Multiple Equilibria in Open Economy Models with Collateral Constraints: Overborrowing Revisited

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Starting Point

- Open-economy models with collateral constraints typically display a pecuniary externality.
- The externality originates in the fact that the price of pledgable objects is endogenous to the model but exogenous to individual agents.
- A result stressed in the literature is that these economies overborrow, that is, they borrow more than they would if agents internalized the pecuniary externality.
- Open-economy models with collateral constraints are prone to multiple equilibria. This issue has been little explored (indeed avoided) in the related literature.
- An exception is Jeanne and Korinek (2010), who present an analysis of sufficient conditions for uniqueness in a model with a stock collateral constraint.

This Paper

- Characterizes multiple equilibria in models with flow collateral constraints.

- Shows that open economies with flow collateral constraintshave equilibria featuring self-fulfilling financial crises.

- The possibility of multiple equilibria causes the economy to underborrow in equilibrium.

- **Intuition:** Agents are aware that the economy is prone to self-fulfilling crises, so they engage in extra precautionary savings, driving down the aggregate level of debt. With less debt, the frequency of crises is low.

A Model with a Flow Collateral Constraint

Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_t = \left[a {c_t^T}^{1-1/\xi} + (1-a) {c_t^N}^{1-1/\xi} \right]^{1/(1-1/\xi)}$$

$$c_t^T + p_t c_t^N + d_t = y^T + p_t y^N + \frac{d_{t+1}}{1+r}$$

$$d_{t+1} \leq \kappa (y^T + p_t y^N)$$

where c_t =consumption; c_t^T, c_t^N =consumption of tradables, nontradables; d_t = debt due in t; d_{t+1} = debt assumed in t and maturing in $t+1$; y^T, y^N = endowments of tradables, nontradables; p_t =relative price of nontradables; r = interest rate.

Observations

(1) The last equation is the flow collateral constraint (CC). It says that the amount of debt issued in period t, d_{t+1}, cannot exceed a fraction κ of income, $y^T + p_t y^N$.

(2) From the individual agent's point of view, the CC is well behaved: the larger is d_{t+1}, the closer he gets to hitting the collateral constraint. This is because he takes as exogenous all of the objects on the RHS of the collateral constraint (in particular p_t).

(3) Also, from the perspective of the individual agent, the collateral constraint defines a convex set of feasible debt levels: if d' and d'' satisfy the collateral constraint, then so does the debt level $\alpha\, d' + (1-\alpha)d''$, for any $\alpha \in [0, 1]$.

(4) As we will see shortly, (2) and (3) do not hold from an aggregate perspective.

Three Equilibrium Conditions of Interest

$$d_{t+1} \leq \kappa(y^T + p_t y^N)$$

$$p_t = \frac{1-a}{a}\left(\frac{c_t^T}{y^N}\right)^{1/\xi}$$

$$c_t^T + d_t = y^T + \frac{d_{t+1}}{1+r}$$

These three conditions give rise to the following **equilibrium collateral constraint**

$$\boxed{d_{t+1} \leq \kappa\left[y^T + \left(\tfrac{1-a}{a}\right)\left(y^T + \tfrac{d_{t+1}}{1+r} - d_t\right)^{1/\xi} {y^N}^{1-1/\xi}\right]}$$

Observations

(1) d_{t+1} appears on both sides of the equilibrium collateral constraint.

(2) Because $\xi > 0$, the equilibrium value of collateral increases with the level of debt, giving rise to the possibility that the higher is d_{t+1} the less tight is the collateral constraint.

(3) Moreover, collateral (i.e., the RHS of the collateral constraint) is in general nonlinear in d_{t+1}, giving rise to the possibility that the set of debt levels that satisfy the equilibrium collateral constraint is nonconvex, that is, if d' and d'' satisfy the equilibrium collateral constraint, then $\alpha\, d' + (1-\alpha)d''$ may not for some $\alpha \in (0, 1)$.

Some Simplifying Assumptions

(1) $\sigma = 1/\xi = 2$.

(2) $\beta(1+r) = 1$.

(3) $a = 0.5$.

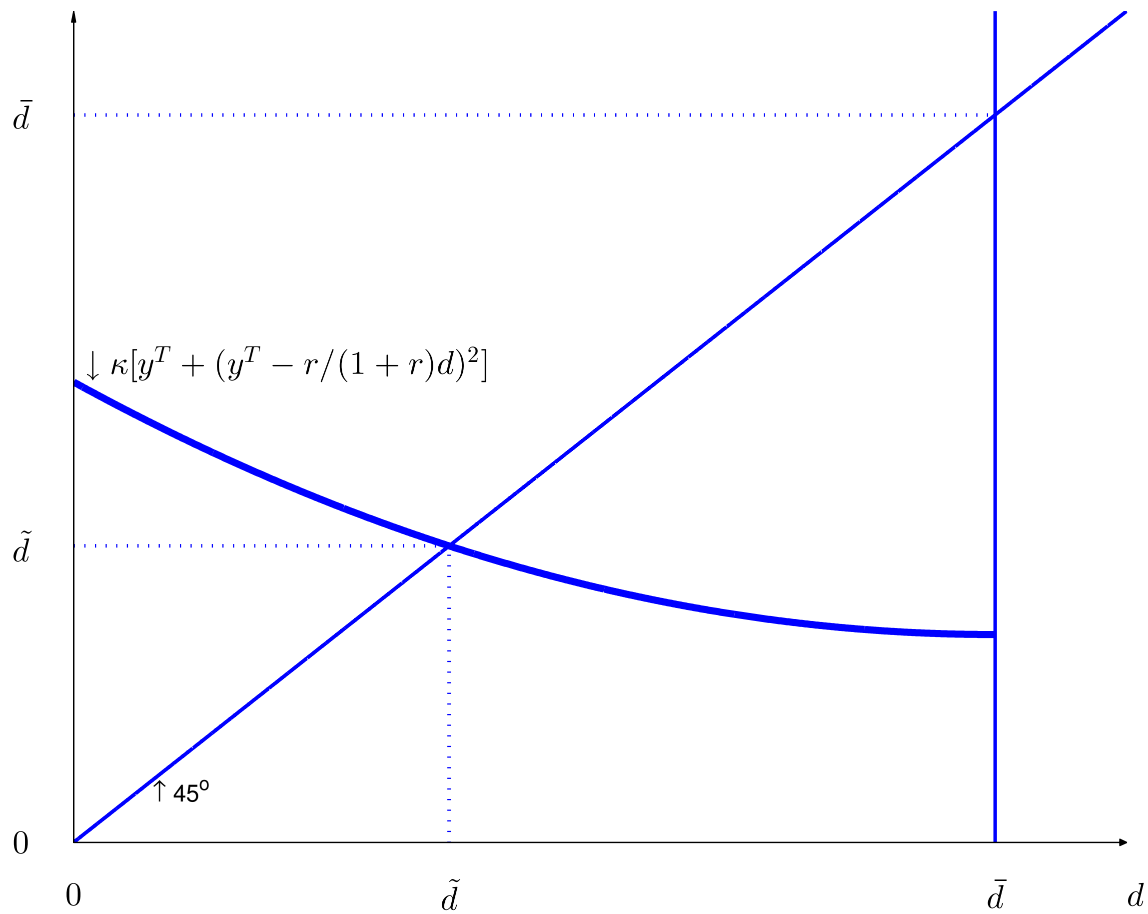
(4) $y^N = 1$. A normalization.

The equilibrium collateral constraint then becomes

$$\boxed{d_{t+1} \leq \kappa \left[y^T + \left(y^T + \frac{d_{t+1}}{1+r} - d_t \right)^2 \right]}$$

The Long-Run Equilibrium Collateral Constraint

$$d \leq \kappa \left[y^T + \left(y^T - rd/(1+r) \right)^2 \right]$$



Observations

(1) the expression under the power of 2 is steady state consumption, $y^T - rd/(1+r)$.

(3) The LR collateral constraint achieves a minimum when long-run consumption is 0, that is, at the natural debt limit.

(4) This means that:
(a) for all relevant values of debt (i.e., all values below the natural debt limit), the LR collateral constraint is well behaved, that is, the larger is debt the tighter it gets.
(b) The LR collateral constraint binds at $\tilde{d}$, and is violated at any level of debt larger than $\tilde{d}$. No steady state equilibrium is possible to the right of $\tilde{d}$.
(b) For any initial debt $d_0 < \tilde{d}$, the constant debt path $d_t = d_0$ for all t satisfies the resource constraint with a positive and constant level of consumption, $c_t = c_0 > 0$ for all t, and ensures that the collateral constraint never binds. Is this an equilibrium? It only remains to show that the Euler equations holds.

The Euler Equation in the Unconstrained Equilibrium

When the collateral constraint does not bind, the Euler equation is

$$\frac{c_{t+1}^T}{c_t^T} = \beta(1+r)$$

Recalling the assumption $\beta(1+r) = 1$, we have that

$$c_{t+1}^T = c_t^T$$

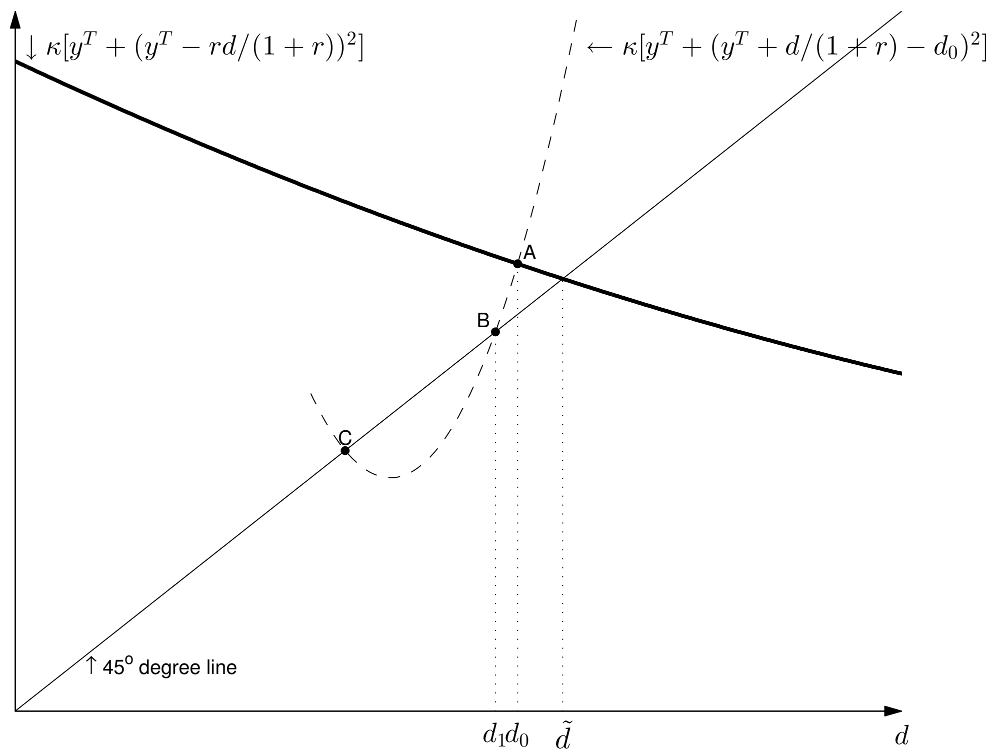
So a constant path of consumption satisfies the Euler equation when the collateral constraint does not bind.

Thus, we have shown that if $d_0 < \tilde{d}$, then $d_t = d_0$ and $c_t = y^T - r/(1+r)d_0 > 0$ for all t is an equilibrium. In this equilibrium, the collateral constraint never binds.

Are there more equilibria?

The Short-Run Equilibrium Collateral Constraint and Multiple Equilibria

$$d \le \kappa \left[y^T + \left(y^T + \frac{d}{1+r} - d_0 \right)^2 \right]$$



Observations

(1) In the figure, $d_0 < \tilde{d}$. Thus, from the previous analysis we have that point A is an equilibrium featuring $d_t = d_0$ for all $t \geq 0$.

(2) The slope of the short-run (SR) CC is proportional to c_0^T. So an equilibrium must be on an upward sloping range of the SR CC. Thus, point C is not an equilibrium.

(3) Consider point B. There, consumpiton is positive in period 0, because the SR CC is upward sloping. Also, at B the SR CC holds with equality (binding), since it is on the 45-degree line. But we must still check (a) that at B the Euler equation holds in period 0, and (b) that all equilibrium condiitons are satisfied after period 1.

We wish to consider an equilibrium in which the economy jumps from A to B in period 0 and then stays at B forever thereafter.

Note first that deleveraging from A to B and then staying at B forever requires that $c_0^T < c_1^T$. Knowing this makes it easy to that at B the Euler equation holds in period 0, as we show next.

The Euler Equation in Period 0

When the collateral constraint binds, like at point B in period 0, the Euler equation is

$$\frac{c_1^T}{c_0^T} = \frac{1}{\sqrt{1 - \mu_0(1+r)}}$$

where μ_0 is the multiplier on the collateral constraint. Is $\mu_0 \geq 0$? Yes, because $c_1^T > c_0^T$.

From period 1 on, we can use the previous analysis to state that point B represents an equilibrium with $d_t = d_1$ for all $t \geq 1$.

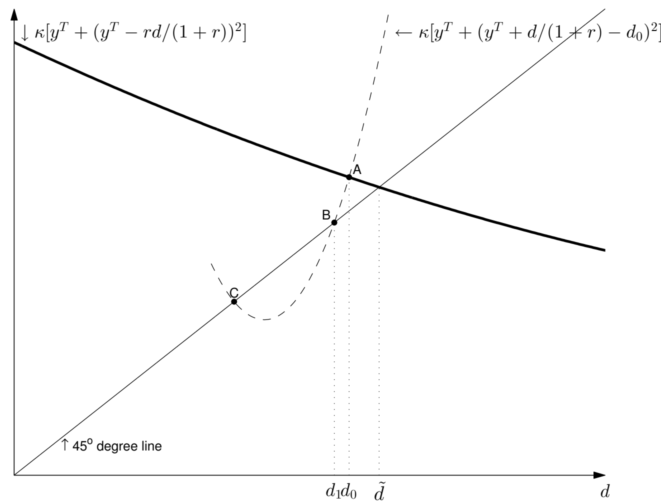
We have therefore shown that B is indeed an equilibrium.

At point B, the economy experiences a self-fulfilling financial crisis, caused by an arbitrary, generalzied desire to deleverage.

The equilibrium at point B is welfare inferior to equilibrium at A, because the former features a drop in consumpiton in period 0 and a recovery in period 1, whereas the latter ensures perfect consumpiton smoothing.

Are Points Between A and B Equilibria?

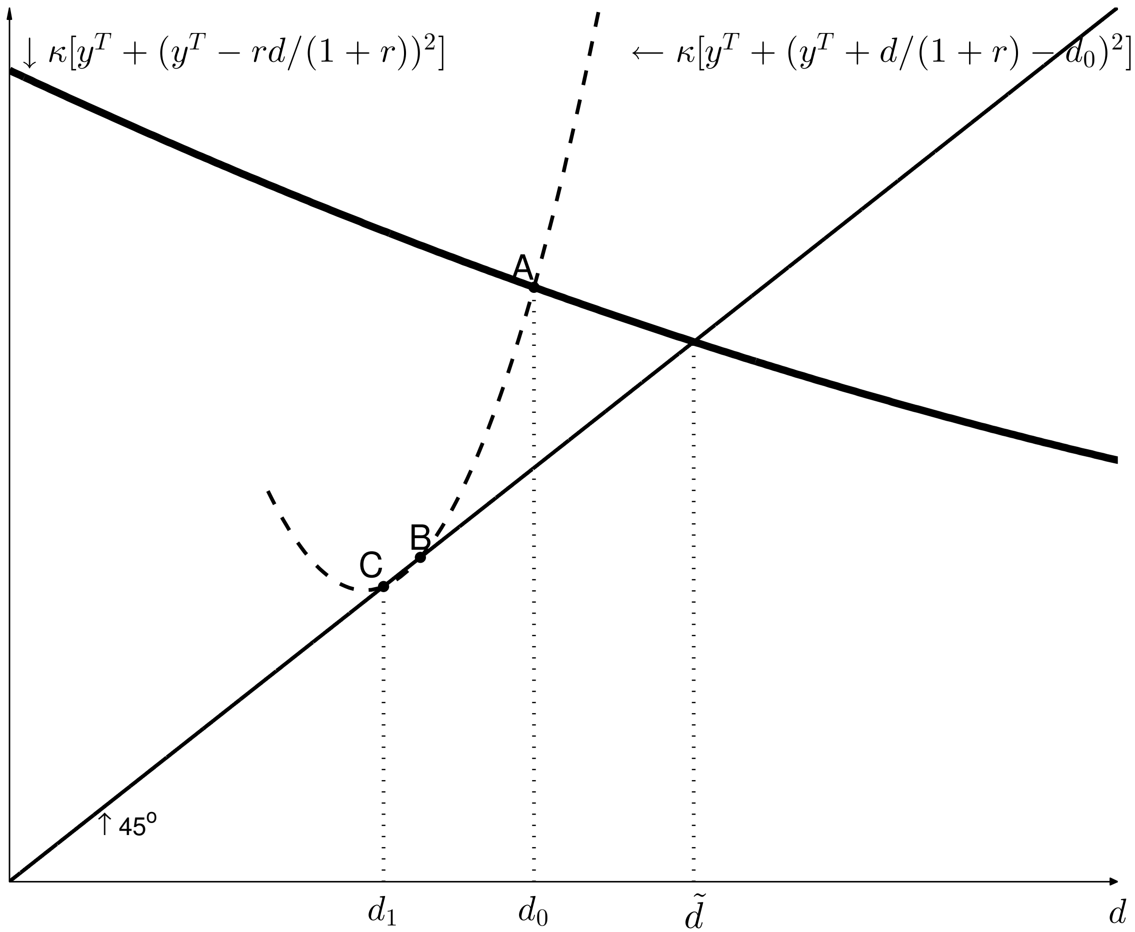
Take another look at the figure



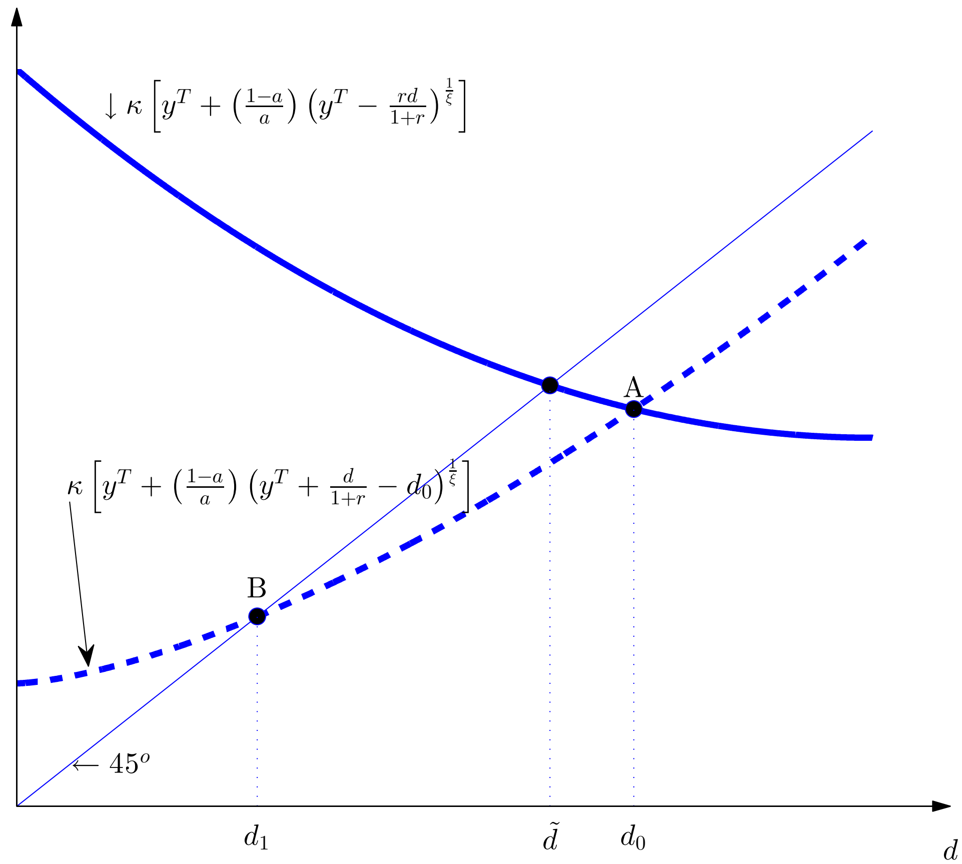
At any such point: (a) The collateral constrain is satisfied (indeed it is not binding); (b) the resource constraint is satisfied with positive consumption.

However, because at any such point $c_0^T < c_1^T$, the Euler equation is satisfied only if $\mu_t > 0$. But this violates the slackness condition, which requires $\mu_0 = 0$, because the collateral constraint doesn't bind in period 0.

Three Equilibria



A Unique Equilibrium



Ramsey-Optimal Capital Control Policy

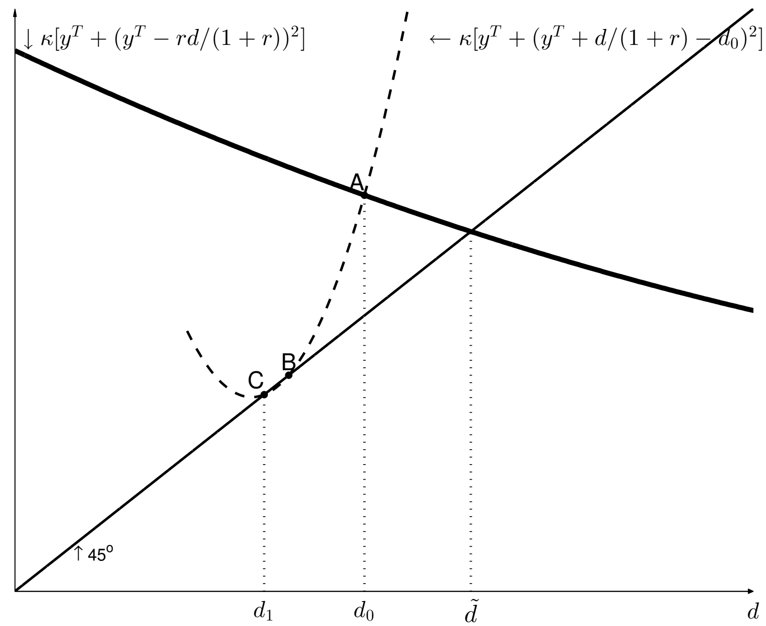
The government imposes a proportional tax on debt, τ_t. The budget constraint of the household becomes

$$c_t^T + p_t c_t^N + d_t = y^T + p_t y^N + \frac{1 - \tau_t}{1 + r} d_{t+1}.$$

Suppose that capital control taxes (subsidies if $\tau_t < 0$) are rebated (financed) lump-sum or through income taxes (transfers).

From all the competitive equilibria that can be supported with capital controls, pick the one that maximizes welfare.

Underborrowing



The competitive equilibrium at point A can be supported with $\tau_t = 0$ for all t. It is also the first-best allocation, so it must be Ramsey optimal.

Thus, if agents coordinate on equilibrium A, there is neither overborrowing nor underborrowing.

But if agents coordinate on equilibrium C, the economy suffers underborrowing.

Implementation

The Ramsey optimal tax rate in this economy is $\tau_t = 0$ at all times.

However, announcing the policy $\tau_t = 0$ for all t does not guarantee that the Ramsey optimal equilibrium will emerge. Indeed, this tax policy also supports the deleveraging equilibrium C.

What capital control policy can induce the Ramsey-optimal equilibrium? Consider a debt-dependent capital control policy:

$$\tau_t = \tau(d_{t+1} - d_t)$$

satisfying $\tau(0) = 0$ and $\tau' < 0$.

Implementation (continued)

Under this tax-policy rule, the Euler equation in period 0 becomes:

$$\frac{c_1^T}{c_0^T} = \frac{1}{\sqrt{1 - \tau(d_1 - d_0) - (1+r)\mu_0}}$$

(1) In the intended (Ramsey) equilibrium, $c_1^T/c_0^T = 1$, $d_1 = d_0$, and $\mu_0 = 0$, so the Euler equation holds and $\tau(d_1 - d_0) = 0$.

(2) In the unintended equilibrium (point C), $c_1^T/c_0^T > 0$, and $d_1 - d_0 < 0$. Make $\tau(d_1 - d_0)$ so large that μ_0 has to be negative for the Euler equation to hold. Since μ_0 must be nonnegative, this capital-control policy rules out the unintended equilibrium.

Quantitative Analysis

Time unit: one quarter.
The economy is driven by endowment and interest-rate shocks.
Estimate a bivariate AR(1) process for (y_t^T, r_t) using Argentine data over the period 1983:Q1 to 2001:Q4.

$\kappa = 1.2$ ($\Rightarrow$upper limit on debt = 30 percent of annual output).
$r = 0.0316$
$\beta = 0.9635$
$\sigma = 1/\xi = 2$
$a = 0.26$
$y^N = 1$.

Discretization: 501 points for d, 21 for y^T, and 11 for r.

Numerical Algorithm

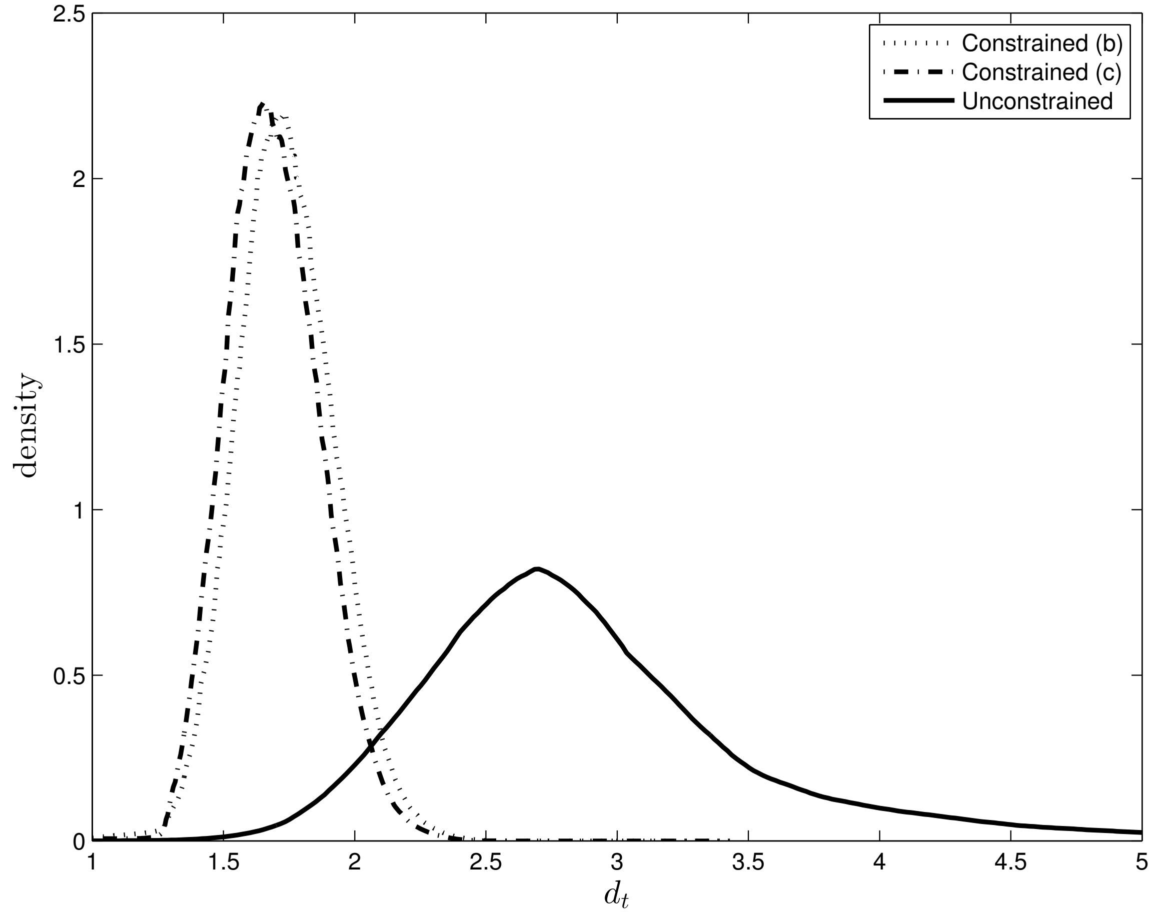
- The numerical solution must take a stance on how to handle the possibility of indeterminacy.

- Failing to address this issue may result in nonconvergence.

- In searching for an equilibrium we favor ones with a binding constraint, as follows:

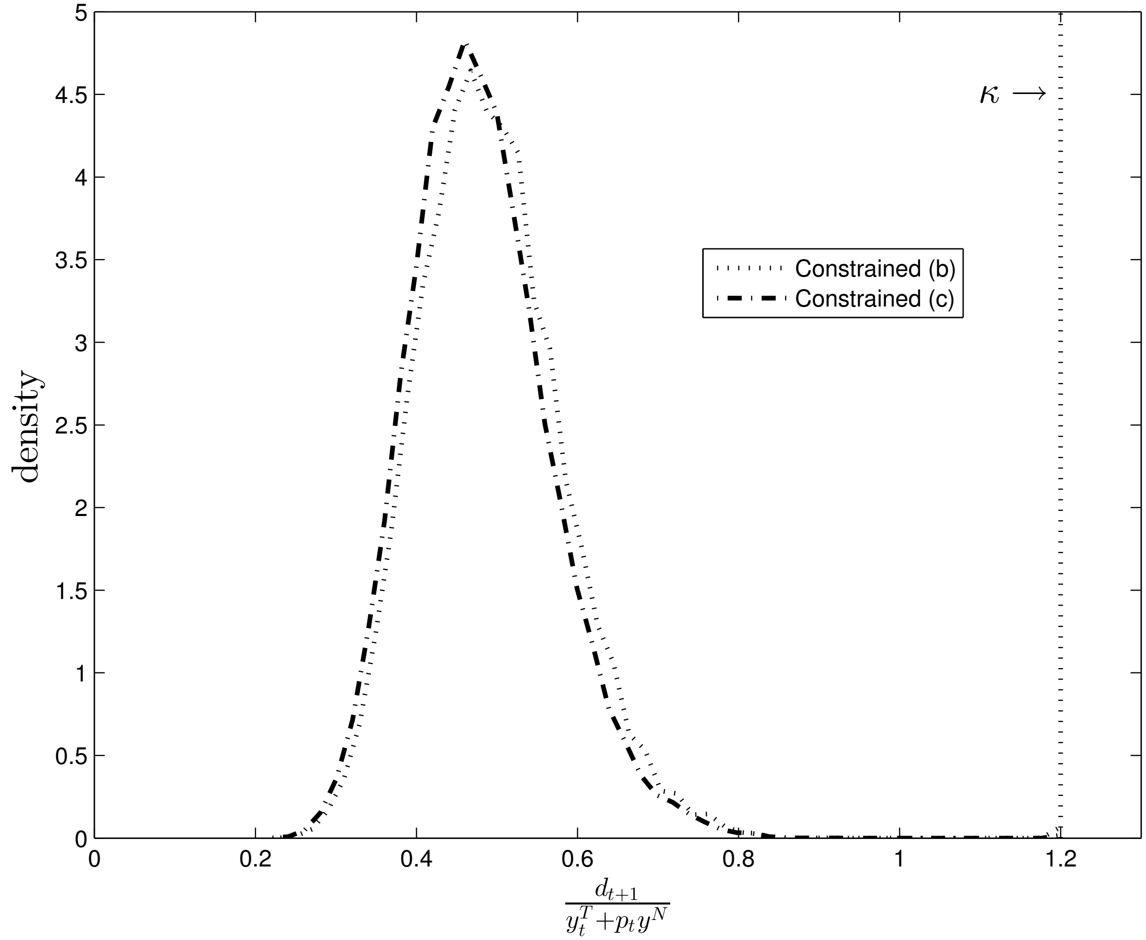
- if for the current state (y_t^T, r_t, d_t) there are one or two values of d_{t+1} for which all equilibrium conditions are satisfied and the collateral constraint is binding, pick the smaller debt value.

External Debt Densities With And Without Collateral Constraints



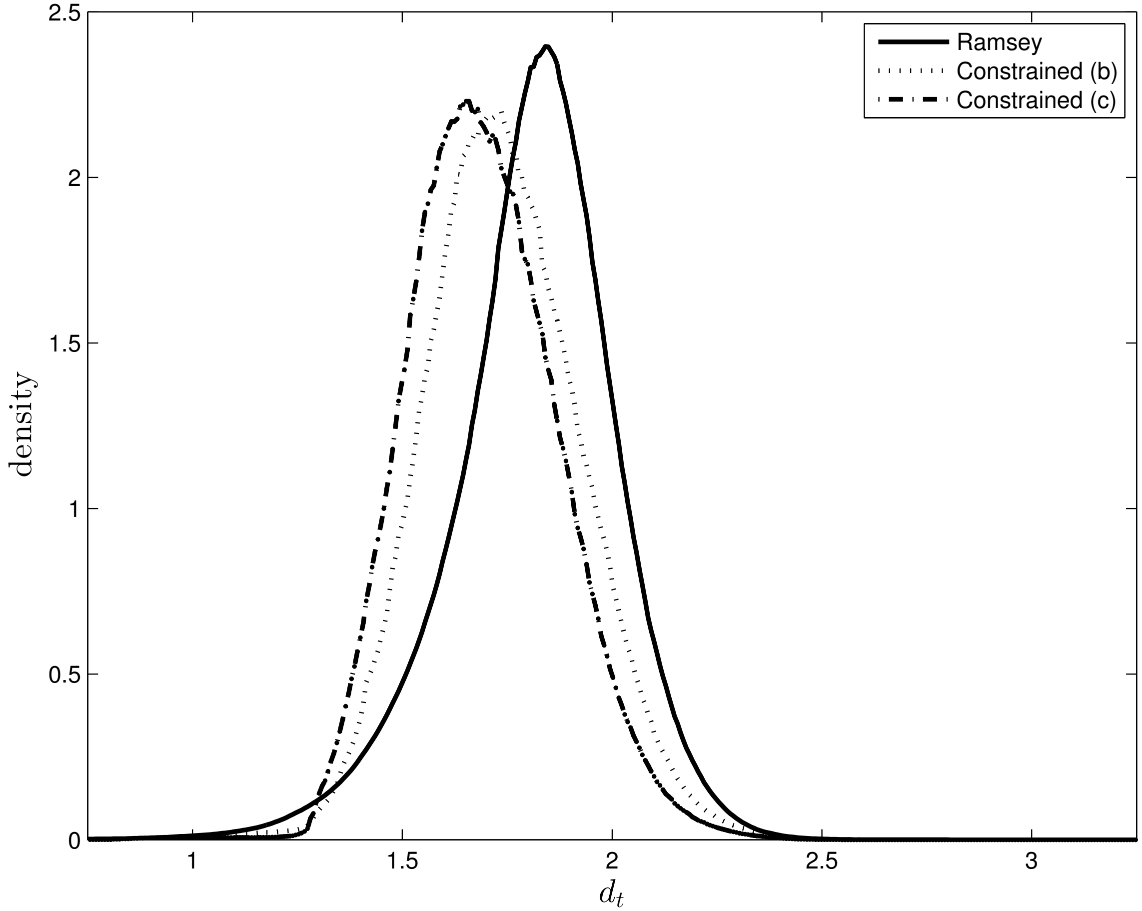
The collateral constraint severely limits the country's ability to borrow. More importantly, it highly compresses the debt distribution (i.e., lowers the unconditional std. dev.).

The Distribution of Leverage



The unregulated economy is less leveraged than the Ramsey economy.

Underborrowing



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The debt distribution under Ramsey-optimal policy lies to the right of those associated with the unregulated equilibria.

Conclusion

• This paper characterizes multiple equilibria in models with stock and flow collateral constraints.

• It shows that economies prone to self-fulfilling crises underborrow. Under plausible calibrations, debt is 30 percent below its optimal value.

• The underborrowing equilibrium may display less frequent financial crises than the Ramsey-optimal equilibrium.

• Averting self-fulfilling crises requires the threat of increasing capital control taxes in such events.

Impatient Consumers, $\beta(1+r) < 1$

. Suppose $d_0 < \tilde{d}$. We believe there is no equilibrium with $d_t \to \tilde{d}$. The idea is as follows. As long as $d_t < \tilde{d}$, the collateral constraint is never binding. Thus, as long as $d_t < \tilde{d}$, the dynamics obey the following laws of motion:

$$c^T_{t+1}/c^T_t = \beta(1+r)$$

$$d_{t+1} = (1+r)[d_t + c^T_t - y^T]$$

given d_0. In addition, we have the terminal conditions

$$d_T = \tilde{d}$$

and

$$c_T = \tilde{c}^T$$

for some T, where $\tilde{c}^T = y^T - r/(1+r)\tilde{d}$. These terminal conditions are necessary, because, in order for the system to remain at $\tilde{d}$, tradable consumption must equal $\tilde{c}^T$. But we have three conditions (one initial condition and two terminal conditions) for a system of only 2 equations in two unknowns. Now the terminal date T is also a free variable, which adds a degree of freedom. But it has the restriction that it has to be an integer.