Bank-Runs, Contagion and Credit Easing

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Motivation

- Bank-runs are a common feature of financial crises
  - Friedman-Schwartz; Kindleberger; Bernanke; Gorton
- Central lesson from Diamond-Dybvig:
  - Solvent banks can be subject to *self-fulfilling* runs
- Banks-runs are typically not isolated events
  - Can be the outcome of general-equilibrium forces
  - ...and in turn, have aggregate general-equilibrium impact

★ General equilibrium model essential to understand feedback
  - What are the implications for government policy?

What we do

- Tractable dynamic general equilibrium model of bank-runs
  - Banks make dynamic portfolio, equity and default decisions
  - Asset values determined in general equilibrium
- Analytical characterization:
  - When a bank faces a run in partial equilibrium
  - Dynamics of asset prices and fraction of banks facing runs
    - Panic/systemic-run one possible outcome (Gertler-Kiyotaki)
- Normative analysis:
  - Study role of credit easing policies

Key Normative Result

Desirability of policies depend on whether crisis is driven by poor fundamentals or self-fulfilling runs

- If crisis driven by poor fundamentals
  - Credit easing is de-stabilizing and welfare reducing
- If crisis driven by self-fulfilling runs
  - Credit easing is stabilizing and welfare improving

★ Key distinction: repaying banks are *net buyers* when crises are driven by fundamentals but are *net sellers* when driven by runs

Outline of the Talk

1. Basic environment without runs
    - Bank problem in partial equilibrium
    - General equilibrium
2. Introduce bank-runs
3. Credit easing

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Environment

- Discrete time, infinite horizon, deterministic
- Continuum of banks, preferences $\sum_{t=0}^{\infty} \beta^t \log(c_t)$.
- Risk-neutral foreign creditors, discount rate R
  - Small open economy
- Technology
  - Production of consumption good: $y = zk$
  - Capital in fixed supply $\overline{K}$
- Competitive market for assets and deposits
- No commitment to repay deposits

Banks: budgets and decisions

- Banks starts period with deposits b and assets k

<u>If repay:</u>

- Budget constraint

$$c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k'.$$

where q is the price of deposits, p is the price of capital

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If default

- Productivity loss $y = \underline{z}k$ and permanent exclusion: $b' = 0$
- Budget constraint

$$c = (\underline{z} + p_t)k - p_t k'$$

Banks Optimization

$$V_t(b,k) = \max\left[V_t^R(b,k), V_t^D(k)\right]$$

Value of repayment:

$$V_t^R(b,k) \quad = \quad \max_{k',b',c} \log(c) + \beta V_{t+1}(b',k')$$

subject to budget constraint & No-Ponzi

Value of default:

$$V_t^D(k) \quad = \quad \max_{k',c} \log(c) + \beta V_{t+1}^D(k')$$

subject to budget constraint

Default Decision

The optimal default rule follows

$$d_t(b,k) = \begin{cases} 1 & \text{if } V_t^R(b,k) < V_t^D(k), \\ 0 & \text{if } V_t^R(b,k) > V_t^D(k), \\ \end{cases}$$

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- Equilibrium bond price is of the following form

$$q_t(b',k') = \begin{cases} 0 & \text{if } b' > \bar{b}_t(k') \\ 1 & \text{if } b' \leq \bar{b}_t(k') \end{cases}$$

The Value of Default

$$V_t^D(k) = A + \frac{1}{1-\beta}\log(k(\underline{z} + p_t)) + \frac{\beta}{1-\beta}\sum_{\tau \geq t}\beta^{\tau - t}\log\left(R_{\tau+1}^D\right),$$

where the return on capital under default

$$R_{t+1}^D = \frac{\underline{z} + p_{t+1}}{p_t}$$

Policy functions:

$$\mathcal{C}_t^D(k) = (1-\beta)\left(\underline{z} + p_t\right)k$$

$$\mathcal{K}_{t+1}^D(k) = \beta\frac{(\underline{z} + p_t)k}{p_t},$$

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Evolution of capital

$$k' = \beta R_{t+1}^D k$$

The Value of Repayment: Prelude

- Define net worth:

$$n \equiv k(\bar{z} + p) - bR$$

- Return on capital

$$R^k_{t+1} \equiv \frac{\bar{z} + p_{t+1}}{p_t}$$

- Guess a linear borrowing constraint $\overline{b}_t(k') = \gamma_t p_{t+1} k'$

Lemma. If $p_t < \gamma_t p_{t+1}$ and $R^k_{t+1} > R$, then $k'_{t+1} = \infty$ in period t for *any* level of networth (and so is its value function).

The Value of Repayment

$$V_t^R(n) = A + \frac{1}{1-\beta}\log(n) + \frac{\beta}{1-\beta}\sum_{\tau \geq t}^{\infty} \beta^{\tau - t}\log(R^e_{\tau+1}),$$

where the equity return is denoted by

$$R^e_{t+1} = R^k_{t+1} + (R^k_{t+1} - R)\frac{\gamma_t p_{t+1}}{p_t - \gamma_t p_{t+1}}$$

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Policy functions:

$$\mathcal{C}_t^R(n) = (1-\beta)n$$

For all $t \geq 0$ such that $R^k_{t+1} > R$.

$$\mathcal{B}^R_{t+1}(n) = \gamma_t p_{t+1}\mathcal{K}^R_{t+1}(n)$$

$$\mathcal{K}^R_{t+1}(n) = \frac{\beta n}{p_t - \gamma_t p_{t+1}}$$

Equilibrium Consistent Borrowing Limits

Proposition. A bank is indifferent between repayment and default at $t+1$ if $\{\gamma_t\}$ is such that:

$$\frac{\bar{z} + p_{t+1}(1 - \gamma_t R)}{\underline{z} + p_{t+1}} = \left(1 - \gamma_{t+1} \frac{p_{t+2}}{p_{t+1}}\right)^{\beta} \qquad \text{(G)}$$

Definition. Given $\{p_t\}_{t=0}^{\infty}$, we say a sequence of borrowing limits $\{\gamma_t\}_{t=0}^{\infty}$ is equilibrium-consistent if (G) is satisfied for all $t \geq 0$.

- With a sequence of γ_t, we can then construct value functions and policy functions for any $n_0 \geq 0$
  - Next: how to find $\{\gamma_t\}$?

How to find $\{\gamma_t\}$?

For a constant price, borrowing limit satisfies

$$\gamma_{t+1} = 1 - \left(\frac{R^k/R - \gamma_t}{R^D/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)$$

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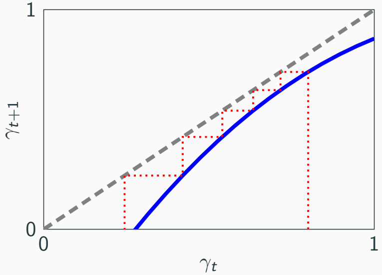

- If R^k is too high, no equilibrium borrowing limit

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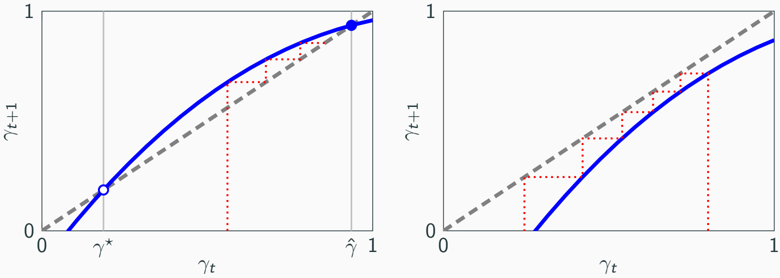

- Two fixed points $\gamma^\star$ but only smallest satisfies No-Ponzi
- Smallest fixed point unstable $\Rightarrow \gamma_t = \gamma^\star$ for all t.

Comparative Statics

- Equilibrium γ is increasing in $(\beta, \bar{z})$ and decreasing in $(R, \underline{z})$
- Equilibrium γ is also decreasing in p, reflecting two forces
  - Higher price reduces return under repayment and default
  - ...but ability to lever up under repayment implies that the value V^R is more sensitive than V^D

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General Equilibrium

- So far, individual bank problem in partial eqm. (given p_t)
- General eqm. requires market clearing for capital

Consider possibility that $\phi \in [0,1]$ banks default

- Fraction ϕ must be consistent with optimal default decision

$$\phi = \begin{cases} 1 \text{ if } B_0 > \gamma_{-1} p_0 \bar{K}, \\ 0 \text{ if } B_0 < \gamma_{-1} p_0 \bar{K}, \\ \in [0,1] \text{ otherwise.} \end{cases}$$

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- Market clearing

$$\phi K_t^D + (1-\phi) K_t^R = \bar{K}$$

Definition of Equilibrium.

Given B_0, an equilibrium is a sequence of $\{p_t\}_{t=0}^{\infty}, \{\gamma_t\}_{t=-1}^{\infty}$, aggregate debt and capital, $\{B_t, K_t^R, K_t^D\}_{t=0}^{\infty}$, and an initial share of defaulting banks, ϕ, such that

(i) Evolution of aggregate debt and capital levels consistent with bank optimality given $\{\gamma_t, p_t\}$

$$B_{t+1} = \mathcal{B}_{t+1}((\bar{z} + p_t)K_t^R - RB_t)$$
$$K_{t+1}^R = \mathcal{K}_{t+1}^R((\bar{z} + p_t)K_t^R - RB_t)$$
$$K_{t+1}^D = \mathcal{K}_{t+1}^D((\underline{z} + p_t)K_t^D)$$

(ii) Borrowing limits are equilibrium consistent

(iii) Market for capital clears

(iv) ϕ is consistent with banks' optimal default decision

Stationary Equilibrium: Two Types

1. Default equilibrium:

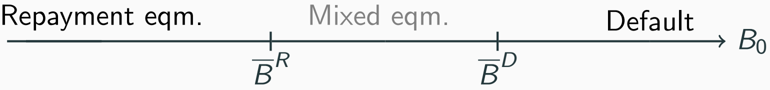
$$p^D = \frac{\beta}{1-\beta}\underline{z}$$
$$\gamma^D = H(\gamma^D, p^D)$$

2. Repayment equilibrium:

$$p^R = \frac{\beta \bar{z}}{1-\beta-(1-\beta R)\gamma^R}$$
$$\gamma^R = H(\gamma^R, p^R)$$

- Higher asset price under repayment
  - Higher dividend and higher "collateral" value

Transitional Dynamics: 3 Regions depending on B_0

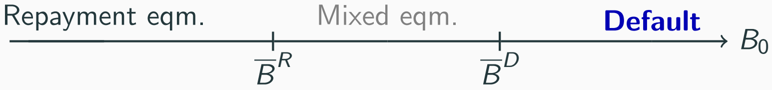


$\overline{B}^R$: threshold at which banks are indifferent while facing p^R

$\overline{B}^D$: threshold at which banks are indifferent while facing p^D

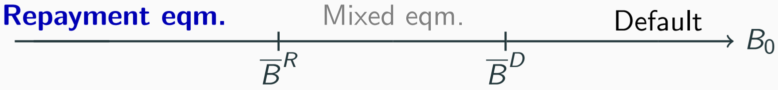
Conjecture that $\overline{B}^D > \overline{B}^R$

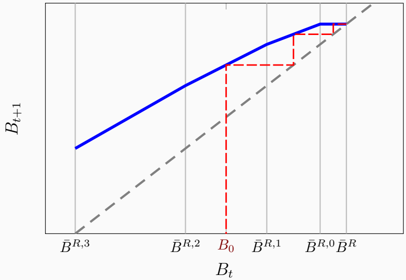
Transitional Dynamics: High B_0



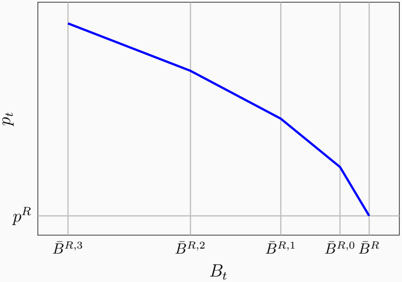
No transition $\Rightarrow$ economy converges immediately to (γ^D, p^D)

Transitional Dynamics: Low B_0



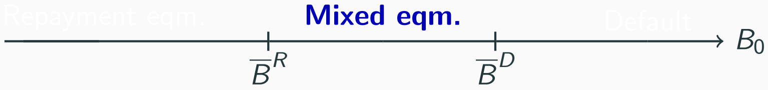


(a) Transition map for B_t



(b) Associated price p_t

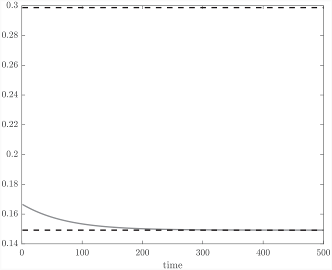
Transitional Dynamics: Intermediate B_0



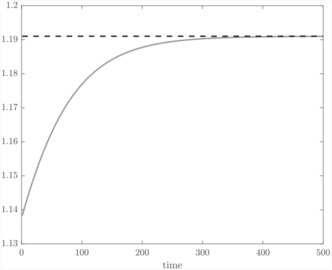
- Equilibrium must be non-degenerate $0 < \phi < 1$
  - A bank would find optimal to deviate under any equilibrium with all banks repaying or defaulting
  - Mixed equilibrium: some banks default and some banks repay

Transitional Dynamics: Intermediate B_0

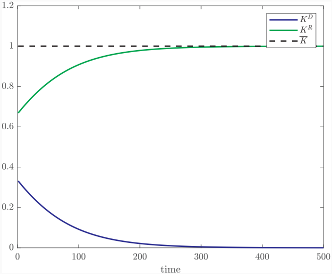
Borrowing Limit γ_t



Price of Capital

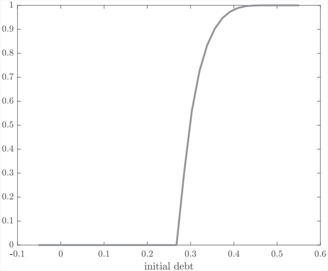


Capital Holdings

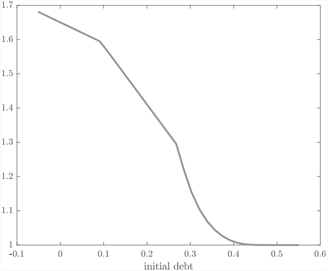


Summary Transitions as a fraction of B_0

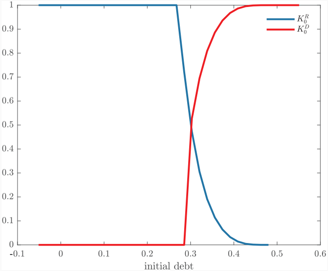
(a) Fraction of banks defaulting



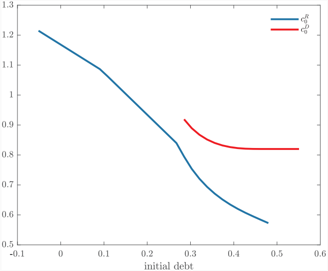
(b) Price of Capital p_0



(c) Allocation of Capital



(d) Consumption



Outline of the Talk

1. Basic environment without runs
    - Bank problem in partial equilibrium
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2. Introduce bank-runs
3. Credit easing

Model with Runs

- Coordination problem between lenders a la Cole-Kehoe
  - Investors may panic and refuse to rollover deposits

- A bank choosing to repay in the event of a run solves

$$V_t^{Run}(n) = \max_{k' \geq 0, c \geq 0} \log(c) + V_{t+1}^{Safe}\left((\bar{z} + p_{t+1})k'\right)$$

subject to:

$$c = \underbrace{(\bar{z} + p_t)k - bR}_{n} - p_t k'$$

- Repayment of b must come from sales of k

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★ If $V_t^{Run}(n) < V_t^D(k) < V_t^{Safe}(n)$, a bank is vulnerable

★ If $V_t^{Run}(n) > V_t^D(k)$, a bank is safe

  - We assume that if a bank is vulnerable, a run happens

The Effects of Bank-Runs

1. Partial equilibrium: tighter borrowing constraint $\gamma^{Run} < \gamma$
   - $V_{t+1}^{Run}(n') \geq V_{t+1}^{D}(k')$
2. General equilibrium: lower price of capital
   - Lower γ, imply lower demand by repaying banks
   - More banks defaulting, which have lower demand for capital

Complementarity effects:

Runs cause more banks to default, which imply a $\Downarrow p_0$

$\Rightarrow$ Lower p_0 hurts banks facing a run

$\Rightarrow$ more defaults

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Credit Easing

- Government purchases of assets financed with debt and lump sum taxes on banks
- Assumptions:
  - Government is less efficient than defaulting banks $z_g \leq \underline{z}$
    - Focus on $R^g \equiv \frac{z^g + p_1}{p_0} < R \Rightarrow$ govt. loses money
  - No taxes/subsidies after $t > 0 \Rightarrow$ government cannot bypass borrowing constraint

Q1: How does credit easing affect the fraction of banks defaulting?

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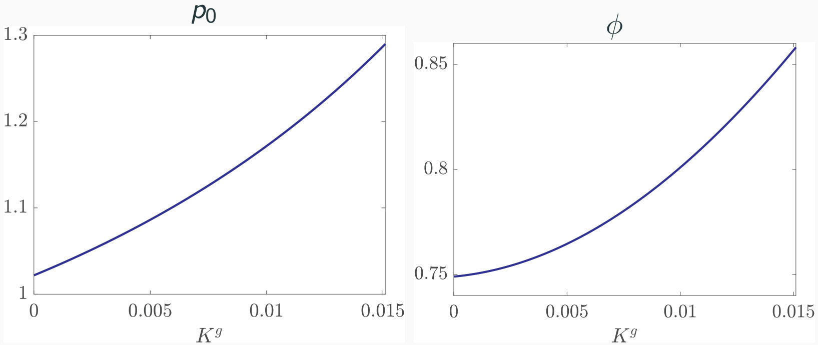
Q1: How does credit easing affect the fraction of banks defaulting?

Q2: What are the welfare implications?

Credit Easing: Fundamental Driven

Fraction of defaulting banks increases absent runs

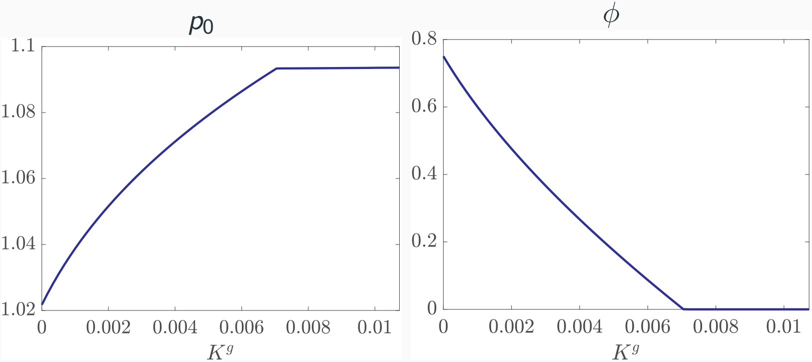


- Banks that default are net sellers of capital

  $\Rightarrow$ credit easing raises p_0 and increases V^D relative to V^R

Credit Easing: Run Driven

Fraction of defaulting banks decreases with runs



- Banks facing run are net sellers of capital (need to pay back b)

  $\Rightarrow$ Credit easing raises p and increases V^{Run} relative to V^D

  $\Longrightarrow$ Deters investors from running

Credit Easing: Taking Stock

★ Credit easing is only welfare improving if crisis is driven by runs

- Given asset prices, credit easing can only reduce welfare because government needs to tax banks to cover the losses
- In equilibrium, asset prices rise:
  - Without runs: more banks default and welfare falls
  - With runs: less banks default and welfare improves by avoiding inefficient defaults

Other policies: Controlling Default

★ Government picks at $t = 0$ banks that default

- Let all markets clear competitively

Absent runs:

$$\frac{dW}{d\phi} = \cancelto{0}{\left[V^D(p_0^E) - V^R(p_0^E)\right]} - (1-\phi)\left[u'(c^R(p_0^E)) - u'(c^D(p_0^E))\right](k^R(p_0^E) - \bar{K})\frac{dp_0}{d\phi} > 0$$

- Optimal to increase share of defaulting banks
  - More defaults reduce the price of capital and helps repaying banks, which have high marginal utility

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With runs:

$$\frac{dW}{d\phi} = \left[V^{Safe}(p_0^E) - V^D(p_0^E)\right] - (1-\phi)\left[u'(c^R(p_0^E)) - u'(c^D(p_0^E))\right](k^R(p_0^E) - \bar{K})\frac{dp_0}{d\phi}$$

- May be optimal to reduce defaults

Other Policies

★ Tax on purchases of capital at $t = 0$ rebated lump sum

- Irrelevant: after-tax price remains constant and has no effects

★ Deposit insurance

- Can deter investors from running, but leads to inefficiently large bank borrowing
    - Banks can borrow at risk-free rate $\Rightarrow$ borrow a lot and default

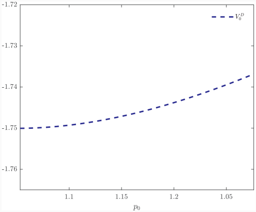
  $\Rightarrow$ Government needs to impose, in addition, borrowing limits

Conclusions

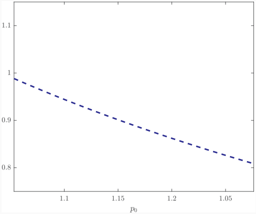
- Presented a tractable macroeconomic model of self-fulfilling bank-runs
- General equilibrium effects induce contagion effects through price of capital and are crucial to assess government policies
- Credit easing is desirable if and only if crisis are driven by runs
- Framework can be used to study other policies
    - Next: macroprudential policy

Credit Easing: Fundamentals vs. Runs

(a) Value Functions Fundamental



(c) Demand for Capital Fundamental



Values and Net Positions as a function of Initial Price

Note: Vertical line indicate equilibrium price

Credit Easing: Fundamentals vs. Runs

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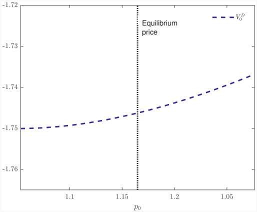
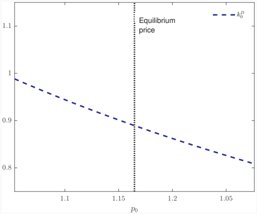


(c) Demand for Capital Fundamental

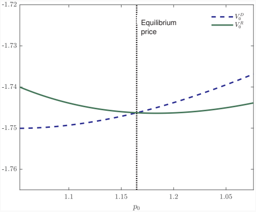


Values and Net Positions as a function of Initial Price

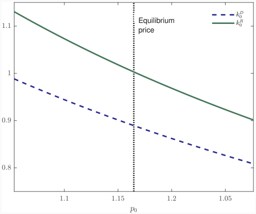
Note: Vertical line indicate equilibrium price

Credit Easing: Fundamentals vs. Runs

(a) Value Functions Fundamental



(c) Demand for Capital Fundamental



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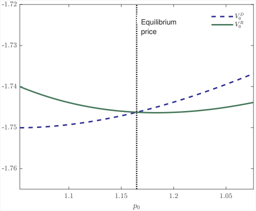
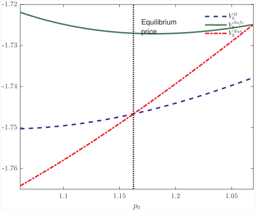
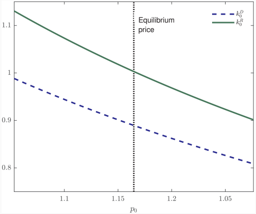


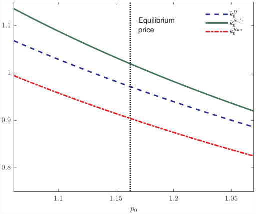
(b) Value Functions Run



(c) Demand for Capital Fundamental



(d) Demand for Capital Run



Values and Net Positions as a function of Initial Price

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Credit Easing: Fundamentals vs. Runs

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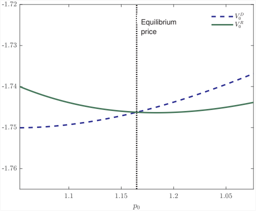
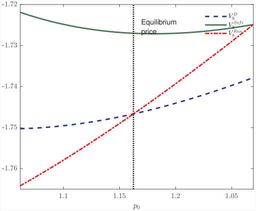


(b) Value Functions Run



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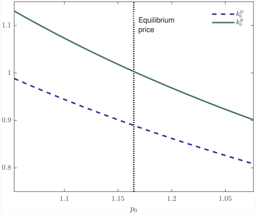
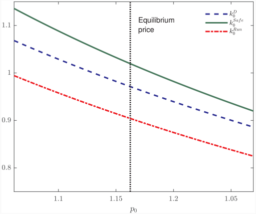


(d) Demand for Capital Run



Values and Net Positions as a function of Initial Price

Note: Vertical line indicate equilibrium price

Definition of Equilibrium.

Given B_0, an equilibrium is a sequence of $\{p_t\}_{t=0}^{\infty}, \{\gamma_t\}_{t=-1}^{\infty}$, aggregate debt and capital, $\{B_t, K_t^R, K_t^D\}_{t=0}^{\infty}$, and an initial share of defaulting banks, ϕ, such that

(i) Evolution of aggregate debt and capital levels consistent with bank optimality given $\{\gamma_t, p_t\}$

$$B_{t+1} = \mathcal{B}_{t+1}((\bar{z} + p_t)K_t^R - RB_t)$$

$$K_{t+1}^R = \mathcal{K}_{t+1}^R((\bar{z} + p_t)K_t^R - RB_t)$$

$$K_{t+1}^D = \mathcal{K}_{t+1}^D((\underline{z} + p_t)K_t^D)$$

(ii) Borrowing limits are equilibrium consistent

(iii) Market for capital clears

(iv) ϕ is consistent with banks' optimal default decision