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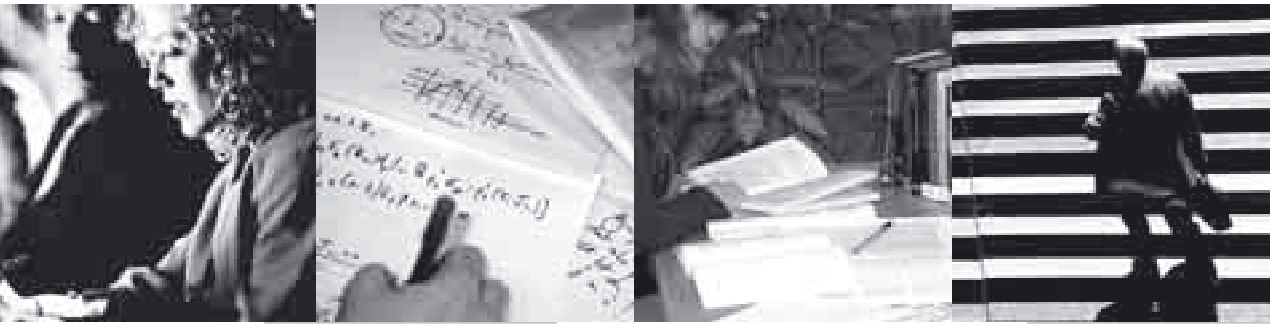

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A MULTI-FACTOR MODEL FOR THE VALUATION AND RISK MANAGEMENT OF DEMAND DEPOSITS

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Abstract

How should we value and manage deposit accounts where deposits have a zero contractual maturity, but which, in practice, remain stable through time and are remunerated below market rates? Does the economic value of the deposit account differ from the face value and can we reliably measure it? To what extent is the economic value sensitive to yield curve changes? In this paper, we try to answer the above questions. The valuation is performed on yield curve, deposit rate and deposit balance data between December 1994 and June 2005 for a sample of Belgian bank retail savings deposits accounts.

We find that the deposits premium component of Belgian savings deposits is economically and statistically significant, though sensitive to assumptions about servicing costs and outstanding balances average decay rates. We also find that deposit liability values depreciate significantly when market rates increase, thereby offsetting some of the value losses on the asset side. The hedging characteristics of deposit accounts depend primarily on the nature of the underlying interest rate shock (yield curve level versus slope shock) and on the average decay rate. We assess the reliability of the reported point estimates and also report corresponding duration estimates that results from a dynamic replicating portfolio model approach more commonly used by large international banks.

JEL-code : G12, G21.

Keywords: Demand deposits, ALM, risk management, arbitrage free pricing, flexible-affine term structure model, interest rate risk, IFRS 39, fair value accounting.

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1 Introduction

Banks create a mismatch between the maturity of their assets and liabilities by taking short-term demand deposits and granting long-term loans. Deposit accounts are hence at the core of banks' maturity transformation function. This paper analyzes the valuation and risk management of deposits.

In general deposit accounts can be divided into two classes. A first class consists of deposit accounts for which the effective time to maturity is identical to the contractual time to maturity and for which the deposit rate dynamics are close to the corresponding-maturity market rate dynamics. Term or time deposit accounts are typical examples of this class of deposit accounts. No special measurement and valuation problems arise for this class of deposits as they resemble simple zero coupon bonds. A second class of deposit accounts is characterized by the fact that the effective time to maturity may substantially exceed the contractual time to maturity and is remunerated at a (fluctuating) margin or spread below market rates. In the remainder of this paper we refer to the latter deposit accounts as nonmaturity or demand deposit accounts (NMAs, DDAs).

From the viewpoint of a depositor, DDAs are relatively straightforward financial instruments. However, from the viewpoint of a deposit-issuing bank or bank supervisor, DDAs are notoriously difficult to value and manage. Three important complexities arise. First, DDAs are subject to important behavioral relationships. More specifically, individual depositors are free to withdraw part or all of their balances and deposit issuing banks have the discretion to change the rate that is paid to the deposit holder. Depositor and bank behavior make the management of savings deposits' interest rate risk challenging, in particular because their exercise time are not independent. For example, if banks were not, or only partially, to raise deposit rates in response to an increase in market rates, depositors might withdraw their balances or part of them in order to invest their funds at the higher market rates. This makes clear that DDA's interest, repricing, and volume risks should best be studied jointly within one coherent framework. A second complexity is that DDAs are not actively traded on a liquid market. Their value is only occasionally disclosed in the rare event that a deposit issuing bank is taken over by another or when deposit bank branches are sold. Hence, valuation models are required without the ability to backtest or calibrate them. Third, DDAs are retail products. They supply depositors with liquidity and payment services. As such, their valuation and pricing may be affected by these services and by the collection and servicing costs which the bank incurs to issue deposits.

Banks and supervisors have been and are still very much interested in the development of a best practice approach for the difficult issue of the valuation and risk management of demand deposits. The reason is that DDAs play a prominent role in the funding structure of deposit issuing banks and that the interest rate sensitivity of deposit accounts is a key issue in banks' asset and liability management (ALM).[1] When market interest rates increase, deposit liabilities typically depreciate in value (in other words, "deposit premiums"[2] appreciate in value), and vice versa. However, given the complexities sketched above it is no surprise that this issue still is actively discussed among bankers and academic researchers. An international consensus is also actively promoted in the context of Pillar II of the New Basel Accord and by the European Commission (EC).[3] Despite all

[1] Kuritzkes and Schuermann (2006) find that structural asset/liability risk accounts for about 20% of total earnings volatility. Market and credit risk account for 50% of total earning volatility and nonfinancial risks for the remaining 30%. The authors plead for future risk management developments to focus upon the risks that are the least known and that have a significant impact on earnings volatility.

[2] "Deposit premium" will be defined below. Intuitively, it can be understood as the difference between the par value of deposits and the "economic value" of the deposit liability. Loosely, the fact that a bank is able to finance itself by a relatively cheap and stable source of funding may represent a kind of "deposit premium" or "net asset value".

[3] The Basel Committee issued principles for the management and supervision of interest rate risk (BIS (2004)). The New Capital Accord states: "To facilitate supervisors' monitoring of interest rate risk exposures across institutions, banks must provide the results of their internal measurement systems, expressed in terms of the threat

efforts, no uniformly accepted modelling approach exists to date and demand deposit management remains a challenge to practitioners and academics alike. A variety of approaches exist, which are briefly reviewed below.

Demand deposit valuation and risk management has also been a point of controversy when International Financial Reporting Standards (IFRS) have been introduced, more specifically IFRS 39 *Financial Instruments: Recognition and Measurement*. The IASB (International Accounting Standards Board), the international accounting rule-maker located in London, states very clearly that "the fair value of a financial liability with a demand feature is not less than the amount payable on demand ...". The assumed equality between fair (model-implied) and par value of DDAs implies that DDA fair values are completely interest rate insensitive. This stance is quite controversial and is inconsistent with banks' current risk management practices, attaching relatively long durations to their DDAs grounded on the factual experience of behavioral stability of deposits. The issue has been discussed among accounting setters, bankers and bank supervisors in the runup to the adoption of IFRS by the EC. In the end, the EC decided to adopt IFRS, but only after "carving out" or deleting the IAS 39 hedge accounting rules. EU Member Countries are effectively free to either apply the carve out with respect to hedge accounting or to apply the hedge accounting provisions as originally devised by the IASB.

In this paper we present a model to estimate the economic value of demand deposits. The valuation model we propose is a no-arbitrage multi-factor flexible-affine term structure model. The idea is to value the deposit liability as the discounted value of all expected cash outflows under the appropriate probability measure, just as is the case for any security in finance theory. Our main contributions with respect to the existing literature can be summarized as follows. First, all existing general equilibrium models in the deposit valuation literature assume that a single risk factor drives all uncertainty in yield curve dynamics. However, multi-factor term structure models outperform single factor models in terms of yield curve fit and forecasting performance and are by now computationally feasible (Dai and Singleton (2000)). Moreover, the extant models also impose restrictions on the risk premium dynamics while these can be relaxed without loss of tractability by using a flexible-affine yield curve model (Dai and Singleton (2001), Duffee (2002), Duarte (2004)). As the simulation of the yield curve into the future is the key driver in the valuation exercise, it is important to use a yield curve model that results in plausible yield curve dynamics. The model estimation is performed in a single step and for all considered banks at once. Second, instead of assuming zero decay rates for deposit balances over the simulation horizon (i.e. constant deposit balances), as is often assumed in the extant literature, we analyze the implications of a number of alternative assumptions about deposit balance dynamics. Third, we compare our estimates with estimates that follow from dynamic replicating portfolio models, which are more commonly used by large international banks.

The remainder of the paper is structured as follows. Section 2 presents the model. Subsection 2.1 presents the general framework of no-arbitrage valuation of economic rents, while Subsection 2.2 describes the specific building blocks of our joint yield curve deposit rate model. Subsection 2.3 close the section with a literature review. The empirical implementation of the model is elaborated in Section 3. The data are presented and discussed in Subsection 3.1. The estimation method is presented in Subsection 3.2 and estimation results are presented and interpreted in Subsection 3.3. Section 3.4 presents duration estimates based on the quite different but in practice more commonly used replicating portfolio model approach. Section 4 discusses the policy implications of our work and Section 5 concludes.

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to economic value, using a standardized interest rate shock." The EC also issued the following recommendation "Systems shall be implemented to evaluate and manage the risk arising from potential changes in interest rates as they affect a credit institution's non-trading activities" (Internal Capital Adequacy Assessment Process, Annex V).

2 The Model

2.1 Framework: no-arbitrage valuation of economic rents

The value of a security is determined by discounting the expected cash flows that stem from its payment schedule under the appropriate probability measure. This general principle is also applied to value a DDA. The traditional valuation approach is to set the deposit liability value L_0 equal to the sum of expected discounted cash outflows C_t:

$$L_0 = E_0 \left[\sum_{t=1}^{\infty} \frac{C_t}{\prod_{j=1}^{t} (1 + R_j) + \Phi_t} \right] \tag{1}$$

where R_t is the time-t 1-period (risk free) spot rate and Φ_t the possibly time-varying risk premium that accounts for the riskiness of future cash flows. E_0 denotes the expectation operator evaluated at time $t = 0$ and under the historical probability measure. An equivalent valuation approach is to discount the expected cash flows under a risk-neutral probability measure $\mathcal{Q}$ which can be shown to exist when arbitrage opportunities are assumed to be absent:

$$L_0 = E_0^{\mathcal{Q}} \left[\sum_{t=1}^{\infty} \frac{C_t}{\prod_{j=1}^{t} (1 + R_j)} \right] \tag{2}$$

The risk-adjustment is reflected in the switch from the historical to the risk-neutral probability measure.[4]

From the viewpoint of the bank, deposit liability cash flows, C_t, consist of three components: (i) interest payments, equal to the deposit rate times outstanding balances, (ii) non-interest payments for servicing the account net of any fees paid by depositors, and (iii) net changes in deposit balances. More specifically, the cash flows to the deposit account are assumed as in Table 1, where inflows (outflows) of funds are denoted with a plus sign (minus sign), where D_t is the deposit balance at time t, R_t^d the deposit rate set at the beginning of the period and paid out at the end of the period, and R_t^c the cost of servicing the account *net* of fees paid by the depositor and expressed as a percentage of the deposited funds. We assume here in all generality that the client withdraws its initial deposit at the end of the period and reinvests a new amount, possibly the same, larger, or smaller, at the beginning of the next period.

Table 1: **Cash flows related to issuing a deposit account - Stylized discrete time example**

Time	0	1	2	...	T	...
	$+D_0$	$-D_0$	$-D_1$		$-D_{T-1}$	
		$+D_1$	$+D_2$		$+D_T$	
		$-\left\{R_0^d + R_0^c\right\} D_0$	$-\left\{R_1^d + R_1^c\right\} D_1$		$-\left\{R_{T-1}^d + R_{T-1}^c\right\} D_{T-1}$	

The deposit net cash outflow at time t is the opposite of the net cash inflow and is defined as $C_t = (D_{t-1} - D_t) + R_{t-1}^d D_{t-1} + R_{t-1}^c D_{t-1}$. Our basic valuation equation (2) then becomes:

$$L_0 = E_0^{\mathcal{Q}} \left[\sum_{t=1}^{\infty} \frac{(D_{t-1} - D_t) + \left(R_{t-1}^d + R_{t-1}^c\right) D_{t-1}}{\prod_{j=1}^{t} (1 + R_j)} \right] \tag{3}$$

By adding and subtracting the face value D_0 to the right-hand side of equation (3), the deposit

[4] The switch from the historical to the risk-neutral probability measure requires the specification of a (set of) market price(s) of risk. The risk-neutral probability measure $\mathcal{Q}$ can be shown to be unique when markets are assumed to be complete (see Duffie (2001) for a formal treatment). This valuation technique is often used in no-arbitrage option pricing and yield curve modelling.

liability value can be rewritten as:

$$L_0 = D_0 - \underbrace{E_0^Q \left[\sum_{t=1}^{\infty} \frac{\left(R_t - R_{t-1}^d - R_{t-1}^c\right) D_{t-1}}{\prod_{j=1}^{t} (1 + R_j)} \right]}_{P_0} = D_0 - P_0 \tag{4}$$

In other words, we find that $D_0 = L_0 + P_0$, or that D_0, the "par value" or "face value" or "nominal value" of deposits can be decomposed into L_0, the "deposit liability value" or "economic value" or "fair value", and P_0, the "deposit premium" or "net asset value", being defined by equation (4) as the discounted sum of all expected net cash inflows or *economic rents* to the bank, $\left(R_t - R_{t-1}^d - R_{t-1}^c\right) D_{t-1}$. The economic rents drive a wedge between the deposit liability value and its face value. It turns out that positive economic rents are reaped by the bank when the collected funds, at a cost of the deposit rate plus collection and servicing cost net of fees, are invested without risk at a higher market interest rate. For it to be a truly risk free investment strategy, we should use the market rate with an instantaneous time to maturity, as deposit balances can be withdrawn on demand.

For mathematical ease but also for theoretical consistency the above framework is translated and estimated in continuous time. In the discrete-time framework of equation (4), the time to maturity of the risk free rate corresponds to the frequency of the time horizon (say, one month or one quarter), which introduces an inconsistency in the valuation, as deposits may be withdrawn on demand. Hence, we need to simulate market rate, deposit rate, deposit balance, and economic rent dynamics in continuous time, such that the risk free short rate indeed is the risk free rate with an instantaneous time to maturity. The continuous-time equivalent of our basic valuation equation (4) can be expressed as:

$$L_0 = D_0 - P_0 = D_0 - E_0^Q \left[\int_0^{\infty} \left(r(s) - r^d(s) - r^c(s)\right) D(s) \left(e^{-\int_0^s r(u)du} \right) ds \right] \tag{5}$$

with $r(t)$, $r^d(t)$, $r^c(t)$, and $D(t)$ the continuous-time equivalents of R_t, R_t^d, R_t^c, and D_t above. The implementation of the general valuation equation (5) requires the specification of three building blocks: (i) a no-arbitrage yield curve model to estimate and simulate the short rate and yield curve dynamics under the risk-free probability measure, (ii) a model for the deposit rate dynamics, which we will assume to depend on one or more yield curve factors and a deposit market spread factor, (iii) a model for the deposit balance dynamics, which will depend on which cash flows are considered in the valuation. Additionally, we will also need to make an assumption about the cost of servicing the account net of fees paid by the depositor, $r^c(t)$. We will consider a range of constant values for this model parameter, i.e. $r^c(t) = c$, for all t.

The above specification allows us to estimate the deposit liability and its corresponding interest rate sensitivity for each simulation path. The remainder of the paper implements the model in equation (5) and applies it to the savings deposits of the eight most important Belgian banks, which jointly account for 86% of outstanding savings deposits.

2.2 The joint yield curve deposit rate model

Absence of arbitrage is a reasonable assumption to impose on bond markets in equilibrium, given their depth and liquidity. It has been the key building block for the affine equilibrium term structure models pioneered by Vasicek (1977) and Cox, Ingersoll, Ross (1985) and categorized by Dai and Singleton (2000). To the best of our knowledge, the yield curve models in the existing deposit valuation literature are part of the affine class and assume that a single risk factor drives all uncertainty in yield curve dynamics. For example, Hutchison and Pennacchi (1996) assume one-factor Vasicek (1977) dynamics, Jarrow and van Deventer (1998) and Janosi, Jarrow and Zullo (1999) a one-factor Heath-Jarrow-Morton (1992) model, O'Brien (2000) and Selvaggio (1996) a one-factor Cox, Ingersoll, Ross (1985) model. The Office of Thrift Supervision assumes a deterministic single path that is implied by the current forward rate curve (OTS (2001)). In all these papers, the short rate is taken to be the sole risk factor and there is no explicit modelling of how deposit rates depend on yield curve shape changes. To the best of our knowledge, Kalkbrener and Willing (2004) are the only authors that estimated a three-factor term structure model, but they assumed a Heath, Jarrow and Morton no-arbitrage model which does not belong to the class of equilibrium term structure models. The advantage over the Heath, Jarrow and Morton type of models is that the latter are not equilibrium models and basically its parameters need to be re-estimated at each point in time to guarantee a perfect fit of the initial yield curve.

As our main aim is a realistic simulation of interest rates over a long horizon and not the exact fit to current market prices of plain vanilla instruments, we propose to filter N term structure factors, $f(t) = (f_1(t), ..., f_N(t))'$, using an *essentially-affine* no-arbitrage equilibrium term structure model. The advantage of essentially-affine models over the original (multifactor) affine Vasicek and Cox, Ingersoll, Ross models is the less restrictive specification of the market prices of risk, which can be an affine function of the term structure factors, instead of being constant or a function of the factor volatilities. Duffee (2002), Dai and Singleton (2001), and Duarte (2004) document and illustrate the superiority of the essentially-affine multifactor models over single factor and multi-factor affine models.

Latent factor term structure model Let $(\Omega, \mathcal{F}, \mathbb{F}, P)$ be the probability space on which the $N+1$ vector of Brownian motions $W(t) = (W_1(t), ..., W_N(t), W_{N+1}(t))'$ is defined and where $\mathbb{F} = \{\mathcal{F}_t\}_{0\leq t\leq T}$. Denote the first N components of $W(t)$ by $\bar{W}(t)$, $\bar{W}(t) = (W_1(t), ..., W_N(t))'$ with corresponding filtration $\{\bar{\mathcal{F}}_t\}_{0\leq t\leq T}$. Let us assume that zero coupon bond yields depend only on $\bar{W}(t)$ and that bond markets are complete. Absence of arbitrage opportunities implies that $p(t,T)$, i.e. the price at time t of a zero-coupon default-free bond maturing at time T, is defined as:

$$p(t,T) = E_t^{\bar{\mathcal{Q}}}\left[\exp\left(-\int_t^T r(s)\,ds\right)\right], \tag{6}$$

where $r(t)$ is the instantaneous interest rate and where $E_t^{\bar{\mathcal{Q}}}$ denotes the expectation operator under the *unique* risk-neutral probability measure $\bar{\mathcal{Q}}$. In general, this unique risk-neutral probability measure is unobserved and can only be specified by assuming some specification for the market price of risk (see Duffie (2001) for a textbook treatment):

$$\bar{\mathcal{Q}} = \exp\left(\int_t^T \bar{\xi}(t)d\bar{W}(t) - \frac{1}{2}\int_t^T \left|\bar{\xi}(t)\right|^2 dt\right) \tag{7}$$

where $\bar{\xi}(t)$, $\bar{\xi}(t) = (\xi_1(t), ..., \xi_N(t))'$, denotes the vector of possibly time-varying market prices of risk, adapted to $\{\bar{\mathcal{F}}_t\}_{0\leq t\leq T}$. The short rate is typically assumed to be the sum of the N latent term structure factors $\bar{f}(t)$, $\bar{f}(t) = (f_1(t), ..., f_N(t))'$:[5]

$$r(t) = \sum_{i=1}^{N} f_i(t) \tag{8}$$

[5] Recently many macro-finance models have been developed to interprete the latent factors, often labelled as "level", "slope" and "curvature", as a function of observed and unobserved macroeconomic variables (see Ang and Piazzesi (2003), Hördahl, Tristani and Vestin (2006), Rudebusch and Wu (2004), Dewachter and Lyrio (2006), and Dewachter, Lyrio, and Maes (2004, 2006)).

In order to explain deposit rates, we assume that an additional latent factor, $f_{N+1}(t)$, exists, which we label as the "deposit spread factor", for reasons that will become apparent below. The augmented $(N+1)$ x 1 vector of latent factors now becomes $f(t) = (f_1(t), ..., f_N(t), f_{N+1}(t))'$ and its joint dynamics are modeled as follows:

$$df(t) = K(\Theta - f(t))\,dt + SdW(t), \tag{9}$$

where Θ groups together the constant unconditional means to which each of the factors converges:

$$\Theta = (\theta_1, ..., \theta_N, \theta_{N+1})', \tag{10}$$

and where the covariance matrix of the $N+1$ factors is defined as the square of the $(N+1)$ x $(N+1)$ diagonal matrix S with constant elements on the diagonal:

$$S = diag(\sigma_1, ..., \sigma_N, \sigma_{N+1}). \tag{11}$$

The assumption of factor orthogonality is made for ease of interpretation and can be relaxed. Finally, we assume that the matrix that describes the speed of mean reversion of the factors, K, looks as follows:

$$K = \begin{bmatrix} \kappa_1 & 0 & ... & 0 \\ 0 & ... & 0 & 0 \\ 0 & 0 & \kappa_N & 0 \\ \gamma_1 & ... & \gamma_N & \kappa_{N+1} \end{bmatrix}, \tag{12}$$

As such, we allow for the term structure factors to affect the deposit rate setting behavior of banks. The joint yield curve-deposit rate model is incomplete and there exist infinitely many equivalent risk-neutral probability measures. In order to identify a specific martingale measure, we follow Kalkbrener and Willing (2004) and use the so-called *variance-minimizing* risk-neutral probability measure (see also Schweizer (1995) and Delbaen and Schachermayer (1996)).[6] The variance-minimizing risk-neutral probability measure is the probability measure Q that is "closest" to the given measure $\bar{\mathcal{Q}}$, in the following sense:

$$\mathcal{Q} = \exp\left(\int_t^T \boldsymbol{\xi}(t)d\bar{W}(t) - \frac{1}{2}\int_t^T |\boldsymbol{\xi}(t)|^2\,dt\right) \tag{13}$$

where $\boldsymbol{\xi}(t)$ is defined as $\boldsymbol{\xi}(t) = (\boldsymbol{\xi}_1(t), ..., \boldsymbol{\xi}_N(t), 0)'$. Under the variance minimizing probability measure, we can obtain unique prices for arbitrary securities defined on $(\Omega, \mathcal{F}, \mathbb{F}, P)$.

Following Duffee (2002), time variability in the prices of risk can be captured by specifying prices of risk as an affine function of the factors. The vector containing the time-varying prices of risk, $\xi(t)$, is defined as:

$$\xi(t) = S\Lambda + S^{-1}\Xi f(t), \tag{14}$$

where $\Lambda = (\lambda_1, ..., \lambda_N, 0)'$ and Ξ a $(N+1)$ x $(N+1)$ matrix containing the sensitivities of the prices of risk to the levels of the factors $\bar{f}(t)$ (with zeros in row and column $N+1$). Changing probability measures is then performed by means of the Girsanov theorem:

$$dW(t) = d\tilde{W}(t) - \xi(t)\,dt, \tag{15}$$

where $\tilde{W}_i(t)$ constitutes a martingale under measure $\mathcal{Q}$. Using equation (15), we can infer the implications for the real world by changing from the risk-neutral measure, $\mathcal{Q}$, to the historical one, $\mathcal{P}$. The factor dynamics under this risk-neutral metric $\mathcal{Q}$ are given by:

[6] Potential alternative methods for the pricing of deposits in incomplete markets are amongst others super-replicating portfolios, utility-based valuation, and good deal bounds.

$$df(t) = \tilde{K}\left(\tilde{\Theta} - f(t)\right)dt + Sd\tilde{W}(t), \tag{16}$$

with

$$\begin{aligned} \tilde{K} &= K + \Xi \\ \tilde{\Theta} &= (K+\Xi)^{-1}\left(K\Theta - S^2\Lambda\right). \end{aligned} \tag{17}$$

A functional form for bond prices can be obtained by assuming that bond prices are time homogeneous functions of the term structure factors $\bar{f}(t)$ and the time to maturity $\tau \equiv T - t$:

$$p(t,\tau) = p\left(\bar{f}(t),\tau\right) = \exp\left(-a(\tau) - b(\tau)'\bar{f}(t)\right), \tag{18}$$

where $a(\tau)$ is a scalar and $b(\tau)$ is a N x 1 vector. Imposing the no-arbitrage condition in the bond markets implies:

$$\mathcal{D}^{\mathcal{Q}}\left(p\left(\bar{f}(t),\tau\right)\right) = r(t)\,p\left(\bar{f}(t),\tau\right), \tag{19}$$

where $\mathcal{D}^{Q}$ denotes the Dynkin operator under the probability measure $\mathcal{Q}$. The intuitive meaning of the latter condition is that, once transformed to a risk-neutral world, instantaneous holding returns for all bonds are equal to the instantaneous risk free interest rate. Equations (18) and (19) determine the solution for the functions $a(\tau)$ and $b(\tau)$ in terms of a system of ODEs that, in the flexible-affine case, can only be solved numerically:

$$\begin{aligned} \frac{\partial a(\tau)}{\partial \tau} &= a_0 + \left(\tilde{K}\tilde{\psi}\right)' b(\tau) - \frac{1}{2}\sum_{i=1}^{N} b_i^2(\tau)\, S_{ii}^2, \\ \frac{\partial b(\tau)}{\partial \tau} &= b_0 - \tilde{K}' b(\tau). \end{aligned} \tag{20}$$

The no-arbitrage restrictions restrict the shape of $a(\tau)$ and $b(\tau)$ throughout the maturity spectrum. A particular solution to this system of ODEs is obtained by specifying a set of initial conditions on a and b. Inspection of equation (18) shows that the relevant initial conditions are $a(0) = 0$ and $b(0) = 0$. The vectors of constants a_0 and b_0 in (20) are defined by the interest rate definition in (8) and, therefore, equal to $a_0 = 0$ and b_0 a N x 1 vector of ones. The bond pricing solution derived here differs in important ways from the ones implied by standard independent multi-factor term structure models presented in the literature. Allowing for interrelations among the factors (*i.e.* non-zero off-diagonal elements in $\tilde{K}$) generates a coupled system of ODEs instead of a set of uncoupled ODEs. The bond pricing solution for the a and b functions, therefore, is not reduced to the standard multi-factor result presented in, for instance, de Jong (2000).

Deposit rate dynamics Deposit rates are modelled as follows:

$$r_j^d(t) = a_j^d + \left(b_j^d\right)' f(t) + \varepsilon_i^d(t),\ j = 1, ..., J \tag{21}$$

where a_j^d is a scalar for all j, $b_1^d = (1, ..., 1, -1)'$ for bank 1 and $b_j^d = (1, ..., 1, b_j)'$ for all other banks, $j \neq 1$. We choose one of the big banks as Bank 1, so that the deposit factor captures the dynamics of the *spread* between the short rate and the deposit rate of this big bank. Indeed, given that the loadings on all term structure factors are fixed to unity, we are equating the deposit rate to the instantaneous rate minus a positive deposit spread factor. All other banks are then allowed to have a higher or lower sensitivity to this unique, common, big bank spread factor. We have tried to estimate a series of alternative deposit rate dynamics specifications, but were unable to outperform the simple version above in the description of actual deposit rate dynamics. Ideally, we would have liked to include asymmetry in the deposit rate dynamics (see O'Brien (2000)), but our sample period does not allow statistically significant inference, as it does not contain a full cycle of deposit and interest rate dynamics.[7]

[7] Ausubel (1990) and Neumark and Sharpe (1992) show that the market structure affects the deposit rate setting behavior of banks. For example, both the equilibrium level and the speed of adjustment of deposit rates are found

Deposit balance dynamics In valuing deposit accounts, it is crucial to clearly define the relevant and irrelevant deposit cash flows (Barth (2005). Indeed, the valuation of *current outstanding* deposits will differ significantly from the valuation of *current and future expected* deposits. Moreover, future expected deposits refer to both new deposits of existing depositors as well as new deposits of new depositors. In our eyes, both existing deposits and future expected deposits need to be included, but the latter only to the extent that they can be identified in a verifiable way. For example, if a client can only take out a mortgage if he signs a contract that he promises that his salary will be cashed out on his deposit account, the bank should also include the expected future new deposits based on the monthly salary payments of its existing depositor. In contrast, the arrival of new depositors is not truly verifiable and hence should be excluded from the exercise to value deposits.

Because of data limitations, we choose to perform the valuation of existing deposits only. Our primary aim is thus to reliably estimate the economic value of the current volume of DDAs and its sensitivity to market interest rates, disregarding future new deposits. An important problem is that we can not assess the extent to which outstanding deposits react to, say, opportunity cost changes by simply regressing our sample of deposit balances on the spread between market and deposit rates. The reason is that the observed ex post historical deposit dynamics involve a mixture of both existing and newly-collected deposits and the latter need to be excluded from the regression analysis. In the absence of reliable data on outstanding balance dynamics (which banks typically do have), we can not endogenize outstanding deposit balance dynamics and rely upon a more *ad hoc* approach. Specifically we assume that *(i)* outstanding deposit balances grow continuously (pro rata) at the deposit rate $r^d(t)$, i.e. we assume that people capitalize the deposit rent that is paid out to them on their account, and that *(ii)* aggregate outstanding deposit balances are withdrawn at a *constant* annualized withdrawal or decay rate r^w, i.e.:

$$dD(t) = \left(r^d(t) - r^w\right) D(t)\, dt \tag{22}$$

O'Brien (2000) and Hutchison and Pennacchi (1996) report their main results under the assumption of constant deposit balances (i.e. zero decay rates, 100 percent retention rates, or infinite halving times). We report results for a range of plausible, constant, annual decay rates and allow r^w to vary between 10% and 50%.[8] We also present estimation results where we simply assume constant deposit balances.

Note that the valuation of current *and* expected future deposits is a more challenging exercise[9]. First, the expected new deposits that we can measure in a verifiable way need to be estimated. Second, the valuation exercise may raise technical problems as nothing guarantees that discounted economic rents converge to zero over a finite horizon when deposit balances are expected to grow. A convergence problem arises whenever verifiable deposit balance growth rates exceed the discount rate that applies in computing the present value of economic rents. Some bankers have reported that they use cutoff horizons, say 10 year, after which earned economic rents are neglected. The cutoff horizon basically becomes another model parameter. As mentioned above, we have opted not to present estimates for this kind of exercise, given a lack of data and the higher discretion and estimation uncertainty that it entails.

to depend on market concentration. They find that deposit rates are on average higher in less concentrated markets, in line with expectations.

[8] Based on equation (22), we can compute outstanding deposits halving times, $s - t$ $(s > t)$, as a function of the decay rate parameter. For an average deposit rate of 2.75%, it follows approximately that the halving time is 9.6 year for a decay rate of 10% and only 1.5 year for a 50% decay rate.

[9] O'Brien (2000) reports and discusses convergence problems when extrapolating model-implied log deposit balance dynamics that are assumed to be a function of the spread between the short rate and the deposit rate, lagged log deposit balances, and a measure of income. O'Brien's benchmark results based on constant deposit balances do not raise these convergence complexities. Likewise, Hutchison and Pennacchi (1996) also report the bulk of their results under a zero deposit balance growth rate assumption.

2.3 Literature review

The literature suggests that economic rents exist. Empirical surveys that look at deposit issuing bank takeovers and deposit bank branch sales typically find that substantial deposit premiums are being paid, i.e. that prices paid for DDA funding exceed the nominal amount of funding (Berkovec, Mingo, and Zhang (1997), Jarrow and van Deventer (1998, p. 257), Selvaggio (1996, p.365)). Potential theoretical sources of economic rents include regulatory barriers to entry leading to market concentration (Jarrow and Van Deventer (1998), Hannan and Berger (1991)), clients accepting low deposit rates because they benefit from other services, for example more advantageous mortgage financing (Jarrow and Van Deventer (1998)), costs to consumers of switching banks (Ausubel (1992), Sharpe (1997)), and limited memory of depositors (Kahn, Pennacchi, and Sopranzetti (1999)). It can also simply be because depositors are more concerned with service and convenience than with opportunity costs.

The *no-arbitrage discounted cash flow valuation approach* of equation (1) is implemented by Selvaggio (1996), OTS (2001), and de Jong and Wielhouwer (2003). The equivalent risk-neutral approach of equation (2) is the one followed by Hutchison and Pennacchi (1996), Jarrow and van Deventer (1998), O'Brien (2000), Janosi, Jarrow and Zullo (1999), and Kalkbrener and Willing (2004).

Most studies listed above use US bank data. de Jong and Wielhouwer (2003) and Kalkbrener and Willing (2004) are exceptions and use Dutch and German bank data, respectively, but limit themselves to a single bank in the latter case or to a virtual bank which aggregates deposit balances across the banking system in the former case. This limitation is unfortunate as studies on US Negotiable Orders of Withdrawal (NOWs) and Money Market Deposit Accounts (MMDAs) show that there is significant variation across individual banks in premium and duration estimates. Moreover, deposit valuation and risk management is perhaps even more important for European banks, as Continental-European economies are much more bank-intermediated compared to the US. For example, banking sector liabilities add up to roughly 330% of GDP, while comparable numbers for the more market-intermediated UK and US economies are close to 140% and 70%, respectively. Finally, note that in some European countries regulation is applicable to DDAs with the aim of further stabilizing the deposited volumes.

In Table 2, we single out two benchmark studies that report estimates for a selection of banks. Hutchison and Pennacchi (1996) estimate deposit premiums and durations for more than 200 commercial banks and report (i) median NOW and MMDA deposit premiums of 6.6% and 7.9%, and (ii) median NOW and MMDA durations of 6.7 years and 0.4 years. Variation across banks is huge, however, as 90% of all bank premium estimates lies within the ranges [-5%,25%] and [-2.5%,18%], respectively (own estimates based on the histograms in their Figures 4 to 7). Similarly, 90% of all NOW and MMDA durations are reported to lie within the ranges [2 year,9 year] and [-2 year,2 year]. Their robustness analysis shows that the noninterest cost parameter and the deposit demand growth rate substantially affect the point estimates for a given bank. O'Brien (2000) reports substantially higher premium estimates based on a sample of between 75 to 100 banks: a 21.1% median estimate with 80% range [8.0%, 36.6%] for NOW accounts and 12.2% median estimate and [-0.1%,21.8%] range for MMDAs. Duration estimates are substantially smaller, with a median estimate of 1.09 (0.70) year for NOW accounts and for a 100bp upward (downward) shock, and a median estimate of 0.5 (0.25) year for MMDAs.

Premium and duration estimates vary a lot across banks and across different studies, but look economically significant. Of course, underlying both Hutchison and Pennacchi (1996) and O'Brien (2000) estimates is the assumption that volumes are constant over the simulation horizon (alternatively zero decay rate, 100% retention rate, infinite halving times). As a result, the premium estimates should be considered to be upper bounds for the case when only existing deposit cash flows are considered.

Table 2: **Deposit premium and duration estimates in the literature**

Literature	Hutchison and Pennacchi (1996) 200 banks, 1986-1990	O'Brien (2000) > 75 banks, 1983-1994
NOW median premium	6.6%	21.1%
[10%; 90%] quantile	[-5%; 25%]	[8.0%; 36.6%]
MMDA median premium	7.9%	12.2%
[10%; 90%] quantile	[-2.5%; 18%]	[-0.1%; 21.8%]
NOW median duration	6.7 years	1.09 & 0.70 years[a]
[10%; 90%] quantile	[2 year; 9 year]	
MMDA median duration	0.4 years	0.5 & 0.25 years[a]
[10%; 90%] quantile	[-2 year; 2 year]	

[a] Duration estimates for a 100bp upward and 100bp downward shock, respectively, for the median bank, and where deposit rates react asymmetrically to market rate changes. See O'Brien (2000) for duration estimates where the shocks differ in size and/or where deposit rate dynamics are assumed to adjust symmetrically.

3 Empirical implementation and estimation results

3.1 Data

The valuation and risk management of deposits is of particular relevance for continental European countries with their relatively large and stable deposit volumes, and even more for Belgium, given that there is specific price regulation in place that aims to stabilize the volume of deposits. Total savings deposits balances in Belgium have grown in absolute numbers from 60 billion euro in December 1994 to 158 billion in June 2005, roughly 50% of Belgian 2005 GDP.[10] The share of savings deposits in total bank liabilities also increased from 10.3% to 15.5% in the last decade, and households allocated as much as 20% of their wealth in this type of account. Substantial variation exists across banks in the importance of this balance sheet post, according to bank specialization and size. For example, whereas savings deposits account for 11% of liabilities on average for the 4 largest Belgian banks in 2004, this average proportion reaches 43% for the medium-size banks specialized in the distribution of this product.

The Belgian financial system is characterized by a highly concentrated banking sector. Basically four conglomerates dominate the banking and insurance sector. We present estimation results for a sample of eight Belgian banks, the four conglomerates (Dexia, Fortis, ING, KBC) and four medium-sized banks that rank as most important issuers of savings deposits after the four conglomerates, and based on the period 1994:Q4-2005:Q2 (43 quarterly observations). Taken together, the selected banks have a 86% market share in June 2005 (135 billion euro of savings deposits). A number of specific issues arise in the Belgian context, as Belgian law introduces regulation affecting the pricing, remuneration structure, and fiscal treatment of savings deposits (Royal Decree of August 27 1993 (KB/WIB 1992)).[11]

[10] Sight deposit accounts, being the actual transaction or checking account, are characterized by a low quasi-constant remuneration and amount to another 117 billion euro in June 2005. Although these accounts can also be regarded as DDAs and although banks attach relatively long durations to these deposits, at least for the core part of the balances, we leave their treatment for future research.

[11] First, savings deposits in Belgium enjoy a favorable fiscal treatment, as interest earned on savings deposits is exempt from the withholding tax (currently 15 p.c.) up to an amount of EUR 1520 of annual interest income per taxpayer. Second, *strictu sensu*, savings deposits are not true demand deposits as Belgian law allows deposit-issuing banks to legally restrict withdrawals to EUR 2500 in any two-week period and to require a 5-day notice for withdrawals exceeding EUR 1250. Third, the remuneration of savings deposits is at the discretion of the banks, but must always consist of a base rate and a growth or loyalty premium. While the base rate is paid out proportionally with time (pro rata), the payout of the premium component is relatively complex and not fully transparant and hinges upon the time the (new) deposits remain on the account (typically 6 or 12 months is required) and on intricate daycount conventions. Both the base rate and the premiums are legally capped. Since April 1990, the legal ceiling has remained unchanged at 6 p.c., i.e. a base rate ceiling of 4 p.c. and a premium ceiling of 2 p.c.. Between December 1983 and April 1990, the ceiling has been changed on various occasions by the Minister of Finance, typically in line with the general interest rate market environment. Fourth, savings deposits can be thought to be risk free, as balances are guaranteed, up to a certain amount, by the Belgian Protection Fund for Deposits and

We retrieve quarterly individual bank 1994:Q4-2005:Q2 savings deposit balance data from the Belgian supervisory reporting scheme (the so-called Schema A, unconsolidated numbers). The balances of institutions that merged in our time sample have been added up before the merger to avoid structural breaks in the times series. To respect confidentiality of individual bank data, we report key results averaged over groups of banks. Specifically, average results are reported for the Big 4 conglomerate banks and the group of four medium-sized banks, respectively.

Instead of using advertised deposit rates, that is advertized base rates plus growth or loyalty premiums, we have extracted *implicit deposit rates*, defined as the ratio of the interest that is paid out by the bank over a specific quarter divided by the average outstanding balances over the same quarter. Compared to advertised rates, being the sum of base and premium rates, the use of implicit deposit rates has a number of advantages. They better reflect the true cost and cash outflows of the bank, given the sometimes intricate day count rules that apply to the advertised rates and given the presence of the growth and loyalty premium rules. Furthermore, they make it easier to integrate pre- and post-merger deposit rate data.

Implicit deposit rate dynamics averaged across banks are reflected in the upper panels of Figure 1. Deposit rates of both groups of banks have experienced a downward trend in the last decade. However, deposit rates of the medium-sized banks have been set above Big 4 bank deposit rates. Indeed, Table 3 shows that there is approximately a 50bp difference between average deposit rates of the two groups (2.6% versus 3.1%). The medium-sized bank deposit rates are slightly less volatile than those of the large banks, but, overall, deposit rate volatility is very low. Deposit rates are characterized by positive skewness and fat tails (excess kurtosis), especially the deposit rates which are set by the Big 4 banks. A formal test statistic (Jarque and Bera (1980)) rejects the hypothesis that the latter are normally distributed at conventional levels of statistical significance.

Table 3: **Summary statistics of Belgian implicit deposit rates (1994:Q4-2005:Q2)**

	Average deposit rates			**Weighted average deposit rates**[a]		
	All banks	**Big 4**	**Medium-sized**	**All banks**	**Big 4**	**Medium-sized**
Mean	2.814	2.602	3.026	2.711	2.616	3.112
Std. deviation	0.491	0.499	0.497	0.482	0.496	0.450
Autocorrelation	0.915	0.860	0.936	0.877	0.856	0.937
Skewness	1.021	1.407	0.701	1.248	1.430	0.315
Kurtosis	3.870	5.047	3.209	4.636	5.146	2.831
JB test (p-value)	0.012	0.000	0.165	0.000	0.000	0.684

Mean and standard deviation are expressed in annualized percentage points. Autocorrelation estimated on a quarterly horizon.
[a]: The ratio of the deposit balance over the sum of all balances across banks is used to compute the weighted average deposit rate.

We now turn to the deposit balance dynamics. Average deposit balance dynamics are plotted in the lower panels of Figure 1, and corresponding summary statistics are reported in Table 4. We observe a relatively stable upward trend (2.3% average quarterly growth rate) in average balances for both groups of banks, although Big 4 banks have experienced slower growth than medium-sized banks (growth rate approximately half as large, 2.0% versus 3.9%). Minimum quarterly growth rates are negative, meaning that aggregate deposits have decreased at some point in our 1994:Q4-2005:Q2 sample, particularly in the period 2000-2002. This period is characterized by stable deposit rates but sharply increasing market rates, increasing the opportunity cost of deposit holders (see Figures 1 and 2), and a sharp stock market decline.[12]

Table 5 presents summary statistics of Belgian market rates, which are plotted in Figure 2. For our analysis we have used monthly Belgian zero coupon bond yield data from 1994:12 until 2005:06 for

Financial Instruments.

[12]See Maes and Timmermans (2005) and van den Spiegel (1993) for a more elaborate description of the determinants of historical Belgian savings deposits balance dynamics.

Table 4: **Summary statistics of selected Belgian savings deposits balances (1994:Q2-2005:Q2)**

	Balance characteristics		
	All banks	**Big 4**	**Medium-sized**
Average balance (EUR bn.)	16.931	27.403	6.460
Average growth rate	2.275	2.018	3.850
Minimum growth rate	-3.335	-3.918	-1.353
Maximum growth rate	9.327	9.904	18.117

Growth rates in percentage points and on a quarterly horizon.

maturities 1m, 2m, 3m, 6m, 1yr, 2yr, 3yr, 4yr, 5yr, and 10yr (Belgostat data, Treasury Certificate rates for maturities below 1 year and zero coupon bond yields derived from linear bonds (OLOs) for the longer maturities). The average term structure has been upward sloping, with an average spread of approximately 2%, and the yield curve did not invert over the sample period. Strong autocorrelation is observed in all univariate time series over the sample period. There is evidence against normality in most yield series with maturities 1 year and longer in terms of skewness and excess kurtosis. The one-month Treasury Certificate on average exceeds the deposit rate by 54bp (64bp relative to the average Big 4 bank deposit rate and only 14bp relative to the average medium-sized bank deposit rate).

Table 5: **Summary statistics Belgian yield curve (1994:12-2004:09)**

	1m	**2m**	**3m**	**6m**	**1yr**	**2yr**	**3yr**	**4yr**	**5yr**	**10yr**
Mean	3.242	3.270	3.291	3.332	3.468	3.801	4.100	4.355	4.565	5.200
Standard deviation	0.907	0.924	0.937	0.968	1.030	1.087	1.081	1.089	1.087	1.131
Autocorrelation (q.)	0.973	0.976	0.976	0.973	0.954	0.951	0.953	0.954	0.955	0.968
Skewness	0.343	0.357	0.370	0.443	0.715	0.782	0.707	0.661	0.683	0.710
Kurtosis	2.580	2.567	2.557	2.721	3.462	3.923	3.900	3.798	3.694	3.161
JB test (p-value)	0.181	0.158	0.140	0.102	0.003	0.000	0.001	0.002	0.002	0.005

Mean and standard deviation are expressed in percentage points. Autocorrelations on a quarterly horizon.

3.2 Estimation method

Given the Gaussian (discrete time) properties of the joint yield curve deposit rate model in Section 2.2 we can estimate all its parameters consistently by means of maximum likelihood estimation, using the Kalman filter algorithm to construct the loglikelihood function and after having rewritten the model in state space notation.

The transition equation is given by equation (9). Since the matrix K is not diagonal, it is not straightforward to obtain closed form equations for the expectation of the level and of the covariance matrix of the factors. These concepts are, nevertheless, of great importance in order to forecast the future evolution of the state of the economy.[13] The final goal of the procedure is to maximize the (multivariate normal) likelihood of the prediction errors of the model.

For the measurement equation, we construct a vector $z(t)$ containing M time-t zero-coupon bond yields $(\hat{y}_m(t, \tau_m))$ for maturities τ_1 through τ_M and J time-t deposit rates $(r_j^d(t))$ for bank 1 to J. Based on the theoretical model, we can write this vector $z(t)$ in terms of the factors $f(t)$ as

[13] A procedure to generate the conditional means and the conditional covariance matrix of the factors is presented in Fackler (2000).

follows:

$$z(t) = \begin{pmatrix} \hat{y}_1(t,\tau_1) \\ \vdots \\ \hat{y}_M(t,\tau_M) \\ \hat{r}_1^d(t) \\ \vdots \\ \hat{r}_J^d(t) \end{pmatrix} = \begin{pmatrix} a \\ a^d \end{pmatrix} + \begin{pmatrix} B \\ B^d \end{pmatrix} f(t) + \varepsilon_t \ , \tag{23}$$

where ε_t is an $(M+J)$ x 1 vector of measurement errors and

$$a = (a(\tau_1)/\tau_1, ..., a(\tau_M)/\tau_M)' \qquad a^d = \left(a_1^d, ..., a_J^d\right)'$$
$$B = \begin{pmatrix} b(\tau_1)'/\tau_1 \\ \vdots \\ b(\tau_M)'/\tau_M \end{pmatrix} \qquad B^d = \begin{pmatrix} -1 \\ b_2^d \\ ... \\ b_J^d \end{pmatrix}' \tag{24}$$

3.3 Estimation results

3.3.1 Joint yield curve deposit rate model estimates

Table 6: **Estimated parameters for the joint yield curve deposit rate model (1994:Q2-2005:Q2)**

	Factor			
	1	**2**	**3**	**4**
$\kappa_{i,i}$	0.0478	0.5130	0.9831	111.2809
	(0.0254)	(0.2438)	(0.6607)	(0.1739)
$\kappa_{4,i}$	70.5667	96.5726	99.8469	111.2809
	(3.5187)	(1.7657)	(2.6909)	(0.1739)
θ_i	0.0358	0	0	0
	(0.0231)			
σ_i^2	0.000065	0.000376	0.000373	0.008000
	(0.000016)	(0.000091)	(0.000099)	(0.0103)
λ_i	3.4133	-37.3483	29.1506	0
	(26.6932)	(12.8555)	(27.4887)	
$\Xi_{i,i}$	-0.0247	-0.0768	0.4759	
	(0.0299)	(0.2476)	(0.6212)	
R_{1m}	0.2020	R_{2yr}	0.5076	
R_{2m}	0.0100	R_{3yr}	0.2578	
R_{3m}	0.0858	R_{4yr}	0.4339	
R_{6m}	0.3821	R_{5yr}	0.0797	
R_{1yr}	0.6989	R_{10yr}	0.3171	
R_{bank_1}	1.0750	R_{bank_5}	8.6435	
R_{bank_2}	0.6942	R_{bank_6}	1.1804	
R_{bank_3}	0.5900	R_{bank_7}	7.0844	
R_{bank_4}	0.5128	R_{bank_8}	19.6918	

Maximum likelihood estimates with standard errors underneath them between brackets. The table entries for the measurement error covariance matrix (R) are multiplied by 10^6. The loglikelihood is on average equal to 110.6187 (excluding the constant in the loglikelihood). Factors 1 to 3 are the term structure factors. Factor 4 represents the deposit spread factor (see main text).

We have estimated three versions of the joint yield curve deposit rate model presented in Section 2.2. Table 6 presents the parameter estimates of our no-arbitrage model with 3 term structure factors and one deposit spread factor (parameters θ_2, θ_3, θ_4 and λ_4 are set to zero and are not

estimated). Corresponding tables with the parameter estimates for the 1 and 2 term structure factor model are presented in an Appendix.
Let us compare the quality of the fit of the different single- and multiple term structure factor models. Figure 3 plots the fit of the observed average yield curve (the corresponding numbers can also be retrieved from the first columns of Table 7). Except for the short end, the 1-factor model is unable to closely fit the observed average yield curve. The 2-factor model already performs remarkably better, but it is the 3-factor model that results in a fit of the average yield curve that is visually almost indiscernible from the actual sample statistics. This is also apparent from Table 7. Means and volatilities are typically fitted with an accuracy of about 1 or 2 basis points in the 3-factor yield curve model, while the fitting errors become an order of magnitude larger for the 2-factor and 1-factor models. Figure 4 plots the fit of selected yields at each data point, instead of the fit of the average yield curve. From this Figure, it is clear that the 3-factor model outperforms the 2-factor model, which in turn outperforms the 1-factor model. The single-factor model performs particularly bad at the long end of the yield curve, and long yields are often under- or overestimated by more than 100 basis points. As our main aim is a realistic simulation of interest rates over a long horizon, we will retain the 3-factor model estimates for our benchmark simulation results.

Table 7: **Comparison fitted average and volatility of yields across different multifactor term structure models**

	Mean				Volatility			
	Data	**1f-model**	**2f-model**	**3f-model**	**Data**	**1f-model**	**2f-model**	**3f-model**
$\mathbf{yield}_{1m}$	3.236	3.242	3.228	3.250	0.932	0.947	0.944	0.932
$\mathbf{yield}_{2m}$	3.263	3.262	3.261	3.262	0.946	0.947	0.947	0.946
$\mathbf{yield}_{3m}$	3.278	3.283	3.295	3.277	0.956	0.947	0.949	0.960
$\mathbf{yield}_{6m}$	3.325	3.345	3.391	3.331	0.997	0.947	0.959	0.999
$\mathbf{yield}_{1yr}$	3.471	3.468	3.571	3.474	1.079	0.947	0.983	1.058
$\mathbf{yield}_{2yr}$	3.801	3.713	3.883	3.800	1.143	0.947	1.039	1.114
$\mathbf{yield}_{3yr}$	4.106	3.957	4.148	4.106	1.137	0.947	1.089	1.136
$\mathbf{yield}_{4yr}$	4.364	4.199	4.376	4.366	1.152	0.947	1.129	1.149
$\mathbf{yield}_{5yr}$	4.581	4.439	4.577	4.582	1.156	0.947	1.158	1.160
$\mathbf{yield}_{10yr}$	5.214	5.615	5.306	5.216	1.197	0.947	1.198	1.189

All table entries are annualized and expressed in percentage points. Column headings "1f-model", "2f-model", and "3f-model" refer to the models with 1, 2, and 3 term structure factors respectively.

Figure 5 plots the loading vectors $b(\tau)/\tau$ for the factors in the 1-, 2-, and 3-factor models. Factor 1, $f_1(t)$, can be labelled a "level factor" as it shifts the entire yield curve approximately in a parallel way when it moves up or down. Factors 2 and 3, $f_2(t)$ and $f_3(t)$, resemble "slope factors" as they shift the short end of the yield curve up or down, while having a smaller impact on the long end of the yield curve.
Table 8 compares the explanatory power of the different factors in the different models. While the single term structure factor in the 1-factor term structure model explains on average only 57.7% of total variability in yields, the three term structure factors taken together explain on average 99.4% of total yield variability.
Figure 6 plots fitting errors for the deposit rates under consideration. Fitted deposit rates are almost identical, irrespective of whether we are using a single- or multi-factor yield curve model. Fitting errors are relatively large overall and remarkably larger for the medium-sized banks. Figure 7 plots the loadings on the deposit spread factor. It turns out that the loadings do not differ dramatically across banks and across different multi-factor models, implying that there is a large amount of comovement in deposit rates across banks. Table 9 reports the explanatory power of the (priced) term structure factors as a percentage of total deposit rate variability. It turns out that the term structure factors explain on average 47%, 54%, and 54% of the total variability in deposit

Table 8: **Explanatory power yield curve factors**

	1f-model	2f-model		3f-model		
	f_1	f_1	f_1+f_2	f_1	f_1+f_2	$f_1+f_2+f_3$
yield$_{1m}$	97.882	56.829	99.762	9.847	59.421	99.875
yield$_{2m}$	99.915	57.851	99.990	10.499	61.584	99.993
yield$_{3m}$	99.369	58.674	99.894	11.142	63.551	99.940
yield$_{6m}$	89.706	60.431	98.575	13.095	68.811	99.682
yield$_{1yr}$	57.918	61.904	93.667	17.076	76.871	99.234
yield$_{2yr}$	38.622	68.355	92.179	25.186	86.345	99.160
yield$_{3yr}$	31.114	77.172	96.105	33.275	91.035	99.424
yield$_{4yr}$	25.164	83.596	98.528	40.731	92.760	98.785
yield$_{5yr}$	23.427	88.055	99.871	48.334	95.028	99.729
yield$_{10yr}$	13.700	86.424	90.466	72.112	96.238	98.202
Average	57.682	69.929	96.904	28.130	79.164	99.402

All table entries expressed in percentage points. Column headings "1f-model", "2f-model", and "3f-model" refer to the models with 1, 2, and 3 term structure factors respectively.

rates across all banks and corresponding to the 1-factor, 2-factor and 3-factor models respectively. When the spread factor is added, these percentages increase to 83%, 80%, and 79%, respectively. Note that these averages conceal relatively large differences across banks. Looking back to the upper panels of Figure 1, it is a stylized fact that the deposit rate data is relatively stable and to a certain extent uncorrelated with market interest rates.

Table 9: **Explanatory power different factors as a percentage of total deposit rate variability**

	1-factor model		2-factor model			3-factor model			
Bank #	f_1	f_1+f_s	f_1	f_1+f_2	$f_1+f_2+f_s$	f_1	f_1+f_2	$f_1+f_2+f_3$	$f_1+f_2+f_3+f_s$
No. 1	52.17	93.95	53.79	59.56	92.01	23.01	44.24	57.19	89.72
No. 2	52.87	96.52	54.93	60.39	94.98	24.10	44.63	56.81	92.81
No. 3	53.19	96.97	55.38	60.93	95.65	24.31	45.41	58.00	93.88
No. 4	55.18	97.16	58.03	65.81	96.11	23.52	49.77	66.85	95.43
No. 5	38.14	65.64	39.16	45.93	62.65	13.83	33.98	48.14	61.06
No. 6	52.68	93.59	55.22	62.37	91.92	22.38	46.12	61.31	89.72
No. 7	41.22	70.50	41.84	48.88	67.17	14.89	35.22	49.25	64.20
No. 8	27.17	46.01	26.43	30.94	42.37	9.32	23.58	33.72	41.97
Average	46.58	82.54	48.10	54.35	80.36	19.42	40.37	53.91	78.60

The column headings refer to the number of term structure factors included (in each model, an additional deposit spread factor is added). Table entries are in percentage points. f_i stands for term structure factor i, while f_s stands for the deposit spread factor (see Section 2.2 for more explanation).

3.3.2 Deposit premium and interest rate elasticity estimates

We now report and discuss the premium and duration estimates that follow from our model specification as presented in Section 2.2 above. For ease of comparability, we standardize the economic value, L_0, and deposit premium, P_0 (see equations (4) and (5)) by expressing them as a percentage of outstanding deposits. Hence, in the remainder of our analysis economic values and deposit premiums are reported as L_0/D_0 and P_0/D_0, respectively.

The estimated factor dynamics under the risk-neutral probability measure in equation (16) drive the Monte Carlo simulation valuation exercise. The intuition is as follows. A number of daily 40-year simulation paths are being generated[14], resulting in simulation paths for the factors, the

[14] The simulation horizon in years is chosen such that the discounted economic rents become negligibly small at even longer horizons. Put differently, the horizon is chosen such that the estimated deposit premium value, being the cumulative sum of these discounted economic rents, converges to a fixed amount.

short rate (being the sum of the term structure factors), the yield curve (being an affine function of the term structure factors), the deposit rates of the eight banks (each being an affine function of the term structure and deposit spread factor), and deposit balance dynamics (depending on the deposit rate dynamics and having a deterministic decaying component). As a result, a number of daily 40-year simulation paths for the economic rents and discounted economic rents result under the risk-neutral probability measure. The deposit premium is then set equal to the cumulative sum of discounted economic rents, averaged over all performed simulations. Additionally, the different corresponding dynamics for all the above variables are being estimated after having shocked the term structure factors separately, each of them resulting in a different average deposit premium estimate. The interest rate elasticity for the shock under consideration is then set equal to the change in deposit premium value over the shock that is imposed.

For illustration purposes, Figure 8 shows the discounted economic rents earned by the bank over the next 40 years, as a percentage of outstanding deposits and averaged over all simulation runs, where (i) the servicing cost is fixed at 0% of outstanding balances[15], (ii) the decay rate parameter is set to 15% per annum (corresponding to a halving time of 5.7 year), and where (iii) each term structure factor starts from its average value at each of the Monte Carlo simulation runs. Figure 8 gives an idea about the timing at which economic rents are earned, evaluated at their present value and averaged across all simulation paths. The discounted rents decrease quasi-monotonically over time, which is in line with the pattern observed and reported by O'Brien (2000). The bulk of all discounted rents is earned in a time span of 20 years (80 quarters). Figure 9 reports the same information as Figure 8, but now expressed in a cumulative way. Quarter t discounted economic rents in Figure 8 can be interpreted as the quarter t slope of the curve in Figure 9. Observe that discounted economic rents converge to zero as we move further in time, due to both the increasing discount rate and decaying balances, which is of course equivalent to the flattening out of the cumulative discounted economic rents. The value to which cumulative discounted economic rents eventually converge (approx. 24% and 21% for Big 4 and medium-sized banks, respectively, in this illustration) corresponds to our definition of the deposit premium, P_0/D_0.

Table 10 reports premium estimates P_0/D_0 for individual banks, averaged over all simulations, where results are presented for the case where the decay rate is fixed at 15% and a 0% servicing cost. The average premium across all banks is 22.7% of outstanding deposits.

The last rows in Table 10 report estimated interest rate elasticities (IREs) with respect to each of the term structure factors, defined as:

$$IRE_{factor\ i} = \frac{dL_0}{L_0 df_i(0)} \tag{25}$$

i.e. the change in economic value that occurs when yield curve factor i is shocked with $x$bp, for each bank (x is taken to be 100bp here, unless stated otherwise). For example, averaged across banks, deposit liability values are estimated to decrease by 3.8% for every 100bp increase in the level of the yield curve (see the IRE estimate for factor 1 for the average bank in Table 10). This is equivalent to saying that the deposit premium or net asset value of the deposit liability is estimated to increase by 3.8% when interest rates go up by 100bp. It is in this latter sense that deposit accounts are said to offset value losses on the asset side when interest rates increase. For the level factor (factor 1), the IRE (with a minus sign) can be understood as a proxy for the modified duration.[16]

[15] See below where a vector of different values is considered for the servicing cost parameter. Servicing costs are usually expressed as a proportion of the deposit balance. O'Brien (2000) shows that the changes in the annual costs per deposit are small and unrelated to the deposit rate. He estimates $r^c(t) = c$ to be 1.2 p.c., 1.31 p.c. and 1.48 p.c. for small, medium-sized and large NOW accounts and 0.75 p.c., 0.83 p.c. and 0.88 p.c. for MMDAs.

[16] This is not fully correct, as the imposed shock dies out and hence is not a truly permanent shock. However, the mean reversion of the level factor is weak and the IRE can thus be interpreted as a modified duration effect.

Table 10: **Estimates of deposit premiums and interest rate elasticities for a base case**

Bank	aver	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8
DDA premium	0.226	0.241	0.239	0.242	0.236	0.193	0.231	0.218	0.206
DDA value	0.774	0.759	0.761	0.758	0.764	0.807	0.769	0.782	0.794
IRE factor 1	-3.771	-3.907	-3.948	-3.939	-3.791	-3.579	-3.823	-3.640	-3.539
IRE factor 2	-2.300	-2.425	-2.445	-2.449	-2.341	-2.095	-2.345	-2.198	-2.106
IRE factor 3	-1.507	-1.611	-1.621	-1.629	-1.548	-1.325	-1.543	-1.427	-1.352

Number of Monte Carlo simulations: 500; horizon in years: 40; servicing cost 0%, withdrawal rate 15%. DDA premium and DDA value are expressed as percentage of outstanding deposits, i.e. P_0/D_0 and L_0/D_0, respectively.

Table 11: **NMA premium and IRE estimates for different values of servicing cost and withdrawal rate parameters**

Panel A: Deposit premium (average across banks)						
		Decay rate r^w				
Net servicing cost	**constant**[a]	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**
0.000	0.478	0.290	0.188	0.133	0.097	0.079
0.005	0.401	0.274	0.163	0.115	0.089	0.071
0.010	0.362	0.233	0.139	0.096	0.079	0.065
0.015	0.301	0.199	0.123	0.083	0.066	0.053
0.020	0.251	0.165	0.103	0.070	0.054	0.044

Panel B: IRE factor 1 ("level")						
		Decay rate r^w				
Net servicing cost	**constant**[a]	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**
0.000	-3.424	-3.654	-3.766	-3.626	-3.450	-3.263
0.005	–3.183	-3.667	-3.765	-3.629	-3.446	-3.263
0.010	–3.268	-3.661	-3.766	-3.632	-3.444	-3.259
0.015	–3.284	-3.673	-3.768	-3.633	-3.445	-3.261
0.020	-3.229	-3.658	-3.762	-3.629	-3.447	-3.261

Panel C: IRE factor 2 ("slope")						
		Decay rate r^w				
Net servicing cost	**constant**[a]	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**
0	-1.662	-1.379	-2.749	-3.060	-3.107	-3.048
0.005	-0.306	-1.470	-2.735	-3.059	-3.104	-3.049
0.010	-0.443	-1.434	-2.726	-3.054	-3.103	-3.047
0.015	-0.199	-1.478	-2.749	-3.066	-3.102	-3.048
0.020	-0.015	-1.414	-2.728	-3.056	-3.104	-3.048

Panel D: IRE factor 3 ("slope")						
		Decay rate r^w				
Net servicing cost	**constant**[a]	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**
0.000	-0.807	-0.469	-2.045	-2.487	-2.626	-2.641
0.005	-0.634	-0.570	-2.030	-2.485	-2.625	-2.642
0.010	-0.504	-0.532	-2.018	-2.478	-2.624	-2.642
0.015	-0.782	-0.578	-2.044	-2.491	-2.624	-2.642
0.020	-1.157	-0.510	-2.021	-2.481	-2.624	-2.642

Table entries represent averages across the eight banks. Number of Monte Carlo simulations per table entry: 250; horizon in years: 40; servicing cost and withdrawal rate specified in row and column headers. DDA premium expressed as percentage of outstanding deposits, i.e. P_0/D_0. Interest rate elasticities computed as: $IRE_{factor\ i} = \frac{dL_0}{L_0 df_i(0)}$.

[a]: Table entries represent results for the case where deposit balances are simply assumed to be constant (no rent capitalization and a zero decay rate).

The results in Table 10 show that premium and IRE estimates vary across banks, although not to the same extent as the cross-sectional variability reported by Hutchison and Pennacchi (1996) and O'Brien (2000) for US NOW and MMDA data (recall Table 2 in Section 2.3). Of course, the premium and IRE estimates in Table 10 are just point estimates that hold for specific assumptions about the servicing cost and decay rate parameters and starting from the unconditional factor averages. It would be desirable to know *(i)* to what extent deposit premium and IRE estimates change when the two main model parameters, servicing costs and decay rates, change, and *(ii)* how certain we are about the reported point estimates, i.e. what range of estimates one gets when one randomly draws starting values from the yield curve factor distributions for a given set of model parameters.

With respect to the first question above, Table 11 reports premium and IRE estimates for a matrix of servicing cost and average decay rate model parameter assumptions. For a given servicing cost, premiums are expected to be smaller when decay rates are higher. Likewise, for a given decay rate, premiums are expected to be smaller when servicing costs are higher. Table 11 Panel A reports premiums averaged across all banks (and simulations).[17] As expected, estimated premiums decrease for a given decay rate with increasing servicing cost and for a given servicing cost with increasing decay rates. For example, when the servicing cost is 1%, a decay rate of 10% (halving time of approx. 10 years) leads to a premium of 23%, while a decay rate of 30% (halving time of approx. 2.5 years) gives rise to a much smaller 10% premium. We see that premiums are never smaller than 6.5% when the servicing cost does not exceed 1% (for decay rates below 50%). Alternatively, for servicing costs up to 2% premiums always exceed 4.4% for all considered decay rates. Of course, if both servicing costs and decay rates are set at high levels, deposit premiums become arbitrarily small and may even become negative. Table 11 Panels B-D give corresponding results for IRE estimates. IREs are relatively insensitive to different servicing cost assumptions, but more strongly affected by average decay rate assumptions. IREs differ dramatically across the different yield curve factors. The sensitivity of the IRE estimates to different withdrawal rates is particularly dependent on the type of IRE (slope versus level). Recall that the level factor IREs can be interpreted as proxies of modified duration. They are relatively stable throughout Panel B and roughly lie in the [-3.8, -3.2] range. With respect to the size of these estimates, the reader is reminded that Belgian savings deposits accounts are somewhat special in the sense that specific price regulation applies to them. Specifically, the remuneration of savings deposits is at the discretion of the banks, but must always consist of a base rate on the one hand and a growth or loyalty premium on the other hand. While the base rate is paid out proportionally with time (pro rata), the payout of the premium components hinges upon the time the (new) deposits remain on the account (typically uninterrupted 6 to 12 month periods are required) and on intricate daycount conventions. These premia are therefore partially responsible for the relative stability of Belgian banks' savings deposits balances.

Figures 10 and 11 address the second question by presenting evidence about the distribution of average deposit premiums and IREs when factors start from different vectors randomly drawn from the factors' multivariate normal distribution, where we again fix the servicing cost to 0% and the decay rate to 15%. From Figure 10, deposit premiums can be seen to roughly vary between 22% and 25% for the Big 4 banks and between 20% and 22% for medium-sized banks, which can be considered fairly narrow and thus accurate. Corresponding ranges for IRE estimates are relatively wide and seem to imply that IREs are more difficult to measure accurately. For example, the level factor IRE seems to vary between -4.5 and -2.5, while similarly wide distributions result for factors 2 and 3. This seems to confirm that IREs cannot be measured with great accuracy. Notwithstanding the implied uncertainty around the point estimate, these findings seem to confirm

[17] Sets of results that distinguish between average big 4 and average medium-sized banks are not reported here but are available upon request. Big 4 bank premiums exceed those of medium-sized banks in all cases, ceteris paribus.

the economic and statistical significance of the nonzero hedging characteristics of deposits.
For convenience, the tabulated results in Table 11 are also plotted as 3D-bar graphs in Figures 12, 13, and 14.

3.4 Results based on a dynamic replicating portfolio approach

Many large internationally active banks seem to use static or dynamic versions of so-called *replicating portfolio models* (RPMs) for their day-to-day management of DDAs, rather than the no-arbitrage discounted cash flow approach presented in this paper.[18] Hence this section will compare our estimates above with the results that follow from a replicating portfolio model approach. Examples of the replicating portfolio approach can be found in Wilson (1994), Frauendorfer and Schürle (2003), Zenios and Ziemba (1992), and Wielhouwer (2003). The idea underlying the most simple static replicating portfolio models is to do an optimization exercise where the available volume of deposits is invested into a portfolio of assets, say, government bonds, such that a specific objective criterion is optimized, and subject to the restriction that the optimal portfolio with fixed weights attached to the set of assets mimics the actual volume dynamics of the DDA, such that volume risk is incorporated. For example, one possible application is to invest the deposited funds in a portfolio of zero coupon bonds such that the resulting replicating portfolio return yields the most stable margin over the deposit rate, i.e. the objective criterion is to minimize the standard deviation of the spread between the replicating portfolio return and the deposit rate, over the given sample of market and deposit rate dynamics (typically 5 to 10 years). The vector of optimal portfolio weights then determines the portfolio duration which is interpreted as the duration of the DDA. Of course, variations along the same lines can be developed. One other popular duration measure is the one that follows from maximizing the Sharpe ratio of the margin instead of minimizing its standard deviation.
The class of *dynamic* RPMs tries to tackle an important shortcoming of the class of *static* RPMs, namely the absence of any kind of uncertainty in the optimization exercise. Indeed, the static models basically run one single optimization based on the last, say, 5 or 10 years of data, without taking into account that rates and balances may move (very) differently in the future. Dynamic RPMs estimate dynamic characteristics (volatility, mean-reversion, etc.) on the available sample of market rates, deposit rates, and deposit balances (taking into account possible correlations among them), but then simulate forward a large number of scenarios of market rates, deposit rates, and deposit balances, very much like in the no-arbitrage models described above, but now typically under the actual and not risk free probability measure. For each of the simulated scenarios, the optimization exercise is repeated, such that the estimated replicating portfolio and duration estimate better reflects the variety of scenarios that may materialize in the future.
Below we present duration estimates based upon such a dynamic replicating portfolio approach. We assume constant portfolio weights and include 1m, 12m, 3yr, 5yr, 7yr, and 10yr discount bonds in the replicating portfolio. We present results for two different model setups. One set of results is based on the assumption that the portfolio investments are sold at the end of each month, after which new portfolios are bought. Hence we need to use zero coupon bond 1m holding period returns. We also assume that we only invest in the bonds that are included in our portfolio. A second set of results depends on a different set of assumptions. Here we assume that all zero coupon bond investments are held until maturity, earning the yield to maturity at inception in each of the

[18] We do not discuss or consider other employed techniques. One of the most simple *ad hoc* approach is to simply assume a discretionary distribution of deposits across a number of repricing or duration buckets (see for example Uyemura and van Deventer (1993), Chapter 9). One could also implement a strictly statistical approach and infer customer withdrawal behavior from recording entry and exit of deposits on an individual account basis (see Anderson and McCarthy (1986) and Sheehan (2004)). This approach is interesting but requires databases that are typically not available at most deposit-issuing banks and is also vulnerable to the critique that a forward-looking assessment of risk is completely absent.

future months it does not mature (i.e. an earnings approach). Moreover, instead of only investing in the bonds that are included in the portfolio, we instead assume that the amount to be invested in each bond is spread out in equal amounts over all intermediary maturities. More explicitly, when for example 50% of outstanding deposits is invested in the 5-year zero coupon bond, effectively 0.833% of deposits is invested in the 1-month zero coupon bond, 0.833% of deposits in the 2-month zero coupon bond, ..., 0.833% in the 60-month zero coupon bond. Of course, such a spreading of the investment implies that the resulting duration is half as large, relative to the case where no spreading of investments occurs.

For both setups, we assume that the bank either wants to minimize the volatility of the spread between the replicating portfolio return and the deposit rate, or that it wants to maximize the Sharpe ratio between the replicating portfolio return and the deposit rate. Finally, notice that we assume away volume effects (deposit withdrawals and growth) for convenience.

In a first step, a large number of plausible monthly yield curve scenarios for the next 60 months is generated, based on our estimated three-factor yield curve model under the risky probability measure. For each yield curve scenario, the implied term structure of yields and one month log holding period returns, $r(t+1,\tau)$, can be derived. One month log holding period returns can be derived from the log yield curve dynamics in each scenario, as follows:

$$r(t+1,\tau) = y(t,\tau) - (\tau-1)(y(t+1,\tau-1) - y(t,\tau)) \tag{26}$$

where $r(t+1,\tau)$ denotes the log return of holding the τ-month bond between months t and $t+1$. In a second step and for each of the scenarios, we then optimize the margin or spread between the replicating portfolio holding period return on the one hand and the respective deposit rate return on the other hand, resulting in a vector of optimal portfolio weights. The optimal portfolio weights, averaged over the scenario runs, are then used to back out an implied duration, using the fact that the duration of a discount bond is equal to its remaining time to maturity.

Table 12: **Duration estimates across banks, based on a dynamic replicating portfolio approach**

Holding period	**Monthly reinvestment (returns)**		**Hold to maturity (yields)**	
Optimization criterion	**Maximize Sharpe ratio**	**Minimize standard deviation**	**Maximize Sharpe ratio**	**Minimize standard deviation**
Bank no. 1	2.320	0.218	4.530	3.547
Bank no. 2	2.520	0.220	4.546	3.592
Bank no. 3	2.336	0.220	4.545	3.583
Bank no. 4	2.262	0.214	4.470	3.435
Bank no. 5	4.163	0.206	4.392	3.243
Bank no. 6	2.603	0.215	4.493	3.482
Bank no. 7	2.826	0.208	4.409	3.293
Bank no. 8	3.139	0.205	4.367	3.200
Average across banks	**2.771**	**0.213**	**4.469**	**3.422**

Table entries reflect duration estimates in years, averaged across 100 scenario runs (i.e. 100 simulated 5-year yield curve dynamics, with the replicating portfolio optimization performed for each bank and for each simulated yield curve scenario). See main text for the main assumptions behind the dynamic replicating portfolio exercises performed here.

Table 12 entries represent duration estimates for all banks in our sample and averaged across the 100 scenario runs. Table 12 shows that discretionary choices about the optimization criterion or the holding period are not innocuous. When performing the optimization with holding period returns, we find that the estimated duration averaged across the banks is 2.8 years when the Sharpe ratio of the margin is optimized, while only 0.2 years when the standard deviation of the margin is minimized. When performing the optimization with yields, we find that the estimated duration averaged across the banks is 4.5 years when the Sharpe ratio of the margin is optimized, while only 3.4 years when the standard deviation of the margin is minimized. The 0.2 year result is atypical and due to the fact that long bond holding period returns are relatively volatile and hence punished severely when their higher return is disregarded. When comparing these replicating portfolio estimates with those that we estimated based on the no-arbitrage discounted cash flow approach (panel B of Table 11 on page 17), we find that the criterion of minimizing the standard deviation and holding bonds until maturity results in estimates that are closest to the [-3.8,-3.2] range we found in our DCF framework (recall that the IRE of the level factor (with a minus sign) can be interpreted as a proxy for the modified duration).

Although the class of dynamic replicating portfolio models is attractive, it has a number of important disadvantages. First, these models are inconsistent with the central no-arbitrage principle that underlies the bulk of asset pricing in finance theory and practice. Second, one is simply unable to estimate a DDA economic value when using replicating portfolio models, in contrast to our no-arbitrage discounted cash flow framework. Finally, in our experience the optimization exercises may result in quite different duration estimates when the objective criterion is changed to equally reasonable criterions or when the vector of available assets is expanded (yields are highly correlated throughout the maturity spectrum, and optimal portfolio weights may not be stable when different sets of assets are being considered). Moreover, it is unclear which setup and criterion should be used and how changes in duration are to be interpreted. The clear advantage of replicating portfolio models is that it is more intuitive to explain and easier to implement compared to the no-arbitrage DCF approach. Indeed, the absence of a valuation framework implies that there is no need to switch to a risk-neutral pricing framework, which avoids the difficulties with market incompleteness.

4 Policy implications

DDA valuation has been a point of controversy with respect to the introduction of International Financial Reporting Standards (IFRS), and more specifically IFRS 39 *Financial Instruments: Recognition and Measurement*. With respect to demand deposits, the IASB (International Accounting Standards Board), the international accounting rule-maker located in London, states very clearly that "the fair value of a financial liability with a demand feature is not less than the amount payable on demand ...".[19] The assumed continuous equality between fair (model-implied) and par value of DDAs also implies that DDA fair values are completely interest rate insensitive, i.e. their interest rate risk can only be hedged with the continuous roll-over of short term assets. Succinctly, deposit accounts with a demand feature have a zero duration. Consistent with this stance, demand deposits are explicitly excluded by the IASB from hedge accounting provisions.

The IASB stance is quite controversial, as it is inconsistent with banks' current risk management practices, attaching relatively long durations to their DDAs grounded on the factual experience of behavioral stability of deposits. Bankers also claim that the recording of DDAs at par effectively injects artificial volatility in reported equity if they are not allowed to recognize that outstanding balances effectively hedge the interest rate risk of medium-term assets. The issue has been discussed among accounting setters, bankers and bank supervisors in the runup to the adoption of IFRS by the European Commission. In the end, the EC decided to adopt IFRS, but only after "carving out" or deleting the IAS 39 hedge accounting rules. EU Member Countries are effectively free to either apply the carve out with respect to hedge accounting or to apply the hedge accounting provisions as originally devised by the IASB.[20]

[19] Already in 1999, the Joint Working Group of Standard Setters mentions "...the IASC/CICA Discussion Paper's conclusion that the appropriate fair value measure of demand deposits is likely to be close to their face value, and that, unlike fixed rate loan assets, the fair value of demand deposits is not likely to vary much with changes in interest rates" (JWG 1999).

[20] Importantly, the issue is also intimately linked to the discussion the IASB is having with insurers about the

Most of the controversy can be linked to the different opinions about the proper definition of what a deposit liability really is. For example, Barth (2005), representing the accounting rule-makers view, defines liabilities as "the present obligation of an entity arising from past events, the settlement of which is expected to result in an outflow of resources from the entity". In the case of deposits, the IASB believes that the present obligation arising from a past event are the *actual deposited funds* by a depositor and the obligations that follow from that deposit, i.e. rent due on outstanding balances. This definition has important valuation implications, as existing deposits typically remain unused for a relatively short period (perhaps a few days, weeks, or months, depending on the type of deposit account). Correspondingly, the value discount will indeed be negligible, as short term zero coupon bonds trade close to their par values.

In contrast, bankers and risk managers typically argue that the past event is not the actual deposit but rather the *establishment of the relationship with the depositor*, such that new future replacement deposits are expected based on past experience that is likely to persist in the future. For example, when the bank gives a mortgage loan to a new client, it may ask the client to open a deposit account on which its salary needs to be paid. It is obvious that the implied deposit balance behavior is radically different in this case than in the more narrowly defined case followed by the IASB, leading to potentially very different valuation and hedging results. When the bankers' view is followed, deposit balances resemble more a perpetuity, valuation discounts are potentially large and interest rate sensitivities are relatively long.

Above we already set out our opinion in this difficult debate. We think it would be best to include existing deposits and future expected deposits *to the extent that the latter can be identified in a verifiable way.* So, our view lies closer to the bankers' view in the sense that we would also want to consider some future expected cash flows (though not all), next to existing deposits only. Moreover, we do not in general support the *a priori* accounting rule-makers' belief that deposits typically remain unused for a short period only, and hence that valuation and hedging problems become irrelevant if one is to restrict oneself to existing deposits only. Such a belief may indeed be justified for transaction deposit accounts, but need not hold in total generality. For example, Belgian savings deposits account balances may be relatively stable, even if we consider outstanding balances only, due to the fact that *(i)* they are not true transactions accounts and *(ii)* specific fiscal and price regulation applies. By law, their renumeration should always consist of a base rate on the one hand and a loyalty or growth premium on the other hand. While the base rate is paid out pro rata of the number of days the deposits have been sitting in the account, growth and loyalty premiums are *only reaped when balances have remained in the account without interruption for relatively long periods* (6 months and 1 year, typically). In return, depositors enjoy a fiscal advantage as the earned rent is fiscally exempt up to a certain limit.

Finally, note that Euro System central banks are typically reluctant to record nonzero deposit premiums in the balance sheet. From a financial stability point of view, this reluctance is understandable, as such a recording may trigger a deposit run when unsophisticated depositors observe that the bank values the debt that is owed to them at less than the par value. In fact, many technical and supervisory working groups, amongst others the European Financial Reporting Advisory Group (EFRAG) and the Basel Committee on Banking Supervision (BCBS), also agree with the IASB position that demand deposits should be recorded in the balance sheet at face value. However, the IASB position that the fair value of demand deposits is continuously equal to the face value, even when interest rates change, remains a controversial stance. In our view, the idea that verifiable future expected deposits must be taken into account may bridge the gap between supervisors, risk managers and standard setters.

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proper accounting rules for insurance products. Similar controversies arise here between the contractual date IASB stance and the more behavioral date stance of the insurers, as one-year contracts are typically being renewed quasi-automatically for the bulk of the customer base, leading to substantially longer effective time to maturities.

5 Conclusions

We propose a framework to value demand deposit accounts and to assess their interest rate risk. We add to the literature by proposing a multi-factor flexible-affine joint yield curve deposit rate model, in which bank deposit rates depend on both term structure and a deposit spread factor. The multifactor term structure model is shown to outperform the single factor yield curve model and results in a superior fit of actual yield curve dynamics. Describing actual yield curve dynamics in a no-arbitrage framework well is a crucial feature of the model as the yield curve needs to be simulated forward far in the future to perform the valuation of deposit accounts. We present estimates of deposit premiums and how they are affected by shifts in the yield curve, for a series of different model parameters. We also compare our estimates with duration estimates that are derived from dynamic replicating portfolio approaches, which are currently more popular among large international banks.

Based on our model specification and Belgian bank data between December 1994 and June 2005, our estimates imply that deposit premiums are statistically and economically significant, but sensitive to the assumed average deposit decay rate. Estimated interest rate elasticities depend to a large extent on the nature of the yield curve shock (level versus slope shock). For a 0% servicing cost and a 15% (40%) decay rate assumption, we find that the estimated deposit premium averaged across banks is 23% (10%). The interest rate elasticity of our first factor, which we label a level factor, is -3.8 years (-3.5 years) and relatively insensitive to changes in withdrawal rate parameters. We find that, while deposit premiums can be measured relatively accurately, there is more uncertainty around the point estimates of estimated interest rate elasticities. Nevertheless, we find that deposit liability values depreciate when market rates increase, thereby offsetting some of the value losses on the asset side. The precise hedging characteristics depend on the decay rate assumptions and to a large extent on the nature of the assumed interest rate shock (level versus slope shock). Variation in deposit premium and IRE estimates across Belgian banks is relatively modest compared to the reported variation in multi-bank premium estimates based on US NOW and MMDA accounts. Keeping model parameters constant across banks, Big 4 bank deposit premiums seem to slightly exceed those of the medium-sized more aggressive banks, but the impact remains limited. As we do not have bank-specific information about our two main model parameters, we opted to report a matrix of premium and IRE estimates for different combinations of servicing cost and decay rate assumptions.

Discounted cash flow models also need to address the difficult issue of identifying the relevant cash flows to include and exclude from the valuation exercise. Our proposal is to put forward *verifiability* as dominant principle to identify cash flows that need to be incorporated in the valuation exercise. As such, a bank, that can show in a verifiable way that its existing depositor will deposit new funds that replace the ones that have run off, must take these expected inflows into account. Due to a lack of data, we are unable to estimate the detailed implications of alternative (more encompassing) definitions of deposit liabilities. However, our results about the impact of different average withdrawal rates on deposit liability values already suggest that such a wider scope may significantly influence value and value sensitivity estimates. Moreover, and importantly, even when the valuation exercise is strictly limited to existing funds only, as the IASB proposes, we find that for certain DDAs in certain countries the fair value may still differ substantially from the face value and that the duration may hence be nonzero. We argue that the valuation and risk management of DDAs should rely on an empirical assessment of deposit stability and not based on *ex ante* beliefs. We also present duration estimates based on the alternative dynamic replicating portfolio approaches that are popular in large internationally active banks. While we argue that no-arbitrage discounted cash flow approaches are preferable to replicating portfolio techniques because of the underlying no-arbitrage principle and because a DDA economic value can not be estimated by replicating portfolio models, we find that our interest rate elasticity point estimates for the level factor are closest to the replicating portfolio results where the standard deviation of the margin is minimized.

Similar valuation techniques have been applied to other assets and liabilities that are traded in imperfectly competitive markets (for example loans with prepayment options, credit card loans, etc.). We leave the analysis of alternative methods for the pricing of deposits in incomplete markets (using for example super-replicating portfolios, utility-based valuation, good deal bounds) for future research.

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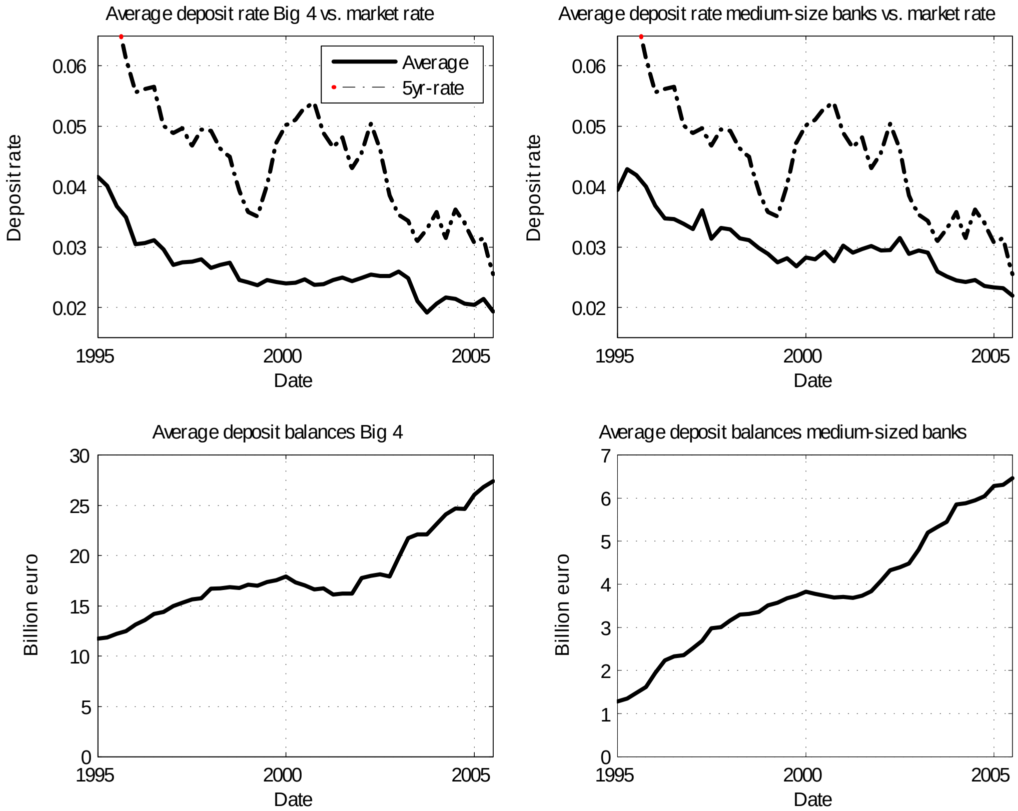

Figure 1: Belgian deposit rate and balance data (1994:Q4-2005:Q2).

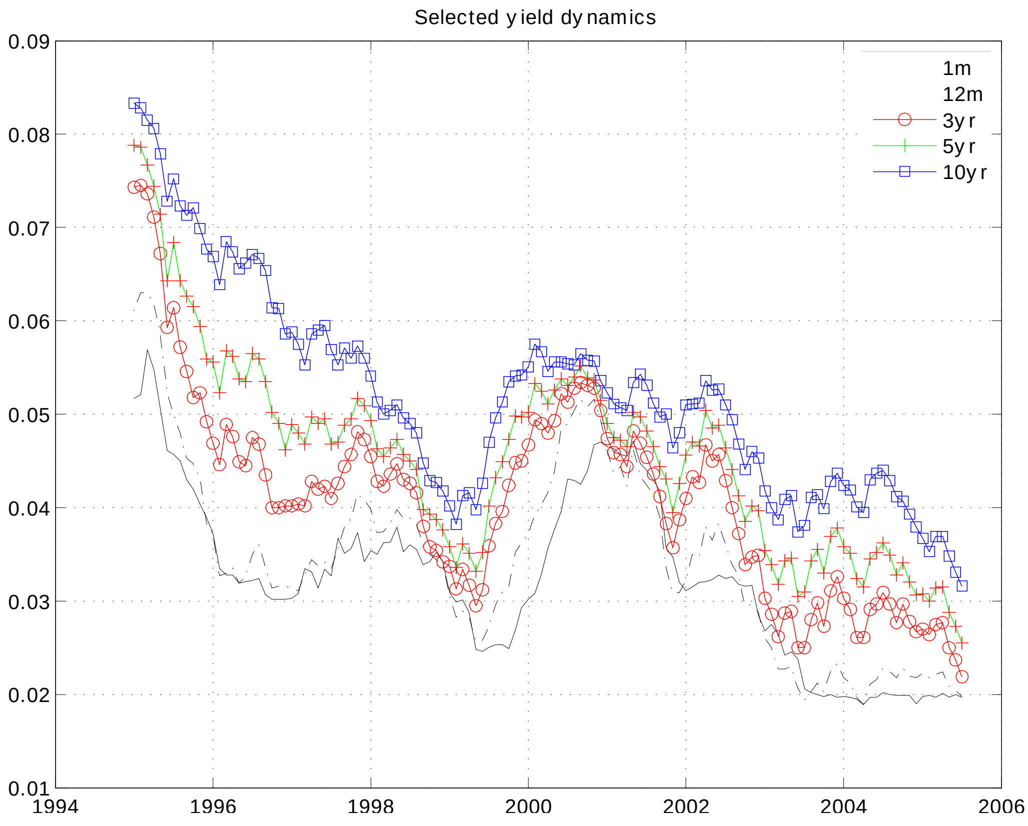

Figure 2: Belgian yield dynamics (December 1994-June 2005).

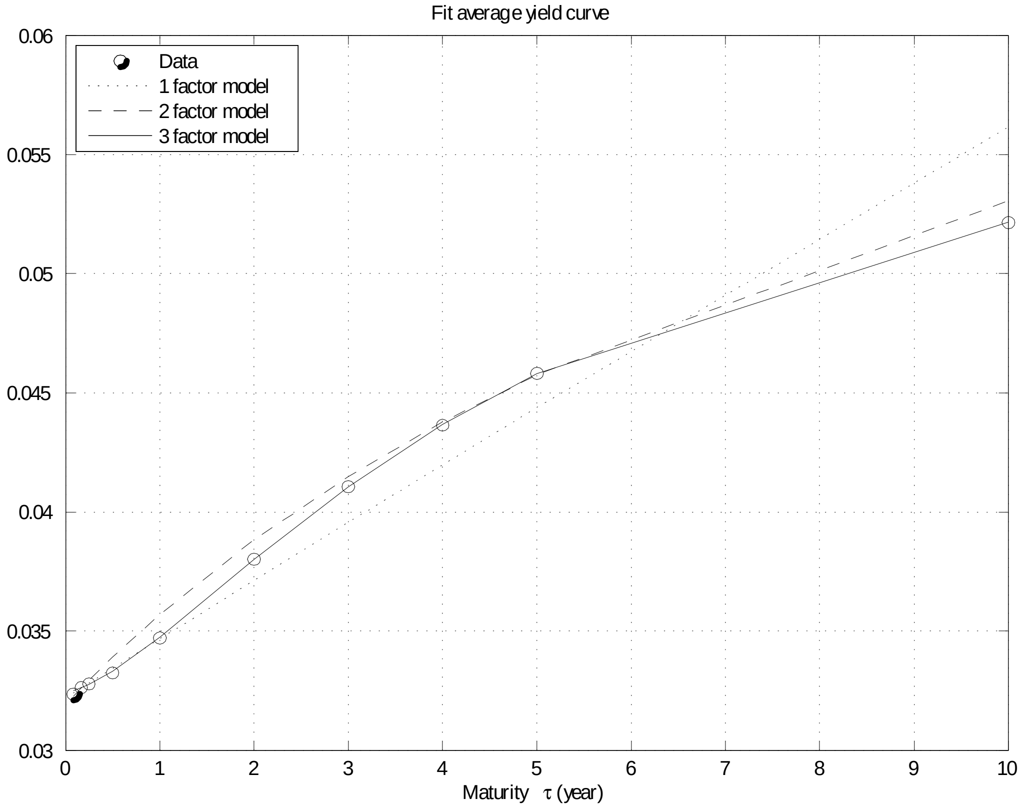

Figure 3: Comparison model-implied fit average yields (1994:Q4-2005:Q2)

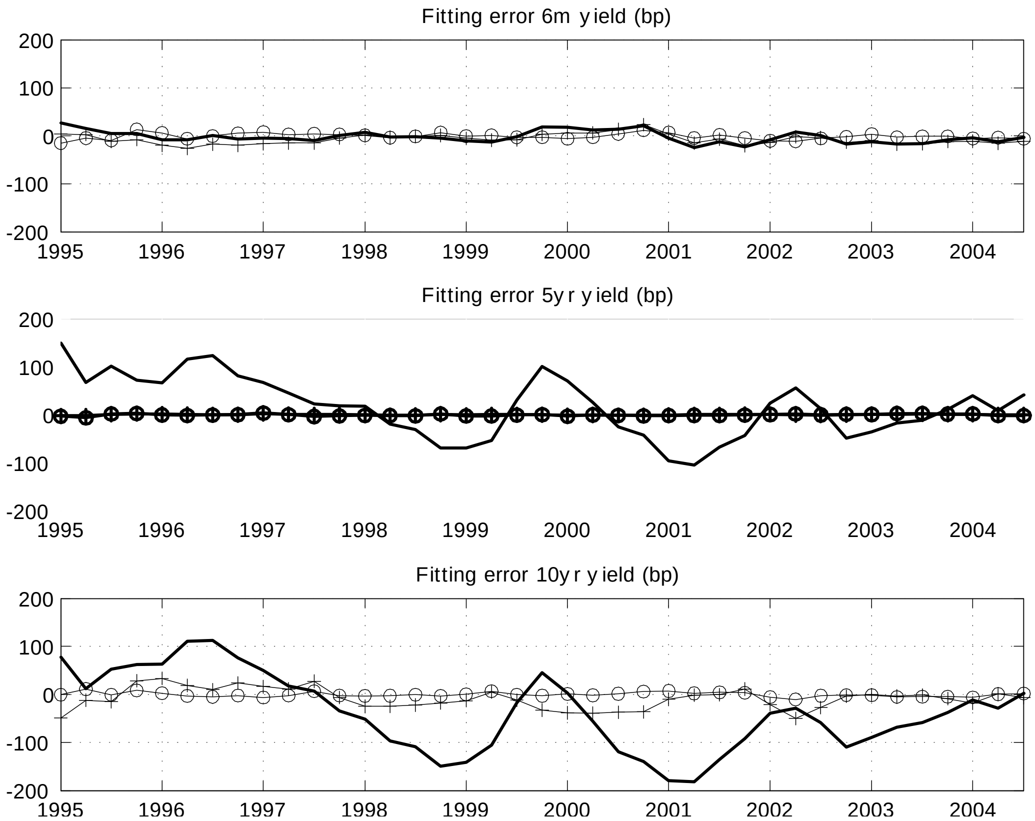

Figure 4: Comparison fitting errors for each yield time series (1994:Q4-2005:Q2)

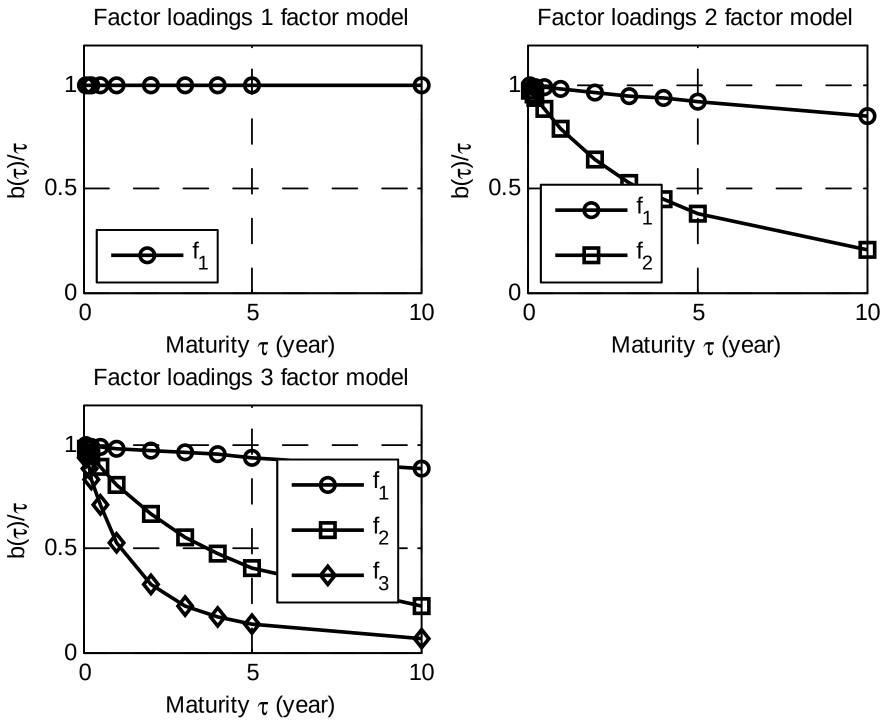

Figure 5: Estimated term structure factor loadings (1994:Q4-2005:Q2)

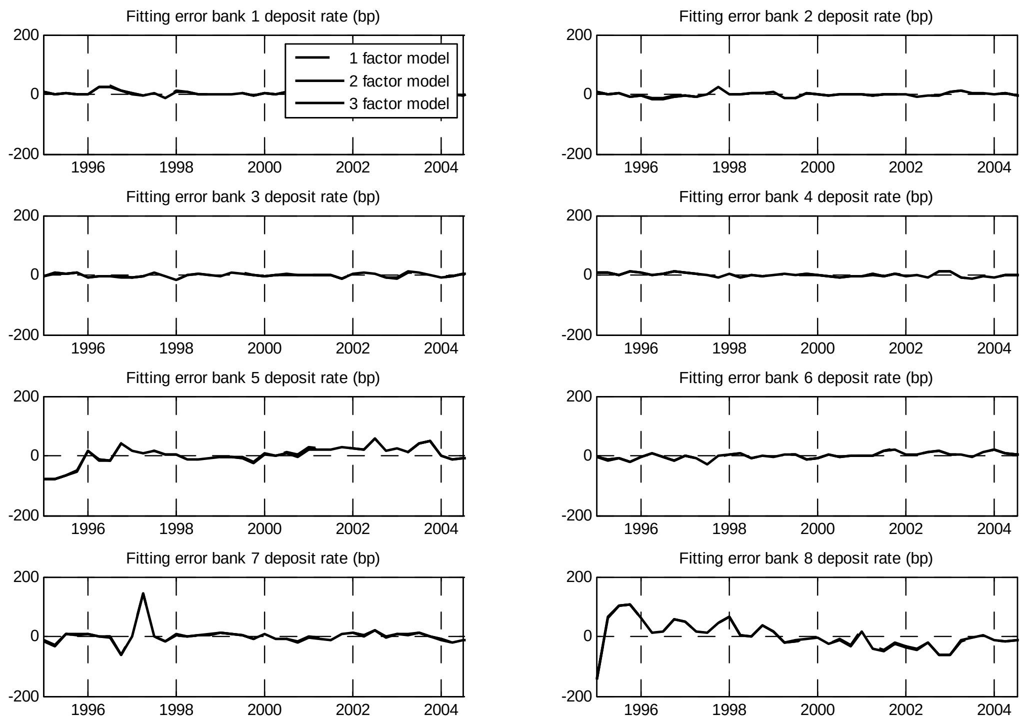

Figure 6: Comparison model fit errors of deposit rate time series of 8 banks considered (1994:Q4-2005:Q2)

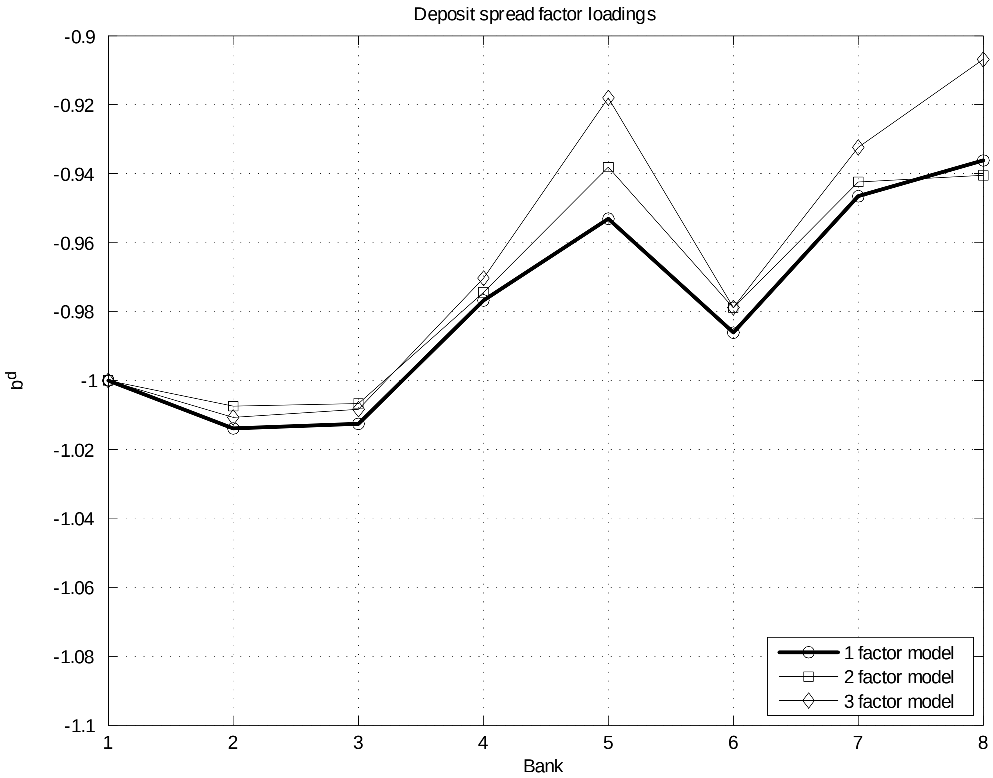

Figure 7: Estimated deposit spread loadings (1994:Q4-2005:Q2)

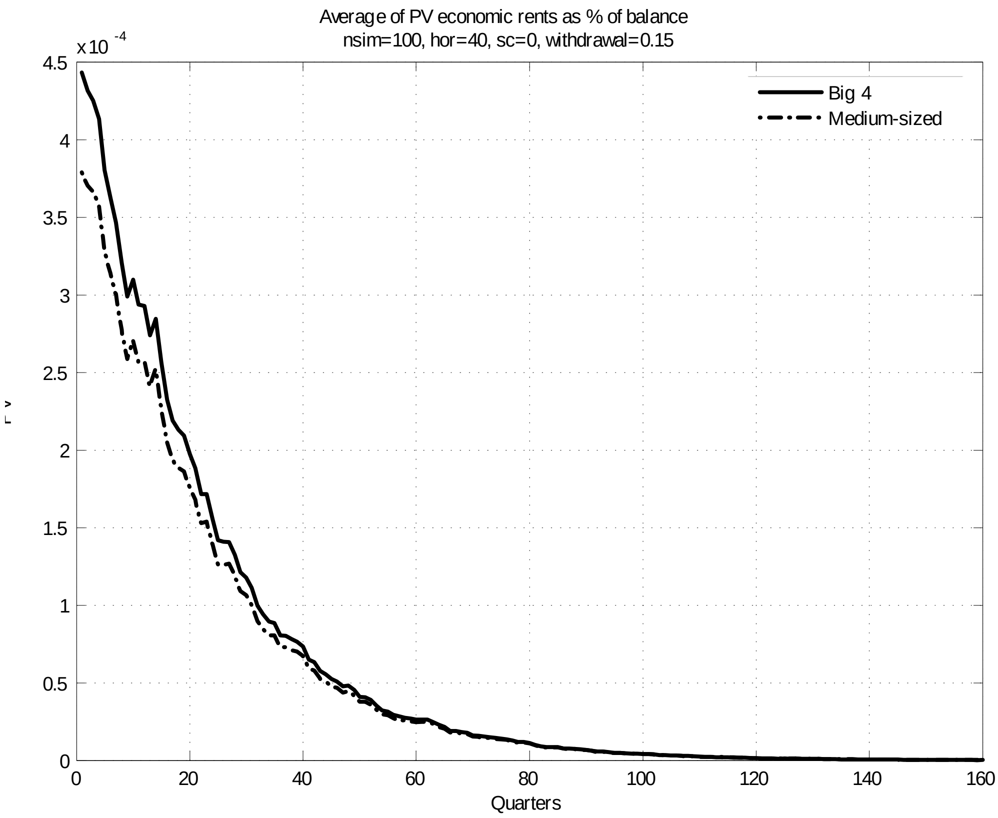

Figure 8: Discounted earned economic rents averaged over all simulation runs and averaged over the banks within a group (servicing cost 0% and decay rate 15%).

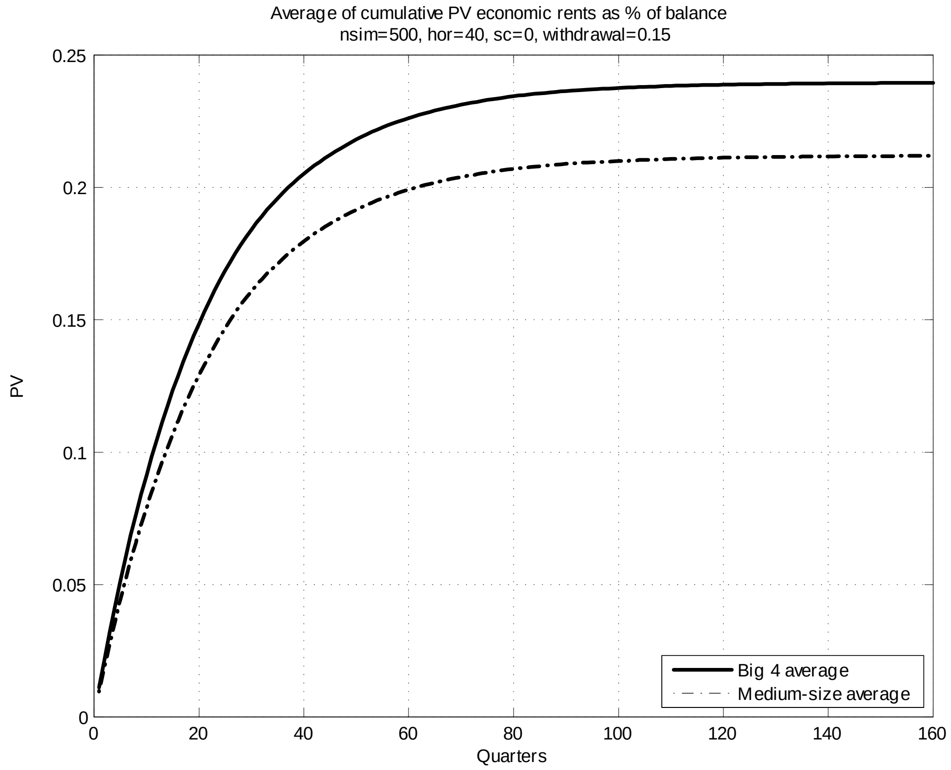

Figure 9: Cumulated discounted earned economic rents, averaged over all simulation runs and averaged over the banks within a group (servicing cost 0% and decay rate 15%).

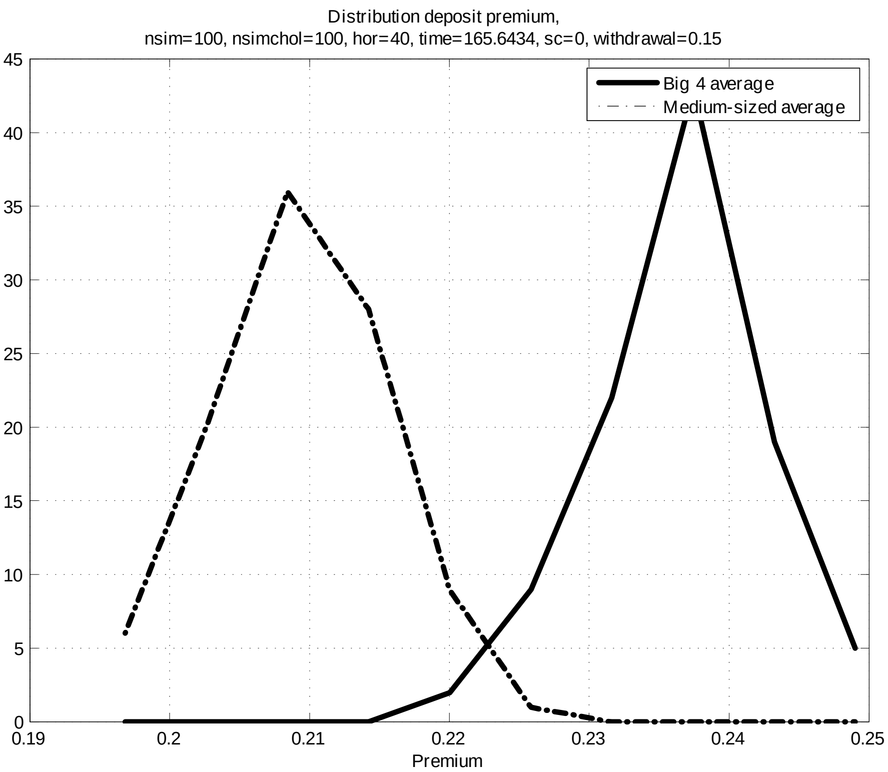

Figure 10: Distribution of premium estimates, when term structure factors start from random draws of their distribution (servicing cost 0%, decay rate 15%).

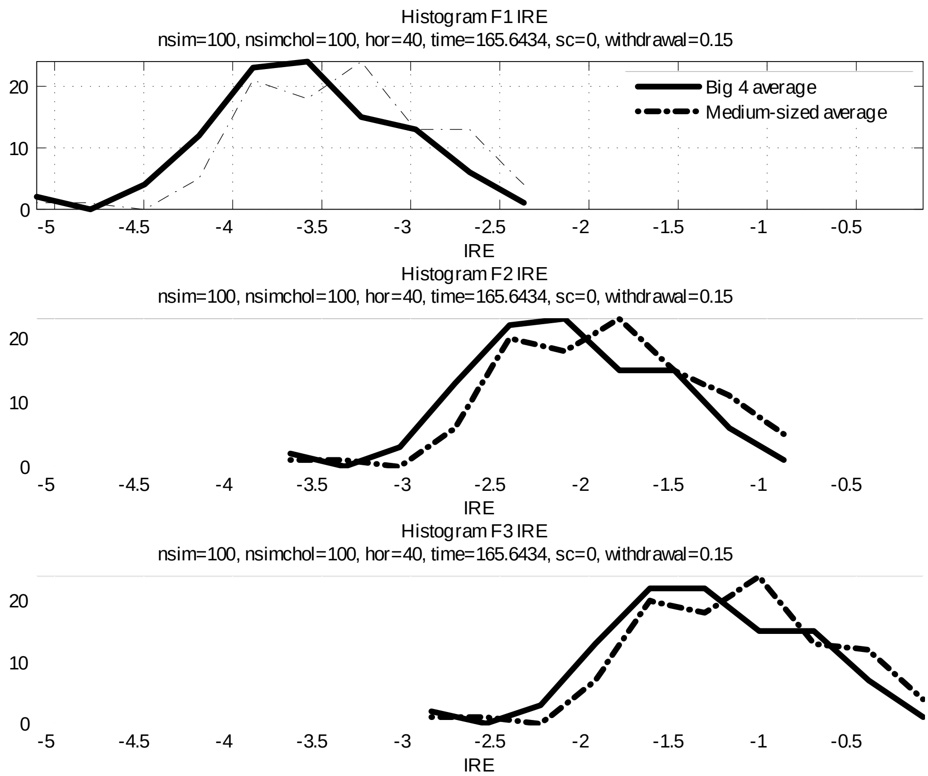

Figure 11: Distribution of IRE estimates, when the term structure factors start from vectors randomly drawn of their distribution (servicing cost 0%, decay rate 15%).

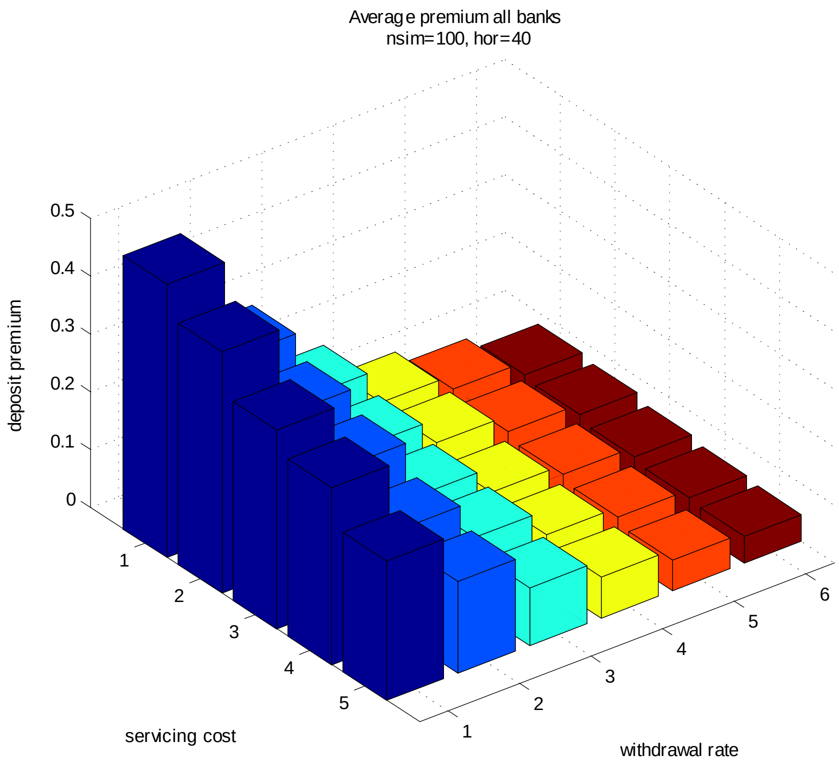

Figure 12: Dependence of deposit premium on different servicing cost and decay rate assumptions (servicing cost from 0-2% in 5 steps of 50bp each and decay rate from 0-50% in 6 steps of 10% each).

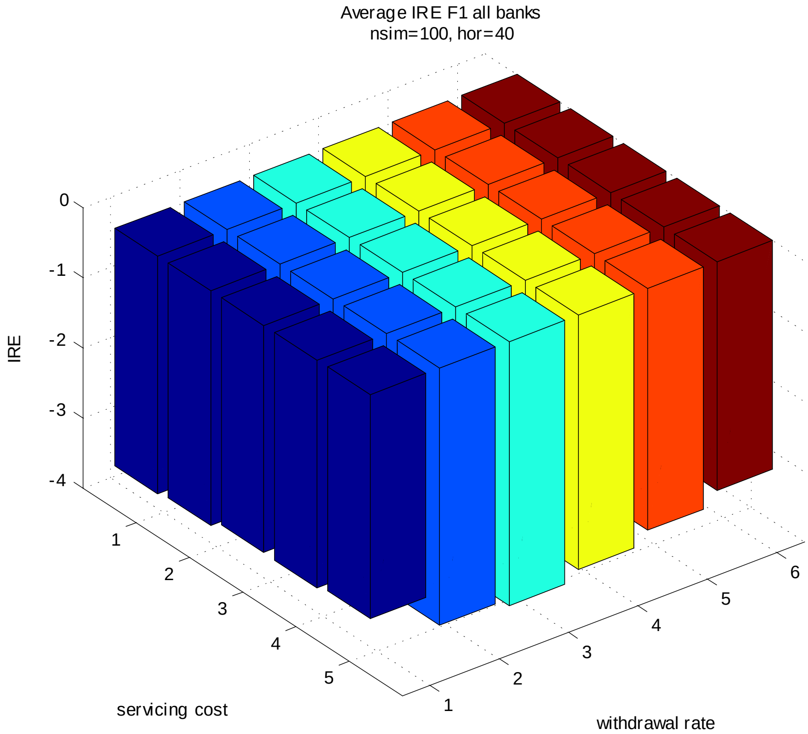

Figure 13: Dependence of interest rate elasticity of level factor on different servicing cost and decay rate assumptions (servicing cost from 0-2% in 5 steps of 50bp each and decay rate from 0-50% in 6 steps of 10% each).

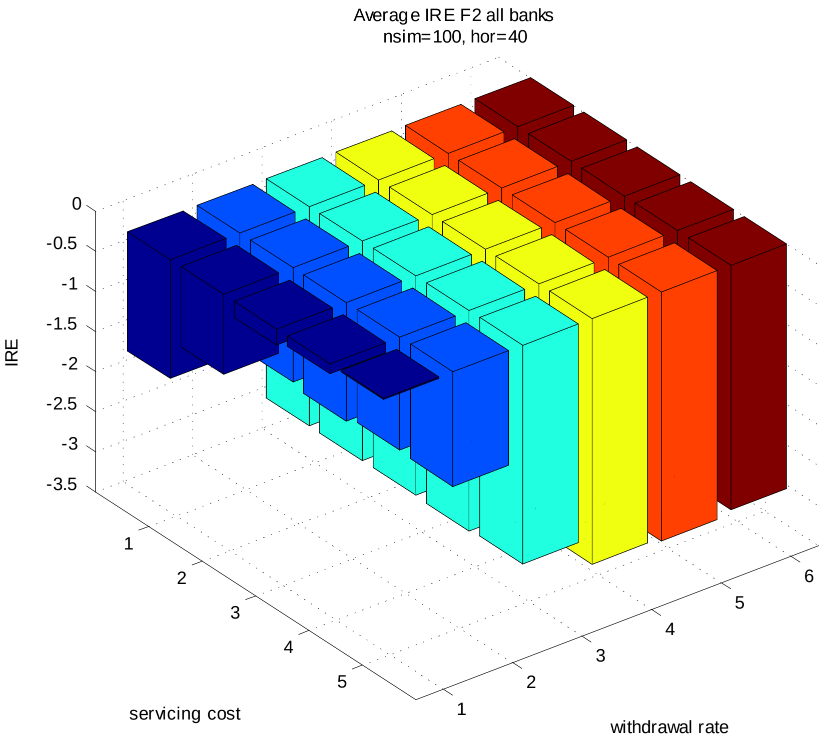

Figure 14: Dependence of interest rate elasticity of slope factor on different servicing cost and decay rate assumptions (servicing cost from 0-2% in 5 steps of 50bp each and decay rate from 0-50% in 6 steps of 10% each).

Appendix: Parameter estimates for the models with 1 and 2 term structure factors

Tables 13-14 present the parameter estimates of our no-arbitrage models with respectively 1 and 2 term structure factors and one deposit spread factor. The results of the three term structure factor model, on which most of our estimates are based are presented in the main text.

Table 13: **Estimated parameters for the joint yield curve deposit rate model with 1 yield curve factor (1994:Q2-2005:Q2)**

	Factor			
	1			**4**
$\kappa_{i,i}$	0.1443			1.4827
	(0.1509)			(0.6926)
$\kappa_{4,i}$	0.7760			1.4827
	(0.4641)			(0.6926)
θ_i	0.0317			0
	(0.0117)			
$\sigma_i^{\mathbf{2}}$	0.000049			0.000044
	(0.000012)			(0.000012)
λ_i	-7.6491			0
	(82.8688)			
$\Xi_{i,i}$	-0.1443			
	(0.1475)			
R_{1m}	0.2559	R_{2yr}	18.7876	
R_{2m}	0.0100	R_{3yr}	26.1733	
R_{3m}	0.0751	R_{4yr}	35.1551	
R_{6m}	1.3568	R_{5yr}	38.6364	
R_{1yr}	8.5902	R_{10yr}	74.4446	
R_{bank_1}	1.1452	R_{bank_5}	8.9753	
R_{bank_2}	0.6491	R_{bank_6}	1.2050	
R_{bank_3}	0.5605	R_{bank_7}	7.1385	
R_{bank_4}	0.5107	R_{bank_8}	19.8657	

Maximum likelihood estimates with standard errors underneath. The values in the measurement error covariance matrix (R) are multiplied by 10^6. The loglikelihood is on average equal to 100.9762 (excluding constant in the loglikelihood). Factor 1 is the single term structure factor. Factor 4 refers to the spread factor (see main text).

Table 14: **Estimated parameters for the joint yield curve deposit rate model with 2 term structure factors (1994:Q2-2005:Q2)**

	Factor			
	1	**2**		**4**
$\kappa_{i,i}$	0.1821	0.4751		111.1279
	(0.3022)	(0.1078)		(9.5656)
$\kappa_{4,i}$	75.0106	100.3997		111.1279
	(5.5887)	(7.9837)		(9.5657)
θ_i	0.0254			0
	(0.0166)			
$\sigma_i^{\mathbf{2}}$	0.000255			0.000215
	(0.000033)			(0.000042)
λ_i	-0.3980	-29.3147		0
	(22.2628)	(24.9554)		
$\Xi_{i,i}$	-0.1500	-0.0008		
	(0.3118)	(0.0876)		
R_{1m}	0.2550	R_{2yr}	6.5426	
R_{2m}	0.0100	R_{3yr}	2.7966	
R_{3m}	0.1096	R_{4yr}	0.9453	
R_{6m}	1.4145	R_{5yr}	0.0761	
R_{1yr}	6.0385	R_{10yr}	4.9220	
R_{bank_1}	1.1042	R_{bank_5}	8.8213	
R_{bank_2}	0.6611	R_{bank_6}	1.1751	
R_{bank_3}	0.5693	R_{bank_7}	7.1537	
R_{bank_4}	0.5468	R_{bank_8}	20.0044	

Maximum likelihood estimates with standard errors underneath. The values in the measurement error covariance matrix (R) are multiplied by 10^6. The loglikelihood is on average equal to 105.8186 (excluding constant in the loglikelihood). Factors 1 and 2 are the term structure factors. Factor 4 refers to the spread factor (see main text).

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