Public debt: Safe at any speed?

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Introduction

The potential causes of a deadly car crash are often many and interrelated (the make of the car, weather conditions, the driver's experience, the type of road, etc.), but speed is always a factor. Yet, roaring at 200 km/h in a brand-new V8 sports car on an empty motorway under clear skies is arguably less risky than pushing a vintage model at 50 km/h on an icy mountain road wrapped in dense fog. Although no car is safe at any speed, it is extremely difficult to determine a limit beyond which the risk of a crash would be too high for comfort.

The same logic applies to sovereign default, be it explicit or implicit. There are many interrelated reasons why countries choose to renege on their financial obligations, but the public debt level is always a factor. Taking the argument to the limit, if debt was zero, there would be nothing to default on. Yet public debt is a useful instrument for policy-makers, and such an extreme form of fiscal rectitude would be highly undesirable. So, what debt level puts a government at excessive risk of default? Simple facts show the magnitude of the challenge. We look for an analytical framework flexible enough to accommodate the fact that a number of advanced economies can thrive with gross debt levels well above their GDP, while Ukraine actually defaulted on a stock of obligations three times smaller than its own GDP[1].

As challenging as this may be, knowing whether a given debt level should ring alarm bells is more important today than at any point in time since the end of World War II. For the last four decades, public debt has been rising in most countries, with two global tail events, the global financial crisis of 2008-9 and the COVID-19 pandemic, adding massive amounts to the already high levels reached at the turn of the 21st century. Should we worry about historically high public debt? Or on the contrary, should we welcome the fact that, at the same time, the debt footprint on the budget (i.e. interest payments) has steadily declined in line with interest rates?[2] After all, if creditors seem happy to reward greater public indebtedness with lower borrowing costs, that leaves policy-makers with ample room to put the State budget to good use.

While there is no reason to panic in the short to medium term, there are reasons to be vigilant. First, no sensible economic theory of *optimal* public debt can possibly rationalise the upward trend in debt ratios seen

1 See Debrun *et al.* (2019) for a survey of public debt sustainability assessments.

2 See Cornille *et al.* (2019).

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since the late 1970s. Of course, the right theory may not have been found yet, but lots of studies show how political agency problems can push well-intended governments towards highly *sub-optimal* debt policies[1]. In other words, excessive public debt might be wired into the political system in a way that could ultimately spark trouble. Second, the global financial crisis was a dramatic reminder that highly indebted advanced economies could be vulnerable to a brand of economic and financial instability usually thought to be confined to the proverbial "basket-case" emerging market. Third, the implication is that the universe of safe assets[2] might shrink, pushing risk-free interest rates ever lower and encouraging excessive risk-taking in a desperate search for yield. Fourth and finally, unless we live in a brave but still uncharted new world, history teaches us that the current public debt levels are strong signals of impending trouble (Sandleris, 2016).

The aim of this article is to develop an operational compass to navigate the high-debt question. We first propose a definition of "safe" public debt. Echoing an approach to sovereign risk assessment developed by Mendoza and Oviedo (2004) for developing countries, a public debt level (in proportion to GDP) is considered safe if it is plausible to expect the government to stabilise or reduce it under most circumstances – including persistently adverse ones – using fiscal policy (i.e. excluding default, inflation or restructuring). The notion rests on three basic elements: (i) the requirement for the government to remain solvent with a high degree of confidence; (ii) the requirement that returning to safety implies a feasible policy path; and (iii) the fact that growth, interest rates, and the budget itself are subject to random shocks that are often correlated and persistent. Taken together, (i) and (ii) form the conventional definition of public debt sustainability, whereas (iii) stresses the importance of not losing control of debt dynamics even under adverse conditions.

To estimate the upper bound of the safe debt zone, we propose three pragmatic and computationally light methods closely related to conventional debt sustainability assessments (stochastic simulations and stress tests). It is important to note upfront that the analysis, although related to the determination of "debt limits", differs from it in important dimensions[3]. A debt limit should be feared because unsustainability lies just beyond with near certainty. In an uncertain world, being close to or at the debt limit is an inherently *unstable* position. By contrast, we conceive a safe debt level as a *stable* position in most circumstances. Since the likelihood of losing control of debt dynamics rises with debt itself, the upper bound of the safe debt zone must be sufficiently low to accommodate adverse shocks without jeopardising the government's capacity to generate the primary balances required to keep debt dynamics in check. Thus, the fundamental practical difference with the notion of debt limit is that, with no binding borrowing constraints and liquidity issues, public indebtedness could rise significantly above the safe zone without triggering notable fears of default.

The pragmatic approaches discussed here can be used to produce useful guideposts to anchor fiscal policy to a specific long-term objective or to pin down the often-elusive notion of fiscal space in the short term. Our main contribution is to link the definition of safe debt boundaries to explicit assumptions about risks to debt dynamics, the intensity of risk aversion and the capacity to generate and sustain primary surpluses. These tools can usefully discipline judgment and help calibrate sensible fiscal policy rules or estimate fiscal space[4].

An important caveat to bear in mind when interpreting our results is that the analysis is premised on the absence of strategic motives to default. Thus, the key consideration underlying our approach is the government's *capacity* to service its debt, not its *willingness* to do so after a cold cost-benefit analysis. That said, we believe that focusing on the capacity to pay is a good approximation in the case of financially sophisticated economies where a sovereign default would be too costly to contemplate (i.e. financial sector meltdown and a massive destruction of wealth).

1 Yared (2019) provides an authoritative survey.

2 Safe or risk-free assets carry no credit risk; they pay off the agreed income stream in all states of the economy. Government bonds are usually considered as the best proxy to a safe asset in a given economy.

3 Wyplosz (2011) and Debrun *et al.* (2019) provide comprehensive surveys of the relevant literature.

4 For instance, Eyraud *et al.* (2018) use two of the methodologies we developed to calibrate the debt "anchor" underpinning well-designed fiscal policy rules according to IMF criteria. Countries may have good reasons to target lower levels, but they would be hard-pressed to justify neglect for debt levels outside the safe zone. Fiscal space is the largest amount of additional funds a government could borrow without jeopardising market access (see IMF, 2016a).

Finally, we cannot emphasize enough that the numbers presented here are mostly illustrative. All the models can be calibrated differently to reflect more recent data or country-specific developments. Moreover, one should always keep in mind the significant uncertainty around the underlying models. Today, the implicit assumption that the past is a good guide for the future is even stronger than usual. The long-term properties of the models, especially regarding the interest-growth differential, warrant close scrutiny before putting them to use. Durably lower differentials are bound to raise the safe debt boundaries obtained with such tools, suggesting that in unchartered territory sound judgment remains an even more essential component of any balanced diagnostic.

The rest of the paper is structured as follows. Section 1 introduces the concept of safe debt and formally compares it to the notion of debt limit. Sections 2 and 3 discuss measures of key inputs, and present the results in the case of known and unknown debt limits respectively.

1. Safe public debt: concept and implementation

1.1 Solvency, sustainability and safety

Any public debt sustainability assessment is rooted in a *sufficient* condition for public sector *solvency*. Applying the no-Ponzi condition to the forward solution of the stock-flow identity linking debt and the primary balance (i.e. the budget balance excluding interest payments on debt), solvency requires public debt not to exceed the net present value of all *future* primary balances. This is a *necessary and sufficient* condition. Operationally, however, the challenge is obvious: assessing solvency is a mere prediction about an unknowable and indefinite future. With an infinite time horizon, even minor changes in parameters have huge effects on the assessment, and uncertainty is considerable. Besides, the very low interest rates – even well below nominal GDP growth rates – that have persisted for more than a decade call into question the relevance of net present values calculated over very long time horizons (Bartolini and Cottarelli, 1994; Blanchard, 2019). Since solvency is not an operational concept offering much concrete guidance over a relevant time window, economists tend to focus on more restrictive criteria that are largely sufficient to meet the solvency requirements[1].

In a celebrated article, Bohn (1998) shows that a systematically positive response of the primary balance to changes in the debt level ensures solvency[2]. Although the Bohn principle is simple and intuitive, its operational relevance remains limited. First, any prospective debt sustainability assessment based on it must assume that future fiscal policy behaviour will replicate historical trends. Second, as the test is defined in marginal terms, the *level* of the primary balance is not bounded, which imposes very little constraint on possible debt trajectories. In other words, while the Bohn criterion rules out explosive debt paths, it cannot exclude trajectories culminating at implausibly high levels.

This explains why debt sustainability frameworks – such as those used at the IMF or the European Commission – rely on a more demanding condition: the *stability of the debt-to-GDP ratio* over a given horizon (see, for example, Blanchard, 1990, or Escolano, 2010). The intuitive rationale behind that condition is that nominal public debt – which is a claim on future tax revenues – should not be allowed to grow faster than the broadest proxy of a country's tax base (GDP) on a permanent basis. As discussed in Bartolini and Cottarelli (1994), that condition is robust to cases where the solvency condition itself becomes fuzzy, such as when the interest-growth differential is persistently negative, which has often been the case since the 2008-2009 great recession (Mauro and Zhou, 2020).

1 Admittedly, deviations from the solvency condition have been used in some cases, including to assess debt overhang in emerging markets (IMF, 2003) or long-term sustainability challenges in the European Union (the solvency gap is at the core of the European Commission's S2 indicator).

2 That condition holds in a fully-fledged general equilibrium model under weak technical assumptions. In practice, the empirical validity of the Bohn test rests on the assumption that both debt and the primary balance are stationary in a statistical sense.

Ensuring a stable debt-to-GDP ratio restricts not only the sign of the primary balance response to debt (positive as shown by Bohn) but also its strength (it must be strong enough). If the primary balance path is deemed politically and economically feasible, there is in principle no reason to question debt sustainability, and the probability of a fiscal crisis – i.e. an incapacity to bring debt dynamics under control through fiscal consolidation only – is negligible. The key question is whether such a benign assessment is robust to most circumstances, including turbulent times. Thus, we define a safe debt level as follows:

Definition. *A public debt level is safe if it does not exceed the largest stock of gross financial liabilities the government could plausibly stabilise or reduce (in proportion to GDP) within a given time frame characterised by persistently adverse conditions for debt dynamics, using fiscal policy only.*

Safe debt calculation thus requires a good understanding of (i) the uncertainty surrounding the main determinants of debt dynamics (interest rate, growth, direct shocks to debt or the budget), (ii) the persistence and correlation of these shocks, and (iii) the government's capacity to generate and sustain primary balances at or above their debt-stabilising level. Clearly, the combination of uncertain debt dynamics and limits to feasible policy response implies that, all else equal, a lower debt ratio is *a priori* "safer" in the sense of the above definition[1].

However, stating that a very low debt level is best to preserve sustainability is not particularly informative for policy-makers, especially if large and protracted fiscal consolidations are expected to cause significant damage to economic activity. As policymakers balance the risks of unsustainability with the potential benefits of wisely using fiscal space – e.g. to plug infrastructure gaps known to undermine long-term growth – there is more value for them in having a sense of the upper bound of the safe debt zone. Symmetrically, if the initial debt level is deemed excessive or unsafe, it is useful to have a sense of the debt reduction required to feel reasonably safe again.

1.2 Safe debt boundary and debt limit

Arithmetically, two opposing factors shape public debt dynamics[2]. The first is the "snowball effect" that arises when interest payments are financed with new debt: the higher the debt, the stronger the effect. The second is the government's offsetting response to the snowball effect: primary surpluses[3]. According to our definition, a debt level is safe if the latter can at some point credibly offset the former even when interest rates are abnormally high compared to GDP growth. By contrast, a debt limit is a level beyond which the government cannot be expected to offset the snowball effect even under *normal* circumstances. At the debt limit, any adverse shock to debt puts it on an explosive path unless there are sufficiently favourable conditions (low interest rates or high growth).

One sufficient condition for the existence of a debt limit is the so-called "fiscal fatigue" (Ostry *et al.* 2010, Bi 2012, Ghosh *et al.* 2013). Because governments cannot be expected to raise the primary surplus indefinitely (or at an ever-increasing speed), there is a point at which keeping debt under control requires impossibly large surpluses. Beyond that point, debt is not sustainable. Even if only the strength of the primary surplus response – but not the level – is bounded, market discipline, in the form of borrowing costs rising with the debt level, eventually ensures that the snowball effect dominates the capacity or willingness to generate primary surpluses, making self-fulfilling prophecies possible.

The difference between conventional debt limits and the safe debt boundary can be illustrated in a diagram (chart 1)[4] describing possible combinations of positive debt levels (d) and primary surpluses (p). To assess whether

1 That statement obviously neglects the fact that some countries make a better use of borrowed resources than others (e.g. to fund growth-enhancing policies), which means that a higher debt level in country X compared to country Y does not necessarily mean that X's sovereign is less safe.

2 Operational aspects of debt sustainability are usually handled outside formal economic models, relying on the simple arithmetic of debt accumulation. See however, Bi (2012) or Battistini *et al.* (2019).

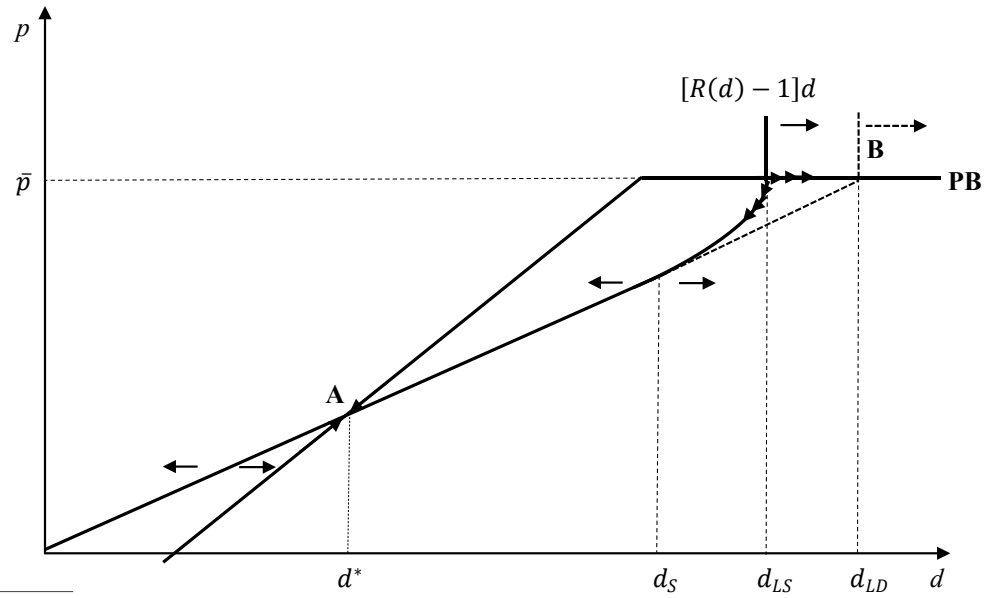
3 A primary surplus ensures that at least part of the interest bill is paid with own revenues.

4 See Ghosh *et al.* (2013) or Fournier and Fall (2015a) for similar diagrams.

these combinations are consistent with falling, stable or rising debt levels, we recall that $\Delta d = (\mathrm{R}(d) - 1)d - p$, where d is the debt-to-GDP ratio[1], Δ designates the discrete time difference operator, p is the primary balance as a percentage of GDP, and $R(d) = \frac{1+r(d)}{1+g(d)} > 1$ denotes the growth-adjusted gross interest rate (which depends on the debt level). These assumptions yield an upward-sloping "demarcation line" along which the primary balance exactly offsets the snowball effect: $p^* = (R(d) - 1)d$, so that $\Delta d = 0$. Above the demarcation line, the primary balance is consistent with a falling debt-to-GDP ratio, whereas below the line, the debt ratio rises.

Chart 1

Public debt dynamics: a simple diagram



Source: Authors' calculations.

Primary balance behaviour can also be depicted in chart 1 assuming a systematic positive response to the debt level up to a point $\bar{p}$ where fiscal fatigue discretely sets in:

$$p = \mathrm{Min}(\kappa + \rho d, \bar{p}), \text{ with } \rho > 0. \tag{1}$$

Equation (1) is represented by the PB schedule in chart 1. The steeper the line (higher ρ), the stronger the fiscal policy response to a change in debt. The PB line captures fiscal (primary balance) behaviour, with adjustment fatigue setting in once a hypothetical upper bound $\bar{p}$ is reached. Beyond $\bar{p}$, the government stops responding to rising indebtedness, violating Bohn's solvency condition and setting the stage for unstable debt dynamics.

Intersections between these two loci define equilibria where the primary balance exactly offsets the snowball effect (debt is constant unless it is hit by a shock). To establish the stability condition for each equilibrium, we use (1) to substitute for the primary balance in the debt accumulation equation and obtain:

$$\Delta d = \left(\frac{r(d) - g(d)}{1 + g(d)} - \rho\right) d - \kappa. \tag{2}$$

1 For simplicity, we abstract from stock-flow discrepancies, liquid assets and uncertainty.

The stability condition directly follows from observing that the debt-to-GDP ratio converges to some finite level (mean-reversion) if and only if the term in brackets is strictly negative. Therefore, a given equilibrium debt level d_0 will be stable if the marginal response of the primary balance to public debt exceeds the growth-adjusted interest rate prevailing at that equilibrium debt level $\left(\rho > \frac{r(d_0)-g(d_0)}{1+g(d_0)}\right)$. In chart 1, an equilibrium debt level is stable if the PB locus is steeper than the demarcation line at the intersection of the two.

Thus, point A determines a dynamically stable equilibrium debt d^*. By contrast, point B identifies another equilibrium debt level d_{LD} that is fundamentally unstable: below d_{LD}, debt converges back towards d^*, but above d_{LD}, the debt ratio is on an explosive path, default is certain, and no finite interest rate can lure market participants into buying government bonds. Point B determines a debt limit like Ostry *et al.* (2010) in *a deterministic setting*.

Matters are technically much more involved in a stochastic setting because the interest rate on government bonds and the probability of explosive debt dynamics (and default) are jointly determined. Ghosh *et al.* (2013) deal formally with this "fixed point" problem. Here, we simply sketch a heuristic argument based on the diagram.

In chart 1, it is easy to imagine that as debt rises, the probability of default turns positive beyond a certain level d_S. At that point, the stable-debt locus departs from the linear/deterministic schedule prevailing when the interest rate on public debt is equal to the risk-free rate. This "stochastic" demarcation line ultimately crosses PB vertically at a debt level d_{LS} which is lower than d_{LD}. This *illustrates* the solution to the "fixed point" problem by Ghosh *et al.* (2013).

According to our definition above, d_S qualifies as a safe debt boundary because it is the highest debt level consistent with a zero probability of default. It could of course be set higher if the government was ready to tolerate a non-zero probability to experience explosive debt dynamics and to pay the corresponding risk premium.

Overall, three broad levels of alert for public debt sustainability can be derived from this discussion: safe $(d \leq d_S)$, risks to debt sustainability $(d_S < d < d_{LS})$, and unsustainable $(d \geq d_{LS})$. The precise boundaries of these zones are country-specific and depend on the joint distribution of shocks affecting the debt trajectory (interest rates, exchange rate, growth, contingent liabilities), the plausible response of fiscal policy to public debt developments, and the probability of default one is ready to tolerate *ex ante*.

The challenge is now to provide an operational meaning to the debt thresholds delineating these three areas. Among them, the determination of debt limits has already received considerable attention. In particular, IMF (2003), Ostry *et al.* (2011), Ghosh *et al.* (2013), and Fournier and Fall (2015a) explore non-linearities in the empirical relationship between public debt and the primary surplus to calculate debt limits. Bi (2012) proposes a dynamic and stochastic general equilibrium (DSGE) model where tax revenues are subject to a Laffer effect and public spending is incompressible below a certain level. Battistini *et al.* (2019) elaborate on the DSGE approach, focusing on interactions between monetary and fiscal policies.

Estimating distributions of debt limits is numerically demanding because it requires a plausible model of the non-linear reaction of the risk premium. In this article, we propose three pragmatic and easily implementable options to capture the notion of "safety" embedded in d_S.[1] None of them pins down d_S as formally illustrated in chart 1, but we believe they provide useful proxies at low computational cost. Close variants of these pragmatic options have now been regularly used since we first developed them (see, e.g., IMF, 2016b; Eyraud *et al.*, 2018).

1 Although Fournier and Fall (2015b) also derive "prudent" debt levels, they use far more restrictive criteria, which, unlike ours, are not exclusively rooted in the idea of preventing a debt crisis depicted in chart 1.

1.3 Operational options

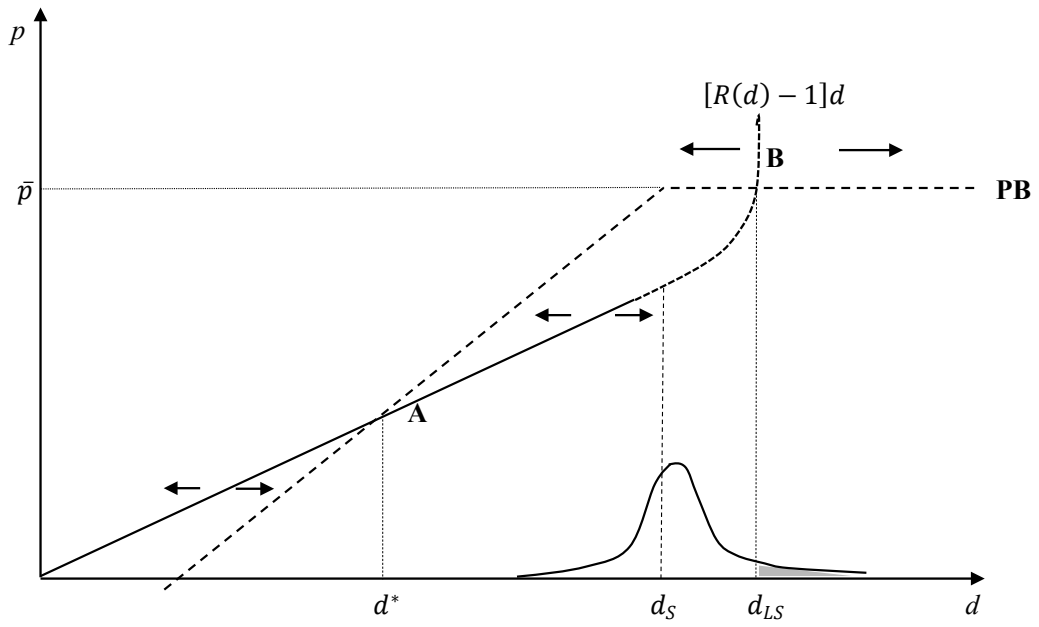
The diagram in chart 1 suggests two alternative avenues to calculate a safe debt boundary similar to d_S, both using simple and readily available simulation tools. The first is to estimate a safety buffer below a *known* debt limit d_{LS}.[1] The second is to assess the highest public debt level a government could plausibly commit to stabilise or reduce under highly adverse conditions.

1.3.1 Safe debt boundary with known debt limit

Reasonable priors about d_{LS} allow defining the safe debt boundary as the difference between the limit and a safety buffer. As illustrated in chart 2, information about the distribution of risks affecting debt trajectories can help to pin down a safe debt boundary. The bell-shaped curve illustrates the uncertainty surrounding the evolution of public debt at a given time horizon, say T years. (The distribution is slightly skewed to the right because the snowball effect amplifies the effect of shocks at higher debt levels.) The safe debt upper bound d_S can then be defined as the initial debt level corresponding to a given probability *not* to exceed d_{LS} over the time horizon T. Note that d_S can be at, below, or above the median of that distribution depending upon whether the median projected debt path is flat, upward or downward sloping, respectively.

Chart 2

Determination of d_S when d_{LS} is known



Note: The bell-shaped curve along the horizontal axis sketches the distribution of debt projections at the end of a given time horizon (period T). The shaded area under the curve symbolizes the probability of exceeding the debt limit at that time. In that representation, the starting value for the debt projection exercise (d_S) is assumed to be smaller than the median debt projection in T, which is consistent with a growing median debt path over the projection window.

Source: Own calculations.

1 If we had a tool to easily estimate plausible distributions of debt limits, the safe debt boundary could be defined as an arbitrarily low percentile of the debt limit distribution (say 1 or 5 percent). See Bi (2012) and Battistini *et al.* (2019).

One practical issue in estimating the safety buffer below the limit is that public debt is typically subject to two very different types of shocks. The first type consists of high-frequency disturbances affecting GDP growth, borrowing costs, the fiscal balance, and, if some government liabilities are denominated in foreign currency, nominal exchange rates. The joint distribution of such shocks is straightforward to estimate and can be used to randomly generate many simulations of future debt outcomes. This approach implies that, all else equal, more volatile economies face lower safe debt boundaries. Similarly, countries with rising median debt trajectories over time would have lower safe debt boundaries.

Low-frequency but large shocks hitting the economy and the public sector balance sheet constitute the second source of uncertainty around public debt projections. For the most part, they are related to the realisation of contingent liabilities (banking crises, natural disasters, bail-out of subnational governments or state-owned enterprises). Because these events are scarce and highly country-specific, a stress-test approach is advisable (Clements *et al.*, 2016). The stress scenario typically assumes the realisation of a fraction of a country's contingent liabilities.

A relevant but tricky issue is whether high and low frequency shocks are correlated. Event studies in Bova *et al.* (2015) show that large shocks often coincide with adverse economic and financial conditions affecting debt dynamics. That coincidence suggests adding up two buffers: one obtained with the stochastic method described above and another capturing the potential fiscal costs of contingent-liability stress. Thus, the safe debt boundary d_S^* could be calculated as:

$$d_S^* = d_{t_0}^*(\pi), \text{ where } d_{t_0}^*(\pi) \text{ is such that } Pr\left(d_{t_0+T}^F \leq (d_{LS} - \bar{C}) | d_{t_0}^F = d_{t_0}^*(\pi)\right) = \pi. \quad (3)$$

$\bar{C}$ is an *ad-hoc* buffer allowing for large but low-frequency shocks to public debt, $d_{t_0}^*(\pi)$ is the starting value of a debt forecasting exercise such that the projected debt level at horizon T (denoted by $d_{t_0+T}^F$) has a probability π of being below $d_{LS} - \bar{C}$, the debt limit d_{LS} adjusted down by an amount $\bar{C}$. This uniquely defines $d_{t_0}^*(\pi)$ because there is a one-to-one mapping between the distribution of debt forecasts at any given point in time and the initial value of debt at time t_0 of the corresponding forecasting exercise.

In the calculations reported later in this article, we set π =0.95 and T=6. However, π can be adjusted upwards (downwards) to reflect a lower (higher) tolerance for risk. Although the time horizon is also a matter of preference, it should ideally balance two considerations: (i) the time needed for sustainable fiscal policy measures to durably influence debt trajectories and (ii) the rapid increase in uncertainty around debt forecasts at longer projection horizons. For instance, long horizons (say 10 years or more) could yield implausibly large buffers without conveying enough sense of urgency to policy-makers.

1.3.2 Safe debt boundary when the debt limit is unknown

When the debt limit is unknown, we can still identify the highest debt level a government could commit to stabilising or reducing over a given time frame under persistently bad circumstances. Hence, a public debt level could be considered safe if the highest sustainable primary surplus ($\bar{p}$) suffices to prevent explosive debt trajectories even after adverse conditions emerge and persist.

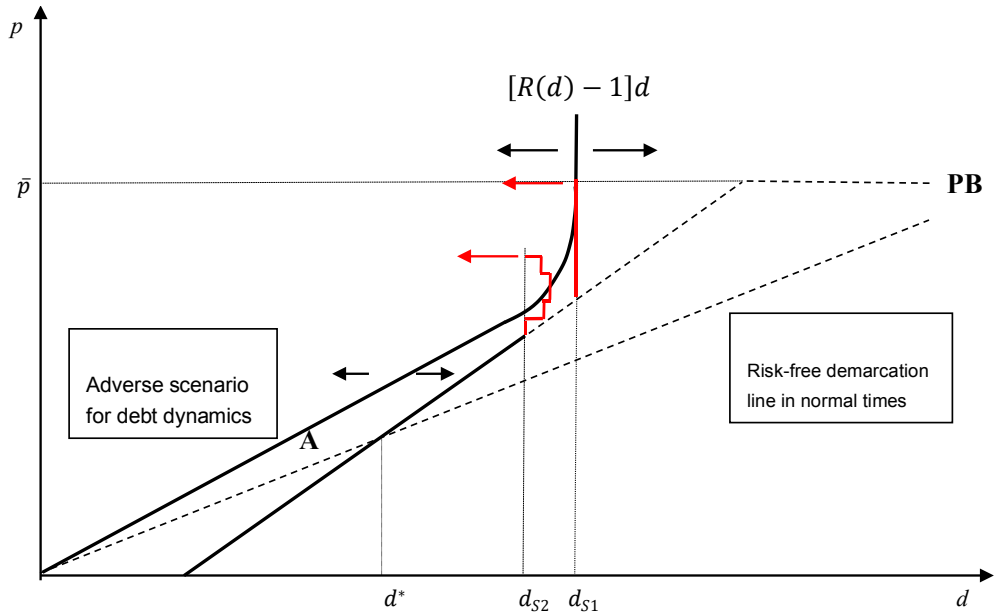
In terms of our diagram, the safe debt upper bound can be identified using either stress-testing or stochastic simulations to characterise adverse circumstances. Under the first approach, we must define a hostile, yet plausible scenario of abnormally high growth-interest differentials and correspondingly jittery market sentiment. Under the second approach, we focus on the probability of having to bring the primary surplus above $\bar{p}$ to stabilise the debt ratio, a policy which, by definition, lacks credibility.

Chart 3 illustrates the proposed deterministic method. The bad state of the world for debt dynamics is described by a steeper "risk-free" demarcation line and the possibility of markets pricing in a default. One question to settle when implementing this approach is how quickly the government could deviate from its normal reaction

function and implement an extraordinary fiscal consolidation in response to the bad state of the world. In the "cold turkey" case, the primary balance could jump to $\bar{p}$ before debt dynamics turns unstable. In a gradual case, only a stepwise fiscal consolidation could be implemented. Under the cold-turkey scenario, the government could stabilise the debt ratio when it asymptotically reaches its limit under the bad scenario (denoted by $d_{s,1}$). With gradual debt consolidation, there is an upper limit to the feasible fiscal adjustment (measured in terms of the annual improvement in the primary balance). The government would then incrementally raise the primary balance until debt goes back to its initial, pre-shock position after a certain time (in the chart, 5 years). Both approaches define a unique safe debt boundary: it is the debt limit (minus an arbitrarily small number) if the maximum primary surplus is attainable in one go, and the highest debt level that the government can commit to returning to over a given time horizon after the bad state of the world materialises. Note that for simplicity, we assume that market beliefs in the adverse scenario for debt dynamics are not affected by the assumed fiscal consolidation path (cold-turkey versus gradual).

Chart 3

Determination of d_S when d_{LS} is unknown: deterministic case



Source: Own calculations.

This reasoning directly echoes the "natural debt limit" concept proposed by Mendoza and Oviedo (2004), which combines the uncertainty affecting government revenues with an assessment of the capacity to cut primary spending to the bare minimum when revenues are the lowest. These authors define their "natural debt limit" as the perpetuity value of the highest primary balance a government could credibly commit to sustain in a state of "fiscal crisis"[1]. What makes the natural debt limit a genuine limit in the sense discussed above is that the government would violate the solvency condition with certainty in a "fiscal crisis". Unsurprisingly, this method points to very low public debt limits – of around 30 % of GDP for selected Latin American countries.

1 Mendoza and Ovideo (2004) define a fiscal crisis as "a sufficiently long sequence of adverse shocks to public revenues", forcing the government to adjust "its [primary] outlays to minimum admissible levels".

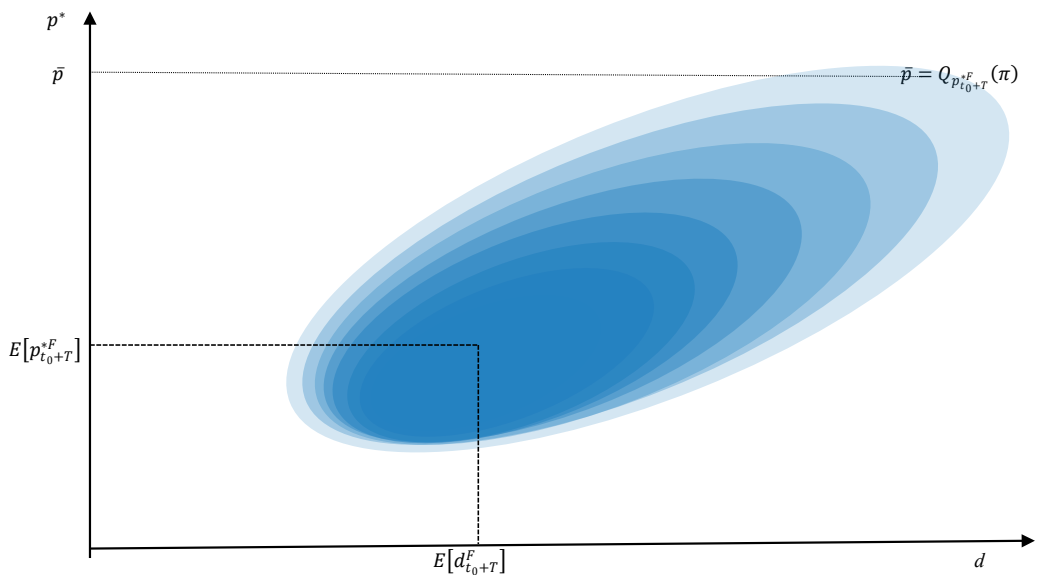
Stochastic methods can also be used when the debt limit is unknown. Once again, the only known critical threshold is $\bar{p}$, the point at which fiscal fatigue sets in. Thus, one pragmatic option to get a sense of the safe debt boundary is to assess the risk of the primary balance required to at least stabilise the public debt ratio exceeding $\bar{p}$. As in the first stochastic approach discussed above, that risk rises with the debt level set as the starting value for the simulation exercise. It should be recalled that, all else equal, the debt-stabilising primary balance p^* is proportional to the debt level (i.e. $p^* = \left(\frac{r-g}{1+g}\right) d_{-1}$, omitting time subscripts). It is therefore possible to identify the starting value for a debt forecasting exercise $d_{t_0}^{**}(\pi)$ such that $Pr\left(p_{t_0+T}^{*F} \leq \bar{p} | d_{t_0}^{F} = d_{t_0}^{**}(\pi)\right) = \pi$, where $p_{t_0+T}^{*F}$ denotes the projected debt-stabilising primary balance at horizon T.

It is worth noting that reliance on model-based stochastic simulations raises the question of the underlying steady-state equilibrium of the model. Should the latter exhibit a strongly negative interest-growth differential, the circumstances under which a 4 % primary surplus might be required to stabilise debt could be so rare that the safe debt boundary might be implausibly high or not even exist. Clearly, that approach is irrelevant for countries with a negative equilibrium interest-growth differential.

For the sake of comparison with the other two methods, chart 4 proposes a stylised visual of the approach, showing the joint distribution of debt forecasts (at a given horizon *T*) and the corresponding debt-stabilising primary balance. That specific distribution yields a probability π of avoiding unfeasible primary balances to realisation the national debt (i.e. to remain at or below $\bar{p}$) and corresponds to a unique initial debt level at the beginning of the simulation exercise.

Chart 4

Determination of d_S when d_{LS} is unknown: stochastic case



Note: $p^* = [R-1]d$ and $Q_{p_{t_0+T}^{*F}}(\pi)$ is the value for a given probability π of the quantile function associated with the marginal distribution of $p_{t_0+T}^{*F}$. Note that the empirical distributions of debt and primary balance forecasts are positively skewed, as the debt impact of a given shock to growth and interest rates is proportional to the debt level itself (see section 2).

Source: Own calculations.

1.3.3 Stochastic versus deterministic: pros and cons

A key advantage of stochastic approaches is to allow assigning probabilities to the adverse situations one wants to insure against. The degree of risk aversion embedded in the safe debt boundary is explicitly measured. Their main disadvantage is to be relatively data intensive as they require estimating a reasonable dynamic forecasting model (typically a VAR) with desirable properties (i.e. stable dynamics and a well-defined steady state). As for any model-based approach, structural breaks in the basic relationships between growth and interest rates, or between the latter and fiscal variables, are bound to have first-order effects on the assessment. Finally, existing estimates of debt limits based on past fiscal behavior and the evidence of fiscal fatigue exist mostly for advanced economies. Debt limits remain difficult to pin down for emerging market and developing economies.

Symmetrically, deterministic approaches are much more parsimonious in terms of country-specific data and can be customised across a broader range of countries. They also offer the flexibility to capture relevant developments – e.g. a permanent shift in growth-adjusted interest rates – that a model-based methodology could only partly reflect. However, they suffer from the drawback associated with any stress-testing exercise: the plausibility of the selected adverse scenario is always debatable because technically speaking, its probability of occurrence is exactly zero.

Adopting a combination of approaches could be a sensible course of action. However, the range of estimates of the safe debt boundary might be too wide to provide useful guidance to policy-makers. In the end, as in any debt sustainability assessment, a serious dose of judgment is needed, depending on the availability of forecasting models and their properties (stability, steady-state, etc.). We now turn to the implementation of each approach.

2. Safe debt with known debt limit

This section calculates the safe debt boundary by identifying desirable buffers under a known debt limit. After a brief discussion of the main inputs, we present results for selected advanced economies and assess the sensitivity of the estimates to variations in key assumptions.

2.1 Main inputs

2.1.1 Distributions of debt forecasts

Any computer routine producing distributions of public debt forecasts can be used to estimate $d^*_{t_0}(\pi)$ in equation (3). In this article, we use a variant of the tool developed by Celasun, Debrun and Ostry (2007, hereafter CDO). That stochastic model incorporates three sources of risk: fiscal policy shocks – reflecting for instance any unexpected expenditure slippages, revenue shortfalls, or calls on guarantees – the budget's sensitivity to macroeconomic (growth) and financial (interest and exchange rates) developments,[1] and the direct impact of macroeconomic shocks on debt dynamics through growth and borrowing costs. The empirical link between the primary balance, the output gap, and public debt being a core component of CDO, we re-estimate it, using a sample of (mostly) advanced economies covered here[2].

The main advantage of the CDO tool is that it operates even under rather limited data availability, allowing applications to many different countries. Indeed, a major practical hurdle in applying stochastic simulations to public debt is the need for consistent higher-than-annual frequency time series for key fiscal aggregates.

1 Since the countries considered here have most of their obligations denominated in domestic currency, the exchange rate is not part of the VAR models.

2 Celasun, Debrun and Ostry (2007) estimate a reaction function on a large panel of emerging economies.

This often limits applicability to the few countries that consistently publish such data, i.e. based on actual budget flows (and not interpolations) and with broad coverage of the public sector (general government) and consistent accounting methodology[1].

CDO requires only annual data for debt and the primary balance. This comes at the cost of separating the analysis of economic dynamics – requiring quarterly data – from the fiscal block. The silver lining of this two-block approach is the ability to use a well-tested specification of the fiscal reaction function instead of the one mechanically imposed by the VAR model at the heart of the economic block.

Two main drawbacks need to be kept in mind when interpreting the results. First, the estimates ignore relevant information on the interaction between the budget and economic conditions. Second, the simulations drawn from the model implicitly assume that although fiscal policy responds to cyclical developments, it does not affect the economy in return. Annex I describes the main technical features of the algorithm.

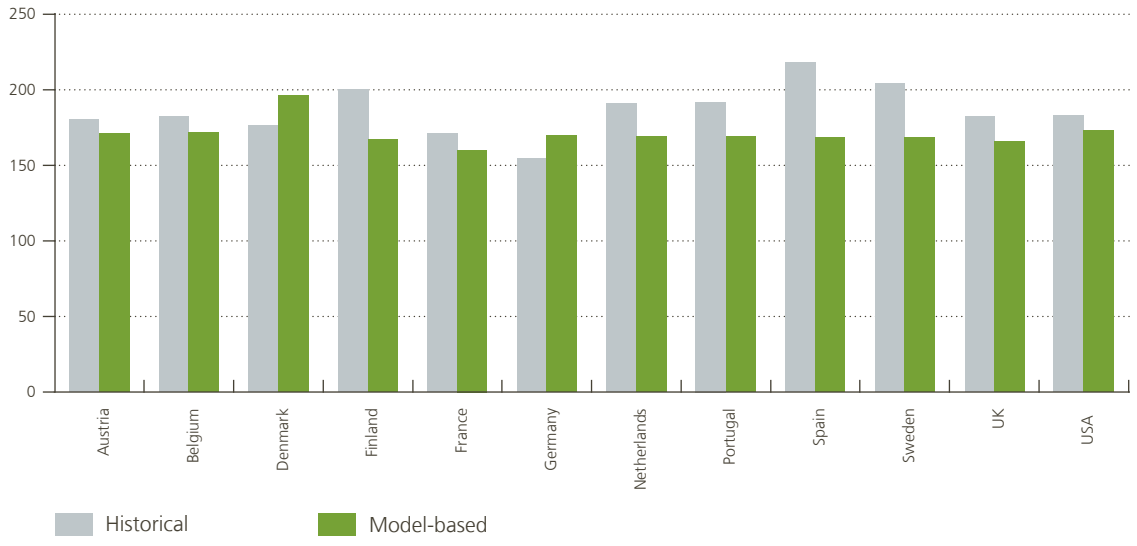
2.1.2 Debt limits

The debt limits $\bar{d}$ are based on a framework similar to chart 1 and borrowed from Ostry *et al.* (2010). Using a panel of advanced economies, they first estimate a non-linear reaction function exhibiting fiscal fatigue – as the PB schedule in chart 1. The debt limit is obtained by intersecting the reaction function with an interest-rate schedule. To construct the latter, they take into account either current market interest rates on government debt or a model-based projection that considers the likely response of borrowing costs to debt as it approaches the limit[2].

Chart 5

Selected Advanced Economies: Debt limits

(in % of GDP)



Source: Ostry *et al.* (2010).

1 High frequency budget data (quarterly or monthly) serve cash management purposes and are often not reported with the same classification or using the same accounting methods as annual data supporting policy analysis. Brazil is one exception (Garcia and Rigobon, 2005, Penalver and Thwaites, 2003, and Tanner and Samake, 2005). However, fiscal policy remains subject to an annual decision process, suggesting that even high-frequency fiscal data of good quality may suffer from a high noise-to-signal ratio. Wyplosz (2005) shows that reaction functions like that estimated in this paper are a very poor match for monthly data for Brazil.

2 As Ostry *et al.* (2010, p.10) note, "the drawback of [this] approach is that it requires various assumptions about the risk-free interest rate, the distribution and support of the shocks to the primary balance, and the recovery rate in the event of default".

Under both assumptions for interest rates, debt limits are generally well above 150 % of GDP (chart 5). Assuming an endogenous response of the interest rate to debt tends to reduce the estimated $\bar{d}$ by about 15 percentage points on average. Note that the methodology proposed by Ostry *et al.* (2010) cannot guarantee that a solution to the debt-limit problem as presented in chart 1 always exists. As a matter of fact, neither a stable solution (d^*) nor a debt limit could be pinned down for both Italy and Japan. In the numerical simulations below, we will therefore ignore the case of Japan. For Italy, we will assume a debt limit equal to the average level in the other countries of the sample.

2.1.3 Contingent liabilities: calibrating the debt impact of a tail event

The CDO simulation tool allows for direct shocks to the primary balance reflecting the history of deviations between actual and average fiscal realisation. Regardless of the assumed distribution of the shocks underlying the stochastic simulations (joint-normal or empirical), the likelihood of large and disruptive realisations of contingent liabilities would be too low for comfort[1]. The recent financial crisis highlighted the strong connections between the health of the banking sector and public finances (Amaglobeli *et al.*, 2015), calling for any notion of safe debt to be resilient to a high-impact tail event. Bova, Toscani and Ture (2015) confirm that the realisation of large contingent liabilities was mostly related to financial sector stress.

To incorporate financial sector risk into the safety margin, we use data collected by Laeven and Valencia (2013) to calibrate the country-specific impact of banking stress on public debt. Debt shocks following banking crises ranged from 9 % of GDP in Italy to 44 % of GDP in Finland, with an average of 25 % of GDP in our sample of advanced economies. As a percentage of total banking assets, the public debt effect lies between 4 and 22 %, with an average of 10 % (table 1). Thus, we calibrate the additional safety buffer $\bar{c}$ to represent 10 % of the total bank assets in the country. In our sample, $\bar{c}$ averages 30 % of GDP with differences across countries reflecting

Table 1

Public debt dynamics after banking crises and size of banking sector

Austria	15	378	4
Belgium	19	360	5
Denmark	25	270	9
Finland	44	202	22
France	17	357	5
Germany	18	261	7
Italy	9	217	4
Netherlands	27	326	8
Portugal	34	277	12
Spain	31	285	11
Sweden	36	230	16
United Kingdom	24	496	5
United States	24	104	23
Mean	25	289	10
Median	24	277	8

Source: Laeven and Valencia (2013).

1 The CDO template assumes normal distributions. Drawing the shocks on their empirical distribution would still attribute a very low probability to very large shocks.

the importance of the financial sector. We will refer to the adjusted debt limits as "debt ceilings" since they are not *limits* in the sense defined in chart 1.

2.1.4 Additional assumptions

We assume that fiscal fatigue uniformly sets in at a primary surplus of 4 % of GDP. That corresponds to the maximum five-year moving average of country-specific primary balances in the IMF historical fiscal database (Mauro *et al.*, 2012) and the 75th percentile of the historic distribution of positive primary balances, observed in the panel of advanced economies. An alternative would be to calibrate fiscal fatigue based on each country's history. However, this would lead our framework to effectively confirm the higher historical debt levels reached by each country as the safe boundary. Concluding that governments with historically low debt also have low safe debt boundaries is too close to a tautology to add value. Moreover, a country that has never had to raise its primary balance by very much to keep debt dynamics under control is unlikely to have experienced fiscal fatigue in the first place.

Safe debt boundaries will differ across countries depending on the macroeconomic volatility prevailing in the economy and the stock of contingent liabilities. To contrast these two sources of heterogeneity, we derive the safety margins in two steps. We first assume that the same debt limit applies to all countries, ignoring the risk from contingent liabilities. We then consider country-specific debt limits and incorporate an additional safety buffer for financial sector contingent liabilities.

Finally, we exclude the last five years of data from all our statistical models' estimates. The reason is that the protracted period of extremely low interest rates since the global financial crisis is long enough to seriously contaminate the steady-state (i.e. long-term) properties of the statistical models. While we might well be in a new normal of permanently negative interest-growth differential $(r - g)$, we have opted to preserve estimated models with long-term properties broadly consistent with positive differentials for most countries. First, this is a technical requirement to have a well-defined intertemporal budget constraint, a feature which is worth preserving in this kind of exercise. Second, as noted by Mauro and Zhou (2020), while negative differentials are not infrequent historically, they can rise abruptly; and the likelihood of such reversals appears to rise with the debt level itself (Lian *et al.*, 2020), suggesting that policy-makers should not indulge in low-rate sedation. At the same time, we could not ignore the post-2008 period entirely and be in denial of the possibility of permanently negative $r - g$.

By truncating the estimation sample at end-2014, we obtained a broad range of steady-state interest-growth differentials, enabling us to discuss the implications of such conditions on debt limits. The median steady-state interest-growth differential embedded in our country-specific VAR models is 0.8 of a percentage point, with two countries in negative territory (the US, at –0.1 % and Austria, at –0.2 %), whereas Italy and Portugal are on the high end, at 2.9 and 3.3 % respectively. With an estimated long-term differential of 0.7 %, Belgium is slightly below the median.

2.2 Results

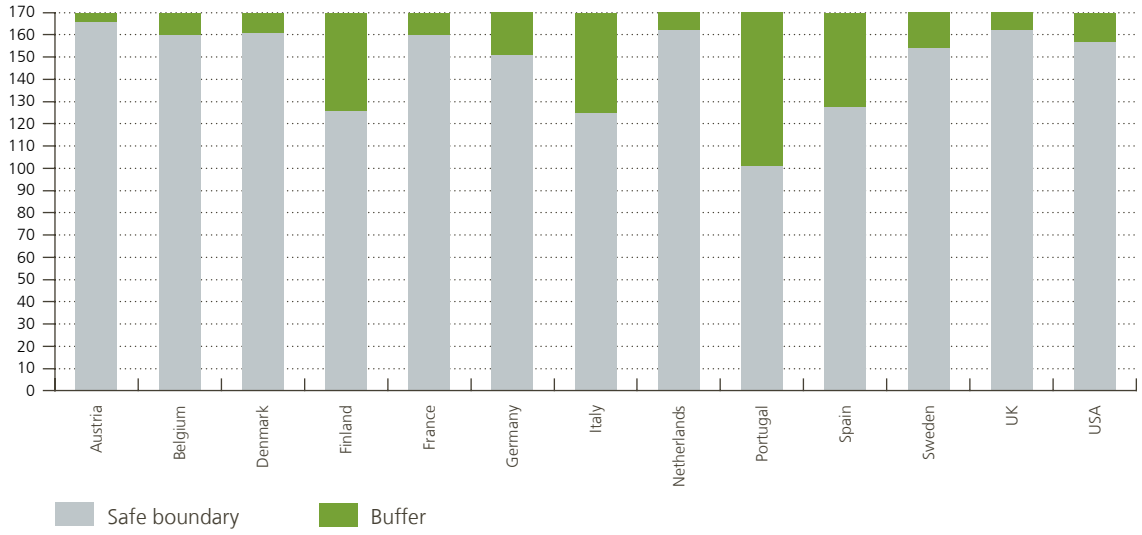
2.2.1 Common debt limit across countries

The common debt limit is set at 170 % of GDP, which is the average of the model-based limit estimated by Ostry *et al.* (2010) for our country sample. The tolerated probability for exceeding the debt limit over a 6-year horizon is set at 0.05. Thus, for each country i, the safe debt boundary is given by $d^*_{i,t_0}(0.95)$ such that $Pr\left(d^F_{i,t_0+6} \leq 170 | d^F_{i,t_0} = d^*_{i,t_0}(0.95)\right) = 0.95$. The results are reported in chart 6. The range of safe debt boundaries spans from 101 % of GDP (Portugal) to 166 % (Austria).

The stochastic simulations behind the numbers in chart 6 can be described with fan charts. For each period of the forecasting horizon, those charts depict the distribution of debt outcomes (see chart 7, top panel). Each coloured band represents a probability mass of 10 %, except for the two outer bands, which cover a 5 %

Chart 6

Safe debt boundaries–uniform debt limits and no adjustment for contingent liabilities



Note: We apply the uniform debt limit of 170 percent of GDP to Italy, even though Ostry et al. (2010) failed to identify such a limit for that country.

Sources: Authors' estimates.

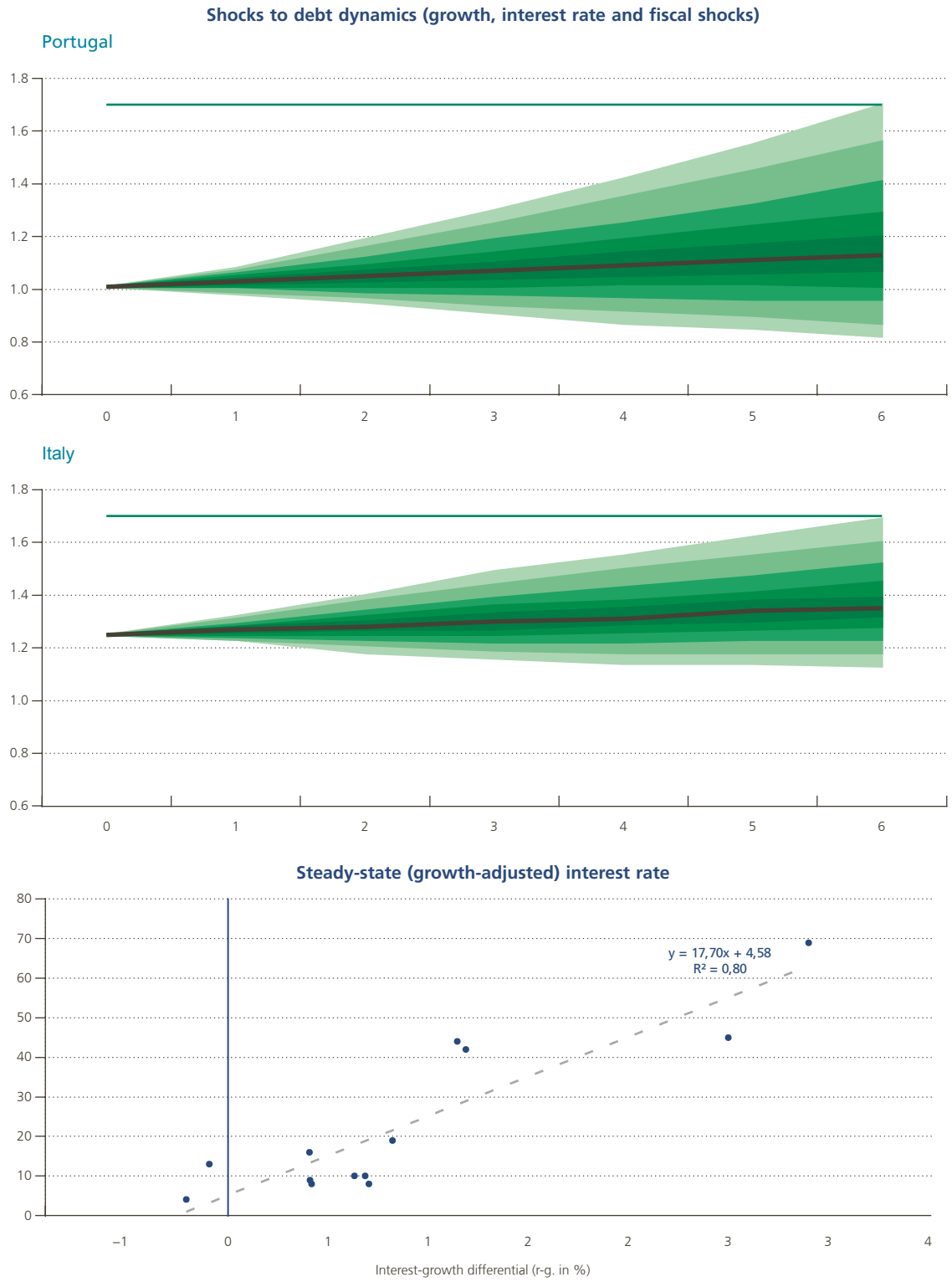
mass each. The width of the debt fan charts can vary substantially across countries, reflecting the volatility of their public debt outcomes. Some countries exhibit narrow fans so that only minimal buffers below the limit appear necessary to feel safe. Other countries with more volatile debt outcomes (wider fan charts) need sizeable room below the limit to secure a low probability of exceeding it.

These differences reflect the intrinsic volatility of interest rates, growth and the primary balance, as well as the average magnitude of the snowball effect. Volatility alone matters a great deal. For illustrative purposes, we look at the cases of Italy and Portugal. These economies face similar steady-state growth-interest differentials, with Portugal having higher exposure to economic, financial and fiscal shocks. The fan charts reflect these differences (chart 7, top panel). The more volatile profile of Portugal's public debt explains why it has a smaller safe debt boundary (101 % of GDP) than Italy (125 % of GDP). The bottom panel of chart 7 also illustrates the important role of the steady-state or equilibrium interest-growth differential in determining the required safety buffer below the limit. In our sample, a change in the equilibrium differential by 100 basis points is on average associated with a change in the safe debt boundary of about 18 % of GDP.

To the extent that the interest-growth differential reflects a government's credibility in keeping debt stable, underlying patterns of policy behaviour matter a great deal in the determination of safe debt boundaries. A key feature of the CDO simulation tool used in this article is that it incorporates the endogenous fiscal policy response to public debt developments (see Annex 1). Thus, the country's history in terms of its "revealed preference" for stabilising the debt ratio (by raising the primary balance in response to higher debt and vice versa) has a first-order effect on the safe debt upper bound. Although the simulations reported here rely on panel estimates of the fiscal reaction function – so that all countries have the same average policy behaviour – the VARs used to simulate interest rates and growth are country-specific. As a result, the debt ratio in countries with a low equilibrium $r - g$ will exhibit stronger mean reversion than countries with a higher $r - g$. At very high debt ratios, mean reversion entails downward trends in most circumstances.

Chart 7

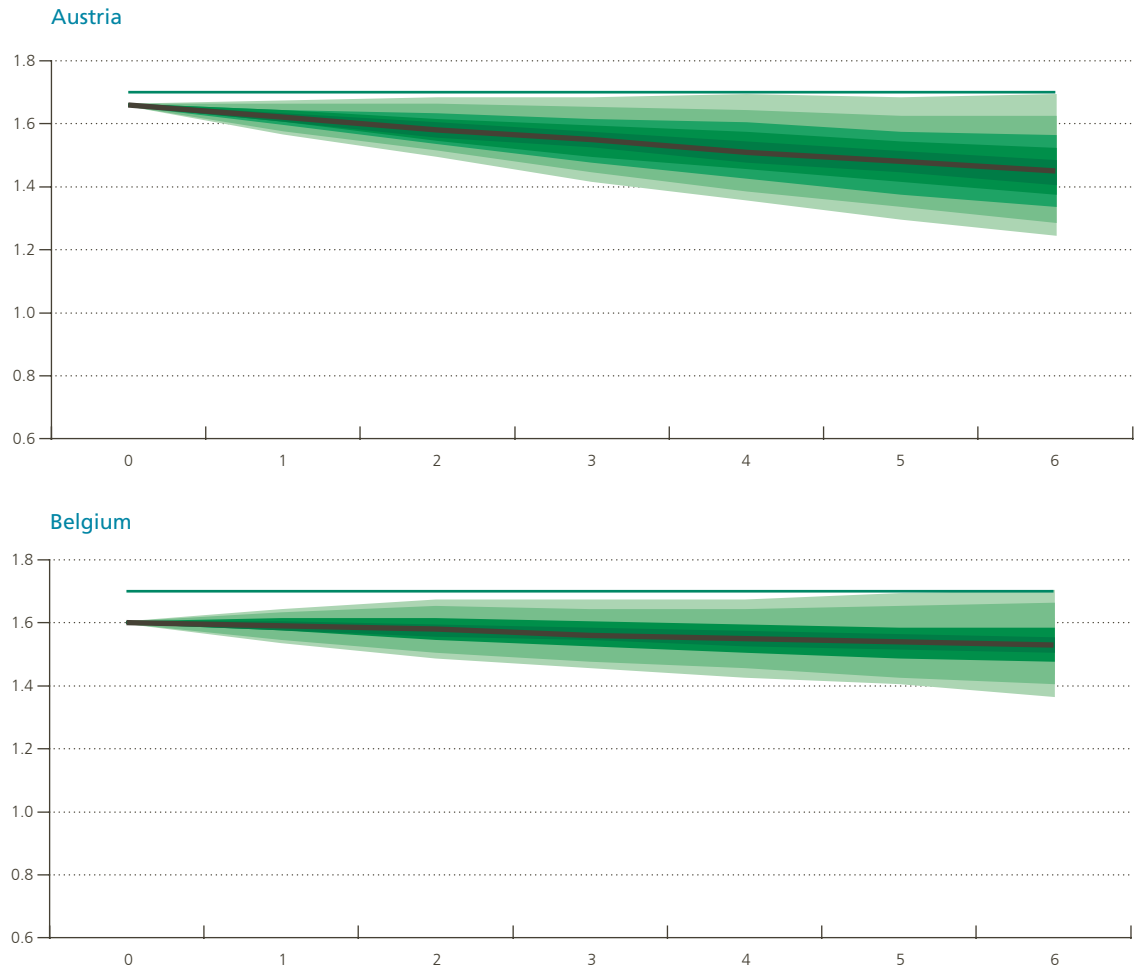
Drivers of safety buffers



Sources: Authors' estimates.

Chart 8

Fiscal behaviour, mean reversion and safe debt boundary



Sources: Authors' estimates.

Chart 8 illustrates that feature with the fan charts for Austria and Belgium. Unlike the cases of Portugal and Italy displayed above, the median debt trajectory shows a clear downward trend over the simulation horizon, allowing the safe debt upper bound to be much closer to the assumed debt limit. This shows the value of fiscal credibility: a country with a consistently sound pattern of policy behaviour can safely sustain much higher debt levels (in this example, close to an estimated debt limit) than countries with weaker credentials in terms of debt stabilisation.

2.2.2 Country-specific debt limits and adjustment for contingent liabilities

We now replicate the exercise using the country-specific debt limits estimated by Ostry *et al.* (2010) and augmenting the safety buffer to accommodate the risk associated with contingent liabilities. First, we subtract from the limit an amount equivalent to 10 % of the country's total banking sector assets (obtained from Laeven and Valencia, 2013)). Second, we identify d^*_{i,t_0} (0.95) such that $Pr\left(d^F_{i,t_0+6} \leq \left(d_{i,LS} - \bar{C}\right) | d^F_{i,t_0} = d^*_{i,t_0}(0.95)\right) = 0.95$. Although the size of the banking sector now matters for the definition of safe debt boundaries, macroeconomic conditions (magnitude of the snowball effect, growth and interest rates volatility, prevalence of fiscal policy shocks) remain a key factor behind cross-country differentiation.

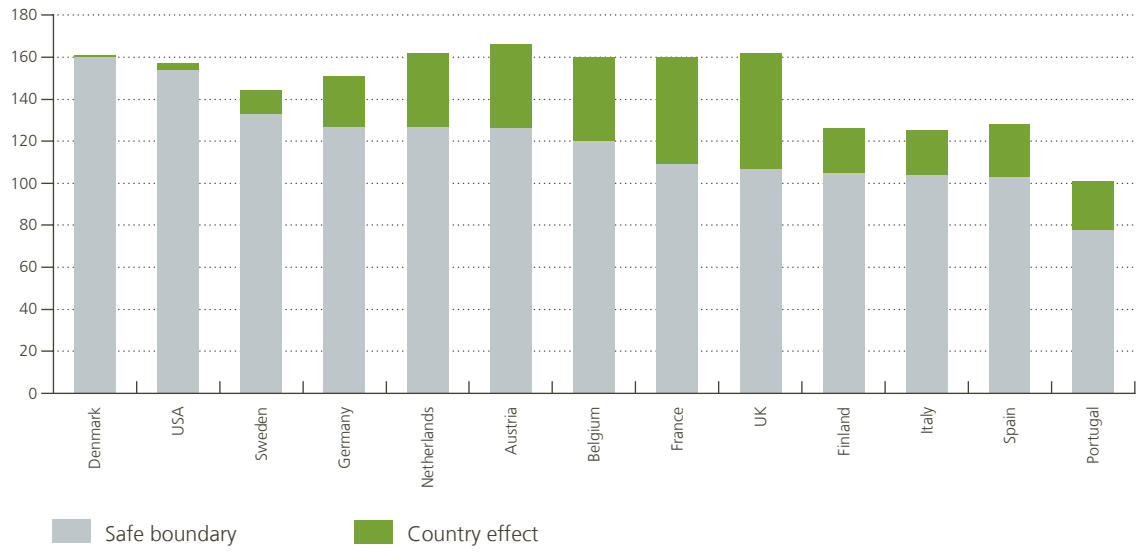
Safe debt boundaries are now more than 25 % of GDP (on average) lower than in the previous scenario. Compared to the numbers reported in chart 6, the safe debt upper bounds (grey bars) displayed in chart 9 are generally lower – sometimes substantially – due to the country-specific safety margin (green blocks) covering contingent liability risk (10 % of the total banking sector assets as reported by Laeven and Valencia (2013)) and some modest differentiation in the estimated debt limits. In the case of Denmark, however, the safe debt boundary is largely unchanged because the extra buffer for banking sector risk is almost fully covered by the higher debt limit estimated by Ostry *et al.* (2010) – i.e. 196 % of GDP instead of the uniform 170 % applied in the previous exercise. As for Belgium, the country-specific safety buffer is 40 percentage points of GDP lower, bringing the safe debt boundary to 120 % of GDP, mostly on account of the country's sizeable banking sector.

Two secondary effects of using a debt ceiling much lower than the debt limit are worth noting. First, because the impact of shocks to growth and interest rates on debt are proportional to the debt level itself, the fan charts below the ceiling will be narrower than those below the limit. This tends to reduce the precautionary buffer needed to handle high-frequency economic and financial disturbances. Second, all else equal, navigating fiscal policy at lower debt levels weakens any downward trend associated with mean reversion in the debt ratio. This contributes to a higher precautionary buffer below the limit.

Chart 9

Safe debt boundaries considering country-specific limits and contingent liabilities

(in % of GDP)



Sources: Authors' estimates.

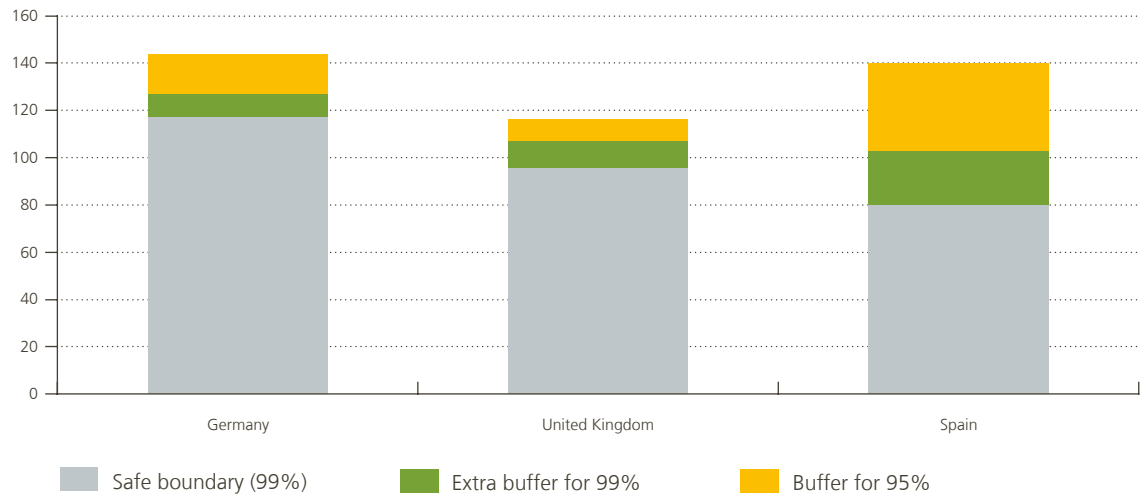
2.3 Lower risk aversion

To assess the sensitivity of our safe debt boundaries to risk aversion, we can lower the tolerance for exceeding the debt ceiling in six years from 0.05 to 0.01, which means raising π from 0.95 to 0.99. Since the impact depends only on the range of the fan chart, the reduction in the safe debt boundary is in general more pronounced in countries with more volatile public debt profiles (chart 10). While the extra buffer to secure a 99 % probability of not exceeding the ceiling (instead of a 95 % probability) is estimated at around 10 % of GDP for Germany and the United Kingdom, it reaches twice that amount for Spain.

Chart 10

Extra buffers to secure a higher probability of not exceeding the debt ceiling

(in % of GDP)



Note: For each country, the sum of the grey and green blocks yields the safe boundary reported in chart 9, while the sum of all blocks corresponds to the debt limit adjusted for the tail-risk buffer (here defined as 10% of the banking sector assets.)

Sources: Authors' estimates.

3. Safe debt with unknown debt limit

This section implements the deterministic and stochastic approaches discussed earlier to estimate safe debt boundaries based solely on the strongest plausible fiscal behaviour a country could adopt in the case of highly adverse conditions for debt dynamics.

3.1 Deterministic approach

3.1.1 Methodology

As discussed in section 2, the underlying methodology relies on building a stress scenario. For that purpose, we define "adverse conditions" during which a "crisis-mode" fiscal policy deviating from normal fiscal behaviour can be credibly implemented. Probability distributions of primary balances and $r - g$ can help define such a combination of bad realisations of the interest-growth differential and good realisations of the primary balance. Here, striking a good compromise between the exceptional and the plausible is of the essence because, like in Mendoza and Oviedo (2004), the stress scenario is conceived as a "permanent" state of the economy. For that reason, we select less extreme realisations than the conventional 95th percentiles, preferring instead the 75th percentiles. Of course, plausibility is in the eye of the beholder, and all the calculations below can be implemented for any other quantile.

As regards the primary balance, the 75th percentile is selected from the distribution of the entire panel of countries in our sample and it is used uniformly as the fiscal fatigue threshold for all countries. The reason for doing so is that not all countries in our sample have faced situations forcing them into "crisis-mode" fiscal

behaviour. Uniformity presumes that if it were pushed into crisis mode, *any* country in our sample would be able to behave as its sample peers that faced very adverse public debt dynamics in the past.

By contrast, the statistical properties of $r-g$ arguably reflect country-specific structural factors moving slowly over time. In the calculations of the safe debt boundary, the uncertainty surrounding the growth-adjusted interest rate is therefore country-specific. In addition, the likely existence of a positive relationship between the growth-adjusted interest rate and the debt level suggests for conditioning the distribution on debt itself. The conditioning relationship is estimated with a simple panel regression reported in Annex 2.

Denoting by $\bar{p}$ the highest primary surplus a country can plausibly achieve (and presumably sustain for some time) the safe debt boundary can be found as follows:

1. Assume a distribution for $r-g$. In the calculations below, we assume a normal distribution with a mean equal to the predicted value of $r-g$ obtained from the model in Annex 2 (and therefore dependent on debt) and a variance equal to the *country-specific* conditional sample variance.

2. Take the 75th percentile of the distribution defined in step 1, denoted by $\overline{r-g}$. Under our assumptions, it is a positive function of the debt level.

3. Select a sufficient condition ruling out explosive debt trajectories under $\overline{r-g}$. In line with the discussion around chart 3, we consider two alternative conditions for step 3.

Condition 3A seeks to identify d_{S1} as the debt level the government can stabilise under $\overline{r-g}$ by keeping the primary balance at $\bar{p}$. This simply requires solving $d_{S1}=\frac{\bar{p}}{f(d_{S1})}$ which has in general multiple roots depending on the specific form of $f(d)$. In the calculations below, it has only one positive root, which gives the upper bound for safe debt. While this formula has the advantage of simplicity, it presumes the feasibility of a discreet jump to $\bar{p}$ as soon as the bad state for $r-g$ materialises. Like in Mendoza and Oviedo (2004), d_{S1} is conceptually a debt limit (hence an unstable equilibrium as depicted in chart 3) contingent on adverse conditions.

Condition 3B determines the safe debt boundary as a stable equilibrium. We define it as the highest debt level to which the government can credibly commit to returning after stress conditions occur. The fiscal consolidation is considered credible if it follows historical patterns of large annual improvements in the primary balance taking place over a given time frame (five or ten years in the calculations below). The feasible fiscal adjustment in one year is calibrated on the 75th percentile of the first-differenced primary balance observed in the sample.

The safe debt boundary calculated in these ways will in any instance:

- decrease with the average growth-interest differential,
- decrease with the variance of that differential,
- increase with the quantile defining the plausibly high primary surplus (or its yearly adjustment),
- decrease with the quantile defining adverse circumstances in terms of growth-interest differential.

3.1.2 Results

For the sake of brevity, and because all the calculations reflect the same fundamentals as above, we only comment on the range of estimated safe debt boundaries under the two conditions 3A and 3B[1].

1 Country-by-country results are available upon request to the corresponding author.

3.1.2.1. Condition 3A: the primary balance jumps to its "fatigue" level

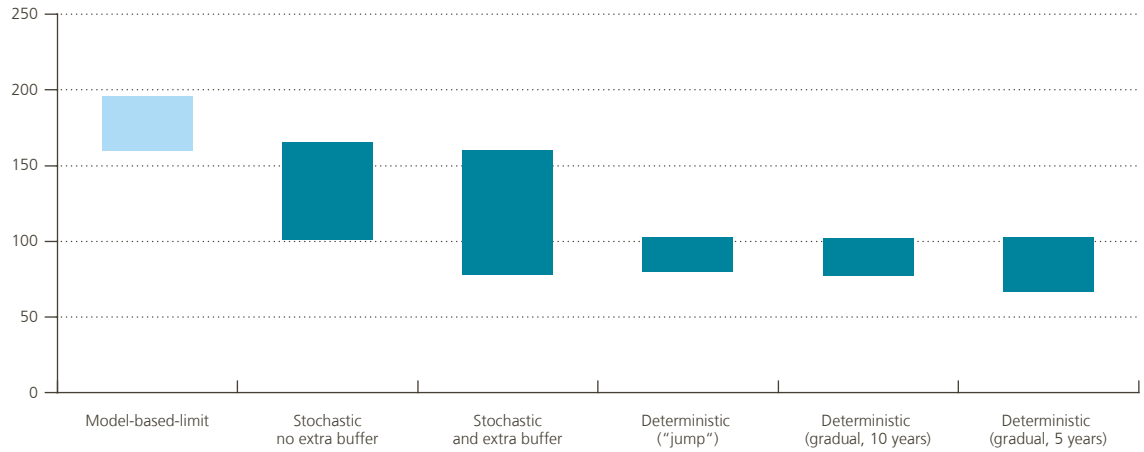
The maximum attainable primary balance of 4 % of GDP apply to all countries, whereas the increase in $r - g$ is permanent and country specific. Countries manage to jump to that threshold during the year of the emergence of adverse conditions for debt dynamics. The median safe debt boundary obtained in that way amounts to about 95 % of GDP, which is significantly lower than the median safe debt upper bound found under the known debt-limit hypothesis (i.e. 157 % without extra buffer for contingent liability and 120 % with such a buffer). Of course, that gap says as much about the methodological difference between the two approaches (stress testing versus stochastic simulations) as about the estimated debt limits themselves (i.e. are those limits "too high" ?).

The range of estimates is also narrower than in section 2, spanning from less than 80 % of GDP to slightly more than 100 %. Heterogeneity across countries is mainly driven by the conditional variance of $r - g$. An intuitive conjecture explaining the narrower cross-country range is that the deterministic stress scenario focuses on the 75th percentile of distributions (instead of the 95th in the stochastic simulations framework). Relatedly, the technical assumption making the adverse scenario a permanent state of the economy is intrinsically more conservative than letting estimated VAR models generate a large number of forecasts reflecting the degree of persistence in the data. Chart 11 compares the ranges of safe debt boundaries under the various methodologies applied in this article.

Chart 11

Range of country-specific safe debt boundaries under alternative methodologies

(in % of GDP)



Sources: Authors' estimates.

3.1.2.2. Condition 3B: gradual adjustment of the primary balance

We obtain safe debt boundaries under the following assumptions:

- The maximum attainable primary balance level is the same across countries and equal to 4 % of GDP.
- The maximum annual improvement in the primary balance is the same across countries and equal to 0.82 % of GDP. That fiscal contraction can be implemented for a maximum of five or ten consecutive years and it stops as soon as the 4 percent fatigue threshold is reached (it might never be reached though, depending on the starting balance).
- Starting primary balances are country-specific and equal to the debt stabilising primary balance at the safe-debt threshold under normal $r-g$ conditions.
- The adverse scenario for $r-g$ is permanent and country-specific.
- The safe debt boundary is the highest initial debt ratio such that the gradual fiscal consolidation as defined above ensures a return of the debt ratio to that initial "safe haven" under the adverse scenario for $r-g$. (This is the stepwise path illustrated in chart 3.)

Compared to the previous sub-section, safe debt boundaries are on average around 5 percentage points lower, and heterogeneity across countries is slightly greater. Results also show that extending the adjustment period from five to ten years does not matter much for countries with high debt thresholds and correspondingly high starting primary balances since their fiscal adjustment quickly pushes the primary balance to the 4 percent cap. The mirror argument applies for countries with low initial primary balances: limiting their fiscal adjustment period to five years yields lower safe debt boundaries. (In chart 3, $d_{S2} < d_{S1}$.).

3.2 Stochastic approach

3.2.1 Methodology

The modeling blocks underlying the computation of the safe debt boundary are the same as in the case of a known limit (see Annex 1). The difference is that the boundary is now determined by the fiscal fatigue threshold for the primary balance. In practice, we select the highest starting debt level of a projection exercise such that, over a given time horizon, the projected *debt-stabilising primary surplus* (i.e. $p^* = \left(\frac{r-g}{1+g}\right)d_{-1}$) exceeds the fiscal fatigue threshold with a low probability (say 5 percent). In the universe of positive interest-growth differentials, setting a higher debt level at the beginning of the projection horizon increases the likelihood that p^* exceeds the threshold. Thus, fixing a tolerance level for unfeasible primary balances determines a unique safe debt boundary.

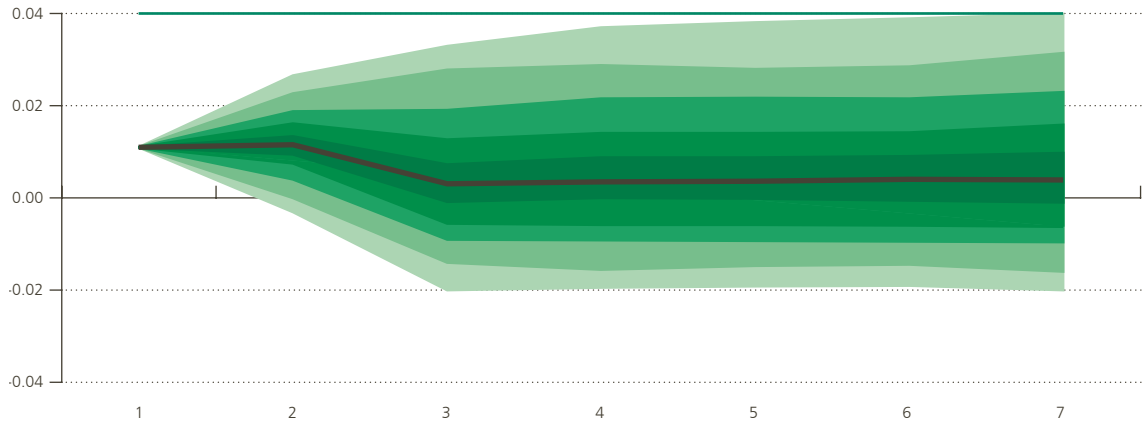
A practical limitation to this approach, however, has to do with the steady-state (or equilibrium) level of the interest-growth differential. If the latter gets close to zero or even plunges into negative territory, we might well never find any p^* above the 4 percent threshold, or at least not for plausible initial debt levels of the projection exercise.

3.2.2 An example: Spain

Chart 12 displays the fan chart for Spain's debt-stabilising primary balance. We see that in year 6, the 95th percentile of p^* is equal to 4 percent of GDP. Chart 12 mirrors a public debt fan chart with a starting value equal to the safe debt boundary. The approach yields sensible results because for the sample period considered here, Spain features an interest-growth differential that is both positive in equilibrium and relatively volatile. In this example, the estimated boundary is 75 percent of GDP. That number is lower than the outcome of the deterministic approach calibrated with the same data (90 percent of GDP) because stochastic simulations consider the full distribution of projected debt paths (and the corresponding p^*). In this case, the distribution is skewed to the right (i.e. higher values of debt and p^*). The safe debt boundary estimated in this way would rise if the equilibrium $r-g$ (and/or its variance) were to fall.

Chart 12

Spain: Fan charts for the debt-stabilising primary balance



Sources: Authors' estimates.

Conclusions

Public debts are historically high, with no significant decline in sight. Many people worry, but many others don't. In the first group are those pointing out that, in the past, current debt levels have tended to predict costly accidents, including sovereign defaults, high inflation, and financial meltdown (e.g. Sandleris 2016). In the second group are those arguing that because negative differentials between interest rates and growth are here to stay, public debt is essentially a fiscal free lunch (Blanchard, 2019). Yet, the two groups converge on one basic reality: no one can have their cake and eat it indefinitely[1]. In plain language, the laws of economic and financial gravity still apply, and for public debt, this means that the sky is *not* the limit.

Thus, more than ever, an important question for policy-makers is the following: when does accumulating additional public debt become unsafe? This is the question we tackle in this article.

Using the standard toolkit of debt sustainability analysis, our goal is to provide pragmatic approaches that can usefully inform an answer to that question. In this context, a debt level (in proportion of GDP) is considered safe if it is plausible to expect the government to stabilise or reduce it under most circumstances (including adverse ones) using fiscal policy. This definition combines the uncertainty surrounding growth and interest rates, and the plausibility of primary surpluses required to at least keep the debt ratio constant. The practical problem we aim to solve is to determine the highest possible debt level beyond which the risk of losing control of debt dynamics could be deemed unreasonable.

Because we rely on well-tested tools of debt sustainability analysis, the common denominator to the different methods reviewed here is the medium-term prospects for debt dynamics. We use simple forecasting models to determine safety buffers below a known "limit" to debt (i.e. a threshold beyond which policy-makers cannot

1 Blanchard (2019) himself cautions his readers: "My purpose in the lecture is not to argue for more public debt. It is to have a richer discussion of the costs of debt and of fiscal policy than is currently the case".

credibly commit to continue to service the debt indefinitely)[1]. These safety buffers reflect the intrinsic riskiness of debt trajectories in a country (due to economic and financial volatility) and the typical response of fiscal policy to debt developments (i.e. strongly stabilising or not). When the debt limit is unknown, we determine the safe debt boundary by reference to a practical limit to the primary budget surplus a government can be expected to generate and sustain.

A fundamental caveat applying to all those forecasting-based methods is the always questionable role of the past as a reliable predictor of the future. The Lucas critique thus applies with full force and should be kept in mind when interpreting the specific numbers coming out of these exercises: they reflect a hypothetical "equilibrium" observed in the (recent) past but that may not properly reflect today's or tomorrow's circumstances. Blanchard (2019) suggests nothing else when he reviews the reasons why public debt is currently less costly than one might think. The safe debt boundaries derived here may thus appear exceedingly conservative to many analysts. We have no qualms about such a judgment and would simply point out that all the tools considered here can be calibrated or re-estimated to stick more closely to present conditions.

Another important property of the safe debt boundaries is that they can be exceeded by significant margins without causing any tangible risk of default or accident. This is especially true if central banks cover Treasuries' backs by ruling out self-fulfilling sovereign debt crises that could be inflicted on fundamentally solvent governments. During tail events, like the global financial crisis of 2008-9 or the COVID-19 pandemic, such backing can and should create significant fiscal space to enable the adequate fiscal policy response (see Bartsch *et al.* 2020).

Taken at face value, our results suggest that public debt levels in advanced economies are currently flirting with the safe debt upper bound or already go well beyond it. While this warrants careful monitoring, it should be no cause for undue concern as long as the "equilibrium" interest-growth differential remains durably lower than the historic norm. That said, the costs of public debt "accidents" are so high that there should be no room for complacency. Today's differential is low because equilibrium interest rates are low, reflecting a persistent excess of savings over investment. This, in itself, is far from a healthy or desirable situation, and we would expect (and actually hope for) normalisation in the longer run. This is when we should remember that based on the estimates discussed in this article, a 100-basis-point increase in the interest-growth differential is on average associated with a reduction in the safe debt boundary by close to 20 percentage points of GDP.

1 An alternative approach is to produce distributions of debt limits as in Battistini *et al.* (2019) and Bi (2012).

Annexes

Annex 1. The stochastic simulation model

The economic block

For each country, we estimate an unrestricted VAR describing the joint dynamics of the non-fiscal variables needed to project public debt, namely real interest rates (foreign and domestic), real GDP growth, and the real exchange rate. Formally, we write:

$$Y_t = A_0 + \sum_{k=1}^{p} A_k Y_{t-k} + \varepsilon_t \ , \qquad \text{(A.1)}$$

with $Y_t = (r_t^{us}, r_t, g_t, z_t)$. The A's are vectors of coefficients, while r^{us} symbolises the real foreign interest rate, r, the real domestic interest rate, g, the real GDP growth rate, z, the (log of the) real effective exchange rate, and ε_t is a vector of normally distributed error terms: $\varepsilon_t \sim N(0, \Omega)$.. Estimation uses quarterly data over 1990-2014.

The VAR plays two roles. First, the estimated variance-covariance matrix of residuals Ω is the basis to calibrate the generation of random, non-fiscal shocks. Specifically, we produce a sequence of shock vectors $\hat{\varepsilon}_{t+1}, \ldots, \hat{\varepsilon}_T$ such that $\forall \tau \in [t+1, T]$, $\hat{\varepsilon}_\tau = \Lambda \mu_t$, where Λ is the Choleski factorisation of Ω ($\Omega = \Lambda' \Lambda$) and $\mu_t \sim N(0,1)$. Second, we feed the simulated shock sequence in the estimated VAR to obtain consistent forecasts of Y.. As shocks occur each period, the VAR produces joint dynamic responses of all elements in Y. In practice, it is important to check that the VAR's coefficients rule out explosive paths for any of the variable.

After annualising all relevant projections, the fiscal block of the model projects the primary balance and the debt-to-GDP ratio reflecting the simulated shocks and the corresponding dynamic response of the economy.

The fiscal block

The fiscal block consists of a reaction function and a stock-flow identity to project debt-to-GDP ratios. The reaction function follows a well-established specification aimed at capturing the main features of average fiscal policy patterns. In line with Bohn (1998), it describes the link between the primary fiscal balance that is related to current economic conditions (the business cycle) and to solvency concerns, as reflected in the positive impact of inherited public debt. We use an unbalanced panel of 29 advanced economies over the period of 1990-2014. Table A.1 shows estimation results using robust country-fixed effects estimator (LSDV) for specifications that differentiate between good and bad times by separately estimating the impact of negative and positive output gaps. Full sample results are reported in column 1. In order to gauge the influence of the financial crisis and the ensuing great recession, we drop one observation at a time until we eliminate the post-crisis observations and report the results in the latter columns.

The reported magnitudes are in line with the empirical literature[1]. The primary balance tends to be persistent, and sensitive to economic conditions, but in an asymmetric fashion. When the output gap is negative, a widening of the output gap by 1 percentage point deteriorates the primary balance by 0.48 % of GDP on average, pointing to a fairly large countercyclical response in bad times. By contrast, any widening of an already positive output gap is not reflected in a statistically significant improvement of the primary balance, in line with a well-documented tendency to spend revenue windfalls in good times. Persistence of primary balance and response of primary balance to debt is robust to the inclusion of post-crisis observations. Finally, countries have tended to react strongly and in a stabilising fashion to public debt developments.

1 Among others, see for example, Celasun, Debrun and Ostry (2007) and Mauro *et al.* (2013, table 7).

Table A.1

Fiscal "reaction function" in advanced economies

	(1) pbal 1990-2014	(2) pbal 1990-2013	(3) pbal 1990-2012	(4) pbal 1990-2011	(5) pbal 1990-2010	(6) pbal 1990-2009	(7) pbal 1990-2008	(8) pbal 1990-2007
pbal t-1	0.687*** (0.0449)	0.694*** (0.0463)	0.694*** (0.0472)	0.692*** (0.0496)	0.685*** (0.0530)	0.663*** (0.0658)	0.655*** (0.0725)	0.672*** (0.0604)
ygap pos	0.00301 (0.0426)	0.00714 (0.0429)	0.000985 (0.0462)	–0.00554 (0.0467)	–0.00411 (0.0472)	–0.0158 (0.0481)	0.00215 (0.0369)	–0.00854 (0.0503)
ygap neg	0.436*** (0.123)	0.440*** (0.127)	0.470*** (0.126)	0.487*** (0.133)	0.489*** (0.138)	0.551*** (0.128)	0.427*** (0.126)	0.478*** (0.124)
debt t-1	0.0494*** (0.00815)	0.0511*** (0.00893)	0.0531*** (0.0102)	0.0544*** (0.0113)	0.0533*** (0.0127)	0.0581*** (0.0134)	0.0555*** (0.0124)	0.0458*** (0.0120)
constant	–2.196*** (0.392)	–2.293*** (0.452)	–2.357*** (0.539)	–2.383*** (0.605)	–2.326*** (0.678)	–2.474*** (0.711)	–2.262*** (0.624)	–1.545** (0.619)
N adj.R-sq	620 0.552	591 0.555	562 0.565	533 0.553	505 0.535	477 0.506	448 0.507	419 0.601

Sources: Authors' estimates.

Accordingly, we choose the asymmetric output gap specification that includes only pre-crisis observations summarised in equation (A.2).

$$\hat{p}_{i,t} = \hat{\alpha}_i + 0.67p_{i,t-1} - 0 \times ygap_{i,t}D_{i,t} + 0.48ygap_{i,t}(1 - D_{it}) + 0.046d_{i,t-1} \qquad \text{(A.2)}$$

where $p_{i,t}$ is the ratio of the primary fiscal balance to GDP in country i and year t; $d_{i,t-1}$ is the gross public debt-to-GDP ratio at the end of the previous year; $ygap_{i,t}$ is the contemporaneous output gap; $D_{i,t}$ are dummy variables equal to 1 when the output gap is non-negative (actual output above or equal trend) and 0 otherwise; and $\hat{\alpha}_i$ are the country fixed effects.

To account for the possibility that fiscal policy can itself be a source of shocks, the primary balance is subject to a fiscal policy shock $\varphi_{\mathrm{i,t}} \sim \mathrm{N}(0, \sigma^2_{\varphi_\mathrm{i}})$, where $\sigma^2_{\varphi_\mathrm{i}}$ is calibrated on the country-specific variance of the reaction function's residuals. The overall algorithm can generate a large number of random shock sequences and their corresponding debt paths. For each year of projection, frequency distributions of debt-to-GDP ratios enable a probabilistic analysis of debt sustainability.

We also introduce the fiscal capacity ceiling $\bar{p}$ on the primary balance to guarantee that the fiscal reaction function remains plausible given country experience. Thus, the primary balance in each period is equal to the minimum of the primary balance predicted by the fiscal reaction function and the fiscal surplus maximum capacity (min $\{\hat{p}_{i,t}, \bar{p}\}$). The ceiling is defined as a cross-country average of the maximum rolling average of country-specific primary balances over five-year time periods. Based on the IMF historical database of fiscal variables (Mauro *et al.*, 2012), this number is 4 % of GDP for our country sample. The same ceiling is applied to all countries to avoid that the safe debt level merely reflects the country's own history of fiscal behaviour. Although a country's history may not contain many episodes where the government was constrained to run very high surpluses, it says little about the country's capacity to do so when the need arises.

To project the debt path corresponding to a given set of shocks, we translate the annualised VAR projections into simulated output gaps and use equation (A.2) and the conventional stock-flow identity (A.3) recursively:

$$d_t \equiv \frac{(1+r_t)d_{t-1}}{(1+g_t)} - pb_t + s_t \qquad \text{(A.3)}$$

where r_t is the average effective real interest rate, g is the real GDP growth rate, and s_t – stock-flow adjustments, for instance due to the call of government guarantees or privatisations. Higher moments remain determined by history, although all VARs used in the paper incorporate the latest available quarterly data (end-2014) and capture in part the increased economic and financial volatility since 2007[1]. One aspect that the CDO model cannot capture, however, is the emergence of fat tails or asymmetries since shocks are all drawn from normal distributions.

1 We compared VARs used in the remainder of the paper to estimates obtained on shorter pre-crisis samples. Significant differences – notably in the underlying steady-state values for growth and interest rates – suggest that crisis time observations have already had a meaningful impact on estimated economic dynamics.

Annex 2. Inputs to the deterministic approach

This Annex describes the main statistical properties of the growth-adjusted interest rate and primary balance, the two main ingredients in the calibration of the model.

Descriptive statistics

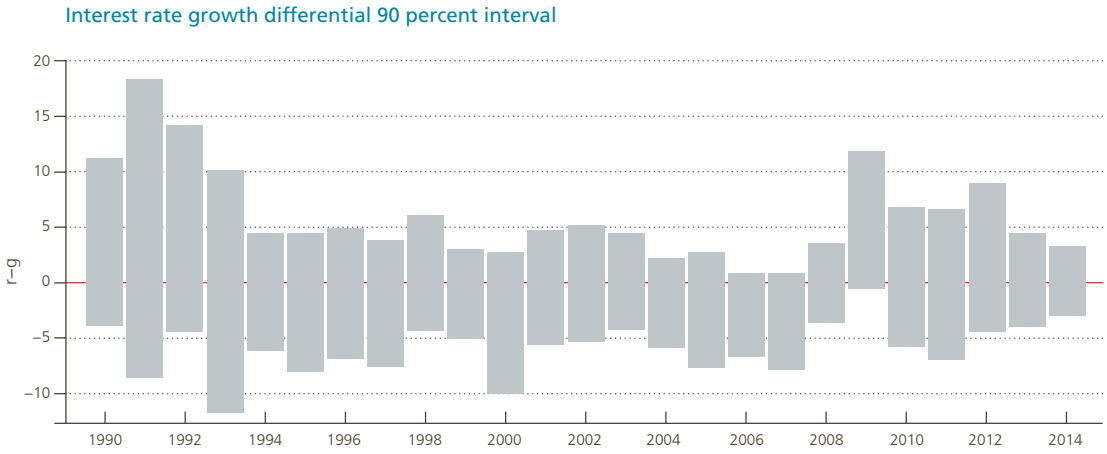
In the years prior to the great financial crisis, most advanced economies experienced very low real interest rates. In fact, two-thirds of advanced economies had on average negative interest-growth differentials in 2000-2007 (chart A.1). With the average growth rate staying close to what it was in the 1980s and 1990s, around 3 percentage points, historically low real interest rates were the key drivers of this trend. However, unlike in the 1980s when the low real interest rates were caused by high inflation and financial repression, the trend in the 2000s mostly reflected nominal interest rates.

The crisis was a powerful reminder that such loose financing conditions could not last. The average interest rate growth differential went up by around 3 percentage points in 2008-2014, which together with sharply deteriorating budget balances called debt sustainability into question for many advanced economies. Faced with sluggish growth and low inflation, record low nominal interest rates are unlikely to ensure a return to significantly and persistently negative $r - g$.

Chart A.1

Range of growth-adjusted interest rates in advanced economies

(1990-2014)



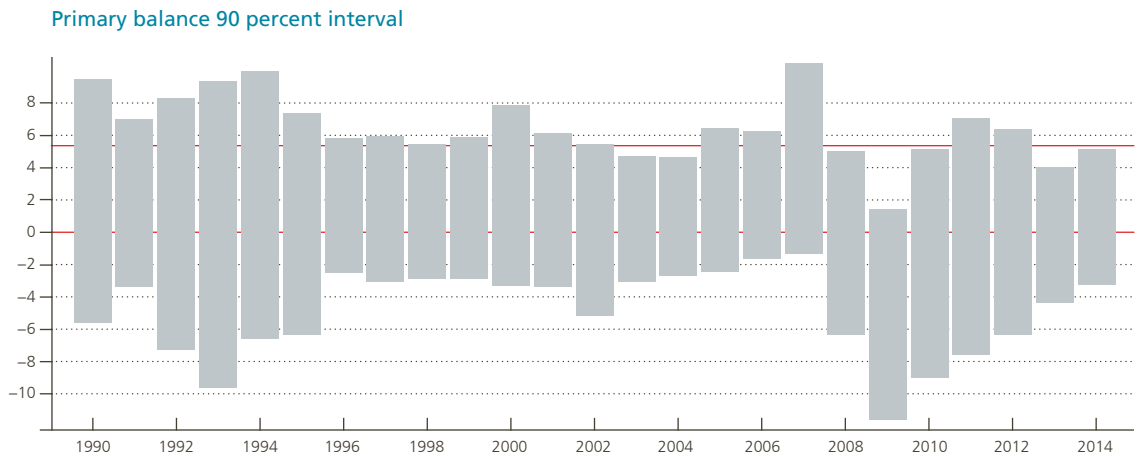
Sources: Authors' calculations.

Looking at primary balances, many advanced economies have exhibited primary surpluses for only a short time in the last two decades. In fact, the average primary balance in advanced economies has been close to zero. However, this number conceals significant variations across countries (chart A.2).

Chart A.2

Range of primary balances in advanced economies

(1990-2014)



Sources: Authors' calculations.

The growth-interest-debt nexus

A simple regression explaining linking the interest-growth differential to potential determinants is used to calculate its conditional variance, which is in turn crucial for the calculation of safe debt levels. That relationship can be viewed as the merger of two separate fields in the literature: one investigating the relationship between debt levels and interest rates and the other looking at impact of debt on growth. We proceed by estimating the empirical model (A.4):

$$r - g_{it} = \alpha + \beta d_{it} + \delta_i + \gamma_t + \varepsilon_{it}, \qquad \text{(A.4)}$$

with r, the effective interest rate calculated as total interest payments divided by the previous year level of debt; g, the nominal GDP growth rate; d, current year debt to GDP ratio; the δ_is are country-fixed effects and γ_ts are time-fixed effects. The model is estimated using a sample of advanced economies, and as a first pass, it omits other determinants than debt. Yet, country-fixed effects and time effects alleviate concerns about the omitted variable bias. To take into account the effect of joining the euro area on Member States' interest rates, we re-estimate an empirical model linking the growth-adjusted interest rate to the level of the public debt allowing for different $r - g$ sensitivity to debt for euro area countries during the years 1999-2007. The results of the estimation of the above regression are presented in table A.1.

Table A.1

Public debt and growth-adjusted interest rates in advanced economies

	(1)	(2)	(3)	(4)
	1990-2014		1990-2007	
debt	0.0663*** (0.0179)		0.0539** (0.0204)	
debt other		0.0665*** (0.0178)		0.0535** (0.0206)
debt during euro effect time		0.0677*** (0.0181)		0.0566*** (0.0201)
constant	–1.716 (1.400)	–1.717 (1.398)	–1.193 (1.543)	–1.155 (1.587)
N adj.R-sq	639 0.366	639 0.365	436 0.315	436 0.314

Sources: Authors' estimates.

We find a positive and fairly robust relationship between $r-g$ and the debt level. Both coefficients are significant, although there is no statistically significant difference between the two slopes. The coefficients from the second regression are used to obtain conditional variance of the growth-adjusted interest rate.

To quantify the effect of shock to $r-g$, the conditional variance is calculated based on the expression below:

$$r-g\ shock(debt^{*}) = InvNormal(75th\ pcl) * sd_{resid\ r-g\ if\ year>2007} + (mean_{r-g}(debt^{*}) + \text{time effect } 1990-2007) \quad \text{(A.5)}$$

We make the following assumptions when calculating the conditional variance:

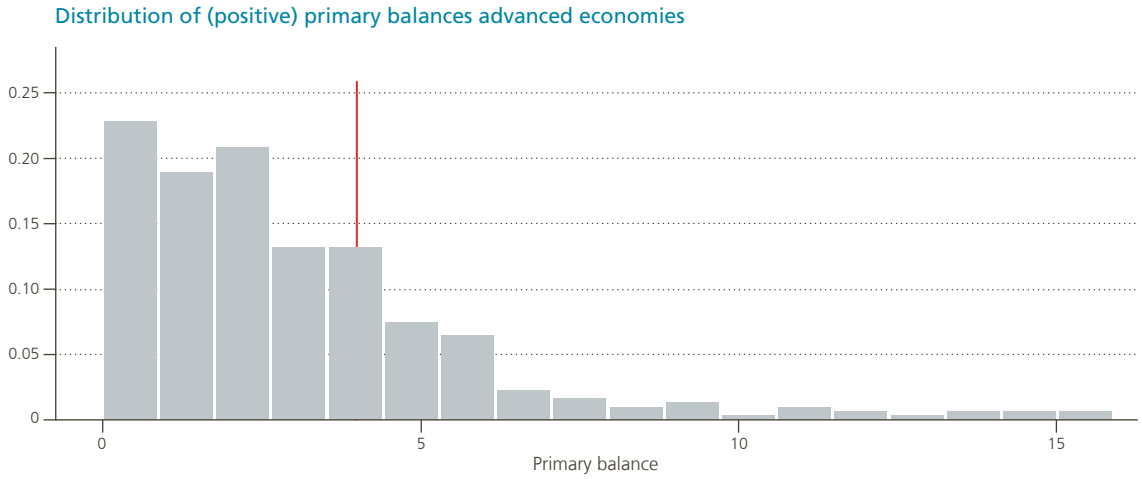
- Country-fixed effects are set to zero to limit the impact of historical high inflation or financial repression episodes.
- We correct for the crisis-time-fixed effects by subtracting average time fixed effect over 2008-2014 from the mean to limit the impact of crisis on the median $r-g$.
- Risk tolerance is set to the 75th percentile of the distribution of conditional variance to gauge the magnitude of shock to $r-g$.

Fiscal behaviour

Fiscal performance that a country is likely to achieve following a shock is determined by maximum attainable primary balance, annual adjustment effort, and the starting primary balance. To estimate the maximum attainable primary balance level that a country is likely to achieve, we take the 75th percentile of the positive primary balance for the underlying sample of advanced economies, that is equal to 4 % of GDP.

Chart A.3

Distribution of primary surpluses



Sources: Authors' calculations.

To quantify the magnitude of annual fiscal effort, we estimate a standard "fiscal reaction function" along the lines of Bohn (1998) based on our underlying sample of 29 advanced economies (equation A.6):

$$pbal_{it} = \alpha + \beta_1 pbal_{it-1} + \beta_2 ygap\_pos_{it} + \beta_3 ygap\_neg_{it} + \rho debt_{it-1} + \delta_i + \varepsilon_{it}, \qquad \text{(A.6)}$$

where *pbal* is the primary balance in percent of GDP, ygap_pos is the positive output gap, ygap_neg is the negative output gap, debt represents the *lagged* debt level as a percentage of GDP. Results are presented in table A.2.

Table A.2

Fiscal reaction function in advanced economies

	(1) pbal 1990-2014	(2) pbal 1990-2007
pbal t-1	0.687*** (0.0449)	0.672*** (0.0604)
ygap pos	0.00301 (0.0426)	–0.00854 (0.0503)
ygap neg	0.436*** (0.123)	0.478*** (0.124)
debt t-1	0.0494*** (0.00815)	0.0458*** (0.0120)
constant	–2.196*** (0.392)	–1.545** (0.619)
N adj. R-sq	620 0.552	419 0.601

Sources: Authors' estimates.

The estimated response of the primary balance to debt conveys useful information on the likelihood of an average country in our panel behaving in a way that ensures mean-reverting debt trajectories. However, we are primarily interested in what the *residuals* of that regression can tell us on the extent of exceptional fiscal efforts. It is the residuals that give an idea of the plausible pace of improvements in the primary balance going over and above what countries normally do in given circumstances. To gauge the extent of the annual fiscal effort that would be perceived by the markets as feasible, we use the 75th percentile of residuals in the Bohn equation, which is equal to 0.82 % of GDP.

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