Price indices with variable mark-ups and changing variety

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Idea

- Price-index variations aim at measuring changes in the standard of living
- i.e. Money needed to obtain the same level of utility given new prices.
- Constructing a (micro-founded) price-index requires both prices, consumed quantities.
- Prices and consumed quantities are influenced by,
  - Change in costs
  - Change in mark-ups
  - Change in demand (preferences)
  - Changes in variety
- Methodological contribution: propose a simple framework that encompasses all these effects.

The problem with estimating price-index changes

- Implicit to the previous definition is that preferences don't change
- and thus that the demand system does not change.
- Assumption easily violated by the data $\longrightarrow$ so demand shocks are added (a.k.a. OLS residuals) ...
- ... which assume that preferences are changing.
- A way to solve this tension is proposed by Redding and Weinstein, 2018.

Redding and Weinstein (2018): reinterpreting demand shocks

- Introduce preference (demand) shifters by introducing vector $\varphi \in \mathbb{R}^n_{++}$.

$$u(q) \to u(\varphi q) \text{ where } \varphi q = \begin{bmatrix} \varphi_1 q_1 \\ \varphi_2 q_2 \\ \vdots \\ \varphi_n q_n \end{bmatrix}.$$

- Expenditure function,

$$\begin{aligned} e(p, u, \varphi) &= \min_q p' q \text{ s.t. } u(\varphi q) \geq u, \\ &= \min_{\tilde{q}} \left(\frac{p}{\varphi}\right)' \tilde{q} \text{ s.t. } u(\tilde{q}) \geq u, \qquad \text{set } (\tilde{q} = \varphi q) \\ &= e(p/\varphi, u). \end{aligned}$$

Demand: introducing variable markups

- Redding and Weinstein (2018) focus on CES-preferences

$$\mathcal{U} = \sum_{i=1}^{i=N} (\varphi_i q_i)^{\frac{\sigma - 1}{\sigma}}$$

- Leads to an iso-elastic demand when N is large
  $\longrightarrow$ ill-equipped to capture variations in markups.

Demand

- We assume instead a translog expenditure function as in Feenstra (2010) and Feenstra and Weinstein (2017).

$$\ln(e(p/\varphi)) = \alpha_0 + \alpha' \ln(p/\varphi) + \frac{1}{2}\ln(p/\varphi)'\Gamma \ln(p/\varphi).$$

- with $\sum_{i=1}^{n} \alpha_i = 1$, Γ is symmetric and each row/column sums to zero.
- Then,

$$s_i(p/\varphi) = \alpha + \Gamma \ln(p/\varphi).$$

Price index

- Exact price index given by an augmented Törnqvist index (Diewert, 1974)

$$\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0)) = \sum_{i=1}^{n} \frac{s_i^1 + s_i^2}{2} (\ln(p_i^1/\varphi_i^1) - \ln(p_i^0/\varphi_i^0)).$$

- Complication w.r.t. CES case: n is the total number of *potential* goods, some of which might not be available in a certain period.
- If i is not available then $s_i = 0$ and p_i/φ^i is equal to the reservation price.
- We can't construct the Törnqvist as we do not observe shadow prices nor the demand shifters φ^i.

Shadow prices

- Reservation prices are a function of the prices of all available varieties through the demand system.
- Notations:
  I set of common goods available in both periods;
  s_i^t expenditure share on i at time t;
  s_I^t total expenditure share on I at time t.
- New expression of our price-index:

$$\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0)) = \frac{1}{\gamma}\sum_{i=1}^{n}\frac{(s_i^0)^2-(s_i^1)^2}{2} - \frac{1}{\gamma}\sum_{i\in I}\frac{(s_i^0)^2-(s_i^1)^2}{S_I^0+S_I^1},$$
$$+\sum_{i\in I}\frac{s_i^0+s_i^1}{S_I^0+S_I^1}\left(\ln(p_i^1/\varphi_i^1)-\ln(p_i^0/\varphi_i^0)\right).$$

  where $S_I^1 = \sum_{i\in I} s_i^1$ and $S_I^0 = \sum_{i\in I} s_i^0$.
- When there is no change in variety, this coincides with the augmented Törnqvist index.
- Still, we don't know γ and φ_i^1/φ_i^0.

Estimation of γ - Price equation

- Oligopolistic competition.
- $\pi_i(p) = D_i(p)(p_i - c_i)$ where $D_i(p)$ is the demand for good i.
- Given the translog specifications, the FOC (taking expenditures as given) are,

$$p_i = c_i \left(1 + \frac{s_i n}{\gamma(n-1)}\right),$$

$$\frac{p_i - c_i}{c_i} = \frac{s_i}{\gamma}\frac{n}{n-1} \approx \frac{s_i}{\gamma}.$$

- Average mark-up is equal to $\frac{1}{\gamma(n-1)}$.
- Mark-up of firm i increases with its share and decreases with γ.

Estimation of γ - Identifying assumption

- Idea, independence between demand and supply shocks (Feenstra 1994)

$$(\varphi_i^1/\varphi_i^0) \perp (c_j^1/c_j^0) \text{ for all } i, j \in I.$$

- Problem: φ_i^1/φ_i^0 is not identified from data (given γ).
- We make a "difference in difference" (time periods + goods)
- With our pricing equation, the identifying assumption is

$$\theta = E\left[\left(\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0)\right)\left(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0)\right)\right] = 0.$$

- Holds if $cov(\ln(\varphi_i^1/\varphi_i^0), \ln(c_j^1/c_j^0)) = 0$ for $i, j \in I$.

Estimation

- Finite sample moment condition,

$$\hat{\theta}_{|I|} = \frac{1}{|I|(|I|-1)} \sum_{i,j \in I} \left[\left(\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0) \right) \left(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0) \right) \right] = 0.$$

- This is a U-statistic (van der Vaart, 1998),

$$\sqrt{|I|}\ (\hat{\theta}_{|I|} - \theta) \sim^a \mathcal{N}(0, 4\sigma^2),$$

where σ^2 is the variance of,

$$E\left[\left(\ln(\varphi_i^1/\varphi_i^0) - \ln(\varphi_j^1/\varphi_j^0) \right) \left(\ln(c_i^1/c_i^0) - \ln(c_j^1/c_j^0) \right) \Big| \varphi_i^1/\varphi_i^0; c_i^1/c_i^0 \right].$$

- We estimate $(1/\gamma)$ by minimizing $(\hat{\theta}_{|I|})^2$ (minimum should be zero).
- σ^2 can be consistently estimated using the finite sample analogue and the estimate of $1/\gamma$.
- Confidence intervals for $\widehat{1/\gamma}$ are derived using the Delta method.

Validity check

Monte Carlo simulation (1000 draws).

n	$\|I\|$	rejection rate (nominal level 5%)
30	10	0.164
90	30	0.087
150	50	0.068
300	100	0.060
900	300	0.056.

Application/Illustration

- ACNielsen homescan panel for Denver area (Aguiar and Hurst, 2007).
- Grocery packed goods purchases for a large number of households from Jan 1993 until March 1995
- Focus on 1993/1994 price indices
- We aggregate over consumers and year. Total value purchased of every good and total quantity purchased of every good.
- Good = UPC code
- Analysis for each product sub-category: 48 in total.
    - example: apple juice (42 upc's); orange juice (60 upc); tomato sauce (131 upc); soup (308 upc); instant (noodles) (157 UPC)...
    - We focus on groups with more than 30 goods common for both periods: 30 groups

- Summary statistics over the 30 groups.

	mean	std	min	max
γ	0.083257	0.070692	0.012714	0.31657
mean mark-up	0.25797	0.098305	0.098714	0.54176
$\lvert I \rvert$	93.7	78.480	30	321

Price decomposition

- Price decomposition

$$
\begin{aligned}
&\ln(e(p^1/\varphi^1)) - \ln(e(p^0/\varphi^0)), \\
&= \frac{1}{\gamma}\sum_{i=1}^{n}\frac{(s_i^0)^2-(s_i^1)^2}{2} - \frac{1}{\gamma}\sum_{i\in I}\frac{(s_i^0)^2-(s_i^1)^2}{S_I^0+S_I^1}, && \text{variety effect} \\
&+ \sum_{i\in I}\frac{s_i^0+s_i^1}{S_I^0+S_I^1}\ln((p_i^1/c_i^1)/(p_i^0/c_i^0)), && \text{mark-up effect} \\
&+ \sum_{i\in I}\frac{s_i^0+s_i^1}{S_I^0+S_I^1}\ln(c_i^1/c_i^0), && \text{cost-shift effect} \\
&- \sum_{i\in I}\frac{s_i^0+s_i^1}{S_I^0+S_I^1}\ln(\varphi_i^1/\varphi_i^0). && \text{demand-shift effect}
\end{aligned}
$$

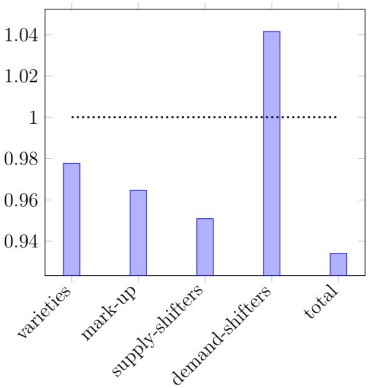
  Given (the estimate of) γ we can identify the first three effects. The fourth part (demand effect) is not identified without additional assumptions.

- We impose that,

$$\sum_{i\in I}\ln(\varphi_i^1/\varphi_i^0) = 0, \text{ then },$$

$$\ln(\varphi_i^1/\varphi_i^0) = \frac{s_i^1-s_i^0}{\gamma} - \frac{S_I^1-S_i^0}{\gamma|I|} + \ln(p_i^1/p_i^0) - \sum_{i\in I}\frac{\ln(p_i^1/p_i^0)}{|I|}.$$

Example



1.04
1.02
1
0.98
0.96
0.94
varieties
mark-up
supply-shifters
demand-shifters
total

Price decomposition

- Summary statistics (in %-points)

	varieties	mark-up	supply	demand	total
mean	-0.64418	-0.74634	0.33683	0.36598	-0.86497
std	1.0592	1.7483	6.3052	3.9010	4.5696
min	-3.5287	-4.8344	-9.9060	-15.8201	-11.3404
max	1.3733	1.8282	24.7810	5.1179	7.8084

- Correlation matrix (in %-points)

	varieties	mark-up	supply	demand	total
varieties	1.000	0.932	0.056	-0.460	0.235
mark-up	0.932	1.000	-0.005	-0.507	0.125
supply	0.056	-0.005	1.000	-0.581	0.782
demand	-0.460	-0.507	-0.581	1.000	-0.140
total	0.235	0.125	0.782	-0.140	1.000

Conclusion

- Importance of mark-ups
- Decomposition of price index into different components
- U-statistic based estimator for γ (for variable number of goods).
- Future research
  - Belgian supermarket data (?)
  - multi-product firms
  - Trade data (mulitple years/countries of origin)
  - Comparison with CES, including a supply-free for identification of γ
  - Beyond translog