# External Instrument SVAR Analysis for Noninvertible Shocks 

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National Bank of Belgium, 4th May 2023

## IV in Macroeconomics

- New and increasingly popular method for Macroeconometrics

$$
z_{t}=\alpha u_{t}^{i}+\eta_{t} \quad \eta_{t} \sim \mathcal{W N}\left(0, \sigma_{\eta}^{2}\right)
$$

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- Wealth of new instruments expanding Macro literature:
- Oil - Hamilton 2003; Kilian, 2008, Känzig, 2021
- Government purchases - Ramey, 2011, Ricco et al., 2016, Ramey and Zubairy, 2018
- Tax - Romer and Romer, 2010, Leeper et al., 2013, Mertens and Ravn, 2012, Mertens and Montiel-Olea, 2018
- Conventional/Unconventional monetary policy - Romer and Romer, 2004, Gürkaynak et al. 2005, Gertler and Karadi, 2015, Jarocinski and Karadi 2020, Swanson, 2020, Miranda-Agrippino and Ricco, forth.,
- Government asset purchases - Fieldhouse et al. 2017, Fieldhouse et al. 2018
- Confidence - Lagerborg et al. 2018
- Technology news - Cascaldi-Garcia and Vukotić 2019


## IV in Macroeconomics

- New and increasingly popular method for Macroeconometrics

$$
z_{t}=\alpha u_{t}^{i}+\underbrace{(\ldots \ldots)}_{\text {contamination }}+\eta_{t} \quad \eta_{t} \sim \mathcal{W} \mathcal{N}\left(0, \sigma_{\eta}^{2}\right)
$$

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## Conditions for External Instrument SVAR

Stock (2008), Stock and Watson $(2012,2018)$ and Mertens and Ravn (2013)

## Reduced-Form VAR

$$
A(L) y_{t}=\varepsilon_{t}
$$

## Conditions - Global Invertibility

(i) $\mathbb{E}\left[u_{t}^{1} z_{t}\right]=\alpha$ (Relevance)
(ii) $\mathbb{E}\left[u_{t}^{2: n} z_{t}\right]=0$ (Contemporaneous Exogeneity)
(iii) $u_{t}=\operatorname{Proj}\left(u_{t} \mid Y_{t}, Y_{t-1}, \ldots\right)$ (Global Invertibility)

## Conditions for External Instrument SVAR

Stock (2008), Stock and Watson (2012, 2018) and Mertens and Ravn (2013), Miranda-Agrippino, Ricco (2023)

## Reduced-Form VAR

$$
A(L) y_{t}=\varepsilon_{t}
$$

## Conditions - Partial Invertibility

(i) $\mathbb{E}\left[u_{t}^{1} z_{t}\right]=\alpha$ (Relevance)
(ii) $\mathbb{E}\left[u_{t}^{2: n} z_{t}\right]=0$ (Contemporaneous Exogeneity)
(iii) $\mathbb{E}\left[u_{t-j}^{m+1: n} z_{t}^{\perp}\right]=0$ for all $j \neq 0$ for which $\mathbb{E}\left[u_{t-j}^{m+1: n} \nu_{t}^{\prime}\right] \neq 0$

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$\Longrightarrow$ Yes, internal instrument SVAR (Plagborg-Møller and Wolf, 2021)

- many additional parameters
- potentially very large information set
- IV and VAR sample have to align
- Lag order fixed by the VAR


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## This Paper

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- many additional parameters
- potentially very large information set
- IV and VAR sample have to align
- Lag order fixed by the VAR
$\Longrightarrow$ Yes, external instrument SVAR to retain flexibility (this paper)
(2) General Representation Result
- invertible/fundamental (Lippi and Reichlin, 1994)
- recoverable (Chahrour and Jurado, 2021)
- non-recoverable
(3) Tests for invertibility and recoverability
(4) Validation in simulated environment \& application to monetary policy

A Representation Result

## The model

- The structural representation (SMA)

$$
\begin{equation*}
y_{t}=B(L) u_{t} \quad u_{t} \sim \mathcal{W N}\left(0, I_{q}\right) \tag{1}
\end{equation*}
$$

$B(L)$ is an $n \times q$ matrix of rational function in the lag operator $L, n \leq q$

- The Wold representation

$$
\begin{equation*}
y_{t}=C(L) \varepsilon_{t} \tag{2}
\end{equation*}
$$

- The VAR representation

$$
\begin{equation*}
A(L) y_{t}=\varepsilon_{t} \tag{3}
\end{equation*}
$$

- What is the relation between the structural shocks $u_{t}$ and the VAR residuals $\varepsilon_{t}$ ?


## Innovations and Shocks

- VAR residuals $\varepsilon_{t}$ are linear combinations of the current and lagged structural shocks $u_{t}$

$$
\begin{equation*}
\varepsilon_{t}=A(L) y_{t}=A(L) B(L) u_{t}=Q(L) u_{t} \tag{4}
\end{equation*}
$$

- Generally, the inverse map is not exact function of the $\varepsilon_{t}$

$$
\begin{equation*}
u_{t}=P\left(u_{t} \mid \mathcal{H}\right)+s_{t}=D^{\prime}(F) \varepsilon_{t}+s_{t} \tag{5}
\end{equation*}
$$

where $P$ is the linear projection operator and $\mathcal{H}=\overline{\operatorname{span}}\left(\varepsilon_{j, t-k}, j=1, \ldots, n, k \in \mathbb{Z}\right)$
The structural IRFs are linked to the Wold representation by

$$
B(L)=C(L) Q(L)=C(L) \Sigma_{\varepsilon} D(L)
$$

In particular, an IRF of interest

$$
b_{i}(L)=C(L) q_{i}(L)=C(L) \Sigma_{\varepsilon} d_{i}(L)
$$

## Invertible shocks

## Invertibility

A shock is invertible if it is a linear combination of the present and past values of the VAR variables, or, equivalently, a contemporaneous linear combination of the VAR residuals

## Proposition - Structural shocks and VAR residuals

If $u_{i t}$ is fundamental for $y_{t}$, then $d_{i}(F)=d_{i 0}=d_{i}$ and $q_{i}(F)=q_{i 0}=q_{i}$, so that

$$
\begin{equation*}
u_{i t}=d_{i}^{\prime} \varepsilon_{t}=q_{i}^{\prime} \Sigma_{\varepsilon}^{-1} \varepsilon_{t} . \tag{6}
\end{equation*}
$$

## Recoverable shocks

## Recoverability

A shock is recoverable if it is a linear combination of the present, past and future values of the VAR variables, or, equivalently, it is a linear combination of the present and future values of the VAR residuals

## Proposition - Structural shocks and VAR residuals

If $u_{i t}$ is recoverable with respect to $y_{t}$,

$$
\begin{equation*}
u_{i t}=d_{i}^{\prime}(F) \varepsilon_{t}=q_{i}^{\prime}(F) \Sigma_{\varepsilon}^{-1} \varepsilon_{t} \tag{7}
\end{equation*}
$$

where $d_{i}(F)=d_{i 0}+d_{i 1} F+d_{i 2} F^{2}+\cdots$ is the $i$-th column of $D(F)$ and $q_{i}(F)=q_{i 0}+q_{i 1} F+q_{i 2} F^{2}+\cdots$ is the $i$-th column of $Q(F)$. Moreover

$$
d_{i}^{\prime}(F) \Sigma_{\varepsilon} d_{i}(L)=q_{i}^{\prime}(F) \Sigma_{\varepsilon}^{-1} q_{i}(L)=1
$$

## A General Representation

Any vector process $y_{t}$ with an SMA and VAR form can be represented as

$$
\begin{align*}
y_{t} & =B^{f}(L) u_{t}^{f}+B^{r}(L) u_{t}^{r}+B^{n}(L) u_{t}^{n} \\
& =C(L) Q^{f} u_{t}^{f}+C(L) Q^{r}(L) u_{t}^{r}+C(L) Q^{n}(L) u_{t}^{n} \\
& =C(L) \Sigma_{\varepsilon} D^{f} u_{t}^{f}+C(L) \Sigma_{\varepsilon} D^{r}(L) u_{t}^{r}+C(L) \Sigma_{\varepsilon} D^{n}(L) u_{t}^{n} . \tag{8}
\end{align*}
$$

where $C(L)$ the Wold representation coefficients and $\Sigma_{\varepsilon}$ is the covariance of $\varepsilon_{t}$

- $u_{t}^{f}$ the fundamental structural shocks
- $u_{t}^{r}$ the recoverable (but nonfundamental) shocks
- $u_{t}^{n}$ of the nonrecoverable ones
- $Q^{h}(L) u_{t}^{h}$, for $h=f, r, n$, is the projection of $\varepsilon_{t}$ onto $u_{t-k}^{h}$, with $k \geq 0$;
- $D^{h}(F) \varepsilon_{t}$ is the projection of $u_{t}^{h}$ onto $\varepsilon_{t+k}$, with $k \geq 0$

Moreover, the following properties hold:
(i) $D^{f}$ and $Q^{f}$ s.t $D^{f \prime} \Sigma_{\varepsilon} D^{f}=Q^{f \prime} \Sigma_{\varepsilon}^{-1} Q^{f}=I_{q_{f}}$, for $q_{f}$ fundamental shocks;
(ii) $D^{r}(L)$ and $Q^{r}(L)$ s.t. $D^{r \prime}(F) \Sigma_{\varepsilon} D^{r}(L)=Q^{r \prime}(F) \Sigma_{\varepsilon}^{-1} Q^{r}(L)=I_{q_{r}}$, for $q_{r}$ recoverable shocks

# Identification 

## A general IV

## The Instrument

The researcher can observe the proxy $\tilde{z}_{t}$, following the relation

$$
\begin{equation*}
\tilde{z}_{t}=\beta(L) \tilde{z}_{t-1}+\mu^{\prime}(L) x_{t-1}+\underbrace{\alpha u_{i t}+w_{t}}_{z_{t}}, \tag{9}
\end{equation*}
$$

where $w_{t}$ is an error orthogonal to $u_{j, t-k}, j=1, \ldots, q$, for any integer $k$ and to $z_{t-k}, x_{t-k}$, $k \geq 0$, and $\beta(L), \mu(L)$ are rational functions in the lag operator $L$

- We consider the 'residual'

$$
\begin{equation*}
z_{t}=\alpha u_{i t}+w_{t} . \tag{10}
\end{equation*}
$$

## The IRFs and the shock

Consider the projection of $\varepsilon_{t}$ onto the present and past of the proxy:

$$
\begin{equation*}
\varepsilon_{t}=\psi(L) z_{t}+e_{t} \tag{11}
\end{equation*}
$$

## Proposition - Relative IRFs

The coefficients of the projection (11) are related to $q_{i}(L)$ by the equation

$$
\begin{equation*}
\psi(L) \sigma_{z}^{2}=q_{i}(L) \alpha \tag{12}
\end{equation*}
$$

Hence the impulse-response functions fulfil the relation

$$
\begin{equation*}
b_{i}(L) \alpha=C(L) \psi(L) \sigma_{z}^{2} \tag{13}
\end{equation*}
$$

## The IRFs and the shock

- Invertible: $\varepsilon_{t}=\psi^{\prime} z_{t}+e_{t}$, and IRFs:

$$
\begin{equation*}
b_{i}(L)=\frac{C(L) \psi}{\sqrt{\psi^{\prime} \hat{\Sigma}_{\varepsilon}^{-1} \psi}} \tag{14}
\end{equation*}
$$

- Recoverable: $\varepsilon_{t}=\psi(L) z_{t}+e_{t}$, and IRFs:

$$
\begin{equation*}
b_{i}(L)=\frac{C(L) \psi(L)}{\sqrt{\sum_{k=0}^{\infty} \psi_{k}^{\prime} \Sigma_{\varepsilon}^{-1} \psi_{k}}} \tag{15}
\end{equation*}
$$

- Non-Recoverable Upper and the lower bounds of $\alpha^{2}$ (Plagborg-Møller and Wolf, 2022)

$$
\begin{align*}
& \alpha^{2} \leq \sigma_{z}^{2}=\bar{\alpha}^{2} \\
& \alpha^{2} \geq \alpha^{2} \sup _{\theta \in(0 \pi]} R_{r}^{2}(\theta)=\sigma_{z}^{4} \sup _{\theta \in(0 \pi]} \psi^{\prime}\left(e^{j \theta}\right) \sum_{\varepsilon}^{-1} \psi\left(e^{-j \theta}\right) . \tag{16}
\end{align*}
$$

## Variance and historical decompositions

- Historical decomposition is easy once the shock is identified
- Variance is difficult...
- The standard forecast error variance decomposition (FVD) only for invertible models
- ... one cannot estimate the denominator without estimating the whole structural model
- Plagborg-Møller and Wolf (2022): denominator with the forecast error variance (FVR)
- Alternative: integral of the spectral density over a frequency band (VD)

$$
\begin{equation*}
\hat{c}_{h}\left(\theta_{1}, \theta_{2}\right)=\frac{\int_{\theta_{1}}^{\theta_{2}} \hat{b}_{i h}\left(e^{-j \theta}\right) \hat{b}_{i h}\left(e^{j \theta}\right) d \theta}{\int_{\theta_{1}}^{\theta^{2}} \widehat{S}_{h}(\theta) d \theta} \tag{17}
\end{equation*}
$$

## Testing for recoverability and invertibility

- Recoverability:

$$
\begin{equation*}
z_{t}=\delta^{\prime}(F) \varepsilon_{t}+v_{t} \tag{18}
\end{equation*}
$$

- If recoverable $\hat{u}_{i t}=\hat{\delta}(F) \hat{\epsilon}_{t}$ (intuition: Plagborg-Møller and Wolf, 2022)
- Ljung-Box Q-test to the estimated projection $\hat{\delta}(F) \hat{\epsilon}_{t}$
- $H_{0}$ is recoverability (serial uncorrelation) vs $H_{1}$ nonrecoverability (serial correlation)
- Invertibility:
- If invertible $\delta_{k}=0$ for all positive $k$
- standard $F$-test for the joint significance of the coefficients of the leads in Eq. (18)
- test $H_{0}$ of fundamentalness vs $H_{1}$ nonfundamentalness
- If not invertibility, the degree of fundamentalness is

$$
\hat{R}_{f}^{2}=\hat{\delta}_{0}^{\prime} \widehat{\Sigma}_{\varepsilon} \hat{\delta}_{0} / \sum_{k=0}^{r} \hat{\delta}_{k}^{\prime} \widehat{\Sigma}_{\varepsilon} \hat{\delta}_{k} .
$$

# Identification in Practice 

## IV Identification in practice

1. Regress $\tilde{z}_{t}$ onto its lags and a set of regressors $x_{t}$, to get $z_{t}$

$$
\begin{equation*}
\tilde{z}_{t}=\beta(L) \tilde{z}_{t-1}+\mu^{\prime}(L) x_{t-1}+\alpha u_{i t}+z_{t} \tag{19}
\end{equation*}
$$

If the $F$-test does not reject the null $H_{0}: \beta(L)=0 \& \mu^{\prime}(L)=0$, step 1 can be skipped
2. Estimate a $\operatorname{VAR}(p)$ with $\operatorname{OLS}$ to obtain $\widehat{A}(L), \widehat{C}(L)=\widehat{A}(L)^{-1}, \hat{\varepsilon}_{t}$ and $\widehat{\Sigma}_{\varepsilon}$
3. Regress $\hat{z}_{t}$ on the current value and the first $r$ leads of the Wold residuals:

$$
\hat{z}_{t}=\sum_{k=0}^{r} \hat{\delta}_{k}^{\prime} \hat{\varepsilon}_{t+k}+\hat{v}_{t}=\hat{\delta}(F) \hat{\varepsilon}_{t}+\hat{v}_{t}
$$

Save the fitted value of the above regression, let us call it $\hat{\eta}_{t}$ Test for invertibility

## IV Identification in practice

4. Invertible shock: Estimate $\delta$ and the unit-variance shock. Estimate

$$
\varepsilon_{t}=\psi^{\prime} z_{t}+e_{t}
$$

and estimate IRFs according to (14). Estimate the variance decomposition
$4^{\prime}$. Invertibility is rejected: Recoverability test
5. Recoverable shock: Estimate the unit-variance shock according. Estimate

$$
\varepsilon_{t}=\psi(L) z_{t}+e_{t}
$$

and IRFs according to (15). Estimate the variance decomposition
$5^{\prime}$. Nonrecoverable shock:

- Either amend the VAR specification and repeat steps 2-4, or
- Estimate

$$
\varepsilon_{t}=\psi^{\prime} z_{t}+e_{t}
$$

Estimate lower and upper bounds according and the corresponding variance contributions

## A Simulated Economy with Fiscal Foresight

## An economy with fiscal foresight

- Leeper et al. (2013) RBC model with log preferences and inelastic labor supply
- Two iid shocks: technology, $u_{a, t}$, and $\operatorname{tax} u_{\tau, t}$

$$
\begin{aligned}
a_{t} & =u_{a, t} \\
\tau_{t} & =u_{\tau, t-2}
\end{aligned}
$$

Tax shocks are announced before implementation: fiscal foresight

- In deviations from the SS capital accumulation is

$$
\begin{equation*}
k_{t}=\alpha k_{t-1}+a_{t}-\kappa \sum_{i=0}^{\infty} \theta^{i} E_{t} \tau_{t+i+1} \tag{20}
\end{equation*}
$$

## An economy with fiscal foresight

- Equilibrium MA representation for capital and taxes:

$$
\binom{\tau_{t}}{k_{t}}=\left(\begin{array}{cc}
L^{2} & 0  \tag{21}\\
\frac{-\kappa(L+\theta)}{1-\alpha L} & \frac{1}{1-\alpha L}
\end{array}\right)\binom{u_{\tau, t}}{u_{a, t}}=B(L) u_{t} .
$$

- Nonfundamental shocks (matrix vanishes for $L=0$ )
- They are recoverable! (The system is square)
- Tax shock is equal to tax two periods ahead: $u_{\tau, t}=\tau_{t+2}$


## An economy with fiscal foresight

- 1000 simulations $\mathrm{T}=240$
- IV simulated as

$$
\tilde{z}_{t}=u_{\tau, t}+0.5 z_{t-1}+0.4 k_{t-1}-0.6 \tau_{t-1}+v_{t},
$$

where $v_{t} \sim \operatorname{iid} \mathcal{N}(0,1)$

- For each dataset, we test for invertibility and recoverability, and estimate the tax shock

$$
p=m=2 \quad r=0 \quad p=m=2 \quad r=4
$$

- Invertibility is correctly rejected in all cases
- Recoverability is (wrongly) rejected at the $5 \%$ level in $10 \%$ of the cases (test is oversized)


## An economy with fiscal foresight










## A comparison with the Internal-Instrument SVAR

- Same model, same IV
- 1000 simulations $T=240$
- The instrument is preliminarily 'cleaned' by setting $x_{t}=y_{t}$ and the number of lags $m$ according to the BIC
- For the Internal-Instrument method, VAR for the vector $\left(\begin{array}{lll}\tilde{z}_{t} & y_{t}^{\prime}\end{array}\right)^{\prime}$
- Estimation error measured as

$$
\begin{equation*}
100 \times \frac{\sum_{h=1}^{n} \sum_{k=0}^{K}\left(\hat{\mu}_{h k}-\mu_{h k}\right)^{2}}{\sum_{h=1}^{n} \sum_{k=0}^{K} \mu_{h k}^{2}} . \tag{22}
\end{equation*}
$$

sum of the squared errors divided by the sum of the squared coefficients of the true IRFs

## A comparison with the Internal-Instrument SVAR

## External IV

VAR order Internal IV $\quad r=B / C \quad r=3 \quad r=4 \quad r=5 \quad r=6 \quad r=7$

$$
p=1 \quad 410.8
$$

$p=2 \quad 34.8$
$\begin{array}{ll}p=3 & 7.6\end{array}$
$p=4 \quad 9.5$
$p=5 \quad 11.2$
$p=6 \quad 12.9$
$p=B I C \quad 7.6$

| $\mathbf{4 . 3}$ | $\mathbf{4 . 3}$ | $\mathbf{5 . 0}$ | $\mathbf{6 . 4}$ | $\mathbf{7 . 9}$ | $\mathbf{9 . 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5.2 | 5.2 | 5.9 | $\mathbf{6 . 4}$ | $\mathbf{7 . 9}$ | $\mathbf{9 . 4}$ |
| 6.0 | 6.0 | 6.8 | 7.3 | 8.0 | 9.5 |
| 7.1 | 7.1 | 7.8 | 8.4 | 9.1 | 9.6 |
| 8.0 | 8.0 | 8.8 | 9.3 | 10.0 | 10.6 |
| 8.9 | 8.9 | 9.6 | 10.2 | 10.9 | 11.5 |

4.3

## A comparison with the Internal-Instrument SVAR

- 3 dynamic relations:
- the IV equation
- the VAR model
- the equation linking VAR residuals and the proxy
- Internal IV approach: they are all fixed at the same lag order
- External-Instrument: they can be independently set optimally


## Monetary policy shocks

## Monthly VAR and High Frequency IV

- Specification I: 1-year gov't bond rate, IP and CPI
- Specification II: Specification I + Gilchrist and Zakrajšek (2012)'s excess bond premium
- Specification III: Specification II + mortgage spread and the commercial paper spread CPI and IP in differences
- Samples: 1983:1-2008:12 (robustness 1979:7/1987:8/1990:1 - 2012:6/2019:6)
- IV: Fed Funds futures (FF4) surprises
... likely to capture both conventional shocks and forward guidance 'Clean' the IV onto its lags and 6 lags variables of Specification I


## Fundamentalness and recoverability

|  | Number of leads $r$ |  |  |  |  |  | Number of leads $r$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $r=4$ | $r=5$ | $r=6$ | $r=7$ | $r=8$ | $r=9$ |  | $r=4$ | $r=5$ | $r=6$ | $r=7$ | $r=8$ | $r=9$ |
| Specification I |  |  |  |  |  |  | Specification I |  |  |  |  |  |  |
| $p=6$ | 0.008 | 0.028 | 0.002 | 0.003 | 0.001 | 0.001 | $p=6$ | 0.619 | 0.662 | 0.251 | 0.469 | 0.037 | 0.060 |
| $p=9$ | 0.016 | 0.051 | 0.003 | 0.003 | 0.002 | 0.001 | $p=9$ | 0.350 | 0.571 | 0.114 | 0.435 | 0.050 | 0.042 |
| $p=12$ | 0.011 | 0.045 | 0.003 | 0.002 | 0.001 | 0.000 | $p=12$ | 0.880 | 0.944 | 0.324 | 0.820 | 0.466 | 0.285 |
| Specification II |  |  |  |  |  |  | Specification II |  |  |  |  |  |  |
| $p=6$ | 0.080 | 0.195 | 0.027 | 0.001 | 0.000 | 0.000 | $p=6$ | 0.441 | 0.473 | 0.308 | 0.777 | 0.394 | 0.357 |
| $p=9$ | 0.180 | 0.351 | 0.034 | 0.002 | 0.000 | 0.000 | $p=9$ | 0.119 | 0.186 | 0.104 | 0.517 | 0.222 | 0.193 |
| $p=12$ | 0.221 | 0.457 | 0.059 | 0.003 | 0.000 | 0.000 | $p=12$ | 0.472 | 0.558 | 0.269 | 0.913 | 0.701 | 0.575 |
| Specification III |  |  |  |  |  |  | Specification III |  |  |  |  |  |  |
| $p=6$ | 0.060 | 0.184 | 0.089 | 0.003 | 0.001 | 0.002 | $p=6$ | 0.034 | 0.315 | 0.446 | 0.608 | 0.738 | 0.546 |
| $p=9$ | 0.184 | 0.362 | 0.220 | 0.020 | 0.002 | 0.003 | $p=9$ | 0.005 | 0.064 | 0.148 | 0.046 | 0.391 | 0.103 |
| $p=12$ | 0.215 | 0.353 | 0.250 | 0.060 | 0.031 | 0.027 | $p=12$ | 0.032 | 0.037 | 0.065 | 0.057 | 0.343 | 0.022 |

(a) Fundamentalness test
(b) Recoverability test

## Small VAR specification: Monetary policy shocks



Figure 1: VAR results: Specification I, p $=12$, GK instrument. Top panels: estimated response functions with $r=0$ (standard method). Bottom panels: estimated response functions with our proposed method $r=6$. Black line: point estimate. Grey area: $68 \%$ confidence bands.

## Medium VAR specifications: Monetary policy shocks



Figure 2: Red line: point estimates for Specification III; blue line: point estimates for Specification II; black line: point estimates for Specification I. Top panels: estimated response functions with $p=12$, $r=0$ (standard method). Bottom panels: estimated response functions with our proposed method, $p=12, r=6$. Pink shaded area: 68\% confidence bands for Specification III.

## Variance decomposition

|  | Waves of periodicity |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $2-18$ months | $18-96$ months | $2+$ months |  |
| Specification I |  |  |  |  |
| CPI inflation | 19.2 | 27.6 | 20.8 |  |
|  | $(13.5-29.1)$ | $(12.8-64.2)$ | $(16.2-35.1)$ |  |
| IP growth | 27.7 | 33.8 | 28.3 |  |
|  | $(19.1-36.4)$ | $(13.1-55.4)$ | $(20.0-37.6)$ |  |
| Specification II |  |  |  |  |
| CPI inflation | 12.3 | 12.9 | 13.2 |  |
|  | $(10.4-23.1)$ | $(9.7-45.1)$ | $(13.4-26.8)$ |  |
| IP growth | 20.3 | 29.5 | 22.5 |  |
|  | $(15.8-28.2)$ | $(11.4-51.5)$ | $(16.7-31.3)$ |  |
| Specification III |  |  |  |  |
| CPI inflation | 12.5 | 10.3 | 12.5 |  |
|  | $(10.2-19.5)$ | $(6.9-34.2)$ | $(11.2-21.5)$ |  |
| IP growth | 16.1 | 5.2 | 13.0 |  |
|  | $(12.2-22.2)$ | $(4.2-22.0)$ | $(11.2-20.7)$ |  |

Table 1: Percentage of variance accounted for by the monetary policy shock, for waves of periodicity 2-18 months (short run), 18-96 months (business cycle), $2+$ months (overall variance). $68 \%$ confidence bands in brackets.

## Variance decomposition

|  | FVR Horizon |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | impact | 3 months | 6 months | 12 months | 24 months | $2+$ months |
| CPI inflation |  |  |  |  |  |  |
| Specification I | 0.5 | 7.2 | 15.3 | 18.4 | 20.7 | 20.8 |
| Specification II | 0.2 | 4.7 | 9.1 | 13.3 | 13.4 | 13.2 |
| Specification III | 0.3 | 5.6 | 7.4 | 12.5 | 12.4 | 12.5 |
| CPI index in levels |  |  |  |  |  |  |
| Specification I | 0.5 | 4.2 | 9.9 | 20.0 | 21.5 |  |
| Specification II | 0.2 | 2.6 | 5.3 | 13.7 | 22.5 |  |
| Specification III | 0.3 | 4.4 | 7.1 | 13.8 | 18.5 |  |

Table 2: Percentage of variance of CPI inflation and prices accounted for by the monetary policy shock, according to the FVR measure of Plagborg-Møller and Wolf (2022), on impact and at 3,6, 12, 24 months horizons.

## Variance decomposition - Subsamples

|  | VD: waves of periodicity |  |  | FVR: horizon |
| :--- | :---: | :---: | :---: | :---: |
| Time span | $2-18$ months | $18-96$ months | $2+$ months | 24 months |
| $1983: 1-2008: 12$ | 10.4 | 22.0 | 16.1 | 15.5 |
| $1990: 1-2012: 6$ | 6.3 | 15.5 | 8.0 | 8.1 |
| $1987: 1-2008: 12$ | 7.3 | 15.4 | 11.3 | 10.6 |
| 1983:1-2012:6 | 10.0 | 24.6 | 12.7 | 12.8 |
| 1979:7-2012:6 | 17.2 | 19.3 | 17.4 | 17.5 |
| 1979:7-2019:6* | 15.7 | 18.2 | 15.3 | 15.1 |

Table 3: Variance decomposition of inflation for different time spans, Specification IV: FFR, CPI inflation, IP growth, EBP. VD: percentage of inflation variance accounted for by the monetary policy shock, for waves of periodicity 2-18 months (short run), 18-96 months (business cycle), $2+$ months (overall variance). FVR: percentage of forecast error variance of inflation accounted for by the monetary policy shock at the 2-year horizon. For the sample 1979:7-2019:6 in place of the EBP series we use three financial variables: the 10-year treasury bond rate, the BAA corporate bond yield and the S\&P500 stock price index.

## Conclusions

- New estimation procedure for structural VARs with an external instrument
- Test for invertibility and a test for recoverability
- The method works well in simulation
- HFI IV policy shocks are not invertible but recoverable
- Standard method produces puzzling results ...
- ... new procedure results in line with textbook effects
- Variance decomposition indicates that monetary policy has sizeable effects

