EUROPEAN CENTRAL BANK
EUROSYSTEM

Simone Manganelli
European Central Bank

Discussion of "Quantile-based Inflation Risk Models" by Ghysels, Iania and Striaukas

National Bank of Belgium conference on "Understanding Inflation Dynamics"

Brussels, 25 October, 2018

Inflation dynamics

Quantile autoregression:

$$\pi_{t+1} = \mu + \rho \pi_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0 | \mathcal{F}_t) = \tau$$

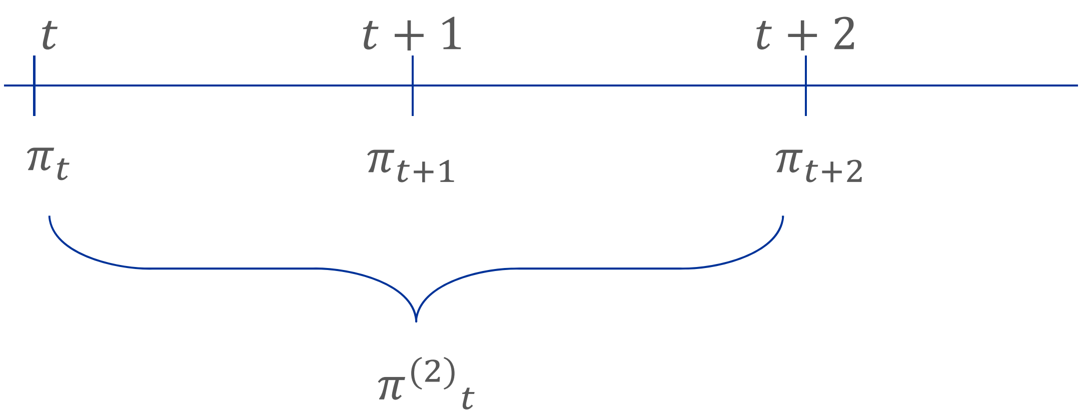
Inflation dynamics

Quantile autoregression:

$$\pi_{t+1} = \mu + \rho\pi_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

Two step ahead forecast:

$$\pi^{(2)}{}_t = \pi_{t+1} + \pi_{t+2}$$



Quantile forecast – Direct approach

Two step ahead forecast:

$$\pi_{t+2} = \mu + \rho\pi_t + \varepsilon_{t+2}$$

$$\mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_t) = \tau$$

$$\pi_{t+2} = \mu + \rho\pi_t + \beta Z_t(\theta) + \varepsilon_{t+2}$$

MIDAS volatility proxy:
$Z_t(\theta) = \Sigma\omega(\theta)|\Delta\pi_{t-m}|$

This is a problem of model selection.

The impact of inflation risk measures

I@R$_t$: $q_{\tau}(\pi_{t+2}|\mathcal{F}_t)$

IQR$_t$: $q_{1-\tau}(\pi_{t+2}|\mathcal{F}_t) - q_{\tau}(\pi_{t+2}|\mathcal{F}_t)$

ASY$_t$: …

The impact of inflation risk measures

I@R$_t$: $q_\tau(\pi_{t+2}|\mathcal{F}_t)$

IQR$_t$: $q_{1-\tau}(\pi_{t+2}|\mathcal{F}_t) - q_\tau(\pi_{t+2}|\mathcal{F}_t)$

ASY$_t$: $\dots$

$$\pi_{t+2} = \beta_0 + \beta_1 IQR_t + \beta_2 I@R_t + \cdots + \varepsilon_{t+2} \qquad \mathrm{E}(\varepsilon_{t+2}|\mathcal{F}_t) = 0$$

The impact of inflation risk measures

I@R$_t$: $q_\tau(\pi_{t+2}|\mathcal{F}_t)$

IQR$_t$: $q_{1-\tau}(\pi_{t+2}|\mathcal{F}_t) - q_\tau(\pi_{t+2}|\mathcal{F}_t)$

ASY$_t$: $\dots$

$$\pi_{t+2} = \beta_0 + \beta_1 IQR_t + \beta_2 I@R_t + \cdots + \varepsilon_{t+2} \qquad \mathrm{E}(\varepsilon_{t+2}|\mathcal{F}_t) = 0$$

$$\pi_{t+2} = \mu + \rho\pi_t + \beta Z_t(\theta) + \varepsilon_{t+2} \qquad \mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_t) = \tau$$

The impact of inflation risk measures

I@R$_t$: $q_\tau(\pi_{t+2}|\mathcal{F}_t)$

IQR$_t$: $q_{1-\tau}(\pi_{t+2}|\mathcal{F}_t) - q_\tau(\pi_{t+2}|\mathcal{F}_t)$

ASY$_t$: $\ldots$

$$\pi_{t+2} = \beta_0 + \beta_1 IQR_t + \beta_2 I@R_t + \cdots + \varepsilon_{t+2} \qquad \mathrm{E}(\varepsilon_{t+2}|\mathcal{F}_t) = 0$$

$$\pi_{t+2} = \mu + \rho\pi_t + \beta Z_t(\theta) + \varepsilon_{t+2} \qquad \mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_t) = \tau$$

- If I@R$_t$ affects the mean, it should also affect the quantile.
- The standard errors of the OLS coefficients are affected by the first stage quantile estimation.

Iterative quantile forecasts

Is quantile multi-step forecast really unfeasible?

$$y_{t+1} = \mu + \rho y_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

Iterative quantile forecasts

Is quantile multi-step forecast really unfeasible?

$$y_{t+1} = \mu + \rho y_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0 | \mathcal{F}_t) = \tau$$

$$y_{t+2} = (1 + \rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

$$\mathrm{P}(\varepsilon_{t+2} < 0 | \mathcal{F}_{t+1}) = \tau$$

Iterative quantile forecasts

Is quantile multi-step forecast really unfeasible?

$$y_{t+1} = \mu + \rho y_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

$$y_{t+2} = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

$$\mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_{t+1}) = \tau$$

$$q_\tau(y_{t+2}|\mathcal{F}_{t+1}) = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1}$$

Iterative quantile forecasts

Is quantile multi-step forecast really unfeasible?

$$y_{t+1} = \mu + \rho y_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

$$y_{t+2} = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

$$\mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_{t+1}) = \tau$$

$$q_\tau(y_{t+2}|\mathcal{F}_{t+1}) = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1}$$

$$q_\tau(q_\tau(y_{t+2}|\mathcal{F}_{t+1})|\mathcal{F}_t) = (1+\rho)\mu + \rho^2 y_t$$

Iterative quantile forecasts

Is quantile multi-step forecast really unfeasible?

$$y_{t+1} = \mu + \rho y_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

$$y_{t+2} = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

$$\mathrm{P}(\varepsilon_{t+2} < 0|\mathcal{F}_{t+1}) = \tau$$

$$q_\tau(y_{t+2}|\mathcal{F}_{t+1}) = (1+\rho)\mu + \rho^2 y_t + \rho\varepsilon_{t+1}$$

$$q_\tau(q_\tau(y_{t+2}|\mathcal{F}_{t+1})|\mathcal{F}_t) = (1+\rho)\mu + \rho^2 y_t$$

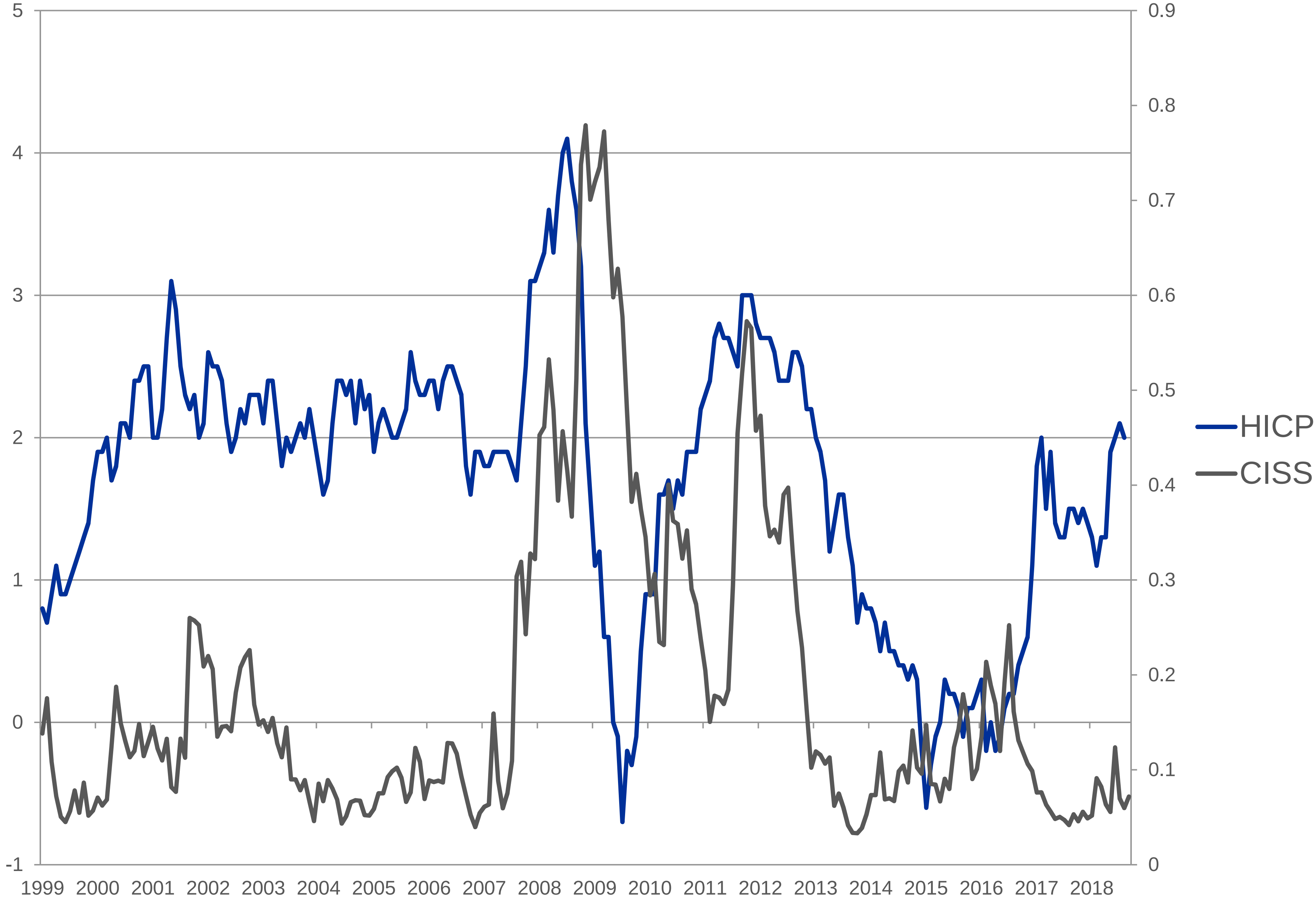
Law of Iterated Quantiles (Chavleishvili and Manganelli 2018)

Quantile VAR

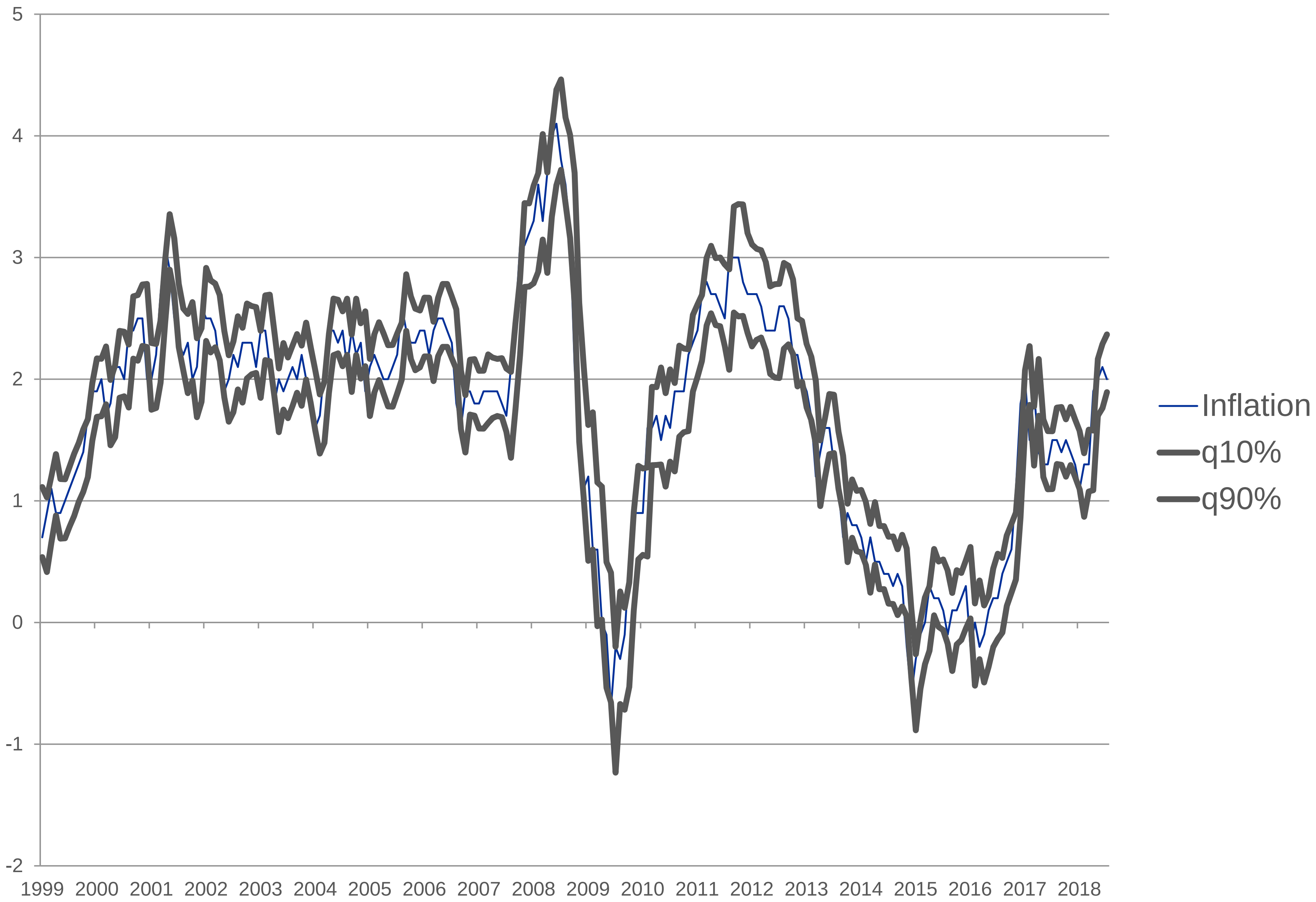
$$y_{t+1} = c + By_t + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|\mathcal{F}_t) = \tau$$

$$y_t = \begin{bmatrix} \pi_t \\ f_t \end{bmatrix} \begin{matrix} \longrightarrow \text{Inflation} \\ \longrightarrow \text{Financial stress (CISS)} \end{matrix}$$

Euro area inflation and CISS



Quantile VAR – One month inflation forecast



Quantile VAR and quantile impulse responses

$$y_{t+1} = c + By_t + \varepsilon_{t+1}$$

$$y_t = \begin{bmatrix} \pi_t \\ f_t \end{bmatrix}$$

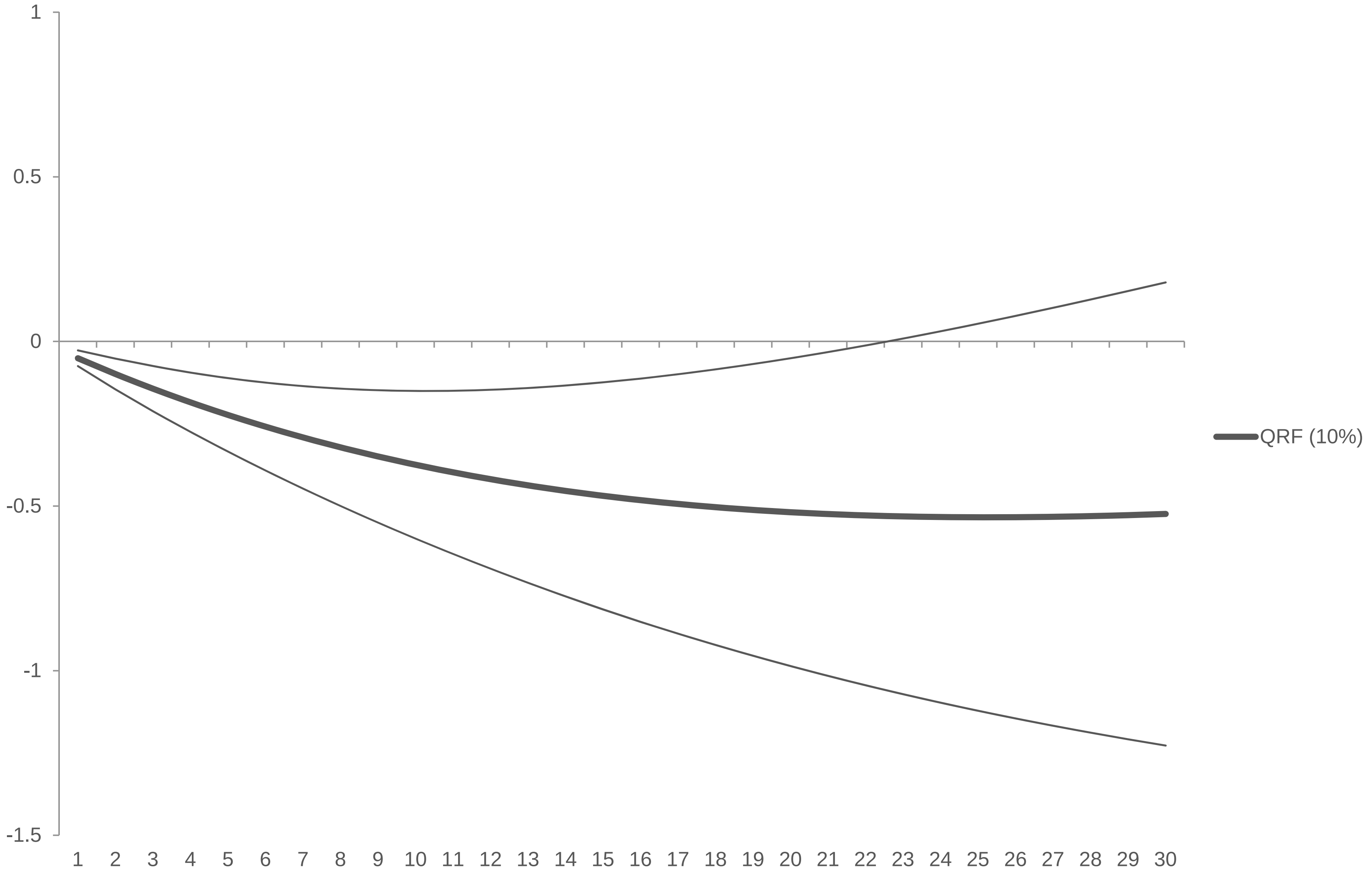
π_t ⟶ Inflation

f_t ⟶ Financial stress (CISS)

Identification: Inflation immediately affects CISS

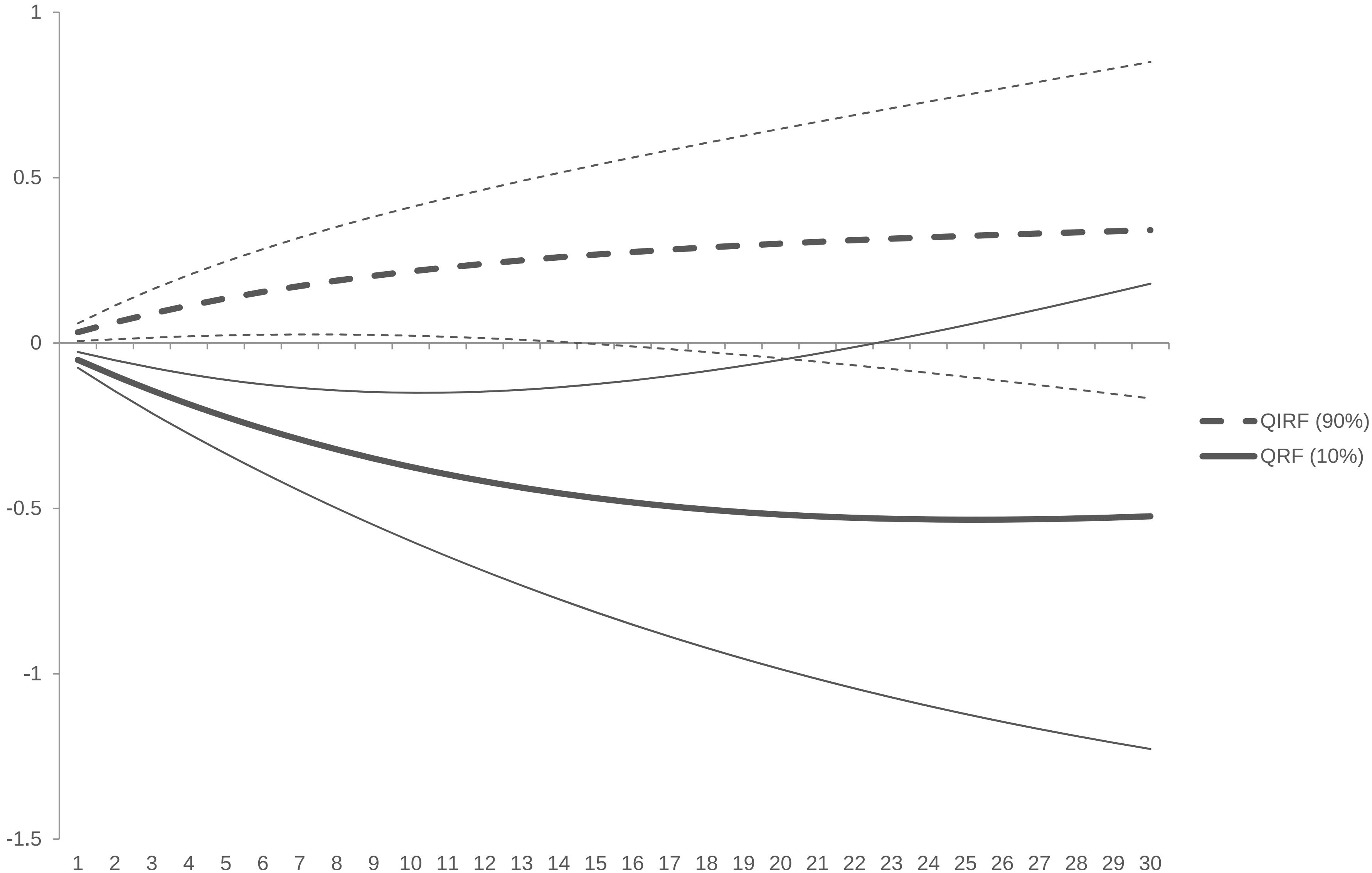
Quantile VAR and quantile impulse responses

Reaction of inflation quantiles to 90% shock to CISS



Quantile VAR and quantile impulse responses

Reaction of inflation quantiles to 90% shock to CISS



Conclusion

- Intriguing application of quantile regression to inflation.
- Construction of inflation risk measures can help policy makers better understand inflation dynamics.
- Usefully complements the idea of Growth-at-Risk (IMF).
- Promising area of research in macro time series, to be extended to quantile VAR.