

Robustifying optimal monetary policy using simple rules as cross-checks



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Abstract

There are two main approaches to modelling monetary policy; simple instrument rules and optimal policy. We propose an alternative that combines the two by extending the loss function with a term penalizing deviations from a simple rule. We analyze the properties of the modified loss function by considering three different models for the US economy. The choice of the weight on the simple rule determines the trade-off between optimality and robustness. We show that placing some weight on a simple Taylor-type rule in the loss function, one can prevent disastrous outcomes if the model is not a correct representation of the underlying economy.

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1 Introduction

There are two main approaches to specifying monetary policy in the literature; *optimal policy* and *simple instrument rules*. By 'optimal policy' we here mean minimizing a specific loss function using all information embedded in the model.¹ 'Simple instrument rules', on the other hand, specify how the monetary policy instrument - the key interest rate - should respond to a subset of the information available to the policy maker. The original Taylor (1993) rule is an example of a simple rule where the central bank responds to a subset of the information set, i.e., the rate of inflation and the output gap. By construction, simple rules lead to higher loss than optimal policy when evaluated in a given model, but the excess loss depends both on how restricted the simple rule is and on the model.

In addition to providing a rough description of actual policy, simple rules have a normative motivation; they are considered more robust to model uncertainty than optimal policy. Taylor and Wieland (2012) provide a survey and discussion of the literature on simple robust rules. In the literature, the model simulations are commonly based on the assumption that the central bank commits to the simple rule in a mechanical way. However, as pointed out by Svensson (2003), full commitment to a simple rule like the Taylor rule is unrealistic, and no central bank does this in practice. Svensson therefore rejects simple rules, both from a positive and from a normative perspective. He advocates instead optimal policy (or 'targeting rules') and argues that this is a more reasonable description of monetary policy, as the central bank is treated as an optimizing agent in the same way as households and firms, and that optimal policy leads to better outcomes than simple rules.

Although Svensson's critique may be justified, the fact that central banks do not commit to following simple instrument rules like the Taylor rule mechanically, does not imply that monetary policy is not influenced by such rules at all. On the

¹The term 'optimal policy' is not restricted to policies that maximize the utility of the representative household, but includes also minimization of *ad hoc* loss functions, which, for example, could represent the objectives for monetary policy as defined by the government.

contrary, we find it reasonable to assume that monetary policy in practice has, at least to some extent, been influenced by the vast literature on simple robust rules. Indeed, Kahn (2012) provides a thorough documentation on how simple Taylor-type rules have influenced the FOMC decisions and how they are used as *cross-checks* to the interest rate decisions in many central banks. An illustrative example is the FOMC meeting in January 31-February 1 1995, where the Greenbook suggested a 150 basis points increase of the federal funds rate to 7 percent. FOMC member Janet Yellen expressed the following concern: *"I do not disagree with the Greenbook strategy. But the Taylor rule and other rules... call for a rate in the 5 percent range, which is where we already are. Therefore, I am not imagining another 150 basis points"*.² Similar references to the Taylor rule can also be found from policy meetings in other central banks.³ We will therefore argue that a realistic description of the monetary policy process is optimal policy using all available information, but where simple rules are used as cross-checks (or guidelines). This approach seems consistent with how policymakers form their interest rate decisions in practice. For example, Yellen (2012) describes the assessments as follows: *"One approach I find helpful in judging an appropriate path for policy is based on optimal control techniques. [...]. An alternative approach that I find helpful [...] is to consult prescriptions from simple policy rules."*

While the existing literature on robustness assumes either optimal policy or full commitment to a simple robust rule, we take an intermediate approach. We introduce a modified loss function extended with a term penalizing deviations of the interest rate from the level implied by a simple rule.⁴ Our approach is inspired

²Kahn notes that, "[a]s it turned out at the meeting, the federal funds rate target was raised 50 basis points to 6 percent, where it stayed until July 1995 when it was cut to 5 $\frac{3}{4}$ percent."

³For example, the Vice-Governor at the Riksbank, Svante Öberg, expressed on the monetary policy meeting December 14, 2010: *"With GDP growth of over 5 per cent, more or less normal resource utilisation, and inflation and inflation expectations at around 2 per cent, it feels slightly uncomfortable to have a repo rate of 1.25 per cent. A traditional Taylor rule would in the present situation result in a repo rate of 3 to 4 per cent."*

⁴Since we first circulated our paper there have been other contributions applying our modified loss function. See Tillmann (2011) and Bursian and Roth (2012).

by Rogoff's (1985) seminal paper on the optimal degree of commitment to an intermediate target, in which he argues that *"it is not generally optimal to legally constrain the central bank to hit its intermediate target (or follow its rule) exactly"* (p.1169). While Rogoff's proposal was aimed to reduce the inflationary bias under discretion, we consider partial commitment to rules aimed to make policy more robust. In other words, we analyze whether the loss across different models tends to be lower if the central bank minimizes a modified loss function with weight on a simple rule. The idea of extending the loss function with a term with a simple interest rate rule is novel, but the idea of robustifying optimal policy through modified loss functions is not new. Orphanides and Williams (2008) show that a loss function with reduced weight on the unemployment gap and on interest rate stability is more robust to wrong assumptions about private agents' expectations formation (i.e., rational expectations vs learning). Our approach of using cross-checks in the modified loss function is also related to Beck and Wieland (2009). They consider a policy where the central bank conducts optimal policy in "normal" times, but extends the loss function with a money growth term when money growth is outside a critical range. Our specification differs in using simple interest rate rules, rather than money growth, as cross-check and by letting the simple rule always enter the operational loss function and not only when the deviation is outside a critical range. The novelty of our modified loss function is that it builds a bridge between the two alternative monetary policy approaches; optimal policy and simple robust rules, making it possible to analyze intermediate solutions.

To analyze the robustness properties of the modified loss function, we consider three alternative models for the US economy: The Smets and Wouters (2007) model, the Rudebusch and Svensson (1999) model, and the Fuhrer and Moore (1995) model. We assume that the Smets-Wouters model is the central bank's reference model due to its influence on models used for policy simulations among central banks in practice. We have re-estimated the other two models on the data set from Smets and Wouters (2007) to get comparable estimates of the variances of the shocks and

thereby losses in the different models. The two alternative models have been thoroughly investigated in the robustness literature, which makes it easier to compare our results to those obtained earlier. Moreover, and importantly, these models represent very different views on issues such as inflation persistence and expectation formation. The Rudebusch-Svensson model is completely backward looking, while the Fuhrer-Moore model is partly forward looking and partly backward looking. As we will show, backward looking models imply very different monetary policy as far as inertia is concerned they and therefore represent a natural alternative to the largely forward looking reference model.

We consider different simple rules, including the classical Taylor rule, an optimal simple Bayesian rule, which minimizes the (weighted) average of the losses in the different alternative models, and a minimax rule, which minimizes the maximum loss in the alternative models. We find that placing a weight on either of the rules, the central bank can insure against very bad outcomes if the reference model is wrong. Even if the simple Bayesian rule and the minimax rule are derived optimally using the alternative models, the classical Taylor rule, with the coefficients of 1.5 on inflation and .5 on the output gap, does surprisingly well and not significantly worse than the simple optimized rules. Another interesting finding is that the weight on a simple rule is always strictly smaller than one. Thus, a robust monetary policy is to *lean towards* simple rules, but not follow them mechanically. We therefore find support for the common view among proponents of simple rules, that they should be used as guidelines, but not as mechanical formulas for the interest setting.

The paper is organized as follows. Section 2 presents the reference model and the two alternative models. Section 3 describes the approach of optimal monetary policy with cross-checking and Section 4 presents the results. Section 5 concludes.

2 A reference model and two alternative models

The analysis uses three distinct estimated macroeconomic models for the US economy. The models are taken from the new model data base described in Taylor and Wieland (2012). The three models are: the Smets and Wouters (2007) model, the Rudebusch and Svensson (1999) model, and the Fuhrer and Moore (1995) model. We briefly discuss each of them in turn. Since the original estimations of the models are based on somewhat different time periods, we re-estimate the Rudebusch-Svensson model and the Fuhrer-Moore model on the same data as the Smets-Wouters model.

2.1 The Smets and Wouters (2007) model

The Smets and Wouters model (SW hereafter) is a medium scale closed-economy dynamic stochastic general equilibrium (DSGE) model estimated on US data from 1966:1 to 2004:4 using a Bayesian estimation methodology. The model is based on Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). Households in the model maximize a nonseparable utility function with goods and labor efforts as arguments. Labor is differentiated by a union, which has monopoly power over wages, and firms use capital and labor to produce differentiated goods. Both prices and wages are sticky and based on the Calvo model, but extended with partial indexation.

Medium scale DSGE models have had a great influence on the model development both in academia and policy institutions, and many central banks use such models as the core model for forecasting and policy analysis. The main advantage of these models is that they combine the property of being structural, which facilitates interpretation and story telling, and having forecasting properties comparable with VARs. Given the influence of these models on monetary policymaking in practice, we let the reference model of the policymaker be represented by SW.

2.2 The Rudebusch and Svensson (1999) model

The model by Rudebusch and Svensson model (RS hereafter) is a small closed-economy model with a Phillips-curve and an IS-curve. The original model was estimated by single-equation OLS on US data from 1961:1 to 1996:2. The model is purely backward-looking, and one may interpret the model as summarizing the traditional Keynesian view on the monetary policy transmission mechanism. The model is summarized by the following two equations:

$$\pi_{t+1} = a_{\pi 1}\pi_t + a_{\pi 2}\pi_{t-1} + a_{\pi 3}\pi_{t-2} + (1 - a_{\pi 1} - a_{\pi 2} - a_{\pi 3})\pi_{t-3} + a_y y_t + \varepsilon_{t+1} \quad (1)$$

$$y_{t+1} = b_y y_t + b_{y1} y_{t-1} - b_r (\bar{i}_t - \bar{\pi}_t) + \eta_{t+1}, \quad (2)$$

where y_t and π_t denote the output gap in period t and the rate of inflation, respectively. The nominal interest rate is denoted i_t , ε_t is a cost-push shock and η_t is a demand shock. The variables with bars are defined as four-quarter averages. We have re-estimated the model over the same sample as the Smets-Wouters model, i.e., 1966:1 to 2004:4, in the presence of the original Rudebusch-Svensson Taylor rule using quarterly time series on inflation, nominal interest rates and real GDP growth, where we used the original parameter estimates as priors. Table 1 shows the new parameter estimates.

[Table 1 about here]

2.3 The Fuhrer and Moore (1995) model

The Fuhrer and Moore model (FM hereafter) is a small, closed-economy model with partly forward-looking and partly backward-looking expectations and is often claimed to provide a good description of inflation persistence. The model assumes overlapping wage contracts, where the price level today, p_t , reflects the contract wages, x_t , negotiated in quarter $t - 1$, i.e.,

$$p_t = \sum_{i=0}^3 f_i x_{t-i}, \quad (3)$$

where $f_i \geq 0$ and $\sum f_i = 1$. The weights on each contract weight is a downward-sloping linear function of contract length according to:

$$f_i = 0.25 + (1, 5 - i)s, \quad 0 < s \leq 1/6. \quad (4)$$

The index of real wage contracts that were negotiated on the contracts currently in effect is given by

$$\nu_\tau = \sum_{i=0}^3 f_i (x_{t-i} - p_{t-i}). \quad (5)$$

Agents set nominal wage contracts so that the current real contract wage equals the average real contract wage index expected to prevail over the life of the contract, but where the degree of pressure in the economy, captured by the output gap y_t , affects the negotiated wage:

$$x_t - p_t = \sum_{i=0}^3 f_i E_t(\nu_{\tau+i} + \gamma y_{t+i}) + \varepsilon_{p,t}, \quad (6)$$

where $\gamma > 0$ and ε_t is a white noise shock. Aggregate demand (the output gap) is given by

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_\rho \rho_{t-1} + \varepsilon_{y,t},$$

where $\rho_t = \frac{D}{1+D} E_t \rho_{t+1} + \frac{1}{1+D} (i_t - E_t \pi_{t+1})$ is the long-term real interest rate and D is a constant approximation to Macaulay's duration

The model is re-estimated using quarterly time series on inflation, nominal interest rates and real GDP growth, where the original parameter values are used as priors, and the results are reported in Table 2.

[Table 2 about here]

3 Monetary policy

In this section we start by describing optimal policy and the objectives of the central bank. We then describe the simple interest rate rules used in the analysis below and proceed with a description of optimal monetary policy with cross-checking by simple rules. This set-up modifies the objective of the central bank to include a penalty for deviations from a simple interest rate rule. We end the section with a description of the simulation set-up.

3.1 Optimal policy

We shall assume that the central bank minimizes the intertemporal loss function

$$E_t \sum_{h=0}^{\infty} \beta^h L_{t+h}$$

where β is the time discount factor. The period loss function L_t is a linear-quadratic function of the variables entering the model. We assume that the central bank has access to a commitment technology, i.e., monetary policy is conducted under commitment. The central bank minimizes the intertemporal central bank loss function subject to the constraints given by the model. The characterization of optimal policy is standard, and we refer to Svensson (2010) for a description. Generally, optimal policy is given by a set of first-order conditions and the Lagrange multipliers associated with the constraints.

The objectives of monetary policy are represented by the following (*ad hoc*) period loss function:

$$L_t^* = \pi_t^2 + \lambda y_t^2 + \delta (i_t - i_{t-1}). \quad (7)$$

The parameters $\lambda \geq 0$ and $\delta \geq 0$ give the central bank's weight on stabilizing the output gap and the change in the interest rate, respectively, relative to stabilizing inflation. If there is no model uncertainty, the central bank should minimize the loss function (7) subject to the constraints given by its reference model. Under imperfect

knowledge of the economy, it might not always be the case that the central bank should minimize (7). We shall thus consider modified loss functions that deviates from (7) with the aim of making optimal policy more robust.

3.2 Simple rules

It is possible to derive an implied instrument rule from the first-order conditions describing optimal policy. However, such rules are usually complicated and very model-specific. Simple rules, on the other hand, are based on a smaller and more restricted set of variables. Although not being optimal, simple rules are often considered more robust than optimal policy. We restrict the attention to the sub-class of Taylor-type rules considered by Taylor and Wieland (2012). Specifically, the simple rules have the following form (where we disregard constant terms):

$$i_t^s = a_i i_{t-1} + a_\pi \pi_t + a_y y_t + a_{y-1} y_{t-1}, \quad (8)$$

where parameters a_i , a_π , a_y , a_{y-1} measure the responsiveness of the nominal interest rate to changes in the corresponding macroeconomic variable.

We consider three types of simple rules. First, we consider optimal simple rules from a *Bayesian* model averaging perspective, i.e.,

$$\min_{(a_i, a_\pi, a_y, a_{y-1})} \sum_{m=1}^M p_m L_m,$$

where p_m is the weight (probability) attached to a given model m and M is the set of alternative models. Second, we consider the optimal *minimax* rule, where the coefficients are optimized to give the minimum loss in the model that implies the maximum loss, i.e.,

$$\min_{(a_i, a_\pi, a_y, a_{y-1})} \max_{m \in M} L_t.$$

One objection to the minimax rule and the Bayesian rule is that they are opti-

mized within a given set of models and may thus be only robust across the models within that particular set. One solution is to use rules that are not optimized within particular set of models, but have “reasonable” parameter values. One such rule is the *classical* Taylor rule, where $a_\pi = 1.5$, $a_y = 0.5$, $a_i = a_{y-1} = 0$, which we therefore include in the set of simple rules.

3.3 Optimal monetary policy with cross-checking

The two alternative approaches to monetary policy are motivated from two distinct perspectives. Optimal policy is designed to give the maximum achievement of the central bank’s objectives given its best knowledge of the functioning of the economy. Simple (robust) instrument rules are designed to avoid bad policy outcomes. Next, we show how the two approaches can be combined in a way that makes optimal policy more robust, or, alternatively, simple rules more optimal. We denote this optimal policy with cross-checking.

The starting point is the central bank loss function, which is extended with a term penalizing deviations from a simple interest rate rule:

$$L_t^m = \pi_t^2 + \lambda y_t^2 + \delta (i_t - i_{t-1})^2 + \gamma (i_t - i_t^s)^2, \quad (9)$$

where γ is the weight on the deviation from the simple instrument rule. For expositional reasons, it is useful to re-write the loss function as a weighted average of the loss function, L_t , and the deviation from the simple rule, i.e.,

$$\hat{L}_t = (1 - \theta)L_t + \theta (i_t - i_t^s)^2, \quad (10)$$

where $\theta = \frac{\gamma}{1+\gamma}$.

Our modified loss function could be interpreted as optimal policy with guidance from simple rules. This seems to be a reasonable description of how simple rules are used as cross-checks in practice, as described in Kahn (2012). An alternative inter-

pretation is that the central bank uses a simple rule as a benchmark, but deviates from it when it finds it appropriate to do so. Our proposal can thus be interpreted as optimal deviations from a simple rule.

Monetary policy with cross-checking has also been considered by Beck and Wieland (2009). They did not, however, consider simple interest rate rules as cross-checks, but instead analysed the case where the monetary policymaker conducts optimal policy in "normal times", but extends the loss function with a money growth term when money growth is outside an estimated critical range.⁵

The most common rationale for modified loss functions is to improve the discretionary solution when the central bank is not able to commit, as initiated by Rogoff (1985). If the central bank is in fact credible and able to commit, this rationale for modified loss function disappears and the central bank should not aim to minimize a modified loss function. However, as mentioned in the introduction, modified loss functions have been suggested also as a way to robustify optimal policy, see Orphanides and Williams (2008). They show that lowering the weight on the unemployment gap and on interest rate smoothing increases robustness if expectations are characterized by adaptive learning instead of rational expectation as in the core model.⁶

3.4 Simulation setup

In order to study robustness of monetary policy, we proceed as follows. We derive optimal monetary policy in the reference model, i.e. SW, and use this policy to compute the implied loss in both the reference model and the alternative models. Optimal policy is derived using the modified loss function, that is, under the assumption of optimal monetary policy with cross-checking. Our rationale for minimizing a

⁵Christiano and Rostagno (2001) also use money growth as a cross-check. They assume that the central bank sets interest rates according to a Taylor rule in "normal" times, but changes to money growth targeting if money growth is outside critical range.

⁶Orphanides and Williams note, however, that their approach can be generalized to include additional variables in the modified loss function, but leave this for future research.

modified loss function is to provide better achievement of the underlying objectives, represented by (7), when the reference model is wrong. When we compute the implied losses to assess the robustness of the (modified) optimal policies, we therefore use the loss function (7).

Since the alternative models do not have the same set of variables, it is not possible to apply the optimal targeting rule from SW to the other models. We will therefore follow Orphanides and Williams (2008) and approximate optimal policy by an interest rate rule based on the three variables which enter all models, namely inflation, the output gap, and the interest rate. To capture the possibly complicated dynamics of optimal policy, we allow for a quite generous lag structure, i.e.,

$$i_t = \sum_{j=1}^4 h_{i,j} i_{t-j} + \sum_{j=0}^4 h_{\pi,j} \pi_{t-j} + \sum_{j=0}^4 h_{y,j} y_{t-j}. \quad (11)$$

The rule which approximates fully optimal policy under commitment thus has 14 parameters, and this rule gives an expected loss that gets very close to the expected loss under optimal policy.⁷

4 Results

The main objective is to analyse robustness of optimal policy with cross-checking. To this end we compute expected losses in each model using policy derived under the modified loss function in the reference model with different weights θ attached to the simple rule. As the benchmark loss function, we use $\lambda = 0.5$ and $\delta = 0.1$.⁸

⁷For simulation purposes we use Dynare, which can be downloaded from the website www.dynare.org. The optimal coefficients are found by a grid search algorithm developed by Junior Mailh. We thank him for providing us with the MatLab code.

⁸As a robustness check of our results, we also consider alternative weights. Those results are available upon request.

4.1 The classical Taylor rule

We consider first the case where the policymaker places weight on the classical Taylor rule. Disregarding the constant terms, the rule is given by

$$i_t = 1.5\pi_t + 0.5y_t. \quad (12)$$

The classical Taylor rule is an interesting benchmark for two reasons. First, there is evidence that monetary policymakers indeed place weight on the Taylor rule, see Kahn (2012) and Ilbas, Røisland and Sveen (2012). Second, it is interesting to analyze the extent to which placing a weight on a simple rule that is not optimized for any of the models considered, still could provide some insurance against model uncertainty.

[Figure 1 about here]

Figure 1 shows the average loss, measured by the loss function (7), evaluated in the alternative models, as a function of the weight θ on the classical Taylor rule. Applying the terminology suggested by Levin and Williams (2003), the figure illustrates the *fault tolerance* of each model with respect to the weight on the Taylor rule. Note that the fault tolerance analysis here is somewhat different than in Levin and Williams, who analyze robustness with respect to parameters in a simple interest rate rule.

Consider first the loss evaluated with the reference model, i.e., SW. The policy that minimizes the *modified* loss function gives, as expected, a higher loss measured by L_t , except for the case $\theta = 0$. Note, however, that the excess loss is quite low for values of θ up to around 0.6. That is, SW is quite fault tolerant for a relatively wide range of weights θ on the Taylor rule. FM is fault tolerant for all admissible values of θ . Even if the loss in FM is lowest when $0 < \theta < 1$, neither optimal policy in SW nor full commitment to the Taylor rule lead to very high losses in this model. This is, however, not the case for RS. With optimal SW policy, RS becomes

dynamically unstable, and hence the expected loss is infinite. The reason for the dynamic instability in RS is that the optimal SW policy implies considerable inertia in policy, which, as shown by Levin and Williams (2003), could lead to dynamic instability in RS. However, by placing a weight on the Taylor rule of about 0.35 or above, dynamic stability is ensured. The loss measured by RS reaches its minimum at $\theta = 0.65$. An interesting result is that in both FM and RS the minimum loss is where $0 < \theta < 1$. This result strongly support Taylor's (1993) advice that central banks should not follow simple rules like the Taylor rule mechanically, but use them as guidelines.

What is the appropriate weight on the Taylor rule? To answer this question we consider two approaches; minimax and Bayesian. The solid line in Figure 2 represents the maximum loss as a function of the weight on the Taylor rule. For values $\theta \in [0, 0.55]$, RS gives the highest loss, and for $\theta \in [0.55, 1]$, SW gives the highest loss. FM is not the worst-case model for any values of θ . The value of θ which minimizes the maximum loss is $\theta = 0.55$.

The dashed line measures the Bayesian loss, where the loss in each model is weighted together with weight 1/2 on SW and 1/4 on RS and FM, respectively. An interesting result is that the Bayesian loss has its minimum also at $\theta \approx 0.55$. Although this depends on the weights attached to the various models, the minimum loss tends to be achieved for θ in the range between 0.4 and 0.6 for any reasonable set of weights on the models. Thus, given this set of alternative models, an appropriate weight on the Taylor rule does not depend too much on whether one takes a minimax or Bayesian perspective. Therefore, the optimal weight on the Taylor rule is in this set of models independent of the degree of ambiguity aversion.

[Figure 2 about here]

4.2 Optimal simple rules

Next we consider the implications of putting a weight on optimal simple rules, where parameters are chosen in an optimal way. As explained above, we consider rules derived under both Bayesian model averaging and minimax policy.

4.2.1 Optimal Bayesian rule

To construct the optimal simple Bayesian rule, we restrict the rule to minimize the weighted loss over the two alternative models, i.e., RS and FM. One could also include SW when minimizing the weighted losses, but since SW is the reference model under which optimal policy is derived, the results become more clear-cut when the robust simple rule is based on only the two alternative models. Placing equal weights on the two alternative models, gives the following optimal simple Bayesian rule:

$$\begin{aligned} i_t^B &= .62i_{t-1} + 2.17\pi_t + 1.89y_t - 0.58y_{t-1} \\ &= .62i_{t-1} + (1 - .62) [5.71\pi_t + 3.44y_t + 0.53\Delta y_t]. \end{aligned} \quad (13)$$

The simple Bayesian rule exhibits a moderate degree of inertia, represented by the coefficients on the lagged interest rate and on the change in the output gap and a quite aggressive response to inflation and the level of the output gap.

[Figure 3 about here]

The next step is to find the optimal policy in the reference model, using the simple Bayesian rule as a cross-check. Figure 3 shows the losses in the three models as a function of the weight on the optimal simple Bayesian rule in the modified loss function. The loss in RS now becomes finite for a much smaller weight on the simple rule than in the case of the classical Taylor rule, which reflects that the simple Bayesian rule is more "taylored" for RS. In the limit where the central bank follows

the simple rule mechanically, i.e., $\theta = 1$, the loss in RS is lower than in FM. The reason is that the FM is generally more fault tolerant than the RS for any parameter values, which implies that on the margin there is more to gain by reducing the loss in RS than the loss in FM. The average loss in the two models is thus minimized for a lower loss in RS than the loss in FM. The loss evaluated with the reference model is, however, high when the central bank follows the Bayesian rule mechanically.

[Figure 4 about here]

Figure 4 shows the maximum loss in the three models and the weighted loss as a function of the weight θ on the Bayesian rule in the modified loss function. We see that the weight $\theta \approx 0.15$ gives the lowest maximum loss, which is the point where the loss in SW crosses the loss in RS in Figure 3. The minimum weighted loss occurs at $\theta \approx 0.2$. Depending on the policymakers' aversion against model uncertainty, the appropriate weight on the Bayesian rule should therefore lie in the interval $[0.15, 0.2]$. Somewhat surprisingly, the maximum loss and the weighted loss are not significantly lower when the central bank places weight on an optimal Bayesian rule than if it places weight on the classical Taylor rule. This suggests that the classical, but in this respect *ad hoc*, Taylor rule could provide almost as good insurance as a simple rule that is optimized over the relevant models, given that the rule has an appropriate weight. Moreover, it is interesting to note that the appropriate weight on the Bayesian rule is lower than the appropriate weight on the classical Taylor rule. The reason is as follows. The Bayesian rule implicitly gives a high weight on RS, since it is the least fault tolerant model. Placing a small weight on this simple rule is therefore sufficient to rule out dynamic instability in RS. Robustness can hence be achieved by deviating less from optimal policy in SW.

It is also interesting to relate to the robust Bayesian rules identified by Levin and Williams (2003). They consider RS in addition to the canonical New Keynesian model and the Fuhrer (2000) model, of which RS is the least fault tolerant. Our results therefore imply that the central bank can achieve a better outcome by

deviating optimally from their robust simple rules.

4.2.2 Optimal minimax rule

The optimal simple minimax rule is given by

$$\begin{aligned} i_t^M &= 0.87i_{t-1} + 0.83\pi_t + 1.37y_t - 0.86y_{t-1} \\ &= 0.87i_{t-1} + (1 - .87)[6.38\pi_t + 3.92y_t + 6.62\Delta y_t]. \end{aligned} \quad (14)$$

Comparing the optimal minimax rule (14) with the optimal Bayesian rule (13), we see that there is considerably more policy inertia, represented by the coefficient coefficients on i_{t-1} and Δy_t , in the minimax rule than in the Bayesian rule. The reason is that the Bayesian rule gives a higher loss in FM than in RS. In order to minimize the maximum loss, the minimax rule implies that the loss in FM is reduced until the two losses are equal. Since high inertia is beneficial in FM, but not in RS, the optimal minimax rule implies more inertia than in the Bayesian rule. However, as the degree of inertia in the minimax rule is only slightly lower than in optimal policy in SW, a larger weight on the minimax rule than on the Bayesian rule is required to make the inertia sufficiently low to give dynamic stability in RS, as seen from figure 5.

[Figure 5 about here]

From figure 6 we see that depending on the degree of ambiguity aversion, where the maximum loss and the Bayesian loss represent the extreme cases, an appropriate weight on the optimal simple minimax rule lies in the range $[0.85, 0.95]$. Although a large weight on the simple rule is required, full commitment to the rule, i.e., $\theta = 1$, is still not advantageous.

[Figure 6 about here]

4.2.3 The trade-off between optimality and robustness

To provide insurance against bad outcomes under model uncertainty, the policy-maker has to accept a higher loss if the reference model is correct. The insurance premium is determined by the trade-off between the loss in the reference model and the losses in the alternative models. We compare the trade-offs implied by the three alternative rules that we consider, i.e., the classical Taylor rule, the optimized Bayesian rule and the minimax rule, by varying the weight on the rule in the modified loss function.

[Figure 7 about here]

Figure 7 shows the trade-off between the loss in the reference model and the *average* loss in the alternative models, and Figure 8 shows the trade-off between the loss in the reference model and the *maximum* loss in the alternative models. One striking result is that the trade-off is generally far less efficient when placing weight on the minimax rule. Only on the part of the trade-off in which the loss in the alternative model is close to the minimum level the minimax rule performs well. For less extreme preferences for robustness, placing weight on either the Taylor rule or the Bayesian rule gives a better trade-off, in the sense that one can achieve both lower loss in the reference model and lower loss in the alternative models than the in case with the minimax rule.

[Figure 8 about here]

Another somewhat surprising result is that the Taylor rule does not in general give a worse trade-off than the Bayesian rule, despite the fact that the Bayesian rule is optimized for the specific models. Placing a weight on a nonsophisticated rule like the classical Taylor rule as a cross-check, albeit not committing to follow it mechanically, thus seems to be a good insurancery policy for a monetary policymaker.

5 Conclusions

In this paper we propose an approach that constitutes a synthesis of the two alternative, common ways of modelling monetary policy - optimal policy and simple rules. Our approach can be summarized as minimizing a loss function that is extended by a weight on deviations of the interest rate from the rate implied by a simple robust rule. By varying the weight on the simple rule, one defines a continuum between standard optimal policy and commitment to a simple rule. Moreover, based on simulations in three different models of the US economy, we find that placing weight on a simple rule like the Taylor rule provides insurance against very bad outcomes if the reference model is wrong. Specifically, we find that a policy that is optimized for the model by Smets and Wouters (2007) leads to dynamic instability in the model by Rudebusch and Svensson (1999). By placing some weight on a Taylor rule, the solution in the Rudebusch-Svensson model becomes dynamically stable at relatively modest costs in terms of increased loss evaluated in the Smets-Wouters model. An optimized simple Bayesian rule improves the outcome relative to the classical Taylor rule, but the gain from an optimized rule is not very large. Both the optimal simple Bayesian rule and the classical Taylor rule tend to perform better than the optimal minimax rule, when used as a cross-check in the loss function.

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Table 1: **Estimation results in the RS model**

| Parameter | Prior Distr. | Prior Mean | Prior St.Dev. | Post. Mode |
|-------------|--------------|------------|---------------|------------|
| $a_{\pi 1}$ | Normal | 0.7 | 0.5 | 0.5519 |
| $a_{\pi 2}$ | Normal | -0.1 | 0.5 | 0.0967 |
| $a_{\pi 3}$ | Normal | 0.28 | 0.5 | 0.1313 |
| a_y | Gamma | 0.14 | 0.5 | 0.2326 |
| b_{y1} | Normal | 1.161 | 0.5 | 1.0894 |
| b_{y2} | Normal | -0.259 | 0.5 | -0.2506 |
| b_r | Gamma | 0.088 | 0.2 | 0.0442 |

Note: The table shows the prior distribution, prior mean and the prior standard errors for the structural parameters and the shock processes (where the standard errors of the shocks are denoted by σ and the AR (1) coefficients of the persistent shocks by ρ). The final column reports the posterior mode estimates.

Table 2: **Estimation results in the FM model**

| Parameter | Prior Distr. | Prior Mean | Prior St.Dev. | Post. Mode |
|-----------|-----------------|------------|---------------|------------|
| a_1 | Normal | 1.34 | 1.0 | 1.1713 |
| a_2 | Normal | -0.37 | 1.0 | -0.2434 |
| a_ρ | Normal | 0.36 | 0.5 | 0.4483 |
| s | Uniform(0, 1/6) | (0.083) | (0.0481) | 0.1110 |
| γ | Gamma | 0.008 | 0.05 | 0.0036 |

Note: The table shows the prior distribution, prior mean and the prior standard errors for the structural parameters and the shock processes (where the standard errors of the shocks are denoted by σ and the AR (1) coefficients of the persistent shocks by ρ). The final column reports the posterior mode estimates.

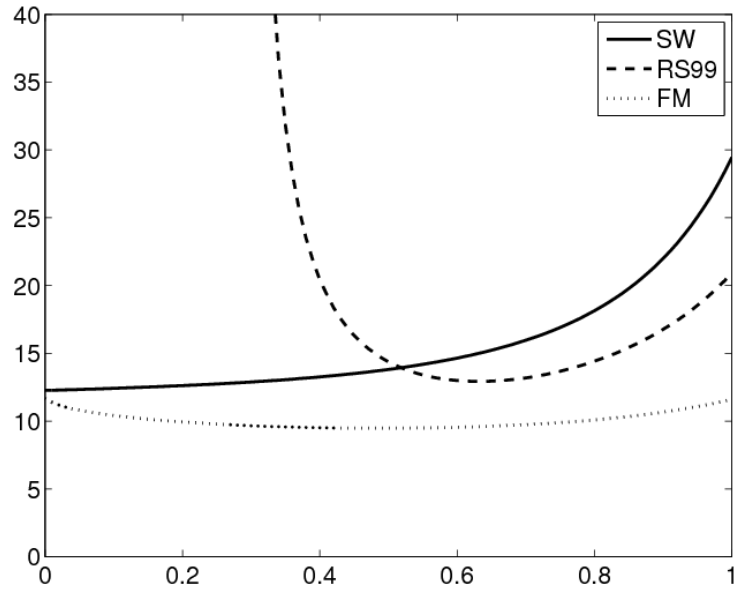


Figure 1: Expected loss in the alternative models as a function of the weight, θ , on the classical Taylor rule in the modified loss function.

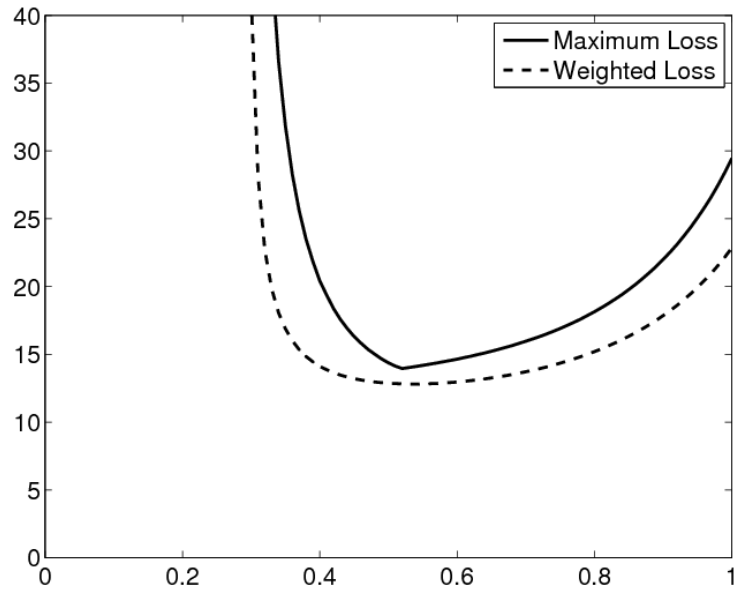


Figure 2: Maximum $E\{L\}$ and weighted $E\{L\}$ loss as a function of the weight, θ , on the classical Taylor rule in the modified loss function.

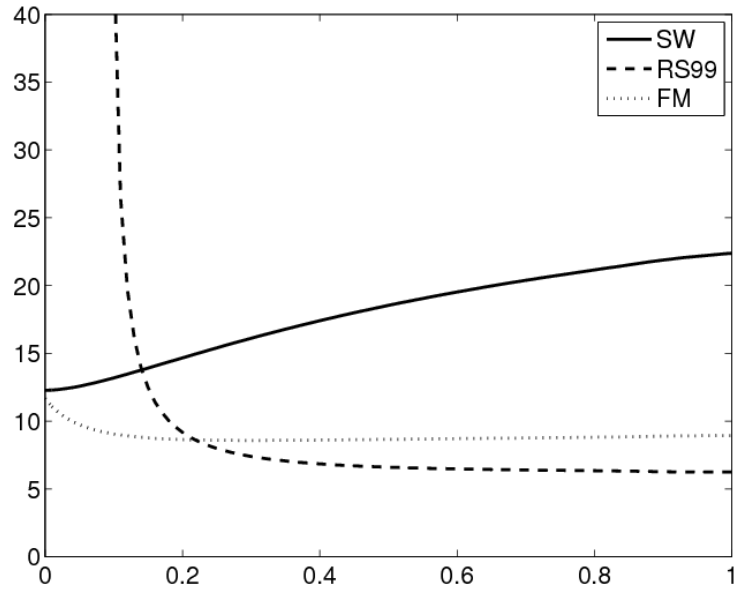


Figure 3: Expected loss in each model as a function of the weight, θ , on the optimal simple Bayesian rule in the modified loss function.

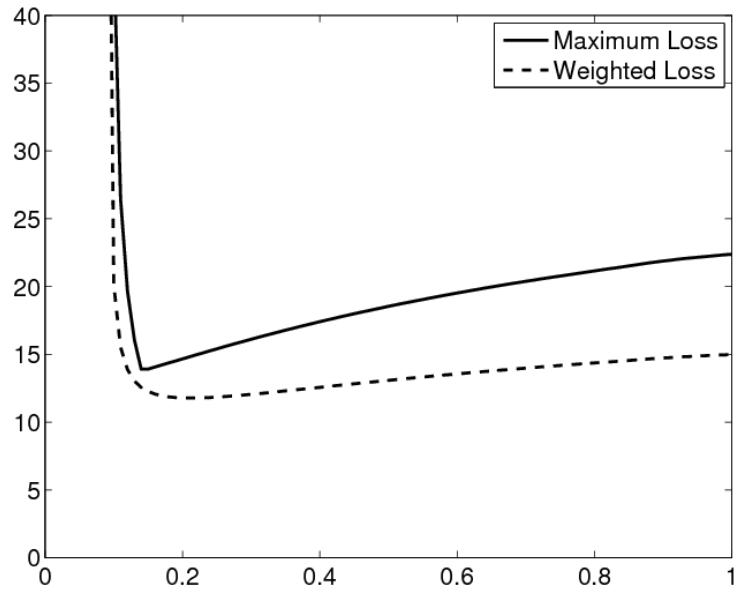


Figure 4: Maximum $E\{L\}$ and weighted $E\{L\}$ loss as a function of the weight, θ , on the optimal simple Bayesian rule.

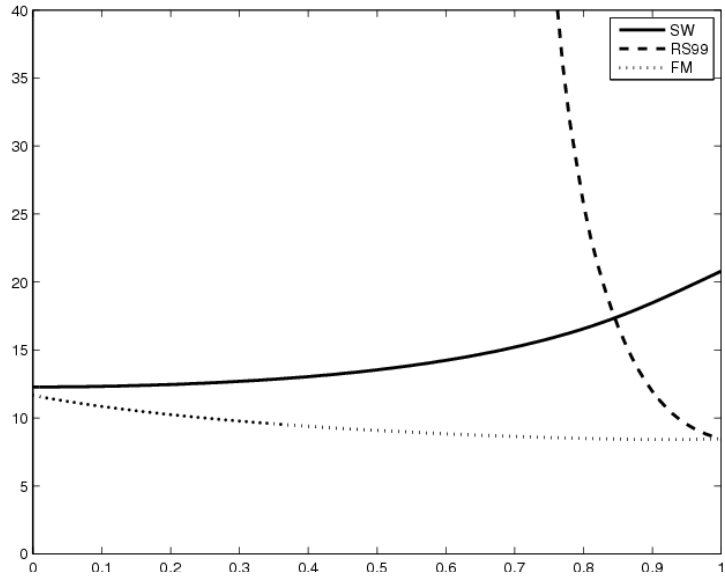


Figure 5: Expected loss in each model as a function of the weight, θ , on the optimal simple min-max rule in the loss function.

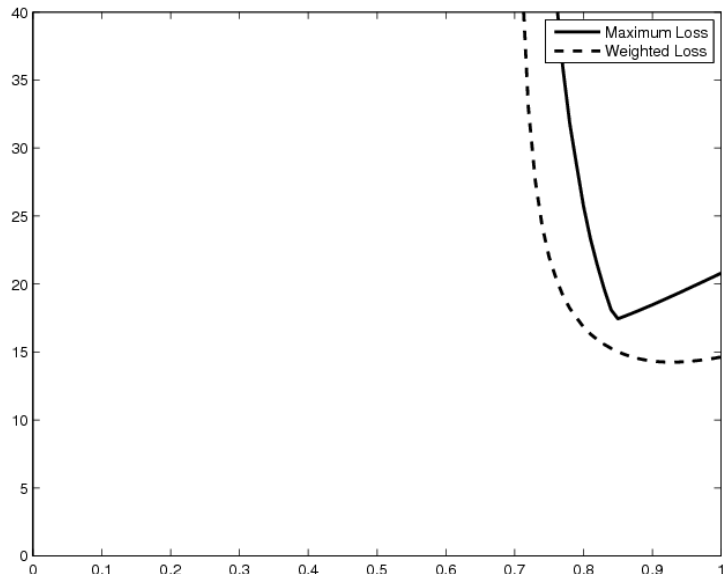


Figure 6: Maximum $E\{L\}$ and weighted $E\{L\}$ loss as a function of the weight, θ , on the optimal simple min-max rule.

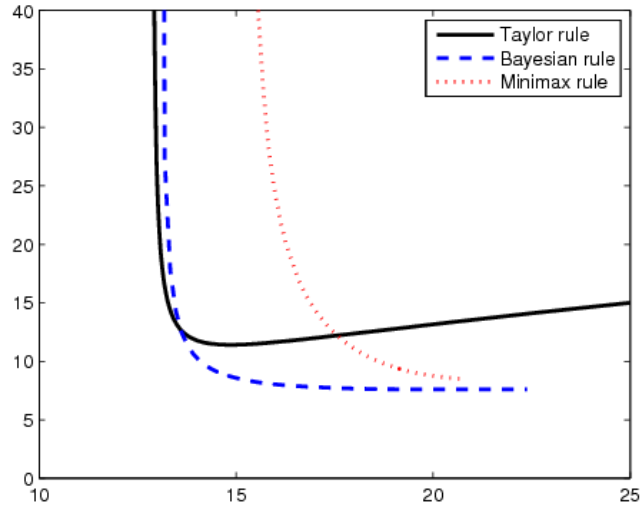


Figure 7: The trade-off between the loss in the reference model (horizontal axis) and the average loss in the alternative models (vertical axis).

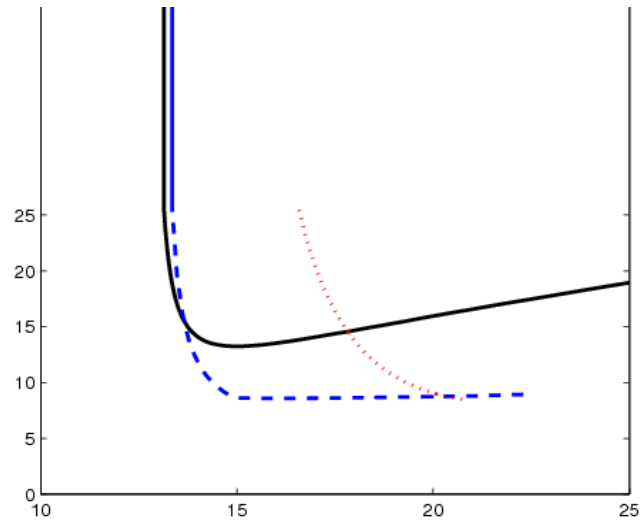


Figure 8: The trade-off between the loss in the reference model (horizontal axis) and the maximum loss in the alternative models (vertical axis).

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