How the Wealth Was Won: Factor Shares as Market Fundamentals

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Sharp Rise in Equity Values in Post-War Period

- Stock market risen sharply in post-war era, driven mostly *last 30 years*.

	Average Annual Growth		
Subsample	**Market Equity**	**Output**	**Earnings**
1989:Q1 - 2017:Q4	6.9%	2.5%	4.8%
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Notes: Variables for the nonfinancial corporate sector (NFCS). Annualized growth rates for the specified sample, in real terms, deflated by the implicit price deflator for NFCS output (net value added).

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- Upshot? Widening **chasm** between **stock market** and **broader economy**.

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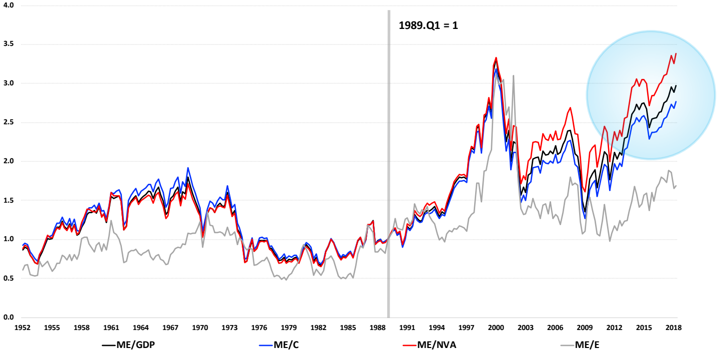
Stock Market v.s Broader Economy

- ME= Total value of market equity of the NFCS.

Notes: **ME:** Nonfinancial Corporate Sector Stock Value. **E:** Nonfinancial Corporate Business After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Net Value Added of Nonfinancial Corporate Sector. The sample spans the period 1952:Q1-2017:Q4.

Stock Market v.s Broader Economy

- ME relative to 3 different measures of agg. economic activity is at or near post-war high.



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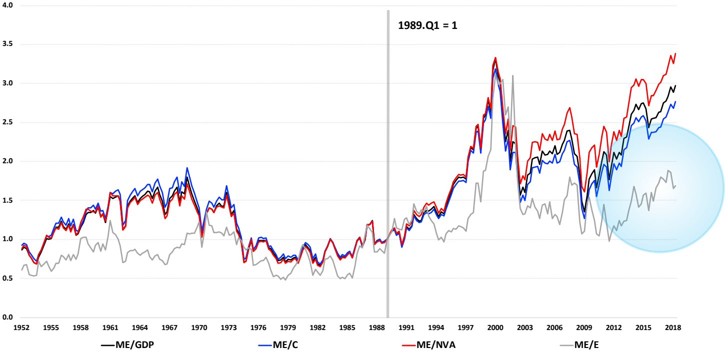
- Notably, ME/E not near post-war high.



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- Figure 1 suggests basic **tenet of macroeconomic** theory not borne out by data.
- *What is responsible for sharply rising equity values over post-war period?*

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  - **Shareholder payout**: Changes in how economic growth expected to be linked to cash payments to shareholders
  - **Discount rates**: Changes in how those payments are discounted back to present (expected path of future short rates, risk premia)
  - **Economic growth**: Could still be key to market's rise over post-war period, even if last 30 years have been a striking exception.

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  - **Infer what values** latent factors must have taken over sample to explain the data.
- Identification of **mutually uncorrelated** components + **loglinear** model => precisely decompose 100% of market's observed growth into **distinct component sources** in the model.

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- Apply model to the NFCS over period 1952:Q1-2017:Q4.

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- **Economic growth** contributed **23%** since 1989, and **50%** over full sample.
- From 1952-1988, **economic growth** accounted for **92%**, but that **37 year** period created *less than half* wealth generated in **29 years since 1989**.

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- **Model & estimates** $=>$ common practice of **averaging** of returns, dividend, payout data over post-war sample to estimate ERP **overstates the true risk premium** by $\approx 50\%$.

Related Literature

- **Drivers of real level of stock market**: Few studies. Lettau & Ludvigson '13, and Greenwald, Lettau, Ludvigson (GLL) '14.
- This paper replaces GLL, differs substantively from both. Neither study did formal estimation of asset pricing model. GLL model is less flexible, less general.
- **Heterogeneous agent, limited partipation** perspective adds **realism**: just 52% households own equity in 2016 (any amt, any form); most own very little: **top 5% of stock wealth dist. owns 76% of market** and earns small fraction of income in form of labor compensation.

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- **Negative correlation returns human wealth and stock market**: Lustig, Van Nieuwerburgh '08; Lettau, Ludvigson '09; Chen et. al., '14.

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- **Aggregate output**:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}$$

  A_t mean zero TFP; N_t labor endowment (hours $\times$ prod. factor).

  - Workers inelastically supply labor; hours fixed, normalized to unity.
  - K_t grows deterministically at gross rate $G \equiv 1 + g => K_t = K_0 G^t$.
  - Labor productivity grows: $N_t = G^t$.

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- Fraction $\boldsymbol{\tau}_t$ of Y_t devoted to **taxes & interest & other**. **Earnings** E_t (after-tax profits):

$$E_t \equiv \mathbf{S}_t Z_t Y_t$$

  $Z_t \equiv 1 - \tau_t$; $S_t \equiv$ AT profit share of AT profit+labor comp.
- Labor compensation

$$W_t N_t \equiv (1 - S_t) Z_t Y_t,$$

- E_t / Y_t "**earnings share**"and $(W_t N_t) / Y_t$ "**labor share**".

Factors Share Shock

- Variable $\mathbf{S_t}$ modeled as exogenous *factors share shock*.
- Captures changes may occur, for any reason, in allocation of rewards between firms and workers under imperfect competition.
- Possible sources include changes in:
  1. **Industry concentration** structure alters labor intensivity of production
  2. **Bargaining power** of US workers (international competition, prevalence of unions, off-shoring)
  3. **Technological factors** alter substitutability of labor for capital.

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- **Cash payments to shareholders** = *net payout* ("cashflows") differs from E_t by **net new investment**.
- Firm reinvests fixed fraction ωY_t each period =>

$$\underbrace{C_t}_{\text{cashflows}} = E_t - \omega Y_t = (S_t Z_t - \omega) Y_t.$$

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- In equilibrium, agg. SH consumption = agg. **net payout** C_t.

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- Distinguished from **representative household** who consumes p.c. **aggregate consumption**.

The Model: SDF

- IMRS of *shareholder* consumption is the **SDF** and takes the form:

$$
\begin{aligned}
M_{t+1} &= \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-x_t}, \qquad \beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)} \\
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- Preference shifter x_t and time varying sub. time-discount factor taken as given by ind. shareholders, driven by market as whole.
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- x_t drives **price of risk** in SDF; latent state variable affects risk premia.
- SDF reflects both preferences and beliefs $=>$ decrease in x_t interpreted as either a decrease in **effective risk aversion** or decrease in **pessimism**.

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$$\begin{aligned} M_{t+1} &= \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-x_t}, \qquad \beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)} \\ \ln M_{t+1} &= -\mathbf{1}'\delta_t - d_t - x_t \Delta \ln C_{t+1} \end{aligned}$$

- More general version SDFs Campbell, Cochrane '99, Lettau, Wachter '07.
- Preference shifter x_t and time varying sub. time-discount factor taken as given by ind. shareholders, driven by market as whole.
- x_t drives **price of risk** in SDF; latent state variable affects risk premia.
- SDF reflects both preferences and beliefs $=>$ decrease in x_t interpreted as either a decrease in **effective risk aversion** or decrease in **pessimism**.
- x_t **positive on average** but may occasionally go negative reflecting occasional risk tolerance or confidence.

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- x_t **positive on average** but may occasionally go negative reflecting occasional risk tolerance or confidence.
- Time varying β_t essential for obtaining stable **risk-free rate** along with volatile equity premium.

Loglinear Model: Earnings

- Work with loglinear approximation solved analytically. $\ln(E_t/Y_t)$ could go above 1, but does so rarely (less than 1% of time in 10,000 period simulation).
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$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \Delta y_{t+1} = g + \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \sim N\, i.i.d. \left(0, \sigma_a^2\right).$$

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- **Earnings**: Since $E_t = S_t Z_t Y_t$, earnings growth

$$\Delta e_t = \Delta s_t + \Delta z_t + \Delta y_t.$$

Loglinear Model: Payout

- **Payout**: Let $Q_t \equiv S_t Z_t$, then $C_t = (Q_t - \omega) Y_t$, or $c_t = \ln (Q_t - \omega) + y_t$.
- Loglinearize to obtain approximate equation for log payout

$$c_t = \bar{c} + \xi (s_t + z_t) + y_t,$$

where $\xi = \frac{\overline{SZ}}{\overline{SZ} - \omega}$ and $\overline{SZ}$ is the average value of $S_t Z_t$.

- **Log payout growth** is given by

$$\Delta c_t = \xi (\Delta s_t + \Delta z_t) + \Delta y_t.$$

Loglinear Model: Dynamics of Cashflows

- Data on earnings share suggests existence of both **low- and higher-frequency components**.
- Allow for this in model. Denote $\mathbf{s}_t = (s_{LFt}, s_{HFt})'$.
- $s_t = \mathbf{1}'\mathbf{s}_t$, where $\mathbf{1}' \equiv (1, 1)$. $s_{LF,t}$ a lower frequency component, $s_{HF,t}$ a higher frequency component.
- Specify dynamics of Δc_t, Δs_t as

$$\begin{aligned}
\Delta c_{t+1} &= \xi \mathbf{1}' \Delta \mathbf{s}_{t+1} + \xi \Delta z_{t+1} + \Delta y_{t+1} \\
\mathbf{s}_{t+1} &= (\boldsymbol{I} - \boldsymbol{\Phi}_s)\bar{\mathbf{s}} + \boldsymbol{\Phi}_s \boldsymbol{s}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \boldsymbol{\varepsilon}_{s,t+1} &\sim N(\mathbf{0}, \boldsymbol{\Sigma}_s) \\
\Delta \mathbf{s}_{t+1} &= -(\boldsymbol{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \widetilde{\mathbf{s}}_t &\equiv \mathbf{s}_t - \bar{\mathbf{s}}
\end{aligned}$$

Loglinear Model: Risk Free Rate

- **Risk-free rate of return** known with certainty at t:

$$R_{f,t+1} \equiv \left(\mathbb{E}_t\left[M_{t+1}\right]\right)^{-1}, \quad \beta_t \equiv \frac{\exp\left(\delta_t\right)}{\exp\left(d_t\right)}.$$

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- Data on short rates suggests **low- and higher-frequency components**.
- Model $\delta_t = \mathbf{1}'\boldsymbol{\delta}_t$, where $\boldsymbol{\delta}_t = (\delta_{LFt}, \delta_{HFt})'$ and

$$\begin{aligned} m_{t+1} &\equiv \ln M_{t+1} = -\mathbf{1}'\boldsymbol{\delta}_t - d_t - x_t \Delta c_{t+1} \\ \boldsymbol{\delta}_{t+1} &= (\boldsymbol{I} - \boldsymbol{\Phi}_\delta)\bar{\boldsymbol{\delta}} + \boldsymbol{\Phi}_\delta \boldsymbol{\delta}_t + \boldsymbol{\varepsilon}_{\delta,t+1},\ \boldsymbol{\varepsilon}_{\delta,t+1} \sim N(0, \boldsymbol{\Sigma}_\delta), \end{aligned}$$

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- Parameter d_t is a compensating factor chosen to ensure

$$r_{f,t} = -\ln \mathbb{E}_t \exp\left(m_{t+1}\right) = \mathbf{1}'\delta_t.$$

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- Gaussian shocks, the SDF is **conditionally lognormal**:

$$r_{f,t+1} = \mathbf{1}'\boldsymbol{\delta}_t + d_t + x_t\left(g - \xi\phi_z\tilde{z}_t - \xi\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t\right) - \frac{1}{2}x_t^2\left(\sigma_a^2 + \xi\left(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}\right)\right).$$

$$d_t = -x_t(g - \xi\phi_z\tilde{z}_t) + \xi x_t \mathbf{1}'\left(I - \boldsymbol{\Phi}_s\right)\widetilde{\mathbf{s}}_t + \frac{1}{2}x_t^2\left(\sigma_a^2 + \xi\left(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}\right)\right).$$

Price of Risk Dynamics

- **Price of risk x_t** follows:

$$x_{t+1} = (1 - \phi_x)\,\overline{x} + \phi_x x_t + \varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \sim N\, i.i.d.\left(0, \sigma_x^2\right).$$

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- **Latent process Z_t**: Data on taxes & interest filtered to infer values of latent stochastic process for Z_t. (Equilibrium asset returns in model depend not only on today's Z_t but also expected future path of Z_t.)

$$z_{t+1} = (1 - \phi_z)\,\overline{z} + \phi_z z_t + \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \sim N\,i.i.d.\left(0, \sigma_z^2\right).$$

Loglinear Model: Equilibrium Stock Market Values

- **Equity return**: Let P_t denote total market equity, with C_t equity payout, return on equity is
$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$
- $pc_t \equiv \ln\left(\frac{P_t}{C_t}\right)$. The log return obeys the following approximate identity:
$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1},$$
where $\kappa_1 = \exp\left(\overline{pc}\right) / \left(1 + \exp\left(\overline{pc}\right)\right)$, and $\kappa_0 = \exp\left(\overline{pc}\right) + 1 - \kappa_1 \overline{pc}$.

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- The first-order-condition for optimal shareholder consumption:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right].$$

- **Conjecture and verify** a solution takes form:

$$pc_t = A_0 + \mathbf{A}_s' \widetilde{\mathbf{s}}_t + \mathbf{A}_r' \widetilde{\boldsymbol{\delta}}_t + A_x \tilde{x}_t + A_z \tilde{z}_t.$$

Loglinear Model Solution

$$pc_t = A_0 + \mathbf{A}_s' \widetilde{\mathbf{s}}_t + \mathbf{A}_r' \widetilde{\boldsymbol{\delta}}_t + A_x \tilde{x}_t + A_z \tilde{z}_t$$

$$\begin{aligned}
\mathbf{A}_s' &= -\xi \mathbf{1}' (\mathbf{I} - \boldsymbol{\Phi}_s)(\mathbf{I} - \kappa_1 \boldsymbol{\Phi}_s)^{-1} \\
A_x' &= -\left[\left(\xi^2 \left(\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1} + \sigma_z^2 \right) + \sigma_g^2 \right) + \xi \kappa_1 \left(\mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{1} \right) \right] (1 - \kappa_1 \phi_x)^{-1} \\
\mathbf{A}_\delta' &= -\mathbf{1}' \left(\mathbf{I} - \kappa_1 \boldsymbol{\Phi}_\delta \right)^{-1} \\
A_z &= -\xi (1 - \phi_z)(1 - \kappa_1 \phi_z)^{-1}
\end{aligned}$$

- All terms LHS are negative.
  - $\mathbf{A}_\delta'$ and $A_x' < 0$: $\uparrow$ risk-free rate or in price of risk increases rate future cash payments **discounted**.
  - $\mathbf{A}_s' < 0$: $\boldsymbol{\Phi}_s < 1$. Equity values rise proportionally less than c_t in anticipation of eventual **mean-reversion in payout**.
  - **Size of effects** depends on magnitudes of $\boldsymbol{\Phi}_\delta$, ϕ_x, and $\boldsymbol{\Phi}_s$.

Loglinear Model Solution

- Model solution implies **log equity premium**:

$$
\begin{aligned}
\mathbb{E}_t[r_{t+1}] - r_{f,t} &= \left[\left(\xi^2\left(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}+\sigma_z^2\right)+\sigma_a^2\right)+\xi\kappa_1\left(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}+A_z\sigma_T^2\right)\right]\mathbf{x_t} \\
&\quad - \frac{1}{2}\mathbb{V}_t(r_{t+1}) \\
\mathbb{V}_t(r_{t+1}) &= \kappa_1^2\left[\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s + A_z^2\sigma_z^2 + A_x^2\sigma_x^2 + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\mathbf{A}_\delta\right] \\
&\quad + \left[\xi^2\left(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}+\sigma_z^2\right)+\sigma_a^2\right] + 2\xi\kappa_1\left[\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1} + A_z\sigma_z^2\right],
\end{aligned}
$$

- **Homoskedastic shocks**: $\mathbb{V}_t$ constant, but risk premium varies with $\mathbf{x_t}$.

Estimation and Data

- **Primitive parameters**

$$\boldsymbol{\theta} = \left(\xi, g, \sigma_a^2, \mathrm{vec}\left(\boldsymbol{\Phi}_s\right), \mathrm{vec}\left(\boldsymbol{\Phi}_\delta\right), \phi_x, \phi_Z, \mathrm{vec}\left(\boldsymbol{\Sigma}_s\right), \mathrm{vec}\left(\boldsymbol{\Sigma}_\delta\right), \sigma_x^2, \sigma_Z^2, \bar{s}, \bar{\delta}, \bar{x}, \bar{z}\right)',$$

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- **Two groups**
  - Small number ($\bar{s}$, ξ, ϕ_x) calibrated (discussed below).
  - Remaining parameters freely estimated.

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- **Two groups**
  - Small number ($\bar{s}$, ξ, ϕ_x) calibrated (discussed below).
  - Remaining parameters freely estimated.
- **Estimation of Parameters**: Bayesian methods with *flat priors*.
- **Estimation of Latent States**: Model linear in logs so can use **Kalman filter**.

Estimation and Data

- Confront model with observations **1952:Q1-2017:Q4** on:

Estimation and Data

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  1. Log output growth Δy_t
  2. Log earnings share $e_t - y_t \equiv ey_t$
  3. Interest rate to proxy $r_{f,t}$
  4. Taxes & interest share z_t
  5. Equity-to-output ratio $p_t - y_t \equiv py_t$

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  4. Taxes & interest share z_t
  5. Equity-to-output ratio $p_t - y_t \equiv py_t$
- **Risk-free rate** 3-Mo T-bill minus fitted $\widehat{\pi}_t$ from regression on lagged π_t.
- **NFCS** observations for all others.
  1. Need y_t, ey_t, py_t etc., to be measured for *same sector* of economy. Otherwise subject to confounding compositional effects.
  2. Corporate sector advantage: $1 - S_t$ not affected by statistical imputation of labor income from total income reported by sole proprietors and unincorporated business.

Estimation and Data

- Forgoing variables are related to $\boldsymbol{\theta}$ and **latent states**:

$$
\begin{aligned}
ey_t &= \mathbf{1}'(\tilde{\mathbf{s}}_t + \bar{\mathbf{s}}) \\
r_{ft} &= \mathbf{1}'(\widetilde{\boldsymbol{\delta}}_t + \bar{\boldsymbol{\delta}}) \\
py_t &= \overline{py} + \left(\mathbf{A}_s' + \xi\mathbf{1}'\right)\widetilde{\mathbf{s}}_t + \mathbf{A}_r'\widetilde{\boldsymbol{\delta}}_t + A_x\tilde{x}_t + (A_Z + \xi)\,\tilde{z}_t \\
\tilde{z}_{t+1} &= \phi_Z\tilde{z}_t + \varepsilon_{Z,t+1} \\
z_t &= \tilde{z}_t + \overline{z} \\
\Delta y_t &= g + \Delta\tilde{y}_t
\end{aligned}
$$

- $\overline{py} \equiv A_0 + \bar{c} + \xi\overline{z_t}$

- Last two are **identities** that exactly pin down values of $\varepsilon_{z,t}$ and $\varepsilon_{a,t}$.

Estimation and Data

- **State space form**:

$$\begin{aligned} \mathcal{Y}_t &= \mathbf{H}'\boldsymbol{\beta}_t + \mathbf{G}'\mathbf{1} && (1) \\ \boldsymbol{\beta}_t &= \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \mathbf{v}_t, && (2) \end{aligned}$$

- **Observation equation**: $\mathcal{Y}_t \equiv \left(ey_t, r_{ft}, py_t, \Delta z_t, \Delta y_t\right)'$

- **Latent states**: $\boldsymbol{\beta}_t \equiv \left(\tilde{s}_{LF,t}, \tilde{s}_{HF,t}, \tilde{\delta}_{LF,t}, \tilde{\delta}_{HF,t}, \tilde{x}_t, \tilde{z}_t, \Delta\tilde{y}_t\right)'$, where

$$\mathbf{v}_t = \left(\varepsilon_{s,LF,t}, \varepsilon_{s,HF,t}, \varepsilon_{\delta,LF,t}, \varepsilon_{\delta,HF,t}, \varepsilon_{x,t}, \varepsilon_{Z,t}, \varepsilon_{a,t}\right)'$$

and **F**, $\mathbf{H}'$, and $\mathbf{G}'$ are matrices of primitive parameters.

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$$

  and $\mathbf{F}$, $\mathbf{H}'$, and $\mathbf{G}'$ are matrices of primitive parameters.

- **Kalman filter** gives *smoothed* estimates of latent states $\boldsymbol{\beta}_{t|T}$.

- **Measurement error effectively zero** in (1) due to flexible loglinear model and use of 7 latent states to match only 5 variables.

Estimation and Data

- **Posterior of θ**: Obtained by computing likelihood using KF and combining with priors.
- **Flat priors**: posterior coincides with likelihood, posterior mode coincides with MLE estimate.
- **Parameter uncertainty**: Characterized using a RWMH algorithm.
- **Latent state uncertainty** Characterized using simulation smoother of Durbin and Koopman (2002).
- **Error bands** therefore reflect both parameter and latent state uncertainty.

Estimation and Data

- **Three parameters are calibrated**: $\bar{s}$, ξ, ϕ_x
- **Mean earnings share variable $\bar{s}$**: forces exactly right mean in ey without error.
- **Payout-earnings growth relation ξ**

$$\Delta c_t = \xi \left(\Delta s_t + \Delta z_t \right) + \Delta y_t.$$

  Calibrated to match relative vol of Δc_t to $\Delta e_t \approx 2$.
- **Persistence of x_t**: No observable series to discipline ϕ_x.
  - If ϕ_x freely estimated with *flat prior*, procedure will choose parameters of FS and RF process to fit s_t, $r_{f,t}$ exactly, set ϕ_x to explain *all* variation in py_t.
  - Implausible implication: RP shocks **very persistent**, since $\widehat{\phi}_x > 0.97$.
  - **Estimates of risk-premium**: cay_t proxy AR1 ≈ 0.9; Martin '17 SVIX proxy: AR1 ≈ 0.8.
  - **Baseline happy medium $\phi_x = 0.85$**; robustness: $\phi_x = 0.80$, $\phi_x = 0.90$.

Parameter Estimates

- Effective mean risk aversion modest reflecting volatility cash payments to shareholders.

Variable	Parameter	Mode	5%	Median	95%
Risk Price Mean	$\bar{x}$	4.4832	3.3174	4.3791	5.8452
Risk Price Vol.	σ_x	3.8086	2.8981	3.8307	5.1905
Risk-Free Rate Mean	$\bar{r}_f$	0.0023	0.0008	0.0027	0.0048
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.1587	0.0290	0.1928	0.4109
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0019	0.0016	0.0019	0.0022
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9321	0.8949	0.9314	0.9558
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0015	0.0012	0.0015	0.0019
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.9250	0.8981	0.9245	0.9455
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0680	0.0633	0.0683	0.0734
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9997	0.9984	0.9996	0.9999
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0179	0.0132	0.0179	0.0230
Tax + Interest Share Pers.	ϕ_Z	0.9545	0.9244	0.9583	0.9875
Tax + Interest Vol.	σ_Z	0.0041	0.0038	0.0041	0.0044
Productivity Vol.	σ_a	0.0160	0.0148	0.0159	0.0171

Notes: The table reports parameter estimates from the posterior distribution. The sample spans the period 1952:Q1-2017:Q4.

Parameter Estimates

- Short rates: $\phi_{\delta,LF} = 0.93 =>$ substantial declines *recently* in $r_{f,t}$ not important impetus for equity boom.

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- Factors share: $\phi_{s,LF} = 0.9997$ estimated to be more persistent.

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Risk Price Mean	$\bar{x}$	4.4832	3.3174	4.3791	5.8452
Risk Price Vol.	σ_x	3.8086	2.8981	3.8307	5.1905
Risk-Free Rate Mean	$\bar{r}_f$	0.0023	0.0008	0.0027	0.0048
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.1587	0.0290	0.1928	0.4109
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0019	0.0016	0.0019	0.0022
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9321	0.8949	0.9314	0.9558
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0015	0.0012	0.0015	0.0019
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.9250	0.8981	0.9245	0.9455
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0680	0.0633	0.0683	0.0734
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9997	0.9984	0.9996	0.9999
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0179	0.0132	0.0179	0.0230
Tax + Interest Share Pers.	ϕ_Z	0.9545	0.9244	0.9583	0.9875
Tax + Interest Vol.	σ_Z	0.0041	0.0038	0.0041	0.0044
Productivity Vol.	σ_a	0.0160	0.0148	0.0159	0.0171

Notes: The table reports parameter estimates from the posterior distribution. The sample spans the period 1952:Q1-2017:Q4.

Asset Pricing Moments

- "Model"numbers from **simulations**. "Fitted"numbers use **estimated latent states** obtained from fitting model to *historical data*.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
Log Price-Payout Ratio	3.507	0.334	3.486	0.456	3.392	0.493
Log Earnings Growth	2.065	11.198	2.450	15.041	2.450	15.041
Log Payout Growth	2.064	21.952	3.095	28.167	4.243	30.558
Log Earnings Share Growth	0.000	10.897	0.405	13.337	0.405	13.337
Log Payout Share Growth	0.000	21.804	1.106	26.607	2.254	28.678

Notes: All statistics are computed for annual (continuously compounded) data. "Model"numbers are averages across 1000 simulations of the model of the same size as our data sample. "Fitted"numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Asset Pricing Moments

- Fitted moments are model's implications *conditional on observed sequence of shocks*; are therefore **directly comparable** to "Data"moments.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
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Notes: All statistics are computed for annual (continuously compounded) data. "Model"numbers are averages across 1000 simulations of the model of the same size as our data sample. "Fitted"numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Asset Pricing Moments

- Fitted moments of Δe_t, Δey_t, and $r_{f,t}$ match exactly b/c observables.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
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Asset Pricing Moments

- Fitted moments of log R, log excess returns, and pc_t match data moments reasonably well.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
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Asset Pricing Moments

- Fitted mean of excess return understates data mean because model understates mean PO growth over the sample (not an estimation target).

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
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Asset Pricing Moments

- Fitted mean log $\mathbb{E}R^e$ (6.4%) > model mean log $\mathbb{E}R^e$ (4.3%) by 2.1 perc. points, attributable to good luck, string of favorable shocks redistributed rents to shareholders.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
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Asset Pricing Moments

- *Fitted means* for Δe_t and Δc_t larger than *model means*.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
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Asset Pricing Moments

- Estimates imply *roughly 2.1 percentage points* of the post-war mean log return on stocks in excess of a T-bill is attributable to this string of **favorable factors share shocks**, rather than to genuine **compensation for bearing risk**.

Variable	Model Mean(%)	Model SD(%)	Fitted Mean(%)	Fitted SD(%)	Data Mean(%)	Data SD(%)
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
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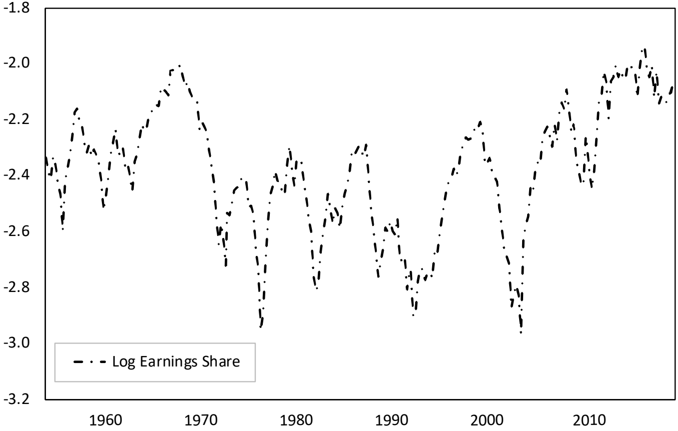
Earnings Share Over Time

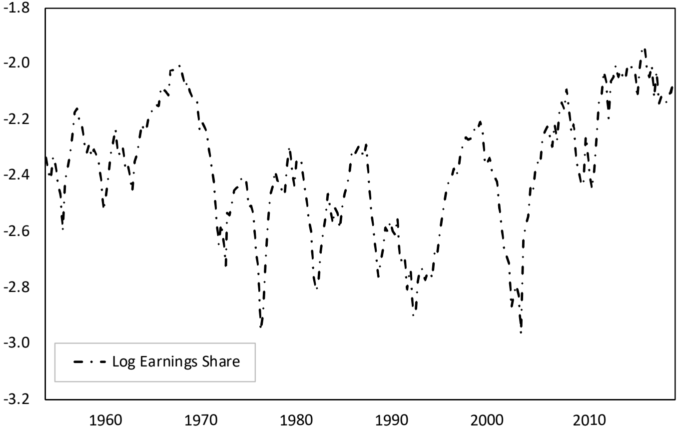
- Look at key data series we match exactly, starting with ey_t.



Notes: The figure exhibits the observed log earnings share series for the nonfinancial corporate sector. The sample spans the period 1952:Q1-2017:Q4.

Earnings Share Over Time

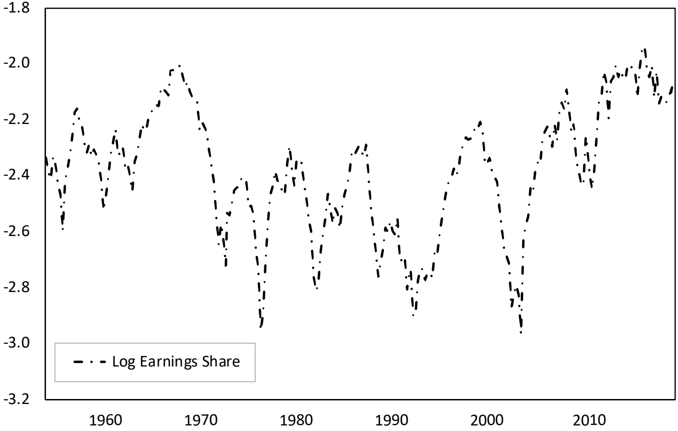
- Increases in ey_t equivalent to declines in *labor share*.



Notes: The figure exhibits the observed log earnings share series for the nonfinancial corporate sector. The sample spans the period 1952:Q1-2017:Q4.

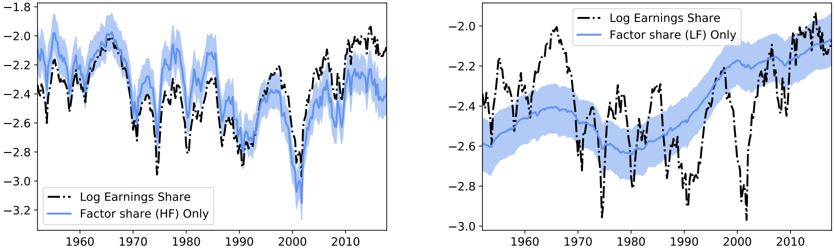
Earnings Share Over Time

- High in 1950s, 1960s, low in 1970s, 1980s, **upward trajectory** since 1990.



Notes: The figure exhibits the observed log earnings share series for the nonfinancial corporate sector. The sample spans the period 1952:Q1-2017:Q4.

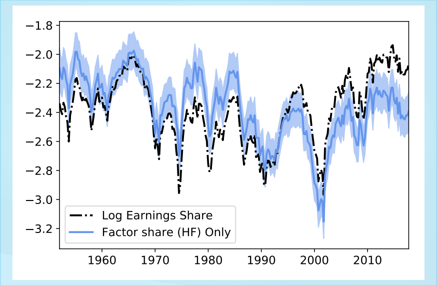
Sources of Variation in Earnings Share Over Time

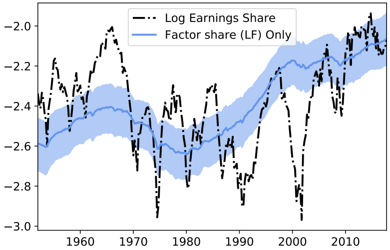


Notes: The figure exhibits the observed earnings share series along with the model-implied variation in the series attributable to the latent factor share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Earnings Share Over Time

- $s_{HF,t}$ captures **transitory variation** in ey_t.

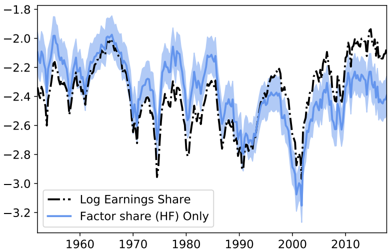


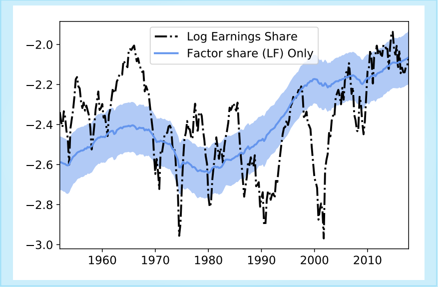


Notes: The figure exhibits the observed earnings share series along with the model-implied variation in the series attributable to the latent factor share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Earnings Share Over Time

- $s_{LF,t}$ captures *longer term trend* in ey_t.

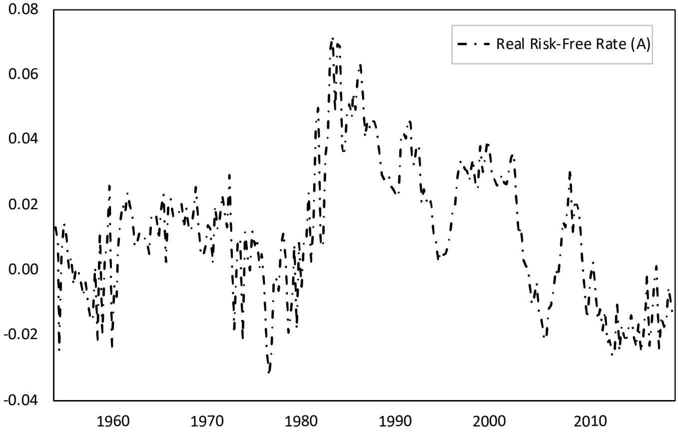




Notes: The figure exhibits the observed earnings share series along with the model-implied variation in the series attributable to the latent factor share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Risk-Free Rate Over Time

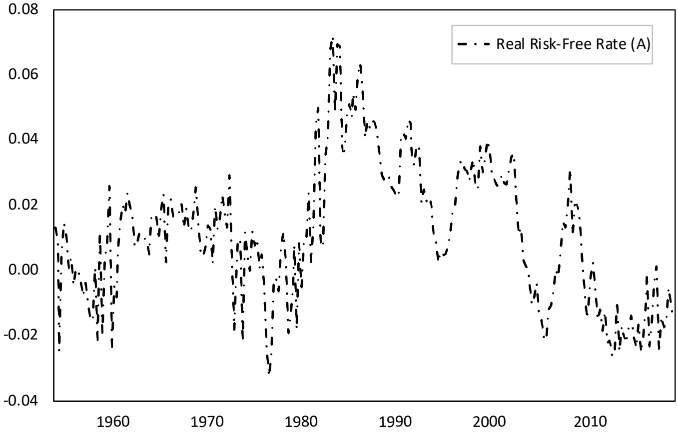
- Real rates **low** in 1950s & late 1970s, **high** during Volcker disinflation and after, **low** post-financial crisis.



Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation. The sample spans the period 1952:Q1-2017:Q4.

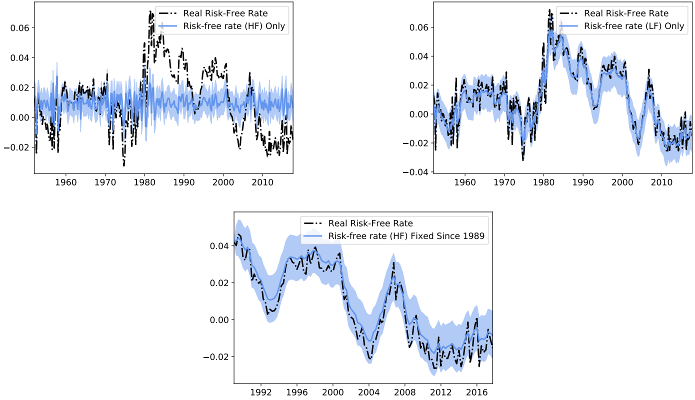
Risk-Free Rate Over Time

- Although rates are low today, they are **not unusually low** by historical standards.



Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation. The sample spans the period 1952:Q1-2017:Q4.

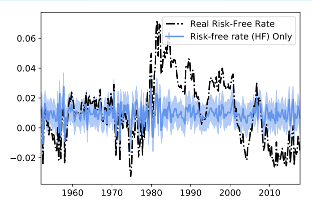
Sources of Risk-Free Rate Variation

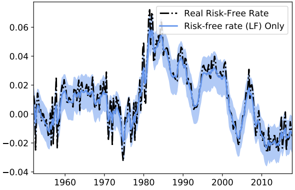


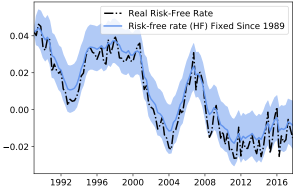
Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhibits the observed risk-free rate series along with the model-implied variation in the series attributable to the latent risk-free rate components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Risk-Free Rate Variation

- Component $\delta_{HF,t}$ picks up transitory variation in $r_{f,t}$.



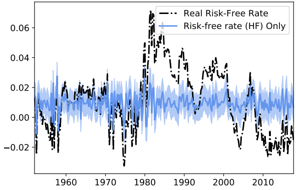


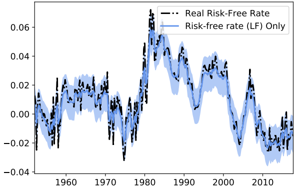


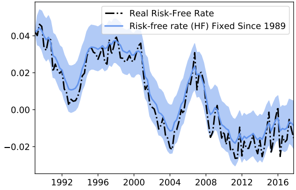
Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhibits the observed risk-free rate series along with the model-implied variation in the series attributable to the latent risk-free rate components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Risk-Free Rate Variation

- **Low-high-low** pattern of $r_{f,t}$ well captured by $\delta_{LF,t}$



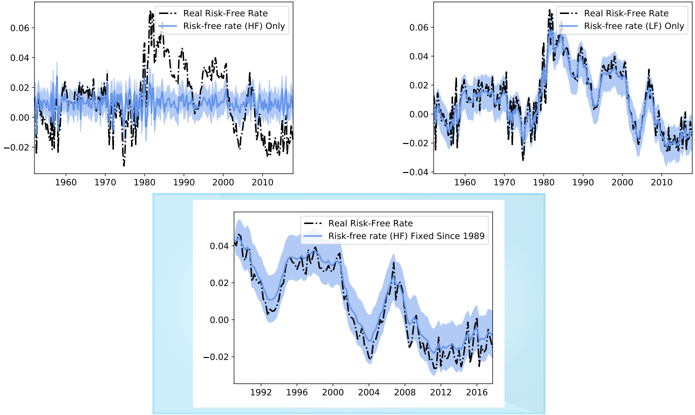




Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhibits the observed risk-free rate series along with the model-implied variation in the series attributable to the latent risk-free rate components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Risk-Free Rate Variation

- LF component shows downward trend since about 1989.



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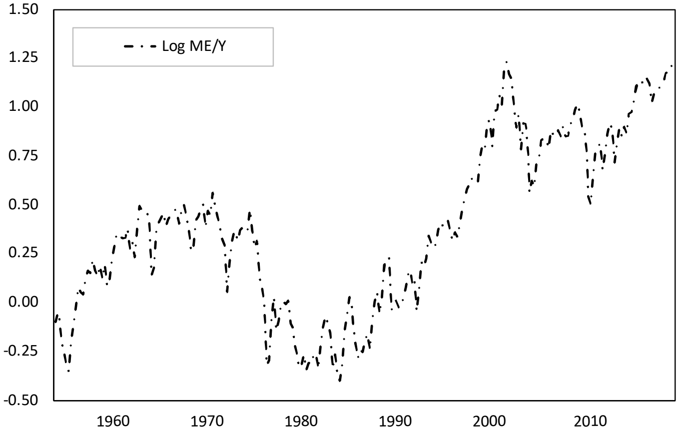
Price-Output Ratio Over Time

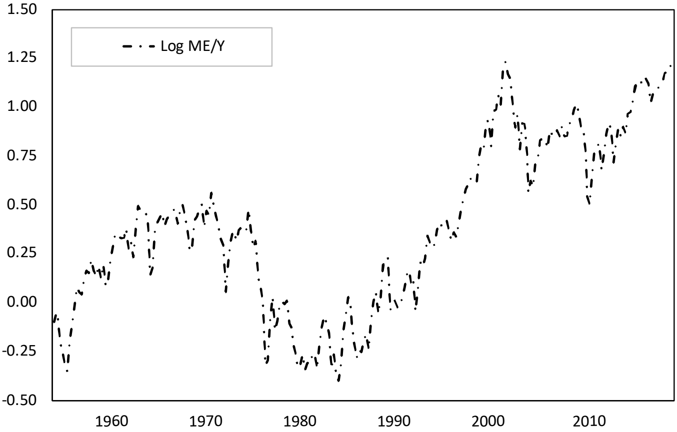
- Equity relative to output has **short-term "wiggles"**, longer-term **trends**.



Notes: The figure exhibits the observed log market equity-to-output series for the nonfinancial corporate sector. The sample spans the period 1952:Q1-2017:Q4.

Price-Output Ratio Over Time

- Over the sample observe an *upward* trend.



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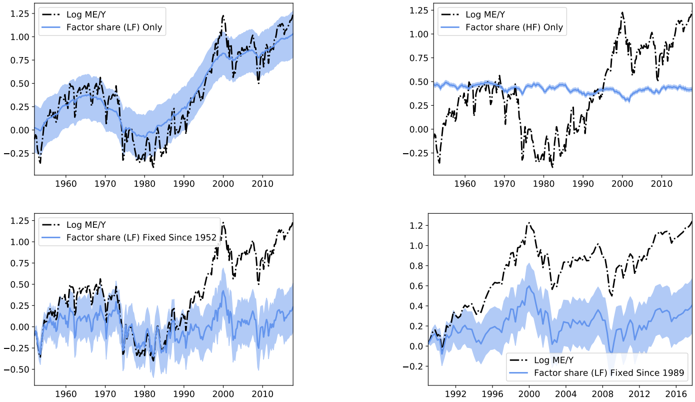
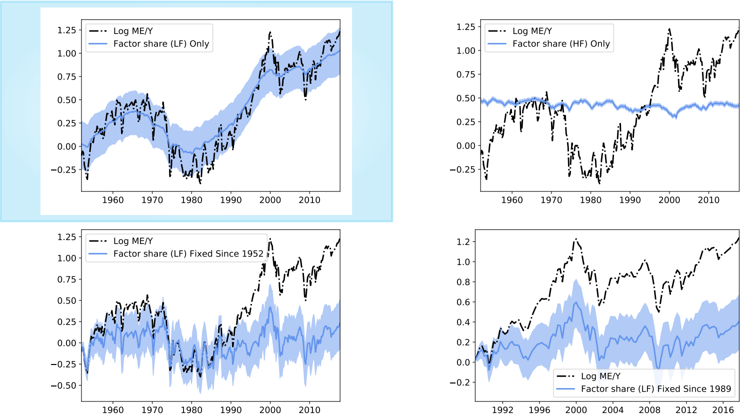
Sources of Variation in Price-Output Ratio



Notes: The figure exhibits the observed log market equity-to-output series along with the model-implied variation in the series attributable to the latent factors share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

- Upward trend well captured by **LF FS factor** $s_{LF,t}$.



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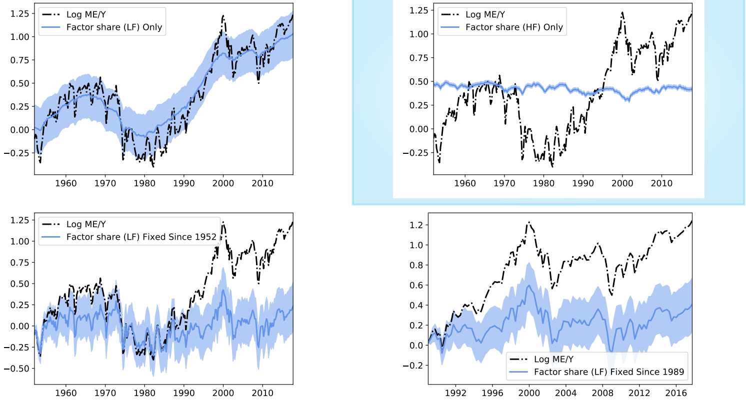
Sources of Variation in Price-Output Ratio

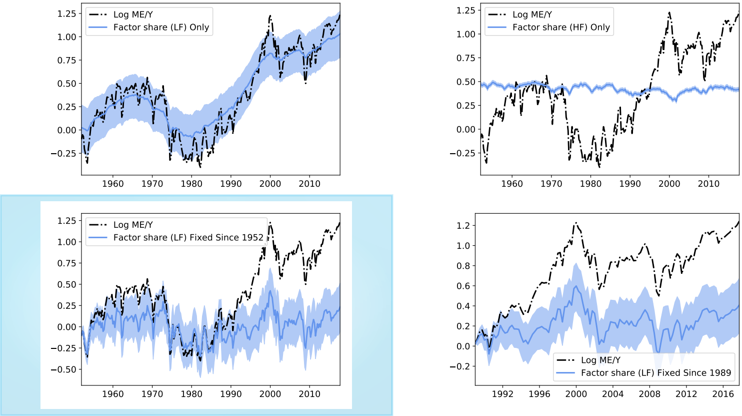
- **HF FS factor** $s_{HF,t}$ captures "wiggles".



Notes: The figure exhibits the observed log market equity-to-output series along with the model-implied variation in the series attributable to the latent factors share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

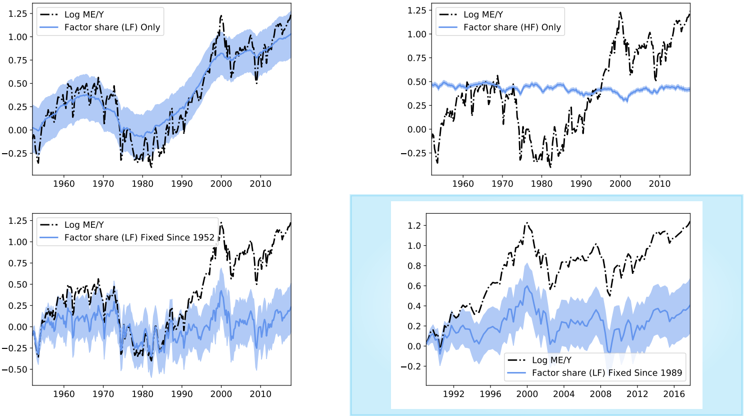
- Fix the LF component, model is unable to capture *any of upward trend* since 1989.



Notes: The figure exhibits the observed log market equity-to-output series along with the model-implied variation in the series attributable to the latent factors share components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

- Zero-in on period post-1989 => **large role for factors share shifts** in driving upward value of ME relative to output.



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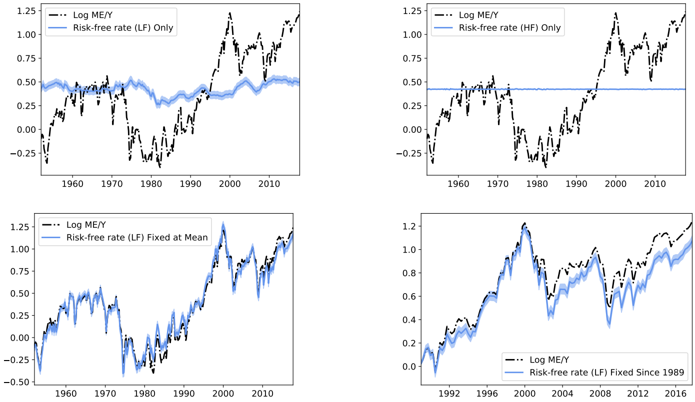
Sources of Variation in Price-Output Ratio

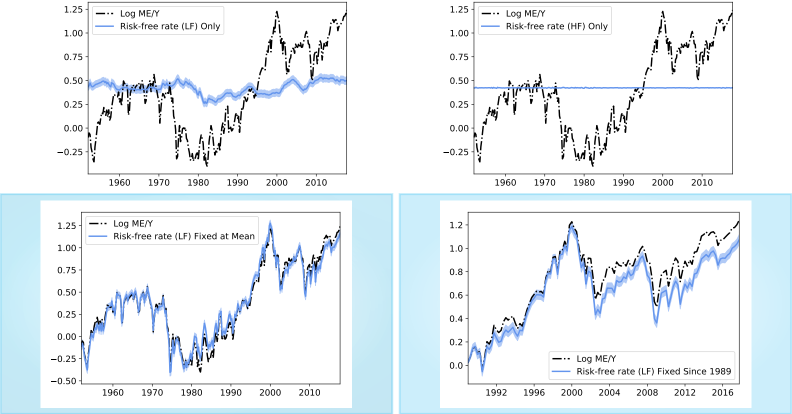
- Role of risk-free rate?



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-free rate component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

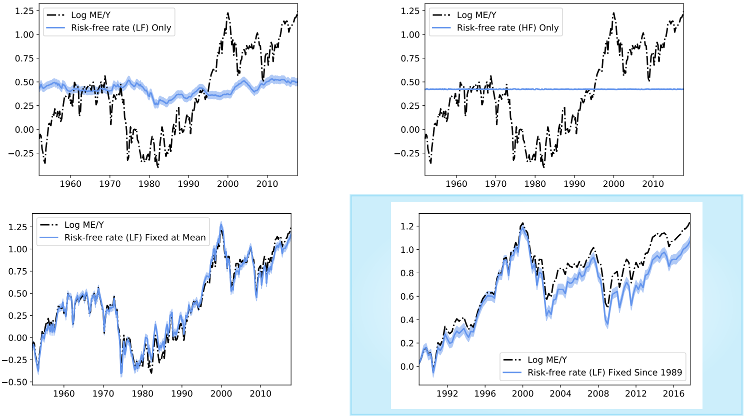
- Shutting down either LF or HF component does little to model's ability to match **trend movements** in py_t.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-free rate component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

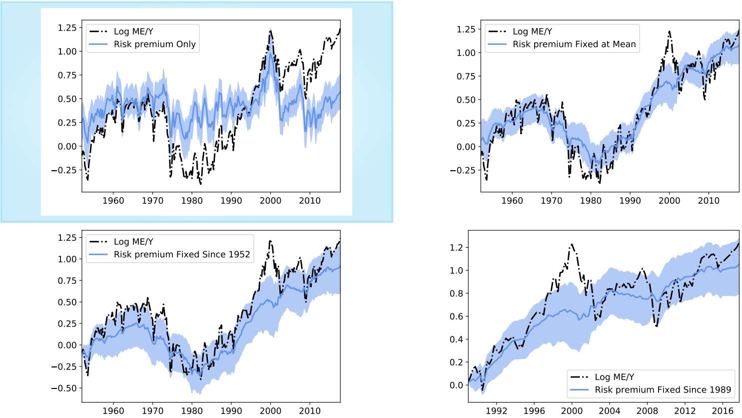
- Modest role since 1989.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-free rate component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

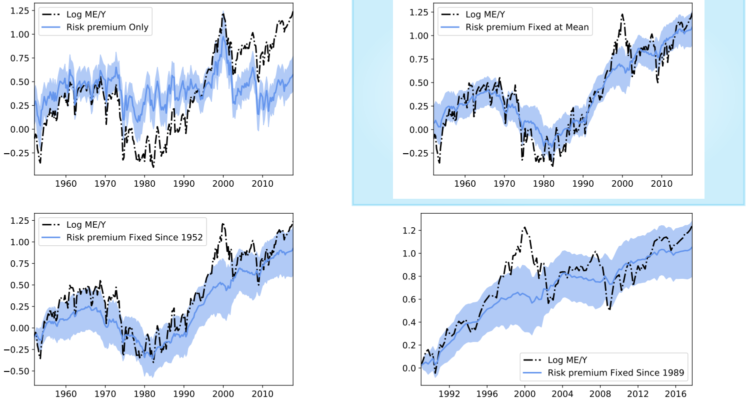
- Risk premium (x_t) variation explains almost all of **transitory booms & busts**.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk premium component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

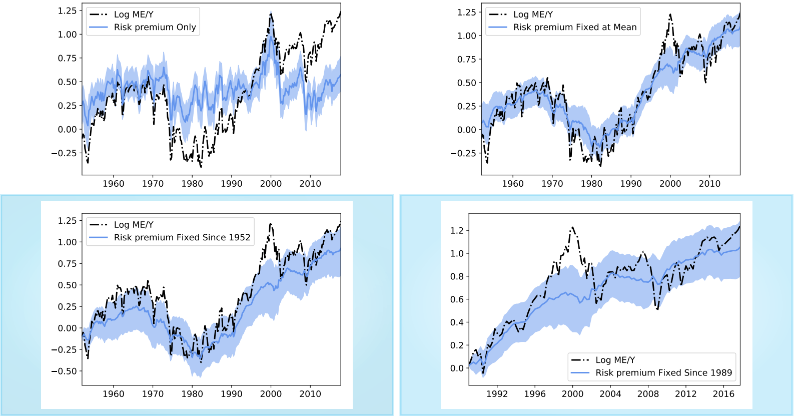
- Does *not* explain **trend component**.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk premium component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

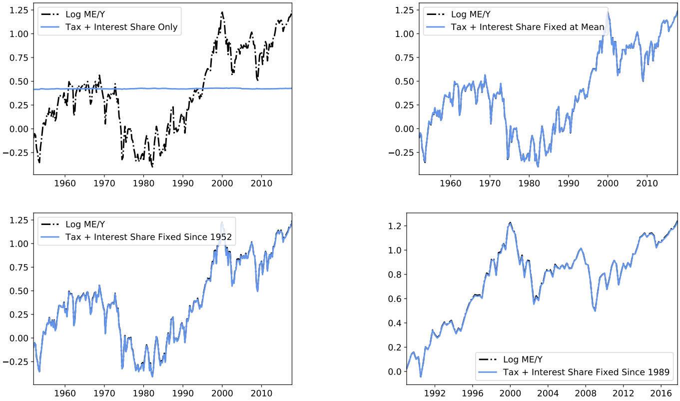
- Small portion of increase in ey_t, esp since 1989, explained by decline in risk premia.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk premium component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Sources of Variation in Price-Output Ratio

- Tax & interest component explains negligible fraction of variation in py_t.



Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the tax/interest component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Growth Decompositions

- Now **quantify importance** of different drives of equity values over time.
- Decompose total growth in equity values into *distinct component sources*.
- Parts attributable to a single source obtained by **fixing all other components** at their values at beginning of sample.
- Components sum to **100% of observed variation**: model + estimated latent components perfectly match time-series on py_t and Δy_t, over sample and **at each point in time**.

Growth Decompositions

- **Market's rise**: **54%** since 1989 and 36% over full sample attributable to $s_{LF,t} + s_{HF,t}$.

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Rep. Agent

Growth Decompositions

- **Other components since 1989**: much smaller roles, e.g., $r_{f,t}$, risk premium.

Contribution	Panel: Market Equity 1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Rep. Agent

Growth Decompositions

- **Economic Growth** contributes **just 23%** since 1989; 50% over full sample.

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Rep. Agent

Growth Decompositions

- **1952-1988**: Δy_t explained **92%** of market's rise. But...

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Rep. Agent

Growth Decompositions

- That **37 year period** created *less than half* wealth created in **29 years** since 1989.

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Rep. Agent

Growth Decompositions: Alternative ϕ_x

- $\phi_x = 0.9$: Declining x_t explains 17% (rather than 11%) of market's rise. $s_{LF,t} + s_{HF,t}$ explain 48% (vs. 54% baseline) since 1989 and 30% (vs. 36% baseline) over full sample.

	Panel A: Market Equity, $\phi_x = 0.80$		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	41.48%	21.16%	55.61%
Factor share (HF)	-2.18%	-5.58%	1.65%
Tax + Interest Share	0.48%	0.54%	0.53%
Risk premium	7.54%	0.16%	8.20%
Risk-free rate (LF)	2.38%	-8.55%	10.19%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real PC Output Growth	50.22%	92.25%	23.69%
	Panel B: Market Equity, $\phi_x = 0.90$		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	30.78%	10.14%	45.07%
Factor share (HF)	-1.35%	-4.68%	2.45%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	17.18%	11.12%	16.99%
Risk-free rate (LF)	2.58%	-9.40%	11.12%
Risk-free rate (HF)	0.09%	0.02%	0.13%
Real PC Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for market equity with persistence parameter of the risk price set to 0.80 (top panel) and set to 0.90 (bottom panel). The sample spans the period 1952:Q1-2017:Q4.

Growth Decompositions: Alternative ϕ_x

- $\phi_x = 0.8$: Declining x_t explains 8% (rather than 11%) of market's rise. $s_{LF,t} + s_{HF,t}$ explain 57% (vs. 54% baseline) since 1989 and 39% (vs. 36% baseline) over full sample.

	Panel A: Market Equity, $\phi_x = 0.80$		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	41.48%	21.16%	55.61%
Factor share (HF)	-2.18%	-5.58%	1.65%
Tax + Interest Share	0.48%	0.54%	0.53%
Risk premium	7.54%	0.16%	8.20%
Risk-free rate (LF)	2.38%	-8.55%	10.19%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real PC Output Growth	50.22%	92.25%	23.69%
	Panel B: Market Equity, $\phi_x = 0.90$		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	30.78%	10.14%	45.07%
Factor share (HF)	-1.35%	-4.68%	2.45%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	17.18%	11.12%	16.99%
Risk-free rate (LF)	2.58%	-9.40%	11.12%
Risk-free rate (HF)	0.09%	0.02%	0.13%
Real PC Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for market equity with persistence parameter of the risk price set to 0.80 (top panel) and set to 0.90 (bottom panel). The sample spans the period 1952:Q1-2017:Q4.

Conclusion and Summary

- **Why has the market risen** over the post-war period? Of importance to financial economists and long-term investors alike.
- We estimate **flexible parametric model** allows influence from several latent components, while **inferring values** components must have taken to explain the data.
- **Finding**: high returns to holding equity due in large part to good luck, attributable to **string of shocks that reallocated rents** toward shareholders away from workers.
- Realizations **added 2.1 percentage points** to mean log excess return, according to estimates (overstating risk premium by $\approx$ 50%).
- Factors share shocks account for most of market's rise since 1989; economic growth and other factors relatively little.
- **For 37 years** from 1952-1989, **economic growth was king** for the equity market.
- But that period was **comparatively lackluster for equity values**, generating less than half as much wealth as the 29 years since 1989.

Appendix

Growth Decompositions: Representative Agent

	Panel: Representative Agent		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%
	Panel B: Baseline Model		
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity. The sample spans the period 1952:Q1-2017:Q4.

Parameter Estimates: Representative Agent

Variable	Parameter	Rep. Agent	Baseline Model
	Parameter Estimates Mode		
Risk Price Mean	$\bar{x}$	56.3120	4.4832
Risk Price Vol.	σ_x	47.8386	3.8086
Risk-Free Rate Mean	$\bar{r}_f$	0.0023	0.0023
Risk-Free (HF) Pers.	$\phi_{delta,HF}$	0.1587	0.1587
Risk-Free (HF) Vol.	$\sigma_{delta,HF}$	0.0019	0.0019
Risk-Free (LF) Pers.	$\phi_{delta,LF}$	0.9321	0.9321
Risk-Free (LF) Vol.	$\sigma_{delta,LF}$	0.0015	0.0680
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.9250	0.9250
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0680	0.0633
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9997	0.9997
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0179	0.0179
Tax + Interest Share Pers.	ϕ_Z	0.9545	0.9545
Tax + Interest Vol.	σ_Z	0.0041	0.0041
Productivity Vol.	σ_a	0.0160	0.0160

Notes: The table reports parameter estimates from the posterior distribution. The sample spans the period 1952:Q1-2017:Q4.

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