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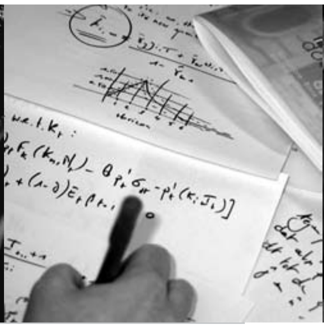

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THE TERM STRUCTURE OF INTEREST RATES IN A DSGE MODEL

Marina Emiris (*)

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Abstract

The paper evaluates the implications of the Smets and Wouters (2004) DSGE model for the US yield curve. Bond prices are modelled in a way that is consistent with the macro model and the resulting risk premium in long term bonds is a function of the macro model parameters exclusively. When the model is estimated under the restriction that the implied average 10-year term premium matches the observed premium, it turns out that risk aversion and habit only need to rise slightly, while the increase in the term premium is achieved by a drop in the monetary policy parameter that governs the aggressiveness of the monetary policy rule. A less aggressive policy increases the persistence of the reaction of inflation and the short interest rate to any shock, reinforces the covariance between the marginal rate of substitution of consumption and bond prices, turns positive the contribution of the inflation premium and drives the term premium up. The paper concludes that by generating persistent inflation the presence of nominal rigidities can help in reconciling the macro model with the yield curve data.

JEL-code : E43; E44, G12.

Keywords: term structure of interest rates, policy rules, risk premia

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1 Introduction

How far from financial data are the new generation fully-fledged dynamic stochastic general equilibrium (DSGE) models? Estimated DSGE models fare very well with macro data but can they do better than standard real business cycle models in solving the equity premium puzzle or in fitting the yield curve? This paper evaluates the implications of the Smets and Wouters (2004) DSGE model for the US yield curve and finds that price and wage rigidities play a key role in reconciling the macro model with the yield curve data. By generating inflation, they introduce additional sources of risk in the economy. However, it seems that what matters from the point of view of the agent which evaluates these risks, is not the properties of the generated inflation per se (i.e. the size of the nominal rigidities) but how the monetary authorities react to inflation. A less aggressive policy will indeed increase substantially the inflation premium. However, it will also decrease the real premium. The reason why the total premium increases enough to match the values in the data is that the decrease in the real premium is substantially smaller than the rise in the inflation premium.

The DSGE model under consideration is the Smets and Wouters (2004) model. This is a micro-founded DSGE model, set up in the lines of Christiano, Eichenbaum and Evans (2001) with real and nominal rigidities. The real rigidities are time non-separable preferences (habit formation), non-separable leisure and consumption in the utility function, and capital utilisation, investment and fixed costs in the production function. The nominal rigidities arise from a price and wage set-up a la Calvo with partial adjustment to past inflation.

This particular choice of model is motivated by several considerations. First, its success in estimation with US and Euro-area macro data. Second, real rigidities, which are present in the model to help match the dynamic properties of the macro data, should also generate large real risk premia and match the US term structure data. For example, habit increases the risk aversion parameter implicit in the model without changing the intertemporal elasticity of substitution of the utility function. This allows for a higher premium while keeping the average interest rate at reasonable levels. Non-separable leisure has also been argued to increase the premium as long as the consumer cannot use leisure as an additional insurance device to smooth consumption (see Uhlig (2006)), i.e. as long as there are rigidities in the labour market. These are introduced in the Smets and Wouters model by wage rigidities. Finally, investment adjustment costs, fixed costs and capital utilisation costs, may be necessary to reduce the smoothness of consumption implied by the higher risk aversion and habit parameters required to generate a high term premium (see e.g. Jermann (1998) and Boldrin, Christiano and Fisher (2001)). With high habit, households would like to smooth consumption more. Since consumption is endogenously determined , its volatility will adjust, becoming smaller, while the cost for the same volatility will increase. The question is then how strongly consumption can actually be smoothed. This will depend on how flexible labour choices are as well as how flexible capital utilisation is. A high capital utilisation cost will then help generate larger term premia.

Finally, the presence of nominal rigidities in the model also motivates the choice of model. Nominal rigidities generate inflation and introduce additional sources of risk like uncertainty about the monetary authorities' inflation target, uncertainty about the realized interest rate, and wage and price mark-up shocks. The agent requires an additional compensation to hold assets whose pay-offs depend on the realization of these shocks. This opens up a new channel for increasing the model-generated term premium and bringing it closer to the data. On the other hand, nominal rigidities also introduce additional free model parameters like the monetary policy stance (the sensitivity of the short interest rate to inflation), the degree of wage and price rigidities measured by the Calvo parameters and parameters for price and wage indexation to past inflation. In the end, the question is whether there are enough free parameters to allow us to jointly match the macro and yield curve data. In any case, the presence of nominal rigidities in the model implies that, contrary to real business cycle models, this type of model can be used to make predictions about the size and sign of the inflation premium, as well as its relative importance with respect to the real premium in explaining the total premium of long maturity bonds.

The $n-$ maturity bond prices are obtained through log-linearisation of the $n-$forward iteration of the DSGE model Euler equation under the assumption of lognormal stochastic shocks.This general equilibrium approach generates bond prices and term premia that are internally consistent: Forward iteration ensures consistency with the weak version of the expectations hypothesis and the absence of arbitrage opportunities in the trade of bonds of different maturities. Up to this point this approach is the same as in the recently developed finance-macro literature in which bond prices are affine in the macro state variables and the no arbitrage restrictions and the dynamics of the macro variables are jointly respected, see e.g. Bekaert, Cho and Moreno (2004), Dewachter and Lyrio (2004), Hordahl, Tristani and Vestin (2004), Ang and Piazzesi (2003). The contribution of this paper is that it takes consistency one step further: By taking a general equilibrium approach, the parameters and the variances of the shocks that enter the Euler equation entirely determine the bond prices and term premia. No additional parameters or stochastic shocks are introduced. The yield curve is therefore entirely consistent with the macro model. The term premium becomes a function of the parameters in the utility function and of the stochastic structure that drives the consumption, investment and monetary policy behaviour in the macro-economy.

The loglinear-lognormal general equilibrium approach delivers and explains a constant term premium, it cannot explain time-variation. The paper therefore concentrates on evaluating the implications of the DSGE model for the average premium. A similar exercise is performed by Hordahl, Tristani and Vestin (2005) who study the term structure implications of the second-order approximate solution of a DSGE model. The advantage of using the loglinear-lognormal approach is that estimation using standard methods remains feasible in this framework and it is therefore straightforward to compare the results to an estimated macro model without financial data.

I estimate the average premium for the model and then perform a calibration exercise to get some intuition about its determinants. Then I re-estimate the model under the restriction that the model-generated premium is close to the observed average premium in 10-year bonds.

The estimated model produces a positive but counterfactually low 10-year term premium and a negative 10-year inflation premium. A calibration exercise indicates that there are several ways to raise the 10-year risk premium: One is to increase risk aversion and habit persistence, but the macroeconomic implications of these values are inconsistent with the data. Another is to substantially reduce the negative contribution of the inflation premium by eliminating the permanent inflation target shock. To explore which solution is best supported by both the macro and yield curve data, the model is re-estimated under the restriction that the implied average 10-year term premium is close to the observed premium. It turns out that to re-concile the macro with the finance stylized facts, the model risk aversion has to rise slightly from 1.8 to 2.7. However, the habit persistence parameter remains at its baseline level. The increase in the term premium is achieved by a drop in the monetary policy parameter that governs the aggressiveness of the monetary policy rule. A less aggressive policy increases the persistence of the reaction of the short interest rate and inflation to any shock, re-inforces the covariance between the marginal rate of substitution of consumption and bond prices, turns positive the contribution of the inflation premium and drives the term premium up.

The next two sections present the macro-finance model and derive the prices of the n-maturity bonds, the associated term premia and the average slope of the nominal term structure as a function of the micro-founded macro model structural parameters. I also derive expressions for the model-implied inflation premia at different maturities n. Section 4 presents the results from the calibration exercise and investigates which characteristics of the macro structural model are essential in matching the average US yield curve. This section also discusses the sensitivity of the term premium to changes in the model stochastic processes and parameters as well as the sign and size of the inflation premia. Then the restricted model is estimated and discussed. The last section summarises and concludes.

2 The description of the DSGE model

In this section, the DSGE model is briefly described. For a thorough discussion of its micro-foundations see Smets and Wouters (2004) and Christiano, Eichenbaum and Evans (2001). The DSGE model contains many frictions that affect both real and nominal decisions of households and firms. Households maximise a non-separable utility function of consumption and labour over an infinite life horizon. Consumption enters the utility function net of a time-varying external habit variable. Labour is differentiated over households, so that there is some monopoly power over wages which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983) with partial indexation to past inflation and a time-varying inflation target. Households rent capital services to firms and decide how much capital to accumulate taking into account capital adjustment costs. Firms produce differentiated goods, decide on labour and capital inputs and set prices according to a Calvo model with partial indexation to past inflation and the time-varying inflation target.

2.1 Households

There is a continuum of households indexed by $\tau \in [0,1]$. The utility of each household is a function of its consumption of final goods C_t^τ net of external habit H_t and its differentiated labour l_t^τ :

$$U_t^\tau(C_t^\tau, H_t, l_t^\tau) = \left(\frac{(C_t^\tau - H_t)^{1-\sigma_c}}{1-\sigma_c} + \varepsilon_t^L \right) \exp\left(\frac{\sigma_c + 1}{1+\sigma_l} (l_t^\tau)^{(1+\sigma_l)} \right) \quad (1)$$

with $log(\varepsilon_t^L) = \rho_L \ log(\varepsilon_{t-1}^L) + \eta_t^L$ where η_t^L is an i.i.d. normal error term, σ_c is the utility parameter that enters the elasticity of substitution for consumption, σ_l is the inverse of the elasticity of work effort with respect to the real wage. The external habit stock H_t, is assumed to be proportional to aggregate past consumption, $H_t = hC_{t-1}$, $0 < h < 1$. ε_t^L is a preference shock to labour supply.

Each household τ maximizes an intertemporal utility function V subject to an (intertemporal) budget constraint. The objective function is given by:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b U_t^\tau \quad (2)$$

where β is the discount rate, ε_t^b is a second preference shock that affects the discount rate, with $log(\varepsilon_t^b) = \rho_b log(\varepsilon_{t-1}^b) + \eta_t^b$ where η_t^b is an i.i.d. normal error term. The household's intertemporal budget constraint is given by:

$$\sum_{n=1}^{N} P_t^n \frac{B_{n,t}^\tau}{P_t} = \sum_{n=0}^{N-1} P_t^n \frac{B_{n-1,t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau \quad (3)$$

Households carry a portfolio of nominal zero-coupon bonds with a maximal maturity of N periods, denoted by $\left\{B_{n,t-1}^\tau\right\}_{n=1}^N$ into period t . The price of the n-maturity bond is P_t^n. Current income and financial wealth brought over from the previous period can be used to consume, invest or buy assets. The household's total income Y_t^τ is given by:

$$Y_t^\tau = (w_t^\tau l_t^\tau + A_t^\tau) + \left(r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau) K_{t-1}^\tau + div_t^\tau \right) \quad (4)$$

Total income is the sum of labour income and income from the state-contingent securities; the return from the real capital stock net of the capital utilisation costs; and the dividends from the firms that produce intermediate goods under monopolistic competition. $\Psi(z_t^\tau)$ is the cost associated with the degree of capacity utilisation of the installed capital and it is assumed (see Christiano,Eichenbaum, and Evans, 2001) that $\Psi(1) = 0$.

The maximization of the objective function subject to the intertemporal budget constraint with respect to consumption C_t^τ and holdings of bonds B_t^τ yields the following first order condition for consumption:

$$E_t \beta \frac{\lambda_{t+1}}{\lambda_t} R_t \frac{P_t}{P_{t+1}} = 1 \tag{5}$$

where, R_t is the gross nominal rate of return on one-period bonds ($R_t = \frac{1}{P_t^1}$) and λ_{t+1} is the marginal utility of consumption given by:

$$\lambda_t \; = \varepsilon_t^b \left(C_t - H_t\right)^{-\sigma_c} \exp\left(\frac{\sigma_c + 1}{1 + \sigma_l} \left(l_t^\tau\right)^{(1+\sigma_l)}\right) \tag{6}$$

Notice that because of the non-separable consumption and leisure preferences in the utility specification, the marginal disutility of work affects the optimal consumption decision.

The labour supply and wage setting equations are modelled as in Smets and Wouters (2004). Households are price-setters in the labour market and, following Calvo (1983), they can optimally set their wage with probability $(1 - \xi_w)$. With probability ξ_w their wage is indexed to past inflation and the central bank inflation objective with respective weights γ_w and $(1 - \gamma_w)$. Optimising households choose the nominal wage $\widetilde{w}_t^\tau$ by maximising the intertemporal objective function (2) subject to its budget constraint (3) and labour demand

$$l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t \tag{7}$$

where the aggregate labour demand L_t and the aggregate nominal wage W_t are respectively

$$L_t = \left[\int_0^1 \left(l_t^\tau\right)^{1/(1+\lambda_{w,t})} d\tau\right]^{1+\lambda_{w,t}} \quad \text{and} \quad W_t = \left[\int_0^1 \left(W_t^\tau\right)^{1+\lambda_{w,t}} d\tau\right]^{-\lambda_{w,t}} \tag{8}$$

Shocks to the wage markup are assumed to be i.i.d. normal around a constant $log(\lambda_{w,t}) = \lambda_w + \eta_t^w$.

Households make decisions about investment and the capital utilisation rate by maximizing their intertemporal objective function (2) subject to its budget constraint (3) and a capital accumulation equation:

$$K_{t+1} = K_t \left(1 - \tau\right) + \left(1 + \varepsilon_t^I - S(\frac{I_t}{I_{t-1}})\right) I_t \tag{9}$$

with I_t the gross investment, τ the depreciation rate and $S(.)$ an adjustment cost function which is a positive function of change in investment and equal to zero at the steady state. The first order conditions for the real value of capital Q_t, investment I_t and the rate of capital utilisation z_t are:

$$Q_t = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(Q_{t+1} (1-\tau) + z_{t+1} r^k_{t+1} - \Psi(z_{t+1}) \right) \right] \tag{10}$$

$$Q_t \left(1+\varepsilon^I_t\right) = Q_t S^{'} \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \beta E_t \left[Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} S^{'} \left(\frac{I_{t+1}}{I_t} \right) \frac{I_{t+1}}{I_t} \left(\frac{I_{t+1}}{I_t} + 1 \right) \right] \tag{11}$$

$$r^k_t = \Psi^{'}(z_t) \tag{12}$$

A shock $log(\varepsilon^I_t) = \rho_I log(\varepsilon^I_{t-1}) + \eta^I_t$ where η^I_t is an i.i.d. normal error term is introduced in the investment cost function. It represents a shock in the relative price of investment vs. consumption goods and plays the role of an investment-specific technological shock.

2.2 Firms

The economy produces an homogeneous final good from a continuum of intermediate goods y^j_t indexed by j, with $j \in [0,1]$. The final good is produced with a CES technology,

$$Y_t = \left[\int_0^1 \left(y^j_t \right)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} \tag{13}$$

where $\lambda_{p,t}$ denotes the time-varying markup in the goods market. It is assumed that $log(\lambda_{p,t}) = \lambda_p + \eta^p_t$ with η^p_t an i.i.d. normal variable that can be interpreted as a cost-push shock to inflation. From cost minimisation, the demand faced by each intermediate producer is:

$$y^j_t = \left(\frac{p^j_t}{P_t} \right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t \tag{14}$$

with p^j_t the price of good j and P_t the price of the final good. Perfect competition in the final good market implies that

$$P_t = \left[\int_0^1 \left(p^j_t \right)^{-1/\lambda_{p,t}} dj \right]^{-\lambda_{p,t}} \tag{15}$$

Intermediate goods y^j_t are produced in a monopolistic competitive sector with a continuum of firms characterised by stick prices. Intermediate goods are produced with a Cobb-Douglas technology:

$$y^j_t = \varepsilon^A_t \left(\widetilde{K}_{j,t} \right)^{\alpha} \left(L_{j,t} e^{\gamma t} \right)^{1-\alpha} - \Phi e^{\gamma t} \tag{16}$$

with $log(\varepsilon^A_t) = \rho_A log(\varepsilon^A_{t-1}) + \eta^A_t$ and η^A_t an i.i.d.-normal error term, where ε^A_t is the productivity shock, $\widetilde{K}_{j,t} = z_t K_{j,t-1}$ is the capital stock effectively utilised, $L_{j,t}$ is an index of various types of labour hired by the firm, γ is the constant rate of technological progress and Φ is a fixed cost introduced to ensure zero profits in steady state. Cost minimisation implies that the capital-labour ratio is identical across intermediate goods produces and equal to the aggregate capital-labour ratio:

$$\frac{W_t L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \tag{17}$$

The marginal cost is given by

$$MC_t^j = MC_t = \frac{W_t^{1-\alpha}\left(r_t^k\right)^{\alpha}}{\alpha(1-\alpha)\varepsilon_t^A e^{\gamma t}} \tag{18}$$

and is also independent of the demand faced by each firm j. Nominal profits are given by

$$\pi_t^j = \left(p_t^j - MC_t\right)\left(\frac{p_t^j}{P_t}\right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t - MC_t\Phi \tag{19}$$

Each firm has market power in the market for its own good and maximizes expected profits using a discount rate (ρ_t) consistent with the pricing kernel for nominal returns used by shareholders-households: $\rho_t = \beta\lambda_{t+k}/\lambda_t P_{t+k}$.

Firms are not allowed to re-optimize their prices unless they receive a random 'price-change signal'. As in Calvo (1983) the probability to receive such a signal is equal to $\left(1-\xi_p\right)$. Prices of firms that do not receive a price signal are indexed to the weighted sum of last period's inflation rate and the inflation objective of monetary policy with respective weights γ_p and $\left(1-\gamma_p\right)$. Profit maximisation of re-optimising firms at time t results in the following first order condition:

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i} \left(\frac{p_t^j}{P_t}\frac{P_t}{P_{t+i}}\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\gamma_p} \left(\overline{\pi}_t\right)^{1-\gamma_p} - \left(1-\lambda_{p,t+i}\right) mc_{t+i} \right) = 0 \tag{20}$$

This equation shows that the price set by firm j at time t is a mark-up over weighted expected future marginal costs. With sticky prices ($\xi_p \neq 0$) the mark-up is variable over time when the economy is hit by exogenous shocks.

2.3 Market equilibrium and monetary policy

The final good market is in equilibrium when production equals the demand by households for consumption and investment and government spending G_t

$$Y_t = C_t + G_t + I_t + \Psi(z_t)K_{t-1} \tag{21}$$

with $\varepsilon_t^G = \rho_G \varepsilon_t^G + \eta_t^G$ and η_t^G an i.i.d.-normal error term.

The capital rental market is in equilibrium when the demand for capital by the intermediate goods firms is equal to the capital supplied by the households.

The labour market is in equilibrium when the firm's demand for labour is equal to the households' labour supply at the wage set by the households.

The model is closed with an empirical reaction function for the short interest rate R_t that describes monetary policy decisions:

$$\begin{aligned}\widehat{R}_t \quad = \quad & \overline{\pi}_{t-1} + \rho\left(\widehat{R}_{t-1} - \overline{\pi}_{t-1}\right) + (1-\rho)\left(r_\pi\left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right) + r_Y\left(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1}\right)\right) + \\ & r_{\Delta\pi}\left[\left(\widehat{\pi}_t - \overline{\pi}_t\right) - \left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right)\right] + r_{\Delta y}\left[\left(\widehat{Y}_t - \widehat{Y}^p_t\right) - \left(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1}\right)\right] + \eta^R_t\end{aligned} \tag{22}$$

Hats denote deviations from the deterministic steady state. The monetary authorities follow a generalised Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective $(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1})$ and the lagged output gap defined as the difference between actual and potential output $\left(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1}\right)$. Potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the three cost-push shocks $(\eta^P_t, \eta^W_t, \eta^Q_t)$. The parameter ρ captures interest rate smoothing. In addition, there is a short run feedback from current changes in inflation and the output gap. Finally , this rule assumes that there are two types of monetary policy shocks: η^R_t, which is a transitory i.i.d.-normal interest rate shock and η^π_t, which is a permanent shock to the inflation objective $(\overline{\pi}_t)$ which is assumed to follow a non-stationary process: $\overline{\pi}_t = \overline{\pi}_{t-1} + \eta^\pi_t$.

To summarize, the model determines nine endogenous sticky-price variables: inflation, the real wage, capital, the value of capital, investment, consumption, the short-term nominal interest rate, the rental rate on capital and employment. The stochastic behaviour of the system of linear rational expectations equations is driven by ten exogenous shocks. Five shocks arise from technology and preference parameters: the total factor productivity shock, the investment-specific technology shock, the preference shock, the labour supply shock and the government spending shock. Three shocks can be interpreted as cost-push shocks: the price mark-up shock, the wage-markup shock and the equity premium shock. Finally there are two monetary policy shocks, the permanent inflation target shock and the temporary interest rate shock.

3 Affine term structure with macro factors

3.1 Bond prices

The DSGE model developed above can be solved by log-linearisation around the deterministic steady state. However, as is well known, a first order approximation of the solution of any rational expectations model, will generate certainty equivalence in the model solution. At the equilibrium, agents will behave as if they were risk neutral and assets with different risk characteristics, in our case different maturity bonds, will have the same expected return. This makes asset pricing trivial and forces us to seek a different approach.

The approach I take is that of Jermann (1998) and Wu (2005)[1]. The macroeconomic model is not affected by the dynamics of the yields. This means that we can solve it without knowing the equilibrium prices of bonds. The first step is to loglinearize the equations of the DSGE model around the nonstochastic steady state and solve the resulting system of linear difference equations. The model solution has a state-space representation with the law of motion for the state variables given by:

$$X_{t+1} = c + AX_t + Bv_{t+1} \tag{23}$$

where c is a vector of constants of size $(np, 1)$, A and B are matrices of coefficients of size (np, np) and (np, k) respectively, X_t is the vector of state variables of size $(np, 1)$ and v_{t+1} the vector of structural shocks of the DSGE model of size $(k, 1)$, np is the number of state variables, k is the number of structural shocks and $E\left(v_{t+1}v'_{t+1}\right) = q$, a (k, k) diagonal matrix.

Under a complete markets hypothesis and in the absence of arbitrage opportunities ($M_{t+1} > 0$, see Harrison and Kreps (1979)), the price P_t^n of any zero-coupon bond of maturity n obeys the Euler equation:

$$P_t^n = E_t M_{t+1} P_{t+1}^{n-1} \tag{24}$$

where M_{t+1} is the nominal stochastic discount factor for pricing nominal assets defined as:

$$M_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$$

with λ_{t+1} the intertemporal marginal rate of substitution of consumption and P_{t+1} the aggregate price level. Replacing λ_{t+1} with its expression from equation (6) yields an expression for the nominal stochastic discount factor as a function of the model's structural parameters σ_c and σ_l and consumption, habit, leisure and the preference shock :

$$M_{t+1} = \beta \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}^b}{\varepsilon_t^b} \left(\frac{(C_{t+1} - H_{t+1})}{(C_t - H_t)} \right)^{-\sigma_c} \exp\left(\frac{\sigma_c + 1}{1 + \sigma_l} \left(\left(l_{t+1}^\tau\right)^{(1+\sigma_l)} - \left(l_t^\tau\right)^{(1+\sigma_l)} \right) \right) \tag{25}$$

Log-linearisation of M_{t+1} around the deterministic steady state yields:

[1] Another approach is to use a second-order approximation to the solution of the model as in Hordahl, Tristani and Vestin (2005). However, because the aim of this paper is to estimate the parameters of the joint macro-finance model, I prefer the log-linear/log-normal approach. Estimation of the second-order approximation of the solution to the macro-finance model would require even more computationally burdenous methods than the Bayesian maximum likelihood estimation used here.

$$\widehat{m}_{t+1} = \frac{\sigma_c(1+h)}{1-h}\widehat{c}_t - \frac{h\sigma_c}{1-h}\widehat{c}_{t-1} - \frac{\sigma_c}{1-h}\widehat{c}_{t+1} - \frac{\sigma_c - 1}{(1+\lambda_w)(1-h)}\left(\widehat{l}_t - \widehat{l}_{t+1}\right) - \frac{1}{1-\rho_b}\left(\widehat{\varepsilon}_t^b - \widehat{\varepsilon}_{t+1}^b\right) - \widehat{\pi}_{t+1} \quad (26)$$

where hats denote deviations of the variables from the non-stochastic steady state, and small-case letters denote logs of capital-letter variables. For later use, we also write down the reduced form dynamics of m_{t+1} obtained from equation (23):

$$m_{t+1} = c_m + A_m^{'} X_t + \Lambda_0^{'} v_{t+1} \quad (27)$$

where c_m is the element in c corresponding to $\widehat{m}_{t+1}$, A_m and $\Lambda_0^{'}$ are the corresponding column vectors of A and B and are functions of the structural parameters of the DSGE model.

To derive bond prices, the next step is to take the log of equation (24) to obtain:

$$p_t^n = E_t\left[m_{t+1} + p_{t+1}^{n-1}\right] + \frac{1}{2} var_t\left[m_{t+1} + p_{t+1}^{n-1}\right] \quad (28)$$

This equation holds exactly when the conditional distribution of bond prices P_{t+1}^{n-1} and the stochastic discount factor M_{t+1} are jointly log-normal variables. It will hold approximately if they are not (see also Campbell, Lo and MacKinlay, 1997).

Then, by substituting (27) into (28) for $n = 1$ and by setting $P_{t+1}^0 = 1$, the one-period bond price is given by:

$$-R_t = p_t^1 = c_m + A_m' X_t + \frac{1}{2}\Lambda_0^{'} q \Lambda_0 \quad (29)$$

The last quadratic term, which would be absent in a strict log-linear model, captures the risk compensation to agents.

Notice that the market price of risk is given by the elements in Λ_0. Any bond of maturity n, carries $B_{n-1}' q^{1/2}$ units of risks:

$$p_{t+1}^{n-1} = E_t\left[p_{t+1}^{n-1}\right] + B_{n-1}' v_{t+1} \quad (30)$$

where B_{n-1}' is a $(k, 1)$ vector. Substituting in equation (28) yields:

$$p_t^n = -R_t + E_t\left[p_{t+1}^{n-1}\right] + B_{n-1}' q \Lambda_0 + B_{n-1}' q B_{n-1}/2$$

and the excess holding period return of an n-period bond over a one-period bond is given by re-writing the above equation:

$$\begin{aligned} ehpr_t^n &\equiv E_t\left[p_{t+1}^{n-1}\right] - p_t^n - R_t \\ &= -B_{n-1}' q \Lambda_0 - B_{n-1}' q B_{n-1}/2 \end{aligned} \quad (31)$$

where $-B_{n-1}'qB_{n-1}/2$ is the (negligible) convexity effect and $-B_{n-1}'q\Lambda_0$ is the compensation for holding $B_{n-1}'q^{1/2}$ units of risk, making -$\Lambda_0 q^{1/2}$ the market price of the risk associated with the macro structural shocks v_{t+1}. Because we have made the assumption of constant second-order moments B and q are time-invariant and, as a result, the *ehpr* is constant: $ehpr_t^n = ehpr^n$.

The last step to price the bonds consists in noticing that the transition equation (23), the short rate equation (29) and the risk price $-\Lambda_0 q^{1/2}$ form a discrete-time Gaussian affine term structure model. If the stochastic discount factor is affine as above in (27) , the log-bond price equation will also be affine (proof and details in *Appendix 3*):

$$p_t^n = a(n) + b(n)^{'} X_t \tag{32}$$

and the coefficients will be defined recursively by:

$$\begin{aligned} a(1) &= c_m + \frac{1}{2}\Lambda_0^{'} q \Lambda_0 && (33)\\ b(1)^{'} &= A_m' && (34)\\ a(n) &= a(n-1) + a(1) + b(n-1)^{'}(c + Bq\Lambda_0) && (35)\\ & \quad + \frac{1}{2} b(n-1)^{'} BqB^{'} b(n-1) \\ b(n)^{'} &= b(1)^{'} + b(n-1)^{'} A && (36) \end{aligned}$$

The continuously compounded yield to maturity y_{nt} for a zero-coupon nominal bond is then given by:

$$y_{nt} = -p_t^n/n = \overline{a}(n) + \overline{b}(n)^{'} X_t$$

with $\overline{a}(n) = -a(n)/n$ and $\overline{b}(n)^{'} = -b(n)/n$.

With the prices of bonds at hand we can proceed to define the excess holding period return as a function of the DSGE model parameters.

3.2 The interaction between the macro shocks and the excess holding period return

Recall that equation (31) expresses the *ehpr* as a function of the reduced-form model parameters, B and q. To establish the link between the *ehpr* and the DSGE model *structural* parameters, we need to replace in equation (28) the expression for the log- stochastic discount factor m_{t+1}(equation (26)) :

$$
\begin{aligned}
ehpr^{n} &\equiv \\
&= \frac{-1}{1-\rho_b} cov\left[\widehat{\varepsilon}^{b}_{t+1}, p^{n-1}_{t+1}\right] + \frac{\sigma_c}{1-h} cov\left[\widehat{c}_{t+1}, p^{n-1}_{t+1}\right] - \\
&\quad - \frac{\sigma_c - 1}{(1+\lambda_w)(1-h)} cov\left[\widehat{l}_{t+1}, p^{n-1}_{t+1}\right] + cov\left[\pi_{t+1}, p^{n-1}_{t+1}\right] \\
&\quad - \frac{1}{2} var_t\left[p^{n-1}_{t+1}\right]
\end{aligned}
$$

Notice again that, because of our assumption that second-order moments are constant, the above variance and covariance terms and the excess holding period return are time-invariant. This expression illustrates how the excess holding period return for any maturity n bond is a function of the preference parameters in the DSGE model σ_c and h and the persistence of the preference shock ρ_b. It is also a function of the covariance terms between the bond prices and next period consumption, leisure, inflation and the preference shock, which depend on the size of the shocks v_{t+1} and the persistence properties of the different variables. As a result, predicting the total effect of a change in the structural parameters on the excess holding period return is not straightforward: any parameter change that also affects the persistence properties of any of the variables will also change the above covariance terms, thereby magnifying or reducing the initial parameter change.

To get an intuition of how this interaction works to change the size and the sign of the *ehpr* , we go back to the its reduced form definition (equation (31) . Recursivity implies that B'_{n-1} is restricted by (36):

$$B'_{n-1} = B^{'} b(n-1)$$

The amount of risk $B'_{n-1} q^{1/2}$ for holding a bond is a function of $b(n-1)$ which in turn depends on the dynamics of the stochastic discount factor A'_m and the dynamics of the macro model A. Ignoring the convexity term $B'_{n-1} q B_{n-1}/2$, we can then re-write the excess holding period return as a sum of rewards to the various macroeconomic sources of risk, i.e. the standard deviations of the structural shocks in the model:

$$
\begin{aligned}
ehpr^{n} &= -\sum_{j=1}^{k} q^{1/2}_{jj} \left[\Lambda^{'}_{0}\right]_{j} \left[B^{'} b\,(n-1)\right]_{j} q^{1/2}_{jj} \\
&\quad - conv.T.
\end{aligned}
\tag{37}
$$

The k market prices of risk are equal to $\sigma\left(m_{t+1}\right) = q^{1/2}_{jj}\left[\Lambda^{'}_{0}\right]_{j}$ and the amount of risk carried in each bond with respect to shock j or the "bond price sensitivity to shock j" is equal to $\left[B^{'} b\,(n-1)\right]_{j} q^{1/2}_{jj}$. The prices of risk are the size of the effect of the shocks on the stochastic discount factor, i.e. the valuation of risk in marginal utility terms, and the bond price sensitivities are the present value of the reaction of the nominal interest rate R_{t+i} to that shock for $i = 1, ..., n$, where n is the maturity of the bond:

$$\begin{aligned} ehpr^n &= -\left(\sum_{j=1}^{k} \sum_{i=1}^{n} \frac{\partial R_{t+i}}{\partial v_{j,t+1}} q_{jj} \left[\Lambda_0 \right]_j \right) \\ &\quad -conv.T. \end{aligned} \tag{38}$$

The above equation illustrates the link between the structural model parameters, persistence and the size of the excess holding period return. The $ehpr^n$ depends essentially on the impulse response functions of the nominal interest rate to the shocks in the model ($\frac{\partial R_{t+i}}{\partial v_{j,t+1}}$) as well as the effect of the shocks on marginal utility of consumption ($\left[\Lambda_0 \right]_j$). Innovations in the expected future interest rate must be large and negatively correlated with the marginal utility of consumption for the model to produce large premia. Then, bonds are a bad hedge and agents require a term premium. The model can produce large premia, only if it generates the appropriate dynamic behaviour in the short term interest rate R_t. In the end, asking the question of whether this model for the economy can generate a term structure that fits the data reduces to asking whether the dynamic behaviour required for R_t and inflation π_t to fit the term structure can be reconciled with the one that is required to fit the macroeconomic data.

3.3 Inflation risk premia

So far, we have used the nominal discount factor $m_{t+1} = \lambda_{t+1} - \lambda_t - \pi_{t+1}$ to price nominal zero-coupon bonds of maturity n through the Euler equation. If we define $\Delta\lambda_{t+1} = \lambda_{t+1} - \lambda_t$ as the real discount factor, we can use the Euler equation to price indexed bonds and obtain an expression for the inflation premium in this model. For example, it is straightforward to see that the real return to a one-period bond is given by:

$$-E_t \left(\Delta\lambda_{t+1} \right) - \frac{1}{2} Var_t \left(\Delta\lambda_{t+1} \right) = R_t^r \tag{39}$$

and the nominal return on the one-period bond R_t by:

$$R_t = R_t^r + E_t \pi_{t+1} - \frac{1}{2} Var_t \left(\pi_{t+1} \right) + cov_t \left(\Delta\lambda_{t+1}, \pi_{t+1} \right) \tag{40}$$

The above expression resembles the Fisher equation except that it includes a term for the inflation premium ($cov_t \left(\Delta\lambda_{t+1}, \pi_{t+1} \right)$). The nominal interest rate on a 1-period bond equals the rate on an indexed bond of the same maturity adjusted for expected inflation (taking into account a Jensen's inequality term), plus an inflation risk premium which arises because some of the model's shocks affect both inflation and the real discount factor. If this creates a positive correlation between $\Delta\lambda_{t+1}$ and π_{t+1}, then the inflation premium on the short term bond will be positive. In any case, nominal rates need not move one-for-one with expected inflation, i.e. the Fisher hypothesis does not have to hold. Longer term bond prices can include not only a real term premium, but also an inflation risk premium. In fact, we can use the excess holding period returns on the nominal and indexed n-period bond to derive an expression for the inflation risk premium.

First, define the excess holding period return on an indexed bond $ehpr_t^{r,n}$, using the real stochastic discount factor $\Delta\lambda_{t+1}$ to price the long term asset:

$$\begin{aligned} ehpr_t^{r,n} &= E_t r_{t+1}^{r,n} + \frac{1}{2} Var_t \left(r_{t+1}^{r,n}\right) - r_t^r \\ &= -cov_t \left(r_{t+1}^{r,n}, \Delta\lambda_{t+1}\right) \end{aligned} \tag{41}$$

where $r_{t+1}^{r,n}$ is the holding period return on an indexed bond. Then write down the excess holding period return on an n-period nominal bond $ehpr^n$ and notice that it can be broken down into two terms, the first one involving the covariance between the holding period return on the nominal bond r_{t+1}^n and the real discount factor $\Delta\lambda_{t+1}$ and the second one involving a covariance with inflation π_{t+1}:

$$\begin{aligned} ehpr_t^{n} &= E_t r_{t+1}^{n} + \frac{1}{2} Var_t \left(r_{t+1}^{n}\right) - r_t \\ &= -cov_t \left(r_{t+1}^{n}, \Delta\lambda_{t+1}\right) + cov_t \left(r_{t+1}^{n}, \pi_{t+1}\right) \end{aligned} \tag{42}$$

Because they are measured relative to different short term assets these excess holding period returns are not directly comparable to each other. However they are very useful because their values can be used to calculate the implicit n-period inflation risk premium as the difference between the nominal and the indexed bond excess holding period returns (also see Campbell and Viceira (2001)):

$$E_t r_{t+1}^{n} + \frac{1}{2} Var_t \left(r_{t+1}^{n}\right) - r_t - E_t r_{t+1}^{r,n} + \frac{1}{2} Var_t \left(r_{t+1}^{r,n}\right) - r_t^r \tag{43}$$

$$= cov_t \left(r_{t+1}^{r,n}, \Delta\lambda_{t+1}\right) - cov_t \left(r_{t+1}^{n}, m_{t+1}\right) \tag{44}$$

$$= \widetilde{b}(n-1)' B' q B_{\Delta\lambda} - b(n-1)' B' q \Lambda_0 \tag{45}$$

As before, we drop the time indices because all conditional second order moments in the model are constant. The last line gives the inflation premium on an $n-$period bond in terms of the reduced form state-space parameters of the DSGE model B, Λ_0 and q, and tildes denote the parameters of the indexed bonds price equations. Using some algebra it is easy to show that:

$$\begin{aligned} \Lambda_0 &= B_{\Delta\lambda} - B_\pi \\ b(n) &= \widetilde{b}(n)' - \sum_{j=1}^{n} A_\pi' A^{(j-1)} \end{aligned}$$

where $A^{(j-1)} = \underbrace{A^0 A ... A}_{j-1}$ and $A^0 = I$. Replacing $b(n-1)$ and Λ_0 in (45) we find:

$$\begin{aligned} ehpr^n - ehpr^{r,n} &= \left(\widetilde{b}(n-1)' - b(n-1)'\right) B'qB_{\Delta\lambda} + b(n-1)'B'qB_\pi \\ &= \left(\sum_{j=1}^{n} A'_\pi A^{(j-1)}\right) B'qB_{\Delta\lambda} \\ &\quad +\widetilde{b}(n-1)'B'qB_\pi \\ &\quad -\left(\sum_{j=1}^{n} A'_\pi A^{(j-1)}\right) B'qB_\pi \end{aligned} \tag{46}$$

To give an intuition of the above expression take e.g. $n = 3$ and rewrite equation (44):

$$\begin{aligned} ehpr^{n=3} - ehpr^{r,n=3} &= cov(\Delta\lambda_{t+1}, E_{t+1}\pi_{t+2}) + cov(\Delta\lambda_{t+1}, E_{t+1}\pi_{t+3}) \\ &\quad +cov(\pi_{t+1}, E_{t+1}(\lambda_{t+3} - \lambda_{t+1})) \\ &\quad -cov(\pi_{t+1}, E_{t+1}\pi_{t+2}) - cov(\pi_{t+1}, E_{t+1}\pi_{t+3}) \end{aligned} \tag{47}$$

The inflation premium per se is given by the sum of the first three terms and involves (1) the covariances between marginal utility and expected inflation at different horizons, and (2) the covariance between inflation and the expected real discount factor for pricing the assets over the n-period. They capture the compensation that households receive for (simultaneous) uncertainty in the valuation of the shocks in marginal utility terms and uncertainty in the future path of inflation. The last two terms are convexity terms due to the autocorrelation of the inflation process. The first three covariances can be positive or negative; this will depend on the effects of monetary policy and the real effects of nominal shocks in the DSGE model. As a result, the above expression can be negative if the convexity term is very large relative to the first three terms, or if the first three terms (the inflation premium per se) is negative. We will explore the implications of a change in the model parameters and the size of the shocks for the sign and size of the inflation premium in the empirical part of the paper.

4 Results

This section of the paper concentrates on three issues. First, can the DSGE model generate term premia that match the ones that we see in the US data? Second, which model parameters need to be hiked up to obtain larger premia? And third, are these values consistent with both the macro and finance data characteristics? The first two questions are answered using calibration, while the third question is answered by estimating the DSGE model under the restriction that the term premium on the 10-year bond generated by the model matches the one in the data.

4.1 Descriptive statistics

Table 1 presents some descriptive statistics for the US yield curve. Quarterly data on zero-coupon yields between $1961 : Q1$ and $2003 : Q3$ are obtained from the BIS database which uses spline interpolations to approximate for the yields of the missing maturity bonds. I compute the average ex-post excess holding period return (ehpr) to approximate for the unconditional mean of the term premium.

(Insert Table 1 here)

Table 1 shows that the average ex-post ehpr for the 10-year bond is equal to 100 bps. Notice however, that the associated standard deviation is very large (551 bps !) which reflects the fact that excess holding period returns exhibit a high amount of time-variation. This is confirmed by other papers using different data sets as well. For example, Campbell, Lo and MacKinley (1997) using the McCulloch and Kwon (1993) data set over the period 1952-1991 and monthly data find that the ehpr associated with the 10-year bond equals -4.8 annualized bps with a standard deviation of 3708 bps ! With the same dataset but considering a different period (1960 - 1997) and quarterly frequency Hordahl, Tristani and Vestin (2005) find that the average ehpr of the period was equal to 60 annualized bps. Furthermore, the *ehpr* over subperiods varies very strongly. For example for the period 1960-1978 it equals -164 bps, while for 1983-1997 it equals 460 bps. This evidence indicates that probably a more appropriate model for the term premia would have a time-varying mean. For the moment, however, since the log-linear -log-normal approach of this paper generates constant term premia, the calibration will focus on replicating the average 10-year ehpr of 100 bps. When interesting, I will also present the entire model-generated term structure of interest rates. In this respect, it is useful to discuss some more yield curve stylized facts for the US.

The average ex post excess holding period return is increasing and varies from 0.5 bps for the 6-month bond up to 100 bps for the 10-year bond. Average yields over this period are increasing with maturity and vary from 6.14% for the 3-month Treasury bill to 7.18% for the 10-year bond. The spread between the yield of a bond of long maturity and the 3-month risk free rate (i.e. the slope of the term structure) is also increasing with maturity and varies from 12.54 bps for the 6-month bond to 104.07 bps for the 10-year bond. The standard deviation of bond yields during this period is slightly decreasing (from 2.85% for the yield of the 3-month bill to 2.34% for the 10-year bond). The upward slope of the mean term structure of US interest rates and the slightly downward slope of the volatilities are considered by the literature as stylized facts, while the upward slope of the average ehpr's although more controversial can be confirmed for bonds up to a maturity of 5 years. As we saw earlier, whether the average ehpr for 10-year bond declines or increases with respect to the ehpr of the 5-year bond depends very much on the sample used.

4.2 Model-generated premium: Calibration exercise

What is the 10-year term premium[2] generated by the estimated DSGE model? Does it match the 100 bps that we observe in the yield curve data? To answer these questions, I estimate the baseline DSGE model[3] and then I calibrate it at the estimated parameter values (shown in the first column of *Table 5* and discussed later on) to generate the average nominal and real *ehpr* as well as the inflation premium according to equations (37)), (41) and (45).

Figure 1 shows the average *ehpr* in annualized basis points across maturities (in quarters) generated in this way.

(Insert Figure 1 here)

From this figure , we can see that although the nominal *ehpr* produced by the baseline DSGE model is positive, its size is very small compared to the *100bps* in the US data . The small size of the nominal *ehpr* is generated on the one hand by a small real *ehpr*, and on the other by a negative and large inflation premium for longer maturities.

In particular, the nominal excess holding period return produced by the baseline model is positive but is hump-shaped over maturities. It reaches a maximum of 6.8 bps at 10 quarters (2.5 years) and drops to *3.7 bps* for maturities over ten years. The real ehpr is increasing through all maturities, even after 10 quarters, reaching *8.3 bps* at maturities larger than 10 years. The inflation premium can be deduced from this graph by taking the difference between the nominal and the real premium corrected for a convexity term. The inflation premium is positive and decreasing up to 4 years and becomes negative and increasing (in absolute terms) for higher maturities. For the 10-year bond it equals to *-3 bps*.

To better understand the mechanisms in the DSGE model that cause the small size of the nominal and real *ehpr*, and the negativity of the inflation premium, in *table 2, column (3)* I break down the nominal *ehpr* into a sum of compensations to bear the risk associated with each macroeconomic shock separately. I concentrate on the *ehpr* for the 10-year bond.

We can distinguish between two cases according to the sign and the size of compensations. Compensations can be positive or negative depending on whether the shock needs to be hedged (negative sign) or insured (positive sign). They can be large or small depending on whether shocks are persistent or temporary and their variance is, accordingly , very large or small.

[2] I use the terms 'term premium' and 'expected excess holding period return' (ehpr) interchangeably from here on.

[3] The model is estimated with Bayesian estimation techniques using the priors of Smets and Wouters (2004). For the details of the approach see Smets an dWouters (2004).

In this sense, the most persistent shocks that need to be insured, i.e. the productivity shock, the labour shock and the government spending shock, are associated with the largest positive compensations. On the other hand, negative compensations are associated with shocks to the nominal side of the economy, i.e. the wage and price mark-up shock and the inflation target shock. These shocks work like hedges for the household, i.e. their contribution to the total *ehpr* is negative, and the more persistent they are, the larger their contribution. As a result, the price (negative compensation) the household pays for hedging inflation target shocks is as large as the compensation he receives for taking up the risk associated with the productivity, labour and government spending shocks combined. The rest of the shocks play a smaller role because their persistence is smaller.

We can understand better the mechanisms at work within the model by decomposing the above risk compensations into a product of a price of risk (valuation of risk in marginal utility terms (*table (2), column (1)*)) and the amount of risk carried by the bond, or "bond price sensitivities" to the different shocks (*table (2), column (2)*). Bond price sensitivities are the present value of the response of the short interest rate to the shock between today and n quarters ahead (see equation (38)). For reference, the impulse response functions for the short interest rate are shown in *figure (2)*.

(Insert Figure 2 here)

Shocks that affect the short interest rate strongly and persistently make the price of the bonds move. For example, a positive productivity shock raises consumption and reduces the current marginal utility of consumption for a given leisure choice. At the same time the productivity shock produces a decrease in the short term interest rate through the policy rule. This implies that the short term bond pays off an unexpectedly high price at a time when consumption is high. The positive correlation between the marginal utility of consumption and the price of the bond makes the bond a bad hedge and therefore households will require a positive premium to hold it. In terms of table (2), the contribution of the productivity shock to the total nominal premium is positive and large.

The inflation target shock contributes negatively to the premium. A positive shock to the inflation target results in an increase in consumption (*figure 2*) and a decrease in the marginal utility of consumption for a given leisure choice. Disinflation or a negative shock to the inflation target has a real cost in consumption terms. The short term interest rate increases. The short term bond pays off a low price during 'good' (high consumption) times. The negative correlation between the asset's payoff and the state of consumption makes this asset a good hedge. Households will want to hold it and therefore its premium will be negative. As a result, its total contribution to the total premium will be negative.

It appears that in a first step, eliminating the inflation target shock from the model should guarantee that the *ehpr* remains an increasing function of maturity n. Other ways to increase the model-generated *ehpr*, should be to change the structural parameters of the DSGE model. In the next section, I perform a calibration exercise to search through the model parameters and find those ones that effectively increase the risk compensations of the shocks and the model *ehpr*. I leave the task of evaluating whether the required parameter values would make any sense from a macro point of view for the last section of the paper.

4.3 Sensitivity exercise: A change in which parameters can potentially increase the model-generated premium?

4.3.1 Changes in the model stochastic processes

Table 4a shows the effect on the term spread and the *ehpr* of reducing to zero the variance of each of the shocks, one at a time. As expected, the largest effect is produced by shutting down the inflation target shock: The *ehpr* on the ten-year bond increases from 3.71 bps to 12.06 bps. This is still a long way from the 100 bps in the US data. Shutting this shock down will always increase the premium independently of the model we are considering, and whatever the values of the rest of the parameters.

(Insert tables 4a and 4b here)

The lower part of *table 4a* shows the effect on the *ehpr* and the spread from a 10% reduction of the persistence of the exogenous processes. This change is compensated with an increase in the variance of the shock so that the variance of the process remains constant. A decrease in the persistence of the processes combined with a compensating increase in the variance of the shock increases the premium and the spread. It seems that the effect that dominates is the compensating and huge increase in the variance of the shock. As a result, although the shocks are less persistent, because their size is larger, they generate premia that are larger and in some cases even match the ones in the data. This is true in particular for the productivity shock, E_A., labour supply shock E_L and government spending shock E_G. Remember however that, because these shocks were so persistent to begin with, the compensating increase in the variance is large and therefore, even though the small change in persistence achieves the desired increase in the *ehpr*, the large compensating change in the variance should be very costly in marginal likelihood terms.

Finally, the last column of *table 4a* shows the mean risk free rate implied by the changed model. The effect is always very small, and the mean risk free interest rate for the different models is equal or larger than the one in the baseline model. The only exception is for changes in the persistence of the shocks, and especially for the productivity shock. Reducing the persistence of this shock also reduces the mean risk free rate implied by the model.

4.3.2 Changes in the model parameters

To briefly summarise the main results from the sensitivity exercise, we find that the term spread and the *ehpr* are most sensitive to changes in the degree of risk aversion (σ_c), the degree of habit persistence (h) and the aggressiveness of monetary policy (r_pie). A change in the capital utilisation cost enhances the effect of the above parameter changes. Nominal rigidities play a limited direct role.

A note of caution is required here: One should remember that only an estimation exercise as the one in the last section of the paper can tell us which combination of parameter changes is needed to simultaneously fit the macro data and the observed 10-year *ehpr*. The results that we present here in detail, only aim to shed some light on the mechanisms within the DSGE model that can potentially generate higher premia. Estimation of the model will show which of these are in fact supported by the data.

*** The utility parameters σ_c, σ_L, and h** The top of *table 4b* shows that an increase in the curvature of the utility function with respect to consumption up to $\sigma_c = 5$ results in a ten-fold increase in the *ehpr*, which equals 33.94 bps and in an increase by 15 bps of the nominal spread. However both the *ehpr* and the term spread remain very low.

An increase in the habit parameter h from 0.6 to 0.9 only slightly increases the term spread and brings the *ehpr* up to 17.18 bps. The observation that the *ehpr* is not very sensitive to changes in the habit persistence parameter has been confirmed elsewhere in the literature. For example Jermann (1998) and Ferson and Constantinides (1991) have shown that, even though habit forming preferences can generate high risk premia in an exchange economy, they fail to do so when the consumption path is endogenously determined. This happens because the agents choose a smoother consumption path when they are more averse to intertemporal substitution. Consumption volatility will be lower than in the exchange economy case compensating for the increase in habit persistence. The two effects compensate each other leaving the risk premium unchanged. This effect of habit on the term premium is interesting to note particularly in view of the fact that the premium increases substantially when we simultaneously increase the cost of capital utilisation. With higher habit consumers desire to smooth consumption more, but if the costs of capital utilisation are high they cannot do so. As a result, the premium will have to rise.

On the other hand, even though habit-forming preferences alone may not always generate a higher term premium, they may help in generating a higher risk free rate. Indeed in the last column of *table 4a*, we can see that the expected risk free rate is closer to its baseline value when $h = 0.9$ than when $\sigma_c = 5$ (the expected risk free rate is equal to 5.761 for a high h vs. 5.286 for a high σ_c).

To better understand the channels through which a change in σ_c and h works, *Table 5a* decomposes the *ehpr* into a sum of products of prices of risk and bond price sensitivities.

(Insert table 5a here)

Recall that the *ehpr* is defined as the conditional covariance between the one-period ahead marginal utility of consumption and the holding period return r^n_{t+1} of the $(n-1)$-maturity bond. *Table 5a* shows that the shocks that contributed a great deal in the premium for the baseline model contribute even more for higher values of σ_c and h. This is true in particular for the productivity and the government-spending shocks. The contribution of the labour supply shocks remains constant or falls and the contribution of the inflation target shock remains constant. On the other hand all the other shocks which did not contribute much to the premium, still do not do so.

(Insert table 5b here)

Figure 3 considers combined changes of both σ_c and habit persistence. It shows that changing habit helps reach a higher *ehpr* especially if σ_c is high. For example, in the case of $\sigma_c = 5$ and $h = 0.9$ the 10-year term *ehpr* equals 123.35 bps (*table 5b, column (a)*). At the same time, the expected risk free rate is lower than the baseline model (4.398% vs 5.860%, annualized, *table 5b*). Looking at the contributions of the different shocks, as previously the largest contribution come from the productivity shocks and the government spending shock, but also the role of the investment shock is reinforced mainly because the bond price sensitivities have increased. Furthermore, notice that the signs of the contributions of some shocks change. This happens for the labour supply shock, the interest rate shock and the wage mark-up shock. The sign of the price of the shock changes in the first case, while it is the 10-year bond price sensitivity that changes sign in the two other cases.

I also calibrate the model for $\sigma_c = 2$ and $h = 0.8$ and simultaneously shutting down the inflation target shock. From the previous section we know that shutting down the inflation target shock will increase the 10 year *ehpr* by at least 8 bps. The result of this experiment is shown in the last column of *table 5b*. The size of the 10-year premium is still small, approximately 17 bps. This version of the model stays very close to the baseline model; we can see that from the contributions of the different shocks to the term premium, the prices of the shocks which remain virtually unchanged (except for the the productivity and the government spending shocks) and the signs of the bond price sensitivities, which also remain unchanged.

Finally, we have considered the effects on the *ehpr* of an increase in the labour disutility σ_L parameter from 2 to 4.The change leaves both the term spread and the term premium unchanged. This can also be seen in *figure 4* , where the 10-year term premium remains virtually independent of σ_L for any given degree of σ_c. Introducing labour in the utility function in a non-separable way does not help generate a larger term premium.

(Insert figures 3 and 4 here)

In conclusion, only the utility parameters σ_c and h, related to consumption appear to play a role in setting the long term premium. The values for these parameters that are implied by the data are relatively high (in the neighbourhood of $\sigma_c = 5$ and $h = 0.9$). An increase in these parameters boosts the contributions to the premium of shocks that contributed strongly in the baseline parameterisation of the model, i.e. the productivity and the government spending shock. However, these values are so high that they should produce counterfactual IRFs of the risk free rate to other shocks in the model, like the monetary policy and the wage mark-up shock.

*** Real rigidities: investment adjustment costs, fixed cost, and capital utilisation cost** Three additional types of real rigidities are part of the macro model: investment adjustment costs, a fixed cost, and a capital utilisation cost.

(Insert figures 5, 6a, 6b and 7 here)

Figure 5, 6a, 6b and *7* show that changing the degree of real rigidities has a limited effect on the *ehpr*. Increasing the investment cost (*figure 5*) or the fixed cost (*figure 7*), results in a slight increase of the *ehpr*, up to 8(30)bps. Increasing the inverse of the elasticity of the capital utilisation function (*czcap*) decreases the *ehpr* (*figure 6a*): A higher *czcap* means that it is more costly to use more intensively the installed capital. This discourages households from smoothing consumption and the term premium decreases (*figure 6a*).

The relation between the term premium and the cost of capital utilisation will depend on the other parameters of the model. For example, *figure 6b* shows that when $\sigma_c = 5$ and $h = 0.9$, a higher *czcap increases* the term premium. This goes into the direction of what Jermann (1998) finds in the case of a real business cycle model with capital utilisation costs: In order to generate large term premia in a model with high risk aversion and habit parameters, capital utilisation costs need to be high to force households to consume more than they would like. The mechanism is the following: To obtain a large premium households must have a large incentive to smooth intertemporally i.e. the difference between today's and tomorrows marginal utility must be large. We saw that with habit preferences and an endogenous consumption process, the consumption path becomes smoother and the variance of consumption becomes smaller, making the premium too small. Adding costly capital adjustment in the model amounts then to adding a technology that does not allow consumers to smooth. Because it becomes more costly to smooth through a change in the capital stock, the consumer will have to take up more risk in consumption. The volatility of consumption will increase as will the term premium.

*** Nominal rigidities** *Table 4a* shows that shutting down all nominal rigidities, either in the form of price and wage indexation or stickiness, has no effect on the long run *ehpr*. *Figure 8* confirms this.

(Insert figure 8 here)

Furthermore, as is shown in *figure 8*, there is no interaction between nominal rigidities and the utility parameters. However, one should be cautious when evaluating the importance of nominal rigidities for generating high term premia and keep in mind that even though the direct effect of nominal rigidities on the long run *ehpr* does not appear to be strong, the indirect effect of introducing nominal rigidities in the DSGE model is important: Nominal rigidities introduce inflation in the DSGE model, and as we saw previously the persistence of the reaction of inflation to the structural shocks is key for generating a high *ehpr*.

*** Monetary policy and the inflation premium** The last part of *table 4* examines how the *ehpr* varies with a change in the parameters of the monetary policy rule. The parameters only that change here are the monetary policy rule ones, the rest are fixed at the baseline values.

First, a reduction in the degree of interest rate smoothing increases the *ehpr* on the 10-year bond. So does a less aggressive rule (smaller reaction of the interest rate to deviations of inflation from its target) and a rule more sensitive to deviations of output from potential output (the level of output that would prevail in the flexible price economy). Innovations in R_{t+1} must be large and negatively correlated with the marginal utility of consumption to produce large term premia. This implies that any parameter choice that makes monetary policy less predictable will generate a larger *ehpr*.

Figures 1 and *9* depict the *ehpr* on a nominal bond over n maturities, the *ehpr* to an indexed bond and the *ehpr* on a nominal bond corrected for the convexity term induced by the variance of inflation, i.e. the different components of equation (46). We examine the size and sign of the inflation premium for the baseline calibration of the model as depicted in *figure 1* and for the alternative model ($\sigma_c = 2$, $h = 0.8$ and the variance of the inflation target shock set to zero) in *figure 9*.

The reason why I focus on the calibration $\sigma_c = 2$, $h = 0.8$ and $V(E\_PIE\_BAR) = 0$ is the following. First, as we have seen in *table 5b,* the *ehpr* on the 10-year bond in this case is higher than in the baseline calibration of the model and therefore closer to the value in the data. Second, shutting down the inflation target shock ensures that the *ehpr* is increasing over all maturities, and that the convexity term due to inflation (see equation (43)) is not dominated by the variance of this permanent shock. Finally, as we can see from the impulse response functions of consumption, inflation and the short interest rate depicted in *figure 2*, even if this model produces higher *ehprs* it should still fit the macro data well, given that its irfs are very close to those of the baseline model. These features make this particular calibration an interesting benchmark for comparison with the results from the restricted estimation later on.

(Insert figure 9 here)

The baseline model (*figure 1*) produces a positive inflation premium up to 14 quarters. After 14 quarters, the inflation premium becomes negative. Its absolute size increases with maturity, so that the return on an indexed 10-year bond is 8.3 bps whereas the return on the same maturity nominal bond is equal to 5bps.

The humped shape of the return on the nominal bond is due to the negativity of the inflation premium, i.e. the negative correlation between the change in the marginal utility of consumption $\Delta\lambda_{t+1}$ and inflation π_{t+1} or expected inflation $E_{t+1}\pi_{t+2}$ (see equation (46)). Notice that the auto-correlation of inflation produces a convexity term that needs to be substracted from the sum of the real excess return and the inflation premium to obtain the nominal excess return. The convexity term, however is not as big as the inflation premium. This implies that when we calibrate the model with the variance the (permanent) inflation target shock set to zero, we observe that (*figure 9*), as expected, the convexity term disappears and the inflation premium becomes positive and increasing through maturities. A model without the inflation target shock , produces an inflation process that is less persistent and which becomes positively correlated with the change in the real marginal utility of consumption, therefore requiring a positive inflation premium.

Next, I derive the inflation term premium for different monetary policy strategies and evaluate the impact of a change in the monetary policy parameters on the inflation premium (see *column (d)* in *table 5b, and figure 10*).

(Insert figure 10 here)

Figure 10 shows how the total premium on a nominal bond changes when monetary policy fights inflation less aggressively, i.e. the r_pie parameter drops from $r\_pie = 2$ to $r\_pie = 1.4878$, its baseline value. The *ehpr* on a nominal bond increases through all maturities, and the effect is stronger for longer maturities. However, the *ehpr* on the indexed bond decreases, over all maturities and the effect is relatively strong at the short maturities as well. As a result, for maturities larger than $n = 15$, the first effect dominates the second and the inflation premium in the long maturity bonds is larger for the economies which fight inflation less aggressively. A similar result is found by Hordahl et.al (2005) in a model with fewer shocks but similar structure and equilibrium dynamics. One difference is, however, that although in both models the inflation premium is increased by a less aggressive policy, the size of the inflation premium per se generated by this model is larger.

4.4 Restricted estimation: Which parameter values reconcile the yield curve with the macro data?

I estimate the baseline macro model with the additional restriction that the excess holding period return on the 10-year bond derived from the model ($ehpr^{n,\mathrm{model}}$) should fit the size of the average (ex-post) excess holding period return in the data ($ehpr^{n,data}$ =100 bps). Estimation is performed using a Bayesian approach as in Smets and Wouters (2004). Within this framework, the cost of deviations from the observed average excess holding period return is captured by an element of $f(\theta)$, the priors set on the model parameters. From the Bayes rule for densities, $f(\theta \mid y)$, the posterior distribution of the parameters given the data y is given by:

$$f(\theta \mid y) = \frac{f(\theta)L(\theta; y)}{f(y)} \propto f(\theta)L(\theta; y)$$

where $L(\theta; y)$ is the likelihood of the data and $f(y)$ is the marginal density of the data, obtained by the Laplace approximation of the integral:

$$f(y) = \int_{\Theta} f(\theta) L(\theta; y) d\theta$$

The idea is that since the excess holding period return is just a function of the model parameters, one can include this additional information on θ by imposing an additional prior on the function of θ that gives the 10-year ehpr ($ehpr^{n,\text{model}}$). The change in the marginal density of the data $f(y)$ indicates the cost of imposing the restriction that $ehpr^{n,\text{model}} = ehpr^{n,data} = 100$ bps. I show the results for $f(ehpr^{n=40,\text{model}} - ehpr^{n=40,data}) = \kappa N\left(0, \sigma_{ehpr^{n=40,data}}\right)$, where κ is a sensitivity parameter that determines the tightness of the prior.

The results in *table 3* show that the marginal likelihood changes substantially and increases in absolute value the larger the sensitivity parameter. Furthermore, the mode of the posterior distribution for σ_c increases, as does the persistence of the short interest rate, while the coefficient on inflation in the policy reaction function decreases strongly.

Figure 2 shows the impulse response functions of consumption, the short rate and inflation for this model. Most irfs remain very close to those of the baseline model. The main difference is that shocks that had a very persistent effect on the short interest rate and inflation (technology, government spending and labour supply shocks) and therefore contributed strongly and positively to the ehpr, have an even larger and more persistent effect. Furthermore, these variables react less persistently to the inflation target shock. The combination of these two changes generates a higher *ehpr*. This is shown in *figures 11a and 11b.*

(Insert figures 11a and 11b here)

In *figures 11a and 11b* the nominal and real excess holding period return and the inflation premium over maturities is shown for the model estimated with $\kappa = 1$ and $\kappa = 3$. The nominal total ehpr increases substantially from one model to the other, moving from 14*bps* to 65*bps*. This increase is due mainly to a large increase in the inflation premium from 7*bps* to 60*bps*. The real term premium on the other hand reduces slightly from 6 to 4*bps*. Notice that when the model is estimated with the restriction on the long run *ehpr*, as above, the inflation premium obtained is always large and positive and the convexity term due to inflation is negligible.

5 Summary and Conclusion

This paper examines the size, sign and determinants of the term premium implied by the DSGE model with nominal rigidities of Smets and Wouters (2004). The model economy and its utility function pin down the stochastic discount factor. The Euler equation (i.e. the absence of arbitrage opportunities) prices bonds in a way that is consistent with the economy and in particular models term premia as the covariance between the stochastic discount factor and the bond prices.

For baseline estimates of the DSGE model, the macro-finance model produces small but positive and increasing term premia. Some shocks, like the technology, government spending and the labour supply shock generate large and positive contributions to the term premium. For example, a productivity shock that increases consumption allows monetary policy to relax for several periods and makes consumption and bond prices positively correlated. Households will require a large premium to insure against this shock.

Other shocks generate large but negative 'premia', i.e. they work as hedges. For example, a very persistent and exogenous change in the inflation target of the central bank (i.e. an inflation target shock) generates a negative contribution to the term premium: An increase in the inflation objective has an expansionary effect on consumption making long term bonds, whose prices decrease in reaction to this shock, a good hedge against consumption risk. If we exclude this shock from the model, the inflation premium becomes positive and very sensitive to the persistence of the inflation and the interest rate processes.

A calibration exercise examines the sensitivity of the model-produced term premia to changes in the structural parameters of the DSGE model as well as to changes in the stochastic processes that drive the model shocks. We find that there are several ways to generate larger premia: one is a very large increase of risk aversion or habit persistence, but, as expected, the dynamics of the macro variables will not support this. Other ways are to exclude the inflation target shock from the model or to change the parameters of the monetary policy rule.

The final section of the paper estimates the DSGE model under the restriction that the implied average 10-year term premium matches the observed 100*bps* average *ehpr*. It turns out that in this case the model risk aversion has to rise slightly from 1.8 to 2.7, the habit persistence parameter remains at its baseline level, and the model's impulse response functions of the interest rate and inflation remain very close to the those of the baseline model, except in the case of the very persistent shocks (technology, labour supply and government spending, inflation target shock).

The increase in the term premium is achieved by a drop in the monetary policy parameter that governs the aggressiveness of the monetary policy rule. A less aggressive policy increases the persistence of the reaction of the short interest rate and inflation to any shock, re-inforces the covariance between the marginal rate of substitution of consumption and bond prices, turns positive the contribution of the inflation premium and drives the term premium up. In the end, the yield curve data seem to suggest that monetary policy has been less aggressive than what the macro data might let us think.

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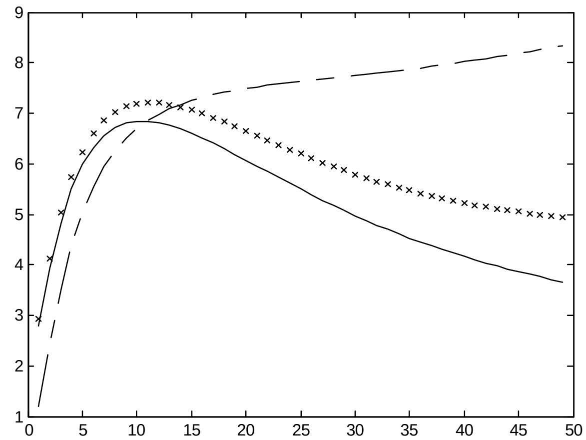
7 Figures and Tables

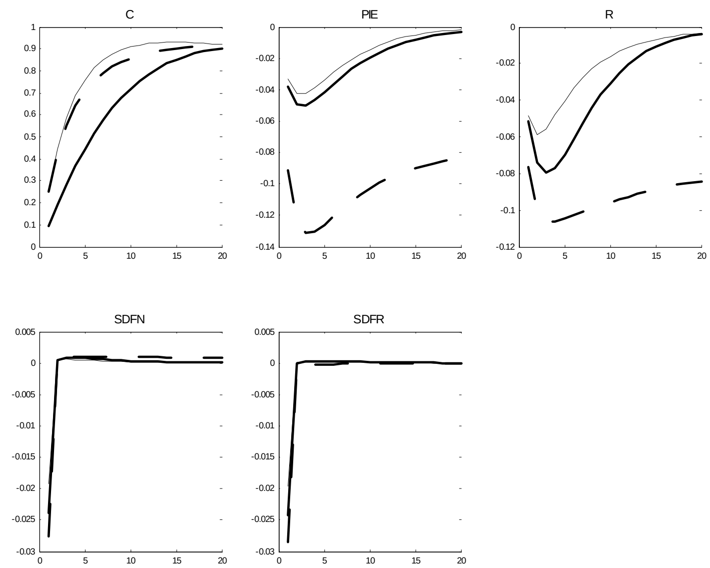
Figure 1: Excess holding period return and decomposition



The graph shows the nominal excess holding period return (line) , the real premium (dash) and the sum of the real and the inflation premium (cross). These values are obtained for the estimated DSGE model (the 'baseline model'). The difference between the nominal excess holding period return and the sum of the real and the inflation premium is equal to the convexity term and is due to auto-correlated inflation. The ehpr and premia are measured in basis points. x-axis is maturities.

Figure 2: Comparing impulse response functions across models

2.1 Technology shock



2.2 Inflation target shock

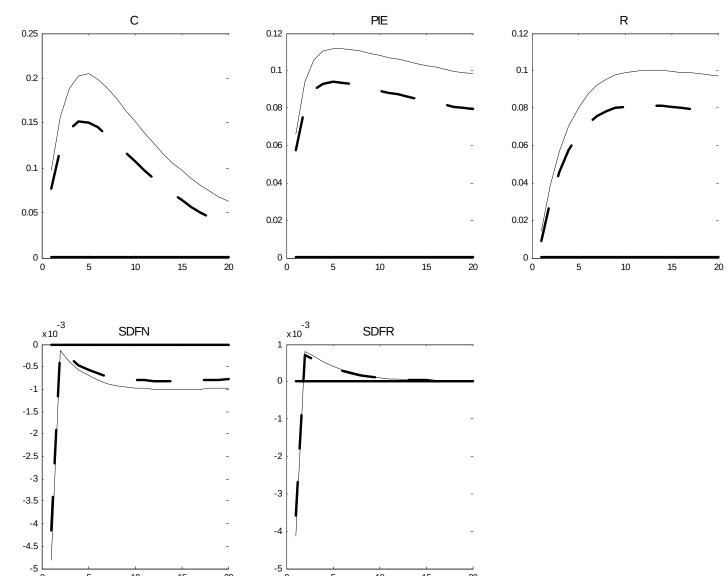
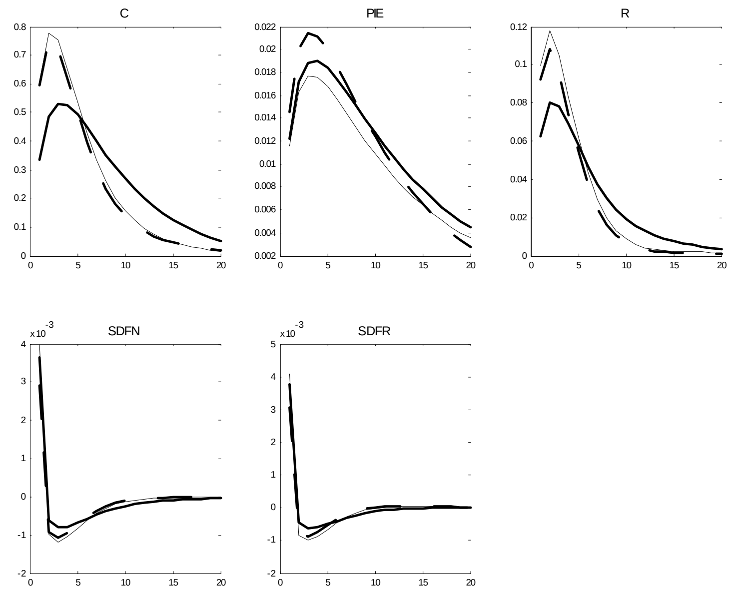


Figure 2 (c'td): Comparing impulse response functions across models

7.3 Preference shock



2.4 Government spending shock

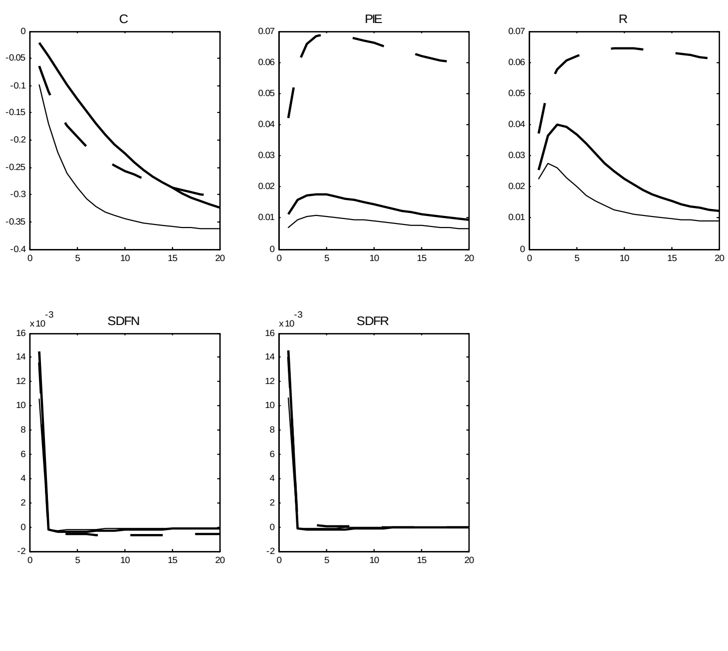
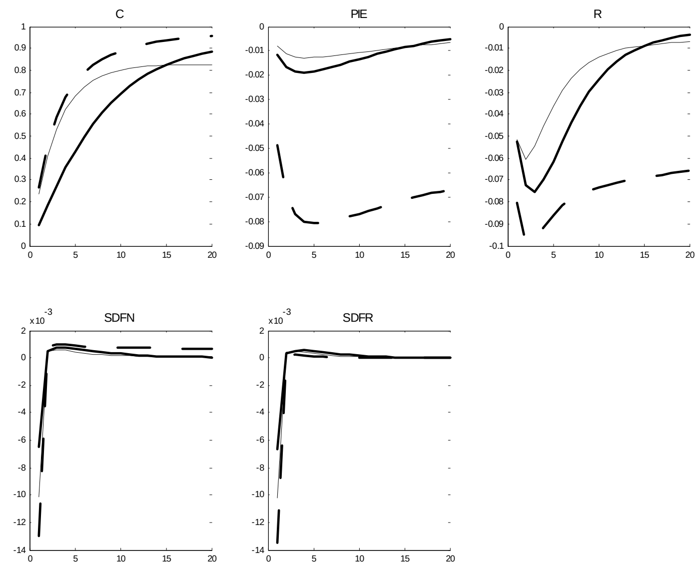


Figure 2 (c'td): Comparing impulse response functions across models

2.5 Labour supply shock



2.6 Investment shock

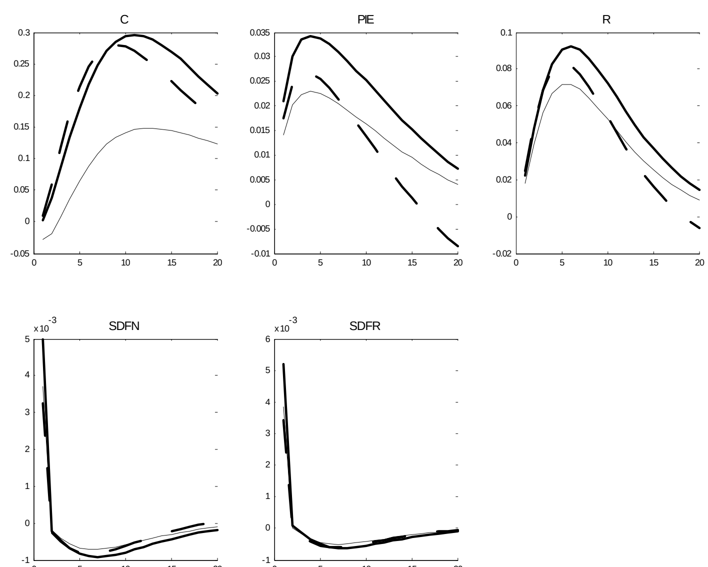
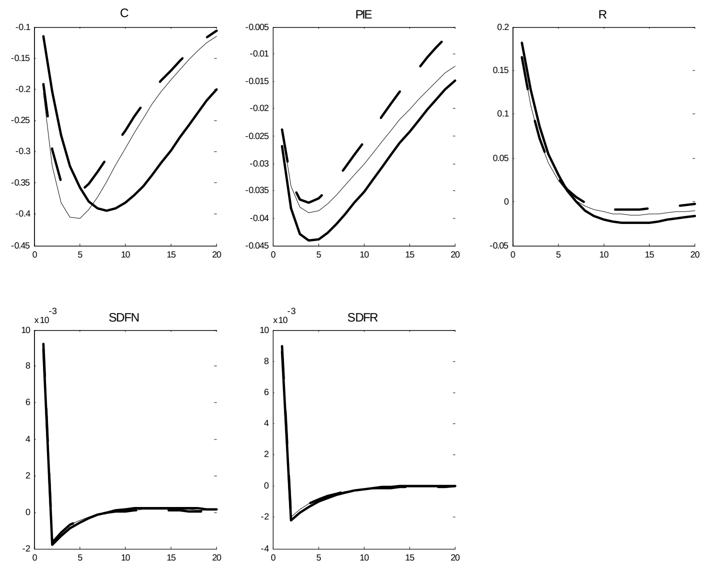


Figure 2 (c'td): Comparing impulse response functions across models

7.7 Monetary policy shock



7.8 Equity premium shock

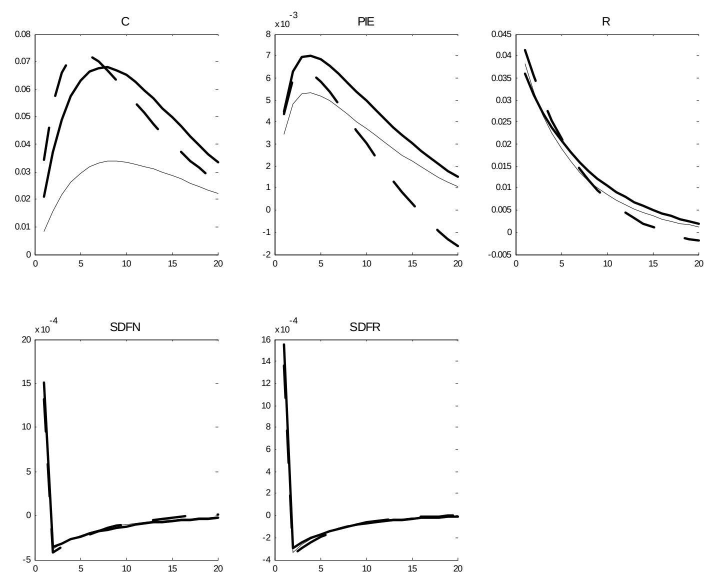
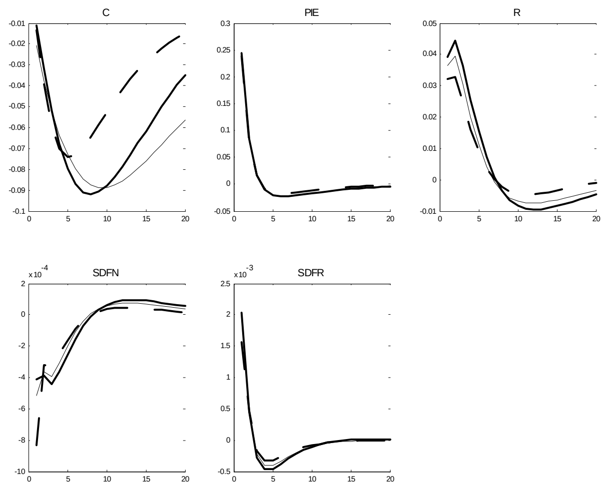


Figure 2 (c'td): Comparing impulse response functions across models

2.9 Price mark-up shock



2.10 Wage mark-up shock

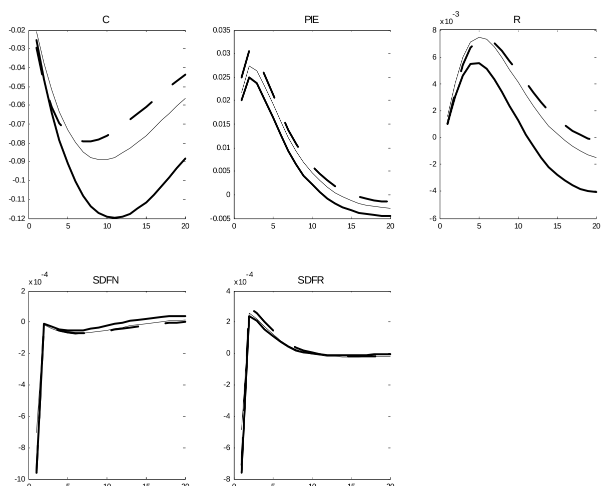


Figure 2 shows the impulse response functions of consumption (C), inflation (PIE), the interest rate (R), the nominal stochastic discount factor (SDFN) and the real stochastic discount factor (SDFR) to the shock of the DSGE model. These are shown for three versions of the model: for the non-restricted estimation (line), the restricted estimation (dashed line) and the calibration exercise for a model with risk aversion=2, habit=0.8, no inflation target shock and the rest of the parameters maintained at their baseline value (bold line).

Figure 3: Ehpr over maturities with consumption risk aversion and habit persistence

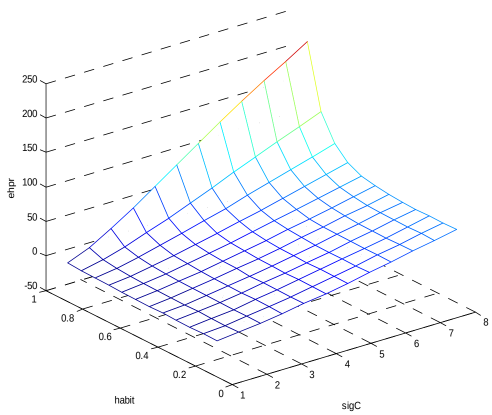
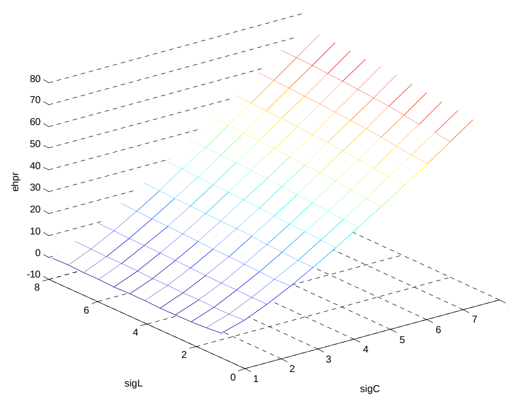


Figure 4: Ehpr over maturities with consumption risk aversion and labour risk aversion



Figures 3 and 4 show the excess holding period return (*ehpr*) as a function of consumption risk aversion (*sigC*) and habit persistence (*hab*) (*Figure 3*) and the excess holding period return as a function of consumption risk aversion (*sigC*) and labour risk aversion (*sigL*) (*Figure 4*). The rest of the model parameters are maintained at their baseline values. The excess holding holding period return is measured in annualized basis points.

Figure 5: Investment cost and ehpr for 10-year bond

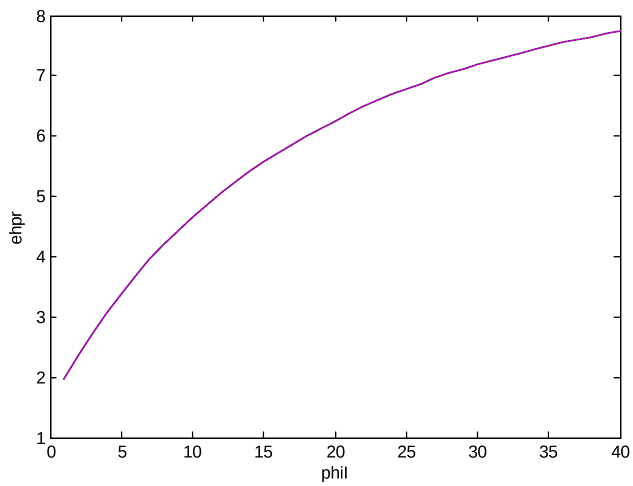


Figure (5) shows the excess holding period return (*ehpr*) for a 10-year bond as a function of the investment cost parameter (*phI*) generated by calibration of the DSGE model. The rest of the model parameters are maintained at their baseline values. The *ehpr* is measured in annualized basis points (bps).

Figure 6a: Cap. utilisation cost and ehpr for 10-year bond

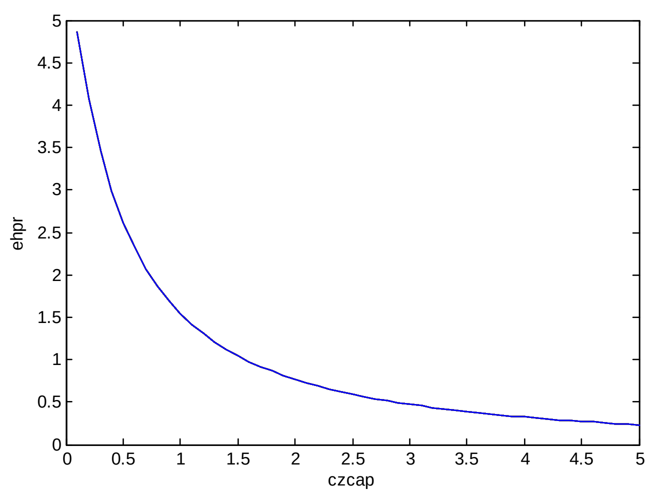


Figure (6a) shows the excess holding period return (*ehpr*) for a 10-year bond as a function of the capital utilisation cost (*czcap*) generated by calibration of the DSGE model. The rest of the model parameters are maintained at their baseline values. The *ehpr* is measured in annualized basis points (bps).

Figure 6b: Cap. utilisation cost and ehpr for 10-year bond, sigC=5, hab=0.9

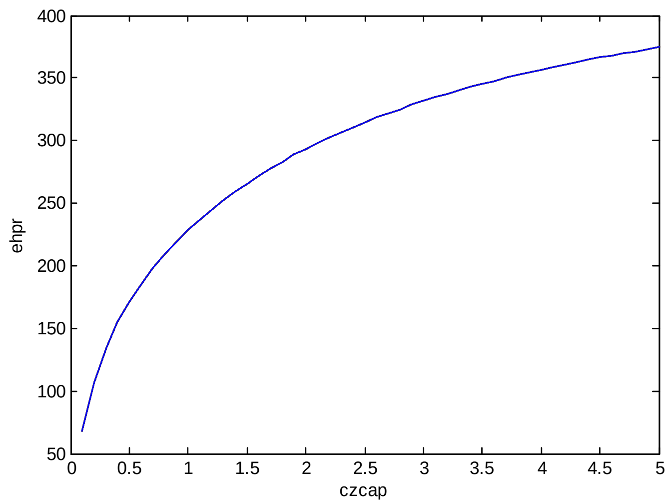


Figure (6b) shows the excess holding period return (*ehpr*) for a 10-year bond as a function of the capital utilisation cost (*czcap*) generated by calibration of the DSGE model. Here, contrary to *Figure 6a*, consumption risk aversion (*sigC*) is set to 5 and habit persistence is set to 0.9 (instead of their baseline values of 1.8 and 0.64 respectively). The rest of the model parameters are maintained at their baseline values. The *ehpr* is measured in annualized basis points (bps).

Figure 7: Fixed prod. cost and ehpr for 10-year bond

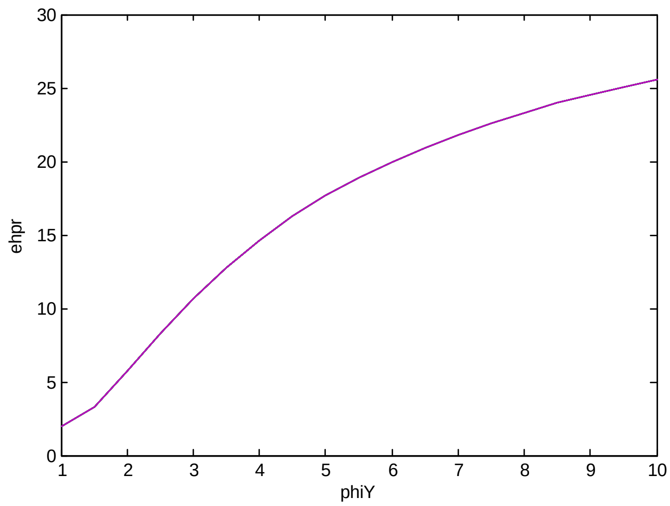


Figure (7) shows the excess holding period return ($ehpr$) for a 10-year bond as a function of the fixed production cost parameter ($phiY$) generated by calibration of the DSGE model. The rest of the model parameters are maintained at their baseline values. The $ehpr$ is measured in annualized basis points (bps).

Figure 8a: Nominal rigidities at baseline

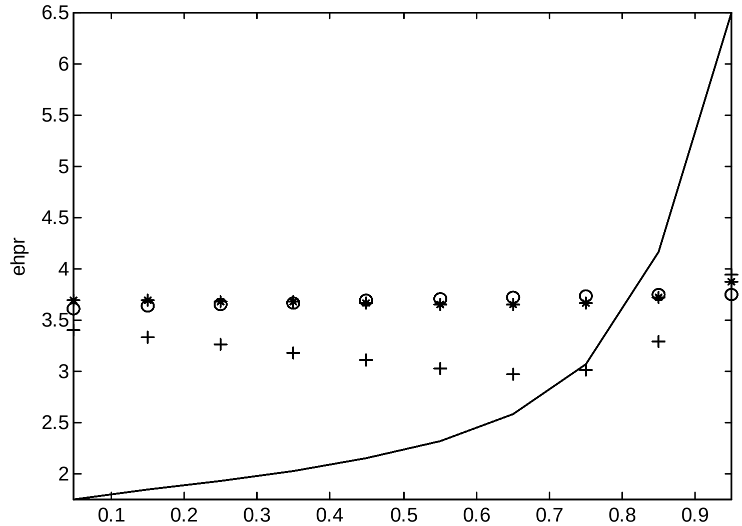


Figure (8a) shows the excess holding period return (*ehpr*) for a 10-year bond as a function of the nominal rigidity parameters generated by calibration of the DSGE model: Calvo wage (xiW) and price (xiP) and partial indexation to past wages (gammaW) and inflation (gamma P). The rest of the model parameters are maintained at their baseline values. The *ehpr* is measured in annualized basis points (bps).

Figure 8b: Nominal rigidities when sigC=5, hab=0.8

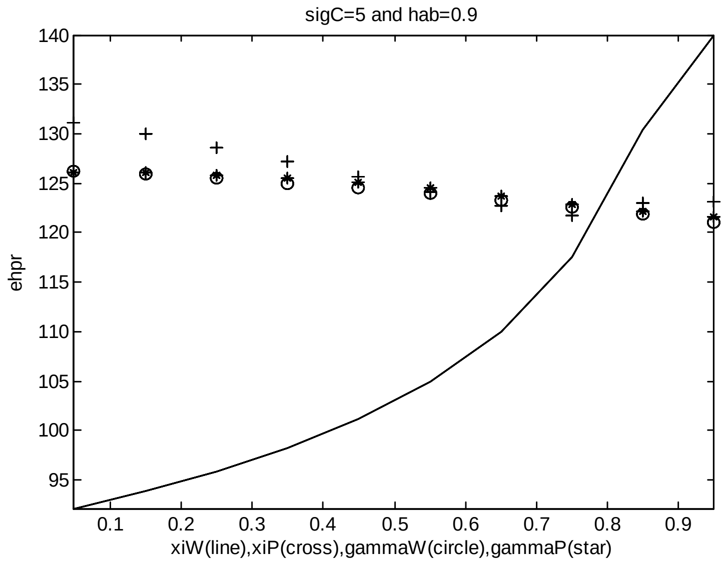


Figure (8b) shows the excess holding period return (*ehpr*) for a 10-year bond as a function of the nominal rigidity parameters generated by calibration of the DSGE model: Calvo wage (xiW) and price (xiP) and partial indexation to past wages (gammaW) and inflation (gamma P). The rest of the model parameters are maintained at their baseline values except for consumption risk aversion set equal to 5 and habit persistence equal to 0.9. The *ehpr* is measured in annualized basis points (bps).

Figure 9: Inflation premium when V(E_PIE_BAR)=0

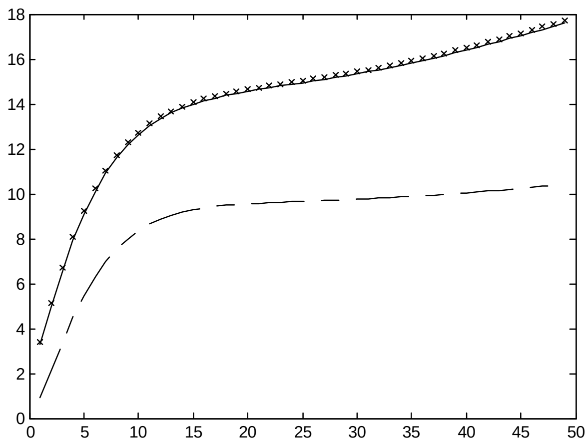


Figure 9 shows the excess holding period nominal return (*ehpr*) (line) , the real premium (dash) and nominal ehpr corrected for the convexity term (cross) over maturities. The ehpr and the premia are in bps and maturities are in quarters. These values are obtained by calibrating the model with the variance of the inflation target shock (*E_PIE_BAR*) set to zero and the rest of the parameters maintained at their baseline values. The convexity term is due to auto-correlated inflation. The nominal ehpr corrected for the convexity term is equal to the sum of the real premium and the inflation risk premium.

Figure 10: Inflation premium for more a aggressive monetary policy

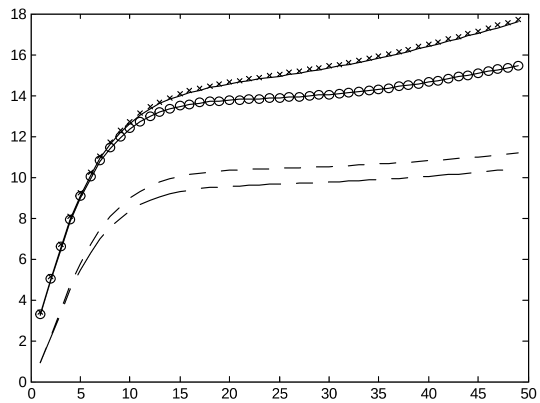


Figure10 shows the excess holding period nominal return (*ehpr*) (line for both model 1 and model 2) , the real premium (dash (model 1) and dot (model 2) and nominal ehpr corrected for the convexity term (cross (model 1) and circle (model 2)) over maturities. *Model 1* sets consumption risk aversion (*sigC*) equal to 2, habit persistence (*hab*) equal to 0.8, the variance of the inflation target shock equal to zero (*V(E_ PIE_ BAR)=0*) , and the rest of the parameters are maintained at their baseline value. *Model 2* sets consumption risk aversion (*sigC*) equal to 2, habit persistence (*hab*) equal to 0.8, the variance of the inflation target shock equal to zero (*V(E_ PIE_ BAR)=0*), the reaction of the short interest rate to deviations from the inflation target (*r_ pie*) equal to 2 and the rest of the parameters are maintained at their baseline value. The ehpr and the premia are in bps and maturities are in quarters. The convexity term is due to autocorrelated inflation. The nominal ehpr corrected for the convexity term is equal to the sum of the real premium and the inflation risk premium.

Figure 11: Nominal, real and inflation premia for 2 estimated models
Figure 11(a) $\kappa = 1$

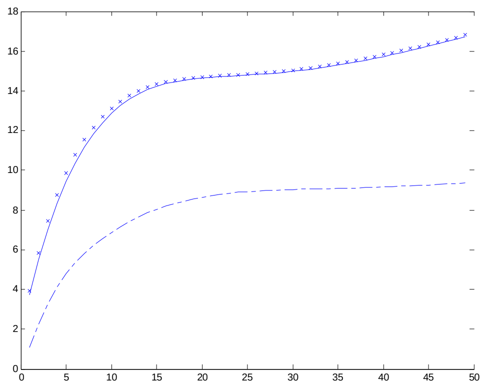


Figure 11(b) $\kappa = 3$

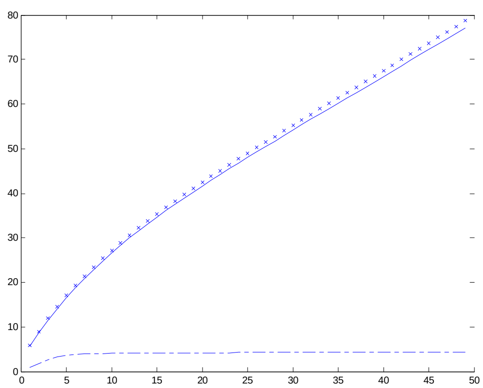


Figure 11 shows the excess holding period nominal return ($ehpr$) for two estimated models under the restriction that the model generated 10-year ehpr equals the average observed ehpr. *Figure 11a* shows the case where $\kappa = 1$, *figure 11b* shows the case $\kappa = 3$ where κ is the sensitivity parameter that determines the tightness of the prior on the ehpr. The nominal ehpr is the continuous line, the real premium is the dashed line and the nominal ehpr corrected for the convexity term is the crossed line. The ehpr and the premia are in bps and maturities are in quarters. The convexity term is due to autocorrelated inflation. The nominal ehpr corrected for the convexity term is equal to the sum of the real premium and the inflation risk premium.

Table 1: Means and standard deviations of term structure variables

Variable	*Bond maturity*				
	3 months	6 months	1 year	5 years	10 years
Yield	6.14	6.26	6.39	6.97	7.18
	(2.85)	(2.79)	(2.74)	(2.48)	(2.34)
Ex-post ehpr		0.01	0.14	0.76	1.000
		(0.48)	(0.96)	(3.51)	(5.51)
Yield spread		12.54	24.94	82.80	104.07
		(21.85)	(41.33)	(114.39)	(142.77)

For each variable the table reports the sample mean and standard deviation (in parenthesis) using quarterly data over the period $1961:Q1-2003:Q4$. 'Yield' is the continuously compounded yield y_{nt}, 'ex-post ehpr' is by the realized log $EHPR_t^n$ calculated using the approximation $y_{n-1,t+1}=y_{n,t+1}$ and 'yield spread' is defined by $y_{nt}-y_{1t}$. The units for the yields and the ex-post excess holding period return are annualized percentage points. The units for the spreads are annualized basis points.

Table 2: The sources of risk and their compensation in a 10-year bond (baseline model)

		1	2	3
Productivity	E_A	1,891	2,260	4,272
Cons. Preference	E_B	-0,376	-2,551	0,959
Government spending	E_G	-1,038	-2,147	2,228
Investment	E_I	-0,354	-2,941	1,040
Labour supply	E_L	1,009	2,669	2,694
Inflation obj.	E_PIE_BAR	0,481	-17,363	-8,347
Price mark-up	ETA_P	0,044	-0,238	-0,011
Equity premium	ETA_Q	-0,134	-0,904	0,121
Interest rate	ETA_R	-0,904	-0,841	0,760
Wage mark-up	ETA_W	0,065	-0,073	-0,005
ehpr				3,713

Table 2 shows the results from the decomposition of the excess holding period return on a 10-year bond into the sum over the 10-year period of the product between the prices of risk $(-q_{jj}^{1/2}\left[\Lambda_0\right]_j)$ and the bond price sensitivities to the shocks $(\sum_{j=1}^{k}\frac{\partial R_{t+i}}{\partial v_{j,t+1}}q_{jj}^{1/2})$ (see equation (37, repeated below for convenience):

$$\begin{aligned}\phi^n &= -b(n-1)^{'}Bq\Lambda_0 \\ &= -\left(\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{\partial R_{t+i}}{\partial v_{j,t+1}}q_{jj}\left[\Lambda_0\right]_j\right)\end{aligned}$$

The prices of risk are shown in column (1), bond price sensitivities are shown in column (2) , their products are shown in column (3) and the sum of the individual terms which equals the excess holding period return (ehpr or ϕ^n in the above equation) is shown in the bottom line of the table. These values are obtained for the non-restricted estimated DSGE model.

Table 3: Estimated parameters and likelihood

		K = 0		*K = 3*	
Standard deviation of shocks		*mode*	*s.d.*	*mode*	*s.d.*
Productivity	E_A	0.481	0.027	0.523	0.032
Inflation obj.	E_PIE_BAR	0.096	0.016	0.101	0.015
Cons. Preference	E_B	1.341	0.610	1.324	0.642
Government spending	E_G	0.602	0.031	0.618	0.034
Labour supply	E_L	2.096	0.520	2.364	0.475
Investment	E_I	0.373	0.096	0.347	0.079
Interest rate	ETA_R	0.213	0.015	0.206	0.015
Equity premium	ETA_Q	0.600	0.156	0.636	0.134
Price mark-up	ETA_P	0.189	0.013	0.185	0.012
Wage mark-up	ETA_W	0.260	0.016	0.265	0.017
Persistence of shocks					
Productivity	rho_a	0.998	0.000	0.998	0.000
Cons. Preference	rho_b	0.581	0.137	0.616	0.147
Government spending	rho_g	0.996	0.003	0.997	0.001
Labour supply	rho_l	0.993	0.003	0.994	0.002
Investment	rho_i	0.708	0.069	0.734	0.062
Preference parameters					
cons. utility	sig_c	1.852	0.264	2.778	0.234
consumption habit	hab	0.645	0.070	0.562	0.072
labour util.	sig_l	2.103	0.599	1.946	0.528
Real rigidities					
Investment adj. cost	phi_i	5.958	1.096	6.048	1.139
cap. util. adj. cost	czcap	0.266	0.062	0.360	0.059
fixed cost	phi_y	1.587	0.067	1.676	0.073
Nominal rigidities					
indexation wages	gamma_w	0.323	0.111	0.399	0.126
indexation prices	gamma_p	0.431	0.099	0.406	0.100
calvo wages	xi_w	0.815	0.032	0.799	0.030
calvo prices	xi_p	0.904	0.013	0.899	0.012
Monetary policy					
r inflation	r_pie	1.487	0.102	1.000	0.000
r d(inflation)	r_dpi	0.186	0.051	0.157	0.053
lagged interest rate	rho	0.870	0.024	0.844	0.022
r output	r_y	0.062	0.027	0.057	0.015
r d(output)	r_dy	0.209	0.024	0.194	0.023
Log data density		-1096.528		-1128.195	

The table shows the estimated parameters for the DSGE model in the unrestricted case ($\kappa = 0$) and the restricted case ($\kappa = 3$). The model is estimated under the restriction that the excess holding period return on the 10-year bond generated by the model matches the one in the data. κ corresponds to κ in

$$f(ehpr^{n=40,\mathrm{model}} - ehpr^{n=40,data}) = \kappa N\left(0, \sigma_{ehpr^{n=40,data}}\right)$$

where κ is a sensitivity parameter that determines the tightness of the prior. Log data density is the Laplace approximation of the marginal posterior density. 'mode' and 's.d.' are the mode and standard deviation of the posterior.

Table 4a: Effect on spread, ehpr and the risk free rate of changes in stochastic processes

		10-year spread	ehpr	E(Rf)
Baseline model		-10.53	3.71	5.860
S.E. of shocks set to 0,001				
Productivity	E_A	-13.79	-0.56	5.931
Inflation obj.	E_PIE_BAR	5.38	12.06	5.864
Cons. Preference	E_B	-11.30	2.75	5.863
Government spending	E_G	-12.23	1.48	5.881
Labour supply	E_L	-12.48	1.02	5.880
Investment	E_I	-11.10	2.67	5.862
Interest rate	ETA_R	-11.69	2.95	5.876
Equity premium	ETA_Q	-11.21	3.59	5.860
Price mark-up	ETA_P	-11.18	3.72	5.860
Wage mark-up	ETA_W	-11.19	3.72	5.860
Persistence of shocks reduced by 10% from baseline				
Productivity	rho_a	63.997	583.26	4.281
Cons. Preference	rho_b	-11.213	3.6489	5.860
Government spending	rho_g	2.335	100.81	5.597
Labour supply	rho_l	-0.138	88.06	5.627
Investment	rho_i	-11.202	3.7419	5.860

Table 4b: Effect on spread, ehpr and the risk free rate of changes in the parameters of the DSGE model

		10-year spread	ehpr	E(Rf)
Preference parameters				
cons. utility	sig_c = 5	4,97	33,94	5,286
consumption habit	hab = 0,9	-6,26	17,18	5,761
labour util.	sig_l = 4	-11,04	3,89	5,855
Real rigidities				
Investment adj. cost	phi_i = 12	-10,25	5,13	5,853
cap. util. adj. cost	czcap = 0,54	-11,62	2,52	5,855
fixed cost	phi_y = 3	-6,45	10,59	5,748
Nominal rigidities				
indexation wages	gamma_w = 0,1	-11,05	3,67	5,860
indexation prices	gamma_p = 0,1	-10,97	3,74	5,859
calvo wages	xi_w = 0,1	-12,43	1,86	5,857
calvo prices	xi_p = 0,1	-12,07	3,42	5,862
Monetary policy				
r inflation	r_pie = 2	-10,95	2,42	5,856
r inflation	r_pie = 1,1	-9,58	12,66	5,872
r d(inflation)	r_dpi = 0,9	-11,32	5,45	5,853
lagged interest rate	rho = 0,9	-13,46	1,64	5,853
lagged interest rate	rho = 0,1	-9,24	7,06	5,878
r output	r_y = 0,125	-8,44	7,06	5,857
r d(output)	r_dy = 0,5	-11,55	5,84	5,840

Table 4a and 4b show the values in annualised basis points (bps) for the 10-year term spread and ehpr, and the expected risk free rate in percentage points (%), for different calibrations of the DSGE model. One parameter is changed at a time, the rest remain at their "baseline" values, equal to the mode of the posterior distribution. These can be found in Table 3, $\kappa = 0$.

Table 5a: Sensitivity of excess holding period return, prices of risk and bond price sensitivities to changes in the DSGE model parameters

I.Prices

		(a)	(b)	(c)	(d)
Productivity	E_A	1,891	5,038	2,714	1,891
Cons. Preference	E_B	-0,376	-0,202	-0,290	-0,376
Government spending	E_G	-1,038	-2,745	-1,779	-1,038
Investment	E_I	-0,354	-0,534	-0,539	-0,354
Labour supply	E_L	1,009	1,137	-0,033	1,009
Inflation obj.	E_PIE_BAR	0,481	0,506	0,460	0,001
Price mark-up	ETA_P	0,044	0,010	0,027	0,044
Equity premium	ETA_Q	-0,134	-0,177	-0,185	-0,134
Interest rate	ETA_R	-0,904	-0,943	-0,873	-0,904
Wage mark-up	ETA_W	0,065	0,102	0,121	0,065

II. Sensitivities

		(a)	(b)	(c)	(d)
Productivity	E_A	2,260	4,506	5,095	2,260
Cons. Preference	E_B	-2,551	-1,365	-2,254	-2,551
Government spending	E_G	-2,147	-4,641	-4,477	-2,147
Investment	E_I	-2,941	-4,587	-4,886	-2,941
Labour supply	E_L	2,669	2,988	2,931	2,669
Inflation obj.	E_PIE_BAR	-17,363	-17,308	-17,740	-0,018
Price mark-up	ETA_P	-0,238	-0,475	-0,275	-0,238
Equity premium	ETA_Q	-0,904	-1,267	-1,427	-0,904
Interest rate	ETA_R	-0,841	-0,941	-0,014	-0,841
Wage mark-up	ETA_W	-0,073	0,304	0,615	-0,073

III. Products

		(a)	(b)	(c)	(d)
Productivity	E_A	4,272	22,703	13,827	4,272
Cons. Preference	E_B	0,959	0,275	0,654	0,959
Government spending	E_G	2,228	12,740	7,966	2,228
Investment	E_I	1,040	2,449	2,636	1,040
Labour supply	E_L	2,694	3,398	-0,096	2,694
Inflation obj.	E_PIE_BAR	-8,347	-8,762	-8,156	0,000
Price mark-up	ETA_P	-0,011	-0,005	-0,007	-0,011
Equity premium	ETA_Q	0,121	0,224	0,264	0,121
Interest rate	ETA_R	0,760	0,888	0,013	0,760
Wage mark-up	ETA_W	-0,005	0,031	0,074	-0,005

IV. Eehpr and 10-year spread

		(a)	(b)	(c)	(d)
exc. hold. p. return	ehpr	3,713	33,941	17,175	12,060
spread		-10,533	4,972	-6,264	5,378

(a) Baseline, (b) sigC=5, (c) hab=0.9, (d) SE(E_PIE_BAR)=0.001

Table 5a shows the results from the decomposition of the yield spread and the *ehpr* (IV) on a 10-year bond into the sum over the 10 year period of the product (III) between the prices of risk (I) and the bond price sensitivities (II) (for details of the decomposition see Table 2). Columns (a) to (d) correspond to different calibrations of the DSGE model. "Baseline" uses the values for the model of the posterior from table 2, $\kappa = 0$, (b) sets risk aversion equal to5, (c) habit equal to 0.9 and (d) the standard error of the inflation target shock equal to 0.001. The rest of the parameters remain at their baseline values.

Table 5b: Sensitivity of spread, risk free rate and excess holding period return, prices of risk and bond price sensitivities to a joint change in the DSGE model parameters

I.Prices

		baseline	(a)	(b)	(c)	(d)
Productivity	E_A	1,891	7,143	7,143	10,307	2,381
Cons. Preference	E_B	-0,376	-0,166	-0,166	-0,444	-0,335
Government spending	E_G	-1,038	-4,669	-4,669	-8,410	-1,426
Investment	E_I	-0,354	-0,961	-0,961	-1,388	-0,448
Labour supply	E_L	1,009	-2,337	-2,337	-3,662	0,617
Inflation obj.	E_PIE_BAR	0,481	0,377	0,000	0,000	0,000
Price mark-up	ETA_P	0,044	-0,003	-0,003	0,156	0,035
Equity premium	ETA_Q	-0,134	-0,319	-0,319	-0,680	-0,160
Interest rate	ETA_R	-0,904	-0,747	-0,747	-0,020	-0,893
Wage mark-up	ETA_W	0,065	0,271	0,271	0,520	0,091

II. Sensitivities

		baseline	(a)	(b)	(c)	(d)
Productivity	E_A	2,260	10,912	10,912	14,641	3,393
Cons. Preference	E_B	-2,551	-1,332	-1,332	-2,647	-2,429
Government spending	E_G	-2,147	-10,182	-10,182	-15,929	-3,197
Investment	E_I	-2,941	-8,931	-8,931	-11,928	-3,896
Labour supply	E_L	2,669	1,794	1,794	7,653	2,673
Inflation obj.	E_PIE_BAR	-17,363	-18,380	-0,019	-0,021	-0,018
Price mark-up	ETA_P	-0,238	-0,479	-0,479	0,654	-0,259
Equity premium	ETA_Q	-0,904	-2,687	-2,687	-4,565	-1,153
Interest rate	ETA_R	-0,841	1,349	1,349	4,956	-0,455
Wage mark-up	ETA_W	-0,073	2,021	2,021	2,662	0,250

III. Products

		baseline	(a)	(b)	(c)	(d)
Productivity	E_A	4,272	77,945	77,945	150,903	8,080
Cons. Preference	E_B	0,959	0,222	0,222	1,175	0,813
Government spending	E_G	2,228	47,541	47,541	133,953	4,560
Investment	E_I	1,040	8,587	8,587	16,557	1,745
Labour supply	E_L	2,694	-4,192	-4,192	-28,025	1,650
Inflation obj.	E_PIE_BAR	-8,347	-6,932	0,000	0,000	0,000
Price mark-up	ETA_P	-0,011	0,001	0,001	0,102	-0,009
Equity premium	ETA_Q	0,121	0,858	0,858	3,105	0,184
Interest rate	ETA_R	0,760	-1,008	-1,008	-0,100	0,406
Wage mark-up	ETA_W	-0,005	0,548	0,548	1,385	0,023

IV. Ehpr, 10-year spread and E(Rf)

		baseline	(a)	(b)	(c)	(d)
exc. hold. p. return	ehpr	3,713	123,570	130,502	279,056	17,452
spread		-10,533	41,245	58,639	113,3814	8,061
E(Rf)	A_R	5,860	4,398	4,400	2,135	5,815

(a) sigC=5 and hab=0.9, (b) sigC=5, hab=0.9 and SE(E_PIE_BAR)=0.001, (c) sigC=5, hab=0.9, czcap=1.8 and SE(E_PIE_BAR)=0.001 , (d) sigC=2 , hab=0.8 and SE(E_PIE_BAR)=0.001

For an interpretation of the symbols in Table 5b, see the legend of Table 5a. The parameter values used for the calibration of models (a) to (d) are shown above. Two, three or four parameters are changed simultaneously and the rest are maintained at their "baseline" value.

8 Appendix

Appendix 1: Summary of linearized model equations

$$\begin{aligned}\widehat{C}_t &= \frac{h}{1+h}\widehat{C}_{t-1} + \frac{1}{1+h}E_t\widehat{C}_{t+1} \\ &+\frac{\sigma_c - 1}{\sigma_c\left(1+\lambda_w\right)\left(1+h\right)}\left(\widehat{l}_t - E_t\widehat{l}_{t+1}\right) \\ &-\frac{1-h}{\sigma_c\left(1+h\right)}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) \\ &+\frac{1-h}{\sigma_c\left(1+h\right)}\frac{1}{1-\rho_b}\left(\widehat{\varepsilon}_t^b - E_t\widehat{\varepsilon}_{t+1}^b\right)\end{aligned} \tag{1}$$

$$\widehat{I}_t = \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1} + \frac{1/\varphi}{1+\beta}\left(\widehat{Q}_t + \varepsilon_t^I\right) \tag{2}$$

$$\begin{aligned}\widehat{Q}_t &= -\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) + \frac{1-\tau}{1-\tau+\overline{r}^k}E_t\widehat{Q}_{t+1} + \\ &+\frac{\overline{r}^k}{1-\tau+\overline{r}^k}E_t\widehat{r}_{t+1}^k + \eta_t^Q\end{aligned} \tag{3}$$

$$\widehat{K}_{t+1} = \widehat{K}_t\left(1-\tau\right) + \tau\widehat{I}_{t-1} + \widehat{\varepsilon}_t^I \tag{4}$$

$$\widehat{Y}_t = \left(1-\tau k_y - g_y\right)\widehat{C}_t + \tau k_y\widehat{I}_t + g_y\gamma t + \widehat{\varepsilon}_t^G \tag{5}$$

$$\begin{aligned}&= \varphi_y\widehat{\varepsilon}_t^a + \varphi_y\alpha\widehat{K}_{t-1} + \varphi_y\alpha\psi\widehat{r}_t^k + \\ &\varphi_y\left(1-\alpha\right)\left(\widehat{L}_t + \gamma t\right) - \left(\varphi_y - 1\right)\gamma t\end{aligned} \tag{6}$$

$$\begin{aligned}\widehat{\pi}_t - \overline{\pi}_t &= \frac{\beta}{1+\beta\gamma_p}E_t\left(\widehat{\pi}_{t+1} - \overline{\pi}_t\right) + \frac{\gamma_p}{1+\beta\gamma_p}\left(\widehat{\pi}_{t-1} - \overline{\pi}_t\right) + \\ &\frac{1}{1+\beta\gamma_p}\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\left(\alpha\widehat{r}_t^k + \left(1-\alpha\right)\widehat{w}_t - \varepsilon_t^\alpha - \left(1-\alpha\right)\gamma t\right) + \eta_t^p\end{aligned} \tag{7}$$

$$\begin{aligned}\widehat{w}_t &= \frac{\beta}{1+\beta}E_t\widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} \\ &+\frac{\beta}{1+\beta}\left(E_t\widehat{\pi}_{t+1} - \overline{\pi}_t\right) - \frac{1+\beta\gamma_w}{1+\beta}\left(\widehat{\pi}_t - \overline{\pi}_t\right) + \frac{\gamma_w}{1+\beta}\left(\widehat{\pi}_{t-1} - \overline{\pi}_t\right) \\ &-\frac{1}{1+\beta}\frac{\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\left(1+\frac{\left(1+\lambda_w\right)\sigma_L}{\lambda_w}\right)\xi_w}\left[\widehat{w}_t - \sigma_L\widehat{L}_t - \frac{\sigma_c}{1-h}\left(\widehat{C}_t - h\widehat{C}_{t-1}\right) + \varepsilon_t^L\right] + \eta_t^w\end{aligned} \tag{8}$$

$$\widehat{L}_t = -\widehat{w}_t + \left(1+\psi\right)\widehat{r}_t^k + \widehat{K}_{t-1} \tag{9}$$

$$\begin{aligned} \widehat{R}_t &= \overline{\pi}_{t-1} + \rho\left(\widehat{R}_{t-1} - \overline{\pi}_{t-1}\right) + (1-\rho)\left(r_\pi\left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right) + r_Y\left(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1}\right)\right) + \\ & \quad r_{\Delta\pi}\left[\left(\widehat{\pi}_t - \overline{\pi}_t\right) - \left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right)\right] + r_{\Delta y}\left[\left(\widehat{Y}_t - \widehat{Y}^p_t\right) - \left(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1}\right)\right] + \eta^R_t \end{aligned} \tag{48}$$

Appendix 2: Yield spread, expected yield spread and the link with $ehpr_t^n$

Define the yield to maturity Y_{nt}:

$$P_{nt} = \frac{1}{(1+Y_t^n)^n}$$

and the yield spread as

$$SP_t^n = \frac{1+Y_t^n}{1+Y_t^1}$$

and the log yield spread as $sp_t^n = \log(1+Y_t^n) - \log\left(1+Y_t^1\right) = -\frac{1}{n}\log(P_{nt}) + \log(P_{1t})$. Replace P_{nt} by its expression from the Euler equation (equation 24):

$$sp_t^n = -\frac{1}{n}\log\left(E_t P_{t+1}^{n-1} M_{t+1}\right) + \log(P_{1t})$$

Assuming joint log-normality of P_{t+1}^{n-1} and M_{t+1} (as before), we can write:

$$\begin{aligned} sp_t^n &= -\frac{1}{n}[E_t p_{t+1}^{n-1} + \frac{1}{2}Var_t\left(p_{t+1}^{n-1}\right) \\ &\quad + E_t m_{t+1} + \frac{1}{2}Var_t(m_{t+1}) + cov_t\left(p_{t+1}^{n-1}, m_{t+1}\right)] + p_t^1 \end{aligned}$$

for $p_{t+1}^{n-1} = \log\left(P_{t+1}^{n-1}\right)$

Taking the unconditional expectation of the above expression and iterating, we obtain the expected log spread:

$$\begin{aligned} E\left(sp_t^n\right) &= \frac{1}{n}\left[-\frac{1}{2}Var\left(p_{t+1}^{n-1}\right) - \frac{1}{2}Var\left(p_{t+1}^{n-2}\right) - ... - \frac{1}{2}Var\left(p_{t+1}^{1}\right)\right] + \\ &\quad + \frac{1}{n}\left[E\left(ehpr_t^n\right) + E\left(ehpr_t^{n-1}\right) + ... + E\left(ehpr_t^2\right)\right] \end{aligned} \tag{49}$$

where we use equation (42) to replace the covariances between bond prices and the stochastic discount factor and $E\left(sp_t^1\right) = 0$. The expected spread equals an average over maturities of the $(n-1)$ $ehpr^n$ and of an average of the $(n-1)$ convexity terms.

By using the fact that the $ehpr^n$ are time-invariant and by iterating using the expression for $a(n)$, we obtain:

$$\begin{aligned} E\left(sp_t^n\right) &\equiv -\frac{a(n)}{n} + a(1) \\ &= -\frac{1}{n}\left[\frac{1}{2}b(n-1)'BqB'b(n-1) + ... + \frac{1}{2}b(1)'BqB'b(1)\right] \\ &\quad -\frac{1}{n}\left[b(n-1)'Bq\Lambda_0 + ... + b(1)'Bq\Lambda_0\right] \end{aligned}$$

This expression shows that the expected spread depends on the transition matrix B from the state-space description of the macro model and $b(n-1)...b(1)$ the bond price sensitivities with respect to the state variables as well as the variances of the structural shocks q and the prices of risk Λ_0 .

Recall from equation (49) that the expected spread $E\left(sp_t^n\right)$ can be written as the sum of an average variance and an average covariance component:

$$E\left(sp_t^n\right) = -\frac{1}{n}\frac{1}{2}\sum_{i=1}^{n} Var_t(p_{t+i}^{i-1}) - \frac{1}{n}\sum_{i=1}^{n} cov_t(m_{t+1}, p_{t+i}^{i-1}) \quad (50)$$

The sign of the expected spread will depend on whether the second term in this sum is positive and larger than the first, which is always negative. The size and the sign of the covariances between the bond prices and the stochastic discount factor will depend on the model parameterisation.

Appendix 3: Bond pricing equations

If the stochastic discount factor is affine, $m_{t+1} = c_m + A'_m X_t + \Lambda_0^{'} v_{t+1}$, the bond price equation will also be affine .

$$p_t^n = a(n) + b(n)^{'} X_t$$

By assumption $P_t^0 = 1$. For $n = 1$,

$$P_t^1 = E_t M_{t+1}$$

which implies that $P_{t+1}^1 = E_{t+1} M_{t+2}$. For $n = 2$,

$$P_t^2 = E_t M_{t+1} P_{t+1}^1$$

Substitue P_{t+1}^1 by its expression in M_{t+2}, and use the law of iterated expectations to obtain:

$$P_t^2 = E_t \left(M_{t+1} E_{t+1} M_{t+2} \right)$$

From eq. (27), M_{t+1} is conditionally lognormal, so taking logs we obtain for $n = 1$:

$$p_t^1 = E_t m_{t+1} + \frac{1}{2} V_t m_{t+1}$$

Replace using $m_{t+1} = c_m + A'_m X_t + \Lambda_0^{'} v_{t+1}$:

$$p_t^1 = c_m + A'_m X_t + \frac{1}{2} \Lambda_0^{'} q \Lambda_0$$

where $q = E(v_{t+1} v'_{t+1})$. This identifies $a(1)$ and $b(1)$ as:

$$\begin{aligned} a(1) &= c_m + \frac{1}{2} \Lambda_0^{'} q \Lambda_0 \\ b(1)^{'} &= A'_m \end{aligned}$$

For $n = 2$,

$$\begin{aligned} p_t^2 &= E_t m_{t+1} + \frac{1}{2} Var_t m_{t+1} + E_t E_{t+1} m_{t+2} + \frac{1}{2} Var_t \left(E_{t+1} m_{t+2} \right) + cov_t(m_{+1}, E_{t+1} m_{t+2}) \\ &= c_m + A'_m X_t + \frac{1}{2} \Lambda_0^{'} q \Lambda_0 + c_m + A_m^{'} \mu + A_m^{'} A X_t + \frac{1}{2} A_m^{'} B q B' A_m + \Lambda_0^{'} q B A_m \end{aligned}$$

This identifies $a(2)$ and $b(2)$ as:

$$\begin{aligned} a(2) &= c_m + \frac{1}{2} \Lambda_0^{'} q \Lambda_0 + c_m + A_m^{'} \mu + \frac{1}{2} A_m^{'} B q B' A_m + \Lambda_0^{'} q B A_m \\ b(2)^{'} &= A'_m + A_m^{'} A \end{aligned}$$

Which can be expressed in terms of $a(1)$ and $b(1)$:

$$\begin{aligned} a(2) &= a(1) + c_m + b(1) \mu + \frac{1}{2} b(1)^{'} B q B' b(1) + \Lambda_0^{'} q B b(1) \\ b(2)^{'} &= b(1)^{'} + b(1)^{'} A \end{aligned}$$

For $n = 3$,

$$P_t^3 = E_t P_{t+1}^2 M_{t+1}$$

$$\begin{aligned} p_t^3 &= E_t p_{t+1}^2 + \frac{1}{2} Var(p_{t+1}^2) \\ &+ E_t m_{t+1} \frac{1}{2} Var_t m_{t+1} + cov_t(p_{t+1}^2, m_{t+1}) \end{aligned}$$

Replace $p_{t+1}^2 = a(2) + b(2)' X_{t+1} = a(2) + b(2)'(c + AX_t + Bv_{t+1})$

$$\begin{aligned} p_t^3 &= a(2) + b(2)'(\mu + AX_t) + \frac{1}{2} b(2)' BqB' b(2) \\ &+ c_m + A_m X_t + \frac{1}{2} \Lambda_0^{'} q \Lambda_0 + b(2)' Bq\Lambda_0 \end{aligned}$$

to obtain

$$\begin{aligned} a(3) &= a(2) + b(2)'(c + Bq\Lambda_0) + c_m + \frac{1}{2} \Lambda_0^{'} q \Lambda_0 + \frac{1}{2} b(2)' BqB' b(2) \\ b(3)^{'} &= A_m^{'} + b(2)' A \end{aligned}$$

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