The Stock Market in an Inflation-targeting Economy

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Find a (*standard*) economic model that accounts for Boudoukh and Richardson (1993)

TABLE 1—STOCK RETURNS AND CONTEMPORANEOUS INFLATION

Period	α_1^* (SE)	β_1^* (SE)	α_5^* (SE)	β_5^* (SE)	$z_{\beta_5^* > \beta_1^*}$ (P value)	MSE_1 R-square	MSE_5 R-square
1802–1990	0.072	0.070[a]	0.333	0.524[a]	4.198	0.027	0.108
	(0.013)	(0.235)	(0.026)	(0.174)	(1.000)	0.001	0.083
1870–1990	0.080	0.131[a]	0.366	0.462[a]	1.045	0.032	0.125
	(0.021)	(0.048)	(0.055)	(0.257)	(0.852)	0.001	0.054
1914–1990	0.094	0.087[a]	0.408	0.432[a]	6.642	0.039	0.154
	(0.014)	(0.221)	(0.035)	(0.050)	(1.000)	0.001	0.041

Notes: The table provides estimates of the regressions of one- and five-year stock returns on one- and five-year inflation rates, respectively. The estimation is conducted using annual continuously compounded data over the sample period 1802–1990 and the subperiods 1870–1990 and 1914–1990. The regressions are from the following systems:

$$R_{t+1} = \alpha_1^* + \beta_1^* \pi_{t+1} + \varepsilon_t^*(1)$$

$$\sum_{i=1}^{5} R_{t+i} = \alpha_5^* + \beta_5^* \sum_{i=1}^{5} \pi_{t+i} + \varepsilon_t^*(5)$$

where R_t denotes the stock return, π_t denotes the inflation rate, and ε_t is the disturbance term. MSE_i is the mean squared error for the i-period inflation equation; $z_{\beta_5^* > \beta_1^*}$ is the test statistic for the hypothesis $\beta_5^* = \beta_1^*$ versus $\beta_5^* > \beta_1^*$.

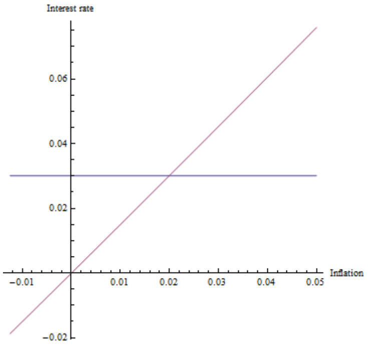
[a]Statistically significant at the 10-percent level for a test of $\beta_j = 1$.

Outline

- Motivation (done)
- Inflation targeting: the Taylor rule
- Brief theoretical background
- Output exogenous: The "finance" side; the IS or "aggregate-demand" schedules
- The "aggregate-supply" side:
    - Output endogenous, prices flexible
    - Output endogenous, prices sticky: the Phillips curve
- Money demand
- The 'Fed model': a comment on stock and bond yields

The Taylor rule: “expectations” form

$$1 + i_t = (1 + \bar{\imath}) \times \left(\frac{\frac{\frac{1}{2} P_{t+1,u} + \frac{1}{2} P_{t+1,d}}{P_t}}{1 + \bar{\pi}} \right)^{\phi}$$



- No monetary shock.
- Zero lower bound comes with money holding.

Theoretical background

Monetary Economics rarely includes stock market considerations. However:

- Sims (1994), Cochrane (2005a, 2005b, 2011) and Niepelt (2004) introduce and study the *Fiscal theory of the price level*: the price level is forward looking
  - Woodford (2003) introduces cashless economy
- Sargent and Wallace (1975) and Leeper (1991) emphasize the need to include distinction between Ricardian and non Ricardian fiscal policy
  - Nakajima and Polemarchaskis (2005) emphasize a basic indeterminacy in case of Ricardian fiscal policy

The "Finance" side

Assume isoelastic utility (power $\gamma < 1$), and t*ake the government surplus s to be exogenous (unrelated to debt (non Ricardian fiscal policy))*

Flow budget constraints of private sector at time $t+1$

$$P_{t+1,j} \times c_{t+1,j} + F_{1,t+1,j} + s_{t+1,j} \times P_{t+1,j} = \theta_{1,t} + P_{t+1,j} \times y_{t+1,j};$$

$$F_{1,T,j} = 0; j = u, d$$

Flow budget constraints of government at time $t+1$

$$F_{2,t+1,j} = \theta_{2,t} + s_{t+1,j} \times P_{t+1,j}; F_{2,T,j} = 0; j = u, d$$

Portfolio-choice or Euler or Fisher condition at time t

$$\frac{1}{1+i_t}\frac{1}{P_t} = \rho \frac{\frac{1}{2}\left(c_{t+1,u}\right)^{\gamma-1}\frac{1}{P_{t+1,u}} + \frac{1}{2}\left(c_{t+1,d}\right)^{\gamma-1}\frac{1}{P_{t+1,d}}}{\left(c_t\right)^{\gamma-1}}$$

Market clearing at time t

$$\theta_{1,t} + \theta_{2,t} = 0$$

Solution of the "Finance" side: "aggregate demand"

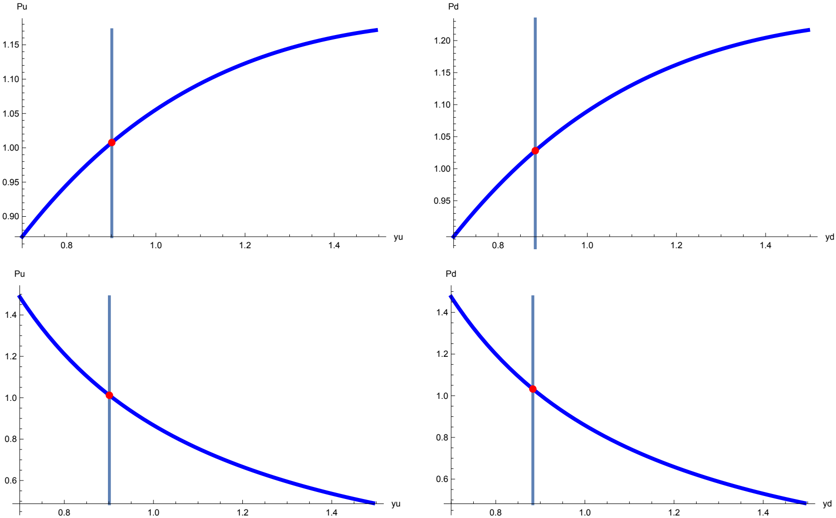
- System made of the above plus Taylor rule (7 equations).
- Solve for $c_{t+1,j}$, $P_{t+1,j}$, i_t, $\theta_{1,t}$ and $\theta_{2,t}$ given $y_{t+1,j}$, c_t and P_t. [Obviously: $c_{t+1,j} = y_{t+1,j}$.]
  - Homogeneity of degree 1 with respect to P_t. So far, value of $c_t = y_t$ (exogenous) at each node.
  - Solve backward for the present value of government debt. Explicit solution.
- The time-0 outstanding nominal debt determines the initial price level

$$f_{2,0} \times P_0 = \theta_{2,-1} + s_0 \times P_0$$

where $\theta_{2,-1}$ is the entering, nominal amount of government debt, s_0 the real surplus and $f_{2,0}$ the real amount of government debt that the government "decides" to carry forward.

Solution of the "Finance" side

Aggregate-demand or "IS" curves: ($\phi > 1$ (top) vs. $\phi < 1$ (bottom))



Solution of the "Finance" side: special case

Assume *IID output growth and proportional taxes*. Define:

$$k \triangleq \rho \times \left[\frac{1}{2} (1+u)^{\gamma} + \frac{1}{2} (1+d)^{\gamma} \right] ; k < 1 \qquad (1)$$

Proposition

The realized rates of inflation are:

$$\frac{P_{t+1,u}}{P_t} = \frac{k}{1+u} \times (1+i_t)$$
$$\frac{P_{t+1,d}}{P_t} = \frac{k}{1+d} \times (1+i_t)$$

independent of the tax rate. Inflation is lower in the u state than in the d state.

Solution of the "Finance" side: special case

Provisionally define the stock market as the present value of output

Proposition

The real gross rates of return on the stock market are:

$$\frac{1+u}{k} \text{ in a } u \text{ state;} \frac{1+d}{k} \text{ in a } d \text{ state}$$

Proposition

Under the IID growth assumption, the realized nominal rate of return on stocks is equal to the nominal interest rate, which is constant.

- the output shock acts both on the stock market and on tax collection.
- The real rate of return on equity being low when inflation is high, it is negatively correlated with the rate of inflation.
- The two offset each other while the nominal rate is constant. That is the source of the nominal character of stocks.

Aggregate supply

We now endogenize output (income y), with exogenous productivity shocks and redefine the stock market as being the claim on profits.

- Households:
  - There exists a continuum $\iota \in [0,1]$ of differentiated varieties of the good. The argument c_t of the households' utility is a composite defined as

$$c_t \triangleq \left(\int_0^1 c_{\iota,t}^{\frac{\sigma-1}{\sigma}} \, d\iota \right)^{\frac{\sigma}{\sigma-1}}$$

    where $\sigma > 1$ is the elasticity of substitution between the separate varieties.
  - They have isoeleastic utility of consumption. They also have disutility of labor (power $\eta > 1$) and supply labor with some elasticity:

$$\frac{l_t^{\eta-1}}{c_t^{\gamma-1}} = \frac{W_t}{P_t}$$

Aggregate supply

- Firms producing variety ι have local monopoly and produce with a production function:

$$y_{\iota,t} = z_t \times l_{\iota,t}$$

- Hire labor. Cost function:

$$\varphi_t = \frac{W_t}{z_t \times P_t} \begin{matrix} > \\ < \end{matrix} 1$$

Aggregate supply

- Flexible-price output is a power function of *current* productivity

$$y_t = \left(\frac{\sigma - 1}{\sigma} z_t^{\eta} \right)^{\frac{1}{\eta - \gamma}}$$

Proposition

Under the assumptions of the IID growth special case and flexible prices, Propositions 1 to 3 remain true with $1 + u$ *replaced by* $(1 + u)^{\frac{\eta}{\eta - \gamma}}$, $1 + d$ *replaced by* $(1 + d)^{\frac{\eta}{\eta - \gamma}}$ *and* k *redefined accordingly (*$k < 1$ *being now assumed anew). In particular, the gross nominal rate of return on stocks remains equal to the gross nominal interest rate, which is constant.*

“New Keynesian” aggregate supply

- If prices were flexible, there would be no relationship between nominal stock return and inflation, as above.
- “Calvo” price setting where a proportion ω of firms are forced to keep their price unchanged:

$$P_t \triangleq \left[(1-\omega)\times\left(P_t^*\right)^{1-\sigma}+\omega\times\left(P_{t-1}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

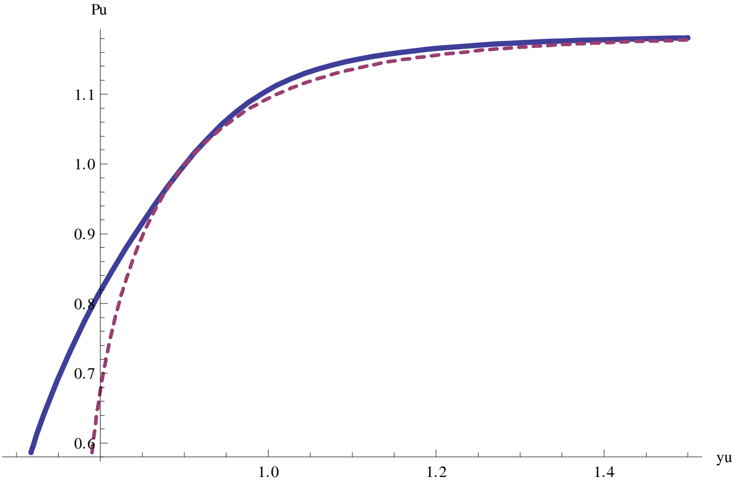
- They choose P^* to maximize:

$$\sup_{P_{\imath,t}}\sum_{i=0}^{T-t}\mathbb{E}_t\left[(\rho\omega)^i\frac{\left(c_{t+i}\right)^{\gamma-1}}{\left(c_t\right)^{\gamma-1}}\left(\frac{P_{\imath,t}}{P_{t+i}}-\varphi_{t+i}\right)\left(\frac{P_{\imath,t}}{P_{t+i}}\right)^{-\sigma}y_{t+i}\right]$$

- Solution for prices $P(y)$ (i.e.: $P_{t+1,u}\left(y_{t+1,u}\right)$ and $P_{t+1,d}\left(y_{t+1,d}\right)$): either explicit but approximate or exact but numerical.

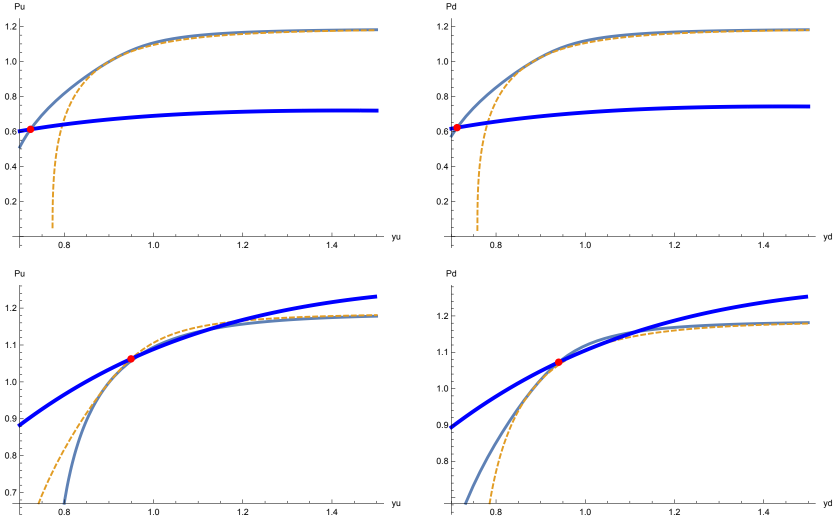
Aggregate supply: the Phillips curve

$\omega = 0.6$ (ignore dashed lines)



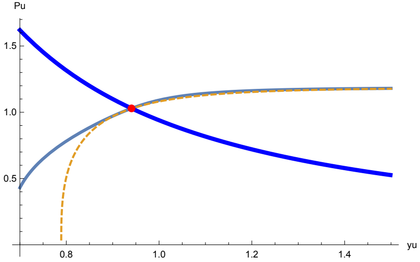
Date T-1: two equilibria

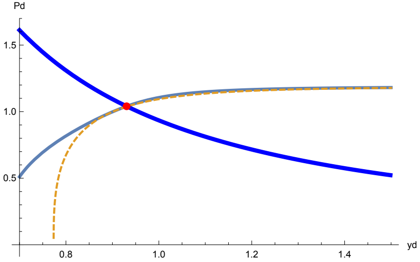
$\phi > 1; \omega = 0.6$. [Current output = 3.55% above flexible-price output.]



Date T-1: one equilibrium

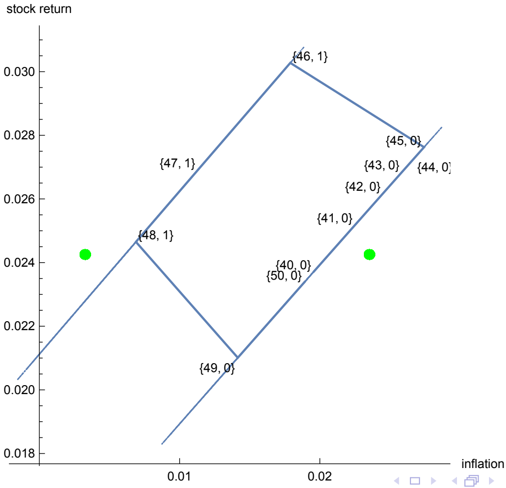
$\phi < 1; \omega = 0.6$. [Current output = 3.55% above flexible-price output.]





Simulation: across paths

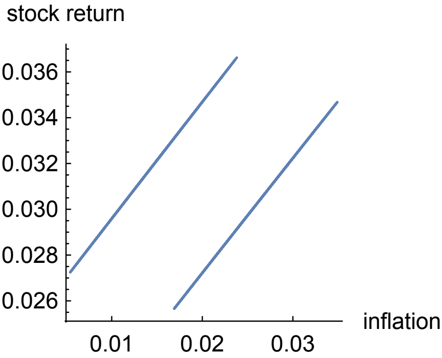
$\phi < 1$; "neutral" target interest rate

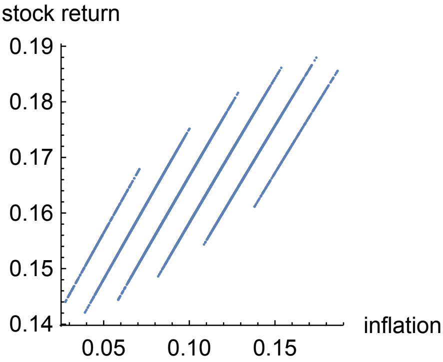


Simulation: across paths

Observation

The relationship between nominal stock return and inflation is steeper over five periods than over one period.





Simulation: Boudoukh and Richardson ex post

$$R_{t \to t+j} = \alpha_j + \beta_j \times \pi_{t \to t+j} + \varepsilon_{t,j}$$

Statistic	α_1	β_1	α_5	β_5
Median	0.023	0.097	0.111	0.243
Upper quintile	0.024	0.137	0.112	0.254
Lower quintile	0.022	0.049	0.11	0.23
Std error				
Median	0.00	0.015	0.001	0.014
Upper quintile	0.00	0.018	0.001	0.017
Lower quintile	0.00	0.012	0.001	0.011

Table: **Stock Returns and Contemporaneous Inflation**

Money: costly trips to the bank, "à la Baumol-Tobin"

- Money is essential feature: crisis = hoarding
- Changes the IS system: money demand restricts $i \geq 0$

Flow budget constraints of private sector

$$P_{t+1,j} \times c_{t+1,j} + F_{1,t+1,j} + P_{t+1,j} \times \sqrt{\frac{1}{2} y_{t+1,j} \times \frac{\nu}{1 - \frac{1}{1+i_{t+1,j}}}} + S_{t+1,j}$$

$$= \theta_{1,t} + P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}} + P_{t+1,j} \times y_{t+1,j}; F_{1,T,j} = 0; j = u, d$$

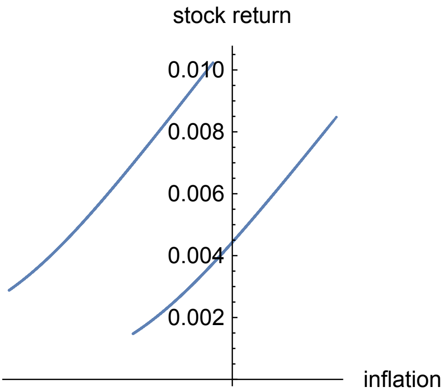
Flow budget constraints of government cum central bank

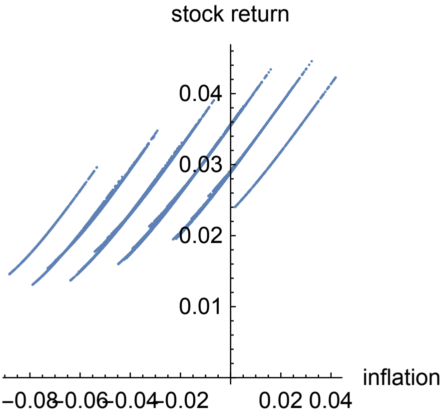
$$F_{2,t+1,j} - P_{t+1,j} \times \sqrt{\frac{1}{2} y_{t+1,j} \times \frac{\nu}{1 - \frac{1}{1+i_{t+1,j}}}}$$

$$= \theta_{2,t} - P_t \times \sqrt{\frac{1}{2} y_t \times \frac{\nu}{1 - \frac{1}{1+i_t}}} + S_{t+1,j}; F_{2,T,j} = 0; j = u, d$$

Simulation: across paths

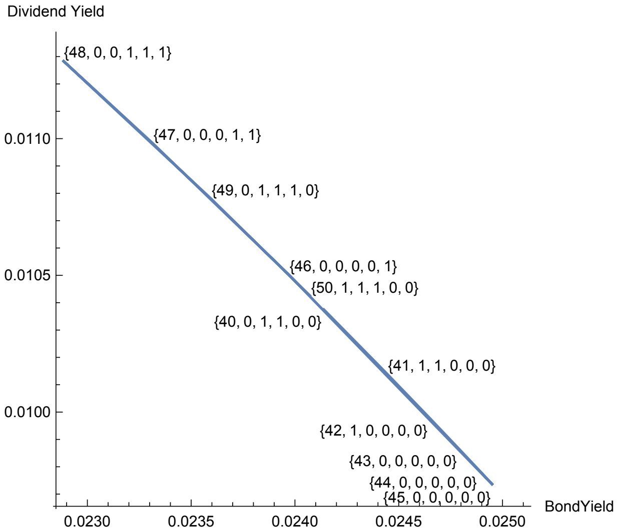
$\phi < 1$; "neutral" target interest rate -1%





The ‘Fed model’

Can one compare yields?



Conclusion

- Equilibrium of an economy with three types of agents:
  - (i) household/investors who supply labor with a finite elasticity, consume a large variety of goods that are not perfect substitutes and trade government bonds;
  - (ii) firms that produce those varieties of goods, setting prices in a Calvo manner;
  - (iii) a government that collects an exogenous fiscal surplus and acts mechanically, buying and selling bonds in accordance with a Taylor policy rule based on expected inflation,
- We tried to take into account the serious non linearities that are rife in the New Keynesian system as opposed to Taylor expansions used by many

Conclusion

- Stock returns have a nominal character
- The slope of the relationship between nominal stock returns and inflation is, indeed, higher over a five-year holding period than over a one-year holding period
- Cashless economy and Baumol-Tobin money-holding economy give similar results
- Bond yield and dividend yield do not move in tango and should not be compared

Supplementary slides

Short empirical survey

- Lintner (1975), Bodie (1976), Jaffee and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), Gultekin (1983), Boudoukh and Richardson (1993), Goto and Valkanov (2000) document a correlation < 0 between real stock returns and inflation at monthly frequency
- Fama (1981) hypothesis of stable money demand and a fixed money supply: a positive *real* shock both increases real stock returns and reduces inflation ("proxy" hypothesis)
  - Geske and Roll (1983): the negative response is due to debt monetization and the resulting counter-cyclical monetary policy
- Goto and Valkanov (2002) and Hagmann and Lenz (2004), using VAR, show attenuation of the negative relationship after Volcker.
- Katz and Lustig (2014) using a panel of countries confirm that stock market is slow to incorporate news about future inflation, while the bond market is not. Puzzled by the nominal character of stocks.
- Modigliani and Cohn (1979), Asness (2003) and Campbell and Vuolteenaho (2004) show evidence of money illusion in the stock market

Short theoretical survey

- The following establish conditions for *real* return on stocks to be negatively correlated with inflation:
    - Danthine and Donaldson (1986): model with money in the utility function;
    - Marshall (1992) with transaction technology.
        - Empirically emphasizes that the model does not fit stock market volatility
- Nístico (2005), De Paoli, Scott and Weeken (2007), Milani (2008), Li and Palomino (2009), Wei (2009), Castelnuovo and Nístico (2010), Challe and Giannitsarou (2014) and Swanson (2013) in *New Keynesian* setting but focus on the effect of *monetary shocks* on the stock market.
- Campbell, Pflueger and Viceira (2013): New Keynesian with habit formation but *ad hoc* connection between the stochastic discount factor and the economy.

$\phi < 1$; "neutral" target interest rate