Endogenous risk in a DSGE model with capital-constrained financial intermediaries

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motivation/objective

- introduce endogenous financial risk in a standard DSGE macromodel:
  Mendoza (2010), Br&Sa (2012) and He&Kr (2012) use specific, small-scale models

- common framework for analysing traditional monetary policy questions (inflation, output gap) and financial stability concerns (risk, volatility, asset pricing, credit supply)

- financial risk is priced by financial intermediaries that are occasionally capital-constrained (following He and Krishnamurthy (2012)):
  see also Gilchrist Zakrajeck (2012) and Adrian et al. (2012)

- use local third-order perturbation approach to solve the model: evaluate the trade-off between approximation errors and computational efficiency of the solution that is also applicable to models with a larger state vector

outline of the paper/presentation

- start from a simplified version of He and Krishnamurthy (2012):
  - continuous-time, global solution with occasionally-binding capital constraint
  - main mechanism behind endogenous risk channel

- local third-order approximation of this simple model:
  - occasionally-binding constraint => non-linear but continuous approximation
  - global dynamics => local solution with third-order perturbation method

- application of this methodology in a standard DSGE macromodel:
  - evaluate the risk channel in a model with additional real and nominal frictions
  - counterfactual simulation illustrating the potential role of the risk channel
  - interaction with monetary policy behavior: risk and risk-free interest rate

Simplified version of He and Krishnamurthy (2012)

- simplification: only productive capital stock (no housing stock)

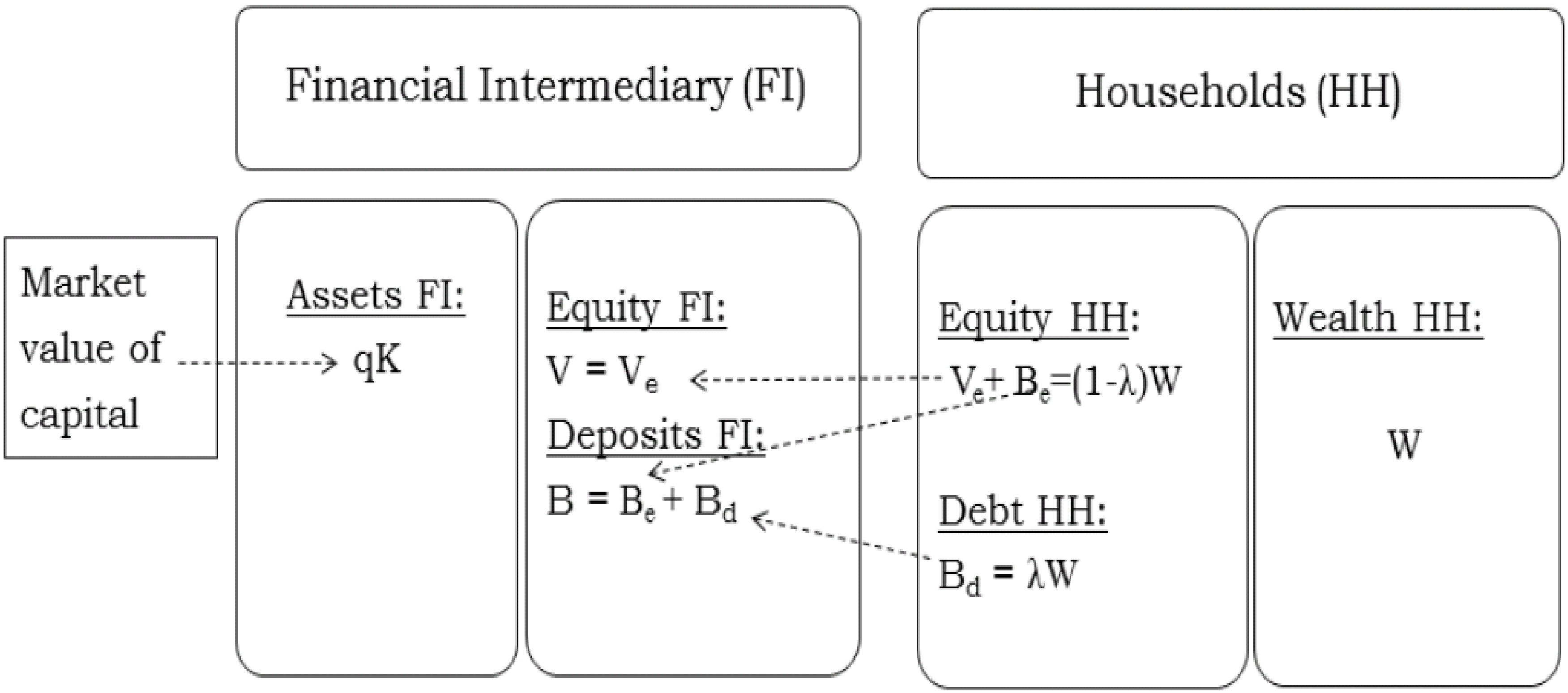
- households: maximize expected utility of consumption and allocate their wealth between risk-free deposits and equity of financial intermediaries

- financial intermediaries: maximize expected utility of reputation (function of past return performance) subject to their balance sheet => mean-variance portfolio optimization

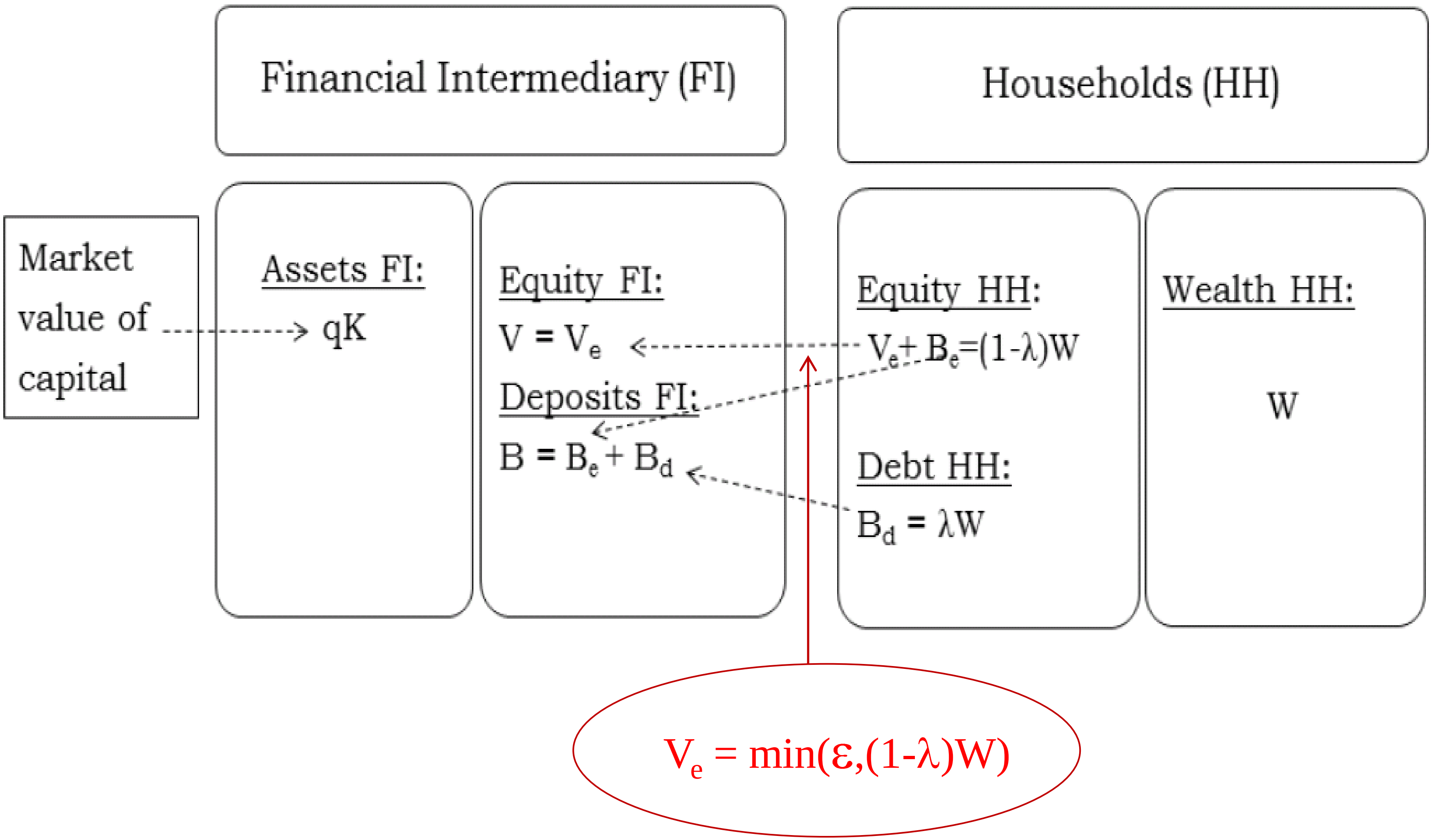
- production: simple AK production technology (Y=AK) and quadratic adjustment costs in capital accumulation

- financial frictions: households have no direct access to the capital market (financial intermediary is the marginal investor in capital), and their equity holdings in FI are restricted by the reputation of the intermediary => FI occasionally capital-constrained

Simplified version of He and Krishnamurthy (2012)



Simplified version of He and Krishnamurthy (2012)



Simplified version of He and Krishnamurthy (2012)

- optimal leverage of financial intermediaries: $\alpha_t^{FI} = \frac{E_t\left[dR_t\right] - r_t}{mVar_t\left[dR_t\right]}$
- capital market equilibrium: $\alpha_t^{FI} V_t = q_t K_t$
- occasional constraint on HH equity position: $V_t = \min(\mathcal{E}_t, (1-\lambda)W_t)$
- reputation dynamics: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = mdR_t^{FI} - \eta dt + d\psi_t$

=> equilibrium required return on capital: $dR_t = \mu_{dR,t} dt + \sigma_{dR,t} dZ_t$

with $\mu_{dR,t}$, $\sigma_{dR,t}$ both time-varying coefficients and dependent on the volatility of reputation and the sensitivity of the asset price to reputation

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dR_t ⬇ => dR_t^{FI} ⬇ => $d\mathcal{E}_t$ ⬇

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=> endogenous risk channel: exog. shock to the efficiency of capital => dZ_t ⬇ =>

dR_t ⬇ => dR_t^{FI} ⬇ => $d\mathcal{E}_t$ ⬇ => V_t ⬇ => α_t^{FI} ⬆

Simplified version of He and Krishnamurthy (2012)

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=> equilibrium required return on capital: $dR_t = \mu_{dR,t} dt + \sigma_{dR,t} dZ_t$

=> endogenous risk channel: exog. shock to the efficiency of capital => $dZ_t \downarrow$ =>

$dR_t \downarrow$ => $dR_t^{FI} \downarrow$ => $d\mathcal{E}_t \downarrow$ => $V_t \downarrow$ => $\alpha_t^{FI} \uparrow$ => $E[dR_t] \uparrow$

Simplified version of He and Krishnamurthy (2012)

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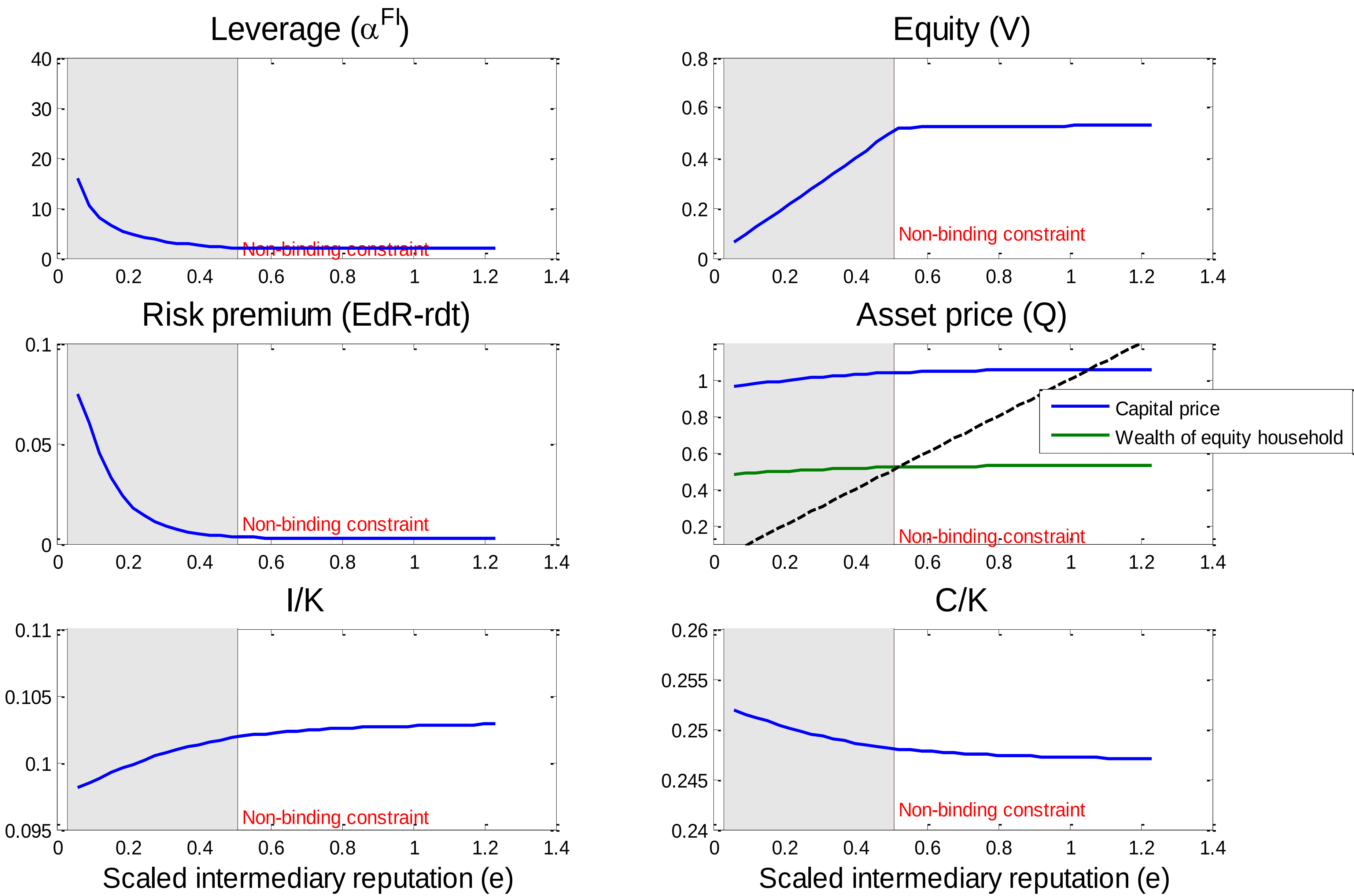
dR_t ⬇ => dR_t^{FI} ⬇ => $d\mathcal{E}_t$ ⬇ => V_t ⬇ => α_t^{FI} ⬆ => $E\left[dR_t\right]$ ⬆ => q_t ⬇ => i_t ⬇

Simplified version of He and Krishnamurthy (2012)

- calibration similar to He and Krishnamurthy (2012) but with lower volatility for the exogenous shock to the efficiency of capital (2% instead of 5%):

Table 1: Calibrated parameters of the model (in per annum terms)

Parameter	**Description**	**Baseline**
Production		
σ	Capital efficiency shock	0.02
δ	Depreciation rate	0.10
κ	Adjustment cost	20
$\bar{A}$	Productivity constant	0.35
ϕ	Productivity price feedback	0
l	Wage share	0.60
Intermediation		
m	Reputation sensitivity	2.50
λ	Debt share	0.50
η	Banker exit rate	0.13
γ	Entry Sharpe ratio	2
Other		
ρ	Discount rate	0.02


Leverage (α^{FI})
Equity (V)
Risk premium (EdR-rdt)
Asset price (Q)
I/K
C/K
Non-binding constraint
Capital price
Wealth of equity household
Scaled intermediary reputation (e)

Simple version of He and Krishnamurthy (2012)

	S-He&Kr		Data*	
	Distress	Non-distress	Distress	Non-distress
Vol(Eq)	42.66	19.72	31.48	17.54
Vol(SR)	23.87	2.30	60.14	12.72
Vol(Q)	1.26	0.92		
Vol(I)	2.23	2.07	8.05	6.61
Vol(C)	1.41	1.66	1.71	1.28
Cov(Eq,I)	0.83	0.37	1.31	0.07
Cov(Eq,C)	0.48	0.28	0.25	0.03
Cov(Eq,SR)	-2.88	-0.01	-6.86	-0.14

Simplified version of He and Krishnamurthy (2012)

- this model generates an important risk channel:
  - "endogenous" risk and volatility in asset prices >> fundamental exogenous productivity risk
  - the role of risk is particular striking when the capital constraint on financial intermediaries becomes binding

- two useful extensions:
  - the interaction and feedback between financial and real variables can be reinforced with a more complex transmission mechanism from asset prices on production or consumption decisions (e.g. credit contraint on working capital)
  - occasionally-binding minimum constraint in HH risk position can be replaced by a non-linear but continuous function => necessary step for local perturbation

Local approximation

- He and Krishnamurthy specify their model in continuous time and solve the global dynamics with shifts between regimes with a binding and non-binding capital constraint

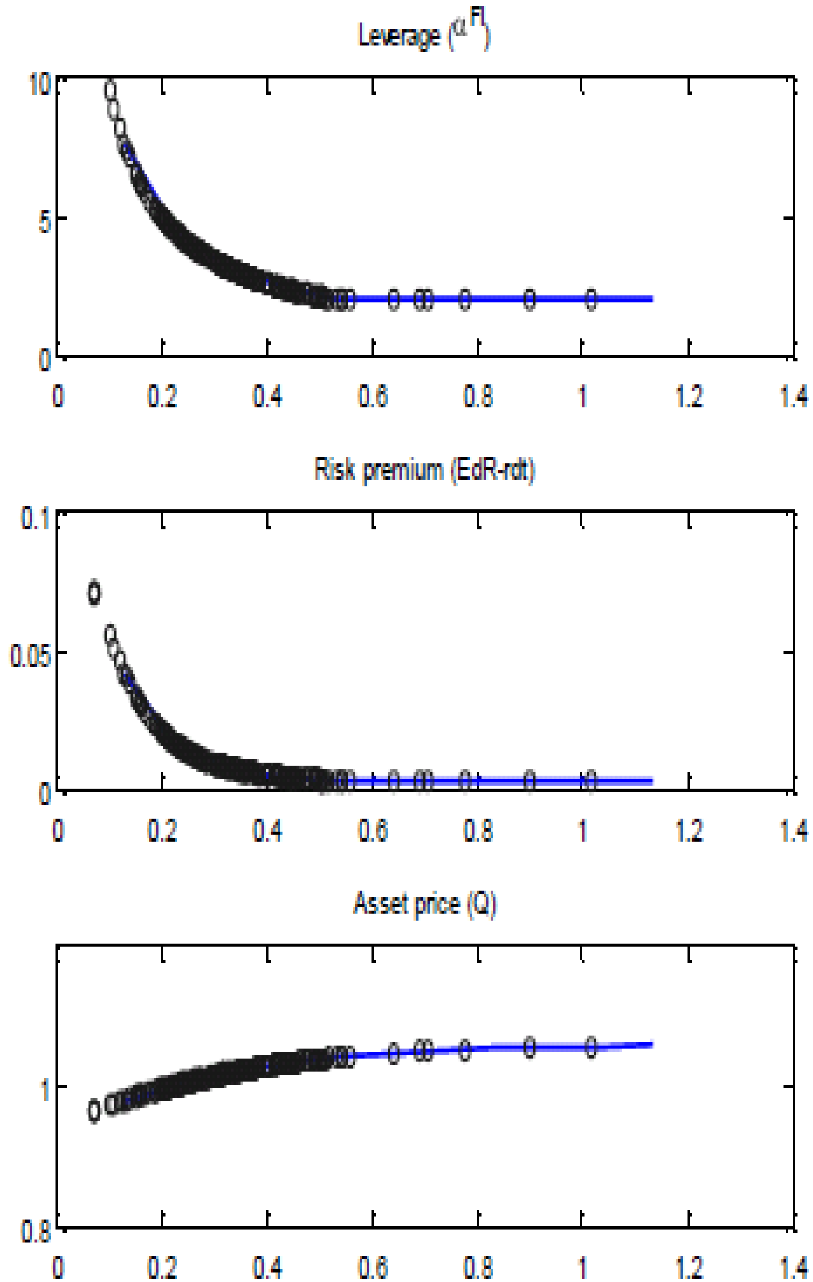
- By replacing the minimum constraint on HH risky position by a continuous portfolio rule, we can solve the model locally with a much more efficient perturbation approach:

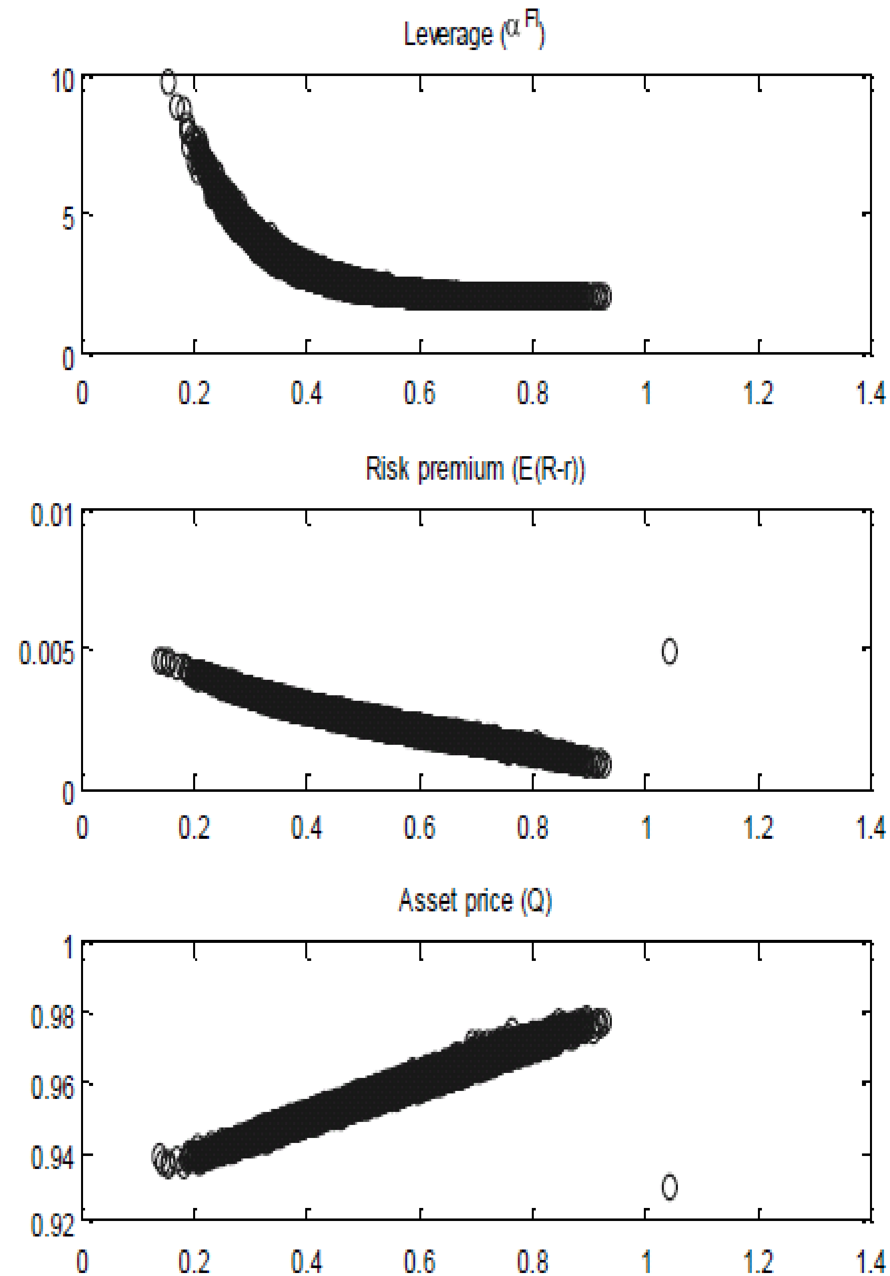
$$\alpha^{FI} = \frac{q_t K_t}{V_t = \min(\mathcal{E}_t, (1-\lambda)W_t)} \quad ==> \quad \alpha^{FI} = \frac{1}{1-\lambda} + \alpha_{\mathcal{E}} \left(\frac{q_t K_t}{\mathcal{E}_t - \mathcal{E}_-} \right)^3$$

- The model, more precisely the reputation of the FI, is indeterminate under a first-order approximation but with a third-order approximation the risk channel generates a stable outcome: $\mathcal{E}_t = \mathcal{E}_{t-1}(1 + m\widetilde{R}_t - \eta)$. The approximation is taken around the center of the stochastic distribution. $\alpha_{\mathcal{E}}$ and $\mathcal{E}_-$ are selected to approximate the min-constraint.

S-HeKr

local approximation of S-HeKr





Local approximation

	Local Approx		S-He&Kr	
	Distress	Non-distress	Distress	Non-distress
Vol(Eq)	23.90	9.44	42.66	19.72
Vol(SR)	2.81	0.58	23.87	2.30
Vol(Q)	0.49	0.50	1.26	0.92
Vol(I)	2.18	2.23	2.23	2.07
Vol(C)	1.86	1.90	1.41	1.66
Cov(Eq,I)	0.40	0.12	0.83	0.37
Cov(Eq,C)	0.36	0.10	0.48	0.28
Cov(Eq,SR)	-1.11	-0.01	-2.88	-0.01

Local approximation

- the local approximation can reproduce quite precisely the dynamics of the leverage and the equity of the FI
- risk aversion and the risk premium are time-varying and linear functions of the reputation-state of the FI
- the convex relation between risk premium and reputation is lost in the approximation

=> we retain a first-order approximation of the risk channel with our local approximation

- the amount of risk (vol) and the price of risk are relatively small because the exogenous volatility is small and because the model has a very weak internal propagation mechanism

=> apply this methodology in a standard DSGE macromodel with real and nominal frictions that reinforces the propagation of the exogenous shock

Application in DSGE model

- model extentions with a more realistic production and distribution specification:
  - endogenous labor supply, habit in preferences and GHH utility function for HH
  - CD production problem with fixed costs that correspond to the price markup
  - monopolistic competition in good and labor markets
  - nominal price and wage stickiness
  - monetary policy rule: simple inflation targeting strategy

=> many of these frictions increase the propagation of exogenous shocks and, important for asset pricing, increase the volatility of dividend returns for the FI

Application in DSGE model

- Calibration and estimation of the productivity shock as single fundamental shock

Table 10: Parameter calibration

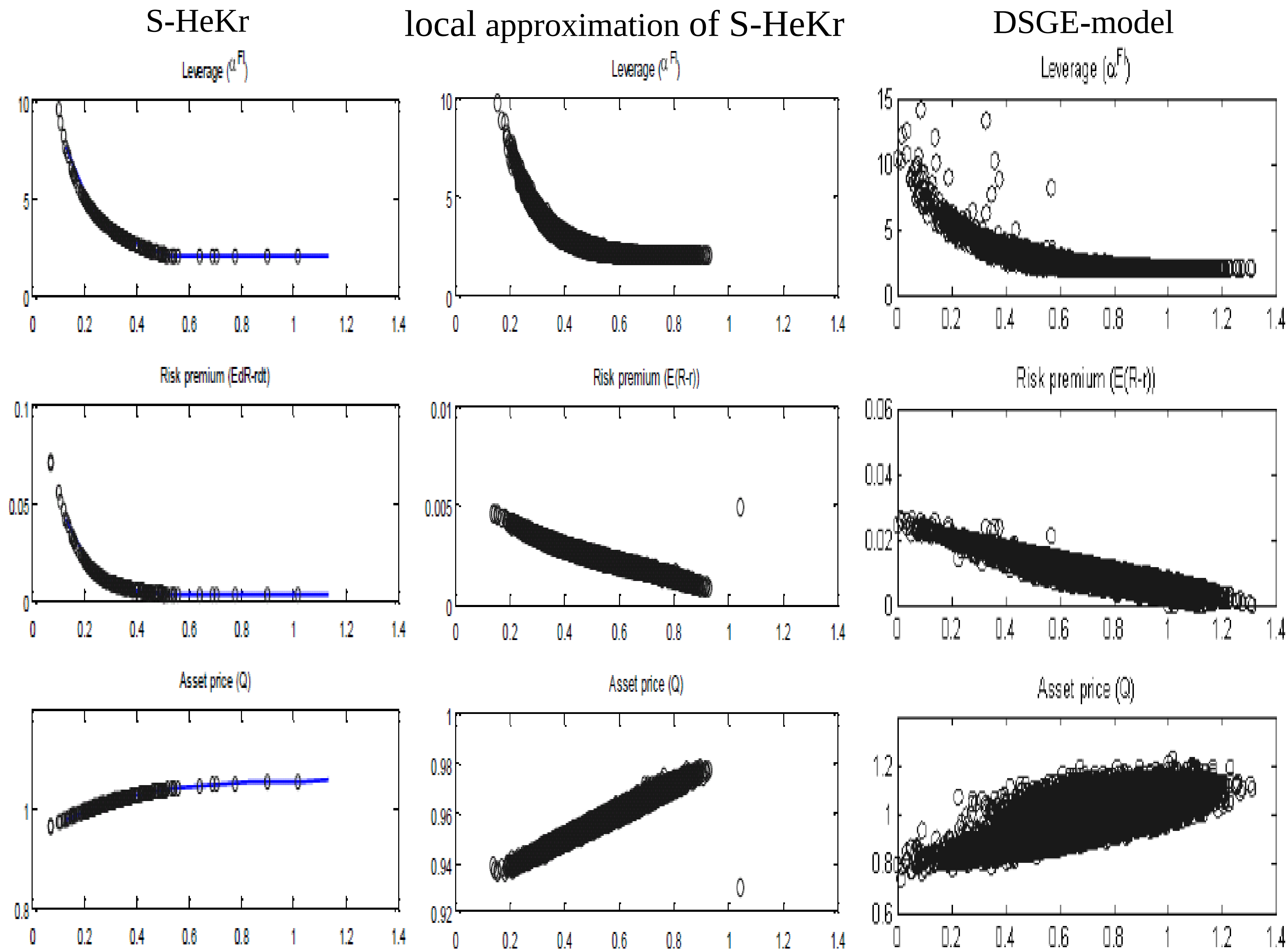
β	δ	σ	h	ϕ	α	κ	κ_p	κ_w
0.96	0.10	1.0	0.3	1.0	0.6	25	0.10	0.02

ι	Ψ	r_π	$\overline{\pi}$	λ	m	$\overline{\varepsilon}$	$\underline{\varepsilon}$
0.5	0.20	1.5	0.0	0.5	2.5	1.0	0.2

Table 11: Estimated standard errors of DSGE model (first-order approximation)

σ_z	ρ_z	σ_{GDP}	σ_{inv}	σ_{cons}	σ_{hours}
0.0075	0.95	0.48	0.58	1.19	0.45

- estimation with four US-data Y, C, I and W/P growth over the period 1955q1-2011q2


S-HeKr
local approximation of S-HeKr
DSGE-model
Leverage (α^{FI})
Leverage (α^{FI})
Leverage (α^{FI})
Risk premium (EdR-rdt)
Risk premium (E(R-r))
Risk premium (E(R-r))
Asset price (Q)
Asset price (Q)
Asset price (Q)
0 0.2 0.4 0.6 0.8 1 1.2 1.4

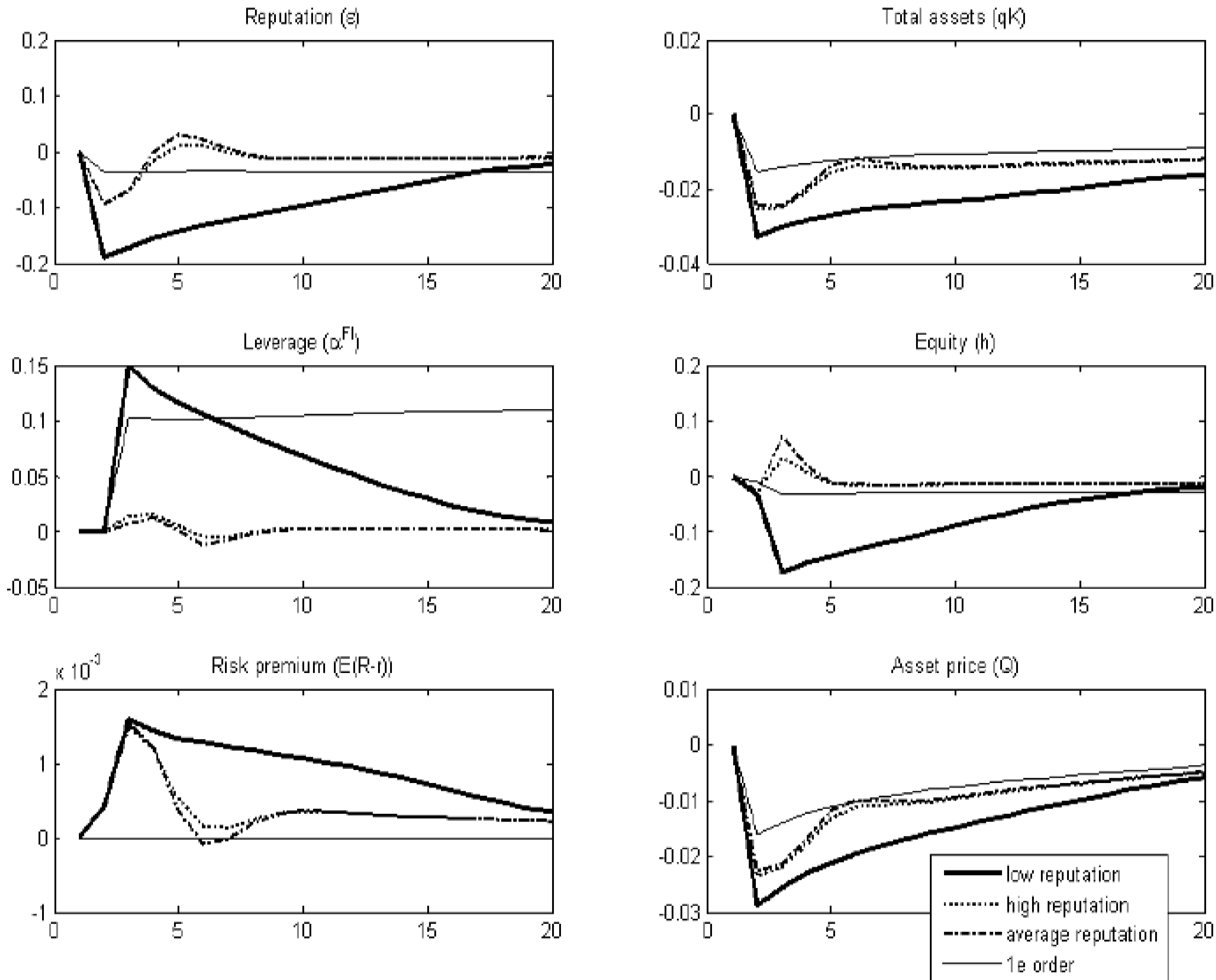
Application in DSGE model

	DSGE Model		Local Approx		Data*	
	Distress	Non-distress	Distress	Non-distress	Distress	Non-distress
Vol(Eq)	24.28	13.43	23.90	9.44	31.48	17.54
Vol(SR)	7.20	1.68	2.81	0.58	60.14	12.72
Vol(Q)	5.42	5.08	0.49	0.50		
Vol(I)	8.90	8.17	2.18	2.23	8.05	6.61
Vol(C)	1.13	1.19	1.86	1.90	1.71	1.28
Cov(Eq,I)	1.50	0.37	0.40	0.12	1.31	0.07
Cov(Eq,C)	0.15	0.08	0.36	0.10	0.25	0.03
Cov(Eq,SR)	-2.07	-0.08	-1.11	-0.01	-6.86	-0.14

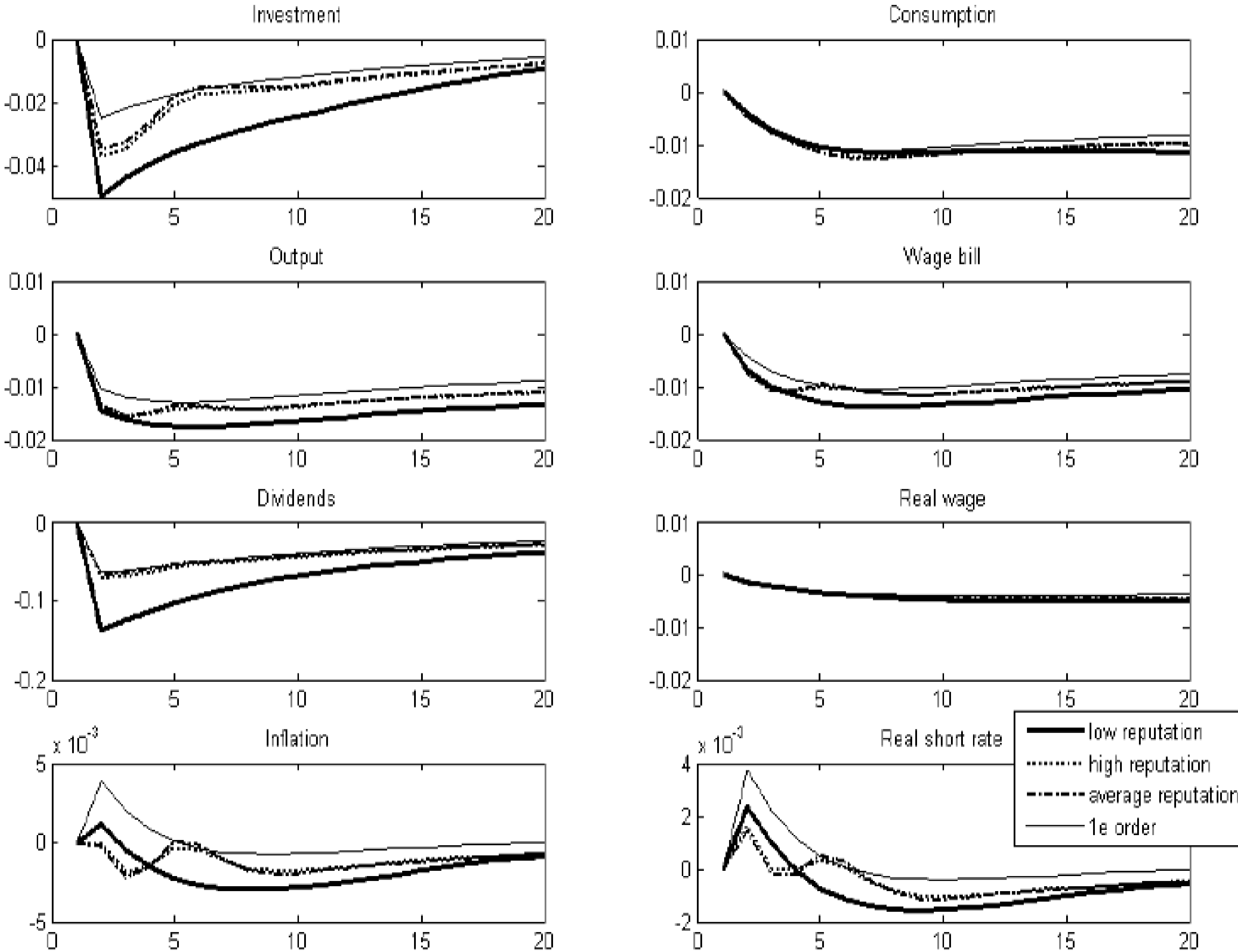
Application in DSGE model

- the impulse response analysis helps to understand the transmission of the productivity shocks:
  - the IRFs are state-dependent: impact is different in states with high, medium and low reputation
  - compare also with the effect under certainty equivalence (first-order approximation): the difference between the third-order and the first-order solution is a proxy for the risk channel in the economy

- graphs plot the IRFs for a one standard error decline in the productivity shock (0.75%)



Reputation (s)
Total assets (qK)
Leverage (α^FI)
Equity (h)
x 10^-3
Risk premium (E(R-r))
Asset price (Q)
low reputation
high reputation
average reputation
1e order



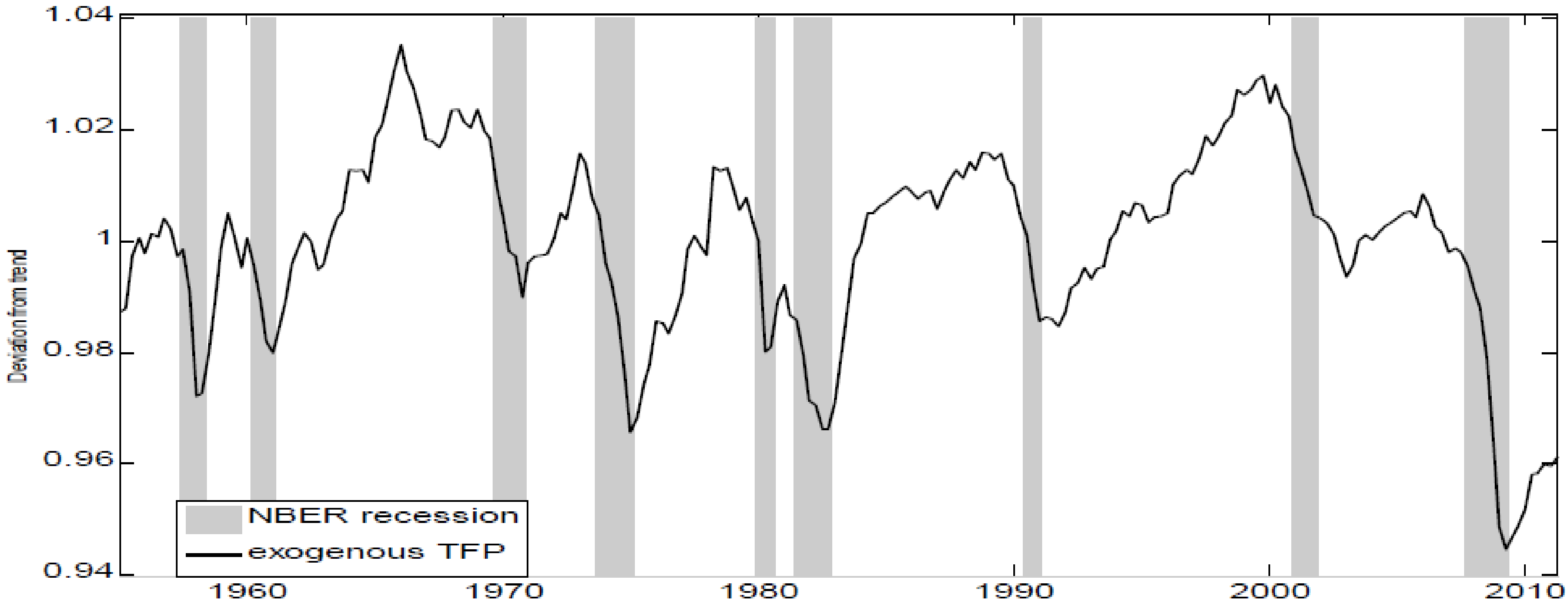
Investment
Consumption
Output
Wage bill
Dividends
Real wage
Inflation
Real short rate
x 10^-3
low reputation
high reputation
average reputation
1e order

Application in DSGE model

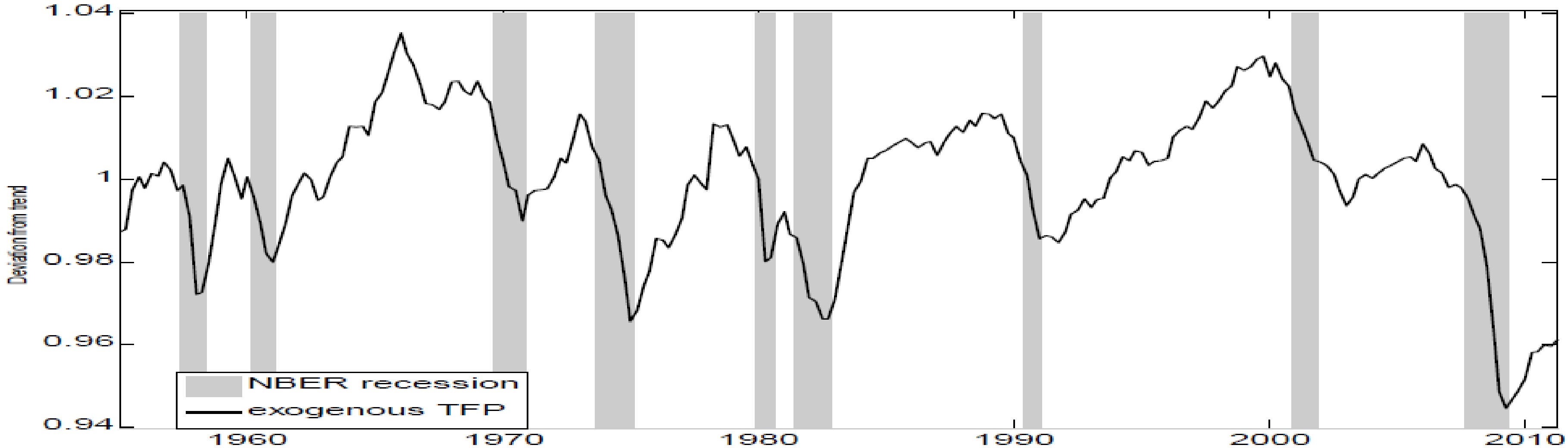
- historical counterfactual simulation: using the model-consistent estimate for the productivity process and the implied transmission mechanism, we obtain an estimate of the potential contribution of the risk channel to the historical macroeconomic realisations by comparing the outcomes under third-order and first-order:

- important contribution of the risk channel in the transmission of fundamental shocks
- strong interaction between monetary policy and risk premium

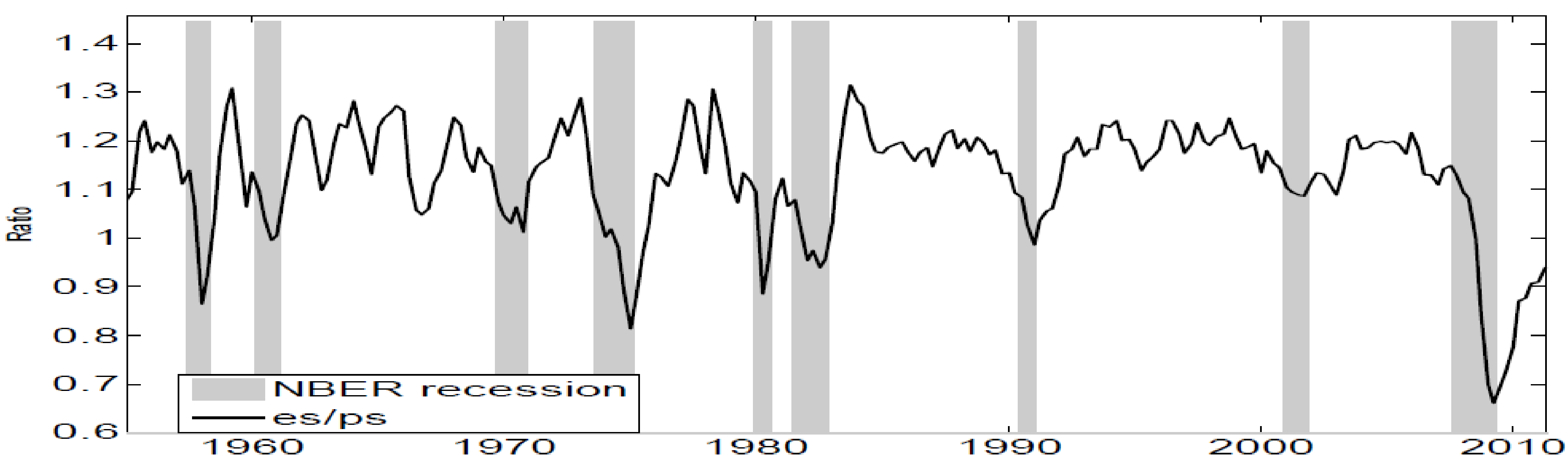
Exogenous total factor productivity process



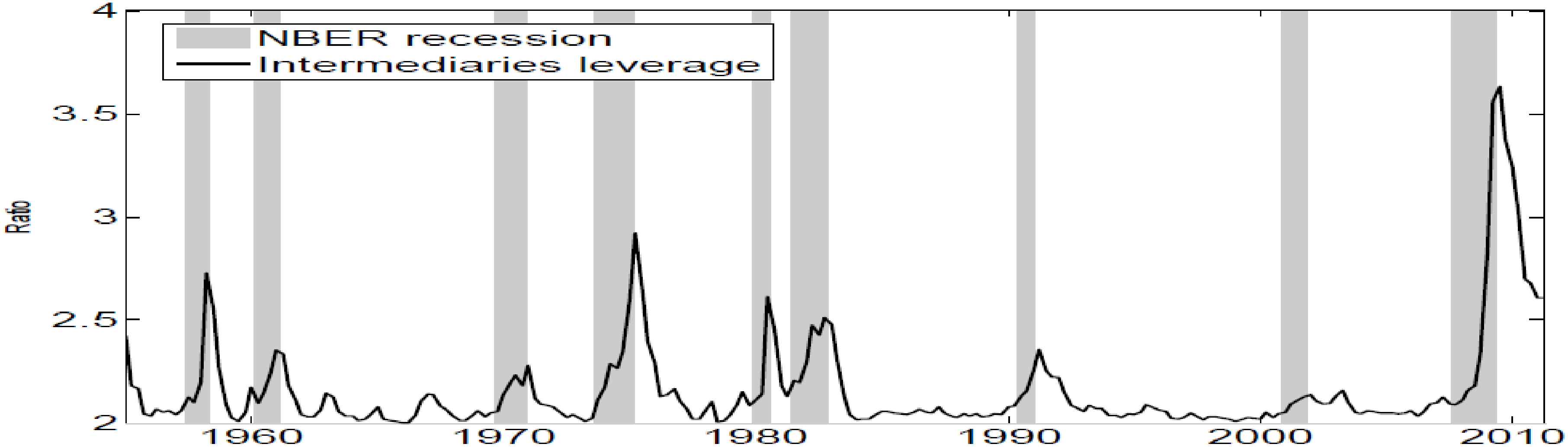
Exogenous total factor productivity process



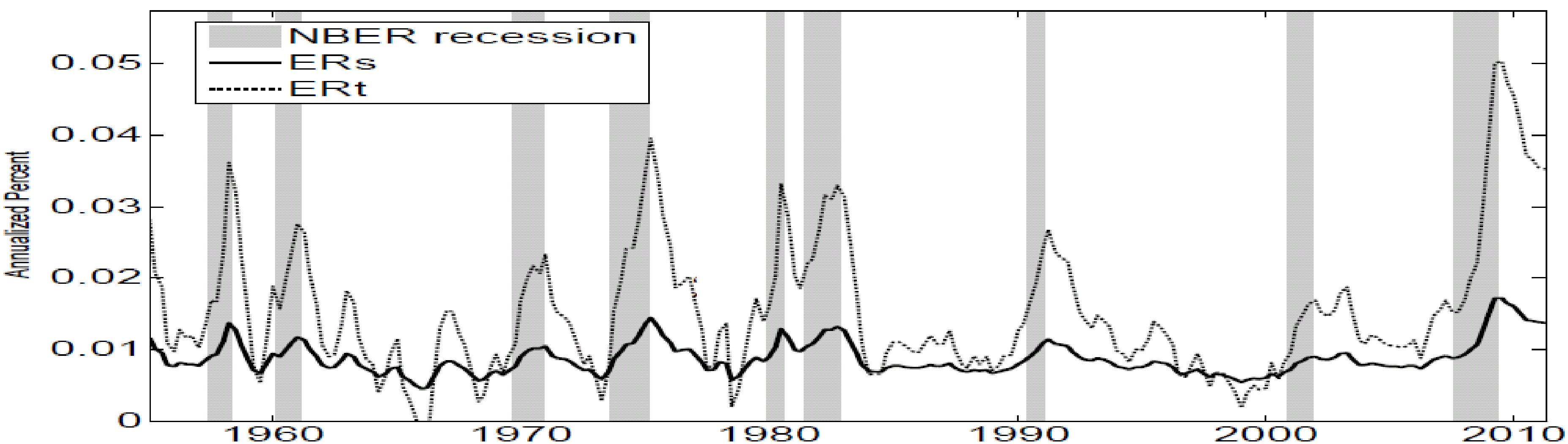
Reputation/total assets

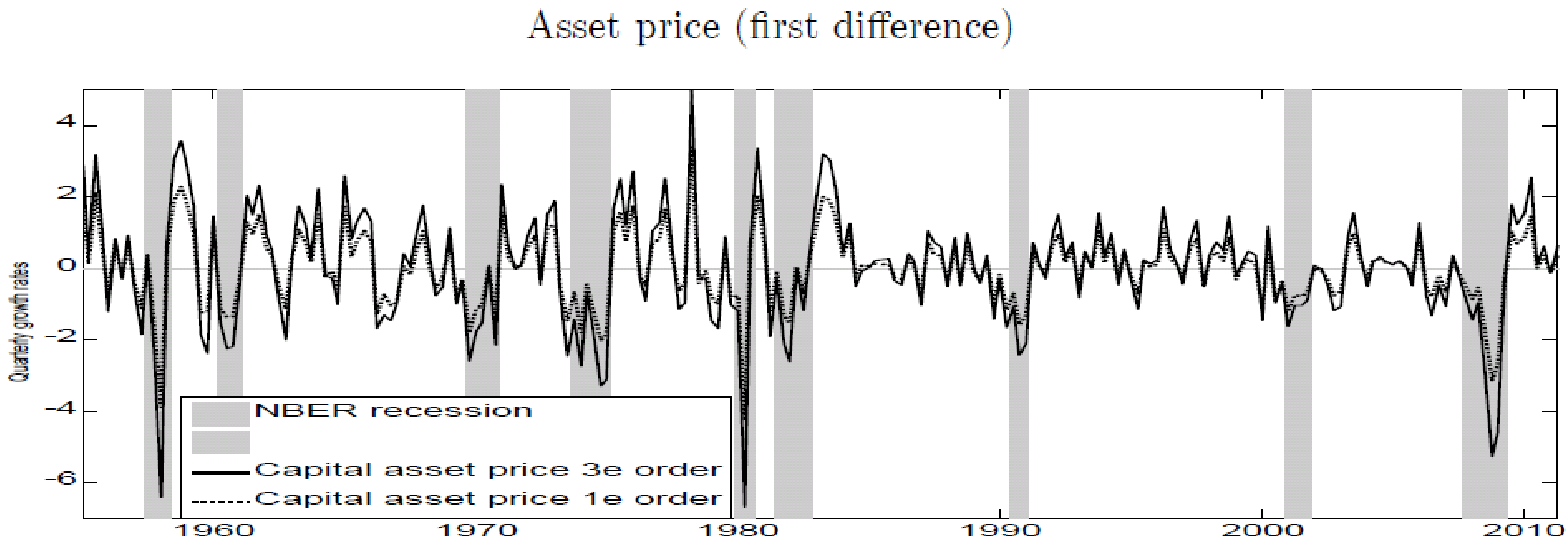


Leverage

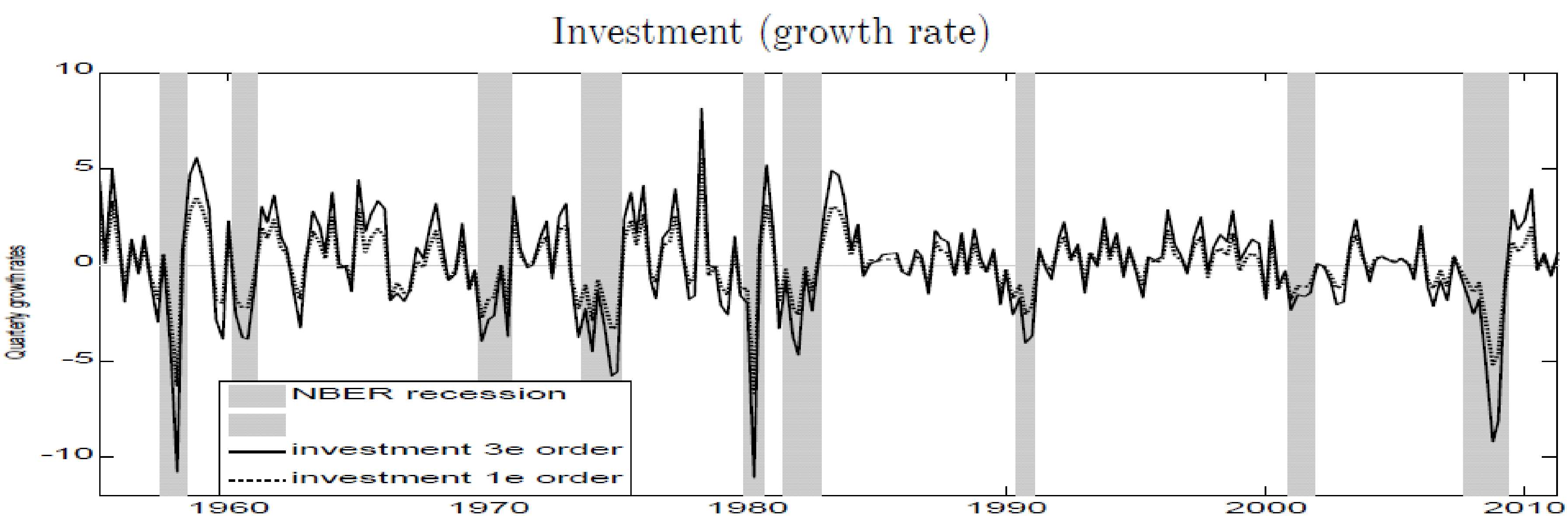


Risk premium

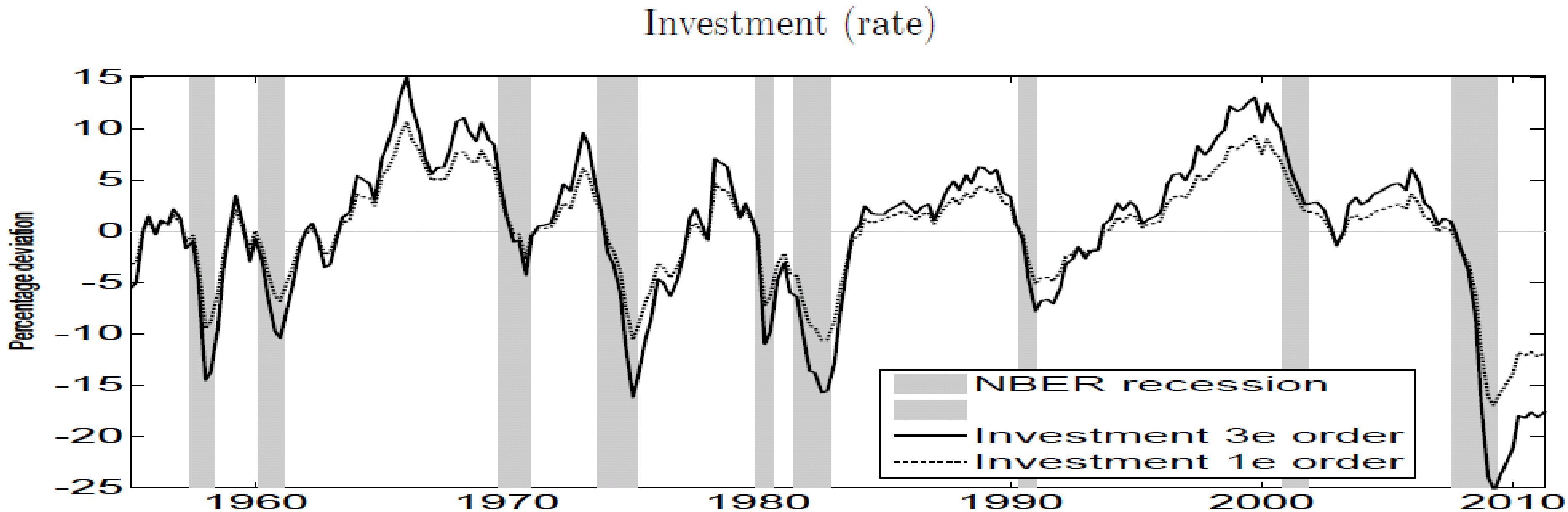


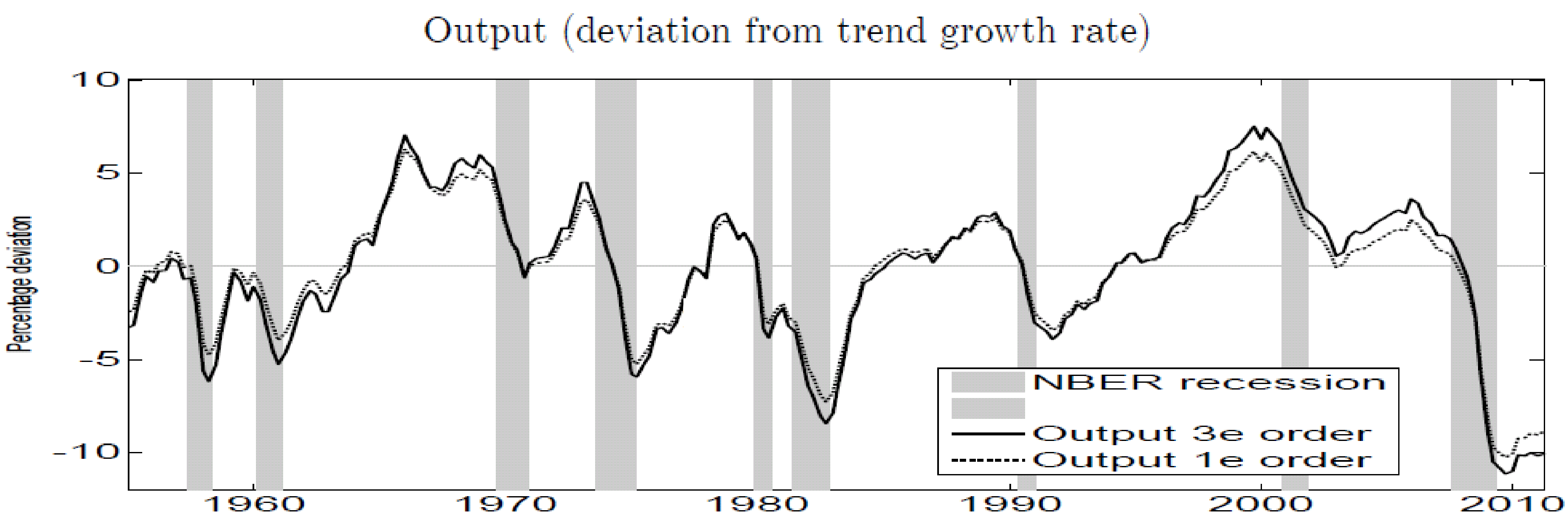


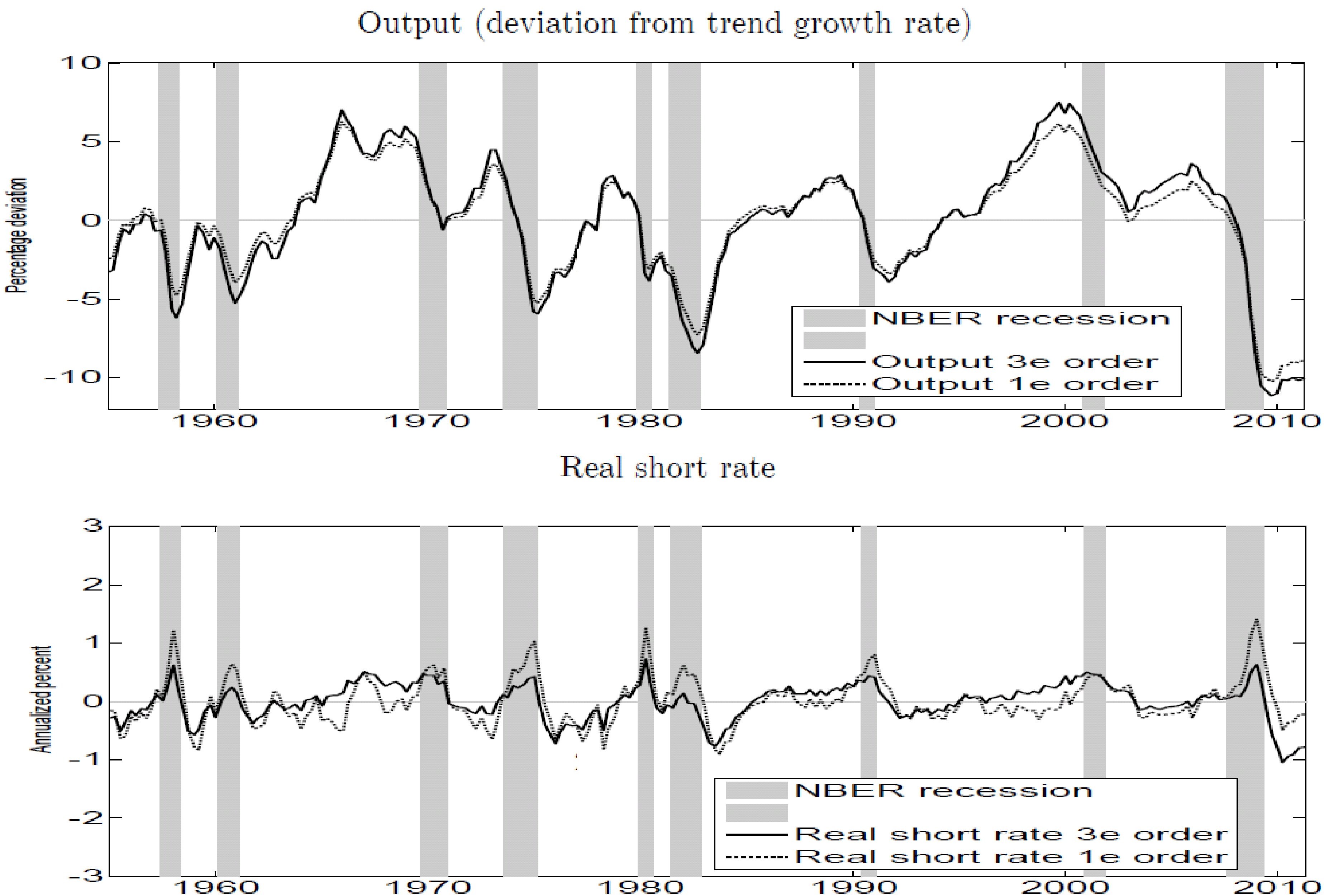
Asset price (first difference)
Quarterly growth rates
NBER recession
Capital asset price 3e order
Capital asset price 1e order



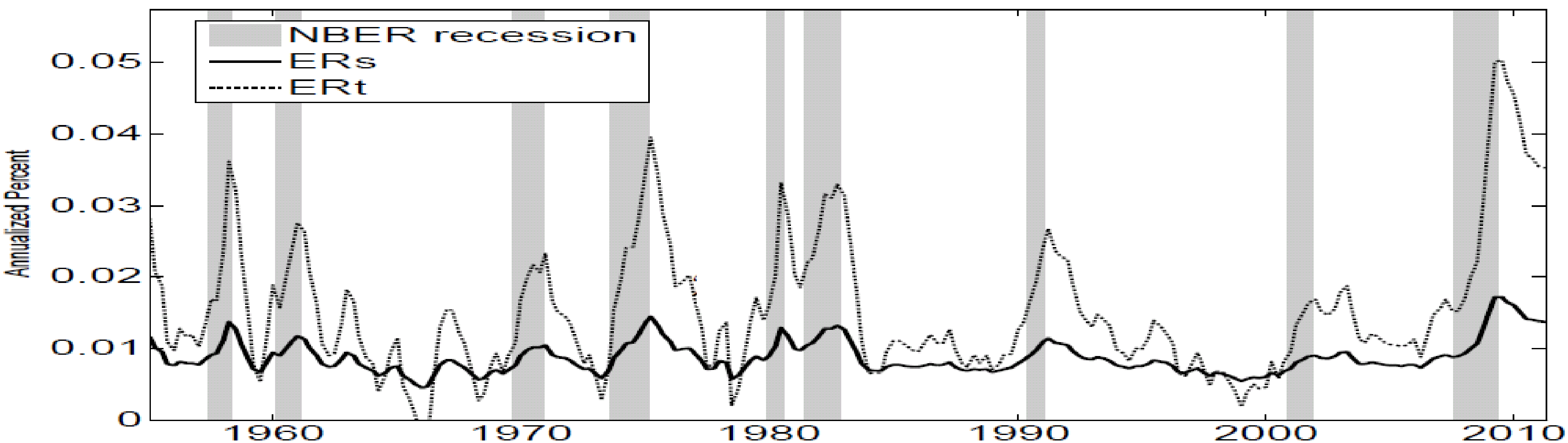
Investment (growth rate)
Quarterly growth rates
NBER recession
investment 3e order
investment 1e order


Investment (rate)
Percentage deviation
15
10
5
0
−5
−10
−15
−20
−25
1960
1970
1980
1990
2000
2010
NBER recession
Investment 3e order
Investment 1e order

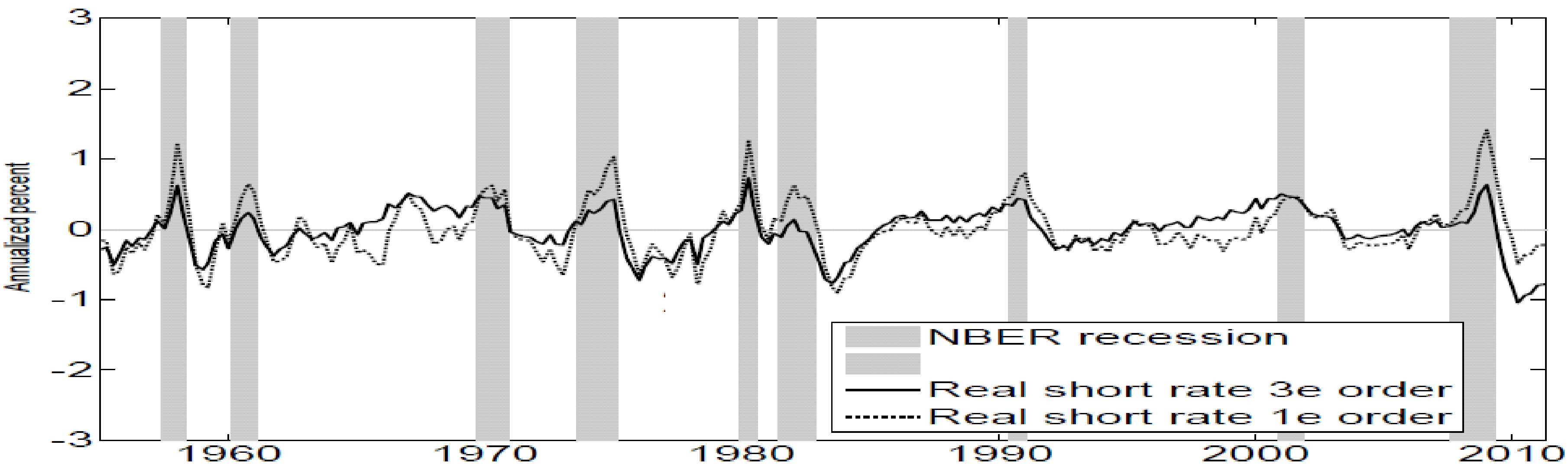

Output (deviation from trend growth rate)
Percentage deviation
10
5
0
−5
−10
1960
1970
1980
1990
2000
2010
NBER recession
Output 3e order
Output 1e order


Output (deviation from trend growth rate)
Percentage deviation
10
5
0
−5
−10
1960
1970
1980
1990
2000
2010
NBER recession
Output 3e order
Output 1e order
Real short rate
Annualized percent
3
2
1
0
−1
−2
−3
NBER recession
Real short rate 3e order
Real short rate 1e order

Risk premium



Real short rate



Application in DSGE model

- historical counterfactual simulation: using the model-consistent estimate for the productivity process and the implied transmission mechanism, we obtain an estimate of the potential contribution of the risk channel to the historical macroeconomic realisations by comparing the outcomes under third-order and first-order:

- important contribution of the risk channel in the transmission of fundamental shocks
- strong interaction between monetary policy and risk premium

- with only fundamental productivity shocks, the financial cycle and the business cycle have a synchronous development:

  => specific shocks to FI, e.g. subprime mortgage shock, can make the financial sector much more vulnerable with important spillover risks to the real economy

Sensitivity analysis

Table 8: Sensitivity analysis: the contribution of risk considerations in the volatility of macroeconomic and financial variables

	SR	Vol(Eq)	Vol(Q)	Vol(inv)	Vol(C)	Vol(Y)
baseline	29.87	20.35	5.20	8.40	1.27	2.78
risk contribution	*100%*	*59%*	*36%*	*38%*	*-3%*	*19%*
risk aversion interm. = 2.75	32.81	37.19	5.58	9.02	1.31	2.88
risk contribution	*100%*	*72%*	*40%*	*41%*	*-2%*	*20%*
representative agent	5.77	-	3.34	5.30	1.34	2.29
risk contribution	*100%*	-	*0%*	*0%*	*0%*	*0%*
constant leverage = 2.5	32.64	6.99	4.46	7.13	1.35	2.69
risk contribution	100%	29%	25%	26%	0%	14%
RBC-version	19.59	7.14	2.14	3.40	1.66	2.15
risk contribution	*100%*	*-12%*	*9%*	*9%*	*-7%*	*0%*
inv.adj.cost = 30	34.25	73.84	6.49	8.66	1.30	2.74
risk contribution	*100%*	*87%*	*42%*	*43%*	*3%*	*22%*
mon.pol.inflation reaction = 3	31.98	24.62	5.45	8.81	1.51	3.10
risk contribution	*100%*	*60%*	*36%*	*37%*	*-19%*	*13%*
financial constraints on firms	30.41	11.70	4.99	8.01	3.47	4.67
risk contribution	*100%*	*15%*	*23%*	*24%*	*12%*	*17%*

Concluding remarks

- our local perturbation approach delivers a first-order approximation to the risk channel implied by the global dynamics of the model:
  - substantial approximation errors
  - but interesting insights on the risk channel in larger models
  - full gain from computational efficiency only realized in empirical validation

- FI are confronted with more complex constraints and firms can also finance themselves directly on the capital market => many extensions are necessary

- many interesting policy applications are possible with this setup:
  - evaluate the traditional inflation objective together with objectives in terms of risk, volatility and strength of the FI
  - introduce specific instruments of macro-prudential regulation