# Discussion of: <br> "Endogenous risk in a DSGE model with capital-constrained financial intermediaries", <br> by Hans Dewachter and Raf Wouters 

Oreste Tristani<br>European Central Bank

Brussels, 12 October 2012

## Summary

- DSGE models with financial frictions contain interesting nonlinearities
- Interesting trade-off between continuous and discrete time models: accuracy vs. size (\# states)
- Larger linear-nonlinear models tend to produce more realistic features on both price and quantities (but what if "all" nonlinearities were introduced?)


## Discussion

- Why He and Krishnamurthy (and not CSV, or "credit constraints")?
- Is there a better approximation to the reputation constraint?
- Why third (and not second) order?


## Why He and Krishnamurthy

- Objective: build a model with appropriate cyclicality of leverage and asset prices and where risk is endogenous and plays a role on allocations
- He and Krishnamurthy has good properties:
- because of the specific financial constraint?
- or because of nonlinear effects are given a chance?


## Why He and Krishnamurthy

- Why not CSV framework (BGG):
- explicit information friction (but restrictions on lending/borrowing - only deposits, no equity, no direct financing)
- fares well on cyclicality; additionally has explicit default risk and actual defaults
- risk neutrality in lending relationships $\rightarrow$ no natural role nonlinearities and changes in price of risk


## Why He and Krishnamurthy

- Why not Gertler and Karadi (2011):
- banker can run away with a fraction $\lambda$ of bank assets; incentive constraint:

$$
q_{t} K_{t+1} \leq \phi_{t} N_{t}
$$

(for given $N$, there is a max value of assets banker can hold; the higher $N$, the less binding constr.)

- Accumulation of net worth

$$
\frac{N_{t+1}}{N_{t}}=\left(R_{k t+1}-R_{t+1}\right) \phi_{t}+R_{t}
$$

(accumulation depends on excess return on equity)

- Constraint assumed as binding and linearised $\rightarrow$ nonlinearities are ruled out


## Why He and Krishnamurthy

- He and Krishnamurthy (2012):
- reputation constraint: intermediary's (leveraged) share of risky assets is either constant $1 / \alpha^{*}$, or constrained from below by its reputation

$$
\alpha_{t}^{F I}=\max \left(\frac{1}{\alpha^{*}}, \frac{q_{t} K_{t}}{\varepsilon_{t}}\right)
$$

- Accumulation of reputation

$$
\frac{\varepsilon_{t+1}}{\varepsilon_{t}}=m \alpha_{t}^{F I}\left(R_{k t+1}-R_{t+1}\right)+m R_{t+1}-\eta
$$

- Account explicitly for nonlinearities


## Why He and Krishnamurthy

- HK vs GK
- Binding

$$
\begin{array}{cc}
H K & G K \\
\alpha_{t}^{F I}=\frac{q_{t} K_{t}}{N_{t}} & \phi_{t}=\frac{q_{t} K_{t+1}}{N_{t}}
\end{array}
$$

- "Non-binding"

$$
\begin{array}{cc}
H K & G K \\
\alpha_{t}^{F I}=\frac{1}{\alpha^{*}} & \phi_{t}=\frac{1}{\alpha^{*}} ?
\end{array}
$$

## Why He and Krishnamurthy

- Why not! It provides a nice benchmark where nonlinearities work in the "right" direction
- Not clear if peculiar type of financial friction is key
- Dewachter and Wouters' results may be very general


## A better approximation of the constraint?

- Well known that kinks are smoothed in the solution of stochastic problems (option pricing): smooth nonlinear approximation to occasionally binding constraint is sensible
- Perturbation methods become feasible, thus applicable to larger models
- Is there a better smoothing function?


## A better approximation of the constraint?

- The approximation of the constraint

$$
\alpha_{t}^{F I}=\max \left(\frac{1}{1-\lambda}, Q^{\varepsilon}\right)
$$

is

$$
\alpha^{F I}=\frac{1}{1-\lambda}+0.1(1-\lambda)^{2}\left(Q^{\varepsilon}\right)^{3}
$$

for $Q^{\varepsilon} \equiv \frac{q_{t} K_{t}}{\varepsilon_{t}}$

## A better approximation of the constraint?



## A better approximation of the constraint?

- Based on option theory, try solution to

$$
\alpha^{F I}=2+\max \left(Q^{\varepsilon}-2,0\right)
$$

ie

$$
\alpha^{F I}=2+N\left(d_{1}\right) Q^{\varepsilon}-2 N\left(d_{2}\right)
$$

for

$$
d_{1}=\frac{\ln \frac{Q^{\varepsilon}}{2}+\frac{\sigma^{2}}{2}}{\sigma}, \quad d_{2}=d_{1}-\sigma
$$

## A better approximation of the constraint?



## A better approximation of the constraint?



A better approximation of the constraint?


A better approximation of the constraint?


## A better approximation of the constraint?



## A better approximation of the constraint?



## A better approximation of the constraint?

- Try a more flexible functional approximation with tuning parameter?
- It should work also when the functional form is approximated to third order
- but higher order perturbation may give wild results away from the approximation point


## Why third order

- Why not start from a second order approximation?
- "Closer" to Ito calculus
- Enough to capture risk (conditional variances and covariances)


## Why third order

- It has to do with the constraint

$$
\varepsilon_{t}=\varepsilon_{t-1}\left(m \widetilde{R}_{t}-\eta\right)
$$

- Note: $\varepsilon$ is indeterminate in non stochastic steady state

$$
m \widetilde{R}=\eta
$$

## Why third order

- To first order

$$
\widehat{\varepsilon}_{t}=\widehat{\varepsilon}_{t-1}+(1+\eta) \widehat{\widetilde{r}}_{t}
$$

- $\widehat{\varepsilon}_{t}$ behaves like a random walk


## Why third order

- Second order

$$
\widehat{\varepsilon}_{t}=\widehat{\varepsilon}_{t-1}+(1+\eta) \widehat{\widetilde{r}}_{t}-\frac{1}{2} \eta(1+\eta) \widehat{\widetilde{r}}_{t}^{2}
$$

- $\widehat{\varepsilon}_{t}$ is still a random walk


## Why third order

- Third order

$$
\widehat{\varepsilon}_{t}=\widehat{\varepsilon}_{t-1}+\ldots-\frac{1}{6} \eta(1+\eta)(\eta+2) \widehat{\tilde{r}}_{t}^{3}-\frac{1}{2} \eta(1+\eta) \widehat{\widetilde{r}}_{t}^{2} \widehat{\varepsilon}_{t-1}
$$

- Minimum approximation order to ensure that the distribution of $\hat{\varepsilon}_{t}$ is well defined


## Conclusion

- Really interesting paper
- It opens the way for many other possible applications

