Quantile-based Inflation Risk Models

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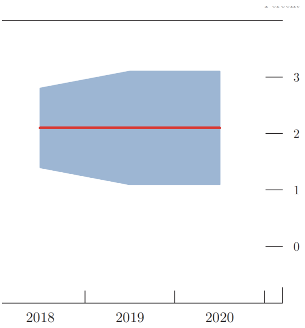
Outline

Motivation

"In assessing the path for the federal funds rate [...] FOMC participants take account of the range of possible economic outcomes, the likelihood of those outcomes, and the potential benefits and costs should they occur."

Source: Minutes of the Federal Open Market Committee. March 20-21, 2018

Figure: Fan chart of PCE Inflation



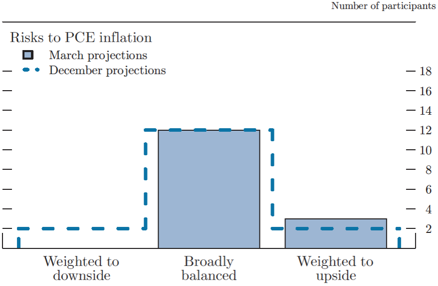
Note: Median projection and confidence interval based on historical forecast errors. Source: Minutes of the Federal Open Market Committee. March 20-21, 2018

Motivation

"Because the fan charts are constructed to be symmetric around the median projections, they do not reflect any asymmetries in the balance of risks that participants may see in their economic projections."

Minutes of the Federal Open Market Committee. March 20-21, 2018

Figure: Risks to PCE inflation



Note: FOMC participants assessments of risks around their economic projections. Source: Minutes of the Federal Open Market Committee. March 20-21, 2018

Motivation

- Zarnowitz and Lambros (1987) propose uncertainty measure using the inter-quantile range of survey data
- Kilian and Manganelli (2008) develop structural model in which monetary authorities can have non-quadratic valuation of inflation costs
- Kilian and Manganelli (2007) estimate several inflation risk measures using GARCH model
- Andrade et al. (2015) use survey data to extract the *expected* inflation asymmetry and document its impact on realized inflation and changes in monetary policy

In this (ongoing) project...

- We focus on quantile forecasting...
- ... in a mixed-frequency setting...
- ...and we construct quantile-based inflation risk measures

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- ...and we construct quantile-based inflation risk measures
  - $\text{I@R}^{\tau}_{t|t-h}$ = Inflation-at-risk
  - $\text{IQR}^{\tau}_{t|t-h}$ = Interquantile range
  - $\text{ASY}^{\tau}_{t|t-h}$ = Robust asymmetry

Outline

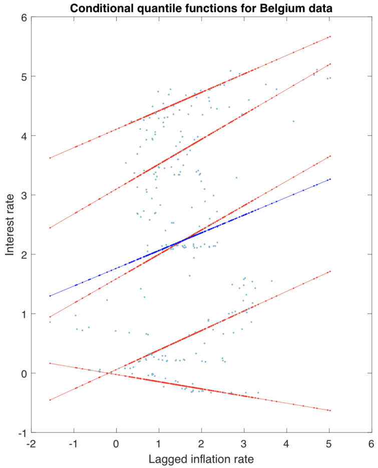
Regression quantiles: intuition

Linear regression (Blue line)

$$E(r_t|\pi_{t-1}) = \mu + \rho\pi_{t-1}$$

Quantile regression (Red lines)

$$q_\tau(r_t|\pi_{t-1}) = \mu(\tau) + \rho(\tau)\pi_{t-1}$$



Models

Quantile autoregressive model (QAR)

$$q^{\tau}_{\pi^h_{(t+h\,|\,t)}} = \mu_\tau + \sum_{j=0}^{p-1} \alpha_{\tau,j}\pi_{t-j} \equiv \mu_\tau + \rho_\tau \pi_t + \sum_{j=0}^{q-1} \beta_{\tau,j}\Delta\pi_{t-j}$$

QAR distributed lag model with MIDAS component (QADL-MIDAS)

$$q^{\tau}_{\pi^h_{(t+h\,|\,t)}} = \mu_\tau + \rho_\tau \pi_t + \beta_\tau Z_t(\theta)$$

where

$$Z_t(\theta_\tau) = \sum_{m=0}^{h} \omega_m(\theta_\tau)|\Delta\pi_{t-m}|$$

Modeling inflation quantiles: analysis

Setting

1. Based on monthly US CPI inflation (Headlines and CORE)
2. Period: 1960-01 to 2018-05
3. We focus on modeling/forecasting year-on-year inflation
4. Number of lags: 12.

Model comparison

1. In-sample
   - Are the regression coefficients quantile-dependent?
   - Is the MIDAS component informative about inflation quantiles?
2. Out-of-sample
   - Which model forecast quantiles better?
   - At which horizon?

In-sample analysis

$$q_{\pi^{(h)}_{t+h}}(\tau \mid \mathcal{F}_t) = \mu_\tau + \rho_\tau \pi_t + \sum_{j=0}^{q-1} \beta_{\tau,j} \Delta\pi_{t-j}$$

Quantile	0.05	0.25	0.5	0.75	0.95
μ	**-0.454**	**0.569**	**0.952**	**1.471**	**2.597**
	(0.047)	(0.000)	(0.000)	(0.000)	(0.000)
ρ	**0.502**	**0.593**	**0.713**	**0.866**	**1.101**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

$$q_{\pi^{(h)}_{t+h}}(\tau \mid \mathcal{F}_t) = \mu_\tau + \rho_\tau \pi_t + \beta_\tau Z_t(\theta)$$

Quantile	0.05	0.25	0.5	0.75	0.95
μ	0.062	**0.723**	**0.581**	**0.658**	**1.891**
	(0.371)	(0.000)	(0.000)	(0.002)	(0.000)
β	**-1.522**	-0.446	**2.738**	**3.507**	**2.335**
	(0.014)	(0.219)	(0.000)	(0.000)	(0.014)
θ	43.668	35.510	1.124	1.716	1.000
	(0.112)	(0.180)	(0.465)	(0.439)	(0.482)
ρ	**0.459**	**0.564**	**0.678**	**0.928**	**1.168**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Out-of-sample analysis

	CPI									
	12-months					3-months				
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
CW statistic	**2.289****	0.677	**2.304****	**3.127*****	**3.364*****	-2.681	1.288	-3.172	**3.507****	**3.550****
p-Value	(0.015)	(0.259)	(0.014)	(0.002)	(0.001)	(0.995)	(0.109)	(0.999)	(0.000)	(0.000)
ratio	0.796	0.951	0.959	0.886	0.657	1.122	0.958	1.034	0.903	0.829
	CPI CORE									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
CW statistic	-2.311	-1.930	0.346	**2.467*****	**2.779*****	-0.189	**3.240*****	**3.598*****	**3.597*****	**3.618*****
p-value	(0.986)	(0.967)	(0.370)	(0.009)	(0.004)	(0.572)	(0.001)	(0.000)	(0.000)	(0.000)
ratio	1.168	1.081	0.922	0.743	0.730	1.012	0.898	0.964	0.862	0.754

Outline

Inflation risk measures (based on Andrade et al. (2015))

Inflation-at-risk

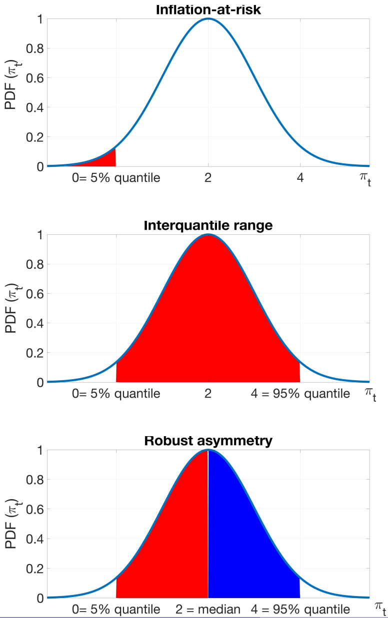
$$I@R^{\tau} = \hat{q}_{\tau}$$

Interquantile range

$$IQR^{\tau} = \hat{q}_{1-\tau} - \hat{q}_{\tau}$$

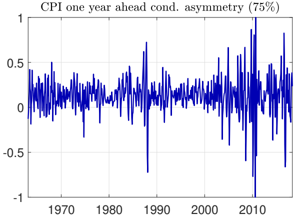
Contitional (robust) asymmetry

$$ASY^{\tau} = \frac{(\hat{q}_{1-\tau} - \hat{q}_{0.50}) - (\hat{q}_{0.50} - \hat{q}_{\tau})}{\mathrm{IQR}^{\tau}}$$

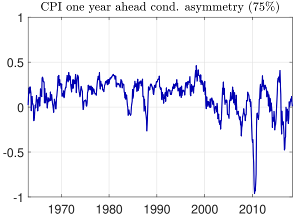


Conditional asymmetry comparison

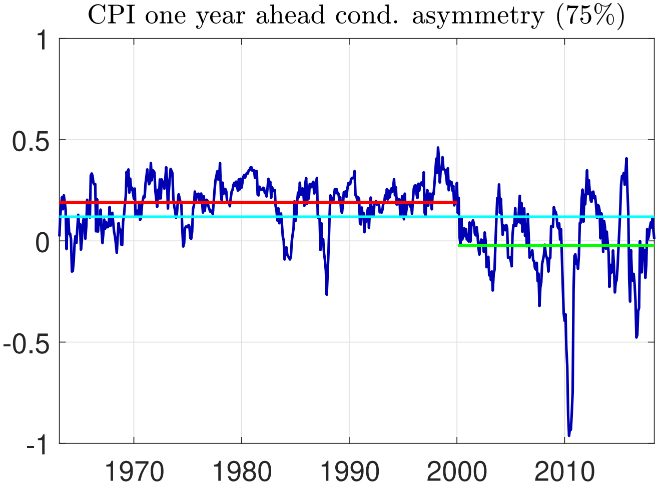
QAR(12) conditional asymmetry



QADL-MIDAS conditional asymmetry



Conditional asymmetry regimes



Note: 12 months ahead conditional asymmetry (75%) for CPI data

Inflation risk measures and monetary policy

We study the relationship between federal fund rates and inflation risk via a two-step procedure:

1. Construct real-time risk measures
2. Regress federal fund rates changes on real-time risk measures and a set of control variables

Step 2: Federal fund rates changes and real-time risk measures

Following Andrade, Ghysels, and Idier (2015), our augmented Taylor Rule-type regression equation is:

$$\Delta i_{t+1} = \beta_0 + \beta_1 \mathrm{IQR}^{\tau}_{t|t-h} + \beta_2 \mathrm{ASY}^{\tau}_{t|t-h} + \rho' C_t + \epsilon_{t+1}$$

where:

- Δi_{t+1} is the change in the effective federal funds rate (EFFR)
- C_t contains
  - Lagged value of the EFFR, Δi_t
  - Headline inflation, π^h_t
  - Commodity inflation, $\pi^h_{t,\mathrm{com}}$
  - A measure of output gap compute using industrial production, u_t.

Table: Parameter estimates and model fit

	01-Mar-1963 to 01-Apr-2018			
	IQR 5%	**ASY 5%**	**IQR 25%**	**ASY 25%**
coeff.	-0.014	**0.192****	-0.033	**0.120****
p-Value	(0.403)	(0.012)	(0.237)	(0.043)
R^2_{adj}		0.184		0.184

Dependent variable is real time change in effective federal funds rate. ***, ** and * refer to 1, 5 and 10 percent significance levels.

$$\Delta i_{t+1} = \beta_0 + \beta_1 \text{IQR}^{\tau}_{t|t-h} + \beta_2 \text{ASY}^{\tau}_{t|t-h} + \rho' C_t + \epsilon_{t+1}$$

Table: Parameter estimates and model fit

01-Jan-1980 to 01-Apr-2018

	IQR 5%	ASY 5%	IQR 25%	ASY 25%
coeff.	0.004	**0.108***	0.001	0.072
p-Value	(0.860)	(0.068)	(0.987)	(0.190)
R^2_{adj}		0.179		0.179

01-Mar-1963 to 01-Dec-1978

	IQR 5%	ASY 5%	IQR 25%	ASY 25%
coeff.	**-0.054*****	**0.297****	**-0.101*****	**0.106****
p-Value	(0.002)	(0.022)	(0.003)	(0.059)
R^2_{adj}		0.364		0.383

01-Mar-1963 to 01-Nov-2008

	IQR 5%	ASY 5%	IQR 25%	ASY 25%
coeff.	-0.007	**0.279****	-0.035	0.136
p-Value	(0.730)	(0.050)	(0.262)	(0.126)
R^2_{adj}		0.190		0.189

Outline

Summary of results

1. We propose a new approach to extract quantile-based inflation risk measures
2. Our model accounts for absolute past inflation changes in quantile modeling and can handle mixed-frequency data sampling
3. Our model performs favorably with respect to a standard QAR model in terms of prediction of conditional quantiles
4. We use our model-based quantiles to construct inflation-risk measures
5. We show that there is a positive and significant relationship of changes in Effective Federal Funds rate and conditional asymmetry
6. Results are in line with survey data based inflation conditional asymmetry, Andrade et al. (2015)

References I

ANDRADE, P., E. GHYSELS, AND J. IDIER (2015): "Tails of Inflation Forecasts and Tales of Monetary Policy," .

KILIAN, L. AND S. MANGANELLI (2007): "Quantifying the risk of deflation," *Journal of Money, Credit and Banking*, 39, 561–590.

——— (2008): "The central banker as a risk manager: Estimating the Federal Reserve's preferences under Greenspan," *Journal of Money, Credit and Banking*, 40, 1103–1129.

ZARNOWITZ, V. AND L. A. LAMBROS (1987): "Consensus and uncertainty in economic prediction," *Journal of Political economy*, 95, 591–621.