Booms and Systemic Banking Crises

F. Boissay, F. Collard and F. Smets

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The views expressed in this presentation are our own and do not necessarily reflect those of the European Central Bank or the Eurosystem.
Motivation/Objective

- Better understand the joint dynamics of regular business cycles and systemic banking crises (SBCs)
- Account for the few features common to SBCs (Reinhart and Rogoff, 2009; Jordà et al., 2011; Claessens et al., 2011; Schularick and Taylor, 2012):
  - Key Fact #1: SBCs are rare events – on average 1 every 40 years in OECD countries
  - Key Fact #2: Recessions that follow SBCs are deeper and last longer – output loss during a SBC is 60% larger
  - SBCs are "credit booms gone wrong"
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  - Key Fact #3: SBCs are "credit booms gone wrong"
Motivation/Objective

- In most DSGE models with financial frictions, banking crises are big negative shocks amplified.

Can explain Key Facts #1 & #2

Cannot explain Key Fact #3

SBCs are not random.

Explaining Key Fact #3 requires modeling the economic dynamics leading to SBCs.

From a policy perspective, our framework is a step forward towards:

- DSGE-based crisis prevention policy analysis
- DSGE-based early warning signals
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- **DSGE–based early warning signals**
Outline

- Stylized facts
- Comparison with the literature
- RBC model with systemic banking crises
- Quantitative analysis
- Concluding remarks
Stylized facts
SBCs are rare and bring about deep and long recessions

Frequency, magnitude, and duration of systemic banking crises

<table>
<thead>
<tr>
<th></th>
<th>Frequency (%)</th>
<th>Magnitude (%)</th>
<th>Duration (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All banking crises</td>
<td>4.49</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Systemic Banking Crises (SBC)</td>
<td>2.42</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>All recessions</td>
<td>10.20</td>
<td>4.86 (5.91)</td>
<td>1.85</td>
</tr>
<tr>
<td>Recessions with SBC (A)</td>
<td>23.86</td>
<td><strong>6.74 (6.61)</strong></td>
<td><strong>2.59</strong></td>
</tr>
<tr>
<td>Recessions w/o SBC (B)</td>
<td>76.13</td>
<td><strong>4.27 (5.61)</strong></td>
<td><strong>1.61</strong></td>
</tr>
<tr>
<td>Test A≠B, p-value (%)</td>
<td>–</td>
<td>2.61</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: Schularik et al. (2011), data for 14 OECD countries, 1870-2008
Crisis defined as in Laeven and Valencia (2008)
Stylized facts

SBCs follow credit booms
Stylized facts

SBCs are not random
Our Framework

- Textbook stochastic optimal growth model (RBC)
- Heterogenous banks endowed with intermediation and storage technologies
- Interbank market subject to MH and AI
- A Systemic Banking Crisis is an inter-bank market freeze
- Spill-over effects between the interbank market, the retail corporate loan market, and the real economy
Model features a (small) financial accelerator in normal times; calibrated to generate financial crises every 40 years.
Main Results

1. Model features a (small) financial accelerator in normal times; calibrated to generate financial crises every 40 years

2. The typical banking crisis follows upon an unusually long sequence of small, positive, transitory productivity shocks — Not a large negative financial shock

Financial recessions follow credit booms. They are deeper and last longer because they come with a credit crunch

The likelihood, depth, and length of a financial recession increase with the intensity of the credit boom that precedes it

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Booms and Systemic Banking Crises
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3. Financial recessions follow credit booms. They are deeper and last longer because they come with a credit crunch.

4. The likelihood, depth, and length of a financial recession increase with the intensity of the credit boom that precedes it.
Gertler-Kiyotaki (2009), Gertler-Karadi (2010):
- Full equilibrium non-linearities, such as sudden bank runs

Bianchi (2009), Bianchi-Mendoza (2010):
- Endogenous interest rates play a key role

Brunnermeier-Sannikov (2012), He-Krishnamurthy (2012):
- Typical crisis follows a rare, long sequence of positive TFP shocks
- Typical crisis identified as a bank run, not as a binding borrowing constraint

Gertler-Kiyotaki (2012)
- Bank run is market based and rationally expected
Model setup
Overview

Financial flows at the end of period t-1 ("core activity")

- Financial flows at the beginning of period t ("core activity")
- Financial flows at the end of period t ("core activity")
- Financial flows at the beginning of period t ("non-core activity")
- Financial flows at the end of period t ("non-core activity")

Banks are heterogeneous w.r.t. their financial intermediation costs
Representative Household and Firm

- Firm: \( \max \{ k_t, h_t \} \pi_t = F(k_t, h_t; z_t) + (1 - \delta) k_t - R_t k_t - w_t h_t \)

- Household:

  \[
  \max_{\{ a_{t+\tau+1}, c_{t+\tau}, h_{t+\tau} \}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau u(c_{t+\tau}, h_{t+\tau})
  \]

  subject to budget constraint

  \[
  c_t + a_{t+1} = r_t a_t + w_t h_t + \pi_t
  \]

- Notice that \( r_t \leq R_t \) (spread) and \( k_t \leq a_t \) (credit crunch)
Banks are atomistic, competitive, and price takers

Heterogeneous 1–period banks

Bank $p$’s net return per unit of corporate loan is $pR_t$

Beneficial to relocate funds: unskilled banks lend to skillful banks on an interbank market. But relocation impaired due to:

Asymmetric information: $p$ is private information

Moral hazard: bank $p$ may borrow $\phi_t$ and run away
The Banking Sector

- Banks are atomistic, competitive, and price takers
- Heterogeneous 1–period banks

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**The Banking Sector**

- **Banks are atomistic, competitive, and price takers**
- **Heterogeneous 1–period banks**

\[ \begin{align*}
  \text{Collect Deposits} & \quad \text{Draw skill} \\
  p \in (0, 1) & \quad \text{Die}
\end{align*} \]

- **Bank** \( p \)’s net return per unit of corporate loan is \( pR_t \)
- Beneficial to relocate funds: unskilled banks lend to skillful banks on an interbank market. But relocation impaired due to:
  - **Asymmetric information:** \( p \) is private information
  - **Moral hazard:** bank \( p \) may borrow \( \phi_t \) and run away
Bank $p$ has 4 options:

1. Lend to other banks on the market $\implies \rho_t$
2. Store goods $\implies \gamma$
3. Raise funds $\phi_t$ from market and lend to firm $\implies pR_t (1 + \phi_t)$
4. Raise funds $\phi_t$ from market and walk away $\implies \gamma (1 + \theta \phi_t)$

Notice that the incentive to divert depends on corporate loan $R_t$

- The higher $R_t$, the lower the incentive to divert
The Borrowing Bank’s Problem

- Borrowing bank $p$ solves:

\[
\max_{\phi_t} r_t(p) = pR_t (1 + \phi_t) - \rho_t \phi_t
\]

**PC:** $pR_t (1 + \phi_t) - \rho_t \phi_t \geq \rho_t \quad \Rightarrow p \geq \bar{p}_t \equiv \frac{\rho_t}{R_t}$

**IC:** $\gamma (1 + \theta \phi_t) \leq \rho_t \quad \Rightarrow \phi_t = (\rho_t - \gamma) / \theta \gamma$

- Profits are fully distributed to household: $r_t \equiv \int_0^1 r_t(p) \, d\mu(p)$
Supply (+)  
\[ \mu(\bar{p}_t) = (1 - \mu(\bar{p}_t)) \times \phi_t \]
"extensive margin" (−)  
"intensive margin" (+)

Demand bends backward (+ or −)

Two opposite effects on aggregate demand of a decrease in \( \rho_t \)

- Risky borrowers switch to demand side
- Lower leverage for skillful borrowers

Interbank Market Equilibrium

Interbank market clearing condition
Interbank Market Equilibrium

Trade takes place when the corporate loan rate is high
Interbank Market Equilibrium

Trade is impossible when the corporate loan rate is low
Interbank Market Equilibrium

Corporate loan rate threshold

[Graph showing demand and supply curves for market rate, with labels and points A, U, E, R, ρE, R_{low}, R_{high}, and \( \mu \left( \frac{\gamma}{R_t} \right) \).]
The Banking Sector

Return on equity and corporate loan supply

- Return on equity:

\[ r_t = \begin{cases} 
R_t \int_{p_t}^{1} p \frac{d\mu(p)}{1-\mu(p)} , & \text{if an equilibrium with trade exists} \\
R_t \left( \frac{\gamma}{R_t} \mu \left( \frac{\gamma}{R_t} \right) + \int_{p_t}^{1} p \, d\mu(p) \right) , & \text{otherwise.} 
\end{cases} \]

- Corporate loan supply

\[ k_t^s = \begin{cases} 
   a_t , & \text{if an equilibrium with trade exists} \\
   \left( 1 - \mu \left( \frac{\gamma}{R_t} \right) \right) a_t , & \text{otherwise} 
\end{cases} \]
Proposition 2 (Interbank loan market freeze): The interbank loan market is at work if and only if \( a_t \leq \bar{a}_t \equiv f_k^{-1}(\bar{R} + \delta - 1; z_t) \), and freezes otherwise.

Proposition 3 (Credit crunch): An interbank market freeze is accompanied with a sudden fall in the supply of corporate loans \( k^s_t \) (i.e. given \( z_t \), \( \lim_{a_t \downarrow \bar{a}_t} k^s_t < \lim_{a_t \uparrow \bar{a}_t} k^s_t \)), as well as by a sudden increase in the interest rate spread \( R_t / r_t \) (i.e. given \( z_t \), \( \lim_{a_t \downarrow \bar{a}_t} R_t / r_t > \lim_{a_t \uparrow \bar{a}_t} R_t / r_t \)).
Interbank market improves efficiency but freezes when $R_t < \bar{R}$

In general equilibrium, $R_t$ is driven by savings ($a_t$) and technology ($z_t$). Hence the interbank market freezes when $a_t > \bar{a}(z_t)$

**Threshold** $\bar{a}(z_t)$ is the banking sector’s "absorption capacity"

A measure of financial imbalances is $\bar{a}_t(z_t) - a_t$
The Banking Sector

Interest rates

Figure 4: Interest Rates

(a) Interest Rates vs Assets

(b) Interest Rates vs Technology Shock

Market freeze and credit crunch
### Bank balance sheets

<table>
<thead>
<tr>
<th>Normal times</th>
<th>Crisis times</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>L</td>
</tr>
<tr>
<td>((1 + \phi_t) a_t)</td>
<td>(a_t)</td>
</tr>
<tr>
<td>(\phi_t a_t)</td>
<td>(\phi_t a_t)</td>
</tr>
<tr>
<td>(p \geq \bar{p}_t)</td>
<td>(p &lt; \bar{p}_t)</td>
</tr>
</tbody>
</table>

Size is \(a_t + (1 - \mu(\bar{p}_t)) \phi_t a_t\)

Size is \(a_t\)
The Banking Sector

Two-way relationship between the retail and the wholesale loan markets

- Whether the interbank market is functioning depends on the corporate loan market equilibrium rate $R_t^*$
- $R_t^*$ depends on whether the interbank market is functioning
- The model must be solved taking these interactions into account:
  1. Conjecture the interbank market operates and solve for $R_t^*$
  2. Verify whether indeed the interbank market operates ($R_t^* \geq \bar{R}$)
  3. In the negative, solve for $R_t^*$ under a credit crunch
Production function: \( F(k_t, h_t; z_t) \equiv z_t k_t^\alpha h_t^{1-\alpha} \) with \( \alpha \in (0, 1) \)

Utility function: \( u(c_t, h_t) = \frac{1}{1-\sigma} \left( c_t - \theta \frac{h_t^{1+v}}{1+v} \right)^{1-\sigma} \)

Cdf of bank skills: \( \mu(p) = p^\lambda \)

Real economy: standard calibration on US (annual) post-WWII data

Financial sector (\( \gamma, \theta, \lambda \)) is calibrated so that:

- Crisis probability is 2.5%
- Average interest rate spread is 1.71%
- Average corporate loan rate of 4.35%
Parameters of the model

<table>
<thead>
<tr>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ 1/1.03</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\sigma$ 4.500</td>
</tr>
<tr>
<td>Frish elasticity</td>
<td>$\vartheta$ 1/3</td>
</tr>
<tr>
<td>Labor disutility</td>
<td>$\vartheta$ 0.944</td>
</tr>
<tr>
<td>Capital elasticity</td>
<td>$\alpha$ 0.300</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$ 0.100</td>
</tr>
<tr>
<td>Standard dev. productivity shock</td>
<td>$\sigma_z$ 0.018</td>
</tr>
<tr>
<td>Persistence of productivity shock</td>
<td>$\rho_z$ 0.900</td>
</tr>
<tr>
<td>Bank distribution; $\mu(p) = p^\lambda$</td>
<td>$\lambda$ 24</td>
</tr>
<tr>
<td>Diversion cost</td>
<td>$\theta$ 0.1</td>
</tr>
<tr>
<td>Storage technology</td>
<td>$\gamma$ 0.936</td>
</tr>
</tbody>
</table>
The model is solved numerically by a collocation method.

Discretize the TFP level (Tauchen and Hussey, 1991).

Decision rule for $a_{t+1}$ is approximated by a function of Chebychev polynomials.

The optimal decision rule is obtained as the fixed point solution to the Euler equation.
Variety of crises: shock-driven (S) and credit boom-driven (U)
Quantitative Analysis
Optimal savings rule: exogenous versus endogenous crises

- Variety of crises: shock–driven (S) and credit boom–driven (U)
- History suggests that credit–boom driven crises prevail
Quantitative Analysis
Intuition behind endogenous SBCs

1. At the beginning, a positive shock brings TFP above its mean
   - Credit demand rises. Return on savings goes up. The household accumulates assets for *consumption smoothing*
   - The **credit boom is initially demand–driven**

2. TFP goes down back to mean but remains above it for a long time
   - Credit demand decreases, while the household keeps on accumulating savings
   - The **credit boom becomes supply–driven**

3. The household accumulates assets for *precautionary motives*, which works to reduce interest rates and to raise further the likelihood of a crisis

4. A SBC breaks out as the corporate loan rate crosses its threshold
Typical path to crisis

Typical path

Technology Shock

Assets

Years

Years

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Booms and Systemic Banking Crises
Quantitative Analysis

Typical path to crisis

Financial variables dynamics along typical path
Quantitative Analysis

Typical path to crisis

Real variables dynamics along typical path

- Output
- Hours Worked
- Consumption
- Investment
Sensitivity of output dynamics to initial conditions

TFP is initially 7.5% above mean

Dynamics in normal times, \textcolor{red}{
Dynamics in a systemic banking crisis,}
long-run average. $\bar{\alpha}(7.5\%)$ denotes the banks' absorption when productivity is 7.5% above average.
Sensitivity of credit dynamics to initial conditions

TFP is initially 7.5% above mean.
Sensitivity of the frequency of SBCs to initial conditions

TFP is initially 7.5% above mean

This figure reports the evolution of the frequency of SBCs during the transition toward the average steady state.
Sensitivity of the frequency of SBCs to initial conditions

TFP is initially 7.5% above mean

(a) Asset

\[ a_0 = 0.8 \bar{a}(7.5\%) \]

\[ a_0 = 0.9 \bar{a}(7.5\%) \]

\[ a_0 = \bar{a}(7.5\%) \]

Years 0 20 40

(b) Credit/Assets

\[ a_0 = 0.8 \bar{a}(7.5\%) \]

\[ a_0 = 0.9 \bar{a}(7.5\%) \]

\[ a_0 = \bar{a}(7.5\%) \]

Years 0 20 40

- Solid black line: Dynamics in normal times in the Solow version (Benchmark Model),
- Dashed red line: Dynamics in a systemic banking crisis in the Solow version (Benchmark Model),
- Dotted line: long-run average. \( \bar{a}(7.5\%) \) denotes the banks' absorption when productivity is 7.5% above average.
Quantitative Assessment

SBCs are rare and bring about deep and long recessions

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**Frequency, magnitude, and duration of systemic banking crises**

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<th>Frequency (%)</th>
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<th>Duration (Years)</th>
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<tbody>
<tr>
<td>Systemic Banking Crises (SBC)</td>
<td><strong>2.69</strong></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>All recessions</td>
<td>10.00</td>
<td>12.08 (7.30)</td>
<td>2.08</td>
</tr>
<tr>
<td>Recessions with SBC (A)</td>
<td>13.00</td>
<td><strong>17.87</strong> (10.50)</td>
<td><strong>2.62</strong></td>
</tr>
<tr>
<td>Recessions w/o SBC (B)</td>
<td>87.00</td>
<td><strong>10.04</strong> (6.73)</td>
<td><strong>1.90</strong></td>
</tr>
</tbody>
</table>

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Booms and Systemic Banking Crises
Quantitative Assessment

SBCs follow credit booms

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**Output**

(% deviation about trend)

---

**Credit**

(% deviation about trend)

---

- **R recessions with a Financial Crisis**
- **Other recessions**
Quantitative Assessment

SBCs are not random
Crisis probabilities for the US

Note: The vertical thin dashed lines correspond to the 1984 Savings & Loans, the 2000 dotcom and 2008 crises.
## Changes in standard parameters

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$\sigma_z$</th>
<th>$\rho_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.20</td>
<td>35</td>
<td>0.02</td>
<td>0.95</td>
</tr>
<tr>
<td>Interbank rate ($\rho$)</td>
<td>0.86</td>
<td>0.23</td>
<td>0.40</td>
<td>1.34</td>
<td>0.89</td>
<td>0.72</td>
</tr>
<tr>
<td>Corporate rate ($R$)</td>
<td>4.35</td>
<td>3.70</td>
<td>5.50</td>
<td>3.70</td>
<td>4.32</td>
<td>4.29</td>
</tr>
<tr>
<td>Return on deposit/equity ($r$)</td>
<td>2.64</td>
<td>1.61</td>
<td>2.61</td>
<td>2.67</td>
<td>2.55</td>
<td>2.59</td>
</tr>
<tr>
<td>Spread ($R - r$)</td>
<td>1.71</td>
<td>2.09</td>
<td>2.89</td>
<td>1.03</td>
<td>1.77</td>
<td>1.70</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>2.43</td>
<td>2.43</td>
<td>4.83</td>
<td>0.41</td>
<td>2.43</td>
<td>2.43</td>
</tr>
<tr>
<td>Probability of a crisis</td>
<td>2.69</td>
<td>5.43</td>
<td>7.34</td>
<td>0.16</td>
<td>3.35</td>
<td>1.90</td>
</tr>
<tr>
<td>Average duration</td>
<td>2.62</td>
<td>4.08</td>
<td>5.06</td>
<td>1.87</td>
<td>2.82</td>
<td>2.92</td>
</tr>
<tr>
<td>Average amplitude</td>
<td>17.87</td>
<td>19.00</td>
<td>16.90</td>
<td>15.80</td>
<td>19.36</td>
<td>16.08</td>
</tr>
</tbody>
</table>
Bank Leverage, Bank Defaults

- Absent frictions between banks and household, bank leverage is undeterminate and bank default is not defined

- Two more assumptions to pin down leverage:
  - Bank deposits are safe assets (non state contingent return)
  - Bank managers are risk neutral (unlike household)

- One more assumption to introduce defaults:
  - Household (bank shareholder) has partial liability
Leverage and bank default dynamics along typical path

Figure 19: Typical Path: Leverage and Default
Concluding Remarks

- Develop a simple DSGE model with SBCs
- SBCs are not caused by large, negative, financial shocks but rather by long sequences of small, positive, productivity shocks
- Highlight the role of financial imbalances, consumption smoothing, and precautionary savings

From a policy making perspective:
- Framework for both crisis management and crisis prevention
- DSGE-based probability of a crisis
Figure C.4: Evolution of Various Corporate Loan Spreads

(a) Spread: Corporate loan rates - Federal Fund Rate

(b) Underlying Real Corporate Loan Rates

Trend line.
The Model in a Nutshell

\[ y_t = z_t k_t^\alpha h_t^{1-\alpha} + (\gamma + \delta - 1) (a_t - k_t) \]
\[ R_t = \alpha k_t \frac{1}{v+\alpha} z_t^{\frac{1-\alpha}{\delta}} (\frac{1}{v+\alpha})^{1-\alpha} + 1 - \delta \]
\[ \left( c_t - \vartheta \frac{h_t^{1+v}}{1+v} \right)^{-\sigma} = \beta \mathbb{E}_t \left[ \left( c_{t+1} - \vartheta \frac{h_{t+1}^{1+v}}{1+v} \right)^{-\sigma} r_{t+1} \right] \]
\[ h_t = \left( \frac{(1-\alpha)z_t}{\vartheta} \right)^{\frac{1}{v+\alpha}} k_t^{\frac{\alpha}{v+\alpha}} \]
\[ \bar{a}_t \equiv ((1 - \alpha) / \vartheta)^{\frac{1}{v}} (\alpha / (R + \delta - 1))^{\frac{v+\alpha}{v(1-\alpha)}} z_t^{\frac{1+v}{v(1-\alpha)}} \]
\[ i_t = a_{t+1} - (1 - \delta) a_t \]

**Normal times**

\[ k_t = a_t \]
\[ \frac{r_t}{R_t} = \int \frac{1}{\rho_t} p d\mu(p) \]
\[ \bar{p}_t = \frac{\rho_t}{R_t} \]
\[ R_t = \frac{\mu^{-1} (\rho_t - \gamma)}{(\rho_t - (1-\theta)\gamma)} \]
\[ y_t = c_t + i_t + (R_t - r_t) a_t \]

**Crisis times**

\[ k_t = a_t - \mu (\gamma / R_t) a_t \]
\[ \frac{r_t}{R_t} = \frac{\gamma}{R_t} \mu (\gamma / R_t) + \int \frac{1}{\gamma / R_t} p d\mu(p) \]
\[ \bar{p}_t = \gamma / R_t \]
\[ \rho_t = \gamma \]
\[ y_t = c_t + i_t + (R_t - r_t) a_t - (R_t - \gamma) (a_t - k_t) \]
The Banking Sector

A reduced form

- **Interest rate spread:**

\[ R_t - r_t = \begin{cases} \Delta_t^n & \text{if } a_t \leq \bar{a}_t(z_t) \\ \Delta_t^c & \text{otherwise} \end{cases}, \text{ with } \Delta_t^c > \Delta_t^n > 0 \]

- **Credit crunch:**

\[ a_t - k_t = \begin{cases} \psi_t^n = 0 & \text{if } a_t \leq \bar{a}_t(z_t) \\ \psi_t^c > 0 & \text{otherwise} \end{cases} \]

- Notice that all this is micro-founded