Multivariate structural time series models with dual cycles: implications for measurement of output gap and potential growth

by Philippe Moës

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Abstract

Structural time series models applied to the factor inputs of a production function often lead to small output gaps and consequently to erratic measures of potential growth. We introduce a dual cycle model which is an extension to the multivariate trend plus cycle model with phase shifts à la Rünstler. The dual cycle model is a combination of two types of models: the trend plus cycle model and the cyclical trend model, where the cycle appears in the growth rate of a variable. This property enables hysteresis to be taken into account. Hysteresis is likely to show up in unemployment but it can also affect the capital stock due to the existence of long investment cycles. In the proposed model, hysteresis may affect all the factor inputs of the production function and phase shifts are extended to the dual cycles. Genuine measures of potential growth can be computed that are hysteresis-free and less prone to volatility. A complementary measure of the output gap that takes hysteresis into account can be derived.

Keywords: Output gap, potential growth, hysteresis, structural time series models.
JEL-code : C32, E32.

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1. **Introduction**

Structural Time Series (STS) models often assume independence between cycle and trend components. This is generally the case when they are applied to the factor inputs of a production function, to deliver measures of the output gap and potential growth. The idea that the cycle could alter the trend is discarded from the outset even though this idea is closely linked to the presence of hysteresis in unemployment. The concept of hysteresis was introduced by Blanchard and Summers (1987). They suggested that a few years of high unemployment could trigger an increase in the mean level of unemployment. In this case, the trend in unemployment is not independent of the cycle.

The advantage of independence between cycle and trend components is the ability to assess the output gap and potential output in a straightforward way. Once trends and cycles are related, the task turns a bit more complicated as we will see. In the standard "trend plus cycle" decomposition of output, the trend and cycle are independent by assumption. This decomposition defines what is termed a "deviation cycle". A competing and very popular definition of the cycle is the cycle found in the growth rate of GDP, the "growth rate cycle", used for instance in the cyclical trend model of Harvey (1989) or to compute the Eurocoin indicator (Altissimo et al., 2001). As discussed in Harding and Pagan (2005), the deviation cycle and the growth rate cycle are closely related but in terms of output levels, the cycle estimated from growth rates is integrated and exerts a permanent impact on the output level, which makes it closer to a trend.

Further evidence on this ambivalent nature of growth rate cycles can be found in Proietti et al. (2007). In their hysteresis model, the growth in factor inputs is a function of the cycle present in capacity utilization (current or lagged). They show that this model, akin to the cyclical trend model, is equivalent to an extended trend plus cycle decomposition of input levels where the long run impact of the cycle remains in the trend, generating hysteresis. The transitory impact of the cycle is responsible for the deviation cycle. Growth rate cycles and deviation cycles are simultaneously present in the model. We will label such a model a "dual cycle" model.

From an economic point of view, the existence of a link between cycle and trend (i.e. hysteresis) looks quite reasonable. At least, it should not be discarded from the outset. In this paper, we will extend the trend plus cycle decomposition to allow for a permanent impact of the cycle on the trend. But in our dual cycle model, capacity utilization does not necessarily define the cycle. Capacity utilization is considered an interesting but imperfect proxy of the business cycle. It is simply added to the vector of factor inputs to deliver extra information. The dual cycle model developed in this paper is also an extension of the multivariate STS model of Harvey and Koopman (1997). Rünstler (2004) introduced phase shifts in the latter model to account for leads and lags between the different cycles. They are incorporated here as well, not only to account for phase shifts between...
cycles on different variables but also to allow for phase shifts between the dual paths taken by a
given cycle: transitory deviation or permanent impact on the level of the variable.

The dual cycle specification is very useful in a production function context. By allowing the transitory
impact of the cycle to be disentangled from its long run impact, output gap and potential growth
measures can be derived that take the difference into account. Potential growth figures that are free
from cyclical influence can be computed, as the cycles present in the trend are explicitly measured
and can be removed, which is not the case for standard measures of potential growth. Conversely,
output gap measures that account for the long run impact of the cycle can be derived.

The rest of the paper is organised as follows. The next section describes the dual cycle model and
its adaptation to the production function approach to the output gap. Section 3 presents the
estimation results for Belgium. In section 4, estimates of the output gap and potential growth are
discussed. Section 5 concludes.

2. THE DUAL CYCLE MODEL

2.1 Specification of the univariate dual cycle model

We assume that the variable $z_t$ is the sum of three components: a trend $\mu$, a deviation cycle $\psi$ and
an irregular component $\varepsilon$.

$$z_t = \mu_t + \psi_t + \varepsilon_t$$

with

$$\varepsilon_t \sim NID(0, \sigma^2_\varepsilon)$$

(1)

The deviation cycle has the standard trigonometric form

$$\begin{pmatrix} \psi_{t+1} \\ \psi^*_t \end{pmatrix} = \begin{pmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{pmatrix} \begin{pmatrix} \psi_t \\ \psi^*_t \end{pmatrix} + \begin{pmatrix} \kappa_{t+1} \\ \kappa^*_t \end{pmatrix} \sim NID(0, \Sigma_\kappa)$$

(2)

with damping factor $0 < \rho < 1$, frequency $0 < \lambda < \pi$ and $\Sigma_\kappa$ diagonal. The cycle periodicity is equal
to $2\pi / \lambda$.

The trend is given by the two equations

$$\begin{align*}
\mu_{t+1} &= \mu_t + \beta_t + m + \alpha_1 \psi_t + \alpha_2 \psi^*_t \\
\beta_{t+1} &= 0 \beta_t + \xi_t
\end{align*}$$

$$\xi_t \sim NID(0, \sigma^2_\xi)$$

(3)
m is the drift constant. The "slope" $\beta$ follows the damped slope specification of Proietti et al. (2007). This specification was the preferred one in a model for Belgian factor inputs (Moës, 2006). $\theta$ is a damping factor with $0 < \theta < 1$. When $\theta$ is close to zero, the genuine trend -the part of the trend not influenced by the cycle- is close to a random walk with drift. If $\theta$ is close to 1, the genuine trend comes close to an integrated random walk (IRW).

The trend will be influenced by the cycle if the trend cycle given by $\alpha_1 \psi + \alpha_2 \psi^*$ is different from zero. This will introduce hysteresis into the model. The $\psi^*$ cycle leads the $\psi$ cycle and the weights allow any lead or lag between deviation and trend cycles. If $\psi^*$ was not present in (3), no phase shift would occur between the deviation and trend cycles.

If no cycle is present in the trend, the model is the standard trend plus cycle model of Harvey (1989) where the cycle is a pure deviation cycle. If the cycle is not present in (1), the model is a cyclical trend model where $\psi$ is only a trend cycle ($\psi^*$ is unnecessary in this case).

When $z_t$ is the GDP, $\psi_t$ corresponds to the standard measure of the output gap and the potential growth rate is given by $\beta_t + m + \alpha_1 \psi_t + \alpha_2 \psi^*_t$. Unfortunately, if a significant cycle is present in the trend, potential growth will be cyclical. In this case, a measure of the potential growth rate free from cyclical impact is given by $\beta_t + m$. We name it "genuine" potential growth. In accordance with this restricted definition of potential growth, it is possible to compute a cyclical component of output that incorporates the cumulated impact of the cycle on the trend. We call it the "full cycle" although the term "cycle" is somewhat improper. It is given by the sum of the output gap and the integral of the trend cycle:

$$\psi_t + \int_0^t \alpha_1 \psi_\tau + \alpha_2 \psi^*_\tau \, d\tau.$$  

Properly centered, this definition will give an extended measure of the output gap that is complementary to the genuine measure of potential growth. It allows to combine in a single measure all the cyclical influences. But it should be used with caution as it is non stationary and depends on the starting point used for the trend cycle. Moreover, the share of the trend cycle in the full cycle clearly increases the longer the time period considered.

After experimenting with the dual cycle model, a constraint was introduced on the phase shift coefficients $\alpha_i$ ($i=1,2$). Without constraint, the trend may turn more volatile than the variable. The (integrated) trend cycle and the deviation cycle may compensate each other resulting in huge deviation cycles accompanied by very counter-cyclical trends. We stick to the common prior that the trend should be smoother than the variable. The integrated trend cycle should not be counter-
cyclical. This requires a lead lower than 25 p.c. of the cycle periodicity (in absolute value). For the trend cycle before integration, a lead lower than 50 p.c. of the cycle periodicity is necessary because the integration induces a lag.\(^2\) To conclude on phase shifts, they were introduced on the trend cycle but they could be introduced on the deviation cycle instead, with the trend cycle used as anchor point. Phase shift restrictions should be modified accordingly.

Now that the basic concepts of the dual cycle model are defined, it is easy to move to the multivariate case.

### 2.2 The multivariate dual cycle model

The vector \(z\) is again the sum of three components: a trend \(\mu\), a deviation cycle \(C\) and an irregular component \(\varepsilon\) where the components are \((N \times 1)\) vectors:

\[
z_t = \mu_t + C_t + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim \text{NID}(0, \Sigma_\varepsilon) \tag{4}
\]

\(C\) is defined in two steps. First, we define \(N\) independent or "structural" cycles \(\psi\) with the standard trigonometric form

\[
\begin{pmatrix}
\psi_{t+1} \\
\psi_{t+1}^*
\end{pmatrix} = \rho \begin{pmatrix}
\cos \lambda & \sin \lambda \\
-\sin \lambda & \cos \lambda
\end{pmatrix} \otimes I_N \begin{pmatrix}
\psi_t \\
\psi_t^*
\end{pmatrix} + \begin{pmatrix}
\kappa_t \\
\kappa_t^*
\end{pmatrix} \sim \text{NID}(0, I_2 \otimes \Sigma_\varepsilon) \tag{5}
\]

with damping factor \(0 < \rho < 1\), frequency \(0 < \lambda < \pi\) and \(\Sigma_\varepsilon\) diagonal to preserve independence between structural cycles. All the structural cycles share a common frequency parameter \(\lambda\) and a common damping factor \(\rho\).

The second step creates the interdependence between the \(C\) cycles:

\[
C_{t+1} = F\psi_t + F^*\psi_t^* \tag{6}
\]

---

\(^2\) Phase shifts are not well defined for the integrated cycle. The restriction holds for deterministic (non stochastic) cycles. Truly deterministic cycles would raise another problem: the cycle and its integral would be one and the same thing, preventing identification between deviation and trend cycles. Fortunately, deterministic cycles seldom appear in multivariate models.
With $F^* = 0$, (5) is equivalent to the "similar cycles" specification of Harvey and Koopman (1997), i.e. $F$ is a Choleski matrix that mixes the independent structural cycles $\psi$ to produce similar C cycles that may be correlated. To allow for phase shifts, it is necessary to give some weight to the cycles $\psi^*$, with an $F^*$ matrix different from zero (see Rünstler, 2004). In this case, (7) is equivalent to the "Choleski decomposition" of Rünstler.

As usual with Choleski decompositions, a different ordering of the variables will produce different structural cycles but the C cycles remain unaffected. "Structural" points to the independence existing between cycles in (5), not to an economic interpretation. In estimation, the number of significant structural cycles may fall below $N$, the number of variables. This would introduce strong commonalities between cycles. In these commonalities and the joint estimation of $\rho$ and $\lambda$ lies the econometric benefit of the multivariate approach. We do not impose a priori restrictions on the true number of structural cycles governing the system. Neither do we define some variables as necessarily leading or lagging. All variables are treated symmetrically and phase shifts will emerge freely from the estimation of the $F^*$ matrix.

The trends are given by the two equations

$$
\begin{align*}
\mu_{t+1} &= \mu_t + \beta_t + m + G\psi_t + G^*\psi^*_t \\
\beta_{t+1} &= \theta \beta_t + \xi_t \\
\end{align*}
$$

m is the vector of drift constants and $\theta$ is a diagonal matrix of damping factors with $0 < \theta_{ii} < 1$. The rank of the matrix $\Sigma_{\xi}$ may be lower than $N$, introducing commonalities between the slopes of the different variables. The trend cycles are now equal to $G\psi + G^*\psi^*$. Their link with the structural cycles replicates the link imposed for the deviation cycles in equation (6). $G$ and $G^*$ are lower triangular but further restrictions are introduced.

**dual cycle restriction:** the deviation and trend cycles on a given variable are identical up to a phase shift and a scale factor.

This restriction comes directly from the univariate model. If a trend cycle and a deviation cycle are present at the same time on a given variable, the deviation cycle should be the one modifying
permanently the trend level. However, deviation cycles are not necessarily independent between variables and this property will be transmitted to the trend cycles.

To understand the implications of the dual cycle restriction, it is easier to use a trigonometric form for \( F \) and \( F^* \). If

\[
F_{ij} = \begin{cases} 
0 & \text{for } j > i \\
1 & \text{for } j = i \\
\delta_i \cos(p_i \lambda) & \text{for } j < i 
\end{cases}
\]

and

\[
F^*_{ij} = \begin{cases} 
0 & \text{for } j \geq i \\
\delta_i \sin(p_i \lambda) & \text{for } j < i 
\end{cases}
\]

then the restriction implies

\[
G_{ij} = \begin{cases} 
0 & \text{for } j > i \\
\delta_i \cos(p_i \lambda) & \text{for } j = i \\
\delta_i \delta_j \cos((p_j + p_i) \lambda) & \text{for } j < i 
\end{cases}
\]

and

\[
G^*_{ij} = \begin{cases} 
0 & \text{for } j > i \\
\delta_i \sin(p_i \lambda) & \text{for } j = i \\
\delta_i \delta_j \sin((p_j + p_i) \lambda) & \text{for } j < i 
\end{cases}
\]

Compared to \( F \) and \( F^* \), the loadings of all the structural cycles \( j \) present in variable \( i \) are affected by a \( \delta_i \) scale factor with \( \delta_i \geq 0 \). The phase shift present on the structural cycles \( j \) is also increased by \( p_i \) periods (\( p_i \) times the one-period rotation \( \lambda \) from (5)).

In terms of matrices, one can verify that the dual cycle restriction is equivalent to imposing

\[
\begin{pmatrix} G \\ G^* \end{pmatrix} = \begin{pmatrix} G_d & G^*_d \\ G^*_d & G_d \end{pmatrix} \begin{pmatrix} F \\ F^* \end{pmatrix}
\]

where \( G_d \) is a diagonal matrix with \( \delta_i \cos(p_i \lambda) \) (\( i = 1, \ldots, N \)) on the main diagonal and \( G^*_d \) is another diagonal matrix with \( \delta_i \sin(p_i \lambda) \) (\( i = 1, \ldots, N \)) on the main diagonal. The pro-cyclicality of the integrated trend cycle in relation to the deviation cycle is now easily implemented: on variable \( i \), 0 \( \leq p_i \leq \pi / \lambda \Leftrightarrow 0 \leq p_i \lambda \leq \pi \). The \( p_i \lambda \) angles used in \( G_d \) and \( G^*_d \) must be positive and lower than \( \pi \). The state-space form of our multivariate dual cycle model is given in appendix 1.

With \( \delta_i \) and \( p_i \) (\( i = 1, \ldots, N \)), the multivariate dual cycle model has 2N more parameters than a pure multivariate trend plus cycle model. This is also the case when it is compared to a pure cyclical trend model. In (9), the \( G \) and \( G^* \) matrices are derived from the \( F \) and \( F^* \) matrices. A symmetric model exists where the \( G \) and \( G^* \) matrices are defined as in equation (7) and the deviation cycles are rescaled and shifted versions of the trend cycles. This model would yield exactly the same
results. So there is no loss of generality involved in generating the trend cycles from the deviation cycles. The cycles are simultaneously defined from their impact as deviation and trend cycles.

The concepts of "genuine potential growth" and "full cycle" introduced for the univariate model still apply per variable (with the trend cycles taken from (8)). For multivariate models, Rünstler (2004) developed the concept of "association". The association is the correlation existing between deviation cycles after removal of the phase shifts. With dual cycles, associations can be computed between trend cycles as well. Associations could even be computed between a combination of deviation and trend cycles. But our dual cycle restriction implies that a perfect association, i.e. an association of 100 p.c., exists between the deviation cycle and the trend cycle of the same variable. As a consequence, the measures of association computed from deviation cycles and the ones computed from trend cycles (or a combination of both) will be the same.

2.3 Application to the production function approach to the output gap

The multivariate model defined by equations (4) to (9) is applied to the factor inputs of a production function (or components thereof). It is assumed that production follows a standard Cobb-Douglas function with constant returns:

\[ Y_t = TFP_t \left( L_t \cdot \text{HOURS}_t \right)^\alpha K_t^{1-\alpha} \]

with \[ L_t = \text{PWA}_t \cdot \text{PART}_t \left( 1 - \text{UR}_t \right) \]

where PWA is the working age population, PART is the participation rate and UR is the unemployment rate. The labour share \( \alpha \) is set at .65, the standard used by the European Commission. TFP, the total factor productivity, is computed as a residual. If a variable is missing in the production function, this will affect TFP.

The multivariate dual cycle model is used to compute the trends and cycles of the different variables:

---

3 The proof of the equivalence is available from the author upon request.
4 This is only true in absolute value. By normalization, the phase shifts are always within bounds equal to \( \pm 25 \text{ p.c.} \) of the cycle periodicity. Slightly above that level, the cyclicality will be reversed to bring the phase shift within bounds and the association will be reversed. If no phase shift was allowed between the deviation cycles and the corresponding trend cycles, the association sign would never change.
5 Cross border workers are ignored in this formula. Implicitly, it is assumed that the cyclical component for domestic, national and cross border workers is -proportionally- the same. In practice, the violation of this assumption would not affect the measure of the output gap.
6 The true labour share changes over time. Once this is properly taken into account in the TFP computation, results hardly differ.
\[
\begin{bmatrix}
\ln \text{HOURS}_t \\
\ln \text{PART}_t \\
\ln \text{TFP}_t \\
\ln(1 - \text{UR}_t) \\
\ln K_t
\end{bmatrix}
\equiv
\begin{bmatrix}
\text{LHOURS}_t \\
\text{LPART}_t \\
\text{LTFP}_t \\
\text{URMIN}_t \\
\text{LKR}_t
\end{bmatrix}
\]

\(\ln(PWA)\) is not included in the vector. It is considered acyclical and consisting of a single trend component. But contrary to common practice in the production function approach to the output gap, the capital stock is kept in the set of variables. With the introduction of cycles into the trends, it should be possible to account for the investment cycle that permeates through the growth rate of the capital stock (see chart 1 for Belgian data) and to compute potential growth measures that are free from this cyclical element.

If hysteresis is present, a trend cycle is expected in \(\text{URMIN}\) as well. In Moës (2006), almost no deviation cycle could be identified on unemployment in an application of the standard STS model to the Belgian production function. However, the author mentions the likely presence of a "long" cycle in the trend slope of \(\text{URMIN}\).

The output gap is the weighted sum of the deviation cycles:

\[
\begin{align*}
\text{C}_t^{\text{LTFP}} + \alpha (\text{C}_t^{\text{LPART}} + \text{C}_t^{\text{URMIN}} + \text{C}_t^{\text{LHOURS}}) + (1 - \alpha) \text{C}_t^{\text{LKR}} &= (\alpha \ 1 \ \alpha \ 1 - \alpha) \text{C}_t. \\
\end{align*}
\]

Usually, it is assumed that \(\text{LKR}\) is fixed and has no cycle component. This can be appropriate in terms of levels but is less likely in terms of growth rates. For the sake of generality, we introduce the deviation cycle of \(\text{LKR}\) in the definition of the output gap.

Potential growth is computed as the growth rate of potential output (the output level minus the output gap). We will also make abstraction from irregular components in its computation. Potential growth is then given by:

\[
(\alpha \ 1 \ \alpha \ 1 - \alpha) \left(\beta_i + m + G\psi_i + G^*\psi_i^*\right) + \alpha \Delta \ln PWA_i. \]

Potential growth can be cyclical if hysteresis (trend cycles) are present in one or more variables. For instance, the trend cycle present in the capital stock may affect the measurement of potential growth although we know that booms and busts in investment do not last forever. It is interesting to compute a measure of potential growth free from cyclicity. To do so, analysts usually average their potential growth measures over 5 or even 10 years. Here, we will be able to retrieve an annual measure of "genuine potential growth". It is equal to
\[(\alpha \ 1 \ 1 - \alpha) (\beta_t + m) + \alpha \Delta \ln PWA_t.\] (12)

The "full cycle" complementary to genuine potential growth will not only include the output gap but also the historical and cumulated impact of the trend cycles. It is given by

\[(\alpha \ 1 \ 1 - \alpha) \left( C_t + \int_0^t G \psi_\tau + G^* \psi_\tau^* \mathrm{d} \tau \right).\] (13)

Capacity utilization (LDUC, taken in logarithm) will be added to the vector \(z_t\) to improve the measurement of cycles. But contrary to what is done in Jaeger and Parkinson (1990) or Proietti et al. (2007), LDUC is treated as any other variable meaning that a stochastic trend may appear in LDUC.\(^7\) Moreover its cycle is not used as a reference cycle although close connections may appear between cycles. One of the conclusions in Moës (2006) was that (deviation) cycles present in the data could not be subsumed in a single measure such as capacity utilization.

**Chart 1: Capital stock and capacity utilization rate**
(respectively, year-on-year growth rate and four-quarter average after linear detrending)

---

\(^7\) In Belgium, capacity utilization may be trending. Repeated Chow breakpoint tests suggest that a break occurred in the level of capacity utilization around 1994.
Another reason to introduce this variable is the link that is usually made between capacity utilization and the investment rate (see chart 1). Hence its introduction could contribute to the identification of the trend cycles. Capacity utilization could be directly introduced into the production function with a proper redefinition of TFP. But it is only measured in the manufacturing sector and it is not clear whether it refers to the level of the capital stock or to the output level. We keep it out of the production function and as a consequence out of the computations of the output gap and potential growth.

3. Estimation results

The model is estimated on Belgian data over the period 1983Q1-2005Q4. We make use of the SSFPack algorithms for Ox 3.0 (Doornik, 1998; Koopman et al., 1998).\(^8\) To initialize the Kalman filter, cycle and slope components are set to zero; trends are set to the value of the first observation. The initial variance-covariance matrix of the state vector is the unconditional variance for stationary variables. For non stationary variables, it is set to a very large value (diffused prior).

Regarding the data used in this paper, the unemployment rate is the harmonized rate of unemployment from Eurostat. Hours are based on published data from the National Accounts Institute over 1995-2005 and on NBB sources before. Quarterly figures for the capital stock are derived from annual data (available over 1980-2003) and extended with the perpetual inventory method. All data are seasonally and working days adjusted and at 2000 price (where applicable).

The results are given in the first subsection. The cycle periodicity is estimated at 3.2 years. Evidence of trend cycles is slim at that frequency. But the trend components suggest the introduction of another frequency into the model to account for low frequency cycles. This is done in the second subsection. Long cycles appear that mainly take the form of trend cycles.

3.1 One-frequency dual cycle model.

The results for the one-frequency model can be found in the first column of table 1. As usual, the goodness of fit statistics are based on the one-step ahead prediction errors. s.e.r. (T) is the standard error of the prediction error at the end of the sample. The Normality test statistic is the Doornik - Hansen statistic (1994), distributed as chi-square with two degrees of freedom. The heteroscedasticity test is the non-parametric test found in Koopman et al. (1995). It is distributed as $F(30,30)$. The Ljung-Box statistic is based on the 16 first autocorrelations.\(^9\)

\(^8\) More information is available on the website http://www.ssfpack.com/ where the package can be downloaded.

\(^9\) We computed "auxiliary" residuals - the smoothed estimates of the shocks - to detect outliers. Applying a benchmark of 3.5 times the standard deviation, the following outliers were removed: LDUC(1999q1), LHOURS(1994q4,1997q2), LPART(1987q3,1988q1), LTFP(1986q4,1992q4,1997q2,2001q3) and LKR(1983q4).
### Table 1: Estimation results

<table>
<thead>
<tr>
<th></th>
<th>one-frequency</th>
<th>two-frequency</th>
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</thead>
<tbody>
<tr>
<td>first cycle</td>
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<td>periodicity</td>
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<td>$\rho$ (damping factor)</td>
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<tr>
<td>second cycle</td>
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<tr>
<td>$\lambda$ (frequency)</td>
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<tr>
<td>periodicity</td>
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<td>168</td>
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<tr>
<td>LDUC</td>
<td>0.0087</td>
<td>0.0081</td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.0023</td>
<td>0.0022</td>
</tr>
<tr>
<td>LPART</td>
<td>0.0031</td>
<td>0.0030</td>
</tr>
<tr>
<td>LTFP</td>
<td>0.0042</td>
<td>0.0043</td>
</tr>
<tr>
<td>URMIN</td>
<td>0.0033</td>
<td>0.0030</td>
</tr>
<tr>
<td>LKR</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>Normality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDUC</td>
<td>0.36</td>
<td>1.45</td>
</tr>
<tr>
<td>LHOURS</td>
<td>4.06</td>
<td>1.26</td>
</tr>
<tr>
<td>LPART</td>
<td>0.90</td>
<td>1.94</td>
</tr>
<tr>
<td>LTFP</td>
<td>6.62</td>
<td>3.25</td>
</tr>
<tr>
<td>URMIN</td>
<td>2.11</td>
<td>6.11</td>
</tr>
<tr>
<td>LKR</td>
<td>4.19</td>
<td>0.70</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDUC</td>
<td>1.09</td>
<td>0.98</td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.92</td>
<td>0.80</td>
</tr>
<tr>
<td>LPART</td>
<td>0.73</td>
<td>0.99</td>
</tr>
<tr>
<td>LTFP</td>
<td>1.56</td>
<td>1.36</td>
</tr>
<tr>
<td>URMIN</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>LKR</td>
<td>0.31</td>
<td>0.77</td>
</tr>
<tr>
<td>Ljung-Box Q(16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LDUC</td>
<td>19.15</td>
<td>23.92</td>
</tr>
<tr>
<td>LHOURS</td>
<td>15.30</td>
<td>12.92</td>
</tr>
<tr>
<td>LPART</td>
<td>22.79</td>
<td>16.01</td>
</tr>
<tr>
<td>LTFP</td>
<td>20.86</td>
<td>19.34</td>
</tr>
<tr>
<td>URMIN</td>
<td>12.91</td>
<td>18.39</td>
</tr>
<tr>
<td>LKR</td>
<td>175.57</td>
<td>23.48</td>
</tr>
</tbody>
</table>
Normality is not a problem but for LTFP where skewness is present in the residuals. There is no evidence of heteroscedasticity after taking into account the higher volatility in LHOURS after 1994 and in URMIN after 1998. In both cases, new data sources are responsible for the change (LPART is also affected as it is based on URMIN). URMIN excepted, there is evidence of autocorrelation among the residuals. The problem is particularly acute on LKR where the model appears to be strongly misspecified.

The length of the cycle is estimated at 3.2 years, not far from the 3.5 years periodicity found in Moës (2006). The deviation cycles and the cumulated impacts of trend cycles are presented on chart 2:

Chart 2: Smoothed cycles (one-frequency model)
The trend cycle is small and dominated by the deviation cycle in LDUC, LHOURS and LTFP. No hysteresis is noticeable in these three variables. The cycle present in LHOURS and LTFP will not affect the potential output ahead. This is not the case for the participation rate. The cumulated trend cycle had as much impact as the transitory deviation cycle over the last 20 years. The $\delta_i$ scale factor of the trend cycle is equal to 0.34, which means that the trend cycle amplitude amounts to 34 p.c. of the deviation cycle amplitude. But this impact is permanent and creates hysteresis. The deviation cycle is almost absent from URMIN and LKR. In the latter case, this is in line with the usual exclusion of the capital stock from the output gap computations. But the cumulated trend cycles are not negligible. Hysteresis is present and the cycles modify the "structural" level of unemployment or the capital stock. However this impact must not be overrated. Over the last 20 years, long-lasting movements of about 4 p.c. are common in both time series (see appendix 2). They are hardly explained by the cumulated trend cycles.

To assess the number of significant structural cycles that hide behind the cycles of the six variables, we may decompose the variance of the latter cycles between the six structural cycles $\psi$ (plus the corresponding cycles $\psi'$). With the dual cycle restriction, the result is the same for the deviation and trend cycles. We get the results in table 2. The table is lower triangular by construction, like the $F$ and $F^*$ matrices in (7). Four structural cycles matter to generate the cycles on the six variables (the fourth one only plays a limited role). They create links between the six deviation/trend cycles. For example, the first structural cycle, which is also the cycle on LDUC, is strongly present in LTFP. But LDUC is not a reference for the other variables, contrary to what is often assumed in the literature.

Table 2: Structural cycle shares in cycle variance

<table>
<thead>
<tr>
<th></th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
<th>$\psi_3$</th>
<th>$\psi_4$</th>
<th>$\psi_5$</th>
<th>$\psi_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.354</td>
<td>0.646</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>LPART</td>
<td>0.120</td>
<td>0.049</td>
<td>0.832</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>LTFP</td>
<td>0.752</td>
<td>0.065</td>
<td>0.108</td>
<td>0.075</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>URMIN</td>
<td>0.174</td>
<td>0.263</td>
<td>0.444</td>
<td>0.119</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>LKR</td>
<td>0.138</td>
<td>0.427</td>
<td>0.213</td>
<td>0.222</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3 gives the associations between the significant deviation cycles. Associations are the correlations measured between cycles after adjusting for phase shifts. Strong associations are always positive. LTFP and LDUC cycles are strongly related as expected. In part, the link could be the result of a measurement error in LTFP since LDUC is absent from the production function when LTFP is computed. LHOURS is positively associated with the two previous cycles. On the contrary, negative (but low) levels of association are found with LPART.

---

10 Computations follow Rünstler (2004).
Table 3: Associations between deviation cycles

<table>
<thead>
<tr>
<th></th>
<th>LDUC</th>
<th>LHOURS</th>
<th>LPART</th>
<th>LTFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>1</td>
<td>0.595</td>
<td>-0.346</td>
<td>0.867</td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.595</td>
<td>1</td>
<td>-0.367</td>
<td>0.383</td>
</tr>
<tr>
<td>LPART</td>
<td>-0.346</td>
<td>-0.367</td>
<td>1</td>
<td>-0.447</td>
</tr>
<tr>
<td>LTFP</td>
<td>0.867</td>
<td>0.383</td>
<td>-0.447</td>
<td>1</td>
</tr>
</tbody>
</table>

Associations with the cumulated trend cycles cannot be computed but it is clear from chart 2 that a kind of negative association exists between the cumulated trend cycle in URMIN and the deviation cycle in LPART. Again LPART is countercyclical. But one must take into account that the normalization used to compute phase shifts and levels of associationfavours low phase shifts. The phase shifts will always be lower than 3.2 quarters in absolute value (see footnote 4). If the phase shifts involving LPART are artificially increased, the association with the other variables will be reversed and LPART will turn procyclical. One should also mention that the current results do not explain the link observed between capacity utilization and investment since further computations reveal an association of only 0.371 between the deviation cycle in LDUC and the trend cycle in LKR.

The phase shifts are given in table 4. LDUC is lagging by 0.7 to 2.8 quarters. Part of the lag may be attributed to a publication lag. LDUC is published in January, April, July and October, making reference to the capacity in the month before. As a consequence, a delay of about 2 months or 0.7 quarter will result. LHOURS is leading. LTFP and LPART (before a possible adjustment) are somewhere between LDUC and LHOURS, depending on the reference variable. Additivity (transitivity) of leads and lags does not hold, reflecting the presence of several structural cycles.

Table 4: Phase shifts between deviation cycles (lag of i with respect to j) in quarters

<table>
<thead>
<tr>
<th></th>
<th>LDUC</th>
<th>LHOURS</th>
<th>LPART</th>
<th>LTFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>0</td>
<td>2.8</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>LHOURS</td>
<td>-2.8</td>
<td>0</td>
<td>-1.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>LPART</td>
<td>-0.7</td>
<td>1.5</td>
<td>0</td>
<td>1.7</td>
</tr>
<tr>
<td>LTFP</td>
<td>-1.0</td>
<td>2.5</td>
<td>-1.7</td>
<td>0</td>
</tr>
</tbody>
</table>

Turning to slopes, the $\theta$ damping coefficients in table 1 are quite lower than 1 for 5 out of 6 variables. LKR is the exception: with a $\theta$ of 0.99, the damped slope specification comes close to an IRW trend. The growth of the capital stock is almost non stationary. On the contrary, with $\theta$ equal to

---

11 Phase shifts can be computed between deviation and trend cycles. But they cannot be computed with cumulated trend cycles. In the limit case of deterministic cycles, assuming that the parameters of the model are known, phase shifts could be computed. The integration would induce an additional lag (lead) between cycles if cyclicality is preserved (reversed).
zero, the genuine trend in LTFP is a random walk, a result also found by Proietti et al. (2007) for the euro area.

A Choleski decomposition can be applied to the slope shocks, similar to the one used to derive the structural cycles and their shares. Only three structural slope shocks matter in the system. As a consequence, close connections exist between genuine trends although different $\theta$ coefficients may blur the picture. In Table 5, high positive correlations appear between the slope shocks of LDUC, LHOOURS and URMIN. There is also a remarkable correlation between the shocks in LKR and LTFP. After correction for the cycle, the shocks to investment and to total factor productivity appear to be one and the same thing. The shocks in LPART are nearly independent. A clear (although imperfect) connection can be made between the three sets of variables and the existence of three significant structural shocks.

Table 5: Correlations of slope shocks

<table>
<thead>
<tr>
<th></th>
<th>LDUC</th>
<th>LHOOURS</th>
<th>LPART</th>
<th>LTFP</th>
<th>URMIN</th>
<th>LKR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>1</td>
<td>0.969</td>
<td>0.214</td>
<td>0.359</td>
<td>0.866</td>
<td>0.262</td>
</tr>
<tr>
<td>LHOOURS</td>
<td>0.969</td>
<td>1</td>
<td>0.059</td>
<td>0.125</td>
<td>0.891</td>
<td>0.043</td>
</tr>
<tr>
<td>LPART</td>
<td>0.214</td>
<td>0.059</td>
<td>1</td>
<td>0.452</td>
<td>-0.288</td>
<td>0.238</td>
</tr>
<tr>
<td>LTFP</td>
<td>0.359</td>
<td>0.125</td>
<td>0.452</td>
<td>1</td>
<td>-0.288</td>
<td>0.971</td>
</tr>
<tr>
<td>URMIN</td>
<td>0.866</td>
<td>0.891</td>
<td>-0.288</td>
<td>0.225</td>
<td>1</td>
<td>0.248</td>
</tr>
<tr>
<td>LKR</td>
<td>0.262</td>
<td>0.043</td>
<td>0.238</td>
<td>0.971</td>
<td>0.248</td>
<td>1</td>
</tr>
</tbody>
</table>

From the trend components, we can derive the potential growth rates per variable. They are presented on chart 3 (for completeness, we apply the concept to LDUC as well). This is not the contribution to the output (growth) potential but the first difference of the trend component present in the variables: $\beta_t + m_t + G\psi_t + \Gamma^*\psi^*_t$. On the same chart, the genuine potential growth is given. It is equal to $\beta_t + m_t$, free from cyclical variations. This does not really matter for LDUC, LHOOURS or LTFP as trend cycles, the difference between the two time series, are small.

It makes a difference for LPART where hysteresis was found. When its temporary impact is removed from potential growth, a much smoother estimate appears. In this case, the introduction of trend cycles into the model makes a difference. Hysteresis was also detected in URMIN and LKR. The variability of genuine potential growth is somewhat lower but hysteresis does not account for the overall movements in the two time series.

12 The trends are given in appendix 2.
Genuine potential growth rates of LDUC, LHOURS and URMIN share a similar pattern due to the high correlation between their slope shocks. LPART is independent as expected. A comparison of the genuine potential growth of LKR with the trend of LTFP in appendix 2 shows the close link existing between LTFP and LKR. The two time series are almost identical. In terms of genuine potential, a 0.09 percentage point increase of the year-on-year capital growth rate corresponds to a 0.14 p.c. increase in TFP.\textsuperscript{13} As documented in appendix 3, the link is already noticeable before correcting for the cycle (and the irregular components). It is not clear where the link comes from. In the neo-classical growth model, an increase in TFP would imply a long-lasting increase in the growth rate of the capital stock but of much lower amplitude. With endogenous growth, the levels of

\textsuperscript{13} The two percentages are the standard errors of the slope shocks. If we assume a perfect correlation between the two shocks, standard errors can be used to estimate their relative impact. A cointegration analysis between DLKR and LTFP gives a similar long run coefficient.
TFP and capital would be connected and in models with capital-embodied technical change, the TFP growth depends on the growth rate of the capital stock.

But the permanent character of some movements in genuine potential growth rates (and trends) could be called into question. What is striking on chart 3 is the presence of long cycles in all the variables but LPART, with peaks in 1988 (1991 for LKR) and around 1997-2000, troughs around 1992-1993 (later again for LKR) and 2001-2003. Making abstraction of the deterministic drifts, long cycles could also be present in the trends of several variables in appendix 2: LDUC, LTFP and URMIN. The long cycles -if present- could modify our assessment of the deviation and trend cycles and of the genuine potential growth rates. In the next section, additional cycles are introduced with another periodicity to see if long cycles can explain some movements in the data.

### 3.2 Two-frequency dual cycle model

With two cycle frequencies, two vectors of deviation cycles are present in the model:

\[
\begin{align*}
    z_t &= \mu_t + C_1^i + C_2^i + \varepsilon_t \\
    \text{with} \\
    C_{i+1}^i &= F_i^i \psi_i^i + F_i^{i*} \psi_i^{i*} \quad \text{for } i = 1, 2 \\
\end{align*}
\]  
(4’)

The structural cycles \( \psi_i^i \) and \( \psi_i^{i*} \) are still given by equation (5) but their coefficients are specific to the frequency \( i \).

In a similar way, the trends include two vectors of trend cycles:

\[
\begin{align*}
    \mu_{t+1} &= \mu_t + \beta_t + m + \left( G_1^i \psi_t^i + G_1^{i*} \psi_t^{i*} \right) + \left( G_2^i \psi_t^2 + G_2^{i*} \psi_t^{2*} \right) \\
    \beta_{t+1} &= \beta_t + \xi_t \\
\end{align*}
\]  
(8’)

The \( F_i^i, F_i^{i*}, G_i^i \) and \( G_i^{i*} \) matrices are given by equations (7) and (9). The coefficients involved are specific to each frequency.

Estimation results are shown in the right part of table 1. The second cycle periodicity amounts to 11.3 years. The likelihood improves markedly compared to the one-frequency model, telling that the second cycle is highly significant. At 3.1 years, the short cycle is hardly affected. The residuals from LTFP are normal but those on URMIN are borderline. There is still no evidence of heteroscedasticity and autocorrelation improves strongly on LKR. The short cycles are close to those found in the one-frequency model (see the chart in appendix 4). Four significant structural
cycles hide behind these cycles. There is now some hysteresis in LTFP and it is stronger in URMIN. We will not discuss the short cycles further and concentrate on the long cycles and the genuine trend components. Some \( \theta \) slope coefficients are strongly affected by the introduction of the second frequency: a large increase in LDUC, an important decrease in LPART and especially in URMIN.

Chart 4: Smoothed long cycles (two-frequency model)

The long cycles can be seen on chart 4. This time, the cumulated impacts of trend cycles tend to dominate the deviation cycles. Hysteresis is widespread and pervasive over 20 years. This is particularly the case in URMIN and LKR (the \( \delta_i \) scale factors of the trend cycles equal 0.83 and 2.22 respectively). Hysteresis is strong in LHOURS and LPART (scale factor around 0.40). In LDUC, the factor drops to 0.12 but over 20 years, the deviation and the cumulated trend cycle have more or less the same impact. The exception is LTFP. With a loading of 0.03, the trend cycle only plays a
limited role. The deviation cycle is much more important. Its shape is well known. It is similar to the trend component found in the one-frequency model although somewhat smaller. So the genuine trend of LTFP and the genuine potential growth rate of LKR from the previous section get another interpretation: for a large part, they are now the result of a long cycle that works as a deviation cycle in LTFP but as a trend cycle in LKR, affecting the growth rate of the capital stock (see chart 5 below). The long cycle has a permanent impact on the capital stock but not on TFP.

In table 6, we decompose the variance of the long cycles between structural cycles. Only 2 significant structural cycles hide behind the six long cycles:

Table 6: Structural cycle shares in cycle variance (low frequency cycles)

<table>
<thead>
<tr>
<th></th>
<th>Ψ1</th>
<th>Ψ2</th>
<th>Ψ3</th>
<th>Ψ4</th>
<th>Ψ5</th>
<th>Ψ6</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.987</td>
<td>0.013</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LPART</td>
<td>0.826</td>
<td>0.174</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTFP</td>
<td>0.916</td>
<td>0.084</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>URMIN</td>
<td>0.604</td>
<td>0.396</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>LKR</td>
<td>0.871</td>
<td>0.129</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The Choleski decomposition is not unique but the table suggests that the second cycle is necessary for the unemployment rate and somewhat for the participation rate. It seems that the two cycles account for different turning points between variables although phase shifts could keep the number of significant structural cycles to a minimum. Moreover, LPART steadily improves throughout the nineties and the last trough in the sample was less damaging for unemployment than it was for the other variables.

Once phase shifts are removed, associations between trend cycles can be computed. They are given in table 7:

Table 7: Associations between trend cycles (low frequency cycles)

<table>
<thead>
<tr>
<th></th>
<th>LDUC</th>
<th>LHOURS</th>
<th>LPART</th>
<th>LTFP</th>
<th>URMIN</th>
<th>LKR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>1</td>
<td>0.994</td>
<td>0.909</td>
<td>0.957</td>
<td>0.777</td>
<td>0.933</td>
</tr>
<tr>
<td>LHOURS</td>
<td>0.994</td>
<td>1</td>
<td>0.865</td>
<td>0.943</td>
<td>0.821</td>
<td>0.911</td>
</tr>
<tr>
<td>LPART</td>
<td>0.909</td>
<td>0.865</td>
<td>1</td>
<td>0.965</td>
<td>-0.723</td>
<td>0.979</td>
</tr>
<tr>
<td>LTFP</td>
<td>0.957</td>
<td>0.943</td>
<td>0.965</td>
<td>1</td>
<td>0.857</td>
<td>0.996</td>
</tr>
<tr>
<td>URMIN</td>
<td>0.777</td>
<td>0.821</td>
<td>-0.723</td>
<td>0.857</td>
<td>1</td>
<td>0.845</td>
</tr>
<tr>
<td>LKR</td>
<td>0.933</td>
<td>0.911</td>
<td>0.979</td>
<td>0.996</td>
<td>0.845</td>
<td>1</td>
</tr>
</tbody>
</table>

14 These results are in line with the euro area results of Proietti et al. (2007): hysteresis is present in LPART and URMIN but absent from LTFP.

15 The deviation cycle is dominant in LTFP but there is no phase shift between its trend cycle and its deviation cycle. So the signs in the table do not change. This is also the case for LDUC.
All associations are very high reflecting the small number of significant structural cycles. The cycles on LDUC and LHOURS are almost identical. This is also the case for the LTFP and LKR cycles. This is reminiscent of correlations found between slope shocks in the one-frequency model. LPART is better associated with the two latter variables. The lowest associations are found on URMIN with even a negative association between URMIN and LPART. Phase shifts can explain this result as we will see. The association between LDUC and LKR is high. It corresponds to the link traditionally made between capacity utilization and the growth of the capital stock (or investment). From table 7, one can also understand why LDUC is often used as a reference cycle. Its association with the other variables is usually on the high side. But this is only true once short cycles (and genuine trends) are properly removed.

The phase shifts between cycles are presented in table 8. By normalization, the shifts will be lower than 11.3 quarters (in absolute value). Given the presence of one dominant structural cycle, a clear picture of leads and lags emerges between variables.

<table>
<thead>
<tr>
<th></th>
<th>LDUC</th>
<th>LHOURS</th>
<th>LPART</th>
<th>LTFP</th>
<th>URMIN</th>
<th>LKR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDUC</td>
<td>0</td>
<td>1.6</td>
<td>-3.6</td>
<td>-1.9</td>
<td>6.3</td>
<td>-2.2</td>
</tr>
<tr>
<td>LHOURS</td>
<td>-1.6</td>
<td>0</td>
<td>-5.5</td>
<td>-3.7</td>
<td>4.2</td>
<td>-4.1</td>
</tr>
<tr>
<td>LPART</td>
<td>3.6</td>
<td>5.5</td>
<td>0</td>
<td>2.3</td>
<td>-10.0</td>
<td>2.0</td>
</tr>
<tr>
<td>LTFP</td>
<td>1.9</td>
<td>3.7</td>
<td>-2.3</td>
<td>0</td>
<td>9.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>URMIN</td>
<td>-6.3</td>
<td>-4.2</td>
<td>10.0</td>
<td>-9.5</td>
<td>0</td>
<td>-10.3</td>
</tr>
<tr>
<td>LKR</td>
<td>2.2</td>
<td>4.1</td>
<td>-2.0</td>
<td>0.5</td>
<td>10.3</td>
<td>0</td>
</tr>
</tbody>
</table>

LTFP and LKR are contemporary. LHOURS is leading by 1.6 quarters on LDUC, about 4 quarters on LTFP and LKR and 5.5 quarters on LPART. The ranking between variables is robust to a change in reference variable. URMIN appears to lead LHOURS by a full year. Its lead on LTFP and LKR is closer to 10 quarters. The lead on LPART would exceed 11.3 quarters but by normalization, URMIN turns countercyclical with a lag close to but lower than 11.3 quarters. The leading character of URMIN with respect to many variables is surprising. One should remember that the lead relates to trend cycles, i.e. cycles modifying the first difference of the trend component. On chart 4, one can see that the deviations cycles in LDUC, LHOURS and LTFP lead their respective (cumulated) trend cycle. This can explain why URMIN is usually perceived as a lagging variable. But from our results, it appears that changes in URMIN convey early information on the long cycles.
The contribution of hysteresis coming from the long cycles to the potential growth rates can be seen on chart 5, per variable (the drift is removed from the potential growth rates to improve comparability). The measure of the potential growth rate may differ from the previous one (on chart 3) because of the extra removal of the long deviation cycles. This is especially the case on LTFP. The trend cycles explain a large part of the movements in URMIN, LKR, LDUC and LHOURS. This is less the case in LPART and LTFP. These contributions will be removed when computing genuine potential growth rates.

**Chart 5: Smoothed potential growth rates (two-frequency model)**

(year-on-year growth rates)

With the new impact of long cycles on trend formation, slope shocks do not stay unaffected. A single structural shock remains. From three significant slope shocks in the one-frequency model, we moved to two significant long cycle shocks and one only slope shock. Genuine trends will differ between variables because of different drifts, different shock variances and different damping
coefficients $\theta$ but shocks behind the trends are perfectly correlated although the correlation sign may change. Genuine trends are shown on chart 6:

**Chart 6: Smoothed genuine trends (two-frequency model)**

(log-levels)

Over 1983-1991, slope shocks are negative (on average) for many variables. The decrease in LHOURS is large, the participation rate grows rather slowly and there is no improvement in the "structural" level of unemployment. On the contrary, the TFP growth is above average because shocks have an opposite effect on LTFP. Things change over the next decade: slope shocks change sign (on average). Participation improves faster, hours are rather stable and there is some improvement in structural unemployment. But the TFP growth rate goes down. The change in the level of LDUC is attributed to the same phenomenon. Since 2002, slope shocks are close to zero (on average) and the genuine trends reflect the drifts in the system. All over the sample period, slope shocks have a very limited impact on the capital stock and its genuine trend is almost linear.
A long run trade-off between hourly labour productivity on the one side, hours worked and employment rate on the other, has been documented by Bourlès and Cette (2007) for 14 OECD countries over 1992-2001. Broersma (2008) also finds a trade-off between hourly labour productivity and employment rate in EU15 and Anglo-Saxon countries over 1971-2004. But we do not know from these studies if the changes in labour productivity correspond to TFP changes and/or alterations to capital deepening. From Dew-Becker and Gordon (2008), it appears that since 1995, not only the capital deepening but also the TFP growth rate were adversely affected by the rising employment rate in the EU15 countries. But its contribution to the TFP slowdown is uncertain. According to EC (2007), only a small part of the slowdown may be attributed to the rising employment rate. In our sample, there is a 1.3 p.c. increase (above drift) in hours worked over 1992-2001 and a 1.5 p.c. increase in the participation rate while the unemployment rate falls by 0.7 percentage point. At the same time, a fall of 1.7 p.c. is recorded on TFP. The rise in production potential would equal 2.3 p.c. without the fall in TFP but is only 0.5 p.c. when it is taken into account. One should notice that our TFP measure still includes changes in human capital. The growth of human capital could be negatively affected by a fall in unemployment or a higher participation rate when less skilled people are involved.

4. OUTPUT GAP AND POTENTIAL GROWTH

The output gap on chart 7 is computed with formula (10), taking both short and long deviation cycles into account. Its amplitude is rather small, between -1.7 p.c. and +1.6 p.c. of GDP, and going down over time. The main driving force behind the output gap is the deviation cycle on LTFP. The correlation between the two time series amounts to 0.97 and the deviation cycle of LTFP has almost the same shape. In Moës (2006), the close connection between the two variables was already noticed but long cycles were absent from the model. Another link observed in that paper was the similarity between the business survey indicator of the National Bank of Belgium and the output gap. It is still valid. The correlation with the gross overall synthetic curve amounts to 0.72. One can also mention that the deviation cycle of LKR has a negligible impact on the output gap, justifying its exclusion from traditional computations.

Because of hysteresis, cycles alter trends in a permanent way and measures of the growth potential are cyclical. We will come back to this in a moment. For the time being, it is interesting to assess the long run impact of the trend cycles jointly with the deviation cycles. The "full cycle" is computed with equation (13). The difference between the two time series on chart 7 comes from the cumulated impact of trend cycles.
Over 20 years, the full cycle fluctuates between -3.5 p.c. and +3.1 p.c. of GDP. Its main determinants are given on chart 8. The full cycles of LTFP and URMIN are the dominant factors. In
LTFP, the full cycle is for a large part the deviation cycle, responsible for the output gap. But in URMIN, hysteresis prevails. Its contribution amplifies the cycle around the years 1990 and 2000, it depresses the cycle in between. The contribution of the capital stock is lower. It has the same origin: the investment cycle creates hysteresis in the capital stock. But the capital stock contribution is lagging the unemployment contribution because of the 10.3 quarters lag found between their (long) trend cycles.

The correlation between the full cycle and the business survey indicator drops to 0.45. The indicator is not influenced by cumulated trend cycles or in other words, by long-lasting movements in capital stock and unemployment. Hysteresis has a strong impact on the output level over 20 years but the business cycle indicator does not measure hysteresis. This does not mean that the indicator is free from influence of the investment cycle and cyclical variations in unemployment. The correlation between the indicator and the cumulated impact of trend cycles is only 0.18 but the correlation with the one-period impact jumps to 0.78. This helps explain why the indicator is usually associated with GDP growth and not with standard measures of the GDP cycle more akin to the full cycle.

The measure of potential growth complementary to the output gap is given by equation (11). It is not free from trend cycles. To get rid of them, we can use equation (12) and compute a measure of genuine potential growth. Both are presented on chart 9. Potential growth is extremely volatile, ranging from a low of 0.3 p.c. in 1993Q3 to a high of 3.7 p.c. in 1989Q1. Genuine potential growth has a much lower volatility, ranging from 1.6 to 2.7 p.c. The main difference comes from the trend cycle in unemployment. In 1989Q1 and 2000Q2, it adds a full 1 p.c. to the growth potential whereas in 1993Q3, an all-sample low of minus 1.4 p.c. strongly depresses the potential. The investment cycle has a more limited impact (lower then 0.45 p.c. in absolute value). Because of hysteresis, rough and genuine potential growth measures can give conflicting signals. The former gives for instance too optimistic growth prospects at the end of the eighties and nineties and too pessimistic prospects soon thereafter. Although hysteresis will alter the production level in a permanent way, the impact on potential growth is temporary and genuine growth gives a better picture of growth to come. On chart 10, its determinants are given in terms of contributions.

Leaving aside the contribution from the capital stock that is fairly constant and the contribution from the population of working age that is exogenous to the model, we can see the counteracting influences of TFP on the one side and of PART, HOURS and unemployment on the other. Positive shocks in TFP are associated with a fall in hours worked and participation rate and with a rise in unemployment. The latter variables dominate after some time but initially, genuine growth is governed by TFP.
The diverging damping coefficients $\theta$ present in the slope components of the variables are responsible for the different speeds of adjustment. Shocks have an instantaneous impact on LTFP (no damping coefficient) but a lasting influence on LHOURS and LPART. For instance, positive shocks over 1988Q3-1992Q1 increased productivity but soon there was an increase in unemployment and both hours and participation rate slowly went down, reverting the initial positive impact from productivity. One should be cautious not to interpret this sequence as a causality running from "technology shocks" to employment. The shock occurs simultaneously in all variables but the adjustment speeds differ. Nevertheless, the different timings are at variance with an explanation in terms of underlying changes to the human capital stock.

The fast adjustment in TFP is responsible for the residual variability of potential growth notwithstanding the removal of hysteresis. Over time, the positive shocks to hours, participation rate and URMIN contributed to a marginally higher potential growth in the nineties (+0.1 percentage point) because of the negative shocks present in TFP. The rise in the population of working age is responsible for the recent improvement, with a genuine potential growth remaining steadily above 2 p.c. over the last years.

5. CONCLUSION

Standard STS models assume that the trend and cycle components are independent. In the context of a production function, this assumption does not allow for hysteresis in factor inputs. We drop this restriction and allow cycles to alter the trend levels in a permanent way. To this end, we develop a multivariate dual cycle model, which is a combination of the trend plus cycle model and the cyclical trend model. The corresponding deviation and trend cycles are closely related for a given factor input but their relative impact is left free and phase shifts are introduced to account for leads and lags.

With independence between trend and cycle components, it is possible to assess the output gap and/or the potential growth rate in a straightforward way but the resulting output gap can be rather small and the potential growth rate rather erratic because hysteresis is not taken into account. We show that in the dual cycle model, the usual statistics can be extended to compute a genuine measure of potential growth, which is free from hysteresis and much less cyclical. Conversely, the output gap can be refined to include the cumulative impact of trend cycles over time (hysteresis).

Applied to Belgian data, the dual cycle model shows that over 20 years, the hysteresis coming from the 3-year cycle present in the data is rather limited. Deviation cycles without long run impact on the output potential dominate in TFP and hours worked. The exception is unemployment but its 3-year cycle only explains a small part of the unemployment fluctuations. When it comes to the long
11-year cycle, things look different. Hysteresis now has a marked impact and the long cycle alters the unemployment rate, capital stock, hours worked and participation rate in a permanent way. The exception is TFP. Here the long cycle is mainly a deviation cycle. It is remarkably close to the investment cycle.

The output gap is essentially the deviation cycle in TFP. If hysteresis is taken into account, the gap becomes twice as large but then it is much less correlated with the NBB business survey indicator. Potential growth is strongly affected by hysteresis in unemployment. The genuine measure of potential growth is much less volatile but volatility does not entirely disappear because of different adjustment speeds to slope shocks: shocks have an instantaneous impact on TFP but they exert a delayed (and opposite) influence on hours, participation and unemployment.

On the whole, the dual cycle model is an improvement on more standard STS models. With the introduction of hysteresis, it may contribute to a better understanding of the long-run impact of output fluctuations. The model also contributes to a better assessment of potential growth. Can the output gap or the full cycle derived from the dual cycle model also provide a better assessment of inflationary pressures in real time? This question requires extensive computations and will be addressed in another paper. The only thing one can say is that the literature on this question is still in its infancy. Preliminary results suggest that the output gap computed in real time from STS models could be more reliable than other measures like the output gap based on the Hodrick-Prescott filter. However, it is not clear that the output gap coming from STS models would contribute to a significantly better assessment of inflationary pressures. Inflation itself is usually a very good predictor of inflation to come and it is very difficult for any output gap measure to constantly improve on this result.
Appendix 1: State space form of the one-frequency dual cycle model

For \( z_t = \begin{pmatrix} LDUC_t \\ LHOURS_t \\ LPART_t \\ LTFL_t \\ UR \min_t \\ LKR_t \end{pmatrix} \), the measurement equation is given by:

\[
 z_t = \begin{pmatrix} I_N & 0_N & 0_N & 0_N & I_N \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \psi_t \\ \psi_t^* \\ C_t \end{pmatrix} + \varepsilon_t.
\]

The transition equation for the state vector is:

\[
 \begin{pmatrix} \mu_{t+1} \\ \beta_{t+1} \\ \psi_{t+1} \\ \psi_{t+1}^* \\ C_{t+1} \end{pmatrix} = \begin{pmatrix} m & 0_N & 0_N \end{pmatrix} \begin{pmatrix} I_N & I_N & G \\ 0_N & 0_N & 0_N \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \psi_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \kappa_t \end{pmatrix}
\]

with \( 0_n \), the \((N \times 1)\) zero vector, \( m \), a vector of drifts and \( \theta \), a diagonal matrix of damped slope coefficients. \( F \) and \( F^* \) are lower triangular matrices:

\[
 F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 \\ F_{41} & F_{42} & F_{43} & 1 \\ F_{51} & F_{52} & F_{53} & F_{54} \end{pmatrix} \quad \text{and} \quad F^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ F_{21}^* & 0 & 0 & 0 \\ F_{41}^* & F_{42}^* & F_{43}^* & 0 \\ F_{51}^* & F_{52}^* & F_{53}^* & F_{54}^* \end{pmatrix}.
\]

The \( G \) and \( G^* \) matrices are related to the \( F \) and \( F^* \) matrices by equation (9). The variance-covariance matrix of shocks from the measurement and transition equations is block diagonal with identical diagonal variance-covariance matrices for \( \kappa_i \) and \( \kappa_i^* \).
Appendix 2: Smoothed trends (one-frequency model)
(centered, drift excluded)
Appendix 3: Capital stock and TFP
(respectively, year-on-year growth rate and four-quarter average after linear detrending)
Appendix 4: Smoothed short cycles (two-frequency model)
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