A Macroeconomic Model with a Financial Sector

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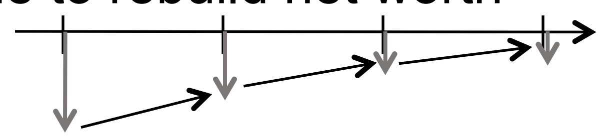
Goals

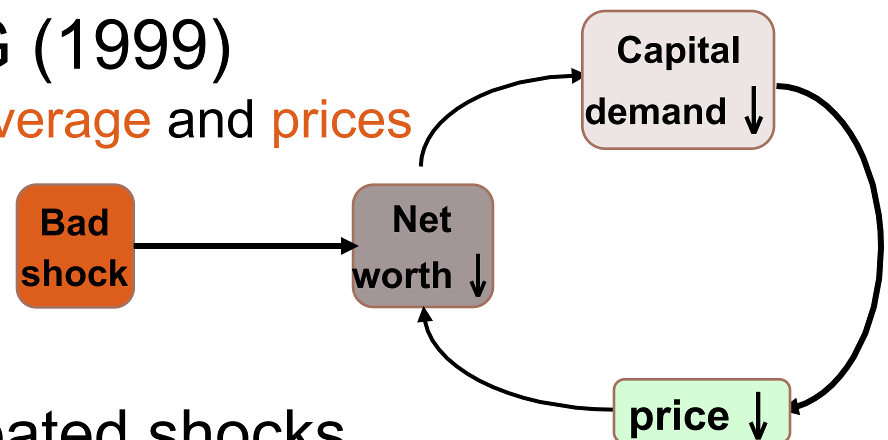
- Build a model capable of generating **crises**
  - a regime different from normal, with high endogenous risk, asset missallocation
- Understanding the resilience of the financial system
  - frequency of crises, level of endogenous risk, speed of recovery
  - role of asset liquidity (market, technological), leverage, asset price level, financial innovations
- How does the system respond to various policies? How do policies affect spillovers/welfare?
  - policies often have unintended consequences, the model finds some of those

Financial Accelerator Models

- Bernanke-Gertler (1989)
  - Temporary shocks can have **persistent** effect on the economy
  - Constrained borrowers (**experts**) need time to rebuild net worth



- Kiyotaki-Moore (1997), BGG (1999)
  - Shocks are **amplified** through leverage and prices



- Kocherlakota (2000): unanticipated shocks near the steady state result in low amplification
  - When an unanticipated shock hits, the system **for sure** recovers, but may be temporarily constrained

Full dynamics

- Agents anticipate shocks
  - map out the path to the worst states, and use backward induction
- Uncertainty (recovery vs. getting trapped in a depressed regime)
  - → huge amplification
  - → endogenous risk
  - → precautionary behavior
- Agents maintain net worth buffers away from this uncertainty
  - low endogenous risk in the normal regime
  - but an unusually large shock can puts the system in crisis.
- Semi-stable stochastic steady state, but volatile crisis regime

Results

- Dynamics
  - nonlinearity (small vs. large shocks)
  - stationary distribution ∪ - shaped (system gets trapped in bad states)
  - asset prices: correlation in crises, fat tails
- Comparative Statics
  - lower exogenous risk → "**volatility paradox**"
  - better hedging/risk management → higher endogenous risk
  - technological / market / funding liquidity and endogenous risk
- Regulation
  - effect on entire dynamics, not just after crisis happens
  - unintended consequences

	asset price level	amplification in crisis	leverage	asset allocation	crisis probability
bounds on leverage	-	-	-	-	-
dividend restrictions	+	+	-	0	-
price floor/recapitalization	+	-	+	+	-

Literature

- Capital structure/financial frictions
  - Towsend (1979), Bolton-Scharfstein (1990), Sannikov (2012) (survey)
  - Diamond (1984), Holmstrom-Tirole (1997), Diamond-Dybvig (1983)
- Prices/collateral values
  - Shleifer-Vishny (1992), Geanakoplos (1997), Brunnermeier-Pedersen
- Infinite-horizon, log-linearization
  - KM, BGG, Carlstrom-Fuerst (1997), Christiano-Eichenbaum-Evans, Gertler-Kiyotaki, Brunnermeier-Eisenbach-Sannikov (survey)
- No log-linearization
  - Basak-Cuoco (1998), Mendoza (2010), He-Krishnamurthy (2012a,b)

Basic Model: Technology

experts $\underline{\delta} \geq \delta$, $\underline{a} \leq a$ less productive households

experts	less productive households
Output $(a - \iota_t)\, k_t$	Output $(\underline{a} - \iota_t)\, k_t$
Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$	Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$
$dk_t = (\Phi(\iota_t) - \delta)\, k_t\, dt + \sigma\, k_t\, dZ_t$	$dk_t = (\Phi(\iota_t) - \underline{\delta})\, k_t\, dt + \sigma\, k_t\, dZ_t$

Basic Model: Preferences

experts	$\underline{\delta} \geq \delta$, $\underline{a} \leq a$	less productive households
Output **$(a - \iota_t)\, k_t$** Investment ι_t creates new capital at rate **$\Phi(\iota_t)\, k_t$** **$dk_t = (\Phi(\iota_t) - \delta)\, k_t\, dt + \sigma\, k_t\, dZ_t$**		Output **$(\underline{a} - \iota_t)\, k_t$** Investment ι_t creates new capital at rate **$\Phi(\iota_t)\, k_t$** **$dk_t = (\Phi(\iota_t) - \underline{\delta})\, k_t\, dt + \sigma\, k_t\, dZ_t$**
risk-neutral, discount rate ρ consumption must be ≥ 0	**$\rho > r$**	risk-neutral, discount rate r may consume negatively

Basic Model: Financial Frictions

experts	$\underline{\delta} \geq \delta$, $\underline{a} \leq a$	less productive households
Output $(a - \iota_t)\, k_t$ Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$ $dk_t = (\Phi(\iota_t) - \delta)\, k_t\, dt + \sigma\, k_t\, dZ_t$		Output $(\underline{a} - \iota_t)\, k_t$ Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$ $dk_t = (\Phi(\iota_t) - \underline{\delta})\, k_t\, dt + \sigma\, k_t\, dZ_t$
risk-neutral, discount rate ρ consumption must be ≥ 0	$\rho > r$	risk-neutral, discount rate r may consume negatively
may issue only risk-free debt + solvency constraint		financially unconstrained

Basic Model: Asset Markets

difference → **market** liquidity

$\underline{\delta} \geq \delta$, $\underline{a} \leq a$

experts		less productive households
Output $(a - \iota_t)\, k_t$ Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$ $dk_t = (\Phi(\iota_t) - \delta)\, k_t\, dt + \sigma\, k_t\, dZ_t$		Output $(\underline{a} - \iota_t)\, k_t$ Investment ι_t creates new capital at rate $\Phi(\iota_t)\, k_t$ $dk_t = (\Phi(\iota_t) - \underline{\delta})\, k_t\, dt + \sigma\, k_t\, dZ_t$
risk-neutral, discount rate ρ consumption must be ≥ 0	$\rho > r$	risk-neutral, discount rate r may consume negatively
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Liquid markets for capital k_t with endogenous price per unit q_t

$$dq_t/q_t = \mu_t^q\, dt + \sigma_t^q\, dZ_t$$

First Best and Autarky

- First-best:
  - experts manage capital forever
  - consume entire net worth at t = 0
  - issue equity to less productive households
  - price of capital

funding liquidity

$$\bar{q} = \max_{\iota} \frac{a - \iota}{r - \Phi(\iota) + \delta}$$

- Autarky:
  - households manage capital forever
  - price of capital

difference is **market** illiquidity

$$\underline{q} = \max_{\underline{\iota}} \frac{\underline{a} - \underline{\iota}}{r - \Phi(\underline{\iota}) + \underline{\delta}}$$

Capital gains/risk

$dk_t/k_t = (\Phi(\iota_t) - \delta)\, dt + \sigma\, dZ_t$

$dq_t/q_t = \mu_t^q\, dt + \sigma_t^q\, dZ_t$ ← endogenous

$d(k_t q_t)/(k_t q_t) = (\Phi(\iota_t) - \delta + \mu_t^q + \sigma \sigma_t^q)\, dt + (\sigma_t^q + \sigma)\, dZ_t$

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Return from investing in capital

$\max_{\iota} \Phi(\iota) - \iota/q_t$

adjustment costs in $\Phi \rightarrow$ **technological** illiquidity

$$dk_t/k_t = (\Phi(\iota_t) - \delta)\,dt + \sigma\,dZ_t$$

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$$dr_t^k = \underbrace{(a - \iota_t)/q_t\,dt}_{\text{dividend yield}} + \underbrace{(\Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q)\,dt + (\sigma_t^q + \sigma)\,dZ_t}_{\text{capital gains}}$$

Return from expert portfolio

$$dk_t/k_t = (\Phi(\iota_t) - \delta)\, dt + \sigma\, dZ_t$$

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$$dn_t/n_t = x_t\, dr_t^k + (1 - x_t)\, r\, dt - dc_t/n_t$$

consumption rate

portfolio weight (>1 if leverage)

Equilibrium Definition

- Equilibrium is a map

histories of shocks $\{Z_s, s \leq t\}$ → prices q_t, allocations (of capital ψ_t, risk-free asset, consumption)

s.t.

- experts, HH solve optimal consumption/portfolio choice (capital vs. risk-free asset) problems (Merton problem)
- markets clear

Equilibrium Characterization

- Equilibrium is a map

histories of shocks
$\{Z_s, s \leq t\}$

prices q_t, allocations
(of capital ψ_t, risk-free asset, consumption)

wealth distribution:
fraction $\eta_t \in (0, 1)$ owned by experts

- since experts are impatient, they consume all net worth when $\eta_t > \eta^* \leftarrow$ endogenous, stochastic steady state
- experts hold all capital when η_t is near η^*

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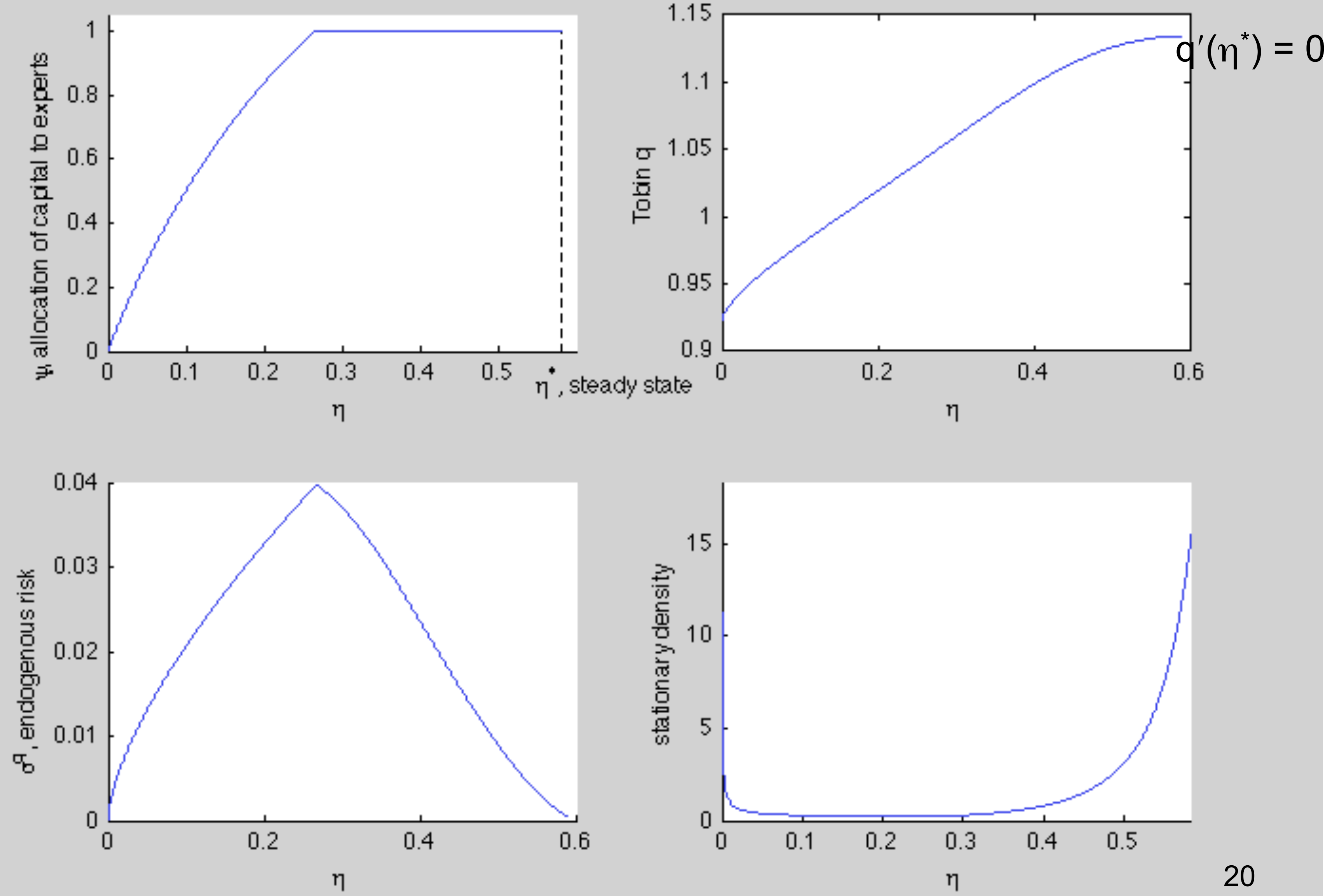
Example: $r = 5\%$, $\rho = 5.2\%$, $a = 11\%$, $\underline{a} = 10\%$, $\delta = 5\%$, $\underline{\delta} = 6\%$, $\sigma = 10\%$, $\Phi(\iota) = ((1 - 2\kappa\iota)^{1/2} - 1)/\kappa$ with $\kappa = 2$ (quadratic adj. costs)

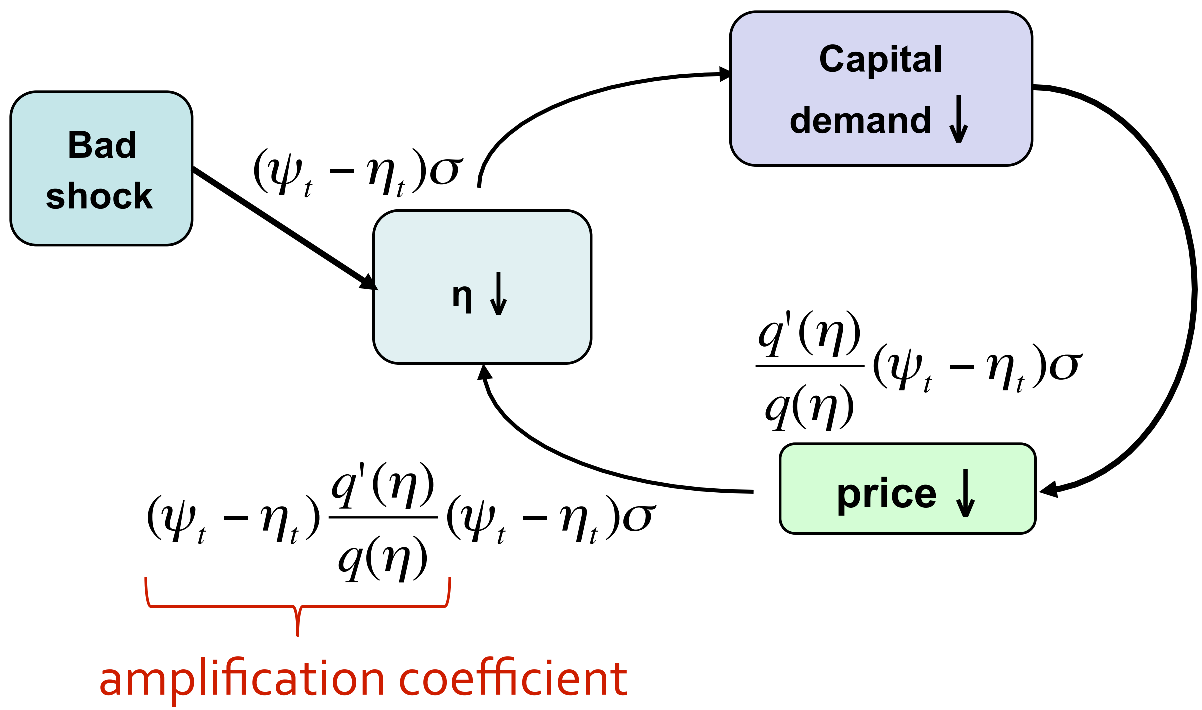


Properties of Equilibrium

Inefficiencies: (1) capital misallocation,
(2) underinvestment, (3) consumption distortion

Amplification:
depends on $q'(\eta)$

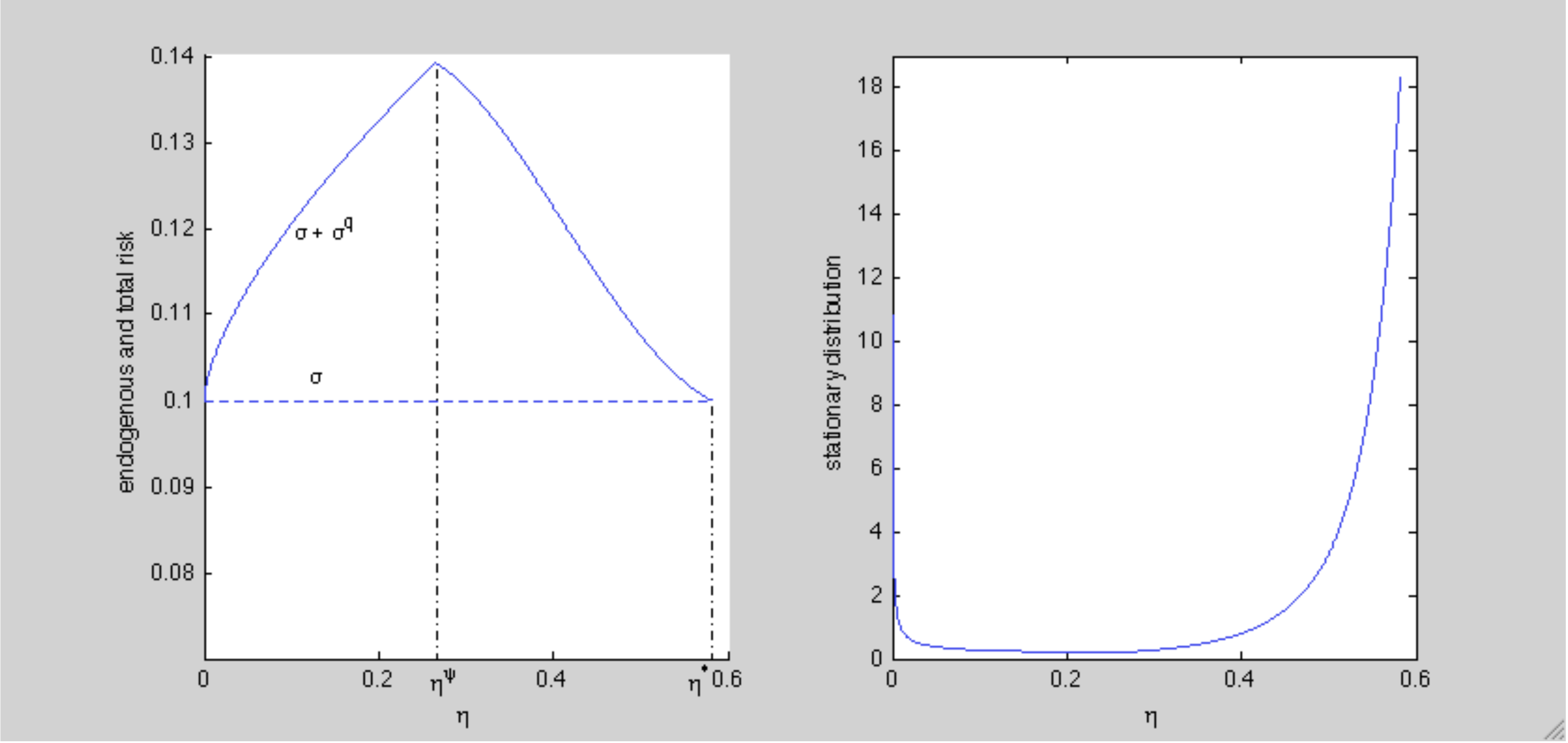
- absent near η^*, $q'(\eta^*) = 0$
- high below η^*



Endogenous risk

$$\sigma_t^q = \frac{q'(\eta)}{q(\eta)} \frac{(\psi_t - \eta_t)\sigma}{1 - (\psi_t - \eta_t)\frac{q'(\eta)}{q(\eta)}}$$

Endogenous Risk and Stationary Density



Proposition. Let $\kappa = (a - \underline{a})/\underline{q} + \underline{\delta} - \delta$ (market illiquidity). If $2(\rho - r)\sigma^2 < \kappa^2$, stationary density exists, converges to ∞ as $\eta \to 0$. If not, the system gets stuck near $\eta = 0$ in the long run (no stationary density).

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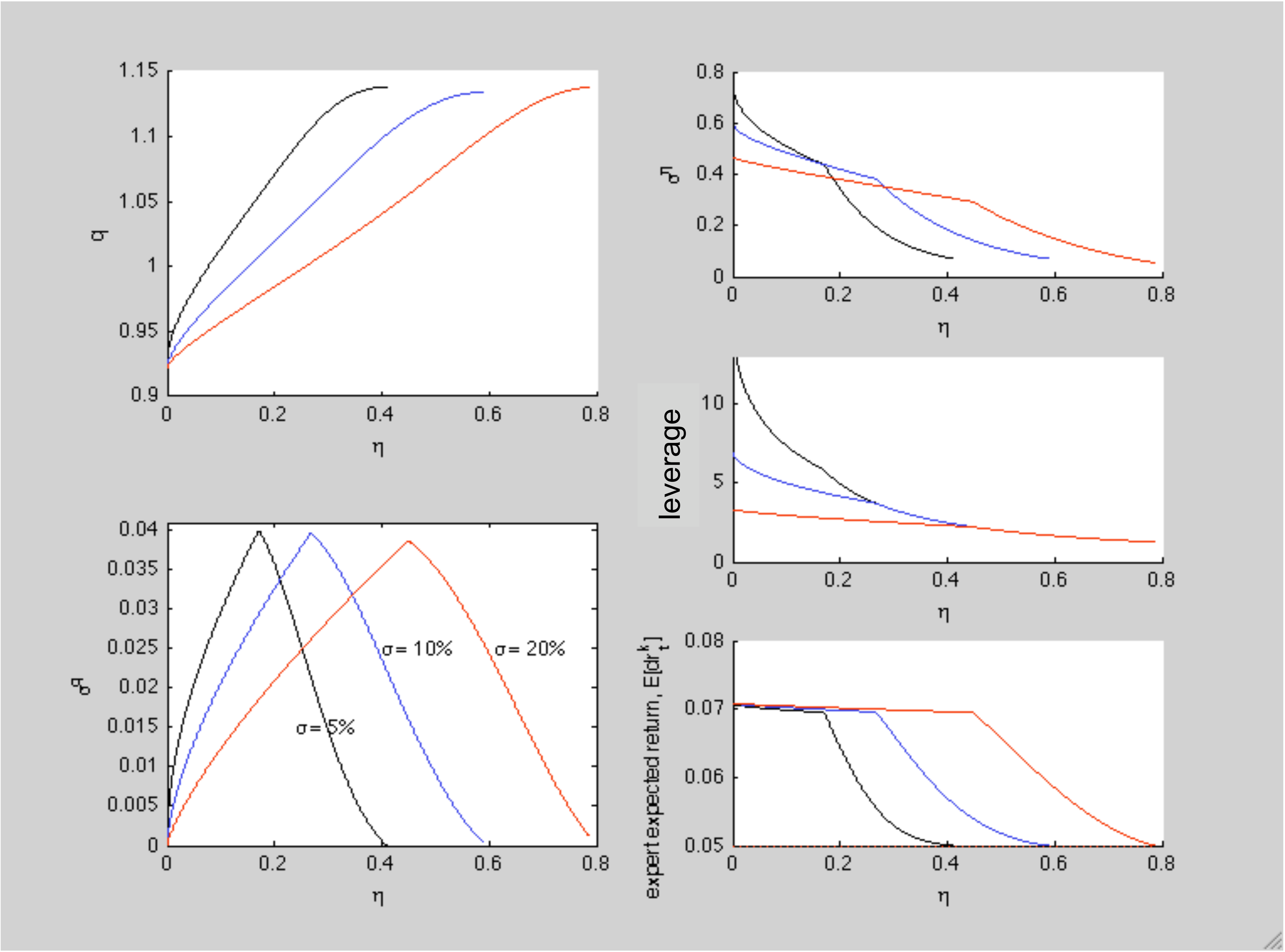
Comparative Statics: σ

- As exogenous risk σ falls, does endogenous risk σ^q also fall?

Comparative Statics: σ

- As exogenous risk σ falls, does endogenous risk σ^q also fall?
- No. **max $\boldsymbol{\sigma^q}$** can actually rise as σ falls - the volatility paradox
- Endogenous risk does not go away because as σ falls, leverage increases (significantly) and price q in boom rises
- **Proposition.** As $\eta \to 0$, $\sigma^\eta \to \kappa/\sigma + O(\sigma)$
- Generally, σ^q and risk premia in crisis are not sensitive to σ


1.15
1.1
1.05
1
0.95
0.9
q
0
0.2
0.4
0.6
0.8
η
0.8
0.6
0.4
0.2
0
σ^η
η
leverage
10
5
0
η
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
σ^q
σ = 10%
σ = 20%
σ = 5%
η
0.08
0.07
0.06
0.05
expert expected return, E[dr^k_t]
η

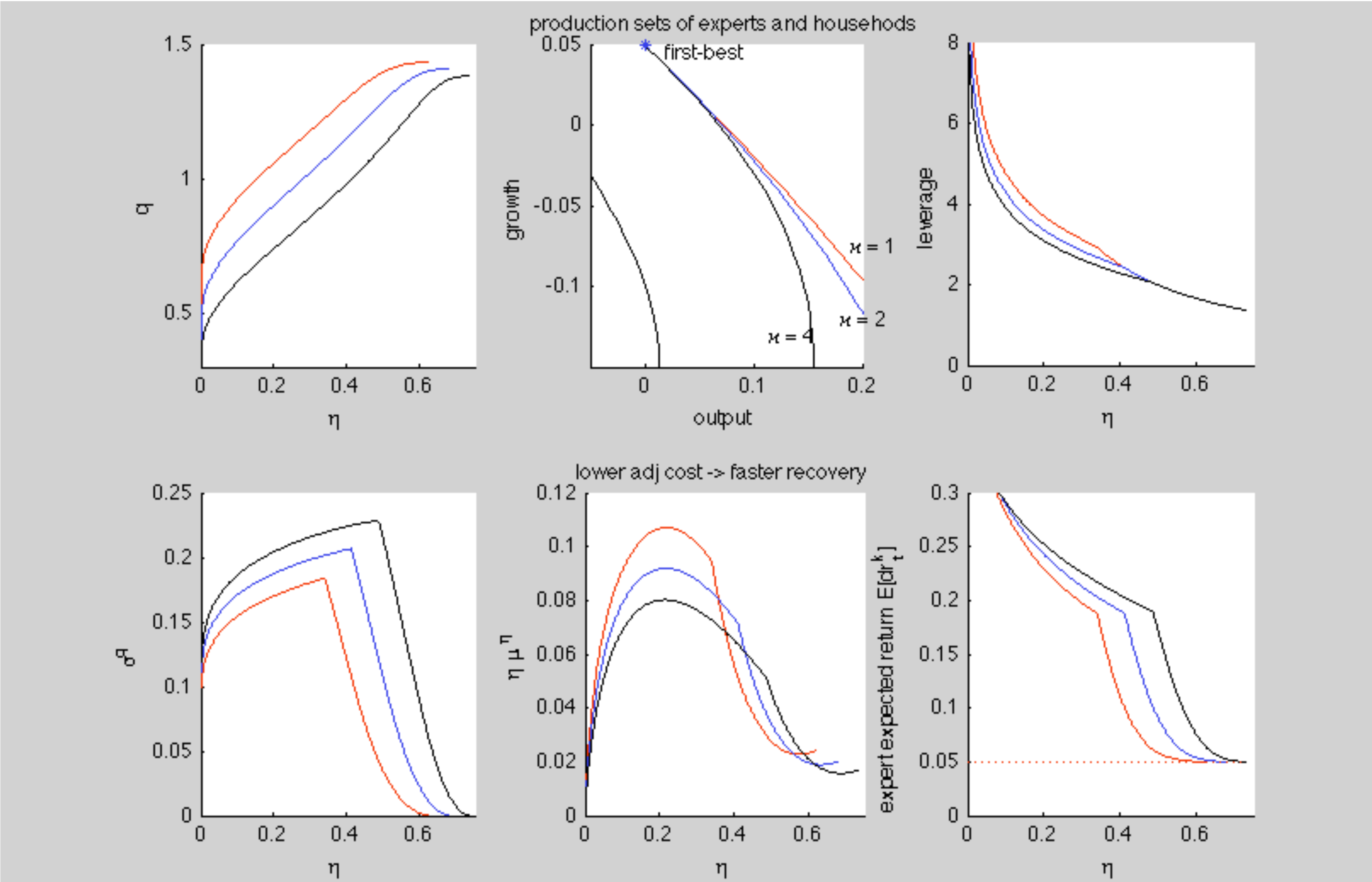
What matters for endogenous risk?

- If exogenous risk σ has little effect on maximal endogenous risk or risk premia, than what does?

Comparative Statics: Liquidity

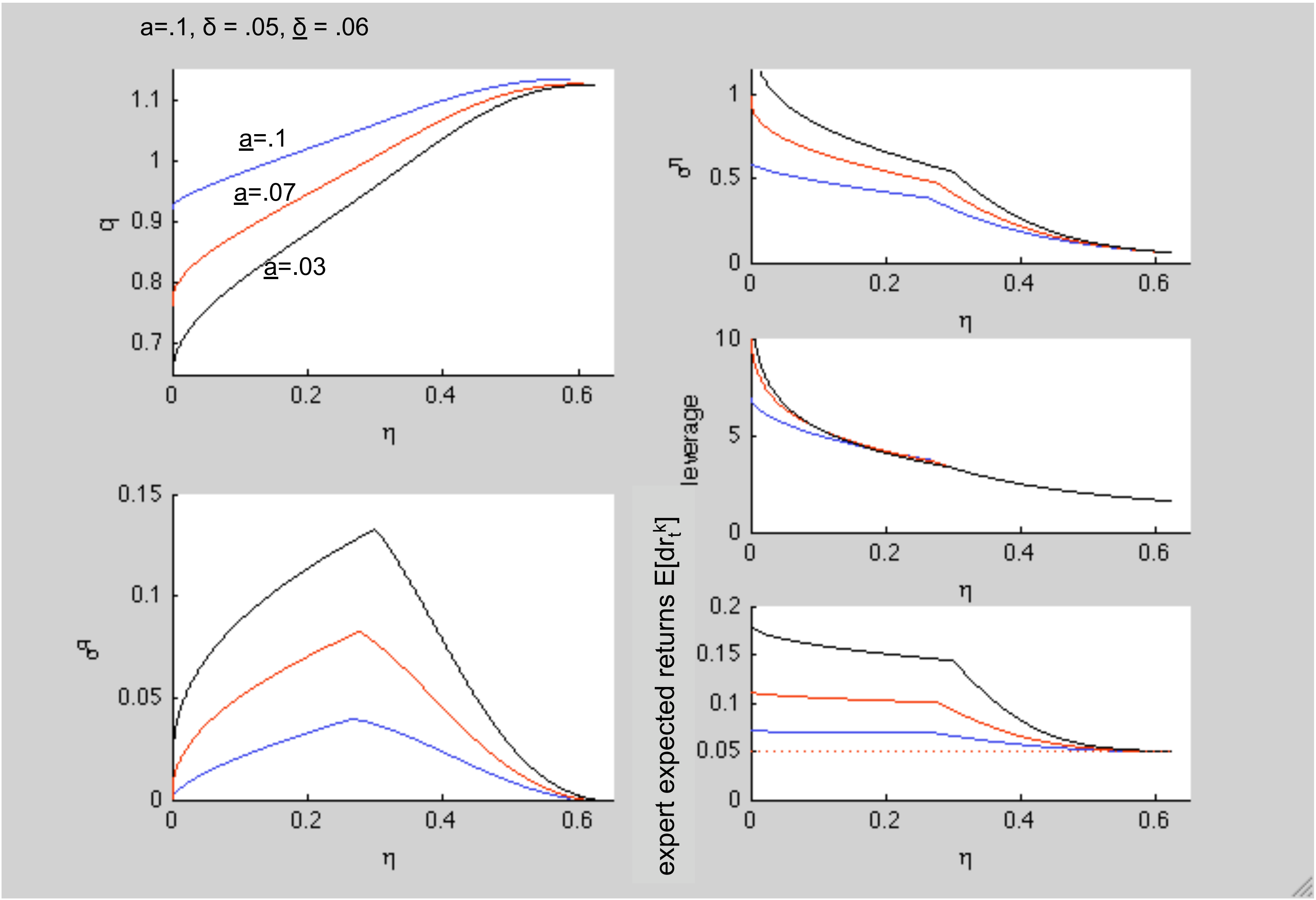
- Technological illiquidity: adjustment costs in function Φ, ability to disinvest
- Market illiquidity: difference between first and second-best uses of assets (between a and $\underline{a}$, δ and $\underline{\delta}$)
- Funding illiquidity: ease with which funding can dry up. Short-term debt (in the model so far) has the worst funding liquidity. Long-term debt, equity are a lot better.

Technological Liquidity



r = 5%, ρ = 5.2%, σ = 10%, adj. cost parameter κ = 1,2,4

Market Liquidity: changing <u>a</u> (and <u>q</u>)



Comparative Statics: Borrowing Costs

Idiosyncratic Poisson shocks cause losses to individual experts that need to be verified (Townsend (1979))

$$dk_t^i = (\Phi(\iota_t) - \delta)\, k_t^i\, dt + \sigma\, k_t^i\, dZ_t + k_t^i\, dJ_t^i$$

compensated (mean 0) process

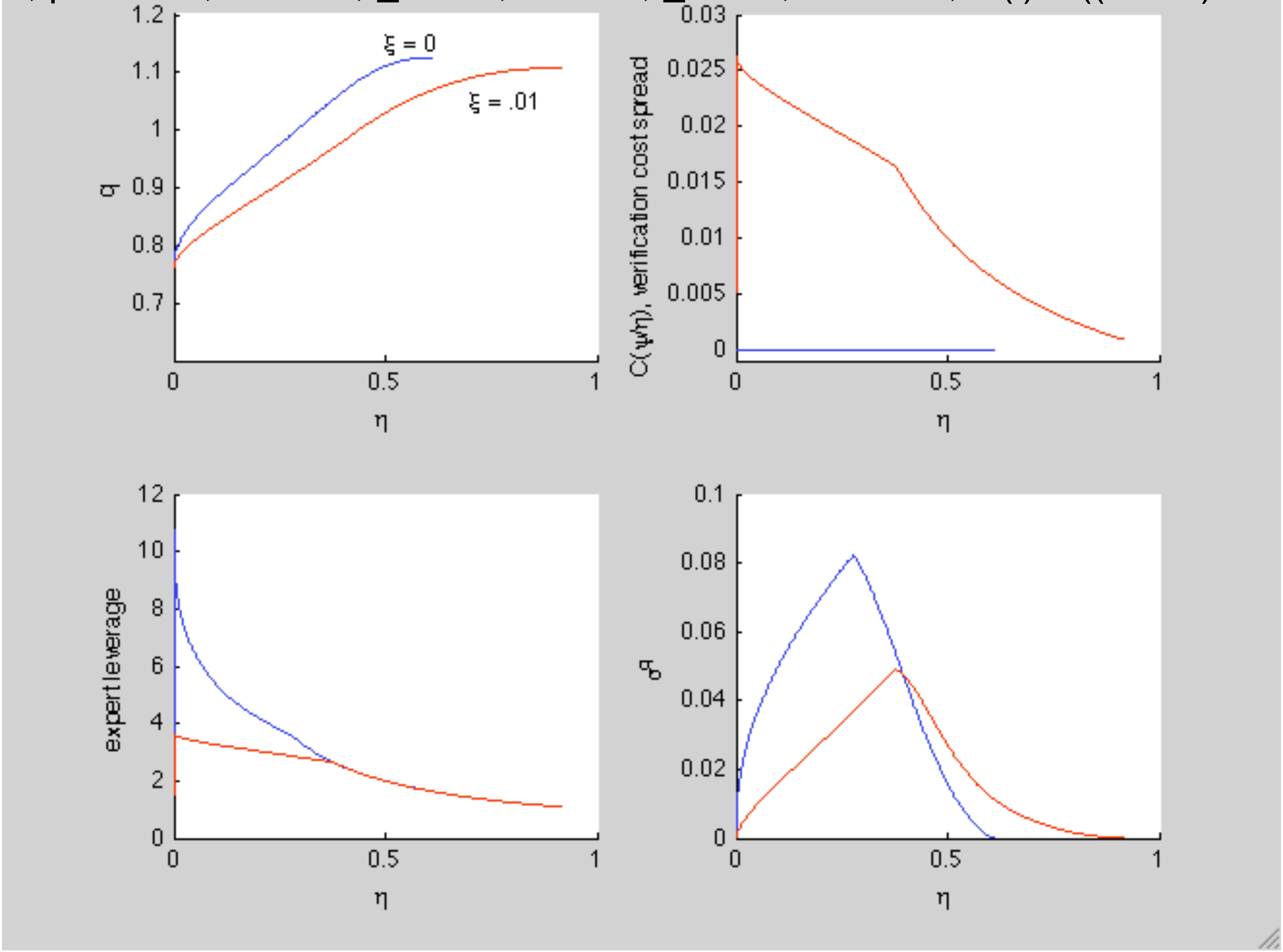
Debt no longer risk-free, experts pay a credit spread

$$E[dn_t/n_t] = x_t\, E[dr_t^k] + (1 - x_t)\,(r + \Lambda(x_t))\, dt - dc_t/n_t$$

spread due to verification costs

Borrowing Costs $\Lambda(x) = \xi(x-1)$, $\xi = 0, .01$

$r = 5\%$, $\rho = 5.2\%$, $a = 11\%$, $\underline{a} = 7\%$, $\delta = 5\%$, $\underline{\delta} = 6\%$, $\sigma = 10\%$, $\Phi(\iota) = ((1 - 2\iota)^{1/2} - 1)$



Risk Management to Reduce Borrowing Costs

- **Proposition.** If experts can hedge idiosyncratic shocks among each other, the solution becomes identical to that with no shocks.

- Thus, while hedging reduces inefficiencies (costly verification), it leads to higher endogenous risk and greater likelihood of crisis

Deterministic vs. Stochastic Steady State

- **Deterministic steady state (BGG, KM):** stationary point of an economy without shocks

- **Proposition.** With borrowing costs $\Lambda(x)$, deterministic steady state η^0 is characterized by

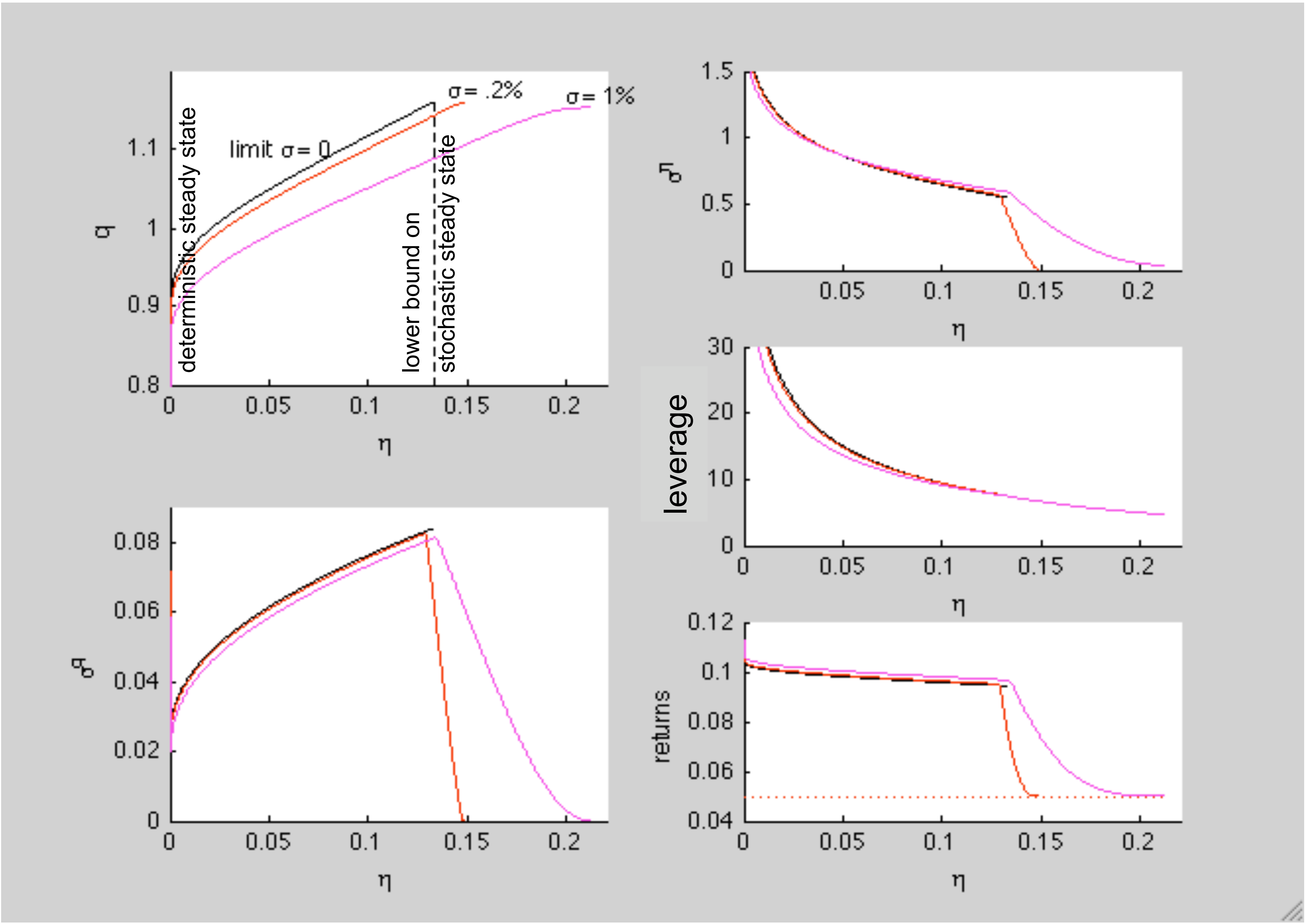
$$\rho - r = (1 - \eta^0)/(\eta^0)^2\, \Lambda'(1/\eta^0) + \Lambda(1/\eta^0)$$

- $\eta^0 \to 0$ as verification costs go to 0.

- **Stochastic steady state:** point where the system stays in place in the absence of shocks, in an economy with anticipated shocks (it is η^*)

Deterministic steady state ≠ stochastic steady state as $\sigma \to 0$

Economy as σ → 0:



Kocherlakota (2000) Critique does not apply

- Unique unanticipated shocks produce little amplification
- Following shock, price recovers for sure, so it drops little
  - if market knows that the recovery is for sure, there is enough demand even if prices drop by a little
- But, fully anticipated shocks can produce a lot of amplification (price may drop further a lot more)
- In fact, as $\sigma \rightarrow 0$, amplification is infinite!

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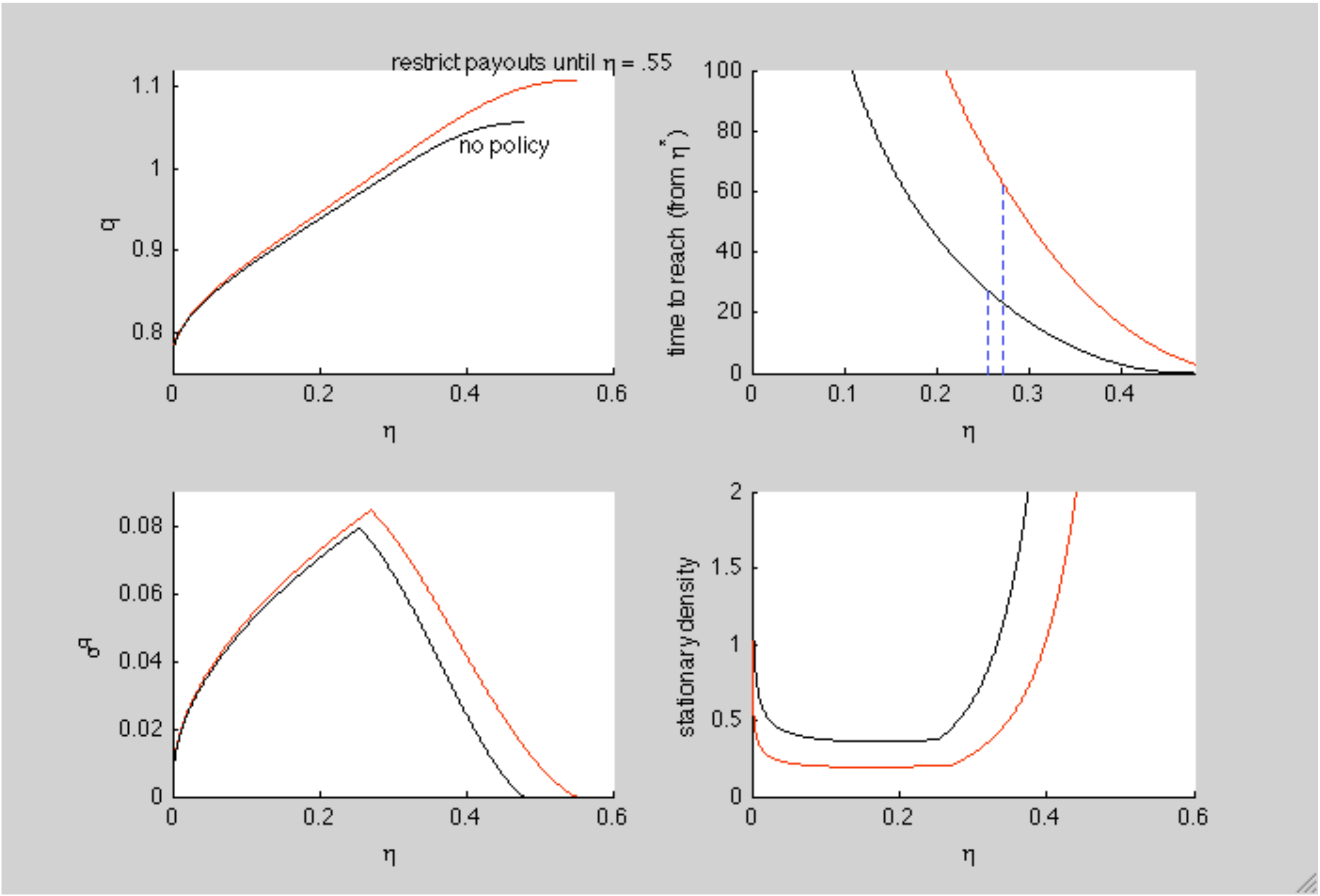
Policies

- **“Micromanaging”**
  - **Proposition:** If a regulator fully controls asset allocation, investment and consumption, subject to resource constraints, based on public information in the market, first-best can be attained
- **Capital requirements/leverage bounds**
  - similar to borrowing costs (but more crude)
  - cost: asset misallocation; benefit: crisis less likely
- **Restriction on dividends/payouts**
  - reduces crisis probability
  - but stimulates prices, i.e. crises become worse
- **Recapitalization in downturns/price floor**
  - improves funding/market liquidity
  - can be decentralized, with freely traded insurance contracts
  - low exogenous, high endogenous risk $\Rightarrow$ low cost to improve welfare

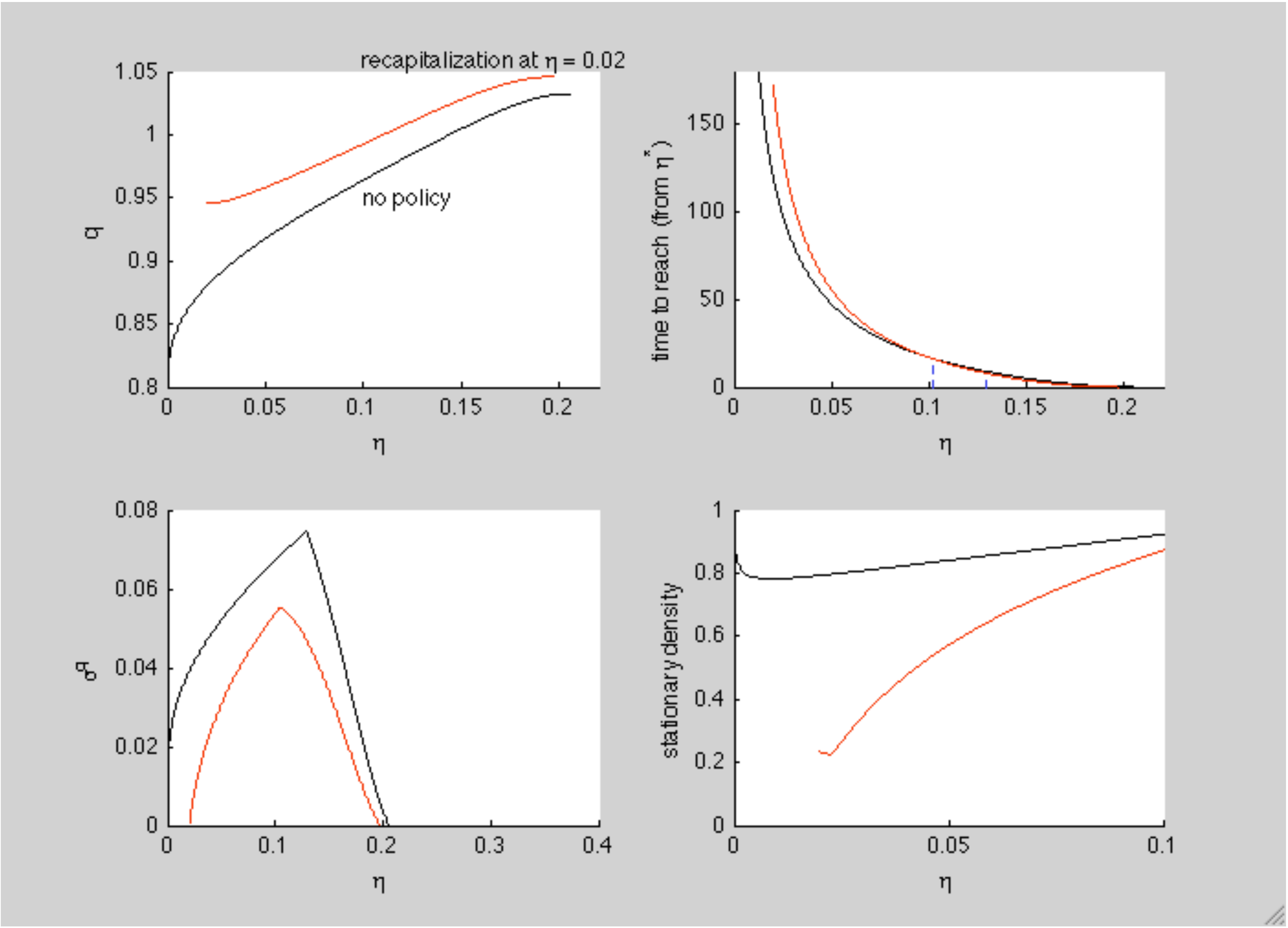
Policy: Restriction on Payouts



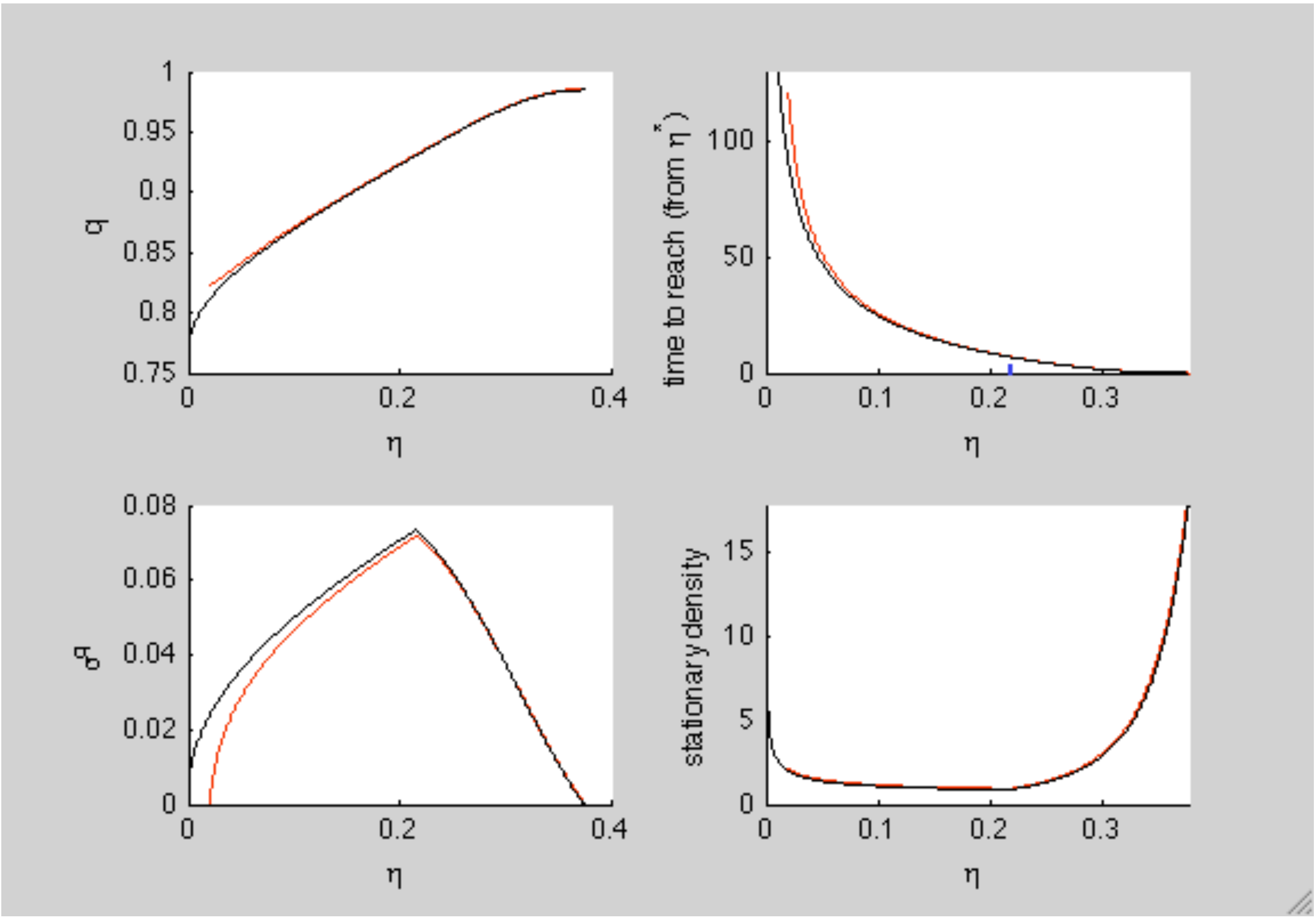
Policy: Restriction on Payouts

- This policy
  - improves experts' net worth buffers
  - reduces frequency of crisis, time spent in depressed regimes
  - stimulates prices, so worse endogenous risk in crisis
  - generally reduces welfare within model, but can improve welfare if there are spillovers

Recapitalizing experts at η = .02, σ = 3%



But with σ = 10%, less impressive effect



Policy: Recapitalization

- This policy
  - works particularly well with low exogenous risk, potentially high endogenous risk, effectively by improving funding liquidity
  - may not reduce the frequency of firesales (endogenous leverage), but reduces time spent in deeply depressed states
  - improves welfare within the model
  - creates little moral hazard if recapitalization is proportional to net worth, i.e. it benefits cautious experts more than risk-takers
  - can be implemented through free trading of insurance securities (rather than an explicit bailout)
  - price support policy has similar effects

Conclusion

- Continuous time offers a powerful methodology to analyze heterogeneous-agent models with financial frictions
- System dynamics: normal times (low amplification) different from crisis times (high amplification/risk premia, correlated asset prices)
- Endogenous risk-taking leads to paradoxes
  - diversification opportunities, hedging instruments, lower exogenous risk may lead to higher endogenous risk in crises
- Regulation
  - model offers a laboratory to study the effects of policies
  - important, because many policies have unexpected consequences

Thank you!