Shadow Interest Rate

Jing Cynthia Wu
Chicago Booth and NBER

Coauthors: Dora Xia (BIS) and Ji Zhang (Tsinghua)

ZLB: monetary policy

Before ZLB

- Central banks lower policy rates to stimulate aggregate demand
- Economists rely on them to study monetary policy

Policy rates at ZLB

- Japan, US, Europe
- Unconventional policy tools
  - large-scale asset purchases (QE)
  - lending facilities
  - forward guidance
- Negative interest rates

ZLB: economic models

Term structure models

- Benchmark models
  - Gaussian ATSM allows undesirably negative interest rates
  - It is especially problematic when interest rates are low
- Our papers:
  - Wu and Xia (JMCB 2016): model US yield curve with ZLB
  - Wu and Xia (2017): model recent negative interest rates in Europe

ZLB: economic models

Term structure models

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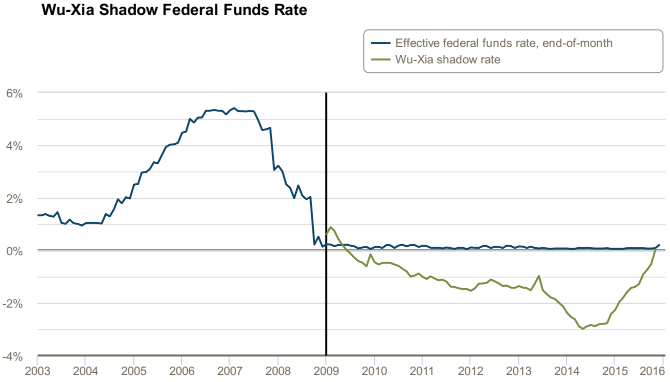
New Keynesian models

- Benchmark models: no unconventional monetary policy
  - ZLB introduces structural break
  - counterfactual economic implications
  - computationally demanding
- Wu and Zhang (2016): incorporate unconventional monetary policy
  - sensible economic implications
  - tractable

Common theme: shadow rate

Black (1995)

$$r_t = max(s_t, \underline{r})$$



Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2015)

Contributions - Wu and Xia (JMCB 2016)

Measuring the Macroeconomic Impact of Monetary Policy at the ZLB

- Develop an analytical approximation for SRTSM
- Shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
- Our shadow rates for US, Euro area, and UK are available at
  - Atlanta Fed
  - Haver Analytics
  - Thomson Reuters
  - Bloomberg
- Wu-Xia shadow rate has been discussed by
  - Policy makers: Governor Powell (2013), Altig (2014) of the Atlanta Fed, Hakkio and Kahn (2014) of the Kansas City Fed
  - Media:The Wall Street Journal, Bloomberg news, Bloomberg Businessweek, Forbes, Business Insider, VOX

Contributions - Wu and Zhang (2016)

A Shadow Rate New Keynesian Model

- Present new empirical evidence relating the shadow rate to
  - private interest rates
  - Fed's balance sheet
  - Taylor rule
- Propose a New Keynesian model with the shadow rate
  - accommodates both conventional and unconventional policies
- Microfoundations
  - QE
  - lending facilities
- Economic implications
  - a negative supply shock decreases output
    - data-consistent
    - contradicts standard models
  - government-spending multiplier is not larger than normal

Outline

1. Wu-Xia shadow rate
2. Motivating shadow rate New Keynesian model
3. Microfoundations
4. Economic implications

Shadow rate

Black (1995):

$$r_t = max(s_t, \underline{r})$$

The shadow rate is affine

$$s_t = \delta_0 + \delta_1' X_t$$

- $\underline{r} = 0.25$, interest on reserves
- X_t: 3 factors

Bond pricing

Factor dynamics:

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I).$$

Pricing kernel

$$\begin{aligned} m_{t+1} &= r_t + \frac{1}{2}\lambda_t' \lambda_t + \lambda_t' \varepsilon_{t+1} \\ \text{where} \quad \lambda_t &= \lambda_0 + \lambda_1 X_t \end{aligned}$$

Pricing equation I

$$P_{nt} = \mathbb{E}_t[\exp(-m_{t+1}) P_{n-1,t+1}]$$

Pricing equation II

$$P_{nt} = \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t) P_{n-1,t+1}]$$

Bond pricing

Factor dynamics under risk-neutral measure $\mathbb{Q}$:

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \overset{\mathbb{Q}}{\sim} N(0, I).$$

where $\mu^{\mathbb{Q}} = \mu - \Sigma\lambda_0$, and $\rho^{\mathbb{Q}} = \rho - \Sigma\lambda_1$

Yield

$$y_{nt} = -\frac{1}{n}\log(P_{nt})$$

Forward rate from $t+n$ to $t+n+1$

$$f_{nt} = (n+1)y_{n+1,t} - ny_{nt}$$

Forward rates

Our approximation

$$f_{nt} = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right)$$

where $g(z) = z\Phi(z) + \phi(z)$.

$$\begin{aligned} a_n &= \delta_0 + \delta_1' \left(\sum_{j=0}^{n-1} \left(\rho^{\mathbb{Q}}\right)^j\right) \mu^{\mathbb{Q}} - \frac{1}{2}\delta_1' \left(\sum_{j=0}^{n-1} \left(\rho^{\mathbb{Q}}\right)^j\right) \Sigma\Sigma' \left(\sum_{j=0}^{n-1} \left(\rho^{\mathbb{Q}}\right)^j\right)' \delta_1 \\ b_n' &= \delta_1' \left(\rho^{\mathbb{Q}}\right)^n \\ (\sigma_n^{\mathbb{Q}})^2 &\equiv \mathbb{V}_t^{\mathbb{Q}}(s_{t+n}) = \sum_{j=0}^{n-1} \delta_1' (\rho^{\mathbb{Q}})^j \Sigma\Sigma' (\rho^{\mathbb{Q}\prime})^j \delta_1 \end{aligned}$$

Comparison to GATSM

SRTSM

$$\begin{aligned} s_t &= \delta_0 + \delta_1' X_t \\ r_t &= max(s_t, \underline{r}) \end{aligned}$$

Forward rate

$$f_{nt} = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right)$$

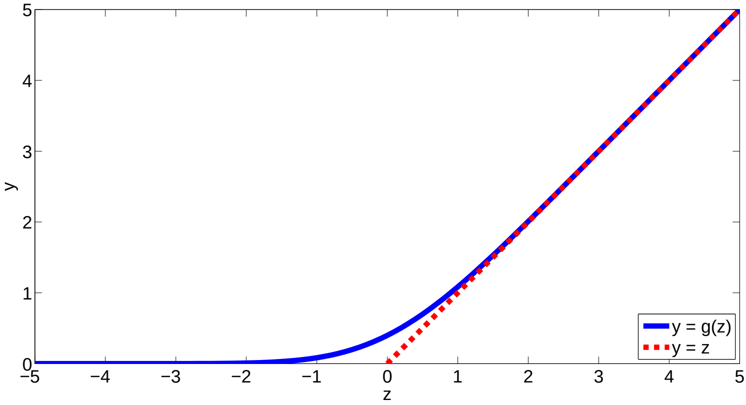
GATSM

$$r_t = \delta_0 + \delta_1' X_t$$

Forward rate

$$f_{nt} = a_n + b_n' X_t$$

Property of $g(.)$



$$f_{nt}^{SR} \begin{cases} \approx \underline{r}, \text{ at the ZLB} \\ \approx a_n + b_n' X_t = f_{nt}^{G}, \text{ when interest rates are high} \end{cases}$$

Extended Kalman filter

State equation

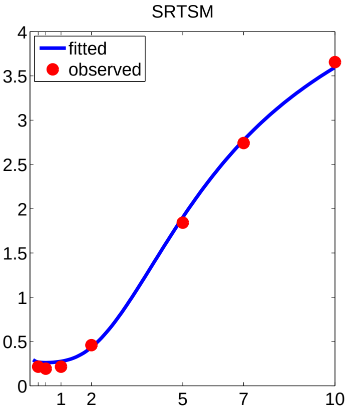
$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I)$$

Observation equation

$$f_{nt}^o = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right) + \eta_{nt}, \eta_{nt} \sim N(0, \omega)$$

Model fit

Figure: Average forward curve in 2012



Log likelihood values

- SRTSM: 850; GATSM: 750

data and normalization

Approximation error

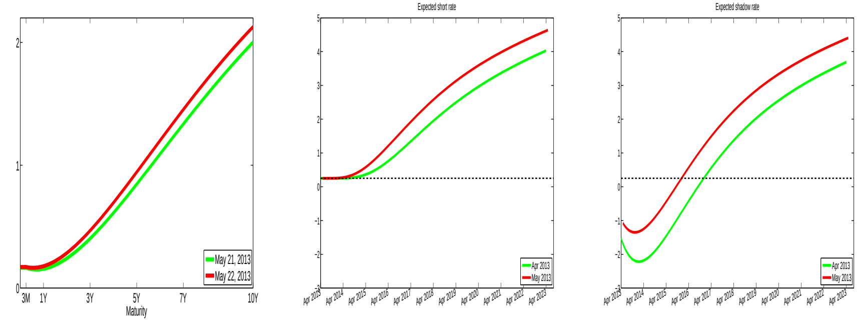
Average absolute approximation error between 1990M1 and 2013M1 (in basis points)

	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error	0.01	0.02	0.04	0.13	0.69	1.14	2.29
forward rate level	346	357	384	435	551	600	636
yield error	0.00	0.01	0.01	0.04	0.24	0.42	0.78

Outline

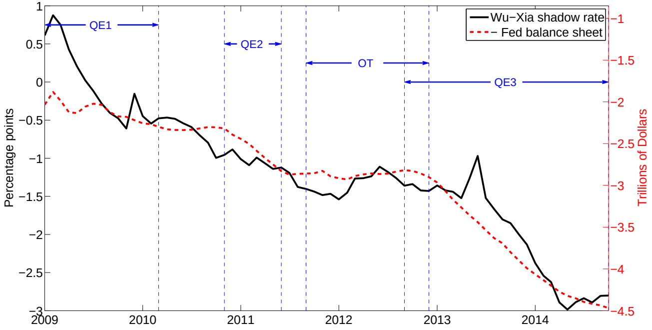
1. Wu-Xia shadow rate
2. Motivating shadow rate New Keynesian model
   - SR as summary of unconventional monetary policy
   - Linear model
3. Microfoundations
4. Economic implications

Evidence 1: taper tantrum



- May 22, 2013: Bernanke told Congress Fed may decrease the size of QE
- shift in shadow rate summarizes this effect

Evidence 2: shadow rate and Fed's balance sheet



Correlation

- QE1 - QE3: -0.94

Evidence 3: structural break test in FAVAR

Replace the fed funds rate with s_t in Bernanke, Boivin, and Eliasz (2005)

$$\begin{aligned} x_t^m &= \mu^x + \rho^{xx} X_{t-1}^m + u_t^x \\ &+ \mathbb{1}_{(t<\text{December 2007})}\rho_1^{xs} S_{t-1} + \mathbb{1}_{(\text{December 2007}\leq t\leq \text{June 2009})}\rho_2^{xs} S_{t-1} + \mathbb{1}_{(t>\text{June 2009})}\rho_3^{xs} S_{t-1} \end{aligned}$$

- monthly VAR(13)
- x_t^m: 3 underlying macro factors

Null hypothesis

$$H_0 : \rho_1^{xs} = \rho_3^{xs}$$

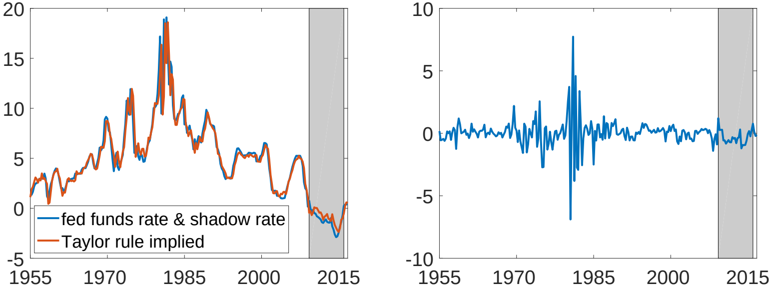
Likelihood ratio test: $\chi^2(39)$

- $p = 0.29$ for s_t
- $p = 0.0007$ for EFFR

model details robustness

Evidence 4: shadow rate Taylor rule

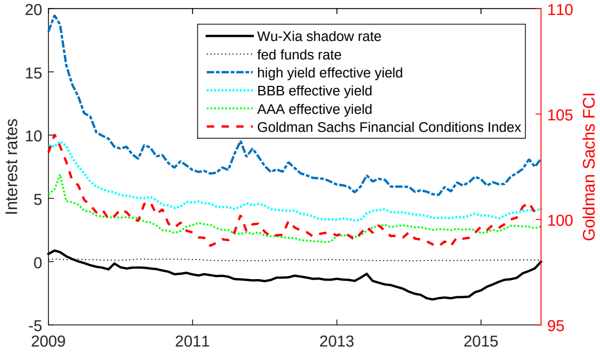
$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 (y_t - y_t^n) + \beta_3 \pi_t + \varepsilon_t$$



Test for structural break

- F statistic $= 2$
- Critical value: 2.37
- No structural break

Evidence 5: shadow rate and private rates



- private rates are the relevant rates for agents and the economy
- correlations: 0.8
- $r_t^B = s_t + rp$

Summary

Shadow rate summarizes unconventional monetary policy

- Taper tantrum
- Fed's balance sheet

There is no structural break in

- relationship between macro variables and the shadow rate
- shadow rate Taylor rule

Private rates

- are the relevant interest rates for economic agents
- respond to unconventional monetary policy
- reflect the overall effect of UMP on the economy
- the shadow rate is a sensible summary

Shadow rate New Keynesian model

Definition

The shadow rate New Keynesian model consists of the shadow rate IS curve

$$y_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1},$$

New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n),$$

and shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1 - \phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right].$$

QE lending facilities

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   - Microfoundations I: QE
   - Microfoundations II: lending facilities
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Microfoundation for SRNKM I: QE

The risk premium channel

- government purchases outstanding loans
- decrease interest rates through reducing risk premium
  - Gagnon et al. (2011) and Hamilton and Wu (2012)
- the same mechanism works for government bonds or corporate bonds

Households' problem

Households' utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t{}^{1+\eta}}{1+\eta} \right)$$

budget constraint

$$C_t + \frac{B_t^H}{P_t} = \frac{R_{t-1}^B B_{t-1}^H}{P_t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta R_t^B \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right]$$

The linear Euler equation

$$y_t = -\frac{1}{\sigma} \left(r_t^B - \mathbb{E}_t \pi_{t+1} - r^B \right) + \mathbb{E}_t y_{t+1},$$

Bond return and policy rate

Define

$$rp_t \equiv r_t^B - r_t$$

- interpreted as convenience yield by Krishnamurthy and Vissing-Jorgensen (2012)
- Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), and Hamilton and Wu (2012) suggest

$$rp_t'(b_t^G) < 0 \Rightarrow rp_t(b_t^G) = rp - \varsigma(b_t^G - b^G)$$

During normal times

- $b_t^G = b^G$
- $rp_t(b^G) = rp \Rightarrow r_t^B = r_t + rp$

At the ZLB

- QE $\rightarrow b_t^G \uparrow \rightarrow rp_t \downarrow \rightarrow r_t^B \downarrow$

Shadow rate equivalence for QE

Euler equation

$$y_t = -\frac{1}{\sigma}\left(r_t^B - \mathbb{E}_t\pi_{t+1} - r^B\right) + \mathbb{E}_t y_{t+1},$$

- During normal times, $b_t^G = b^G, r_t = s_t$

$$r_t^B = r_t + rp = s_t + rp$$

- At the ZLB, $r_t = 0$

$$r_t^B = rp_t = rp - \varsigma(b_t^G - b^G) = s_t + rp$$

if $s_t = -\varsigma(b_t^G - b^G)$

Shadow rate equivalence for QE

Proposition

The shadow rate New Keynesian model represented by the shadow rate IS curve

$$y_t = -\frac{1}{\sigma}\left(s_t - \mathbb{E}_t \pi_{t+1} - s\right) + \mathbb{E}_t y_{t+1},$$

New Keynesian Phillips Curve, and shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1-\phi_s)\left[\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s\right]$$

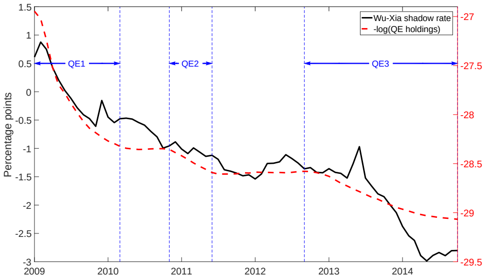
nests both conventional Taylor interest rate rule and QE operation that changes risk premium if

$$\begin{cases} r_t = s_t,\ b_t^G = b^G & \text{for } s_t \geq 0 \\ r_t = 0,\ b_t^G = b^G - \frac{s_t}{\varsigma} & \text{for } s_t < 0. \end{cases}$$

Shadow rate NK model

Quantifying assumption in proposition

$$s_t = -\varsigma(b_t^G - b^G)$$



- linear assumption: correlation $= 0.92$
- $\varsigma = 1.83$
  - Fed increases its bond holdings by 1%, the shadow rate decreases by 0.0183%
  - QE1: 490 billion to 2 trillion $\Rightarrow$ 2.5% decrease in the shadow rate
  - QE3: 2.6 trillion to 4.2 trillion $\Rightarrow$ 0.9% decrease in the shadow rate

Outline

1. Wu-Xia shadow rate
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3. Microfoundations
   - Microfoundations I: QE
   - Microfoundations II: lending facilities
4. Economic implications

Microfoundation for SRNKM II: lending facilities

Lending facilities

- extend loans to the private sector ⇒ change the loan-to-value ratio
- Example: Term Asset-Backed Securities Loan Facility in the US

Combine this with a tax policy

- tax (subsidy) on the interest rate income (payment)

Model features

Entrepreneurs

- produce intermediate goods with labor and capital
- maximize utility
- discount factor $\gamma < \beta$
- borrow from households with a loan-to-value ratio M
- accumulate capital
- use capital as collateral

Government policy at the ZLB

- lending facilities
  - lend directly to entrepreneurs
  - change the loan-to-value ratio from M to M_t
- tax (subsidy) on the interest rate income (payment)

Entrepreneurs' problem

Utility function

$$\max \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t \log C_t^E$$

production function

$$Y_t^E = A K_{t-1}^{\alpha} (L_t)^{1-\alpha}$$

capital accumulation

$$K_t = I_t + (1-\delta) K_{t-1}$$

budget constraint

$$\frac{Y_t^E}{X_t} + \tilde{B}_t = \frac{R_{t-1}^B \tilde{B}_{t-1}}{\mathcal{T}_{t-1} \Pi_t} + W_t L_t + I_t + C_t^E$$

borrowing constraint

$$\tilde{B}_t \leq M_t \mathbb{E}_t \left(K_t \Pi_{t+1} / R_t^B \right)$$

Entrepreneurs' FOCs

Labor demand

$$W_t = \frac{(1-\alpha)AK_{t-1}^{\alpha}L_t^{-\alpha}}{X_t}$$

Euler equation

$$\frac{1}{C_t^E}\left(1 - \frac{M_t \mathbb{E}_t \Pi_{t+1}}{R_t^B}\right) = \gamma \mathbb{E}_t \left[\frac{1}{C_{t+1}^E}\left(\frac{\alpha Y_{t+1}^E}{X_{t+1}K_t} - \frac{M_t}{\mathcal{T}_t} + 1 - \delta\right)\right]$$

Households' problem

Households' utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta} \right)$$

budget constraint

$$C_t + \tilde{B}_t^H = \frac{R_{t-1}^B \tilde{B}_{t-1}^H}{\mathcal{T}_{t-1} \Pi_t} + W_t L_t + T_t$$

Euler equation

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left(R_t^B \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1} \mathcal{T}_t} \right)$$

labor supply

$$W_t = C_t^{\sigma} L_t^{\eta}$$

Sources of funding

Entrepreneurs' borrowing constraint

$$\tilde{B}_t \le M_t \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

Households lend

$$\tilde{B}_t^H \le M \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

- During normal times $\tilde{B}_t = \tilde{B}_t^H$, and $M_t = M$
- At the ZLB $M_t > M$

Government lends the rest

$$\tilde{B}_t^G = (M_t - M) \mathbb{E}_t \left(\frac{K_t \Pi_{t+1}}{R_t^B} \right)$$

Conventional and unconventional policy

Suppose $R_t^B = R_t RP$

Conventional and unconventional policy tools appear in the model in pairs:

- $R_t/\mathcal{T}_t$ – HH Euler equation, HH&E budget constraints
- R_t/M_t – E borrowing constraint, E Euler equation
- $M_t/\mathcal{T}_t$ – E Euler equation

model

Decreasing R_t is equivalent to increasing $\mathcal{T}_t$ and M_t.

Shadow rate equivalence for lending facilities

Proposition

If

$$\begin{cases} R_t = S_t, \mathcal{T}_t = 1, M_t = M & \text{for } S_t \geq 1 \\ \mathcal{T}_t = M_t/M = 1/S_t & \text{for } S_t < 1, \end{cases}$$

then $R_t/\mathcal{T}_t = S_t$, $R_t/M_t = S_t/M$, $M_t/\mathcal{T}_t = M \ \forall S_t$.

- S_t summarizes both conventional and unconventional policies
- Equivalence in the non-linear model

Shadow rate equivalence for lending facilities

Proposition

The shadow rate New Keynesian model represented by the Euler equation

$$c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t\pi_{t+1} - s) + \mathbb{E}_t c_{t+1}$$

the shadow rate Taylor rule, Phillips curve, ..., nests both conventional Taylor interest rate rule and lending facility – tax policy if

$$\begin{cases} r_t = s_t, \tau_t = 0, m_t = m & \text{for } s_t \geq 0 \\ \tau_t = m_t - m = -s_t & \text{for } s_t < 0. \end{cases}$$

Shadow rate NK model Detailed model

Outline

1. Wu-Xia shadow rate
2. Motivating shadow rate New Keynesian model
3. Microfoundations
4. Economic implications
   - Economic implication I: negative supply shock
   - Economic implication II: government spending multiplier

Economic implication I: negative supply shock

Technology shock

$$a_t \downarrow \;=\; \rho_a a_{t-1} + e_{a,t} \downarrow$$

Phillips Curve

$$\pi_t \uparrow \;=\; \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y) - \frac{\kappa(1+\eta)}{\sigma+\eta} a_t \downarrow$$

Economic implication I: negative supply shock

Standard model

Monetary policy

$$r_t \quad = \quad \max\{\phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right], 0\}$$

Real interest rate

$$rr_t = r_t - \mathbb{E}_t[\pi_{t+1}]$$

IS curve

$$y_t \quad = \quad -\frac{1}{\sigma}(rr_t - r) + \mathbb{E}_t y_{t+1}$$

normal times:	$\pi\uparrow$	$\rightarrow$	$r\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$y\downarrow$	
ZLB without UMP:	$\pi\uparrow$	$\rightarrow$	$r=0$	$\rightarrow$	$rr\downarrow$	$\rightarrow$	$y\uparrow$	Counterfactual

Economic implication I: negative supply shock

Shadow rate model

Shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right]$$

Real interest rate

$$rr_t = s_t - \mathbb{E}_t[\pi_{t+1}]$$

Shadow rate IS curve

$$y_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t\pi_{t+1} - r) + \mathbb{E}_t y_{t+1}$$

normal times:	$\pi\uparrow$	$\rightarrow$	$r\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$y\downarrow$	
ZLB without UMP:	$\pi\uparrow$	$\rightarrow$	$r=0$	$\rightarrow$	$rr\downarrow$	$\rightarrow$	$y\uparrow$	Counterfactual
ZLB with UMP:	$\pi\uparrow$	$\rightarrow$	$s\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$y\downarrow$	Data consistent

Extended model

Iacoviello (2005)

- Patient households
  - consume, work, hold house, lend
- Impatient households
  - consume, work, hold house
  - borrow with collateral
- Entrepreneurs
  - consume, invest, hold houses
  - borrow with collateral
  - produce intermediate goods using housing, capital, and labor
- Retailers
  - monopolistically competitive
  - Calvo sticky
- Government
  - Taylor rule

Calibration

Difference from Iacoviello (2005)

- Unconventional policy
  - QE: time-varying risk premium
  - lending facilities: time-varying loan-to-value ratio
  - time-varying tax
- government spending
- preference shock: create ZLB

Models and methodology

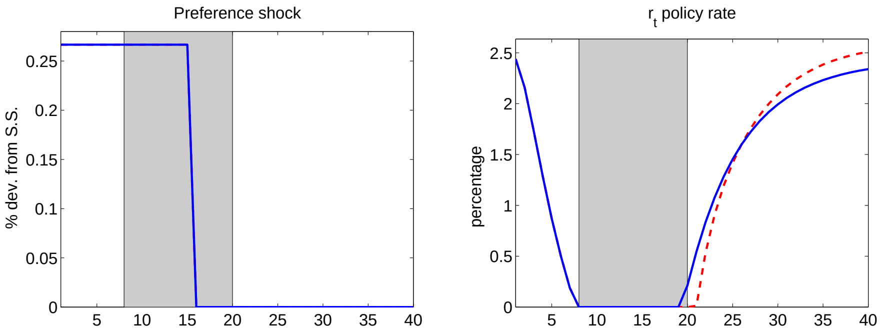
Notation

- Standard model: without unconventional policy: $r_t = 0$
- Shadow rate model: with unconventional policy: $s_t < 0$

Methodology for shadow rate model

- solve linear model with shadow rate
- then use propositions mapping into various UMP

Preference shock and the ZLB



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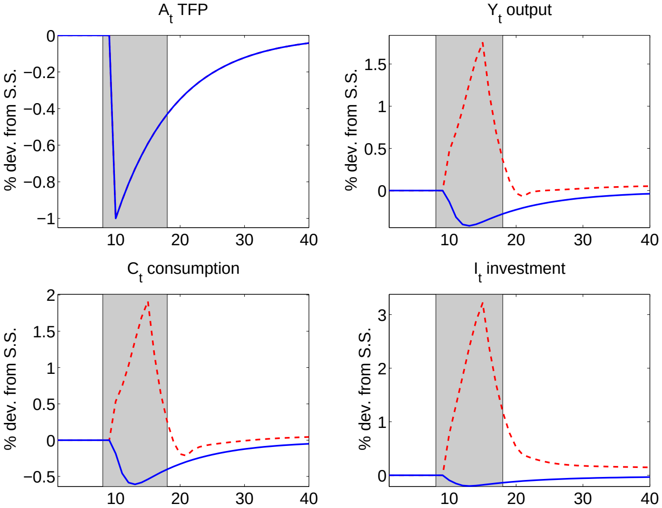
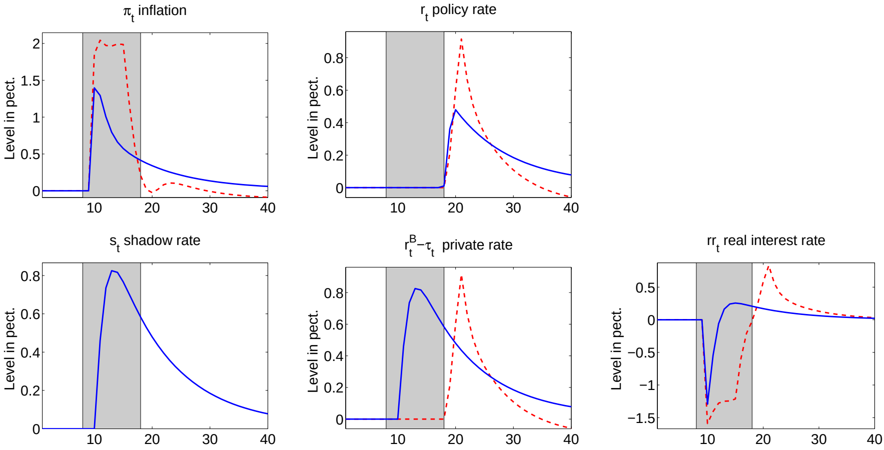
Economic implication I: negative supply shock



Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Economic implication I: negative supply shock



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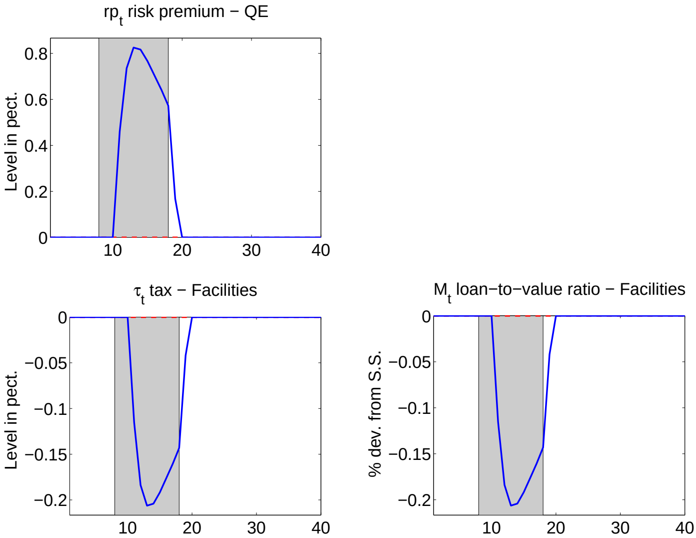
Economic implication I: negative supply shock



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Economic implication II: government spending multiplier

Government spending shock

$$g_t \uparrow \;=\; (1-\rho_g)g + \rho_g g_{t-1} + e_{g,t} \uparrow$$

Market-clearing condition

$$y_t \uparrow \;=\; c_y c_t + g_y g_t \uparrow$$

Phillips Curve

$$\pi_t \uparrow \;=\; \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{\delta+\eta}(\sigma(c_t - c) + \eta(y_t \uparrow - y))$$

Economic implication II: government spending multiplier

Standard model

Monetary policy

$$r_t = \max\{\phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right], 0\}$$

Real interest rate

$$rr_t = r_t - \mathbb{E}_t[\pi_{t+1}]$$

IS curve

$$c_t = -\frac{1}{\sigma}(rr_t - r) + \mathbb{E}_t c_{t+1}$$

normal times:	$\pi\uparrow y\uparrow$	$\rightarrow$	$r\uparrow\uparrow$	$\rightarrow$	$rr\uparrow$	$\rightarrow$	$c\downarrow$	$\rightarrow$	$\Delta y < \Delta g$
ZLB without UMP:	$\pi\uparrow y\uparrow$	$\rightarrow$	$r = 0$	$\rightarrow$	$rr\downarrow$	$\rightarrow$	$c\uparrow$	$\rightarrow$	$\Delta y > \Delta g$

Economic implication II: government spending multiplier

Shadow rate model

Shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1-\phi_s)\left[\phi_y(y_t - y_t^n) + \phi_\pi \pi_t + s\right]$$

Real interest rate

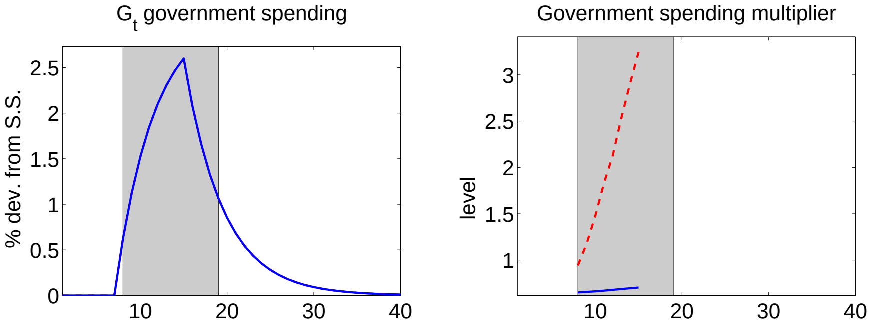
$$rr_t = s_t - \mathbb{E}_t[\pi_{t+1}]$$

Shadow rate IS curve

$$c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t c_{t+1}$$

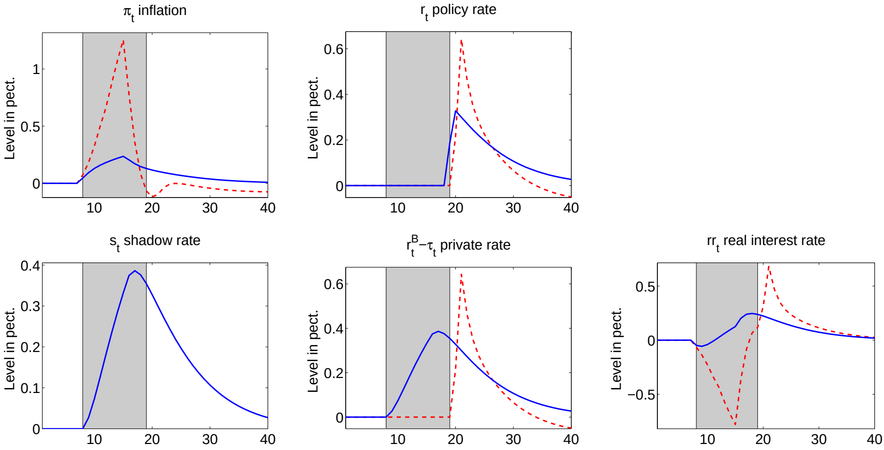
normal times:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r \uparrow\uparrow$	$\rightarrow$	$rr \uparrow$	$\rightarrow$	$c \downarrow$	$\rightarrow$	$\Delta y < \Delta g$
ZLB without UMP:	$\pi \uparrow y \uparrow$	$\rightarrow$	$r = 0$	$\rightarrow$	$rr \downarrow$	$\rightarrow$	$c \uparrow$	$\rightarrow$	$\Delta y > \Delta g$
ZLB with UMP:	$\pi \uparrow y \uparrow$	$\rightarrow$	$s \uparrow\uparrow$	$\rightarrow$	$rr \uparrow$	$\rightarrow$	$c \downarrow$	$\rightarrow$	$\Delta y < \Delta g$

Economic implication II: government spending multiplier



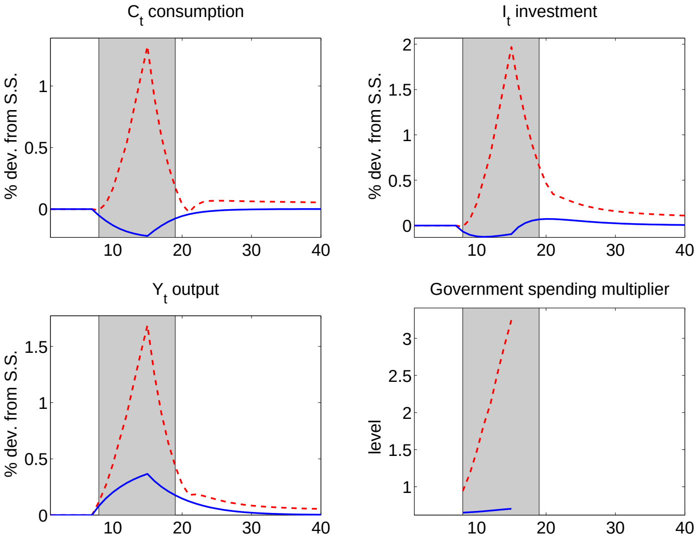
Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Economic implication II: government spending multiplier



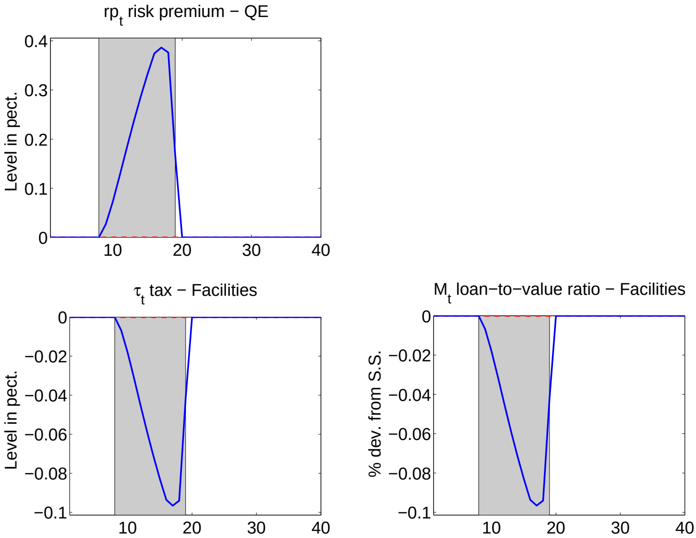
Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Economic implication II: government spending multiplier



Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Economic implication II: government spending multiplier



Note: blue: shadow rate NK model with UMP; red: standard NK model without UMP

Computational advantages

The ZLB imposes one of the biggest challenges for solving and estimating these models

Existing methods

- have undesirable economic implications
- computationally demanding

Shadow rate model

- no structural break
- can rely on traditional methods
- unique solution

Conclusion

Wu and Xia (JMCB 2016)

- provide an analytical approximation for SRTSM
- Wu-Xia shadow rate summarizes unconventional monetary policy

Wu and Zhang (2016)

- propose a New Keynesian model with the shadow rate
- map unconventional policy tools into the shadow rate framework
- make counterfactual results in standard NK model disappear
- computationally tractable

Related work: Wu and Xia (2017)

Time-Varying Lower Bound of Interest Rates in Europe

- A new shadow rate term structure model with $r_t = max(s_t, \underline{r}_t)$
  - $\underline{r}_t = \underline{r}_t^d + sp_t$
    - Discrete policy lower bound $\underline{r}_t^d$
    - Non-constant spread sp_t
  - Agents are forward looking
  - Derive bond prices
- Asymmetry
  - 10 bp drop in the ELB decreases the 10-year rate by 6.5-8.5 bp
  - 10 bp initial rise in the ELB increases the 10-year rate by 9-14 bp

Data and model specification

Gürkaynak, Sack, and Wright (2007) dataset

- monthly, January 1990 - December 2013
- maturities: 3 and 6 months, 1, 2, 5, 7 and 10 years

Normalization with repeated eigenvalues

$$\rho^{\mathbb{Q}} = \begin{bmatrix} \rho_1^{\mathbb{Q}} & 0 & 0 \\ 0 & \rho_2^{\mathbb{Q}} & 1 \\ 0 & 0 & \rho_2^{\mathbb{Q}} \end{bmatrix}.$$

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FAVAR

Replace the fed funds rate with s_t in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

- Y_t^m: 97 economic variables from 1960 to 2013
- x_t^m: 3 underlying macro factors

Factor dynamics:

$$\begin{bmatrix} x_t^m \\ s_t \end{bmatrix} = \begin{bmatrix} \mu^x \\ \mu^s \end{bmatrix} + \begin{bmatrix} \rho^{xx} & \rho^{xs} \\ \rho^{sx} & \rho^{ss} \end{bmatrix} \begin{bmatrix} X_{t-1}^m \\ S_{t-1} \end{bmatrix} + \Sigma^m \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^{\mathrm{MP}} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^{\mathrm{MP}} \end{bmatrix} \sim N(0, I)$$

- monthly VAR(13)
- Σ^m: Cholesky decomposition

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Rubustness

		p-value for $\rho_1^{xs} = \rho_3^{xs}$	p-value for $\rho_1^{sx} = \rho_3^{sx}$
	Baseline	0.29	1.00
A1	estimate $\underline{r}$	0.18	1.00
A2	2-factor SRTSM	0.13	0.97
A3	Fama-Bliss	0.38	1.00
A4	5-factor FAVAR	0.70	1.00
A5	6-lag FAVAR	0.09	0.98
	7-lag FAVAR	0.19	0.97
	12-lag FAVAR	0.22	1.00

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Lending facilities

$$
\begin{aligned}
c_t &= -\frac{1}{\sigma}(r_t^B - \tau_t - \mathbb{E}_t \pi_{t+1} - r - rp) + \mathbb{E}_t c_{t+1} \\
\Rightarrow c_t &= -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - r) + \mathbb{E}_t c_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
C^E c_t^E &= \alpha \frac{Y}{X}(y_t - x_t) + Bb_t - R^B B(r_{t-1}^B + b_{t-1} - \tau_{t-1} - \pi_{t-1}) - Ii_t + \Lambda_1 \\
\Rightarrow C^E c_t^E &= \alpha \frac{Y}{X}(y_t - x_t) + Bb_t - R^B B(s_{t-1} + rp + b_{t-1} - \pi_{t-1}) - Ii_t + \Lambda_1
\end{aligned}
$$

$$
\begin{aligned}
b_t &= \mathbb{E}_t(k_t + \pi_{t+1} + m_t - r_t^B) \\
\Rightarrow b_t &= \mathbb{E}_t(k_t + \pi_{t+1} + m - s_t - rp)
\end{aligned}
$$

Lending facilities

$$
\begin{aligned}
0 &= \left(1-\frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \\
&+ \frac{M}{R^B}\mathbb{E}_t(\pi_{t+1} - r_t^B + m_t) + \gamma M(\tau_t - m_t) + \Lambda_2 \\
\Rightarrow 0 &= \left(1-\frac{M}{R^B}\right)(c_t^E - \mathbb{E}_t c_{t+1}^E) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_t(y_{t+1} - x_{t+1} - k_t) \\
&+ \frac{M}{R^B}\mathbb{E}_t(\pi_{t+1} - s_t - rp + m) - \gamma Mm + \Lambda_2
\end{aligned}
$$

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Calibration

para	description	source	value
β	discount factor of patient households	Iacoviello (2005)	0.99
β^I	discount factor of impatient households	Iacoviello (2005)	0.95
γ	discount factor of entrepreneurs	Iacoviello (2005)	0.98
j	steady-state weight on housing services	Iacoviello (2005)	0.1
η	labor supply aversion	Iacoviello (2005)	0.01
μ	capital share in production	Iacoviello (2005)	0.3
ν	housing share in production	Iacoviello (2005)	0.03
δ	capital depreciation rate	Iacoviello (2005)	0.03
X	steady state gross markup	Iacoviello (2005)	1.05
θ	probability that cannot re-optimize	Iacoviello (2005)	0.75
α	patient households' wage share	Iacoviello (2005)	0.64
M^E	loan-to-value ratio for entrepreneurs	Iacoviello (2005)	0.89
M^I	loan-to-value ratio for impatient households	Iacoviello (2005)	0.55
r_R	interest rate persistence	Iacoviello (2005)	0.73
r_Y	interest rate response to output	Iacoviello (2005)	0.27
r_Π	interest rate response to inflation	Iacoviello (2005)	0.13
$\frac{G}{Y}$	steady-state government-spending-to-output ratio	Fernandez-Villaverde et al. (2015)	0.20
ρ_a	autocorrelation of technology shock	Fernandez-Villaverde et al. (2015)	0.90
ρ_g	autocorrelation of government-spending shock	Fernandez-Villaverde et al. (2015)	0.80
ρ_β	autocorrelation of discount rate shock	Fernandez-Villaverde et al. (2015)	0.80
σ_a	standard deviation of technology shock	Fernandez-Villaverde et al. (2015)	0.0025
σ_g	standard deviation of government-spending shock	Fernandez-Villaverde et al. (2015)	0.0025
σ_β	standard deviation of discount rate shock	Fernandez-Villaverde et al. (2015)	0.0025
ξ_p	price indexation	Smets and Wouters (2007)	0.24
Π	steady-state inflation	2% annual inflation	1.005
B^G	steady-state government bond holdings	no gov. intervention in private bond market	0
$\mathcal{T}$	steady-state tax/subsidy on interest rate income/payment	no tax in normal times	1
rp	steady-state risk premium	3.6% risk premium annually	1.009

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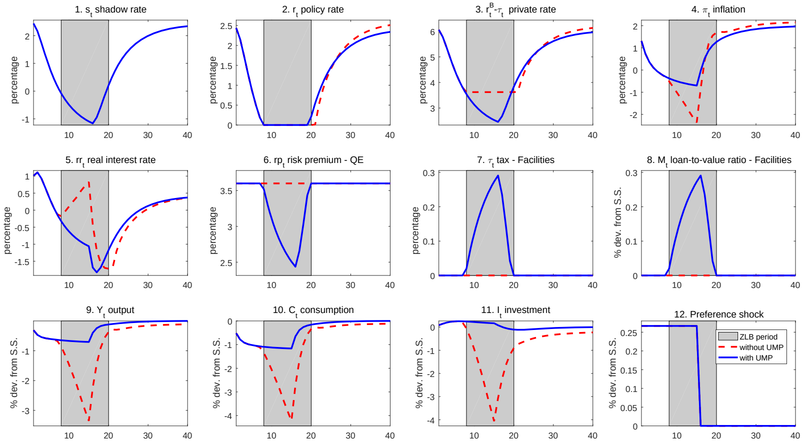
Preference shock and the ZLB



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