A Search and Learning Model of Export Dynamics

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2 sets of relevant issues

- Aggregate/industry level export dynamics
  - What makes export responses to exchange rates vary across countries and time periods?
  - Why are export responses to trade liberalization unpredictable?
  - What are the underlying causes of export booms?

- Trade frictions at the firm level
  - What form and how important?
  - How do frictions interact with firm characteristics to determine micro patterns of exporting—cross sectional and dynamic?

This paper approaches these issues by studying the formation, evolution, and dissolution of international buyer-seller relationships.
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- This paper: Approach these issues by studying formation, evolution, and dissolution of international buyer-seller relationships.
The exercises

- Characterize buyer-seller relationships in decade’s worth of data on individual merchandise shipments from Colombia to the United States.
- Develop a (partial equilibrium) dynamic search and learning model that explains patterns found in shipments.
- Fit the model to the data, and quantify exporting frictions:
  - costs of finding new buyers
  - costs maintaining relationships with existing ones.
  - learning about product appeal in foreign markets
  - network effects
- Perform counterfactual exercises
- Heterogeneity and trade
  - Melitz (2003), etc.

- Beachhead exporting costs:
    Das, Roberts, and Tybout (2008)

- Marketing costs: Arkolakis (2009, 2010); Drozd and Nozal (2011)


- Learning: Rauch and Watson (2002); Albornoz, Calvo, Corcos and Ornelas (2012)
**Stylized facts**

- **Evidence from Colombian customs data**
  - Each transaction has a date, value, product code, firm ID, and destination country.

- **Evidence from U.S. customs records**
  - Population of (legal) import transactions over the course of a decade (1996-2009).
  - Each transaction has a date, value, product code, affiliated trade indicator, exporter country *and* firm ID, and importer firm ID.
  - See also Blum et al, 2009a, 2009b; Albornoz et al, 2010.
As a fraction of total exporters, firms that enter a market and immediately exit are important.
But as a fraction of total export revenue, brand new exporters don’t account for much.
The firms that survive their first year grow exceptionally rapidly (see also Ruhl and Willis, 2008).
Hence young cohorts typically gain market share despite rapid attrition.

Post-1996 entrants account for about half of cumulative export expansion by 2005.
Most new matches fail within a year, but
- Chances of survival are higher for matches with large initial sales
- Survival rates improve and converge for all matches after the first year.
- To sustain or increase exports, firms must continually replenish their foreign clientele.
Matches that start small tend to stay small.

After a match’s first year, there is no systematic tendency for its annual sales to grow.
A seriously Pareto client distribution

- Most firms have a single buyer, but the distribution of client counts across exporters is fat-tailed.
### Table 3: Transition Probabilities, Number of Clients

<table>
<thead>
<tr>
<th>t \ t+1</th>
<th>exit</th>
<th>texit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6-10</th>
<th>11+</th>
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<tbody>
<tr>
<td>enter</td>
<td>0.000</td>
<td>0.000</td>
<td>0.947</td>
<td>0.044</td>
<td>0.007</td>
<td>0.002</td>
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<tr>
<td>texit</td>
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<td>0.896</td>
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<tr>
<td>1</td>
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<td>0.332</td>
<td>0.043</td>
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<td>3</td>
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<td>0.043</td>
<td>0.225</td>
<td>0.282</td>
<td>0.206</td>
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<tr>
<td>4</td>
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<td>0.112</td>
<td>0.226</td>
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<td>0.097</td>
<td>0.078</td>
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<tr>
<td>5</td>
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<td>0.103</td>
<td>0.184</td>
<td>0.197</td>
<td>0.184</td>
<td>0.094</td>
<td>0.197</td>
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<tr>
<td>6-10</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>0.070</td>
<td>0.082</td>
<td>0.114</td>
<td>0.149</td>
<td>0.465</td>
<td>0.066</td>
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<tr>
<td>11+</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>.</td>
<td>.</td>
<td>0.440</td>
</tr>
</tbody>
</table>
Firms engage in costly search to meet potential buyers at home and (possibly) abroad.

Firms new to the foreign market don’t know what fraction of buyers there will be willing to do business with them.

As they encounter potential buyers, firms gradually learn the scope of the market for their particular products, and they adjust their search intensities accordingly (learning).

Search costs fall as firms accumulate successful business relationships (reputation effects).

Maintaining a relationship with a buyer is costly, so sellers drop relationships that yield meager profits.
Three model components

1. A Seller-Buyer Relationship
2. Learning About Product Appeal from Encounters with Potential Buyers
3. Searching for Potential Buyers
Why continuous time?

- Two types of discrete events occur at random intervals, sometimes with high frequency
  - Sellers meet buyers
  - Once business relationships are established, orders are placed

- With a continuous time formulation, we can:
  - allow for an arbitrarily large number of events during any discrete interval
  - allow agents to update their behavior each time an event occurs
1. Relationship dynamics

profits from a shipment

- Define exogenous state variables:
  - $\phi_j$ productivity of seller $j$ (time invariant)
  - $x_t^m$ size of market $m \in \{ h, f \}$ (Ehrenfest jump process)
  - $y_{ijt}^m$ idiosyncratic shock to operating profits from shipment to buyer $i$ by seller $j$ in market $m$ (Ehrenfest jump process)

- Let $\Pi^m$ be a profit function scalar (so that all exogenous state variables can be normalized to mean log zero)

- When buyer $i$ places an order with seller $j$ in market $m$ it generates operating profits:

$$\pi(x_t^m, \phi_j, y_{ijt}^m) = \Pi^m x_t^m \phi_j^{\sigma-1} y_{ijt}^m.$$ 

Superscripts and subscripts mostly suppressed hereafter:

$$\pi_\phi(x, y) = \Pi x \phi^{\eta-1} y$$
1. Relationship dynamics
value of a business relationship

- In active business relationships, buyers place orders with exogenous hazard \( \lambda^b \).

- After each order, sellers must pay fixed cost \( F \) to keep a business relationship active.

- Value to a type-\( \phi \) seller of a relationship in state \( \{x, y\} \):

  \[
  \tilde{\pi}_\phi(x, y) = \pi_\phi(x, y) + \max \{ \tilde{\pi}_\phi(x, y) - F, 0 \}
  \]

- \( \tilde{\pi}_\phi(x, y) \) is the continuation value to a type-\( \phi \) seller of a relationship in state \( \{x, y\} \).

- Continuation values depend negatively on
  - \( \delta \): exogenous hazard of relationship death.
  - \( \rho \): seller’s discount rate.
1. Relationship dynamics

expected value of a new relationship

- Sellers don’t know what $y$ value their next business relationship will begin from.
- Let $\Pr(y^s)$ be the probability of initial shock $y^s$, determined by the ergodic distribution of $y$.
- Expected value of a successful new encounter:

\[
\tilde{\pi}_\phi(x) = \sum_{y^s} \Pr(y^s) \tilde{\pi}_\phi(s, y)
\]
2. Learning about product appeal
the "true" scope of the market

- Let $\theta^m_j \in [0, 1]$ be the fraction of potential buyers in market $m$ who are interested in seller $j$'s product.
- Assume $\theta^m_j$'s are time-invariant, mutually independent draws from a beta distribution:

$$ r(\theta|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1 - \theta)^{\beta-1}, $$

- Expected value:

$$ E(\theta|\alpha, \beta) = \frac{\alpha}{\alpha + \beta}. $$

- Posterior beliefs, after meeting $n^m$ potential clients in market $m$, $a^m$ of whom want to do business:

$$ \theta^m(a^m, n^m) = E[\theta^m|a^m, n^m] = \frac{a^m + \alpha}{n^m + \alpha + \beta} $$
3. Searching for buyers

the cost of search

- Seller continuously chooses the hazard $s$ with which she encounters a potential buyer at a flow cost $c(s, a)$
  - Maintain web site
  - Pay to be near top of web search listings
  - Attend trade fairs
  - Research foreign buyers
  - Send sales reps. to foreign markets
  - Maintain foreign sales office

- The number of successful encounters, $a$, allows for network effects (NYT 2/27/12: Panjiva, ImportGenius).

- Functional form used for estimation (Arkolakis, 2010):

$$c(s, a) = \kappa_0 \frac{(1 + s)^{(1 + 1/\kappa_1)} - 1}{(1 + a)^{\gamma}(1 + 1/\kappa_1)(1 + 1/\kappa_1)}$$

Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}Census Bureau (CES))
3. Searching for buyers
the value of search abroad

- Define the value of continued search for a type-\( \varphi \) firm with \( a \) successes in \( n \) meetings, market state \( x \):

\[
V_{\varphi}(a, n, x)
\]

- The first-order for optimal search abroad is:

\[
c_s(s^*, a) = \bar{\theta}_{a,n}(\bar{\pi}_{\varphi}(x) + V_{\varphi}(a + 1, n + 1, x)) + (1 - \bar{\theta}_{a,n})V_{\varphi}(a, n + 1, x) - V_{\varphi}(a, n, x).
\]
3. Searching for buyers
the value of search in the domestic market

- As $n$ increases, $\bar{\theta}_{a,n}$ converges to the true $\theta$.
- There is no more learning, and the reward to search depends on $a$ and $n$ only through network effects.
- We assume this characterizes the domestic market.
- If network effects are ignored, the first-order condition for optimal search at home is thus:

$$c_s(s^*, a) = \theta_j \bar{\tau}_\varphi(x).$$
Estimation

The exogenous state variables

- Notation refresher: if \( z \) follows an Ehrenfest diffusion process:
  - \( e \in I^+ \) and \( \Delta \in R^+ \) determine support:
    \[
    z \in \{-e\Delta, -(e - 1)\Delta, \ldots, 0, \ldots, (e - 1)\Delta, e\Delta\}
    \]
  - The process jumps with hazard \( \lambda_z \):
    \[
    F[t] = 1 - e^{-\lambda_z t}
    \]

- As the grid becomes finer, this type of random variable asymptotes to an Ornstein-Uhlenbeck process:
  \[
  dz = -\mu z dt + \sigma dW
  \]
- Asymptotically, \( \mu = \frac{\lambda_z}{e}, \sigma = \sqrt{\lambda_z} \Delta \) (Shimer, 2006).
Estimation

The exogenous state variables

- If \( z \) observed at regular intervals, can estimate \( \mu \) and \( \sigma \) by regressing \( z \) on lagged \( z \)
- For \( x^f, x^h \), obtain maximum likelihood estimates of \( \mu \) and \( \sigma \) using logged and de-meaned time series on total real consumption of manufactured goods in each country.
- Recover \( \lambda_z \) and \( \Delta \) using Shimer’s mapping.
- Since \( y \) is unobservable, recover the parameters of its jump processes using the structure of the dynamic model.
### Market-wide Shock Processes \((x^f, x^h)\)

<table>
<thead>
<tr>
<th>Orstein-Uhlenbeck Parameters</th>
<th>Colombia</th>
<th>United States</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu) Mean Reversion</td>
<td>0.171</td>
<td>0.174</td>
</tr>
<tr>
<td>(\sigma) Dispersion</td>
<td>0.003</td>
<td>0.058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ehrenfest Process Parameters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda) Jump Hazard</td>
<td>1.200</td>
<td>1.215</td>
</tr>
<tr>
<td>(\Delta) Jump Size</td>
<td>0.003</td>
<td>0.053</td>
</tr>
<tr>
<td>grid points</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Eaton et al. (\(^a\)Penn State, \(^b\)NBER, \(^c\)U. de lo)
Unidentified preference parameters taken from literature: $\rho = 0.05$, $\sigma = 5$

Remaining parameters identified using indirect inference

$$\Lambda = \left( \Pi^h, \Pi^f, \delta, F, \alpha, \beta, \sigma_\varphi, \lambda_y, \lambda_b, \gamma, \kappa_0, \kappa_1 \right)$$
Indirect inference (Gouriéroux and Monfort, 1996)

**basic idea**

- Using reduced-form auxiliary regressions and/or moments, summarize key relationships in the data using a vector of statistics ($\hat{M}$).
- For a candidate set of parameter values ($\Lambda$), simulate same statistics using the model $\hat{M}^s(\Lambda)$.
- Construct the loss function:

$$Q(\Lambda) = (\hat{M} - \hat{M}^s(\Lambda))^T \Omega (\hat{M} - \hat{M}^s(\Lambda))$$

where $\Omega$ is a positive definite weighting matrix.
- Use a robust algorithm to search parameter space for $\hat{\Lambda} = \arg\min Q(\Lambda)$. 

Indirect inference

identification

- **Profit scaling constants**, \((\Pi^h, \Pi^f)\)
  - means of log home and foreign sales
- **Shipment hazards** \((\lambda^b)\)
  - average annual shipment rates per match
- **Product appeal parameters** \((\alpha, \beta)\)
  - distribution of home and foreign sales
- **Firm productivity dispersion** \((\sigma_\varphi)\)
  - distribution of home and foreign sales
  - covariance of home and foreign sales
- **Search cost parameters** \((\kappa_0, \kappa_1, \gamma)\)
  - match rates
  - client frequency distribution (especially fatness of tail)
  - client transition probabilities
  - fraction of firms that export

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Indirect inference
dentification

- **Idiosyncratic shocks to importers** ($\lambda_Y$)
  - cross-plant variances in home and foreign sales
  - covariation of home and foreign sales
  - autocorrelation, match-specific sales
  - client frequency distribution, client transition probabilities

- **Match maintenance costs** ($F$)
  - client frequency distribution, client transition probabilities
  - sales among new versus established matches
  - age-specific match failure rates

- **Exogenous match separation hazard** ($\delta$)
  - separation rates after first year
  - age-specific match failure rates
  - client frequency distribution

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## Data versus simulated statistics

### Transition probs.,

<table>
<thead>
<tr>
<th>no. clients ($n^c$)</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 0</td>
<td>n^c_{jt} = 1]$</td>
<td>0.618</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 1</td>
<td>n^c_{jt} = 1]$</td>
<td>0.321</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 2</td>
<td>n^c_{jt} = 1]$</td>
<td>0.048</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} \geq 3</td>
<td>n^c_{jt} = 1]$</td>
<td>0.013</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 0</td>
<td>n^c_{jt} = 2]$</td>
<td>0.271</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 1</td>
<td>n^c_{jt} = 2]$</td>
<td>0.375</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} = 2</td>
<td>n^c_{jt} = 2]$</td>
<td>0.241</td>
</tr>
<tr>
<td>$\hat{P}[n^c_{jt+1} \geq 3</td>
<td>n^c_{jt} = 2]$</td>
<td>0.113</td>
</tr>
</tbody>
</table>

### Share of firms exporting

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{E}(1_{X^f_{jt} &gt; 0})$</td>
<td>0.299</td>
<td>0.351</td>
</tr>
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</table>

### Log foreign sales on log domestic sales

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}^hf_1$</td>
<td>0.727</td>
<td>0.515</td>
</tr>
<tr>
<td>$s\hat{e}(e^hf)$</td>
<td>2.167</td>
<td>1.424</td>
</tr>
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</table>
### Match death hazards

<table>
<thead>
<tr>
<th>Death rate, $A^m_{ijt-1} = k$</th>
<th>Data</th>
<th>Model</th>
<th>Exporter exit rate, $A^m_{ijt-1} = 0$</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 0$</td>
<td>0.694</td>
<td>0.857</td>
<td>$k = 0$</td>
<td>0.709</td>
<td>0.748</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>0.515</td>
<td>0.329</td>
<td>$k = 1$</td>
<td>0.383</td>
<td>0.099</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>0.450</td>
<td>0.304</td>
<td>$k = 2$</td>
<td>0.300</td>
<td>0.121</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>0.424</td>
<td>0.281</td>
<td>$k = 3$</td>
<td>0.263</td>
<td>0.055</td>
</tr>
<tr>
<td>$k = 4$</td>
<td>0.389</td>
<td>0.305</td>
<td>$k = 4$</td>
<td>0.293</td>
<td>0.100</td>
</tr>
</tbody>
</table>
### Data versus simulated statistics

**Log sales per client vs. no. clients**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Ave. log sales by cohort age</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^m_1$</td>
<td>2.677</td>
<td>0.842</td>
<td>$\widehat{E}(\ln X^f_{jt}</td>
</tr>
<tr>
<td>$\beta^m_2$</td>
<td>-0.143</td>
<td>0.042</td>
<td>$\widehat{E}(\ln X^f_{jt}</td>
</tr>
<tr>
<td>$s\widehat{\epsilon}(\epsilon^m)$</td>
<td>2.180</td>
<td>1.622</td>
<td>$\widehat{E}(\ln X^f_{jt}</td>
</tr>
</tbody>
</table>

**No. clients, inverse**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\widehat{E}(\ln X^f_{jt}</td>
<td>A^c_{jt} = 3)$ 10.369 10.679</td>
<td></td>
</tr>
</tbody>
</table>

**CDF regression**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\widehat{E}(\ln X^f_{jt}</td>
<td>A^c_{jt} \geq 4)$ 10.473 10.669</td>
<td></td>
</tr>
</tbody>
</table>

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Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de lo)
### Data versus simulated statistics

<table>
<thead>
<tr>
<th>Match death prob regression</th>
<th>Data</th>
<th>Model</th>
<th>Log match sale autoreg.</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}^d_0$</td>
<td>1.174</td>
<td>1.640</td>
<td>$\hat{\beta}^f_1$</td>
<td>0.811</td>
<td>0.613</td>
</tr>
<tr>
<td>$\hat{\beta}^d_1$</td>
<td>0.166</td>
<td>0.203</td>
<td>$\hat{\beta}^f_1$</td>
<td>0.233</td>
<td>0.370</td>
</tr>
<tr>
<td>$\hat{\beta}^d_{1st~year}$</td>
<td>-0.070</td>
<td>-0.100</td>
<td>$s\hat{\epsilon}^f$</td>
<td>1.124</td>
<td>0.503</td>
</tr>
<tr>
<td>$\hat{\beta}^d_{sales}$</td>
<td>0.453</td>
<td>0.395</td>
<td>Log dom. sales</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s\hat{\epsilon}^d$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Match shipments per year**

<table>
<thead>
<tr>
<th>$\hat{E}(n^s)$</th>
<th>Data</th>
<th>Model</th>
<th>$\hat{\beta}^h_1$</th>
<th>0.976</th>
<th>0.896</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s\hat{\epsilon}^h$</td>
<td></td>
<td></td>
<td></td>
<td>0.462</td>
<td>0.683</td>
</tr>
</tbody>
</table>
### Parameters Estimated using indirect inference ($\Lambda$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate of exogenous separation</td>
<td>$\delta$</td>
<td>0.267</td>
</tr>
<tr>
<td>domestic market size</td>
<td>$\Pi^h$</td>
<td>11.344</td>
</tr>
<tr>
<td>foreign market size</td>
<td>$\Pi^f$</td>
<td>10.675</td>
</tr>
<tr>
<td>log fixed cost</td>
<td>$\ln F$</td>
<td>7.957</td>
</tr>
<tr>
<td>First $\theta$ distribution parameter</td>
<td>$\alpha$</td>
<td>0.716</td>
</tr>
<tr>
<td>Second $\theta$ distribution parameter</td>
<td>$\beta$</td>
<td>3.161</td>
</tr>
</tbody>
</table>

- A substantial fraction of matches fail for exogenous reasons.
- Fixed cost of maintaining a relationship: $\exp(7.957) = $2,855, about 35% of the value of a typical shipment.
- Only about $\alpha / (\alpha + \beta) = 0.18$ of the potential buyers a typical exporter meets are interested in doing business.
- Success rates vary across exporters with standard deviation $\sqrt{\alpha \beta / [(\alpha + \beta)^2(\alpha + \beta + 1)]} = 0.176$
### Parameters Estimated using indirect inference ($\Lambda$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand shock jump hazard</td>
<td>$\lambda_y$</td>
<td>0.532</td>
</tr>
<tr>
<td>demand shock jump size</td>
<td>$\Delta y$</td>
<td>0.087</td>
</tr>
<tr>
<td>shipment order arrival hazard</td>
<td>$\lambda_b$</td>
<td>8.836</td>
</tr>
<tr>
<td>std. deviation, log firm type</td>
<td>$\sigma_\varphi$</td>
<td>0.650</td>
</tr>
<tr>
<td>network effect parameter</td>
<td>$\gamma$</td>
<td>0.298</td>
</tr>
<tr>
<td>search cost function curvature parameter</td>
<td>$\kappa_1$</td>
<td>0.087</td>
</tr>
<tr>
<td>search cost function scale parameter</td>
<td>$\kappa_0$</td>
<td>111.499</td>
</tr>
</tbody>
</table>

- Convexity of search cost function is important
- Cost of search at hazard $s = 1$: $5,786$ when $a = 0$; $437$ when $a = 1$.
- Cost of search at hazard $s = 5$: $5.277 \times 10^9$ when $a = 0$; $6,301$ when $a = 20$.  

Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de lo)  
Search and Export Dynamics  
6/5/13 36 / 54
Search intensity over trials and productivity, holding the number of successes constant at 0.
History and the policy function

- Search intensity as a function of past successes and failures, allowing for reputation effects
History and the policy function

- Search intensity as a function of past successes and failures, shutting down reputation effects

network policy

Eaton et al. (Penn State, NBER, U. de los Andes, Census Bureau (CES)) Search and Export Dynamics
A 20% reduction in search costs

Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de lo) Search and Export Dynamics

\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de lo
A 20% reduction in fixed costs

Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}Census Bureau (CES))

Search and Export Dynamics
A 20% increase in foreign market size
Eliminating reputation effects

Eaton et al. (\textsuperscript{a}Penn State, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}Census Bureau (CES)) Search and Export Dynamics
Micro patterns of transactions and buyer-seller relationships through the lens of the model:

- Large volume of small scale exporters explained by large volume of inexperienced firms, searching at a low level.
- High exit rate reflects short lifespan of typical match, combined with low-level search and learning about product appeal.
- Small number of major exporters reflects combination of skewed distribution of product appeal and reputation effects.

Search costs, multi-period matches, learning, and reputation effects combine to provide an explanation for hysteresis in trade.

- Reputation effects appear to be particularly important.
- Since learning is mainly relevant for new, marginal players, probably doesn’t have a big effect on short-run export dynamics.
From the perspective of time 0, let the probability that an event will occur before time $t$ be described by the exponential distribution:

$$F[t] = 1 - e^{-qt}$$

The likelihood of the event happening exactly at $t$ (the "hazard rate" at $t$) is then:

$$\frac{f(t)}{1 - F(t)} = \frac{qe^{-qt}}{e^{-qt}} = q$$

This hazard rate doesn't depend upon $t$. 
Suppose $k$ independent events occur with hazard $q_1, q_2, \ldots q_k$. The probability that none occur before $t$ is:

$$\prod_{j=1}^{k} (1 - F_j(t)) = e^{-t\sum q_j}$$

So by time $t$, at least one event occurs with probability $1 - e^{-t\sum q_j}$, and the likelihood that this happens exactly at $t$ is

$$\frac{\sum q_j \left[ e^{-t\sum q_j} \right]}{e^{-t\sum q_j}} = \sum q_j$$
Any variable $z$ that obeys Ehrenfest process:
- changes value with hazard $\lambda_z$. Next jumps occur within interval $t$ with probability
  \[ F[t] = 1 - e^{-\lambda_z t} \]
- has discrete support, equally-spaced values:
  \[ e \in I^+: z \in \{-e\Delta, -(e-1)\Delta, \ldots, 0, \ldots, (e-1)\Delta, e\Delta\} \]
- jumps only to contiguous values:
  \[
  z' = \begin{cases} 
  z + \Delta & \text{with probability } \frac{1}{2} \left(1 - \frac{z}{e\Delta}\right) \\
  z - \Delta & \text{with probability } \frac{1}{2} \left(1 + \frac{z}{e\Delta}\right) \\
  \text{other} & 0
  \end{cases}
  \]
- As the grid becomes finer ($\uparrow e$, $\downarrow \Delta$), Ehrenfest processes asymptote to Ornstein-Uhlenbeck processes:
  \[ dz = -\mu z dt + \sigma dW \]
Relationship dynamics

Let \( q_{xx'} \) be the hazard of transiting from market state \( x \) to state \( x' \).

Let \( q_{yy'} \) be the hazard of transiting from match-specific state \( y \) to state \( y' \).

\[
\lambda_X^x = \sum_{x' \neq x} q_{xx'}
\]
is hazard of any change in market-wide state \( x \).

\[
\lambda_Y^y = \sum_{y' \neq y} q_{yy'}
\]
is hazard of any change in match-specific state \( y \).

Let \( \lambda^b \) be the hazard of a new purchase order from existing client.

\( \tau_b \) time until the next change in state, which occurs with hazard

\[
\lambda^b + \lambda_X^x + \lambda_Y^y
\]
Continuation value of a business relationship in state \((x, y)\) for a type-\(\varphi\) exporter:

\[
\hat{\tau}_\varphi(x, y) = \mathbb{E}_{\tau_b} \left[ e^{-(\rho + \delta)\tau_b} \frac{1}{\lambda^b + \lambda^X_x + \lambda^Y_y} \right.
\]

\[
\cdot \left( \sum_{x' \neq x} q^{X}_{xx', \hat{\tau}_\varphi(x', y)} + \sum_{y' \neq y} q^{Y}_{yy', \hat{\tau}_\varphi(x, y')} + \lambda^b \hat{\tau}_\varphi(x, y) \right) \right]
\]

\[
= \frac{1}{h} \left( \sum_{x' \neq x} q^{X}_{xx', \hat{\tau}_\varphi(x', y)} + \sum_{y' \neq y} q^{Y}_{yy', \hat{\tau}_\varphi(x, y')} + \lambda^b \hat{\tau}_\varphi(x, y) \right)
\]

where

- \(\delta\) is the exogenous hazard of relationship death.
- \(\rho\) is the seller’s discount rate.
- \(h = \rho + \delta + \lambda^b + \lambda^X_x + \lambda^Y_y\)
Suppress market superscripts to reduce clutter.

The **prior distribution** is:

\[
    r(\theta|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1 - \theta)^{\beta-1},
\]

The **likelihood**: Given \(\theta\), and given that a seller has met \(n\) potential buyers, the probability that \(a\) of these buyers were willing to buy her product is binomially distributed:

\[
    q[a|n, \theta] = \binom{n}{a} [\theta]^a [1 - \theta]^m^{n-a}.
\]
The posterior distribution for $\theta$:

$$p(\theta|a, n) \propto q[a|n, \theta] \cdot r(\theta|\alpha, \beta)$$

The expected success rate after $a$ successes in $n$ trials is thus:

$$\bar{\theta}(a, n) = E[\theta|a, n] = \frac{a + \alpha}{n + \alpha + \beta}$$

Sellers base their search intensity on this posterior mean.
The value of continued search for a type-$\varphi$ firm with $a$ successes in $n$ meetings is:

$$V_{\varphi}(a, n, x) = \max_s E_{\tau_s} \left[ -c(s, a) \int_0^{\tau_s} e^{-\rho t} dt + \frac{e^{-\rho \tau_s}}{s + \lambda^X_x} \cdot \left( \sum_{x' \neq x} q_{xx'}^X V_{\varphi}(a, n, x') \right) + s \left[ \bar{\theta}_{a,n}(\tilde{\pi}_{\varphi}(x)) + V_{\varphi}(a + 1, n + 1, x) + (1 - \bar{\theta}_{a,n}) V_{\varphi}(a, n + 1, x) \right] \right]$$

where:

- $\lambda^X_x = \sum_{x' \neq x} q^X_{xx'}$ is the hazard of any change in the market-wide state $x$.
- $\tau_s$ is the random time until the next search event, which occurs with hazard $s + \lambda^X_x$.
Searching for buyers
the value of search

Taking expectations over $\tau_s$ yields:

$$V_\phi(a, n, x) = \max_s \frac{1}{\rho + s + \lambda_x^X} \left[ -c(s, a) + \sum_{x' \neq x} q_{xx'}^X V_\phi(a, n, x') \right]$$

$$+ s \left\{ \bar{\theta}_{a,n} \left[ \tilde{\pi}_\phi(x) + V_\phi(a + 1, n + 1, x) \right] + (1 - \bar{\theta}_{a,n}) V_\phi(a, n + 1, x) \right\}$$

The first-order condition is thus:

$$c_s(s^*, a) = \bar{\theta}_{a,n} (\tilde{\pi}_\phi(x) + V_\phi(a + 1, n + 1, x))$$

$$+ (1 - \bar{\theta}_{a,n}) V_\phi(a, n + 1, x) - V_\phi(a, n, x).$$
In the domestic market the reward to search depends on \( a \) and \( n \) only through network effects.

The value of search at home is thus simply:

\[
V_{\varphi}(x) = \max_s \frac{1}{\rho + \lambda_x^X} \left[ -c(s, a) + \sum_{x' \neq x} q_{xx'}^X V_{\varphi}(x') + s\theta_j \tilde{\pi}_\varphi(x) \right]
\]

The associated first-order condition is:

\[
c_s(s^*, a) = \theta_j \tilde{\pi}_\varphi(x).
\]