The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

Glenn D. Rudebusch   Eric T. Swanson

Economic Research
Federal Reserve Bank of San Francisco

Conference on Monetary Policy and Financial Markets
National Bank of Belgium
October 16, 2008
Outline

1. Motivation and Background
2. A DSGE Model with Epstein-Zin Preferences
3. Long-Run Risks
4. Conclusions
Why Study Asset Prices in a DSGE Model?

Asset pricing is important:

- DSGE models increasingly used for policy analysis; total failure to explain asset prices may signal flaws in the model
- many empirical questions about asset prices require a structural DSGE model to provide reliable answers
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Equity prices have received much attention in the literature But bond prices are at least as interesting because they:

- apply to a larger amount of securities
- provide an additional perspective on the model
- test nominal rigidities in the model
- model short-term interest rate process, not dividends
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The equity premium puzzle: excess returns on stocks are much larger (and more variable) than can be explained by standard preferences in an RBC model (Mehra and Prescott, 1985).

The bond premium puzzle: excess returns on long-term bonds are much larger (and more variable) than can be explained by standard preferences in an RBC model (Backus, Gregory, and Zin, 1989).

Note:
- Since Backus, Gregory, and Zin (1989), DSGE models with nominal rigidities have advanced considerably
Recent Studies of the Bond Premium Puzzle

- Wachter (2005)
  - can resolve bond premium puzzle using Campbell-Cochrane preferences in endowment economy
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We examine to what extent the Piazzesi-Schneider results generalize to the DSGE model and a production economy
Our Analysis

We incorporate Epstein-Zin preferences in standard DSGE model

The model has three key ingredients:

1. Intrinsic nominal rigidities
   - makes bond pricing interesting

2. Epstein-Zin preferences
   - makes households risk averse

3. Long-run risk (productivity or inflation)
   - introduces a risk households cannot offset
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Can we match the unconditional moments of both bond prices and macroeconomic variables?
Related Strands of the Literature

The Bond Premium in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in an Endowment Economy:

Epstein-Zin Preferences in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in a DSGE Model:
A DSGE Model with Epstein-Zin Preferences

- Standard Preferences
- Epstein-Zin Preferences
- Firms and Government
- Bond Pricing and Measures of the Bond Premium
- Results
Standard Preferences

Representative household with preferences:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \right)$$
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Stochastic discount factor (nominal):

$$m_{t+1} = \frac{\beta(C_{t+1} - bC_t)^{-\gamma}}{(C_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}$$
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Parameters: $\beta = .99$, $b = .66$, $\gamma = 2$, $\chi = 1.5$
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\[ V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1} \]
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Epstein-Zin stochastic discount factor (nominal):

\[ m_{t,t+1} \equiv \left. \frac{\beta u_1}{u_1} \right|_{(c_{t+1}, l_{t+1})} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^\alpha \frac{P_t}{P_{t+1}} \]
Firms and Government

Continuum of differentiated firms:
- face Dixit-Stiglitz demand with elasticity $\frac{1+\theta}{\theta}$, markup $\theta$
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions: $y_t = A_t \bar{k}^{1-\eta} l_t^{\eta}$
- have firm-specific capital stocks
- face aggregate technology: $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters $\theta = .2$, $\rho_A = .9$, $\sigma_A^2 = .01^2$

Perfectly competitive goods aggregation sector
Firms and Government

Government:
- imposes lump-sum taxes $G_t$ on households
- destroys the resources it collects
- $\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \epsilon_t^G$

Parameters $\bar{G} = .17 \bar{Y}$, $\rho_G = .9$, $\sigma_G^2 = .004^2$
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Monetary Authority:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [1/\beta + \pi_t + g_y(y_t - \bar{y}) + g_\pi(\bar{\pi}_t - \pi^*)] + \varepsilon^i_t$$

Parameters $\rho_i = .73$, $g_y = .53$, $g_\pi = .93$, $\pi^* = 0$, $\sigma^2_i = .004^2$
Bond Pricing

Pricing of any nominal asset:

\[ p_t = d_t + E_t[m_{t+1}p_{t+1}] \]
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Zero-coupon nominal bond pricing:

\[ p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}] \]

\[ i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)} \]

Notation: let \( i_t \equiv i_t^{(1)} \)
The Term Premium
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In DSGE framework, convenient to work with a default-free *consol*, a perpetuity that pays $1, \( \delta_c \), \( \delta_c^2 \), \( \delta_c^3 \), \ldots (nominal)
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Price of the consol:

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\tilde{p}_t^{(n)} = 1 + \delta_c E_t m_{t+1} \tilde{p}_{t+1}^{(n)}
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Term premium:

\[
\psi_t^{(n)} \equiv \log \left( \frac{\delta_c \tilde{p}_t^{(n)}}{\tilde{p}_t^{(n)} - 1} \right) - \log \left( \frac{\delta_c \hat{p}_t^{(n)}}{\hat{p}_t^{(n)} - 1} \right)
\]
Solving the Model

We solve the model by perturbation methods.
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- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a *third*-order approximation of the solution around nonstochastic steady state
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The model has a relatively large number of state variables: \( C_{t-1}, A_{t-1}, G_{t-1}, i_{t-1}, \Delta_{t-1}, \bar{\pi}_{t-1}, \varepsilon_t^A, \varepsilon_t^G, \varepsilon_t^i \).

It is difficult to solve, impossible to estimate
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We examine unconditional moments of standard parameters
We also search for over parameter space for the “best fit” set, which minimizes the average deviation of 13 moments
### Definitions of Unconditional Moments Matched

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$sd[C]$</td>
<td>Real consumption*</td>
</tr>
<tr>
<td>$sd[L]$</td>
<td>Labor, total hours worked*</td>
</tr>
<tr>
<td>$sd[w']$</td>
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<td>$sd[\pi]$</td>
<td>Price inflation, Annualized quarterly rate</td>
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<td>$sd[i^{(10)}]$</td>
<td>10-year zero-coupon nominal rate, annualized p.p.</td>
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<td>mean[$\psi^{(10)}$]</td>
<td>Term premium on 10-year zero-coupon bond (affine no-arbitrage estimates)</td>
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<td>Yield curve slope (long - short rate, annualized p.p.)</td>
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<td>mean[$i^{(10)} - i$]</td>
<td>Quarterly excess holding period return (10-year bond, annualized p.p.)</td>
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*deviations from HP trend in percentage points
### Table 2: Empirical and Model-Based Moments

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\]

This is the CRRA \textit{if} labor were held fixed and \textit{if} all shocks were multiplicative with respect to wealth.

Better measures of risk aversion (e.g., thought experiments) are likely to look less risk-averse than the quasi-CRRA would suggest:
- households can self-insure risk by varying labor supply
Long-Run Risks

- Long-Run Real Risk
- Long-Run Inflation Risk
Long-Run Productivity Risk

Following Bansal and Yaron (2004), introduce long-run real risk to make the economy more risky:

Assume productivity follows:

\[ \log A_t^* = \rho_{A^*} \log A_{t-1}^* + \varepsilon_t^{A^*} \]

\[ \log A_t = \log A_t^* + \varepsilon_t^A \]

where \( \rho_{A^*} = .98 \), \( \sigma_{A^*} = .002 \), and \( \sigma_A = .005 \).

- makes the economy much riskier to agents
- increases volatility of stochastic discount factor
### Table 3: Moments with Long-Run Productivity Risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data 1961-2007</th>
<th>EU Preferences</th>
<th>Best Fit EZ Prefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>1.19</td>
<td>0.92</td>
<td>2.95</td>
</tr>
<tr>
<td>sd[L]</td>
<td>1.71</td>
<td>1.03</td>
<td>1.32</td>
</tr>
<tr>
<td>sd[w']</td>
<td>0.82</td>
<td>1.43</td>
<td>1.90</td>
</tr>
<tr>
<td>sd[π]</td>
<td>2.52</td>
<td>1.12</td>
<td>3.14</td>
</tr>
<tr>
<td>sd[i]</td>
<td>2.71</td>
<td>1.17</td>
<td>2.88</td>
</tr>
<tr>
<td>sd[r]</td>
<td>2.30</td>
<td>0.66</td>
<td>1.35</td>
</tr>
<tr>
<td>sd[i(10)]</td>
<td>2.41</td>
<td>0.65</td>
<td>1.84</td>
</tr>
<tr>
<td>mean[ψ(10)]</td>
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<td>.005</td>
<td>.872</td>
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<tr>
<td>sd[ψ(10)]</td>
<td>0.54</td>
<td>.000</td>
<td>.183</td>
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<td>mean[i(10) − i]</td>
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<td>−.018</td>
<td>.758</td>
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<td>sd[i(10) − i]</td>
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<td>0.64</td>
<td>1.15</td>
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<tr>
<td>mean[x(10)]</td>
<td>1.76</td>
<td>.005</td>
<td>.859</td>
</tr>
<tr>
<td>sd[x(10)]</td>
<td>23.43</td>
<td>4.39</td>
<td>11.59</td>
</tr>
</tbody>
</table>

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Long-Run Inflation Risk

Introduce long-run inflation risk to make long-term bonds more risky:

- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary
Motivation

DSGE Model with EZ Preferences

Long-Run Risks

Conclusions

Long-Run Inflation Risk

Fig. 1 10-year Treasury bond yield and inflation expectations

- 10-year zero-coupon yield
- Survey-based 10-year inflation expectations
Suppose:

\[ \pi_t^* = \rho_{\pi} \pi_{t-1}^* + \varepsilon_t^* \]
Long-Run Inflation Risk

Suppose:

$$\pi_t^* = \rho_{\pi} \pi_{t-1}^* + \varepsilon_t^*$$

Then:

- inflation is volatile, but not risky
- in fact, long-term bonds act like insurance:
  - when $\pi^* \uparrow$, then $C \uparrow$ and $p^{(10)} \downarrow$
- result: term premium is negative
Consider instead:

\[ \pi_t^* = \rho_{\pi} \pi_{t-1} + (1 - \rho_{\pi}) \theta_{\pi}^* (\pi_t^* - \pi_t^*) + \varepsilon_t^* \]

Gürkaynak, Sack, and Swanson (2005) found evidence for \( \theta_{\pi}^* > 0 \) in U.S. bond response to macro data releases makes long-term bonds act less like insurance: when technology/supply shock, then \( \pi \uparrow, C \downarrow, \) and \( p_{(10)} \downarrow \) supply shocks become very costly. The term premium is positive, closely associated with \( \theta_{\pi}^* \).
Consider instead:

\[ \pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta_{\pi}^* (\bar{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*} \]

- \( \theta_{\pi}^* \) describes pass-through from current \( \pi \) to long-term \( \pi^* \)
- Gürkaynak, Sack, and Swanson (2005) found evidence for \( \theta_{\pi}^* > 0 \) in U.S. bond response to macro data releases
- makes long-term bonds act less like insurance:
  when technology/supply shock, then \( \pi \uparrow, C \downarrow, \) and \( p^{(10)} \downarrow \)
  supply shocks become very costly
- The term premium is *positive*, closely associated with \( \theta_{\pi}^* \)
## Table 4: Moments with Long-Run Inflation Risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data 1961-2007</th>
<th>EU Preferences</th>
<th>Best Fit EZ Prefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd[C]</td>
<td>1.19</td>
<td>1.92</td>
<td>1.86</td>
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<tr>
<td>sd[L]</td>
<td>1.71</td>
<td>3.33</td>
<td>1.73</td>
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<tr>
<td>sd[w']</td>
<td>0.82</td>
<td>2.55</td>
<td>1.45</td>
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<tr>
<td>sd[π]</td>
<td>2.52</td>
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<tr>
<td>sd[i]</td>
<td>2.71</td>
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<td>2.99</td>
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<td>sd[r]</td>
<td>2.30</td>
<td>2.61</td>
<td>1.48</td>
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<tr>
<td>sd[i(10)]</td>
<td>2.41</td>
<td>3.32</td>
<td>1.94</td>
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<td>.748</td>
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<tr>
<td>sd[ψ(10)]</td>
<td>0.54</td>
<td>.001</td>
<td>.431</td>
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<td>mean[i(10) − i]</td>
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</tr>
<tr>
<td>sd[i(10) − i]</td>
<td>1.33</td>
<td>1.60</td>
<td>1.11</td>
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<tr>
<td>mean[x(10)]</td>
<td>1.76</td>
<td>.003</td>
<td>.737</td>
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<tr>
<td>sd[x(10)]</td>
<td>23.43</td>
<td>16.96</td>
<td>11.83</td>
</tr>
</tbody>
</table>

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Conclusions

1. Epstein-Zin preferences appear to solve bond premium puzzle in DSGE model, as in an endowment economy: agents are risk-averse and cannot offset long-run real or nominal risks.
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Conclusions

1. Epstein-Zin preferences appear to solve bond premium puzzle in DSGE model, as in an endowment economy: agents are risk-averse and cannot offset long-run real or nominal risks.

2. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments.

3. Unresolved issues:
   - Reliance on technology shocks, not $\pi^*$ shocks
   - Fitting more moments, estimation from data
   - Is quasi-CRRA appropriate measure of risk aversion?
   - Little feedback from asset prices to economy