lower the π.

Equation (2) does not relate the risk horizon to the centered moments of the distribution in any particular manner: H does not necessarily increase in variance, decrease in skewness or increase in kurtosis. The absence of any pre-specified relation between the risk measure and unconditional moments is desirable when one considers potential interactions between moments. For instance, it is reasonable to claim that investors generally dislike high kurtosis, as it implies high probabilities of extreme losses. However, if we consider an asset that displays important and positive skewness, some investors (i.e. those who assign significant value to upside potential, hence with an important γ) might indeed find higher kurtosis desirable, as it would be associated with a higher chance of extremely high returns, while extreme losses would be potentially not too heavy due to the positively-skewed distribution. Moreover, within an expected utility framework, Brockett and Kahane (1992) show that when moments are not orthogonal to one another, the effect on utility of increasing one of them does indeed depend on the sensitivities of the others to this increase, and the outcome is therefore ambiguous. Therefore, for any well behaved utility function (that is, for which derivatives alternate in their signs), it is possible to find a returns distribution for which investors might prefer a solution with lower expected returns, high variance and lower skewness. They conclude that any utility-based model should not posit preferences for specific moments. The risk horizon approach complies with Brockett and Kahane's (1992) critique.

An alternative to the Chebyshev-type of bound would be to rely on the Cornish-Fisher approximation (Cornish and Fisher, 1938) to estimate the cumulative probabilities using central moments of the distribution. It is not an upper bound as Mallows's (1956) inequality, and is in this sense more accurate in the approximation of probabilities. In simulations not reported in this paper for brevity reasons, we find that the Cornish-Fisher and the Mallows approaches deliver similar relative measures of risk horizon, i.e. similar rankings for the portfolios in terms of their risk horizons. We

with the *robust optimization* theory in portfolio problem (see for instance Goldfarb and Iyengar, 2003), in which worst-case scenarios for uncertain parameters are used in portfolio optimization.

also find that the Cornish-Fisher method loses some accuracy when important departures from the Gaussian case are considered, and seems to underestimate cumulative probabilities. This may be an important problem in the risk horizon framework, as the underestimation of the probabilities of falling short of a target for expected return may be harmful to investors. In this case, an investor with aversion to downside risk might prefer a more conservative method, such as Mallows's (1956) upper bound.[8]

The characterization of the risk horizon measure presented so far relies on the assumption that financial returns are i.i.d. with unknown distribution. The i.i.d. assumption is required in order to obtain the Chebyshev-type of upper bounds π. Even though it helps to make the framework tractable, this assumption can be restrictive in practice. Though there is little evidence of significant serial correlation in the log returns of many equity portfolios, the presence of time-varying volatility and the so-called "GARCH effects" (see for instance Bollerslev, 1987; Ding et al., 1993; Tauchen, 2001) motivates the need of an alternative way to implement the risk horizon framework for non-i.i.d. financial returns. Let us assume $r_t = \mu + \varepsilon_t$, with $\varepsilon_t = \sqrt{h_t}\eta_t$ where η_t are i.i.d. innovations with expectations 0 and variance 1 from an unknown distribution F. The variance equation writes $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$, where h_t is the conditional variance of returns. Mean returns over n periods ($\bar{R}^n$) have conditional expectation $E_t(\bar{R}^n) = n^{-1} \sum_{i=1}^{n} E_t(r_{t+i}) = \mu$. Conditional variance of mean returns is $Var_t(\bar{R}^n) = n^{-2} \sum_{i=1}^{n} E_t(h_{t+i})$, with $E_t(h_{t+i}) = \alpha_0 + (\alpha_1 + \beta)E_t(h_{t+i-1})$. Therefore, under the stationary conditions $\alpha_1 + \beta < 1$, $E_t(h_{t+n})$ converges to the long term variance $V_L = \alpha_0/(1 - \alpha_1 - \beta)$ as n increases.[9] Accordingly, the risk horizon problem of a financial asset whose returns follow a GARCH(1,1) process is to find H such that:

[8] The Cornish-Fisher method may also suffer from other drawbacks when implemented in order to approximate unknown distribution characterized by asymmetry and fat tails. For instance, Simonato (2011) reports that an important drawback of the Cornish-Fisher approach (as well as an alternative, i.e. the Gram-Charlier approximation) is that it fails to generate valid quantile or density functions for some skewness and kurtosis pairs. In particular, the method may generate densities with negative values or non-monotone quantile functions. Simonato (2011) further examines an alternative with Johnson's (1949) system of distributions, which also uses the first four moments as main inputs but is capable of accommodating all possible skewness and kurtosis pairs. We shall leave the investigation of such alternative methods in the risk horizon framework to future research.

[9] In order to come back to the i.i.d. case, one can impose $\alpha_1 = \beta = 0$ and $\alpha_0 = \sigma^2 = V_L$ and conditional variance of mean returns is equal to the unconditional variance V_L/n.

$$\mathbb{P}\left(\bar{R}^H \leq \mu - \lambda\right) - \gamma \mathbb{P}\left(\bar{R}^H \geq \mu + \lambda\right) \leq \Omega \tag{10}$$

$$\Leftrightarrow \quad \mathbb{P}\left(\frac{\bar{R}^H - \mu}{\sigma_H} \leq \frac{-\lambda}{\sigma_H}\right) - \gamma \left[1 - \mathbb{P}\left(\frac{\bar{R}^H - \mu}{\sigma_H} \leq \frac{\lambda}{\sigma_H}\right)\right] \leq \Omega \tag{11}$$

where $\sigma_H = Var_t(\bar{R}^H)^{1/2} = H^{-1}\left[\sum_{i=1}^{H} E_t(h_{t+i})\right]^{1/2}$ is the conditional standard deviation of mean returns for an horizon H. The assessment of the riskiness of an asset with GARCH-type of returns is therefore equivalent to solving a risk horizon problem for returns standardized by the conditional volatility σ_H. The probabilities in equation (11) can be approximated by the upper bound function π_i described earlier, using moments of the standardized returns and the modified interval boundary λ/σ_H as inputs.

As we can see, the adjustment of the risk horizon framework to deal with GARCH effects can be very easy.[10] In practice, however, GARCH parameters are unknown and the implementation of the risk horizon requires an estimation procedure. Maximum likelihood methods can be used but rely on distributional assumptions. Alternatively, the Generalized Method of Moments (GMM) introduced by Hansen (1982) not only provides a tractable way to estimate GARCH parameters, but is also consistent with the risk horizon framework in the sense that it does not rely on any assumptions on the distribution of the residuals.

In the portfolio implementation of Section 4, we use annual returns from industry portfolios of the Fama and French dataset. These returns are less likely to display GARCH effects as they are collected at a low-frequency level. Moreover, the number of observations in the sample is relatively small, while obtaining reasonable estimates using GMM may require very large sample sizes (Hayashi, 2000). In order to ease the implementation of the risk horizon framework for portfolio selection problems, we therefore make the simplifying assumption of i.i.d. returns, and directly refer to equation (2) for the risk horizon characterization.

[10]Using the family of GARCH-type models, it is also possible to account for time-varying skewness and kurtosis alongside with the time-varying characterization of variance. See for instance Harvey and Siddique (1999) for a GARCHS specification for estimating conditional skewness, and the extension to a GARCHSK structure in Leon et al. (2005) with an additional equation to characterize time-varying kurtosis.

2.3 Portfolio selection

Investors are assumed to adopt risk horizon as the primary criterion for portfolio selection. An investor j chooses his or her portfolio p with a level of risk horizon H_p lower or equal to his or her investment horizon H^j so as to maximize the expected return $E[R_p]$ of the portfolio.

$$\begin{aligned} \max_p \quad & E[R_p] \\ \text{s.t.} \quad & H_p \leq H^j, \text{ with } H_p \equiv \arg\min_{H_p} [\pi_p(-\lambda, H_p) - \gamma\pi_p(\lambda, H_p) \leq \Omega] \end{aligned} \tag{12}$$

where portfolio choice p, through the associated risk horizon H_p, depends on investors' attitudes towards risk reflected by parameters λ, γ and Ω. The formulation of the problem means that investors first select their investment horizon, then choose a portfolio that maximizes their expected returns for that horizon. This is consistent with investors willing to select the portfolio that best fits their needs for liquidity at their personal horizon. This common practice is emphasized in Marshall (1994). The fact that investors select a portfolio and hold it until its risk horizon is reached means that they are indifferent to the behavior of its mean return before this moment in time. In this context, it is natural to adopt risk horizon in the definition of a commonly shared risk measure as it explicitly accounts for the timing of the convergence of the returns distribution towards the expectation.

In practice, higher horizon allows the investor to take on more risk, and implies higher expected returns.[11] This suggests the selection of a portfolio with the maximum risk horizon, i.e. a portfolio such that $H_p = H^j$. This portfolio offers the most attractive expected return while respecting the risk horizon constraint. While the selection of a portfolio with $H_p = H^j$ is what most investors of the risk horizon type would do, some of them, associated with an important γ, are likely to diverge from this principle. High values for γ characterize investors who rely heavily on the upside potential of portfolios in order to respect their horizon constraint. These "high-γ" investors may not be resolved to see the probability of upside potential being diversified away (relative to the probability

[11] Simulations of optimal portfolio choices with reasonable values of the parameters λ, γ and Ω, included the ones used later in this paper, confirm the monotonous property of the relation between expected return and risk horizon of optimal portfolios.

of downside risk) with horizon. They might instead be ready to trade off some expected return in order to obtain a certain level of upside potential for their portfolio. For a given investment horizon, a portfolio with a higher risk horizon might not necessarily display a better upside-downside trade-off. Therefore, "high-γ" investors would not necessarily favor portfolios with $H_p = H^j$, as these portfolios are not necessarily associated with the most attractive upside potential for their investment horizon.

The formulation of the portfolio selection program is flexible enough to encompass fixed as well as sliding investment horizons. An individual with a fixed horizon has the incentive to invest in a life-cycle fund, whose composition is rebalanced according to the remaining distance to the horizon (target date investment). Someone whose risk horizon serves as a pure substitute for their utility function would keep, in our i.i.d. world, a constant mix of financial assets. This buy-and-hold strategy corresponds to a sliding investment horizon: at each point in time, the target investment date is incremented so that the risk horizon is constant over time. Thanks to this flexibility, investors can split their decisions according to various programs, depending on the objective of the investment. For the same investor, multiple programs (12), with potentially different triplets $(\lambda, \gamma, \Omega)$ correspond to evidence of mental accounting, which underlies the goal-based investing framework.

3 Standard portfolio selection programs

The use of risk horizon in (12) does not impose any functional form on investors' utility or returns distribution. However, inconsistency with optimal portfolio compositions derived from a standard parametric characterization of investors' preferences might ultimately cast doubt on the validity of the risk horizon framework. The theoretical framework might appear to be quite far removed from a traditional characterization of investors' behavior in portfolio selection. Nevertheless, it has many degrees of freedom in the characterization of investors' attitude towards risk (parameters λ, γ and Ω), suggesting that a parallel with standard approaches can be drawn. To assess the consistency of these characterizations with standard approaches, we compare optimal allocations derived from risk horizon optimization with those generated by standard portfolio criteria. We describe below the standard approaches to the portfolio problem that we consider. The results of portfolio comparison are reported in the empirical section of the paper.

One of the earliest and most common approaches to the portfolio problem finds its roots in the mean-variance paradigm put forward by Markowitz (1952). The underlying structure imposed on investors' preferences is the assumption that they have quadratic preferences. It can then be shown that investors choose a portfolio that maximizes the Sharpe ratio criterion (Sharpe, 1994). This criterion is based on excess gross returns and introduces a penalty for variance. Excess returns are usually evaluated with respect to a risk-free rate or a benchmark rate (here denoted by r).

$$\max_p \frac{E\left[\tilde{R}_p\right] - r}{\sqrt{Var\left[\tilde{R}_p\right]}} \tag{13}$$

Another common approach to the portfolio problem advocates that investors select the portfolio that maximizes the expected growth. This strategy is known as the growth-optimal strategy, or Kelly's (1956) criterion. It reduces to the following maximization problem.

$$\max_p E\left[R_p\right] \tag{14}$$

Hakansson and Ziemba (1995) further show the connection between the growth-optimal strategy and the expected log utility criterion. This corresponds to investors with a very low degree of risk aversion (i.e. the associated power utility coefficient is equal to 1), and usually generates highly risky allocations in the long term. This has led several researchers to add a downside risk dimension to the strategy, by considering the probability that invested wealth will fall short of investor goals (see for instance Fishburn, 1977; Browne, 1999; Stutzer, 2003; Puhalskii, 2011). In particular, Stutzer's (2003) approach relies on decay rate maximization: investors select the composition that makes the probability of falling short of a target rate R^* decay to zero as fast as possible. One particular interesting aspect of this approach lies in its analogy to the class of power utility functions, of which log utility is a special case. Specifically, decay rate investors are equivalent to expected power utility investors, with a risk aversion coefficient dependent on target returns and on the investment opportunity set. Therefore, the comparison to Stutzer's (2003) decay rate allocation enables us to compare risk horizon optimal allocation to that of investors characterized by power utility.[12] The optimization problem starts with the minimization of the shortfall probability, which turns out to be equivalent to the maximization of the decay rate (see Stutzer, 2003),

$$\max_p \max_\theta \theta \log R^* - \log E\left[\tilde{R}_p^\theta\right] \tag{15}$$

We also consider the comparison of allocation associated with an objective function derived from Bell's (1995) utility (see also Bell, 1988). This framework provides the class of utility functions that corresponds to a reasonable minimal set of requirements: being continuous and increasing in initial wealth; increasing and concave in expected wealth; decreasing in the measure of risk, and exhibiting decreasing absolute risk aversion (DARA). The general form of this class of utility functions is $U(x) = x - be^{-cx}$. Parameters c and b measure respectively the relative riskiness and the degree of aversion to that riskiness, and therefore provide an interesting dichotomization of investors' attitudes towards risk. Using a fourth-order Taylor series expansion,[13] it is easy to express expected utility in terms of the moments of returns distribution:

$$\begin{aligned} E\left[U\left(\tilde{R}_p\right)\right] &= E\left(\tilde{R}_p\right) - be^{-cE\left(\tilde{R}_p\right)} - \frac{bc^2e^{-cE\left(\tilde{R}_p\right)}}{2} E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^2 \\ &+ \frac{bc^3e^{-E\left(\tilde{R}_p\right)}}{6} E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^3 - \frac{bc^4e^{-cE\left(\tilde{R}_p\right)}}{24} E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^4 \end{aligned} \tag{16}$$

Transforming the second term of this equation by dividing by $bc^2e^{-cE\left(\tilde{R}_p\right)}$ and removing a constant term, we obtain Bell's measure of risk as a function of moments of order 2, 3 and 4:

$$Risk(\tilde{R}_p) = \frac{1}{2}E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^2 - \frac{c}{6}E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^3 + \frac{c^2}{24}E\left[\tilde{R}_p - E\left(\tilde{R}_p\right)\right]^4 \tag{17}$$

Equation (17) underlines the importance of parameter c in the definition of Bell's risk: it controls

[12] Furthermore, Haley and Whiteman (2008) relate the i.i.d. version of Stutzer's (2003) decay rate maximization to a generalization of Roy's (1952) safety-first principle. They show that the generalization of the safety-first rule leads to the same objective function and portfolio decision as does Stutzer's.

[13] Hlawitschka (1994) documents the good accuracy of fourth-order Taylor expansions to approximate expected utility.

the degree to which the investor is concerned about asymmetric and fat-tail risk. In their portfolio choice, we consider that Bell's investors seek to maximize expected (excess) returns adjusted for Bell's risk:

$$\max_{p} \frac{E\left[\tilde{R}_p\right] - r}{Risk\left[\tilde{R}_p\right]} \tag{18}$$

Optimal portfolios of investors with low values for parameter c approach the Sharpe ratio type of allocation. By contrast, investors with a large c penalize more importantly portfolios with negative asymmetry and a high probability of extreme outcomes. Using portfolio rule (18), we are therefore able to differentiate between investors with the same consideration for variance but a different emphasis on extreme risks. We expect that the resulting differences in allocation can be matched using a different and meaningful set of risk horizon parameters.

Finally, we include the class of flexible three-parameter utility functions in the analysis (hereafter FTP, see Conniffe, 2007; Meyer, 2010). The FTP form is the most general form for utility in the literature and is remarkably flexible in its capacity to represent varying preferences for risk. While it is a generalization of Xie's (2000) power risk aversion (PRA) utility functions, it also encompasses other types of utility functions including the hyperbolic absolute risk aversion (HARA) family (Merton, 1971). In particular, the FTP form can replicate increasing, constant and decreasing absolute as well as relative risk aversion characterizations (i.e., IARA, CARA, DARA, IRRA, CRRA and DRRA representations). The fact that FTP utility functions are general, very flexible and that, like the risk horizon framework, are characterized by three parameters, makes the FTP approach a serious candidate for a horse race with the risk horizon model. The FTP utility function is as follows:

$$u(x) = \frac{1}{\xi}\left\{1 - \left[1 - k\xi\left(\frac{x^{1-\sigma} - 1}{1-\sigma}\right)\right]^{\frac{1}{k}}\right\} \tag{19}$$

with

$$1 - k\xi \left(\frac{x^{1-\sigma} - 1}{1 - \sigma} \right) > 0 \tag{20}$$

Parameters k, ξ and σ characterize the magnitude and shape of the risk aversion measure. We refer to Conniffe (2007) for the different restrictions that can be made on parameters and the corresponding types of risk aversion that these restrictions characterize. The portfolio selection problem is the maximization of the expected utility of final wealth: $\max_p E\left[u(W_T)\right]$, where $W_T = W_0\tilde{R}_p$ and $\tilde{R}_p$ are gross portfolio returns over the period. Normalizing by W_0, we obtain the following problem: $\max_p E\left[u(\tilde{R}_p)\right]$. In order to evaluate $E\left[u(\tilde{R}_p)\right]$, we take the expectation of a 4-order Taylor series expansion of $u(\tilde{R}_p)$ around $E(\tilde{R}_p)$.

4 Portfolio implementation

4.1 Data

For a practical implementation of the risk horizon portfolio selection, we use the 10 domestic (US) industry portfolios provided by Fama and French. The dataset can easily be downloaded from Kenneth French's website[14] and is widely used in academic research in finance. The choice of this database is also motivated by the comparison with Stutzer (2003) who performs optimal allocations based on these portfolios. The Fama-French industry portfolios contain value-weighted assets and are regularly re-balanced. Each stock from NYSE, AMEX, and NASDAQ exchanges are assigned to an industry portfolio at the end of June of each year in the sample. This association is based on Compustat SIC codes for the previous fiscal year (or CRSP SIC codes whenever Compustat SIC codes are not available). We use an updated version of the database, which might have slightly changed when compared to the one available to Stutzer (2003), and collect annual returns running from 1927 through 2013. Descriptive statistics and cross-correlations of the industry portfolios are reported in Table 1. Consumer durables, denoted *Durbl*, have the highest mean return, but they also make up the leading portfolio in terms of volatility and kurtosis. This asset is thus expected to obtain relatively more weights in more aggressive strategies, as the growth-optimal strategy.

[14]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Note that *Durbl* also displays the highest positive skewness, meaning that a high excess kurtosis implies important upside potential. The consumer non-durables (*NoDur*) and Telecommunication (*Telcm*) portfolios are the least volatile portfolios, with returns close to a normal distribution (i.e. with skewness and excess kurtosis at near zero, and with Chi-square statistics that fail to reject the null of normally distributed data), and might then be favored by more defensive strategies. Under the *Other* portfolio, assets assigned to the following sectors can be found: Mines, Construction, Building Maintenance, Transport, Hotels, Bus Services, Entertainment, and Finance. This portfolio has the worst skewness of the industry portfolios, with relatively important volatility. Cross-correlations among Fama-French's domestic industry portfolios vary between 0.39 and 0.88, indicating diversification potential in the asset universe considered.

Table 1: Summary statistics and cross-correlations of the asset universe. Assets are the 10 domestic industry portfolios provided by Fama and French and available for download from Kenneth French's website. Value-weighted annual returns are downloaded for a period from 1927 to 2013. The last column of the upper panel reports the p-values of a Jarque-Bera test for normal distribution.

Statistics	mean	std	skewness	kurtosis	JB pvalue
NoDur	0.127	0.192	-0.130	2.769	>0.50
Durbl	0.152	0.349	0.853	6.055	0.00
Manuf	0.130	0.242	0.399	5.561	0.00
Enrgy	0.134	0.218	-0.083	3.454	>0.50
HiTec	0.139	0.277	0.092	2.886	>0.50
Telcm	0.113	0.196	-0.050	3.207	>0.50
Shops	0.134	0.244	-0.091	2.712	>0.50
Hlth	0.138	0.217	0.152	2.666	>0.50
Utils	0.113	0.219	0.167	3.869	0.13
Other	0.116	0.237	-0.402	3.243	0.18

Correlations	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
NoDur	100%									
Durbl	69%	100%								
Manuf	77%	87%	100%							
Enrgy	52%	58%	78%	100%						
HiTec	59%	71%	79%	54%	100%					
Telcm	59%	53%	56%	39%	63%	100%				
Shops	87%	78%	79%	50%	72%	60%	100%			
Hlth	74%	47%	61%	39%	59%	49%	72%	100%		
Utils	64%	45%	52%	51%	50%	61%	59%	59%	100%	
Other	83%	78%	88%	76%	72%	64%	81%	65%	64%	100%

4.2 Comparison with standard approaches

In this section, we compare the optimal portfolio allocations of different risk horizon investors to optimal allocations derived from traditional criteria: the Sharpe ratio, the growth-optimal criterion, Stutzer's (2003) decay rate optimization, and the objective function based on Bell's (1995) risk measure. For each traditional allocation, we verify the equivalent risk horizon allocations. The objective is to document the fact that traditional types of investors can be associated with risk horizon investors. In this sense, the risk horizon framework is shown to be consistent with traditional portfolio choice approaches. Moreover, when we compare attitudes towards risk reflected by λ, γ and Ω associated with two different traditional criteria, we observe meaningful differences in risk horizon parameters.

In line with the risk horizon portfolio problem described in the theoretical section above, risk horizon investors select the portfolio that maximizes the expected return for a given level of risk horizon. As in Stutzer (2003), we present the results of portfolio allocations when short sales are allowed, though we draw similar conclusions in a version with a short selling constraint, as presented in the appendix. We fix the risk horizon and allow investors to differ in terms of their attitudes towards risk, reflected by parameters λ, γ and Ω. This provides a comparable basis with traditional criteria that do not explicitly refer to a notion of horizon. In the results presented in Table 2, we use a risk horizon H equal to 30 years.[15]

Each column refers to a traditional allocation and the corresponding best match from risk horizon optimization. The best match is the risk horizon allocation that minimizes the relative pointwise distance in weights normalized by the sum of the absolute values of weights

$$d = \frac{\sqrt{\sum_{i=1}^{N} (w_i - w_i^*)^2}}{\sum_{i=1}^{N} |w_i^*|} \tag{21}$$

where w_i is the weight assigned to asset i in the risk horizon portfolio, and w_i^* its correspondent in the traditional portfolio. The distance metric expressed in (21) is related to the Euclidean distance if one considers weights as the coordinates of a portfolio in an N-dimension plan. Nevertheless, portfolios differ in the absolute values of their exposures. Therefore, the same relative error in

[15] In the appendix to this paper, we present the results for a shorter and a longer risk horizon, i.e. respectively $H = 10$ and $H = 50$ years. The observations underlined in this section for $H = 30$ are robust to changes in investors' risk horizon.

Table 2: Comparison of optimal portfolio allocations from standard and risk horizon approaches. Portfolios are composed of the 10 industry indices of Fama-French, for a period from 1927 to 2013. Each column is associated with the matching of a particular standard approach. The reported statistics correspond to the risk horizon allocation which minimizes the normalized Euclidean distance with respect to the portfolio weights of the standard approach (see equation (21)). Portfolio weights, mean and standard deviations are in percentage. Numbers in parentheses correspond to differences with respect to the portfolio allocation derived under the standard approach. For Bell-type of portfolios, the value used for the coefficient c in equation (17) is reported in the column title. For Stutzer's (2003) type of allocations, the decay rate and associated risk aversion coefficients of the standard portfolio allocation are reported in the last two rows.

	Sharpe	Bell 0.5	Bell 5	Decay 5%	Decay 10%	Decay 15%	Growth
Risk horizon parameters							
λ	0.099	0.055	0.049	0.057	0.049	0.12	0.19
γ	0.00015	0.065	0.32	0.49	0.47	0.47	0.46
Ω	0.010	0.073	0.078	0.048	0.14	0.037	0.093
Portfolio weights							
NoDur	89 (1)	83 (1)	84 (-4)	97 (-8)	117 (-3)	157 (-5)	267 (-9)
Durbl	18 (0)	8 (0)	9 (0)	28 (0)	54 (2)	113 (7)	279 (23)
Manuf	-60 (-1)	-43 (-1)	-47 (1)	-80 (-2)	-119 (1)	-215 (3)	-487 (5)
Enrgy	87 (0)	75 (0)	78 (1)	103 (1)	134 (2)	201 (3)	391 (4)
HiTec	17 (-1)	10 (-2)	8 (0)	19 (0)	36 (-4)	72 (-12)	161 (-34)
Telcm	38 (2)	44 (3)	48 (-1)	42 (-3)	29 (-2)	9 (-5)	-38 (-8)
Shops	-18 (-1)	-17 (-1)	-18 (1)	-24 (3)	-31 (3)	-45 (10)	-82 (27)
Hlth	34 (-1)	27 (0)	29 (3)	40 (6)	54 (3)	92 (4)	206 (6)
Utils	-18 (0)	-13 (-1)	-16 (-1)	-29 (-1)	-46 (-4)	-82 (-8)	-188 (-18)
Other	-88 (0)	-74 (0)	-74 (0)	-96 (4)	-129 (2)	-203 (2)	-410 (5)
distance	0.007	0.010	0.014	0.021	0.012	0.017	0.022
Performance statistics							
Mean	14.7 (-0.1)	13.9 (-0.1)	13.9 (0.1)	15.2 (0.1)	17.1 (0.2)	21.1 (0.3)	32.2 (0.8)
Std	14.5 (0)	14 (0)	14.1 (0)	15.2 (0)	18.5 (0.3)	28.7 (0.8)	62.4 (2)
Skewness	-0.2 (0.04)	-0.02 (0.07)	0.07 (0.13)	-0.06 (-0.05)	-0.08 (0.02)	0.01 (0.02)	0.07 (0.01)
Kurtosis	2.5 (0.01)	2.9 (0.1)	3 (0.12)	2.6 (0.11)	2.8 (0.25)	3.5 (0.52)	3.6 (0.63)
Decay rate				0.19	0.045	0.0031	0
Risk aversion				5.5	2.8	1.3	1.0

weights produces a more important distance for portfolios with more important absolute exposures. For instance, if there is a relative error of 10% for a weight of 0.5, this translates into an increment $(w_i - w_i^*)^2$ of 0.0025, while the same relative error for a weight of 1.5 corresponds to a squared difference of 0.0225. By normalizing by the sum of the absolute values of optimal weights, we obtain a comparable metric across the portfolios. This metric is reported below risk horizon allocations in the tables. It is relatively low, and achieves a maximum value of around 0.02, which corresponds to an average relative error of about 5%. Risk horizon portfolios are thus able to accurately replicate the optimal allocations of traditional portfolios. This result is also underlined by the small differences reported in the performance statistics of risk horizon portfolios and their traditional counterparts.

We start by considering differences across allocations. In the Sharpe and Bell traditional approaches, we assume a zero constant reference rate when computing the optimal portfolios using respectively equation (13) and equation (18). Portfolios of the Sharpe and the Bell type have the lowest volatility among the optimal portfolios. Large long positions can be noticed in assets with the least volatile profiles: Non-durables, Energy, Telecommunications and Health. We observe small exposures to volatile industries such as Durables and High Technology. We consider different target returns in the implementation of decay rate optimization portfolios: 5%, 10% and 15%. As mentioned above, different target returns are associated with different risk aversion coefficients in the power utility function. Typically, higher target returns imply lower risk aversion coefficients. The growth-optimal portfolio corresponds to an extreme case with the smallest risk aversion coefficient (that is, a risk aversion coefficient equal to 1). Portfolio statistics are consistent with this progression in risk aversion. Portfolios associated with decay rate types of investors with lower target returns (hence higher risk aversion) display lower mean returns, volatility and kurtosis. The decay rate portfolio associated with a target return of 5% displays performance statistics close to the Sharpe ratio in terms of mean returns and volatility. The growth-optimal portfolio shows important positions in terms of absolute value and its historical mean and volatility are the highest. Skewness is more favorable in decay rate portfolios than in the Sharpe portfolio. This is consistent with the skewness preference implied by power utilities associated with degrees of risk aversion greater or equal to 1 (see for instance Kraus and Litzenberger, 1976). Indeed, Stutzer (2003) shows that the decay rate increases in odd-order moments (and decreases in even-order moments). This explains the more important weight of Durables in decay rate portfolios, as this asset displays high volatility but strongly positive skewness and significant kurtosis. Moreover, its weight increases with a lower risk aversion coefficient.

Risk horizon parameters associated with the Sharpe portfolio reflect a relatively defensive risk profile, with restrictive values for loss aversion (γ is virtually 0) and for tolerance probability ($\Omega = 0.01$), and with a value of 0.10 for λ. These results are intuitive. Sharpe investors are concerned about the dispersion of the returns distribution, with no consideration of asymmetry or fat-tailness. For a given interval around expectation, a low tolerance probability Ω implies a low dispersion: this ensures that a high probability mass lies within the interval. This explains a tight value of Ω for Sharpe-type investors. Their low interest in extreme gains outside the interval around expectation suggests a very low value of the loss aversion parameter γ.

The way we formulate the portfolio problem in equation (18) provides Bell-optimal portfolios close to those selected by a Sharpe investor. The set of risk horizon parameters that characterize those investors is therefore also very tight, with relatively low values of λ and Ω. The difference lies in the degree of risk awareness concerning asymmetric and fat-tail risks. A Bell-type investor with a high degree of relative riskiness (Bell 5 Table 2) largely penalizes portfolios with unfavorable skewness and kurtosis. The optimal allocation of this investor displays the highest positive skewness among traditional allocations. Consistently, the risk horizon profile associated with this investor reflects a much more important consideration for upside potential relative to downside risk, with a γ equal to 0.32.

The progressivity in the riskiness of allocations of decay rate portfolios is also reflected in their associated risk horizon parameters. Investors with the highest target returns (i.e. the lowest aversion coefficient for power utility) tend to be associated with less restrictive values for risk horizon parameters. The set of parameters associated with the decay rate optimization with a target return of 5% is tighter than for other decay rate optimizations. At the other extreme, the growth-optimal portfolio is associated with the least restrictive profile, i.e. with the largest interval parameter $\lambda = 0.19$, an important consideration for upside potential with $\gamma = 0.46$, and a high tolerance probability ($\Omega = 0.09$). Compared to Sharpe investors, decay rate investors are associated with higher γ and Ω. As these investors are also concerned about moments higher than 2, they put relatively less emphasis on the dispersion of the distribution than Sharpe investors and relatively more on the upside potential of portfolios. This explains the higher values for Ω.

Significant values for γ can be understood through the following example. Consider two assets with the same variance (dispersion), but with one displaying a higher upside potential than the other. The first asset (with the higher upside potential) is likely to be preferred by decay-rate investors, as it would increase the decay rate, i.e. it would have a lower probability of falling short of the target than the second asset. Decay-rate investors thus value upside potential more than do Sharpe investors and this is reflected in a higher associated γ. Stutzer (2003) shows that the risk aversion coefficient of power utility investors is inversely related to the target return. Parameter λ is therefore consistent with the fact that investors associated with higher target returns are also less risk averse as measured by their power utility function. A larger interval around expectation corresponds to the fact that investors allow for more uncertainty. They are less restrictive in their risk horizon constraint, and are able to target greater expected returns for the same horizon. The statistics of their portfolios typically display a riskier profile with a higher mean return.

4.3 Comparison with FTP utility

As mentioned earlier, the FTP class of utility functions, described in Conniffe (2007) and Meyer (2010), offers a very flexible way to characterize a wide range of risk preferences. Though the approach is not popular in the portfolio selection literature (at least in part due to the complexity of its structural form), it is the most general class of utility function suggested so far. As in the risk horizon framework, the FTP form is characterized by three parameters, making the two approaches even as regards the available degrees of freedom. In this section, we implement a two-way comparison between the two approaches. If it can be shown that the risk horizon approach is able to replicate

FTP allocations but that the reverse is not necessarily true, the analysis would provide additional support in favor of the use of the risk horizon framework and the general feature of its allocations. In order to prove this point, we use two analyses. First, in a similar way to the previous section, we try to match FTP optimal allocations with optimal allocations of risk-horizon investors. This is the *matching* application. Second, we try to match risk-horizon-optimal allocations with optimal allocations derived from FTP type of investors. We call this second operation *reverse-matching*.

4.3.1 Matching analysis

The matching application requires the specification of reasonable and representative combinations of the three FTP parameters, k, ξ and σ. The literature is silent on the selection of plausible values for the FTP parameters. In order to be as exhaustive as possible, we implement the following methodology for the selection of parameter ranges. We first proceed parameter per parameter in order to find minimum and maximum values for each of them. We start from a very large range of values for one parameter and check for which values the portfolio allocation significantly changes. For instance, when we select values for k inside $[-30; +30]$ for large fixed ranges of values for ξ and σ, we observe that values for k lower than -15 produce similar allocations. Values for k higher than 10 also yield to similar optimal portfolios. This analysis suggests that $k \in [-15; 10]$. Implementing the same procedure for ξ and σ suggests ranges of $[-10; 10]$ for the two parameters. These intervals limit the range of reasonable values for the three parameters, and enable us to test a reasonable number of representative combinations of the three parameters.

In order to select values inside these min-max intervals, we use the result of the reverse-matching presented below. In the reverse matching, plausible combinations of risk horizon parameters are used, based on those that replicate traditional allocations (i.e. the risk horizon parameters in Table 2. The FTP parameters that deliver similar allocations are stored. Median and percentile values of these stored series are used to select the following range of reasonable values for k, ξ and σ:

$$\begin{aligned} k &\in [-15\ -0.76\ -0.18\ -0.04\ 0.24\ 2.35\ 10]; \\ \xi &\in [-10\ 0.12\ 0.41\ 0.68\ 1.08\ 1.84\ 10]; \\ \sigma &\in [-10\ -3.37\ -0.1\ 0.17\ 0.64\ 1.6\ 10]; \end{aligned}$$

The matching is therefore implemented over a triplet of 7x7x7 possible combinations of the three FTP parameters, that is, a total of 343 combinations. It is important to note that some combinations of the three parameters might not lead to feasible portfolio allocations, for two reasons. First, some combinations might lead to complex values for the expected utility. For instance, if $k \neq 1$ and $1 - k\xi\left(x^{1-\sigma} - 1\right)/(1-\sigma) \leq 0$, the utility function (19) is not well defined and leads to a solution involving imaginary numbers. Second, although we allow for short selling in the portfolio optimization, we impose large lower and upper bounds for weights to avoid unrealistic portfolio compositions. Whenever a combination of parameters produces complex numbers for the utility function or a boundary value for weights, it is disregarded in the matching analysis. Due to these two filters, 162 allocations out of the 343 possible combinations of the FTP parameters are used in the matching analysis.

Table 3 reports the results of the matching analysis on the total sample of 162 allocations. The first four columns report minimum, median, mean and maximum values of portfolio statistics and associated risk horizon parameters. The portfolio statistics indicate that the FTP allocations cover a large range of portfolio profiles, from very defensive to very aggressive profiles. Distance metrics indicate that the risk horizon portfolio optimization is able to closely replicate the FTP allocations. Median and average distance values are around 0.02, and a maximum value of 0.046 is observed. The last column of Table 3 shows average statistics conditional on the distance metric being higher or equal to 0.04. The worst replications seem to be associated with relatively defensive profiles, with a combination of low volatility and kurtosis and small and negative skewness. The worst matching associated with the maximum distance value of 0.046 is reported in Table 4. The high distance value is mostly due to differences in the weights of the Non-durable, Health and Other assets, though the signs of the allocation are respected. The portfolio statistics of the risk horizon optimal allocation are nevertheless very close to the ones of the FTP allocation, meaning that the matching is still reasonable. The results of the matching analysis confirm the good performance of the risk horizon approach in the replication of FTP optimal allocations.

4.3.2 Reverse-matching analysis

In the reverse-matching analysis, we try to match reasonable risk horizon allocations with the FTP portfolio approach. In analogy with the matching analysis, we generate all possible combinations of a triplet of risk horizon parameters, and simulate the optimal portfolios associated with each profile. We then iterate over values of the FTP parameters in order to minimize the distance metric between risk horizon and FTP allocations. The triplet of risk horizon values is selected on the basis of the comparison with traditional approaches, reported in Table 2. This comparison provides us with a reasonably wide range of values for parameters λ and Ω, but only covers a part of the range of possible values for γ (i.e., values lower or equal to 0.49 which is the maximum value for γ found in the replication of traditional portfolios). Therefore, we also consider higher value of the γ parameter. The ranges of risk horizon parameters considered for the reverse-matching analysis are the following:

$$\begin{aligned} \lambda &\in [0.05\ 0.1\ 0.15\ 0.2]; \\ \gamma &\in [0\ 0.2\ 0.4\ 0.6]; \\ \Omega &\in [0.01\ 0.05\ 0.10\ 0.15]; \end{aligned}$$

The ranges of values presented above lead to 4x4x4 = 64 possible combinations of the parameters. Out of these 64 combinations, some may be too restrictive for the Fama-French asset universe. Indeed, combinations of very strict values of the risk horizon parameters activate very defensive risk profiles which require extremely defensive portfolios that cannot be reasonably achieved with the Fama-French assets.[16] Those combinations lead to implausibly high absolute values of weights,

[16]The addition of a relatively less risky asset (e.g. a Treasury bond) to the investment universe could solve this issue, but this is a marginal case in the context of very long-term allocations with potential short sales.

exceeding the lower and upper bounds that we impose for weights, and are therefore ruled out from the simulation. This filter leaves us with 53 effective risk horizon allocations.

Results of the reverse-matching are shown in Table 5 below. In general, the FTP optimization replicates quite closely risk horizon allocations, as indicated by a low median distance metric. However, the distribution of distance metrics is skewed to the right. The maximum distance reaches an extreme value of 0.186, meaning that the FTP allocation is unable to replicate some of the risk horizon allocations. A deeper investigation into the worst FTP matches reveals that 10 out of the 53 replications have a distance metric higher than 0.05. All ten of these allocations are characterized by risk horizon investors with a coefficient γ of 0.6 (i.e. the maximum value considered for this parameter). Their portfolio statistical profiles are also more volatile (average volatility of 0.52) and display important positive asymmetries and fat-tailness (average skewness and kurtosis of respectively 1.54 and 11.87). Table 6 reports the 5 worst matches and their related statistics. A detailed comparison of the moments of optimal portfolios generated from the risk horizon and the FTP utility frameworks is shown in Figure 1. Optimal portfolios generated by the risk horizon approach cover dispersed values of volatility, skewness and kurtosis. While FTP optimal portfolios largely span the volatility dimension, the values of their other moments remain concentrated around values close to the Gaussian law. Values for the skewness of FTP portfolios remain in the interval [-0.26;0.15], and kurtosis values are inside [2.20;3.56]. Despite the use of a four-order Taylor series expansion in order to account for third and fourth moments in expected utility, the restrictions imposed by the FTP approach are such that it posits a focus on volatility risks. Varying parameters enables to vary investors' risk aversion associated with the dispersion dimension, but does not dichotomize investors along the loss aversion dimension (i.e. aversion with respect to asymmetric and fat-tail risks). As a result, FTP portfolios fit risk horizon portfolios associated with moments close to the Gaussian law reasonably well, but are unable to match portfolios with more extreme values for higher order moments. The risk horizon approach does not posit such moment preferences, and the trade-off parameter γ plays an important role in the loss aversion dimension.

These results confirm that the FTP approach has difficulties replicating the optimal allocation of aggressive risk horizon investors who are fond of the upside potential of portfolios. Altogether, the matching and reverse matching analyses demonstrate the capacity of the risk horizon approach to cover relevant risk profiles, which can not necessarily be found in a very flexible and general parametric framework.

Table 3: Matching of FTP optimal allocations with risk horizon optimal allocations. A total of 162 plausible combinations of the three FTP parameters are simulated. Minimum, median, mean and maximum statistics of portfolios, weights distance metric and associated risk horizon parameters are reported. Mean statistics conditional on the distance being higher or equal to 0.04 are reported in the last column. Mean and standard deviation of portfolios are in percentage.

Portfolio statistics	min	median	mean	max	mean if dist ≥ 0.04
Mean	14.4	15.9	18.4	49.4	15.1
Std	14.5	16.2	23.3	117.3	15.2
Skewness	-0.1	-0.1	-0.1	0.1	-0.1
Kurtosis	2.2	2.5	2.6	3.5	2.2
distance	5.1E-10	0.02	0.02	0.046	0.04
Risk horizon parameters					
λ	0.05	0.09	0.10	0.37	0.11
γ	0.22	0.41	0.41	0.70	0.44
Ω	0.00048	0.06	0.05	0.17	0.0094

Table 4: Worst matching of a FTP allocation with the risk horizon approach. Weights, mean and standard deviation of portfolios are in percentage.

	FTP	RH	
	k : -15	λ : 0.12	
	ξ : 10	γ : 0.46	absolute
	σ : 10	Ω : 0.0027	difference
Portfolio weights			
NoDur	111	95	15
Durbl	28	26	2
Manuf	-73	-75	3
Enrgy	93	99	-6
HiTec	25	21	4
Telcm	40	38	2
Shops	-25	-21	-4
Hlth	26	40	-13
Utils	-18	-25	6
Other	-106	-98	-8
distance		0.046	
Performance statistics			
Mean	15.1	15.2	
Std	15.2	15.1	
Skewness	-0.1	-0.1	
Kurtosis	2.2	2.5	

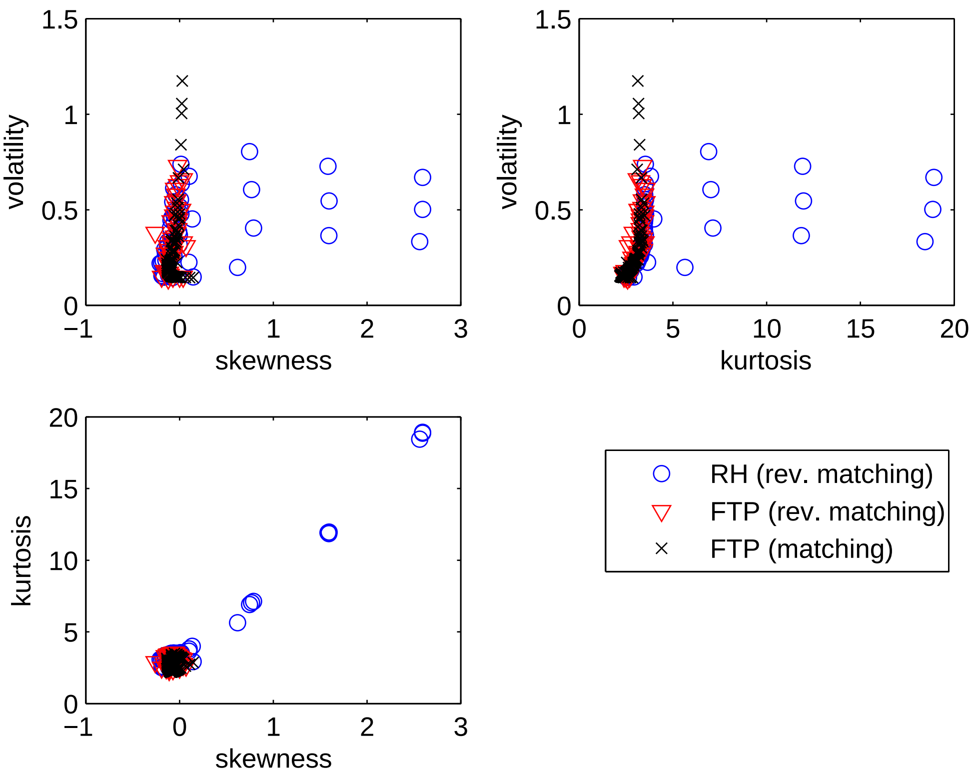
Table 5: Reverse-matching of risk horizon optimal allocations with the FTP approach. A total of 53 plausible combinations of the risk horizon parameters are simulated. Minimum, median, mean and maximum statistics of portfolios, weights distance metric and associated FTP parameters are reported. Mean statistics conditional on the distance being higher or equal to 0.04 are reported in the last column. Mean and standard deviation of portfolios are in percentage.

Portfolio Statistics	min	median	mean	max	mean if dist ≥ 0.04
Mean	14.6	22.8	23.7	36.3	26.0
Std	14.5	35.4	38.5	80.5	51.5
Skewness	-0.2	-0.1	0.2	2.6	1.5
Kurtosis	2.5	3.5	4.9	18.9	11.9
distance	5.31E-07	0.01	0.03	0.19	0.12
FTP parameters					
k	-433.67	-0.05	-10.28	7.53	-55.72
ξ	-8.92	0.65	0.57	7.55	-0.49
σ	-33.87	0.23	-0.34	21.22	-7.18

Table 6: Top 5 of worst matches in the reverse-matching analysis. Risk horizon parameters and the statistical profile of their related optimal portfolios are reported in the first two panels. Associated FTP parameters are reported in the bottom of the table. Mean and standard deviation of portfolios are in percentage.

	1	2	3	4	5
Risk horizon parameters					
λ	0.15	0.2	0.1	0.2	0.1
γ	0.6	0.6	0.6	0.6	0.6
Ω	0.01	0.01	0.01	0.05	0.05
Portfolio statistics					
Mean	23.9	28.0	19.5	32.1	21.9
Std	50.2	66.9	33.3	72.7	36.4
Skewness	2.6	2.6	2.6	1.6	1.6
Kurtosis	18.9	18.9	18.4	11.9	11.8
distance	0.19	0.19	0.18	0.12	0.11
FTP parameters					
k	-0.15	2.71	-23.27	-8.35	-102.06
ξ	0.63	-7.82	2.57	-0.09	1.05
σ	0.75	-4.95	-29.63	1.56	-33.87

Figure 1: Comparison of the moments of optimal portfolios generated by the risk horizon framework and the FTP utility approach. Blue circles (red triangles) show moments of the risk horizon portfolios (respectively the FTP optimal portfolios) generated in the reverse matching analysis. Black crosses highlight the moments of FTP optimal portfolios generated in the matching analysis.



5 Conclusion

The paper proposes a general framework for characterizing investors' behavior in portfolio selection. It takes the realistic view that, in practice, investors do not know their own utility function, and they have incomplete knowledge about asset returns distributions. The keystone of the approach is the introduction of a semi-parametric risk measure called risk horizon. This concept defines an investor's risk as the probability that his or her realized growth rate of wealth over a finite horizon will deviate from its expectation. Accordingly, riskier portfolios are characterized by a longer horizon for which this probability is negligible. Investors are allowed to differ in their assessment of the risk horizon of securities. Their attitudes towards risk are captured by parameters that reflect the accepted level of uncertainty around expectation, the trade-off between downside risk and upside potential and a level of confidence. Variation in this set of parameters is shown to generate important heterogeneity in risk profiles.

A generalized four-moment Chebyshev inequality borrowed from Mallows (1956) is used as a tool to characterize the risk horizon dependence on the first four moments. An alternative characterization of risk horizon in order to capture non-i.i.d. GARCH effects in financial returns is also discussed. In portfolio selection, investors seek to maximize their expected return for a given risk horizon. Comparison of optimal portfolios to allocations derived from traditional criteria, such as the Sharpe ratio, the growth-optimal strategy, an objective criterion derived from Bell's (1995) utility and from Stutzer's (2003) decay rate, provides evidence on the consistency of the risk horizon framework with traditional approaches. Moreover, the risk horizon approach is able to replicate allocations derived from a general utility framework, and produce relevant risk profiles that cannot be obtained in the parametric utility framework.

The application of the risk horizon framework is not limited to portfolio choices and has interesting implications for future research. The framework has the potential to contribute to the literature on multi-moment asset-pricing models, with interesting implications for the derivation of CAPM-type of equilibria (capital market line, security market line) under less restrictive assumptions than traditional models (see, for instance, Kraus and Litzenberger, 1976; Simaan, 1993;

Jurczenko and Maillet, 2001). Moreover, as the risk measure refers to a notion of time, it can be matched with the one on Treasury securities (i.e. maturity or duration), which is estimated with the information contained in multiple points on the yield curve. Information on interest rates ensures the consistency of the framework with a non-constant term structure. Such matching is not easily transposable to other risk measures as it is allowed by the nature of the risk horizon measure. Through an arbitrage argument, a link can be made between the risk and return of treasuries and a market portfolio, and market-wide attitudes towards risk can be extracted. In particular, such a link can be used to endogenously characterize the dynamics of expected market returns and its components. If market-wide attitudes towards risk are a leading indicator of market sentiment, the estimation of these dynamics with stock market and term structure data should enable us to test new predictors of future excess stock returns and financial stress.

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Appendix

A Robustness checks

In this appendix, we present the results of robustness checks for portfolio allocations under different risk horizons. In the main body of the chapter, Section 4.2 compares the optimal portfolio allocations derived from traditional criteria (i.e. the Sharpe ratio, Kelly's (1956) growth criterion, Bell's risk measure and Stutzer's decay rate criterion) with risk horizon allocations for investors with an horizon of 30 years. We consider here the case of a shorter and a longer risk horizon of respectively $H = 10$ and $H = 50$ years. The results are reported in Table for $H = 10$ and in Table 8 for $H = 50$.

Table 9 reports results when no short selling is allowed in optimal portfolio allocation. Optimal allocations among the different types of portfolios are now closer to each other, with no allocation to the assets *Manuf, HiTec, Shops, Utils* and *Others*. The least volatile assets, *NoDur* and *Telcm*, are privileged by defensive strategies such as the Sharpe Ratio, Bell and the decay rate strategy associated with a high risk aversion coefficient. *Durbl* and *Health* obtain significant weights in more aggressive strategies. Note that when short sales are not allowed, the decay rate strategy with a target return of 15% is not feasible, and hence not reported in Table 9. The conclusions made for the baseline case where short sales are allowed, presented in the main text, are robust under short selling constraints. The Sharpe Ratio optimization is associated with very low γ and values for λ and Ω are similar to the baseline case. We also found differences in γ in the two Bell strategies, which differ on the consideration for asymmetric and fat-tail risks. The trade-off coefficient γ is also more important for decay rate investors than in the Sharpe Ratio type of profile. The graduation of risk-taking is also reflected in λ, and to a lesser extent in Ω, when one reads from the decay rate portfolio associated with the highest risk aversion coefficient to the most aggressive strategy.

Table 7: Comparison of optimal portfolio allocations from traditional and risk horizon approaches with $H = 10$ years. Portfolios are composed of the 10 industry indices of Fama-French, for a period from 1927 to 2013. Each column is associated with the matching of a particular standard approach. The reported statistics correspond to the risk horizon allocation which minimizes the normalized Euclidean distance with respect to the portfolio weights of the standard approach (see equation (21)). Portfolio weights, mean and standard deviations are in percentage. Numbers in parentheses correspond to differences with respect to the portfolio allocation derived under the standard approach. For Stutzer's (2003) type of allocations, the decay rate and associated risk aversion coefficients of the standard portfolio allocation are reported in the last two rows.

	Sharpe	Bell 0.5	Bell 5	Decay 5%	Decay 10%	Decay 15%	Growth
Risk horizon parameters							
λ	0.16	0.13	0.15	0.14	0.20	0.21	0.51
γ	0.046	0.055	0.32	0.40	0.43	0.30	0.38
Ω	0.011	0.027	0.0088	0.017	0.0083	0.044	0.024
Portfolio weights							
NoDur	91 (4)	85 (3)	86 (-2)	100 (-5)	119 (-1)	160 (-2)	274 (-1)
Durbl	18 (0)	8 (0)	9 (0)	28 (0)	53 (1)	111 (5)	272 (17)
Manuf	-60 (-1)	-43 (-1)	-47 (0)	-80 (-2)	-121 (0)	-217 (1)	-494 (-2)
Enrgy	87 (0)	74 (0)	77 (0)	103 (1)	133 (1)	201 (2)	389 (2)
HiTec	17 (-1)	11 (-1)	8 (0)	19 (0)	38 (-2)	76 (-8)	174 (-22)
Telcm	40 (4)	44 (3)	48 (0)	43 (-2)	31 (0)	11 (-3)	-33 (-3)
Shops	-19 (-2)	-17 (-1)	-18 (0)	-25 (1)	-32 (2)	-47 (8)	-89 (19)
Hlth	32 (-3)	25 (-2)	27 (1)	38 (4)	52 (1)	89 (1)	199 (-1)
Utils	-18 (0)	-12 (0)	-15 (-1)	-29 (-1)	-44 (-2)	-82 (-7)	-190 (-20)
Other	-88 (0)	-74 (0)	-74 (0)	-97 (3)	-129 (2)	-201 (3)	-403 (12)
distance	0.014	0.012	0.006	0.014	0.006	0.013	0.016
Performance statistics							
Mean	14.6 (-0.1)	13.9 (-0.1)	13.9 (0)	15.2 (0.1)	17 (0)	21 (0.2)	31.9 (0.5)
Std	14.5 (-0.1)	14 (0)	14.1 (0)	15.2 (0)	18.2 (0.1)	28.4 (0.5)	61.7 (1.3)
Skewness	-0.12 (0.07)	-0.03 (0.07)	0.1 (-0.02)	-0.03 (-0.03)	-0.09 (0.01)	0.03 (0.03)	0.09 (0.04)
Kurtosis	2.54 (0.01)	2.8 (0.05)	2.98 (0.01)	2.55 (0.08)	2.68 (0.1)	3.33 (0.38)	3.47 (0.45)
Decay rate				0.19	0.045	0.0031	0
Risk aversion				5.53	2.80	1.31	1

Table 8: Comparison of optimal portfolio allocations from traditional and risk horizon approaches with $H = 50$ years. Portfolios are composed of the 10 industry indices of Fama-French, for a period from 1927 to 2013. Each column is associated with the matching of a particular standard approach. The reported statistics correspond to the risk horizon allocation which minimizes the normalized Euclidean distance with respect to the portfolio weights of the standard approach (see equation (21)). Portfolio weights, mean and standard deviations are in percentage. Numbers in parentheses correspond to differences with respect to the portfolio allocation derived under the standard approach. For Stutzer's (2003) type of allocations, the decay rate and associated risk aversion coefficients of the standard portfolio allocation are reported in the last two rows.

	Sharpe	Bell 0.5	Bell 5	Decay 5%	Decay 10%	Decay 15%	Growth
Risk horizon parameters							
λ	0.077	0.074	0.059	0.045	0.076	0.063	0.18
γ	0.097	0.00018	0.52	0.58	0.64	0.52	0.56
Ω	0.009	0.010	0.012	0.038	0.010	0.11	0.043
Portfolio weights							
NoDur	89 (1)	82 (0)	84 (-4)	97 (-8)	117 (-3)	156 (-5)	267 (-9)
Durbl	18 (0)	8 (0)	9 (0)	28 (0)	54 (2)	114 (8)	279 (23)
Manuf	-60 (-1)	-42 (0)	-47 (1)	-81 (-2)	-119 (1)	-214 (4)	-486 (6)
Enrgy	88 (0)	75 (0)	78 (1)	103 (2)	134 (2)	201 (3)	391 (4)
HiTec	17 (-1)	12 (0)	8 (0)	19 (0)	37 (-4)	71 (-13)	161 (-35)
Telcm	38 (2)	42 (0)	48 (-1)	42 (-3)	30 (-2)	9 (-5)	-38 (-8)
Shops	-18 (-1)	-16 (0)	-18 (1)	-23 (3)	-31 (2)	-44 (11)	-81 (27)
Hlth	34 (-1)	27 (0)	29 (3)	40 (7)	54 (3)	93 (4)	206 (6)
Utils	-19 (-1)	-13 (0)	-16 (-1)	-29 (-1)	-46 (-4)	-83 (-8)	-188 (-18)
Other	-88 (0)	-74 (0)	-74 (0)	-96 (4)	-129 (2)	-203 (2)	-410 (5)
distance	0.006	0.002	0.014	0.022	0.011	0.019	0.022
Performance statistics							
Mean	14.7 (-0.1)	14 (0)	13.9 (0.1)	15.2 (0.1)	17.1 (0.1)	21.1 (0.4)	32.2 (0.8)
Std	14.5 (0)	14 (0)	14.1 (0)	15.2 (0)	18.4 (0.3)	28.7 (0.9)	62.5 (2)
Skewness	-0.15 (0.04)	-0.08 (0.01)	0.07 (-0.04)	-0.06 (-0.06)	-0.08 (0.02)	0.01 (0.02)	0.07 (0.01)
Kurtosis	2.55 (0.02)	2.76 (0.01)	2.99 (0.02)	2.59 (0.11)	2.82 (0.24)	3.53 (0.58)	3.66 (0.64)
Decay rate				0.19	0.045	0.0031	0
Risk aversion				5.53	2.80	1.31	1

Table 9: Comparison of optimal portfolio allocations from traditional and risk horizon approaches with $H = 30$ years, and when short selling is not allowed. Portfolios are composed of the 10 industry indices of Fama-French, for a period from 1927 to 2013. Each column is associated with the matching of a particular standard approach. The reported statistics correspond to the risk horizon allocation which minimizes the normalized Euclidean distance with respect to the portfolio weights of the standard approach (see equation (21)). Portfolio weights, mean and standard deviations are in percentage. Numbers in parentheses correspond to differences with respect to the portfolio allocation derived under the standard approach. For Stutzer's (2003) type of allocations, the decay rate and associated risk aversion coefficients of the standard portfolio allocation are reported in the last two rows.

	Sharpe	Bell 0.5	Bell 5	Decay 5%	Decay 10%	Growth
Risk horizon parameters						
λ	0.075	0.12	0.069	0.066	0.072	0.099
γ	0.00012	0.11	0.52	0.55	0.51	0.10
Ω	0.050	0.0060	0.034	0.043	0.044	0.05
Portfolio weights						
NoDur	13 (-1)	16 (0)	12 (-1)	3 (-2)	0 (0)	0 (0)
Durbl	0 (0)	0 (0)	0 (0)	0 (0)	4 (0)	42 (0)
Manuf	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Enrgy	32 (-1)	30 (-1)	26 (1)	35 (2)	41 (0)	11 (0)
HiTec	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Telcm	24 (1)	30 (0)	34 (1)	18 (1)	0 (0)	0 (0)
Shops	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Hlth	30 (2)	24 (1)	29 (-1)	45 (-1)	56 (0)	47 (0)
Utils	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Other	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
distance	0.027	0.011	0.023	0.032	0.000	0.001
Performance statistics						
Mean	12.9 (0)	12.7 (0)	12.7 (0)	13.2 (0)	13.7 (0)	14.3 (0)
Std	16.5 (0)	16.3 (0)	16.3 (0)	17 (0)	18.3 (0)	22.9 (0)
Skewness	-0.42 (0.02)	-0.43 (0.01)	-0.37 (-0.01)	-0.34 (-0.01)	-0.29 (0)	0.02 (0)
Kurtosis	3.15 (-0.01)	3.2 (-0.01)	3.17 (0.02)	3.01 (0.03)	2.79 (0)	3.32 (0)
Decay rate				0.07	0.0063	0
Risk aversion				3.23	1.64	1

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