A New Keynesian Model with Unemployment

by

Olivier J. Blanchard

and

Jordi Galí

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Two Popular Paradigms

- *The New Keynesian Model*

  - useful tool for monetary policy analysis in the presence of nominal rigidities

  - shortcoming: no attempt to explain unemployment or labor market flows

- *The Search and Matching Model of Labor Market Flows (DMP)*

  - useful tool for the analysis of labor market flows and the effects of policy interventions on unemployment

  - shortcoming: focus on real frictions

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Our Paper

- attempt to integrate both strands of the literature

- tractable framework combining labor market frictions and nominal rigidities

Households

Representative household, continuum of members, $[0, 1]$

$$E_0 \sum \beta^t \left(\log C_t - \chi \frac{{N_t}^{1+\phi}}{1+\phi} \right)$$

where

$$C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$0 \leq N_t \leq 1$$

Budget constraint:

$$\int_0^1 P_t(i)\ C_t(i)\ di + Q_t\ B_t \leq B_{t-1} + P_t W_t\ N_t + \Pi_t$$

Remark: utility specification different from standard DMP model.

Firms

Continuum of firms, each producing a differentiated good, $i \in [0,1]$

Technology:

$$Y_t(i) = A_t \; N_t(i)$$

Employment

$$N_t(i) = (1-\delta) \; N_{t-1}(i) + H_t(i)$$

Labor Market

- Beginning-of-period unemployment (given full participation):

$$U_t = 1 - (1 - \delta)\ N_{t-1}$$

- Aggregate hiring

$$H_t = N_t - (1 - \delta)\ N_{t-1}$$

- Index of *labor market tightness*

$$x_t \equiv \frac{H_t}{U_t} \in [0, 1]$$

Alternative interpretation: job finding rate

- End-of-period unemployment:

$$u_t \equiv 1 - N_t$$

Hiring costs:

- for an individual firm:

$$G_t \; H_t(i)$$

with the cost per hire G_t taken as given.

- aggregate determinant of cost per hire:

$$G_t = A_t \; Bx_t^{\alpha}$$

Implications under staggered price setting

Comparison with DMP Model

Outline

- Constrained Efficient Allocation
- Equilibrium with Flexible Prices

  (i) Nash Bargaining

  (ii) Real Wage Rigidities

- Equilibrium with Sticky Prices

  → Implications for Monetary Policy

Outline

- Constrained Efficient Allocation $\Longrightarrow$ constant unemployment

- Equilibrium with Flexible Prices

  (i) Nash Bargaining $\Longrightarrow$ constant unemployment

  (ii) Real Wage Rigidities $\Longrightarrow$ inefficient unemployment fluctuations

- Equilibrium with Sticky Prices

  $\rightarrow$ Implications for Monetary Policy

  $\Longrightarrow$. emergence of a policy trade-off
  $\Longrightarrow$ role for monetary policy in stabilizing unemployment
  $\Longrightarrow$ partial accommodation of inflation

Models with Labor Market Frictions and Staggered Price Setting

- Examples with Nash Bargaining

  - Chéron-Langot (EL 2000): Tech+MP shocks $\Longrightarrow$ Beveridge + Phillips curves
  - Walsh (RED 05), Trigari (06): NK model + labor market frictions $\Longrightarrow$ greater persistence of effects of MP shocks
  - Andrés-Doménech-Ferri (06): Shimer + sticky prices (+) $\Longrightarrow$ amplified effects of productivity shocks on labor market variables

- Examples with Real Wage Rigidities

  - Blanchard-Galí (05): but no microfounded model of labor market frictions
  - Krause-Lubik (05), Christoffel and Linzert (06): focus on persistence in MP shocks.

Introducing Sticky Prices

Calvo pricing: fraction θ of firms with unchanged prices

Optimal price setting rule:

$$E_t \left\{ \sum_{k=0}^{\infty} \theta^k \; Q_{t,t+k} \; Y_{t+k|t} \; (P_t^* - \mathcal{M} \; P_{t+k} \; MC_{t+k}) \right\} = 0$$

where

$$MC_t = \frac{W_t}{P_t} + Bx_t^{\alpha} - \beta(1-\delta) \; E_t \left\{ \frac{C_t}{C_{t+1}} \; \frac{A_{t+1}}{A_t} \; Bx_{t+1}^{\alpha} \right\}$$

Real Wage Rigidities

Assumed wage schedule:

$$W_t = \Theta \; A_t^{1-\gamma}$$

Limiting cases:

$$\gamma = 1 \qquad \text{(Hall model)}$$
$$\gamma = 0 \qquad \text{(Nash + flexible prices)}$$

Assumptions:

$$\chi C_t N_t^{\phi} \leq W_t \leq \overline{W}_t$$

$\Longrightarrow$ non-forced labor + positive value for all firms

$$W_t > \chi(1 - \delta B) \; A_t$$

$\Longrightarrow$ full participation

$\Longrightarrow$ *involuntary* nature of unemployment

Linearized Equilibrium Dynamics

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \lambda \ \widehat{mc}_t$$

where $\widehat{mc}_t \equiv \log(MC_t/MC)$

Letting $g \equiv Bx^{\alpha}$, $a_t \equiv \log A_t$, and $\Phi \equiv 1 - \mathcal{M}(1 - \beta(1 - \delta))g < 1$

$$\widehat{mc}_t = \alpha g \mathcal{M} \ \widehat{x}_t - \beta(1 - \delta) g \mathcal{M} \ E_t\{(\widehat{c}_t - a_t) - (\widehat{c}_{t+1} - a_{t+1}) + \alpha \ \widehat{x}_{t+1}\} - \Phi\gamma \ a_t$$

$$\delta \ \widehat{x}_t = \widehat{n}_t - (1 - \delta)(1 - x) \ \widehat{n}_{t-1}$$

$$\widehat{c}_t = a_t + \frac{1 - g}{1 - \delta g} \ \widehat{n}_t + \frac{g(1 - \delta)}{1 - \delta g} \ \widehat{n}_{t-1} - \frac{\alpha g}{1 - \delta g} \ \delta \ \widehat{x}_t$$

A Good Working Approximation

Assumption "small" δ and g $\quad (\longrightarrow$ drop terms in $\delta\, \widehat{n}_t$ or $g\, \widehat{n}_t$).

Marginal cost:

$$\widehat{mc}_t = \alpha g \mathcal{M}\ (\widehat{x}_t - \beta\ E_t\{\widehat{x}_{t+1}\}) - \Phi\gamma\ a_t$$

Combined with inflation equation, assuming AR(1) process for productivity:

$$\pi_t = \alpha g \mathcal{M} \lambda\ \widehat{x}_t - \Psi\gamma\ a_t$$

where $\Psi \equiv \lambda\Phi/(1-\beta\rho_a) > 0$.

Letting $\widehat{u}_t \equiv u_t - u$

$$\pi_t = -\kappa\ \widehat{u}_t + \kappa(1-\delta)(1-x)\ \widehat{u}_{t-1} - \Psi\gamma\ a_t$$

where $\kappa \equiv \alpha g \mathcal{M}\lambda/\delta(1-u)$.

wage rigidities $(\gamma > 0) \Longrightarrow$ unemployment/inflation tradeoff (BG 05)

Monetary Policy

Extreme Policy (I): Constant Unemployment

$$\widehat{u}_t = 0$$

$$\pi_t = -\Psi\gamma \; a_t$$

Extreme policy (II): Constant Inflation

$$\pi_t = 0$$

$$\widehat{u}_t = (1-\delta)(1-x) \; \widehat{u}_{t-1} - (\Psi\gamma/\kappa) \; a_t$$

Remark: persistence higher for sclerotic labor markets (low δ, low x)

Optimal Monetary Policy

Central Bank's Problem

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha_u \, \widehat{u}_t^2\right)$$

subject to

$$\pi_t = -\kappa \, \widehat{u}_t + \kappa(1-\delta)(1-x) \, \widehat{u}_{t-1} - \Psi\gamma \, a_t$$

where $\alpha_u \equiv \frac{\lambda(1+\phi)(N^*)^{\phi-1}}{\epsilon} > 0$.

First order conditions:

$$\pi_t = \zeta_t$$

$$\alpha_u \, \widehat{u}_t = \kappa \, \zeta_t - \beta(1-\delta)(1-x)\kappa \, E_t\{\zeta_{t+1}\}$$

Implied "targeting rule"

$$\pi_t = \beta(1-\delta)(1-x)\ E_t\{\pi_{t+1}\} + (\alpha_u/\kappa)\ \widehat{u}_t$$

Equivalently,

$$\pi_t = \left(\frac{\alpha_u}{\kappa}\right)\ \sum_{t=0}^{\infty}(\beta(1-\delta)(1-x))^k\ E_t\{\widehat{u}_{t+k}\}$$

Combined with NKPC,

$$\widehat{u}_t = q\ \widehat{u}_{t-1} + \beta q\ E_t\{\widehat{u}_{t+1}\} - s\ a_t$$

where $q \in (0,1)$ and and $s > 0$.

Stationary solution:

$$\widehat{u}_t = \psi_u\ \widehat{u}_{t-1} - \psi_a\ a_t$$

and

$$\pi_t = \varphi_u\ \widehat{u}_t - \varphi_a\ a_t$$

where $\psi_u \in (0,1)$, $\psi_a > 0$, $\varphi_u > 0$ and $\varphi_a > 0$.

Quantitative Analysis

Calibration

- preferences: $\beta = 0.99 \quad \phi = 1 \quad \epsilon = 6$
- rigidities: $\theta = 2/3 \quad \gamma = 0.5$
- labor markets:

  U.S. : $x = 0.7 \quad u = 0.5 \quad \longrightarrow \quad \delta = ux/((1-u)(1-x)) = 0.12$

  Europe: $x = 0.25 \quad u = 0.1 \quad \longrightarrow \quad \delta = 0.04$

- hiring costs:

  DMP matching function: $H_t = Z\ U_t^{\eta} V_t^{1-\eta}$

  expected cost per hire proportional to $V/H = Z^{\frac{1}{\eta-1}}(H/U)^{\frac{\eta}{1-\eta}}$ (vs $B(H/U)^{\alpha}$)

  $\eta = 0.5 \implies \alpha = 1$

  B : hiring costs 1% of GDP under U.S. calibration.

Impulse Responses

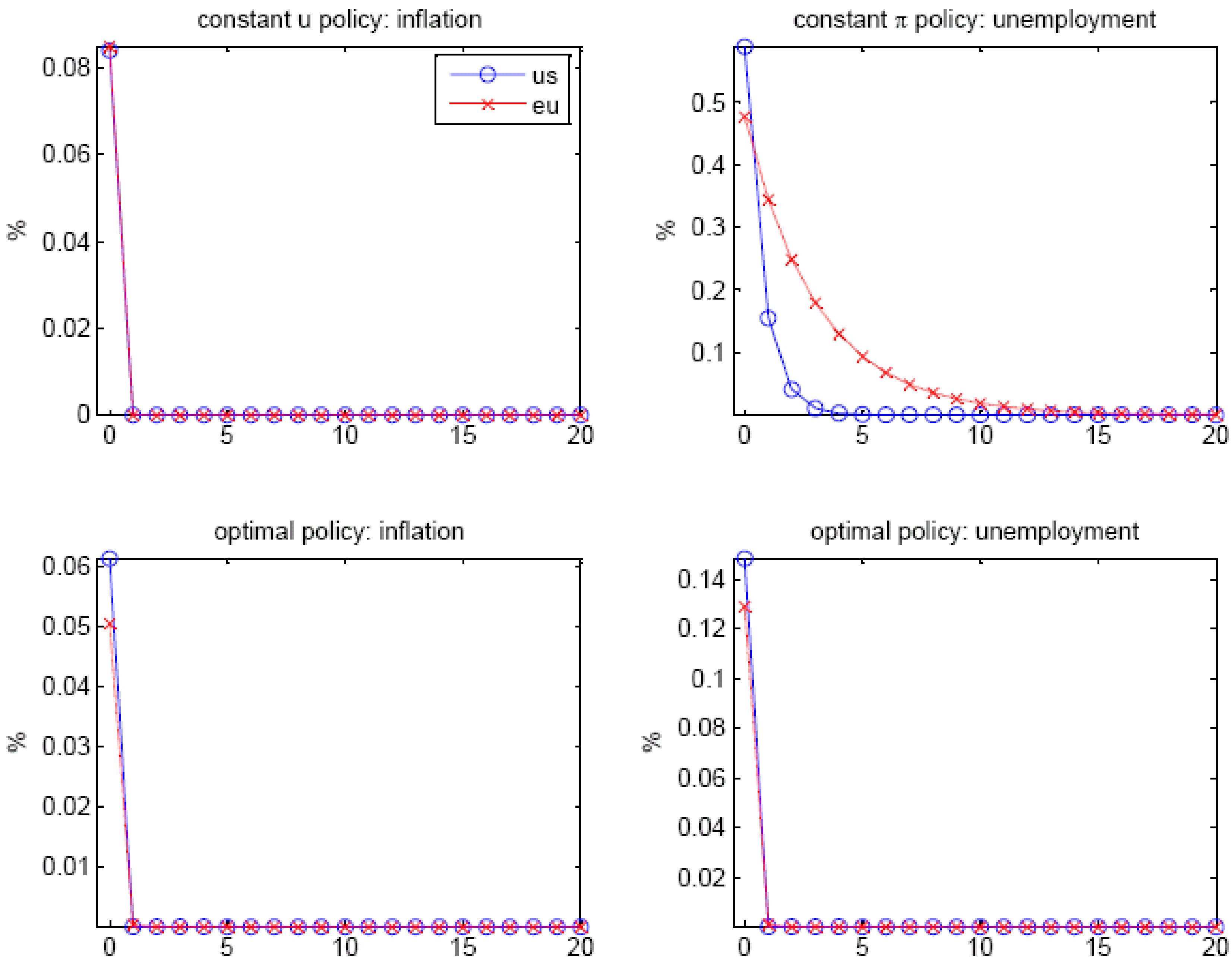

Figure 4. Dynamic Reponses to a Transitory Productivity Shock

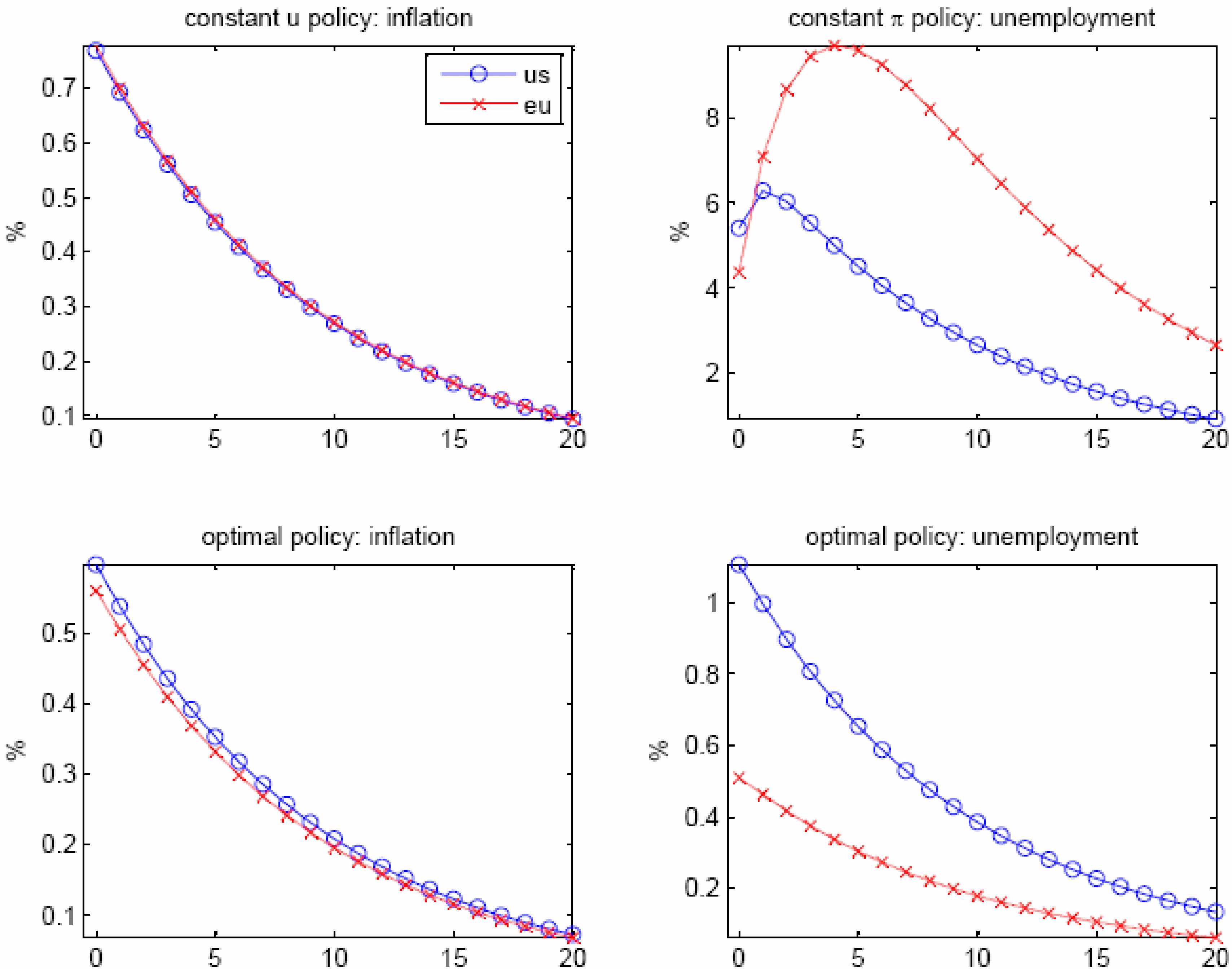

Figure 5. Dynamic Responses to a Persistent Productivity Shock

A Nearly-Optimal Simple Rule

U.S. Calibration

$$i_t = \rho + 1.5\ \pi_t - 0.2\ \widehat{u}_t$$

European Calibration

$$i_t = \rho + 1.5\ \pi_t - 0.6\ \widehat{u}_t$$

Impulse Responses

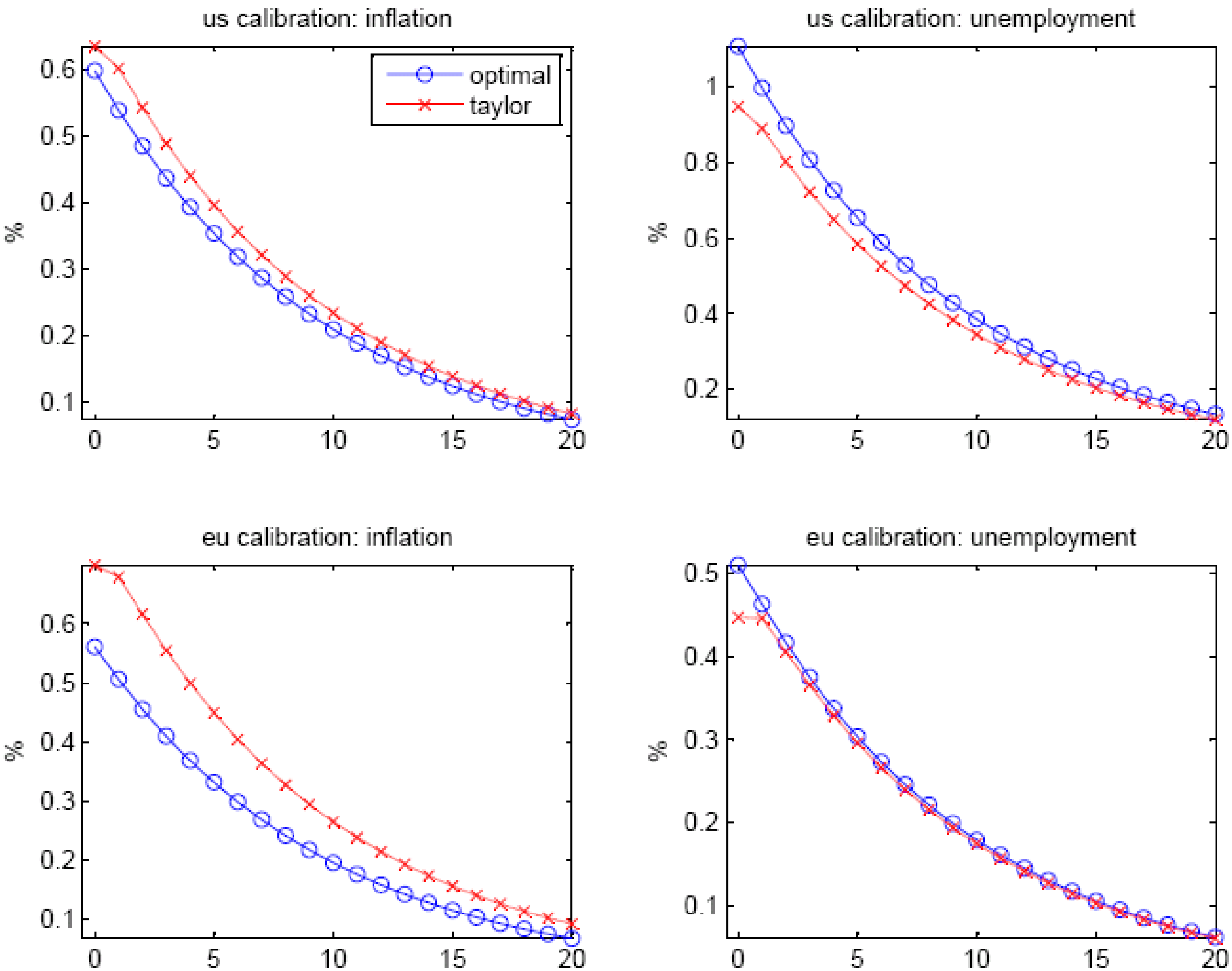

Figure 6. Optimal Policy vs. Taylor-type Rule

Summary and Conclusions

- simple framework to model labor market flows, unemployment, and nominal rigidities

- real wage rigidities: source of excessive fluctuations in unemployment in response to shocks, even in the absence of nominal rigidities

- combined with nominal rigidities: tradeoff between inflation and unemployment instability

- optimal policy: partial accommodation of inflationary pressures, persistent unemployment fluctuations.

Constrained Efficient Allocation

Social planner's problem

$$E_0 \sum \beta^t \left(\log C_t - \chi \frac{N_t^{1+\phi}}{1+\phi} \right)$$

subject to

$$C_t = A_t \ (N_t - Bx_t^{\alpha} \ H_t)$$

Optimality condition (interior solution):

$$\chi C_t N_t^{\phi} = A_t - (1+\alpha) A_t B x_t^{\alpha} + \beta(1-\delta) \ E_t \left\{ \frac{C_t}{C_{t+1}} \ A_{t+1} B \ ((x_{t+1}^{\alpha} - \alpha x_{t+1}^{\alpha}(1 - x_{t+1})) \right\}$$

Interpretation

Solution:

$$N^* = \frac{x^*}{\delta + (1-\delta)x^*} \equiv N(x^*)$$

where $x^* \in (0,1)$ is implicitly determined by:

$$(1 - \delta B x^\alpha)\ \chi N(x)^{1+\phi} \leq 1 - (1 - \beta(1-\delta))(1+\alpha)\ Bx^\alpha - \beta(1-\delta)\alpha\ Bx^{1+\alpha}$$

Implications:

$$C_t^* = A_t\ (1 - \delta B x^{*\alpha}) N^*$$

$$Y_t^* = A_t\ N^*$$

$$u_t^* \equiv 1 - N_t^* = \frac{\delta(1 - x^*)}{\delta + (1-\delta)x^*}$$

Equilibrium under Flexible Prices

Value Maximization (given wage)

$$\max E_t \sum_k Q_{t,t+k} \; [P_{t+k}(i) Y_{t+k}(i) - P_{t+k} W_{t+k} N_{t+k}(i) - P_{t+k} G_{t+k} \; H_{t+k}(i)]$$

subject to

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} (C_t + G_t H_t)$$

where $Q_{t,t+k} \equiv \beta^k \; \frac{C_t}{C_{t+k}} \frac{P_t}{P_{t+k}}$

Optimality condition:

$$P_t(i) = \mathcal{M} \; P_t \; MC_t$$

where

$$\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$$

and

$$MC_t = \frac{W_t}{A_t} + B x_t^{\alpha} - \beta(1 - \delta) \; E_t \left\{ \frac{C_t}{C_{t+1}} \; \frac{A_{t+1}}{A_t} \; B x_{t+1}^{\alpha} \right\}$$

Symmetric Equilibrium

$$MC_t = \frac{1}{\mathcal{M}}$$

$$\frac{W_t}{A_t} = \frac{1}{\mathcal{M}} - Bx_t^{\alpha} + \beta(1-\delta)\ E_t\left\{\frac{C_t}{C_{t+1}}\ \frac{A_{t+1}}{A_t}\ Bx_{t+1}^{\alpha}\right\}$$

Missing element: specification of wage determination

Wage range consistent with positive employment + non-forced labor:

$$\chi C_t N_t^{\phi} \leq W_t \leq \frac{A_t}{\mathcal{M}}$$

Auxiliary assumption:

$$W_t > \chi(1-\delta B)\ A_t$$

$\Longrightarrow$ full participation

$\Longrightarrow$ *involuntary* nature of unemployment

Equilibrium under Flexible Prices (I): Nash Bargaining

Value of an employed member:

$$\mathcal{W}_t^N = W_t - \chi C_t N_t^{\varphi} + \beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[(1 - \delta(1 - x_{t+1}))\, \mathcal{W}_{t+1}^N + \delta(1 - x_{t+1})\, \mathcal{W}_{t+1}^U \right] \right\}$$

Value of an unemployed member:

$$\mathcal{W}_t^U = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[x_{t+1}\, \mathcal{W}_{t+1}^N + (1 - x_{t+1})\, \mathcal{W}_{t+1}^U \right] \right\}$$

$\Longrightarrow$ Household 's surplus from an established relationship

$$\mathcal{W}_t^N - \mathcal{W}_t^U = W_t - \chi C_t N_t^{\varphi} + \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (1 - x_{t+1}) \left(\mathcal{W}_{t+1}^N - \mathcal{W}_{t+1}^U \right) \right\}$$

Firm"s surplus from an established relationship: $\quad G_t$

Nash Bargain

$$\mathcal{W}_t^N - \mathcal{W}_t^U = \vartheta \ G_t$$

Nash wage schedule:

$$\frac{W_t}{A_t} = \frac{\chi C_t N_t^{\varphi}}{A_t} + \ \vartheta B x_t^{\alpha} - \beta(1-\delta) \ E_t \left\{ \frac{C_t}{C_{t+1}} \ \frac{A_{t+1}}{A_t} \ (1 - x_{t+1}) \ \vartheta \ B x_{t+1}^{\alpha} \right\}$$

Equilibrium

$$\frac{\chi C_t N_t^{\phi}}{A_t} = \frac{1}{\mathcal{M}} - (1+\vartheta) \ B x_t^{\alpha} + \beta(1-\delta) \ E_t \left\{ \frac{C_t}{C_{t+1}} \frac{A_{t+1}}{A_t} \ B(x_{t+1}^{\alpha} + \vartheta x_{t+1}^{\alpha}(1 - x_{t+1}^{\alpha})) \right\}$$

Solution:

$$N^{nb} = \frac{x^{nb}}{\delta + (1-\delta)x^{nb}} \equiv N(x^{nb})$$

where x^{nb} is implicitly given by:

$$\chi(1-\delta B x^{\alpha})\ N(x)^{1+\phi} = \frac{1}{\mathcal{M}} - (1-\beta(1-\delta))\ (1+\vartheta)\ Bx^{\alpha} - \beta(1-\delta)\vartheta\ Bx^{\alpha}$$

$$C_t^{nb} = A_t\ (1-\delta B(x^{nb})^{\alpha})\ N^{nb} \qquad ; \qquad Y_t^{nb} = A_t\ N^{nb}$$

$$u_t^* = \frac{\delta(1-x^{nb})}{\delta + (1-\delta)x^{nb}}$$

$$\frac{W_t^{nb}}{A_t} = \frac{1}{\mathcal{M}} - (1-\beta(1-\delta))\ B(x^{nb})^{\alpha}$$

$\Longrightarrow$ Constant unemployment independently of α and ϑ (vs. DMP model).

Efficiency of Equilibrium under Nash Bargaining

$$\mathcal{M} = 1$$

$$\vartheta = \alpha$$

Equilibrium under Flexible Prices (II): Real Wage Rigidities

Assumed wage schedule:

$$W_t = \Theta \ A_t^{1-\gamma}$$

Limiting cases:

$$\begin{array}{ll} \gamma = 1 & \text{(Hall)} \\ \gamma = 0 & \text{(Nash)} \end{array}$$

Equilibrium dynamics

$$\Theta \ A_t^{-\gamma} = \frac{1}{\mathcal{M}} - Bx_t^{\alpha} + \beta(1-\delta) \ E_t \left\{ \frac{C_t}{C_{t+1}} \ \frac{A_{t+1}}{A_t} \ Bx_{t+1}^{\alpha} \right\}$$

Solving forward,

$$Bx_t^{\alpha} = \sum_{k=0}^{\infty}(\beta(1-\delta))^k \; E_t \left\{ \Lambda_{t,t+k} \; \left(\frac{1}{\mathcal{M}} - \Theta \; A_{t+k}^{-\gamma} \right) \right\}$$

where $\Lambda_{t,t+k} \equiv \frac{C_t}{C_{t+k}} \; \frac{A_{t+k}}{A_t}$

$\Longrightarrow$ inconsistent with constant unemployment

Using approximation discussed later:

$$\widehat{u}_t = (1-\delta)(1-x) \; \widehat{u}_{t-1} - (\Psi\gamma/\kappa) \; a_t$$

$\Longrightarrow$ inefficient fluctuations in activity, even in the absence of nominal rigidities