Firms, Failures, and Fluctuations

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Input-Output Linkages and Propagation of Shocks

- Modern economies organized as complex production networks:
    - Expenditure on intermediate goods & services in the U.S. in 2007 $\approx$ 1 GDP.

- A growing literature argues that input-output linkages...
    (i) function as mechanism for propagation & amplification of shocks (micro);
    (ii) can translate micro shocks into aggregate fluctuations (macro).

- Even though linkages are between firms, most models...
    (i) focus on interactions at the industry level;
    (ii) ignore the possibility of firm failures (all the action is at the intensive margin)

Firm-Level Linkages

- In reality, failures of firms' suppliers and customers can be first order.
    - the U.S. auto industry in 2008–09
    - bankruptcies due to spillovers over credit linkages (Jacobson and Von Schedvin, 2015)
    - the aftermath of the Great East Japan Earthquake (Carvalho et al., 2016)
- Important advances in modeling linkages and propagation through input-output networks, but typically focusing on sectoral models and sectoral shocks.
- But if there is a lot of action at the firm-level, the sectoral focus can miss the most important elements.

This "Paper"

- A theoretical model of firm-level interactions with (i) firm-specific relationships, (ii) endogenous bankruptcies, and (iii) market power.
  - Failures are the main channel via which negative shocks propagate.

- Study how firm-level linkages and firm failures shape the nature of how shocks propagate in the economy and impact aggregate fluctuations.

- The aggregated economy at the sectoral level is isomorphic to an industry-level model with distortions, but these distortions are *endogenous* and depend on the extent of firm failures.

- Main take-away: to understand network-originated fluctuations, we may have to go beyond sectoral linkages and study how firm-level interactions cause chains of failures.

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- Main take-away: to understand network-originated fluctuations, we may have to go beyond sectoral linkages and study how firm-level interactions cause chains of failures.

Related Literature

- Production networks and the origins of aggregate fluctuations
  - Long and Plosser (1983); Horvath (1998, 2000); Acemoglu et al. (2012, 2017); Atalay (2017); Baqaee (2018); Baqaee and Farhi (2017), and many more...
  - Jones (2013), Bigio and La'O (2018), Baqaee and Farhi (2018), Liu (2018)

- Endogenous production networks
  - Carvalho and Voigtländer (2014); Oberfield (2018); Acemoglu and Azar (2018)

- Empirical evidence
  - Acemoglu et al. (2016); Barrot and Sauvagnat (2016); Carvalho et al. (2016)

- Models of firm-level interactions
  - Taschereau-Dumouchel (2018); Tintelnot et al. (2018); Kikkawaa et al. (2018)

Model

- An economy with $N + 1$ industries.
  - industries $1, \ldots, N$ produce intermediate goods
  - industry 0 produces the final good.

- Each industry $I \in \{1, \ldots, N\}$ consists of two types of firms
  - a competitive fringe of firms producing a generic variant of the good
  - a unit mass of specialized firms producing specialized/customized inputs

- A unit mass of households
  - log utilities over the final good
  - one unit of labor supplied inelastically

Generic Inputs

- A competitive fringe of firms in each industry produces a generic variant of the good using labor and other generic inputs.

- Constant returns to scale technology:

$$\widetilde{y}_I = F_I(\widetilde{\ell}_I, B_{I1}\widetilde{x}_{I1}, \ldots, B_{IN}\widetilde{x}_{IN})$$

  - generic good producers can be indexed by the industry they belong to
  - $\widetilde{x}_{IJ}$: quantities
  - B_{IJ}: productivity shock

- Production of generic goods can be represented by an *industry-level* network

Customized Inputs

- Specialized firms can produce inputs that are customized to specific customers.
- Firms are matched to potential suppliers via a matching function $\phi_{IJ} : I \to J \cup \{\emptyset\}$

$$\phi_{IJ}(i) := \begin{cases} j & j \in J \text{ is a matched supplier of } i \in I \\ \emptyset & i \in I \text{ is not matched to a supplier in } J \end{cases}$$

- Each firm can be matched to...
    - suppliers in its input-producing industries;
    - at most one supplier in any given industry;
    - at most one customer in the entire economy.
- Not all customized firms may be active.

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Customized Inputs

- Let S denote the set of active firms.
- Production technology of firm $i \in I$:

$$y_i = F_I \left(\ell_i, \{A_{ij} x_{ij}\}_{j \in S}, \{B_{ij} \tilde{x}_{ij}\}_{j \notin S} \right).$$

Assumption

Customized inputs are more productive than generic inputs:

$$A_{ij} \geq B_{ij}.$$

Customized Inputs

- Production of customized goods entails fixed costs, borne by the supplier j.
- z_j units of labor, where $z_j \sim G_J$ and G_J has full support over $[0, \infty)$.
- Costs are sunk once the firm customizes its technology to its matched customer.

- Suppliers that cannot meet this fixed cost “fail”.
  - ▷ The set of active firms S may be different from the set of all firms.
  - ▷ S is determined endogenously

Customized Inputs: Terms of Trade

- A pair of matched firms (i, j).
- Price p_{ij} is determined as the SPNE of the Rubinstein bargaining game:
  - the supplier j makes an offer with probability δ_{ij}
  - the customer i makes an offer with probability $1 - \delta_{ij}$
  - common discount factor $\eta \to 1$
- Commitment by j to deliver as many units demanded by i at price p_{ij}.
- Remarks:
  - Customer i has the outside option of using the generic variant of input J
  - When negotiating, firms take all other (generic and customized) prices as given.

Consumption Good Sector

- A firm combining outputs from various firms in an industry into industry-level bundles, which are then combined into a single consumption good:

$$x_{0I} = H_I\left((x_{0i})_{i\in\phi_{0I}(0)}\right)$$

$$y_0 = F_0(x_{01},\ldots,x_{0N})$$

- Generic variants from industry I are perfect substitutes for all goods produced using the customized technologies
- The supplier has all the bargaining power $\delta_{0i} = 1$.
- Various inputs in H_I are gross complements.

Summary and Timing

- Timing:
  1. Firms are matched with potential suppliers/customers in other industries.
  2. Productivities A_{IJ} and B_{IJ} and customization costs z_i are realized.
  3. Firms decide whether to operate the customized technology.
  4. Customized firm-level p_{ij} prices are determined.
  5. Firms choose their inputs, production occurs.
  6. Customized firms that cannot cover their fixed costs fail.

Production Equilibrium

Definition

Given the set of active firms S and customized prices $\boldsymbol{p}$, a *production equilibrium* is a collection of quantities $x(S, \boldsymbol{p})$ such that

(a) all firms maximize their profits while meeting their output obligations;

(b) households maximize profits taking prices as given;

(c) all markets clear.

- The equilibrium notion treats prices as exogenous.
- The only requirement on the firms is to minimize production costs to meet demand from their customized customers.

Bargaining Equilibrium

Definition

For a set of active firms S, a *bargaining equilibrium* is a collection of prices $\boldsymbol{p}(S) = (p_{ij})_{i,j\in S}$ such that there does not exist a pair of matched supplier-customer firms in E such that one party would rather deviate by entering into a bargaining process with the other, taking all other prices as given.

- For $\boldsymbol{p}$ to be a bargaining equilibrium, no firm would want to unilaterally
  1. (i) renegotiate a price
  2. (ii) terminate an agreement
  3. (iii) enter into a new agreement

Full Equilibrium

Definition

A *full (subgame perfect) equilibrium* consist of a collection of active firms S^*, firm-level prices $\boldsymbol{p}^*(S)$, and quantities $x^*(S, \boldsymbol{p})$ such that

(a) given any S and $\boldsymbol{p}$, the quantities $x^*(S, \boldsymbol{p})$ form a production equilibrium;

(b) given any S, the price vector $\boldsymbol{p}^*(S)$ is a bargaining equilibrium;

(c) no firm in S^* fails and no firm outside of S^* would rather start operating:

$$\begin{aligned} \pi_i(S^*) &\geq 0 \qquad && \forall \boldsymbol{i} \in S^* \\ \pi_i(S^* \cup \{i\}) &< 0 && \forall \boldsymbol{i} \notin S^*. \end{aligned}$$

Generic Inputs

- Generic inputs are produced by a competitive fringe of firms in each industry.
- Competitive sub-economy with constant returns to scale & a single factor of production

  Non-substitution theorem ⟶ generic prices determined irrespective of the matching, relationship-specific productivities, bargaining, etc.

- System of N equations and N unknowns:

$$\widetilde{p}_I = c_I\left(w, \frac{\widetilde{P}_1}{B_{I1}}, \frac{\widetilde{P}_2}{B_{I2}}, \ldots, \frac{\widetilde{P}_N}{B_{IN}}\right).$$

- Generic technologies also pin down the real wage.

Production Equilibrium

- Given the set of active firms S and generic and customized prices, the production equilibrium is determined via cost minimization, market clearing, and household's utility maximization.

- cost minimization:

$$(\ell_i, x_{i1}, \dots, x_{in}) = \arg\max \quad w\ell_i + \sum_{j\in S} p_{ij}x_{ij} + \sum_{j\notin S} \tilde{p}_j x_{ij}$$

$$\text{subject to} \quad y_i = F_i(\ell_i, (A_{ij}x_{ij})_{j\in S}, (B_{ij}\tilde{x}_{ij})_{j\notin S})$$

- market clearing:

$$y_j = x_{ij}$$

$$\tilde{y}_J = \int_{i\in S, j\notin S} \tilde{x}_{ij} di + \sum_I \tilde{x}_{IJ}$$

- Household budget constraint:

$$y_0 = wL + \sum_{J=1}^{N} \int_0^{\infty} \pi_j dj$$

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For a set of active firms S, a *bargaining equilibrium* is a collection of prices $\boldsymbol{p}(S) = (p_{ij})_{i,j \in S}$ such that there does not exist a pair of matched supplier-customer firms in E such that one party would rather deviate by entering into a bargaining process with the other, taking all other prices as given.

Assumption

(Generic or customized) inputs from different industries are gross complements with another and labor in F_i.

$$y_i = F_i \left(\ell_i, \{A_{ij} x_{ij}\}_{j \in S}, \{B_{ij} \tilde{x}_{ij}\}_{j \notin S} \right).$$

Pairwise Bargaining

Lemma

Supplier-customer pair (j, i) *reach an agreement if and only if* $\boldsymbol{p}_{-ij}$ *satisfies*

$$c_i(\boldsymbol{p}_{-ij}, c_j(\boldsymbol{p}_{-ij})) \leq p_i.$$

- It guarantees that there are gains from trade between supplier j and customer i.
- Firm i's marginal cost is smaller than its output price if j sells at marginal cost.
- If violated, there are no gains from trade: the two firms would take the outside option of not trading with one another.

Pairwise Bargaining

Lemma

Suppose $c_i(\boldsymbol{p}_{-ij}, c_j(\boldsymbol{p}_{-ij})) \leq p_i$*. The SPNE of the bargaining game entails the price*

$$p_{ij} = \begin{cases} p_{ij}^{\dagger} & \textit{if} \quad \psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) \geq 0 \\ \\ \min\{\bar{p}_{ij}, p_{ij}^{o}\} & \textit{if} \quad \psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) < 0 \end{cases},$$

where $p_{ij}^{\dagger}$ *is the solution to the equation*

$$\psi_{ij}(p_{ij}^{\dagger}) = \delta_{ij} \frac{\bar{\pi}_i(p_{ij}^{\dagger})}{\bar{\pi}_i'(p_{ij}^{\dagger})} + (1 - \delta_{ij}) \frac{\bar{\pi}_j(p_{ij}^{\dagger})}{\bar{\pi}_j'(p_{ij}^{\dagger})} = 0,$$

$\bar{p}_{ij} = A_{ij}\widetilde{p}_j / B_{ij}$ *is the outside option of firm i, and* p_{ij}^{o} *is the price at which firm i makes zero profits.*

- The price depends on the production functions, but not the quantities.
- $\psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) \geq 0$ is the condition that implies outside options do not bind.
- Otherwise, firm *i* either uses the generic variant or may not produce at all.

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Pairwise Bargaining

Corollary

p_{ij} is increasing in supplier bargaining power δ_{ij}, supplier's marginal cost c_j, customer's output price p_i, and the relationship-specific productivity A_{ij}.

Corollary

p_{ij} is decreasing in the productivity of generic technology B_{ij}.

Bargaining Equilibrium

Theorem

For any given set of active firms S, a bargaining equilibrium $\boldsymbol{p}$ always exists.

Corollary

The bargaining equilibrium $\boldsymbol{p}(S)$ is determined independently of the quantities.

Proposition

An increase in productivity A_{ij}

(a) *weakly increases all prices upstream to j*

(b) *weakly decreases all prices downstream to i*

(c) *weakly increases all prices that are horizontal to the pair (j, i).*

Bargaining Equilibrium: Special Cases

- $\delta_{ij} = 1$ for all supplier-customer pairs (j, i):

$$p_{ij} = \frac{A_{ij}}{B_{ij}} \widetilde{p}_j$$

$$\mu_{ij} = p_{ij} / c_j = A_{ij} / B_{ij}.$$

- Leontief production functions:

$$p_{ij} = \min \left\{ (1 - \delta_{ij}) c_j + \delta_{ij} A_{ij} (p_i - \hat{c}_{ij}), \frac{A_{ij}}{B_{ij}} \widetilde{p}_j \right\},$$

  where $\hat{c}_{ij}$ is the marginal cost of acquiring all other inputs.

Example: Production Chains

- Firms $i_1, \ldots, i_n \in S$ form a production chain:

$$p_{k,k+1} = \min\left\{ (1-\delta)\frac{p_{k+1,k+2}}{A_{k+1,k+2}} + \delta A_{k,k+1} p_{k-1,k}, \frac{A_{k,k+1}}{B_{k,k+1}} \widetilde{p}_{k+1} \right\}$$

with initial conditions:

$$p_{0,1} = 1$$
$$p_{n,\ell} = w.$$

Full Equilibrium

Definition

A *full (subgame perfect) equilibrium* consist of a collection of active firms S^*, firm-level prices $\boldsymbol{p}^*(S)$, and quantities $x^*(S,\boldsymbol{p})$ such that

(a) given any S and $\boldsymbol{p}$, the quantities $x^*(S,\boldsymbol{p})$ form a production equilibrium;

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(c) no firm in S^* fails and no firm outside of S^* would rather start operating:

$$\begin{aligned} \pi_i(S^*) &\geq 0 \qquad && \forall i \in S^* \\ \pi_i(S^* \cup \{i\}) &< 0 && \forall i \notin S^*. \end{aligned}$$

Theorem

A full equilibrium exists.

Firm-Level Analysis

- The matching ϕ induces a distribution over firm-level production trees:

$$Q(I_0, I_1, \ldots, I_k) = \left\{ i_0 \in I_0 : \exists (i_1, \ldots, i_k) \text{ s.t. } i_r = \phi_{I_{r+1} I_r}(i_{r+1}) \text{ and } \nexists j \text{ s.t. } i_k = \phi_{J I_k}(j) \right\}$$

  - ▹ $Q(I_0)$: set of firms in I_0 with no matched customers.
  - ▹ The firm-level trees can be infinitely long ($k = \infty$).
  - ▹ Not sufficient statistics for firm-level variables, but nonetheless useful.

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Failure Propagations

- A specialized firm $i \in I$ survives only if its profits exceed the fixed costs wz_i, where $z_i \sim G_I$.
- Any revenue a firm earns is obtained from sales to its matched customer in a downstream industry.
- Therefore, as long as G_I has no mass point at 0, firm $i \in I$ fails almost surely if its designated customer fails.

→ Implication: failures propagate upstream from a firm to its suppliers
(may also propagate downstream depending on parameters)

Production Trees

Lemma

An intermediate good producing firm $i_0 \in I_0$ fails if either of the following two conditions are satisfied:

(i) $i_0 \in Q(I_0, I_1, \ldots, I_k)$ *for some finite* k *such that* $I_k \neq 0$.

(ii) $i_0 \in Q(I_0, I_1, \ldots, I_k)$ *for* $k = \infty$.

- A firm can only survive if there is a finite production tree connecting it to the consumption good sector.

▷ We can limit our attention to such structures.

Comparative Statics

- How do changes in fixed costs shape the set of active firms?
- Best response function:

$$\psi(S) = \{i \in E : \pi_i(S \cup \{i\}) \geq 0\}.$$

Firm $i \in \psi(S)$ if i finds it optimal to operate the customized technology when the set of active firms is S.

- Full equilibrium is a fixed point of the above mapping:

$$S^* = \psi(S^*).$$

- Need a discrete monotone comparative statics result, applied to the above mapping (generalization of Tarski's fixed point theorem)

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Comparative Statics (detour)

Theorem

Suppose the mapping $\psi : 2^E \to 2^E$ *satisfies the following:*

(i) *if* $S_1 \subseteq \psi(S_2)$, *then* $S_1 \subseteq \psi(S_1 \cup S_2)$.

(ii) *if* $S_r \subseteq \psi(S_r)$ *for* $r \in R$, *then* $\cup_{r \in R} S_r \subseteq \psi(\cup_{r \in R} S_r)$.

Then, $S^* = \cup_{S \in \mathcal{S}} S$ *is a maximal fixed point of* ψ, *where* $\mathcal{S} = \{S \subseteq E : S \subseteq \psi(S)\}$.
Furthermore, if $\psi_1(S) \subseteq \psi_2(S)$ *for all* S, *then* $S_1^* \subseteq S_2^*$.

Lemma

Suppose either

(1) $\delta_{ij} = 1$ *for all* (i, j) *or*

(2) *all production functions are Leontief.*

Then, ψ *satisfies (i) and (ii).*

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Theorem

Suppose either $\delta_{ij} = 1$ *for all* (i, j) *or all production functions are Leontief. Then,*

(a) *the set of full equilibria is non-empty and has a maximal element;*

(b) *the set of active firms in the maximal equilibrium shrinks as fixed costs increase.*

Corollary

If G_I *is replaced by a distribution that first-order stochastically dominates* G_I*, then*

(a) *the likelihood of failure in all industries weakly increases;*

(b) *aggregate output declines;*

(c) *the average length of the production chains decreases.*

- PE effect: an increase in the likelihood of failures in other industries
- GE effect: less demand for all goods in the economy, thus lower (gross) profits

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Decomposition

- So far: existence, (partial) characterization, and (micro) propagations
- Next step: aggregate implications

Partial Equilibrium Decomposition

- $S = \cup_{r \in R} T_r$: partition of the set of firms to various production trees
- $s_r = m(T_r)$: the mass of active firms in tree T_r

- Partial equilibrium decomposition:

$$d \log \mathrm{GDP} = \underbrace{\sum_{(i,j)} \frac{\partial \log \mathrm{GDP}}{\partial \log A_{ij}} d \log A_{ij}}_{\text{productivity}} + \underbrace{\sum_{r \in R} \frac{\partial \log \mathrm{GDP}}{\partial s_r} ds_r}_{\text{extensive margin movements}}$$

- productivity: direct technology effect + reallocation + changes in markups
- extensive margin: changes in the set of active firms changes the production possibility frontier, the total expenditure on fixed costs, household demand.

Partial Equilibrium Decomposition: Productivity

- Keeping the set of active firms constant, the effect of productivity shocks manifests itself as two separate terms:

$$\frac{d\log \mathrm{GDP}}{d\log A_{ij}} = \underbrace{\frac{\partial \log \mathrm{GDP}}{\partial \log A_{ij}}}_{\substack{\text{shifts in technology frontier}\\ \text{\& reallocation}}} + \sum_{k} \underbrace{\frac{\partial \log \mathrm{GDP}}{\partial \log \mu_k} \frac{d\log \mu_k}{d\log A_{ij}}}_{\text{endogenous changes in markup of firm } k}$$

- Productivity shocks shift the production possibility frontier, keeping the allocation of resources unchanged (Hulten's)

- When economy is inefficient, reallocation of resources has first-order effect (Baqaee and Farhi, 2018)

- New term: endogenous shifts in markups in the bargaining equilibrium.
  - ▷ Determines the passthrough of the shocks

General Equilibrium Decomposition

- Productivity shocks impact the set of active firms (by changing firm profits)
- Household's budget constraint:

$$y_0 = w(L - z(s)) + (1 - c(s, A, B))y_0,$$

- $c(s, A, B)$: equilibrium marginal cost of producing one unit of consumption good, which depends on the bargaining equilibrium and set of active firms.
- $z(s)$: total fixed cost expenditure, which only depends on the set of active firms

- Hence,

$$\text{GDP} = w\frac{L - z(s)}{c(s, A, B)}.$$

- Chain rule:

$$\frac{d}{dA_{ij}} \log \text{GDP} = -\frac{1}{c}\frac{dc}{dA_{ij}} - \frac{1}{c}\sum_r \frac{ds_r}{dA_{ij}}\left(\frac{w}{\text{GDP}}\frac{dz}{ds_r} + \frac{dc}{ds_r}\right).$$

General Equilibrium Decomposition

- The set of active firms, however, itself depends on the output.

$$\frac{ds_r}{dA_{ij}} = \frac{\partial s_r}{\partial A_{ij}} + \frac{\partial s_r}{\partial y_0}\frac{d\,\mathrm{GDP}}{dA_{ij}}.$$

  - $\partial s_r / \partial A_{ij}$: higher productivity increases profits
  - $\partial s_r / \partial y_0$: aggregate demand channel. Higher demand increases all firms' profits

- Therefore:

$$\frac{d\log \mathrm{GDP}}{d\log A_{ij}} = -\frac{\frac{\partial \log c}{\partial \log A_{ij}} + \sum_r \left(\frac{\partial s_r}{\partial \log A_{ij}}\right)\left(\frac{w}{c\mathrm{GDP}}\frac{dz}{ds_r} + \frac{d\log c}{ds_r}\right)}{1 + \sum_r \left(\frac{\partial s_r}{\partial \log y}\right)\left(\frac{w}{c\mathrm{GDP}}\frac{dz}{ds_r} + \frac{d\log c}{ds_r}\right)}.$$

General Equilibrium Decomposition

$$\frac{d\log \mathrm{GDP}}{d\log A_{ij}} = -\frac{\overset{\text{partial equilibrium }(-)}{\frac{\partial \log c}{\partial \log A_{ij}}} + \sum_r \left(\overset{\text{cascade effect }(+)}{\frac{\partial s_r}{\partial \log A_{ij}}}\right)\left(\frac{w}{c\,\mathrm{GDP}}\overset{\text{fixed cost effect }(+)}{\frac{dz}{ds_r}} + \frac{d\log c}{ds_r}\right)}{1+\sum_r \left(\underset{\text{aggregate demand effect }(+)}{\frac{\partial s_r}{\partial \log y}}\right)\left(\frac{w}{c\,\mathrm{GDP}}\frac{dz}{ds_r} + \underset{\text{entry effect }(-)}{\frac{d\log c}{ds_r}}\right)}.$$

- Partial equilibrium effect: direct technology effect (Hulten's), reallocation effect, and movements in markups, holding the set of active firms constant
- Cascade effect: shocks change the set of active firms
- Entry effect: changes in the set of active firms changes aggregate productivity
- Aggregate demand effect: more active firms increases households' demand, which then translates into higher profits

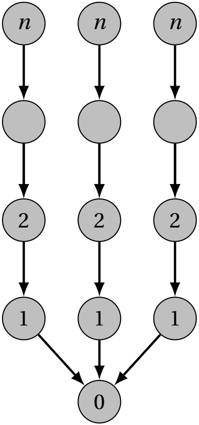
General Equilibrium Decomposition

- Suppose the distribution of fixed costs G_k is parameterized by a parameter ζ_k, with an increase in ζ_k corresponding to a first-order stochastic dominance shift in the distribution G_k.

$$\frac{d\log \mathrm{GDP}}{d\log \zeta_k} = -\frac{\frac{w}{c\,\mathrm{GDP}} \overset{\text{partial equilibrium }(+)}{\frac{\partial z}{\partial \zeta_k}} + \sum_r \left(\overset{\text{cascade effect }(-)}{\frac{\partial s_r}{\partial \zeta_k}} \right) \left(\frac{w}{c\,\mathrm{GDP}} \frac{dz}{ds_r} + \frac{d\log c}{ds_r} \right)}{1 + \sum_r \left(\underset{\text{aggregate demand effect }(+)}{\frac{\partial s_r}{\partial \log y}} \right) \left(\frac{w}{c\,\mathrm{GDP}} \frac{dz}{ds_r} + \underset{\text{entry effect }(-)}{\frac{d\log c}{ds_r}} \right)}.$$

Example: Production Chains

- Suppose there are n industries, with industry I_k supplying industry I_{k-1}.
- Supplier has all the bargaining power $\delta_{k-1,k} = 1$.



Failures and Aggregate Output

- m_k: measure of active firms in industry I_k.
- Aggregate output:

$$\text{GDP} = \frac{L - \bar{z}}{1 - \sum_{k=1}^{n} m_k (A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n})^{-1} \left(\frac{A_{k-1,k}}{B_{k-1,k}} - 1 \right)}$$

- failure cascades:

$$m_{k+1} = m_k G_{k+1} \left(\frac{(1 - B_{k,k+1} / A_{k,k+1}) \text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right)$$

- expenditure on fixed costs:

$$\bar{z} = \sum_{k=1}^{N} \int_0^{\infty} z g_k(z) \mathbf{1} \left\{ z \leq \frac{(1 - B_{k,k+1} / A_{k,k+1}) \text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right\} dz.$$

Failures and Aggregate Output

$$\text{GDP} = \frac{L - \bar{z}}{\sum_{k=1}^{n} (m_k - m_{k+1})(A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n})^{-1}}$$

$$m_{k+1} = m_k G_{k+1} \left(\frac{(1 - B_{k,k+1}/A_{k,k+1}) \text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right)$$

- Compare the output to the economy with endogenous set of active firms (GDP^*) to that of an economy with exogenous set of active firms (GDP):

$$\lim_{A_k \to \infty} \lim_{A_1 \to B_1} \frac{\text{GDP}^*}{\text{GDP}} = \infty.$$

Summary and Next Steps

- A firm-level model of input-output linkages that takes firm-specific relationships and failures into account. Failures are the main channel via which negative shocks propagate

- Expressions for the failure rates and aggregate output as a function of firm-level production chains.

- Aggregated industrial-level variables (Domar weights, sectoral markups) not sufficient statistics for understanding
  (i) the propagation of the shocks
  (ii) how various shocks shape aggregate output

- Next steps:
  - ▷ more detailed comparative statics
  - ▷ numerical estimates for the various forces in a more realistic economy.
  - ▷ measuring the various terms in the data?