A Macroeconomic Model with a Financial Sector

Markus K. Brunnermeier and Yuliy Sannikov
Goals

• Build a model capable of generating crises
  – a regime different from normal, with high endogenous risk, asset missallocation

• Understanding the resilience of the financial system
  – frequency of crises, level of endogenous risk, speed of recovery
  – role of asset liquidity (market, technological), leverage, asset price level, financial innovations

• How does the system respond to various policies? How do policies affect spillovers/welfare?
  – policies often have unintended consequences, the model finds some of those
Financial Accelerator Models

• Bernanke-Gertler (1989)
  – Temporary shocks can have **persistent** effect on the economy
  – Constrained borrowers (**experts**) need time to rebuild net worth

  – Shocks are **amplified** through leverage and prices

• Kocherlakota (2000): unanticipated shocks near the steady state result in low amplification
  – When an unanticipated shock hits, the system **for sure** recovers, but may be temporarily constrained
Full dynamics

• Agents anticipate shocks
  – map out the path to the worst states, and use backward induction
• Uncertainty (recovery vs. getting trapped in a depressed regime)
  → huge amplification
  → endogenous risk
  → precautionary behavior
• Agents maintain net worth buffers away from this uncertainty
  – low endogenous risk in the normal regime
  – but an unusually large shock can puts the system in crisis.
• Semi-stable stochastic steady state, but volatile crisis regime
Results

• Dynamics
  – nonlinearity (small vs. large shocks)
  – stationary distribution U-shaped (system gets trapped in bad states)
  – asset prices: correlation in crises, fat tails

• Comparative Statics
  – lower exogenous risk “volatility paradox”
  – better hedging/risk management higher endogenous risk
  – technological / market / funding liquidity and endogenous risk

• Regulation
  – effect on entire dynamics, not just after crisis happens
  – unintended consequences

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• Capital structure/financial frictions

• Prices/collateral values
  – Shleifer-Vishny (1992), Geanakoplos (1997), Brunnermeier-Pedersen

• Infinite-horizon, log-linearization
  – KM, BGG, Carlstrom-Fuerst (1997), Christiano-Eichenbaum-Evans,
    Gertler-Kiyotaki, Brunnermeier-Eisenbach-Sannikov (survey)

• No log-linearization
  – Basak-Cuoco (1998), Mendoza (2010), He-Krishnamurthy (2012a,b)
Basic Model: Technology

\[ \text{Experts: } \delta \geq \delta, \ a \leq a \]

Output: \((a - \ell_t) k_t\)

Investment \(\ell_t\) creates new capital at rate \(\Phi(\ell_t) k_t\)

\[ dk_t = (\Phi(\ell_t) - \delta) k_t \, dt + \sigma k_t \, dZ_t \]

\[ \text{Less productive households: } \]

Output: \((a - \ell_t) k_t\)

Investment \(\ell_t\) creates new capital at rate \(\Phi(\ell_t) k_t\)

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## Basic Model: Preferences

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Basic Model: Financial Frictions

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Basic Model: Asset Markets

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Output $(a - I_t) k_t$

Investment $I_t$ creates new capital at rate $\Phi(I_t) k_t$

$dk_t = (\Phi(I_t) - \delta) k_t \, dt + \sigma k_t \, dZ_t$

risk-neutral, discount rate $\rho$ consumption must be $\geq 0$

may issue only risk-free debt + solvency constraint

Liquid markets for capital $k_t$ with endogenous price per unit $q_t$

$\frac{dq_t}{q_t} = \mu_t^q \, dt + \sigma_t^q \, dZ_t$

$\rho > r$

Output $(a - I_t) k_t$

Investment $I_t$ creates new capital at rate $\Phi(I_t) k_t$

$dk_t = (\Phi(I_t) - \delta) k_t \, dt + \sigma k_t \, dZ_t$

risk-neutral, discount rate $r$ may consume negatively

financially unconstrained
First Best and Autarky

- **First-best:**
  - experts manage capital forever
  - consume entire net worth at $t = 0$
  - issue equity to less productive households
  - price of capital

$$q = \max_i \frac{a - \iota}{r - \Phi(i) + \delta}$$

- **Autarky:**
  - households manage capital forever
  - price of capital

$$\bar{q} = \max_i \frac{a - \iota}{r - \Phi(i) + \delta}$$

The difference is market illiquidity.
Capital gains/risk

\[ \frac{dk_t}{k_t} = (\Phi(l_t) - \delta) \, dt + \sigma \, dZ_t \]

\[ \frac{dq_t}{q_t} = \mu_t^q \, dt + \sigma_t^q \, dZ_t \quad \leftarrow \text{endogenous} \]

\[ \frac{d(k_t q_t)}{(k_t q_t)} = (\Phi(l_t) - \delta + \mu_t^q + \sigma \sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t \]
Capital gains/risk

dk_t/k_t = (Φ(l_t) - δ) dt + σ dZ_t

dq_t/q_t = μ_t^q dt + σ_t^q dZ_t \quad \text{← endogenous}

d(k_tq_t)/(k_tq_t) = (Φ(l_t) - δ + μ_t^q + σσ_t^q) dt + (σ_t^q + σ) dZ_t

risk
Capital gains/risk

dk_t/k_t = (\Phi(i_t) - \delta) \, dt + \sigma \, dZ_t

dq_t/q_t = \mu_t^q \, dt + \sigma_t^q \, dZ_t

d(k_tq_t)/(k_tq_t) = (\Phi(i_t) - \delta + \mu_t^q + \sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t

risk
Return from investing in capital

\[ \frac{d k_t}{k_t} = (\Phi(i_t) - \delta) \, dt + \sigma \, dZ_t \]

\[ \frac{d q_t}{q_t} = \mu_t^q \, dt + \sigma_t^q \, dZ_t \]

\[ \frac{d(k_t q_t)}{(k_t q_t)} = (\Phi(i_t) - \delta + \mu_t^q + \sigma \sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t \]

\[ dr_t^k = (a - i_t)/q_t \, dt + (\Phi(i_t) - \delta + \mu_t^q + \sigma \sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t \]

\[ \max_i \Phi(i) - i/q_t \]

adjustment costs in \( \Phi \rightarrow \)
technological illiquidity

risk

dividend yield
capital gains
Return from expert portfolio

\[ \frac{dk_t}{k_t} = (\Phi(l_t) - \delta) \, dt + \sigma \, dZ_t \]

\[ \frac{dq_t}{q_t} = \mu_t^q \, dt + \sigma_t^q \, dZ_t \]

\[ \frac{d(k_tq_t)}{(k_tq_t)} = (\Phi(l_t) - \delta + \mu_t^q + \sigma\sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t \]

\[ dr_t^k = \left(a - l_t\right)/q_t \, dt + (\Phi(l_t) - \delta + \mu_t^q + \sigma\sigma_t^q) \, dt + (\sigma_t^q + \sigma) \, dZ_t \]

\[ \frac{dn_t}{n_t} = x_t \, dr_t^k + (1 - x_t) \, r \, dt - dc_t/n_t \]
Equilibrium Definition

• Equilibrium is a map

histories of shocks \{Z_s, s \leq t\} \rightarrow \text{prices } q_t, \text{ allocations}

(of capital } \psi_t, \text{ risk-free asset, consumption)

s.t.
• experts, HH solve optimal consumption/portfolio choice (capital vs. risk-free asset) problems (Merton problem)
• markets clear
Equilibrium Characterization

• Equilibrium is a map

histories of shocks \( \{Z_s, s \leq t\} \)

prices \( q_t \), allocations
(of capital \( \psi_t \), risk-free asset, consumption)

wealth distribution:
fraction \( \eta_t \in (0, 1) \) owned by experts

• since experts are impatient, they consume all net worth when \( \eta_t > \eta^* \) ← endogenous, stochastic steady state
• experts hold all capital when \( \eta_t \) is near \( \eta^* \)
Results

- **Dynamics**
  - nonlinearity (small vs. large shocks)
  - stationary distribution \(\cup\) - shaped (system gets trapped in bad states)
  - asset prices: correlation in crises, fat tails

- **Comparative Statics**
  - lower exogenous risk → "volatility paradox"
  - better hedging/risk management → higher endogenous risk
  - technological / market / funding liquidity and endogenous risk

- **Regulation**
  - effect on entire dynamics, not just after crisis happens
  - unintended consequences

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Example: \( r = 5\%, \ \rho = 5.2\%, \ a = 11\%, \ \alpha = 10\%, \ \delta = 5\%, \ \delta = 6\%, \]
\( \sigma = 10\% \), \( \Phi(\eta) = \frac{(1 - 2\kappa \eta^{1/2} - 1)}{\kappa} \) with \( \kappa = 2 \) (quadratic adj. costs)
Properties of Equilibrium

Inefficiencies: (1) capital misallocation, (2) underinvestment, (3) consumption distortion

Amplification: depends on $q'(\eta)$
  - absent near $\eta^*$, $q'(\eta^*) = 0$
  - high below $\eta^*$

Endogenous risk

$$\sigma_q^t = \frac{q'(\eta)}{q(\eta)} \frac{(\psi_t - \eta_t)\sigma}{1 - (\psi_t - \eta_t)\frac{q'(\eta)}{q(\eta)}}$$
Endogenous Risk and Stationary Density

**Proposition.** Let $\kappa = (a - \bar{a})/q + \delta - \delta$ (market illiquidity). If $2(\rho - r)\sigma^2 < \kappa^2$, stationary density exists, converges to $\infty$ as $\eta \to 0$. If not, the system gets stuck near $\eta = 0$ in the long run (no stationary density).
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Comparative Statics: $\sigma$

- As exogenous risk $\sigma$ falls, does endogenous risk $\sigma^q$ also fall?
Comparative Statics: $\sigma$

- As exogenous risk $\sigma$ falls, does endogenous risk $\sigma^q$ also fall?
- No. $\max \sigma^q$ can actually rise as $\sigma$ falls - the volatility paradox
- Endogenous risk does not go away because as $\sigma$ falls, leverage increases (significantly) and price $q$ in boom rises
- **Proposition.** As $\eta \to 0$, $\sigma^\eta \to \kappa/\sigma + O(\sigma)$
- Generally, $\sigma^q$ and risk premia in crisis are not sensitive to $\sigma$
What matters for endogenous risk?

• If exogenous risk $\sigma$ has little effect on maximal endogenous risk or risk premia, than what does?
Comparative Statics: Liquidity

- **Technological illiquidity**: adjustment costs in function $\Phi$, ability to disinvest
- **Market illiquidity**: difference between first and second-best uses of assets (between $a$ and $\bar{a}$, $\delta$ and $\bar{\delta}$)
- **Funding illiquidity**: ease with which funding can dry up. Short-term debt (in the model so far) has the worst funding liquidity. Long-term debt, equity are a lot better.
Technological Liquidity

$r = 5\%, \ \rho = 5.2\%, \ \sigma = 10\%, \ \text{adj. cost parameter} \ \kappa = 1, 2, 4$
Market Liquidity: changing $a$ (and $q$)

$a = 0.1, \delta = 0.05, \bar{\delta} = 0.06$

Graphs showing the relationship between $q$ and $\eta$ for different values of $a$. The graphs illustrate how expert expected returns $E'[\eta]$ change with $\eta$. The plots display the leverages and expert expected returns for various $\eta$ values.
Idiosyncratic Poisson shocks cause losses to individual experts that need to be verified (Townsend (1979))

$$dk_t^i = (\Phi(i_t) - \delta) k_t^i dt + \sigma k_t^i dZ_t + k_t^i dJ_t^i$$

Debt no longer risk-free, experts pay a credit spread

$$E[dn_t/n_t] = x_t E[dr_t^k] + (1 - x_t) (r + \Lambda(x_t)) dt - dc_t/n_t$$

Comparative Statics: Borrowing Costs

[Diagram with annotations: compensated (mean 0) process, spread due to verification costs]
Borrowing Costs $\Lambda(x) = \xi(x-1)$, $\xi = 0, .01$

$r = 5\%$, $\rho = 5.2\%$, $a = 11\%$, $\alpha = 7\%$, $\delta = 5\%$, $\delta = 6\%$, $\sigma = 10\%$, $\Phi(i) = ((1 - 2i)^{1/2} - 1)$
Risk Management to Reduce Borrowing Costs

• **Proposition.** If experts can hedge idiosyncratic shocks among each other, the solution becomes identical to that with no shocks.

• Thus, while hedging reduces inefficiencies (costly verification), it leads to higher endogenous risk and greater likelihood of crisis.
Deterministic vs. Stochastic Steady State

• **Deterministic steady state (BGG, KM):** stationary point of an economy without shocks

• **Proposition.** With borrowing costs $\Lambda(x)$, deterministic steady state $\eta^0$ is characterized by

$$\rho - r = (1 - \eta^0)/(\eta^0)^2 \Lambda'(1/\eta^0) + \Lambda(1/\eta^0)$$

• $\eta^0 \to 0$ as verification costs go to 0.

• **Stochastic steady state:** point where the system stays in place in the absence of shocks, in an economy with anticipated shocks (it is $\eta^*$)

*Deterministic steady state ≠ stochastic steady state as $\sigma \to 0$*
Economy as $\sigma \to 0$: 
Kocherlakota (2000) Critique does not apply

- Unique unanticipated shocks produce little amplification
- Following shock, price recovers for sure, so it drops little
  - if market knows that the recovery is for sure, there is enough demand even if prices drop by a little
- But, fully anticipated shocks can produce a lot of amplification (price may drop further a lot more)
- In fact, as $\sigma \to 0$, amplification is infinite!
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Policies

• “Micromanaging”
  – Proposition: If a regulator fully controls asset allocation, investment and consumption, subject to resource constraints, based on public information in the market, first-best can be attained

• Capital requirements/leverage bounds
  – similar to borrowing costs (but more crude)
  – cost: asset misallocation; benefit: crisis less likely

• Restriction on dividends/payouts
  – reduces crisis probability
  – but stimulates prices, i.e. crises become worse

• Recapitalization in downturns/price floor
  – improves funding/market liquidity
  – can be decentralized, with freely traded insurance contracts
  – low exogenous, high endogenous risk \(\Rightarrow\) low cost to improve welfare
Policy: Restriction on Payouts

restrict payouts until $\eta = 0.55$

-no policy-

stationary density

0.02 0.04 0.06 0.08
0 0.2 0.4 0.6

stationary density

0 0.5 1 1.5 2
0 0.2 0.4 0.6

time to reach (from $\eta^*$)

0 100
0 0.1 0.2 0.3 0.4

$q$

0 1 1.1
0 0.2 0.4 0.6

$q$

0.8 0.9 1 1.1
0 0.2 0.4 0.6

$\eta$

$\eta$

$\eta$

$\eta$

Policy: Restriction on Payouts

• This policy
  – improves experts’ net worth buffers
  – reduces frequency of crisis, time spent in depressed regimes
  – stimulates prices, so worse endogenous risk in crisis
  – generally reduces welfare within model, but can improve welfare if there are spillovers
Recapitalizing experts at $\eta = .02$, $\sigma = 3\%$
But with $\sigma = 10\%$, less impressive effect
Policy: Recapitalization

- This policy
  - works particularly well with low exogenous risk, potentially high endogenous risk, effectively by improving **funding liquidity**
  - may not reduce the frequency of firesales (endogenous leverage), but reduces time spent in deeply depressed states
  - improves welfare within the model
  - creates little moral hazard if recapitalization is proportional to net worth, i.e. it benefits cautious experts more than risk-takers
  - can be implemented through free trading of insurance securities (rather than an explicit bailout)
  - price support policy has similar effects
Conclusion

- Continuous time offers a powerful methodology to analyze heterogeneous-agent models with financial frictions.
- System dynamics: normal times (low amplification) different from crisis times (high amplification/risk premia, correlated asset prices).
- Endogenous risk-taking leads to paradoxes:
  - diversification opportunities, hedging instruments, lower exogenous risk may lead to higher endogenous risk in crises.
- Regulation:
  - model offers a laboratory to study the effects of policies.
  - important, because many policies have unexpected consequences.
Thank you!