Inference in Bayesian Proxy-SVARs*

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Introduction

- Proxy-SVARs aim to identify one or more structural shocks by augmenting SVARs with instruments which must be correlated with the shocks of interest and uncorrelated with all other shocks
- Proxy-SVARs have become influential in empirical macro
  - See e.g. Stock and Watson (12), Montiel-Olea, Stock and Watson (12), Mertens and Ravn (13), Mertens and Montiel-Olea (18)
- Most studies work in the frequentist paradigm. Exceptions are
  - Bahaj (14), Drautzburg (16), and Caldara and Herbst (16)

Introduction

- This paper makes two novel methodological contributions
  - Efficient algorithms to independently draw from the family of NGN over the structural parameterization of a proxy-SVAR
  - Proxies may not be enough to identity a proxy-SVAR
  - Set identification
    - Combine external instruments with sign and zero restrictions
    - Restrictions unrelated to the proxies can be added
- We revisit Mertens and Montiel-Olea (QJE '18)
  - Some conclusions change when using less restrictive assumptions

Setup and Notation

- Consider the SVAR

$$\tilde{\mathbf{y}}_t'\tilde{\mathbf{A}}_0 = \sum_{\ell=1}^{p} \tilde{\mathbf{y}}_{t-\ell}'\tilde{\mathbf{A}}_\ell + \tilde{\mathbf{c}} + \tilde{\varepsilon}_t' \quad \text{for } 1 \leq t \leq T$$

- $\tilde{\boldsymbol{y}}_t' = [\boldsymbol{y}_t' \;\; \boldsymbol{m}_t']$ is a vector of endogenous variables and instruments
  - Let the $n \times 1$ vector $\boldsymbol{y}_t$ denote the endogenous variables
  - Let the $k \times 1$ vector $\boldsymbol{m}_t$ denote the instruments
- $\tilde{\mathbf{A}}_\ell$ is a $(n+k) \times (n+k)$ matrix of parameters for $0 \leq \ell \leq p$
- $\tilde{\mathbf{A}}_0$ invertible, $\tilde{c}$ is a $1 \times (n+k)$ vector of parameters
- $\tilde{\varepsilon}_t' = [\varepsilon_t' \;\; \boldsymbol{\nu}_t']$, where ε_t is $n \times 1$ and $\boldsymbol{\nu}_t$ is $k \times 1$
- $\tilde{\varepsilon}_t$ is conditionally standard normal
- p is the lag length, and T is the sample size

Setup and Notation

- The SVAR can be written as

$$\tilde{\mathbf{y}}_t' \tilde{\mathbf{A}}_0 = \tilde{\mathbf{x}}_t' \tilde{\mathbf{A}}_+ + \tilde{\varepsilon}_t' \quad \text{for } 1 \leq t \leq T$$

where $\tilde{\boldsymbol{x}}_t' = [\tilde{\boldsymbol{y}}_{t-1}' \ \cdots \ \tilde{\boldsymbol{y}}_{t-p}' \ 1]$ and $\tilde{\boldsymbol{A}}_+' = [\tilde{\boldsymbol{A}}_1' \cdots \tilde{\boldsymbol{A}}_p' \ \ \tilde{\boldsymbol{c}}']$

- We will begin by imposing two types of restrictions on $(\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_+)$
  - Block restrictions
  - Exogeneity restrictions
- We turn to this next

Block Restrictions

- We assume that $\mathbf{y}_t$ evolves according to

$$\boldsymbol{y}_t' \boldsymbol{A}_0 = \boldsymbol{x}_t' \boldsymbol{A}_+ + \boldsymbol{\varepsilon}_t',$$

- This implies that

$$\tilde{\boldsymbol{A}}_\ell = \begin{bmatrix} \boldsymbol{A}_\ell & \boldsymbol{\Gamma}_{\ell,1} \\ \mathbf{0}_{k,n} & \boldsymbol{\Gamma}_{\ell,2} \end{bmatrix}, \text{ for } 0 \leq \ell \leq p$$

  where $\boldsymbol{\Gamma}_{\ell,1}$ is $n \times k$ and $\boldsymbol{\Gamma}_{\ell,2}$ is $k \times k$

- We call these zero restrictions on $\tilde{\mathbf{A}}_\ell$ the *block restrictions*

A Proxy-SVAR

- SVAR + *block restrictions (bc)* = Proxy-SVAR
- $(\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_+)$ + *bc* = Proxy-SVAR structural parameters
- The proxy-SVAR structural parameters $(\bar{\boldsymbol{A}}_0, \bar{\boldsymbol{A}}_+)$ and $(\hat{\boldsymbol{A}}_0, \hat{\boldsymbol{A}}_+)$ are observationally equivalent if and only if $\bar{\boldsymbol{A}}_0 = \hat{\boldsymbol{A}}_0 \boldsymbol{Q}$ and $\bar{\boldsymbol{A}}_+ = \hat{\boldsymbol{A}}_+ \boldsymbol{Q}$, for some matrix $\boldsymbol{Q} \in \mathcal{Q} \subset \mathcal{O}(\tilde{n})$, where $\mathcal{O}(\tilde{n})$ is the set of $\tilde{n} \times \tilde{n}$ orthogonal matrices and $\mathcal{Q}$ is defined by $\mathcal{Q} = \{\boldsymbol{Q} \in \mathcal{O}(\tilde{n}) | \boldsymbol{Q} = \text{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2), \boldsymbol{Q}_1 \in \mathcal{O}(n), \text{and } \boldsymbol{Q}_2 \in \mathcal{O}(k)\}$
- A proxy-SVAR is not identified
  - If $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$ are proxy-SVAR structural parameters then $(\tilde{\boldsymbol{A}}_0 \boldsymbol{Q}, \tilde{\boldsymbol{A}}_+ \boldsymbol{Q})$ for $\boldsymbol{Q} \in \mathcal{Q}$ are observationally equivalent proxy-SVAR structural parameters
- Proxy-SVARs are identified using *exogeneity restrictions*

Exogeneity Restrictions (I/II)

- We assume that $\boldsymbol{m}_t$ are proxies for the last k shocks in ε_t and that they are uncorrelated with the first $n-k$ shocks in ε_t

$$\mathbb{E}[\boldsymbol{m}_t \varepsilon_t'] = [\mathbf{0}_{k\times(n-k)} \quad \boldsymbol{V}_{k\times k}]$$

- If $(\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_+)$ are proxy-SVAR structural parameters

$$\left(\tilde{\boldsymbol{A}}_0^{-1}\right)' = \begin{bmatrix} \left(\boldsymbol{A}_0^{-1}\right)' & \mathbf{0}_{n,k} \\ -\left(\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_{0,1}\boldsymbol{\Gamma}_{0,2}^{-1}\right)' & \left(\boldsymbol{\Gamma}_{0,2}^{-1}\right)' \end{bmatrix}$$

- Multiplying the proxy-SVAR by $\tilde{\boldsymbol{A}}_0^{-1}$, the last k equations are

$$\boldsymbol{m}_t' = \tilde{\boldsymbol{x}}_t'\tilde{\boldsymbol{A}}_+ \begin{bmatrix} -\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_{0,1}\boldsymbol{\Gamma}_{0,2}^{-1} \\ \boldsymbol{\Gamma}_{0,2}^{-1} \end{bmatrix} - \varepsilon_t'\tilde{\boldsymbol{A}}_0^{-1}\boldsymbol{\Gamma}_{0,1}\boldsymbol{\Gamma}_{0,2}^{-1} + \boldsymbol{\nu}_t'\boldsymbol{\Gamma}_{0,2}^{-1}$$

- Thus

$$\mathbb{E}[\boldsymbol{m}_t \varepsilon_t'] = -\left(\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_{0,1}\boldsymbol{\Gamma}_{0,2}^{-1}\right)'$$

Exogeneity Restrictions (II/II)

- Hence

$$\left(\tilde{\boldsymbol{A}}_0^{-1}\right)' = \begin{bmatrix} \left(\boldsymbol{A}_0^{-1}\right)' & \mathbf{0}_{n,k} \\ \mathbb{E}[\boldsymbol{m}_t \varepsilon_t'] & \left(\boldsymbol{\Gamma}_{0,2}^{-1}\right)^{-1} \end{bmatrix} = \begin{bmatrix} \left(\boldsymbol{A}_0^{-1}\right)' & \mathbf{0}_{n,k} \\ [\mathbf{0}_{k\times(n-k)} \quad \boldsymbol{V}_{k\times k}] & \left(\boldsymbol{\Gamma}_{0,2}^{-1}\right)' \end{bmatrix}$$

- *Exogeneity restrictions*: the lower left-hand $k \times (n-k)$ block of $\left(\tilde{\boldsymbol{A}}_0^{-1}\right)'$ must be zero
- We also need $\boldsymbol{V}_{k\times k}$ to be full rank
  - Following the literate we refer to this as the relevance condition

Our Prior

- The proxy-SVAR structural parameters have a prior density proportional to $NGN_{(\nu,\boldsymbol{\Phi},\boldsymbol{\Psi},\boldsymbol{\Omega})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$, where

$$NGN_{(\nu,\boldsymbol{\Phi},\boldsymbol{\Psi},\boldsymbol{\Omega})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+) \propto |\det(\tilde{\boldsymbol{A}}_0)|^{\nu-n} e^{-\frac{1}{2}\operatorname{vec}(\tilde{\boldsymbol{A}}_0)'\boldsymbol{\Phi}\operatorname{vec}(\tilde{\boldsymbol{A}}_0)}$$
$$e^{-\frac{1}{2}(\operatorname{vec}(\tilde{\boldsymbol{A}}_+)-\boldsymbol{\Psi}\operatorname{vec}(\tilde{\boldsymbol{A}}_0))'\boldsymbol{\Omega}^{-1}(\operatorname{vec}(\tilde{\boldsymbol{A}}_+)-\boldsymbol{\Psi}\operatorname{vec}(\tilde{\boldsymbol{A}}_0))}$$

- This is a restricted NGN density over the structural representation of the proxy-SVAR because it is a normal-generalized-normal density over $\mathbb{R}^{\tilde{n}^2+\tilde{m}\tilde{n}}$ conditional on the block restrictions, where $\tilde{m} = p\tilde{n} + 1$

- Conjugate family of priors commonly used in the literature
  - Sims and Zha (1998)

Our Posterior

- Since our prior is conjugate, the posterior is also a restricted NGN distribution over proxy-SVAR structural parameters and it is proportional to

$$NGN(\tilde{\nu}, \tilde{\mathbf{\Phi}}, \tilde{\mathbf{\Psi}}, \tilde{\mathbf{\Omega}})$$

where

$$\begin{aligned}
\tilde{\nu} &= T + \nu \\
\tilde{\mathbf{\Omega}} &= (\boldsymbol{I}_{\tilde{n}} \otimes \tilde{\boldsymbol{X}}'\tilde{\boldsymbol{X}} + \mathbf{\Omega}^{-1})^{-1} \\
\tilde{\mathbf{\Psi}} &= \tilde{\mathbf{\Omega}}(\boldsymbol{I}_{\tilde{n}} \otimes \tilde{\boldsymbol{X}}'\tilde{\boldsymbol{Y}} + \mathbf{\Omega}^{-1}\mathbf{\Psi}) \\
\tilde{\mathbf{\Phi}} &= \boldsymbol{I}_{\tilde{n}} \otimes \tilde{\boldsymbol{Y}}'\tilde{\boldsymbol{Y}} + \mathbf{\Phi} + \mathbf{\Psi}'\mathbf{\Omega}^{-1}\mathbf{\Psi} - \tilde{\mathbf{\Psi}}'\tilde{\mathbf{\Omega}}^{-1}\tilde{\mathbf{\Psi}} \\
\tilde{\boldsymbol{Y}} &= [\tilde{\boldsymbol{y}}_1 \ \cdots \ \tilde{\boldsymbol{y}}_T]' \text{ and } \tilde{\boldsymbol{X}} = [\tilde{\boldsymbol{x}}_1 \ \cdots \ \tilde{\boldsymbol{x}}_T]'
\end{aligned}$$

- One may choose another **prior/posterior** . . . not necessarily an issue

Simulation

- ARRW showed how to independently draw from the restricted normal-generalized-normal posterior distribution over the structural representation for the case of a SVAR identified with sign and zero restrictions
- One would like to use ARRW algorithm. But, the techniques of that paper cannot be directly applied here
- Example: $n = 3$, $k = 1$ and $p = 1$
  - Maximum number of zero restrictions using ARRW $= 6$
  - Number of proxy-SVAR restrictions ARRW $= 8$
- Even so, the techniques can be adapted for proxy-SVARs

Simulation

- The key insight in ARRW
  - Draw the *reduced-form* parameters from NIW of choice
  - Draw from set of orthogonal matrices such that zero restrictions hold
  - Map the draws to the structural parameterization and take appropriate care of the volume elements
  - IS to draw structural parameters from a NGN of choice
- This paper
  - Draw the *triangular-block* parameters from a NGN of choice
  - Draw from set of orthogonal matrices such that exogeneity restrictions hold
  - Map the draws to the structural parameterization of the Proxy-SVAR and take appropriate care of the volume elements
  - IS to draw structural parameters from a NGN of choice

Orthogonal-Triangular-Block Parameterization (I/III)

- This parameterization is characterized by four matrices
  1. An $(n+k) \times (n+k)$ upper-triangular matrix $\tilde{\boldsymbol{\Lambda}}_0$
  2. A $(p(n+k)+1) \times (n+k)$ matrix $\tilde{\boldsymbol{\Lambda}}_+$
     - $\tilde{\boldsymbol{\Lambda}}_+' = [\tilde{\boldsymbol{\Lambda}}_1' \cdots \tilde{\boldsymbol{\Lambda}}_p' \;\; \boldsymbol{d}']$, where $\tilde{\boldsymbol{\Lambda}}_i$ is $(n+k) \times (n+k)$ with the lower left-hand $k \times n$ block equal to zero, and $\boldsymbol{d}$ is $1 \times (n+k)$
  3. An $n \times n$ orthogonal matrix $\boldsymbol{Q}_1$
  4. A $k \times k$ orthogonal matrix $\boldsymbol{Q}_2$
- $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$ maps to structural parameters satisfying the block restrictions by

$$(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2) \xrightarrow{f} (\underbrace{\tilde{\boldsymbol{\Lambda}}_0 \operatorname{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2)}_{\tilde{\boldsymbol{A}}_0}, \underbrace{\tilde{\boldsymbol{\Lambda}}_+ \operatorname{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2)}_{\tilde{\boldsymbol{A}}_+}),$$

Orthogonal-Triangular-Block Parameterization (II/III)

- The mapping f has an inverse

$$(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+) \xrightarrow{f^{-1}} (\underbrace{\tilde{\boldsymbol{A}}_0 \boldsymbol{P}}_{\tilde{\boldsymbol{\Lambda}}_0}, \underbrace{\tilde{\boldsymbol{A}}_+ \boldsymbol{P}}_{\tilde{\boldsymbol{\Lambda}}_+}, \underbrace{\boldsymbol{P}_1'}_{\boldsymbol{Q}_1}, \underbrace{\boldsymbol{P}_2'}_{\boldsymbol{Q}_2}).$$

where

  - $\mathbf{P}$ is such that $\tilde{\boldsymbol{A}}_0^{-1} = \boldsymbol{P}\boldsymbol{R}$ is the QR-decomposition of $\tilde{\boldsymbol{A}}_0^{-1}$
  - The lower left-hand $k \times n$ block of $\tilde{\boldsymbol{A}}_0^{-1}$ is zero, and hence
    - $\boldsymbol{P} = \mathrm{diag}(\boldsymbol{P}_1, \boldsymbol{P}_2)$, where $\boldsymbol{P}_1$ is $n \times n$ and $\boldsymbol{P}_2$ is $k \times k$
  - $\tilde{\boldsymbol{\Lambda}}_0$ is upper triangular because $\tilde{\boldsymbol{A}}_0 \boldsymbol{P} = \boldsymbol{R}^{-1}$
  - The lower left-hand $k \times n$ block of each $\tilde{\boldsymbol{\Lambda}}_i$ is zero because
    - $\boldsymbol{P}$ is block diagonal
    - The lower left-hand $k \times n$ block of $\tilde{\boldsymbol{A}}_i$ is zero

Orthogonal-Triangular-Block Parameterization (III/III)

- We can write the proxy-SVAR as

$$\tilde{\boldsymbol{y}}_t' \tilde{\boldsymbol{\Lambda}}_0 = \tilde{\boldsymbol{x}}_t' \tilde{\boldsymbol{\Lambda}}_+ + \tilde{\mathbf{u}}_t',$$

  where $\tilde{\mathbf{u}}_t' = \tilde{\varepsilon}_t' \operatorname{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2)'$ are conditionally normal

- Our ultimate goal is drawing the proxy-SVAR structural parameters from a NGN posterior of choice

  1. We draw orthogonal-triangular-block parameters from a posterior distribution
  2. We map them to the structural parameterization
  3. We calculate the volume element of the mapping
  4. Note that (1)-(3) imply we will be drawing from a posterior over the structural parameterization of the proxy-SVAR, but not the NGN posterior of choice
  5. Because we know the volume element we can use an important sampling to draw from the NGN posterior of choice

The Algorithm

- We independently draw triangular-block parameters $\left(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+\right)$ from a NGN posterior distribution
- Given a draw of $\left(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+\right)$ the exogeneity restrictions are linear restrictions on the columns of the orthogonal matrix $\boldsymbol{Q}_1$
  - Thus, we use ideas in ARRW to independently draw orthogonal matrices $(\boldsymbol{Q}_1, \boldsymbol{Q}_2)$ such that the exogeneity restrictions hold
- Then, we use f to map $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$ into $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$
- These draws are not from the NGN posterior of choice
  - But, we will be able to numerically compute the density associated with the implied distribution
- We can use those draws as an intermediate step in an importance sampler to draw from the desired distribution

Independent Draws of the Triangular-Block Parameters

- We use the Gibbs sampler of Waggoner and Zha ('03) to independently draw from a restricted normal-generalized-normal posterior distribution over the triangular-block parameters characterized by $NGN(\hat{\nu}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Omega}})$
- The triangular and block restrictions on $(\tilde{\mathbf{\Lambda}}_0, \tilde{\mathbf{\Lambda}}_+)$ satisfy the conditions to use the sampler
- Since $\tilde{\mathbf{\Lambda}}_0$ is upper triangular, the Gibbs draws are independent
- Often, it suffices to chose $(\hat{\nu}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Omega}})$ to be equal to $(\tilde{\nu}, \tilde{\mathbf{\Phi}}, \tilde{\mathbf{\Psi}}, \tilde{\mathbf{\Omega}})$
  - However, sometimes this can lead to small effective sample sizes in our sampler. Hence, a more tailored choice of $(\hat{\nu}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Omega}})$ is needed in such cases

Exogeneity Restrictions Linear over $\boldsymbol{Q}_1$

- Let $\boldsymbol{J} = [\mathbf{0}_{k\times n} \;\; \boldsymbol{I}_k]$, the exogeneity restrictions are of the form

$$\boldsymbol{J}(\tilde{\boldsymbol{A}}_0^{-1})'\boldsymbol{e}_{\tilde{n},j} = \mathbf{0}_{k\times 1} \text{ for } 1 \leq j \leq n-k,$$

- This is equivalent to

$$\boldsymbol{J}(\tilde{\boldsymbol{\Lambda}}_0^{-1})'\,\mathrm{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2)\boldsymbol{e}_{\tilde{n},j} = \underbrace{\boldsymbol{J}(\tilde{\boldsymbol{\Lambda}}_0^{-1})'\boldsymbol{L}'}_{\boldsymbol{G}(\tilde{\boldsymbol{\Lambda}}_0)}\boldsymbol{Q}_1\boldsymbol{e}_{n,j} = \mathbf{0}_{k\times 1}$$

  for $1 \leq j \leq n-k$, where $\mathbf{L} = [\;\mathbf{I}_n \quad \mathbf{0}_{n,k}\;]$

- We will denote the number of exogeneity restrictions on the jth column of $\boldsymbol{Q}_1$ by $\tilde{z}_j$, which is k if $1 \leq j \leq n-k$ and is zero if $n-k < j \leq n$

Algorithm 1

1. Draw $(\tilde{\mathbf{\Lambda}}_0, \tilde{\mathbf{\Lambda}}_+)$ independently from the restricted $NGN(\hat{\nu}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Omega}})$ distribution
2. For $1 \leq j \leq n$, draw $\boldsymbol{x}_{1,j} \in \mathbb{R}^{n+1-j-\tilde{z}_j}$ from a $\mathcal{N}(0,1)$, set $\boldsymbol{w}_{1,j} = \boldsymbol{x}_{1,j} / \parallel \boldsymbol{x}_{1,j} \parallel$
3. For $1 \leq j \leq k$, draw $\boldsymbol{x}_{2,j} \in \mathbb{R}^{k+1-j}$ from a $\mathcal{N}(0,1)$, set $\boldsymbol{w}_{2,j} = \boldsymbol{x}_{2,j} / \parallel \boldsymbol{x}_{2,j} \parallel$
4. Define $\boldsymbol{Q}_1 = [\boldsymbol{q}_{1,1} \ \cdots \ \boldsymbol{q}_{1,n}]$ recursively by $\boldsymbol{q}_{1,j} = \boldsymbol{K}_{1,j}\boldsymbol{w}_{1,j}$ for any matrix $\boldsymbol{K}_{1,j}$ whose columns form an orthonormal basis for the null space of the $(j-1+\tilde{z}_j) \times n$ matrix

$$\boldsymbol{M}_{1,j} = \begin{cases} \begin{bmatrix} \boldsymbol{q}_{1,1} & \cdots & \boldsymbol{q}_{1,j-1} & \boldsymbol{G}(\tilde{\mathbf{\Lambda}}_0)' \end{bmatrix}' & \text{for } 1 \leq j \leq n-k \\ \begin{bmatrix} \boldsymbol{q}_{1,1} & \cdots & \boldsymbol{q}_{1,j-1} \end{bmatrix}' & \text{for } n-k+1 \leq j \leq n \end{cases}$$

5. Define $\boldsymbol{Q}_2 = [\boldsymbol{q}_{2,1} \ \cdots \ \boldsymbol{q}_{2,k}]$ recursively by $\boldsymbol{q}_{2,j} = \boldsymbol{K}_{2,j}\boldsymbol{w}_{2,j}$ for any matrix $\boldsymbol{K}_{2,j}$ whose columns form an orthonormal basis for the null space of the $(j-1) \times k$ matrix

$$\boldsymbol{M}_{2,j} = \begin{bmatrix} \boldsymbol{q}_{2,1} & \cdots & \boldsymbol{q}_{2,j-1} \end{bmatrix}' \text{ for } 1 \leq j \leq k.$$

6. Set $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+) = f(\tilde{\mathbf{\Lambda}}_0, \tilde{\mathbf{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$.
7. Return to Step 1 until the required number of draws has been obtained.

Density Implied by Algorithm 1 (I/II)

- Step 1 independently draws $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+)$ from a restricted NGN
- Step 2 draws $\boldsymbol{w}_{1,j}$ from the uniform distribution on the unit sphere in $\mathbb{R}^{n+1-j-\tilde{z}_j}$
- Step 3 draws $\boldsymbol{w}_{2,j}$ from the uniform distribution on the unit sphere in $\mathbb{R}^{k+1-j}$
- The density implied by Steps 1-3 is $\propto$ a restricted NGN
- Step 4 and 5 map $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{w}_{1,1}, \cdots, \boldsymbol{w}_{1,n}, \boldsymbol{w}_{2,1}, \cdots, \boldsymbol{w}_{2,k})$ to $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$
- Step 6 maps $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$ into $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$ using f

Density Implied by Algorithm 1 (II/II)

- The composite mapping implied by Steps 1 to 6 together with Theorem 3 in ARRW will be used to compute the density implied the Algorithm
- Let $\mathcal{Z}$ denotes the set of all proxy-SVAR structural parameters that satisfy the exogeneity restrictions
- By Theorem 3 in ARRW, the posterior density over proxy-SVAR structural parameters subject to the exogeneity restrictions implied by the algorithm is proportional to

$$NGN_{(\hat{\nu},\hat{\mathbf{\Phi}},\hat{\mathbf{\Psi}},\hat{\mathbf{\Omega}})}(\tilde{\mathbf{\Lambda}}_0,\tilde{\mathbf{\Lambda}}_+)v_{(g\circ f^{-1})|_{\mathcal{Z}}}(\tilde{\boldsymbol{A}}_0,\tilde{\boldsymbol{A}}_+),$$

where $(\tilde{\mathbf{\Lambda}}_0,\tilde{\mathbf{\Lambda}}_+,\boldsymbol{Q}_1,\boldsymbol{Q}_2)=f^{-1}(\tilde{\boldsymbol{A}}_0,\tilde{\boldsymbol{A}}_+)$

Algorithm 2

1. Use Algorithm 1 to independently draw $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$
2. Set its importance weight to

$$\frac{NGN_{(\tilde{\nu},\tilde{\boldsymbol{\Phi}},\tilde{\boldsymbol{\Psi}},\tilde{\boldsymbol{\Omega}})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)}{NGN_{(\hat{\nu},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Omega}})}(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+) v_{(g\circ f^{-1})|_{\mathcal{Z}}}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)},$$

   where $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2) = f^{-1}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$ and $\mathcal{Z}$ denotes the set of all proxy-SVAR structural parameters that satisfy the exogeneity restrictions
3. Return to Step 1 until the required number of draws has been obtained
4. Re-sample with replacement using the importance weights

The Need of Additional Restrictions

- Exogeneity restrictions separate the structural shocks into two blocks
- If we only use the exogeneity restrictions, we have an identification problem within the set of the structural shocks that are correlated with the proxies unless $k = 1$
- The same problem occurs within the set of the structural shocks that are not correlated with the proxies
- Let $(\bar{\boldsymbol{A}}_0, \bar{\boldsymbol{A}}_+)$ and $(\hat{\boldsymbol{A}}_0, \hat{\boldsymbol{A}}_+)$ be proxy-SVAR parameters that satisfy the exogeneity restrictions and the relevance condition, then $(\bar{\boldsymbol{A}}_0, \bar{\boldsymbol{A}}_+)$ and $(\hat{\boldsymbol{A}}_0, \hat{\boldsymbol{A}}_+)$ are obs equivalent iff there exists a matrix $\boldsymbol{Q} \in \mathcal{X} \subset \mathcal{Q} \subset \mathcal{O}(\tilde{n})$ such that $\bar{\boldsymbol{A}}_0 = \hat{\boldsymbol{A}}_0 \boldsymbol{Q}$ and $\bar{\boldsymbol{A}}_+ = \hat{\boldsymbol{A}}_+ \boldsymbol{Q}$, where $\mathcal{X}$ is defined by $\mathcal{X} = \{\boldsymbol{Q} \in \mathcal{Q} | \boldsymbol{Q} = \text{diag}(\boldsymbol{Q}_3, \boldsymbol{Q}_4, \boldsymbol{Q}_5)$, $\boldsymbol{Q}_3 \in \mathcal{O}(n-k)$, $\boldsymbol{Q}_4 \in \mathcal{O}(k)$, and $\boldsymbol{Q}_5 \in \mathcal{O}(k)\}$

Additional Zero Restrictions (I/II)

- Let $\boldsymbol{Z}_j$ be a $z_j \times r$ matrix of full row rank
- Assume that the zero restrictions in the proxy-SVAR structural parameterization are $\boldsymbol{Z}_j \boldsymbol{F}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)\boldsymbol{e}_{\tilde{n},j} = \boldsymbol{0}_{z_j,1}$ for $1 \leq j \leq \tilde{n}$
- The zero restrictions in the orthogonal-triangular-block parameterization are

$$\begin{aligned} &\boldsymbol{Z}_j \boldsymbol{F}(f(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2))\boldsymbol{e}_{\tilde{n},j} \\ &= \boldsymbol{Z}_j \boldsymbol{F}(f(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{I}_n, \boldsymbol{I}_k)) \operatorname{diag}(\boldsymbol{Q}_1, \boldsymbol{Q}_2)\boldsymbol{e}_{\tilde{n},j} \\ &= \boldsymbol{0}_{z_j,1} \text{ for } 1 \leq j \leq \tilde{n} \end{aligned}$$

Additional Zero Restrictions (II/II)

- The zero restrictions can be written as

$$\underbrace{\boldsymbol{Z}_j \boldsymbol{F}(f(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{I}_n, \boldsymbol{I}_k)) \begin{bmatrix} \boldsymbol{I}_n' & \boldsymbol{0}_{k,n}' \end{bmatrix}'}_{\boldsymbol{G}_{1,j}(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+)} \boldsymbol{Q}_1 \boldsymbol{e}_{n,j} = \boldsymbol{0}_{z_j,1} \text{ for } 1 \leq j \leq n$$

- Again, linear restrictions on $\boldsymbol{Q}_1$

Algorithm 3

1. Draw $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+)$ independently from the restricted $NGN(\hat{\nu}, \hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{\Psi}}, \hat{\boldsymbol{\Omega}})$
2. For $1 \leq j \leq n$, draw $\boldsymbol{x}_{1,j} \in \mathbb{R}^{n+1-j-\tilde{z}_j-z_j}$ from a $\mathcal{N}(0,1)$, set $\boldsymbol{w}_{1,j} = \boldsymbol{x}_{1,j} / \parallel \boldsymbol{x}_{1,j} \parallel$
3. For $1 \leq j \leq k$, draw $\boldsymbol{x}_{2,j} \in \mathbb{R}^{k+1-j-z_{n+j}}$ from a $\mathcal{N}(0,1)$, set $\boldsymbol{w}_{2,j} = \boldsymbol{x}_{2,j} / \parallel \boldsymbol{x}_{2,j} \parallel$
4. Define $\boldsymbol{Q}_1 = [\boldsymbol{q}_{1,1} \ \cdots \ \boldsymbol{q}_{1,n}]$ recursively by $\boldsymbol{q}_{1,j} = \boldsymbol{K}_{1,j}\boldsymbol{w}_{1,j}$ for any matrix $\boldsymbol{K}_{1,j}$ whose columns form an orthonormal basis for the null space of the $(j-1+\tilde{z}_j+z_j) \times n$ matrix

$$\boldsymbol{M}_{1,j} = \begin{cases} \begin{bmatrix} \boldsymbol{q}_{1,1} & \cdots & \boldsymbol{q}_{1,j-1} & \boldsymbol{G}(\tilde{\boldsymbol{\Lambda}}_0)' & \boldsymbol{G}_{1,j}(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+)' \end{bmatrix}' & \text{for } 1 \leq j \leq n-k \\ \begin{bmatrix} \boldsymbol{q}_{1,1} & \cdots & \boldsymbol{q}_{1,j-1} & \boldsymbol{G}_{1,j}(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+)' \end{bmatrix}' & \text{for } n-k+1 \leq j \leq n \end{cases}$$

5. Define $\boldsymbol{Q}_2 = [\boldsymbol{q}_{2,1} \ \cdots \ \boldsymbol{q}_{2,k}]$ recursively by $\boldsymbol{q}_{2,j} = \boldsymbol{K}_{2,j}\boldsymbol{w}_{2,j}$ for any matrix $\boldsymbol{K}_{2,j}$ whose columns form an orthonormal basis for the null space of the $(j-1+z_{n+j}) \times k$ matrix

$$\boldsymbol{M}_{2,j} = \begin{bmatrix} \boldsymbol{q}_{2,1} & \cdots & \boldsymbol{q}_{2,j-1} \end{bmatrix}' \text{ for } 1 \leq j \leq k.$$

6. Set $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+) = f(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2)$
7. Return to Step 1 until the required number of draws has been obtained

Density Implied by Algorithm 3

- We proceed as when computing the density implied by Algorithm 1
- Let $\mathcal{X}$ denote the set of all proxy-SVAR structural parameters that satisfy the proxy and the zero additional restrictions
- The density over the structural parameterization of the proxy-SVAR subject to the proxy and the zero additional restrictions implied by the algorithm is proportional to

$$NGN_{(\hat{\nu},\hat{\mathbf{\Phi}},\hat{\mathbf{\Psi}},\hat{\mathbf{\Omega}})}(\tilde{\mathbf{\Lambda}}_0,\tilde{\mathbf{\Lambda}}_+)v_{(g\circ f^{-1})|\mathcal{X}}(\tilde{\boldsymbol{A}}_0,\tilde{\boldsymbol{A}}_+),$$

where $(\tilde{\mathbf{\Lambda}}_0,\tilde{\mathbf{\Lambda}}_+,\boldsymbol{Q}_1,\boldsymbol{Q}_2)=f^{-1}(\tilde{\boldsymbol{A}}_0,\tilde{\boldsymbol{A}}_+)$

Additional Sign and Zero Restrictions

- Let $\boldsymbol{S}_j$ be an $s_j \times r$ matrix of full row rank
- $\boldsymbol{S}_j$ defines the additional sign restrictions on the jth structural shock. In particular,

$$\boldsymbol{S}_j \boldsymbol{F}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)\boldsymbol{e}_{\tilde{n},j} > \boldsymbol{0}_{s_j,1}$$

- $\boldsymbol{F}$ continuous
- At least one parameter value satisfy the sign restrictions

Algorithm 4

1. Use Algorithm 3 to independently draw $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$.
2. If $(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$ satisfies the sign restrictions, set its importance weight to
$$\frac{NGN_{(\tilde{\nu},\tilde{\boldsymbol{\Phi}},\tilde{\boldsymbol{\Psi}},\tilde{\boldsymbol{\Omega}})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)}{NGN_{(\hat{\nu},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Omega}})}(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+) v_{(g\circ f^{-1})|\mathcal{X}}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)},$$
where $(\tilde{\boldsymbol{\Lambda}}_0, \tilde{\boldsymbol{\Lambda}}_+, \boldsymbol{Q}_1, \boldsymbol{Q}_2) = f^{-1}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$ where $\mathcal{X}$ denotes the set of all proxy-SVAR structural parameters that satisfy the proxy and the additional zero restrictions.
3. Return to Step 1 until the required number of draws has been obtained.
4. Re-sample with replacement using the importance weights.

The Dynamic Effects of TFP Shocks

- The proxy-SVAR features 5 endogenous variables and 2 proxies. Quarterly data: 1947Q2-2015Q4
- The endogenous variables are
  1. Real GDP growth
  2. Employment growth
  3. Inflation
  4. Real consumption (non-durables and services) growth
  5. Real investment (equipment + durables consumption) growth
- The proxies are
  1. A proxy for consumption TFP shocks
  2. A proxy for investment TFP shocks
- Prior: $NGN_{(\nu,\mathbf{\Phi},\mathbf{\Psi},\mathbf{\Omega})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$. We set $\nu = \tilde{n}$, $\mathbf{\Phi} = \mathbf{0}_{\tilde{n},\tilde{n}}$, $\mathbf{\Psi} = \mathbf{0}_{\tilde{m}\tilde{n},\tilde{n}^2}$ and $\mathbf{\Omega}^{-1} = \mathbf{0}_{\tilde{m}\tilde{n},\tilde{m}\tilde{n}}$. And, we set $\hat{\nu} = \nu$, $\hat{\mathbf{\Phi}} = \mathbf{\Phi}$, $\hat{\mathbf{\Phi}} = \mathbf{\Phi}$ and $\hat{\mathbf{\Omega}}^{-1} = \mathbf{\Omega}^{-1}$.

The Dynamic Effects of TFP Shocks

Identification Restrictions

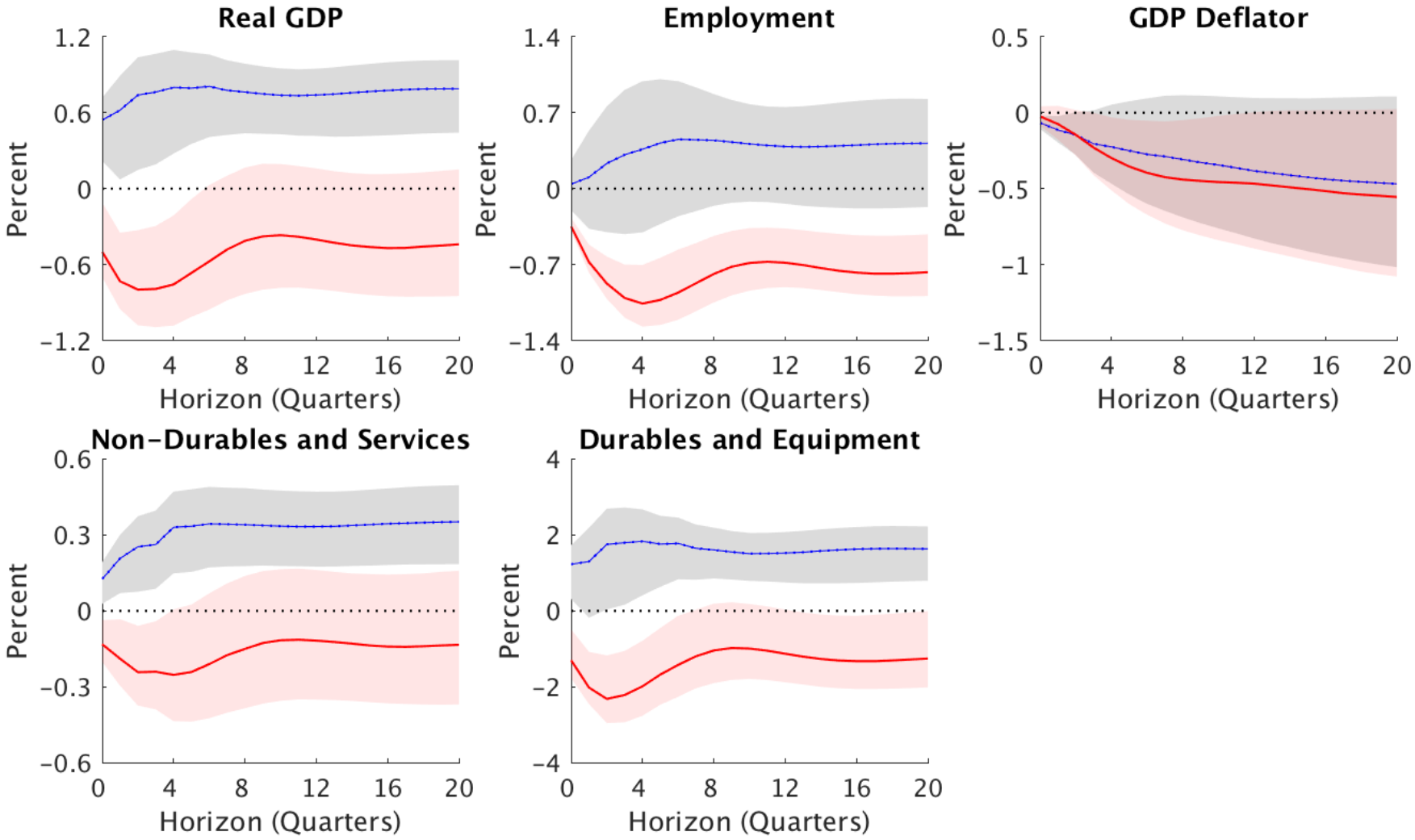
- The relevance condition and the exogeneity restrictions are

$$\mathbb{E}\left[\boldsymbol{m}_t \varepsilon_t^{TFP\prime}\right] = \boldsymbol{V} \neq \boldsymbol{0}_{2\times 2} \text{ and } \mathbb{E}\left[\boldsymbol{m}_t \varepsilon_t^{O\prime}\right] = \boldsymbol{0}_{2\times 3}$$

  where $\boldsymbol{m}_t' = [\boldsymbol{m}_{C,t} \quad \boldsymbol{m}_{I,t}]$ are the proxies for the consumption and investment TFP shocks.

- We also impose the following sign restrictions $\mathbb{E}\left[\boldsymbol{m}_{C,t}\varepsilon_{C,t}\right] > 0$, $\mathbb{E}\left[\boldsymbol{m}_{I,t}\varepsilon_{I,t}\right] > 0$, $\mathbb{E}\left[\boldsymbol{m}_{C,t}\varepsilon_{C,t}\right] > \mathbb{E}\left[\boldsymbol{m}_{C,t}\varepsilon_{I,t}\right]$ and $\mathbb{E}\left[\boldsymbol{m}_{I,t}\varepsilon_{I,t}\right] > \mathbb{E}\left[\boldsymbol{m}_{I,t}\varepsilon_{C,t}\right]$ on the entries of $\boldsymbol{V}$.

The Dynamic Effects of TFP Shocks



The Dynamic Effects of Personal Tax Rates Shocks

Roadmap

1. Replicate Mertens and Montiel-Olea ('18) using our algorithm when there is only one proxy
2. Replicate Mertens and Montiel-Olea ('18) with both AMTR and ATR proxies assuming AMTR does not respond to ATR shock
3. Combine Mertens and Montiel-Olea ('18) instruments with sign restrictions
4. Compare effects of income specific marginal tax rate shocks using Mertens and Montiel-Olea ('18) instruments and identification scheme versus sign restrictions.

Specification to Study AMTR Shocks (I/II)

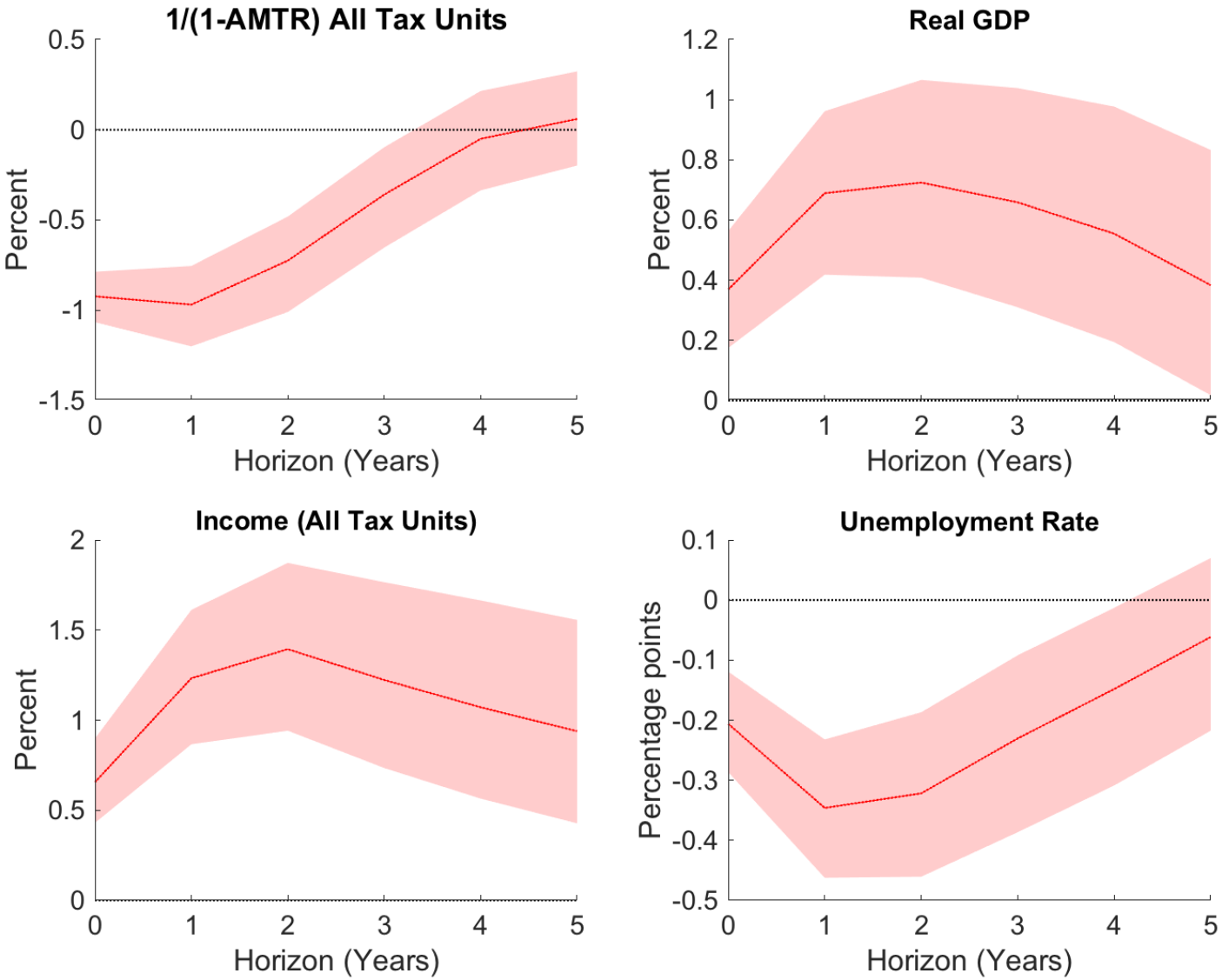
- The proxy-SVAR features 9 endogenous variables, 2 exogenous variables, and 1 proxy. Yearly data: 1946-2012
- The endogenous variables are
  1. The negative of log net of-tax-rate
  2. Log of reported income level
  3. Log of real GDP per tax unit
  4. The unemployment rate
  5. Log of real stock market index
  6. Inflation
  7. The federal funds rate
  8. Log of real government spending per tax unit
  9. Change in log real federal government debt per tax unit

Specification to Study AMTR Shocks (II/II)

- Exogenous variables: Dummy variables for 1949 and 2008
- Proxy: Collection of instances of variation in marginal tax rates considered to be contemporaneously exogenous changes in AMTR
- Prior: $NGN_{(\nu,\mathbf{\Phi},\mathbf{\Psi},\mathbf{\Omega})}(\tilde{\boldsymbol{A}}_0, \tilde{\boldsymbol{A}}_+)$. We set $\nu = \tilde{n}$, $\mathbf{\Phi} = \mathbf{0}_{\tilde{n},\tilde{n}}$, $\mathbf{\Psi} = \mathbf{0}_{\tilde{m}\tilde{n},\tilde{n}^2}$ and $\mathbf{\Omega}^{-1} = \mathbf{0}_{\tilde{m}\tilde{n},\tilde{m}\tilde{n}}$
- We normalize the sign of the AMTR shock by assuming that the IRF of the AMTR is negative in response to a negative AMTR shock, thus we use Algorithm 2 to obtain an effective sample size of 10,000

AMTR shock (rate cut)

One proxy for AMTR shock. Replicates Figure 5 from Mertens and Montiel-Olea ('18)



Specification to Study AMTR and ATR Shocks (I/II)

- The proxy-SVAR features 10 endogenous variables, 2 exogenous variables, and 2 proxies. Yearly data: 1946-2012

- The endogenous variables are

  1. The negative of log net of-tax-rate
  2. Log of reported income level
  3. Log of real GDP per tax unit
  4. The unemployment rate
  5. Log of real stock market index
  6. Inflation
  7. The federal funds rate
  8. Log of real government spending per tax unit
  9. Change in log real federal government debt per tax unit
  10. Log of ATR

Specification to Study AMTR and ATR Shocks (II/II)

- Proxies: Mertens and Montiel-Olea ('18) identify AMTR and ATR shocks using two proxies. Analogously to the case of the AMTR proxy, the new proxy (which we call the ATR proxy) is a collection of instances of variation in average tax rates that the authors reasonably consider to be contemporaneously exogenous changes in the ATR

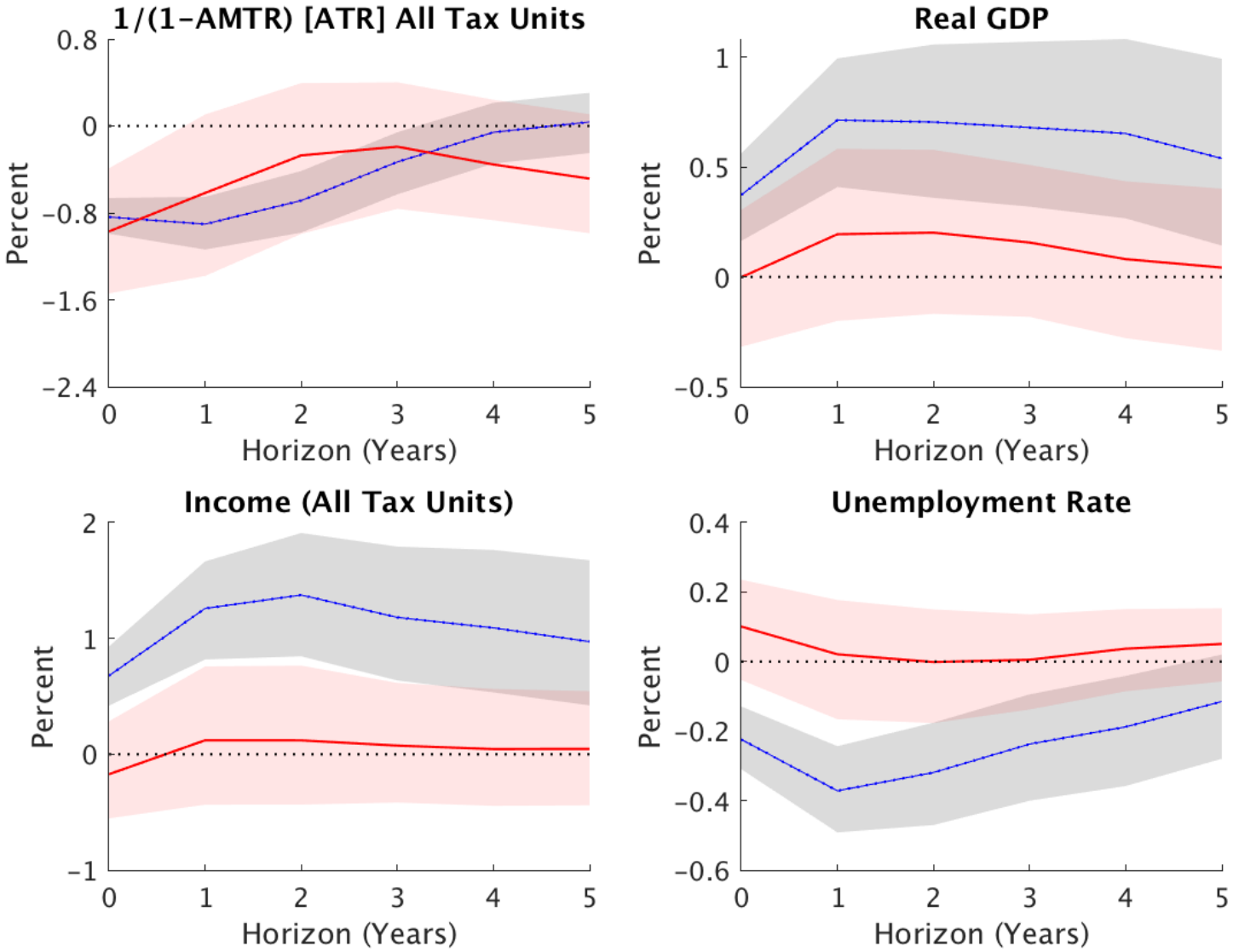
- Mertens and Montiel-Olea ('18) introduce an additional zero restriction

  1. $e_2' \boldsymbol{A}_0 e_1 = 0$ (AMTR ordered first)
  2. $e_1' \boldsymbol{A}_0 e_2 = 0$ (ATR ordered first)

- If AMTR ordered first, this means AMTR cannot react contemporaneously to the ATR

Effects of AMTR and ATR shocks

Replicates Panels (A) and (B) in Figure 10 of Mertens and Montiel-Olea ('18)



Effects of AMTR and ATR shocks

- These figures clearly justify sentences like these one
  - *There is, on the other hand, no evidence for any effect on incomes when average tax rates decline but marginal rates do not (See Mertens and Montiel-Olea ('18) page 2.)*
  - *The main finding is that, in sharp contrast to the results for marginal tax rate changes after controlling for average tax rates, there is no evidence that income responds strongly to average tax rate changes once marginal rate changes are controlled for. The point estimates are in fact slightly negative, although they are not statistically significant at any horizon. (See Mertens and Montiel-Olea ('18) page 35.)*

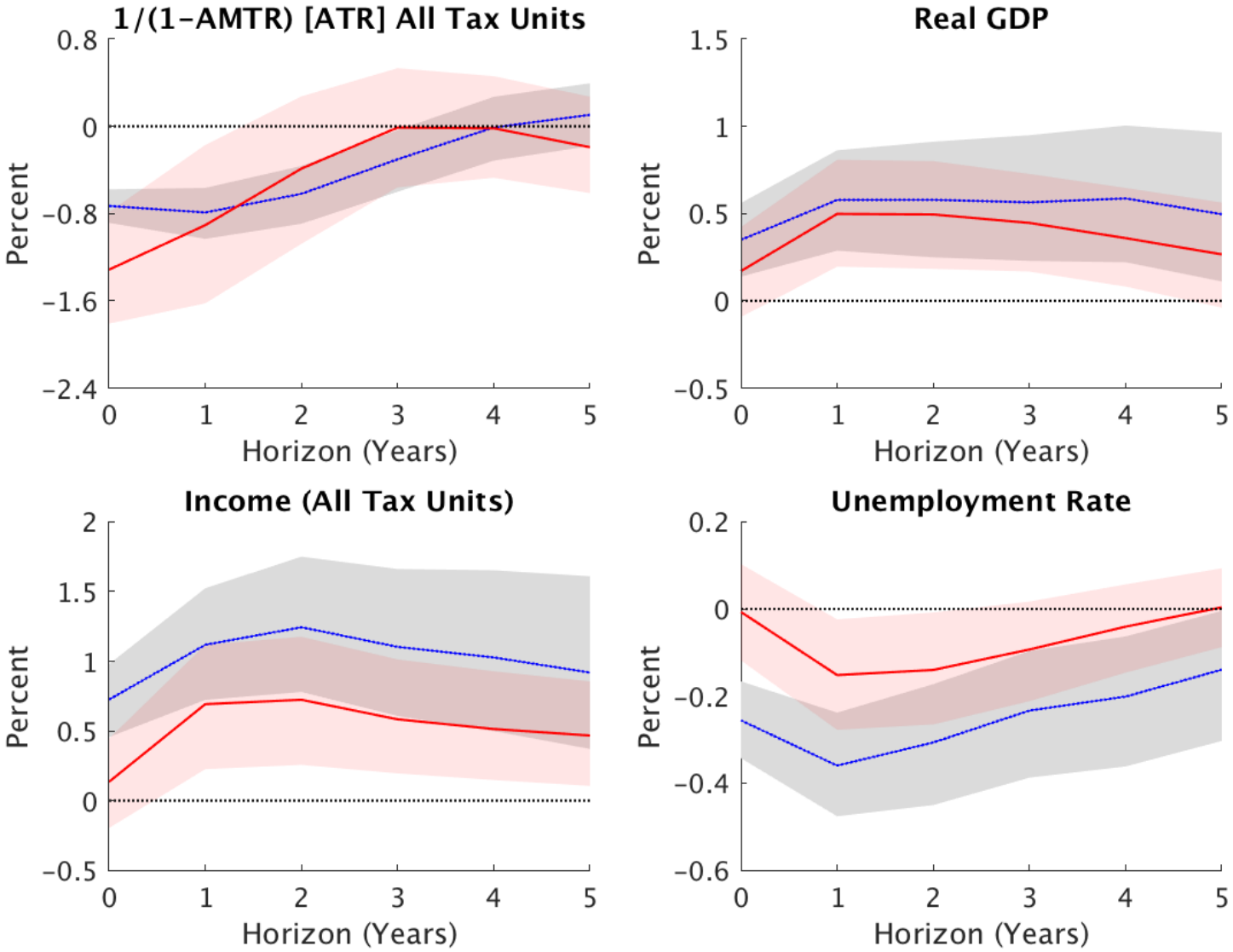
Effects of AMTR and ATR shocks

Identification

- Proxies are only correlated with AMTR and ATR shocks
- To separate the shocks, we use sign restrictions
  - AMTR proxy is positively correlated with AMTR shock
  - ATR proxy is positively correlated with ATR shock
  - Covariance between AMTR shock and proxy larger than covariance between ATR shock and AMTR proxy
  - Covariance between ATR shock and proxy is larger than covariance between AMTR shock and ATR proxy

Effects of AMTR and ATR shocks

Mertens and Montiel-Olea ('18) without Additional Exclusion Restriction



Specification to Study Income Specific AMTR Shocks (I/II)

- The proxy-SVAR features 11 endogenous variables, 4 exogenous variables, and 2 proxies. Yearly data: 1946-2012

- The endogenous variables are

  1. The negative of log net of-tax-rate for the top 1%
  2. The negative of log net of-tax-rate for the bottom 99%
  3. Log of reported income level for the top 1%
  4. Log of reported income level for the bottom 99%
  5. Log of real GDP per tax unit
  6. The unemployment rate
  7. Log of real stock market index
  8. Inflation
  9. The federal funds rate
  10. Log of real government spending per tax unit
  11. Change in log real federal government debt per tax unit

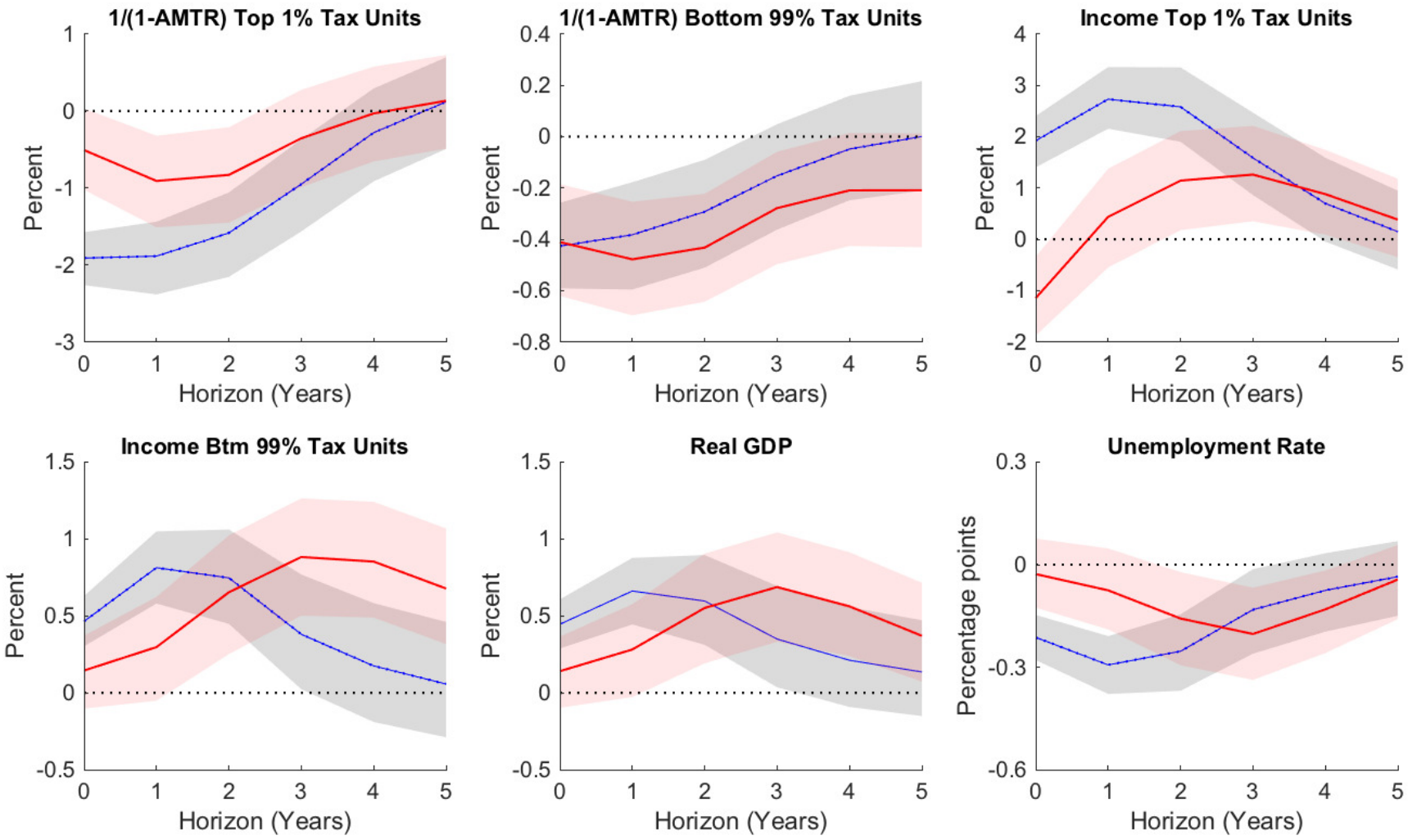
Specification to Study Income Specific AMTR Shocks (II/II)

- Exogenous variables: Dummy variables for 1949 and 2008 as well as a linear and a quadratic trend to capture long trends in income inequality
- Proxies: Newly built disaggregated measures of exogenous variation in tax rates across the income distribution as proxies for top and bottom marginal tax rate shocks
- Mertens and Montiel-Olea ('18) identification

  1. $\mathbf{C}_{21}$: Proxy restriction + $e_2' \boldsymbol{A}_0 e_1 = 0$ (AMTR Bottom 99% ordered first)
  2. $\mathbf{C}_{12}$: Proxy restriction + $e_1' \boldsymbol{A}_0 e_2 = 0$ (AMTR Bottom 99% ordered first)

Effects of income specific MTR shocks

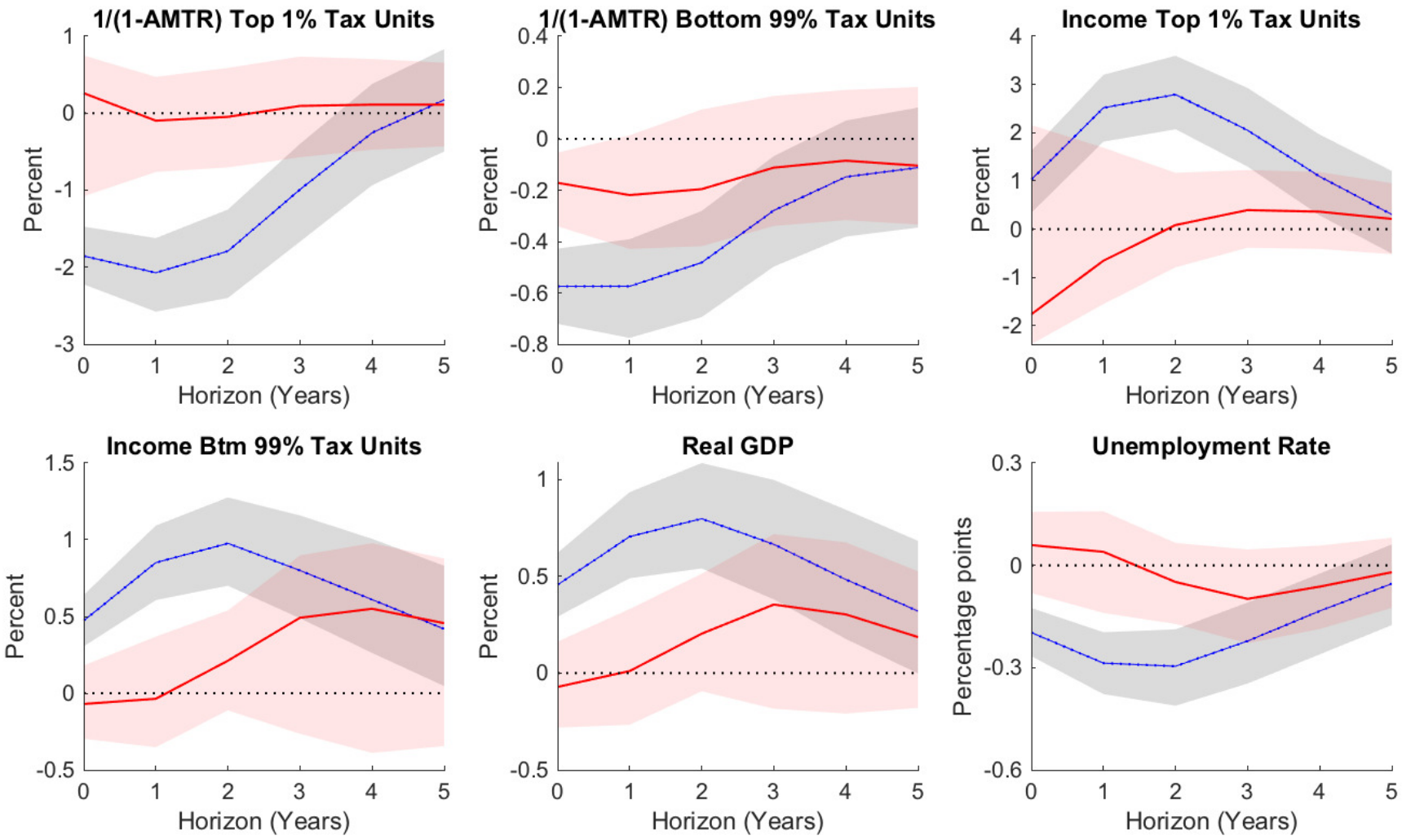
IRFs to a one std dev MTR shock of the Top 1% (Gray) and Bottom 99% (Red)



Replicates Figures 11 and 12 from Mertens and Montiel-Olea ('18)

Effects of income specific MTR shocks

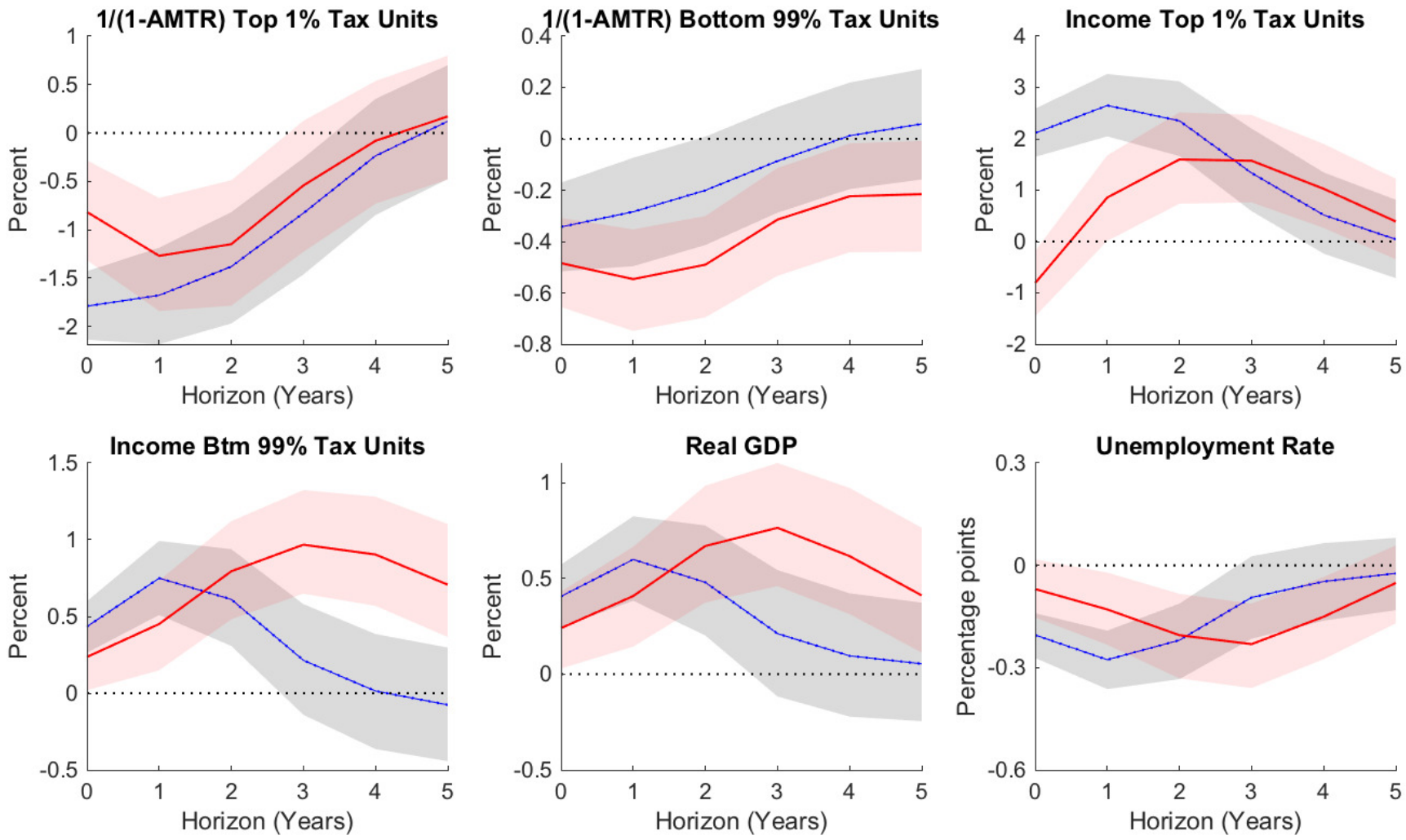
IRFs to a one std dev MTR shock of the Top 1% (Gray) and Bottom 99% (Red)



Replicates Figures 11 and 12 from Mertens and Montiel-Olea ('18)

Effects of income specific MTR shocks

IRFs to a one std dev MTR shock of the Top 1% (Gray) and Bottom 99% (Red)



Mertens and Montiel-Olea ('18) without Additional Exclusion Restriction

Conclusions

In this paper we develop:

- An efficient algorithm for Bayesian inference
- The algorithm gives independent posterior draws of the structural parameters
- Can be efficiently applied with priors from the NGN family, which is an often used conjugate family of priors
- Perhaps most importantly the external instruments can easily be combined with sign and zero restrictions