Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria

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What We Do

- ZLB on nominal interest rate has become empirically relevant for U.S.

- Once the ZLB is explicitly included in a New Keynesian DSGE model, it has many equilibria. We consider three:
  - "the" equilibrium near the targeted-inflation steady state;
  - a minimal state-variable equilibrium near the deflation steady state;
  - a sunspot equilibrium that switches between a targeted-inflation and a deflation regime.

- Solve for equilibria using piece-wise smooth, global approximations of decision rules.

- Analyze the dynamics of these three equilibria, especially near ZLB.

- Conditional on a set of model parameters, we
  - extract sequence of exogenous shocks that rationalize U.S. data from 2000:Q1 to 2010:Q3
  - conditional on the filtered states, we conduct policy experiments

- Conclusion: Effectiveness of fiscal and monetary policy significantly different under the different equilibria.
Some Related Work

- As in Judd, Maliar, and Maliar (2011), Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2012) and in Gust, Lopez-Salido and Smith (2012), we are using projection methods to solve a stochastic model with 2 endogenous and 3 exogenous states:
  - JMM2011, FVGGQRR2012 and GLSS2012 solve for targeted-inflation equilibrium; we also solve for two alternative equilibria.
  - Some differences in the implementation of solution method.

- Benhabib, Schmitt-Grohe and Uribe (2001a,b): ZLB causes multiple equilibria. Mertens and Ravn (2012) also consider a sunspot equilibrium, but
  - our model contains a full set of shocks in addition to the sunspot shock;
  - we conduct policy experiments conditional on states extracted from data.

Multiple equilibria in a two-equation model

A New Keynesian DSGE model with ZLB constraint
  - Multiple steady states
  - Local dynamics near deflation and targeted-inflation steady states
  - Nonlinear solution

Quantitative Analysis
  - Extract model state variables from U.S. data
  - Model dynamics
  - Conduct policy experiments
Two-Equation Model

Adapted from Benhabib, Schmitt-Grohe, and Uribe (2001) and Hursey and Wolman (2010).

- Fisher equation:
  \[ R_t = rE_t[\pi_{t+1}] \]

- Monetary policy rule
  \[ R_t = \max \left\{ 1, r\pi_\ast \left( \frac{\pi_t}{\pi_\ast} \right)^\psi \exp[\sigma \epsilon_t] \right\} \]
  \[ \epsilon_t \sim iidN(0, 1), \psi > 1 \]

- Combine:
  \[ E_t[\pi_{t+1}] = \max \left\{ \frac{1}{r}, \pi_\ast \left( \frac{\pi_t}{\pi_\ast} \right)^\psi \exp[\sigma \epsilon_t] \right\} \]

- Model has two steady states. (\( \sigma = 0 \))
  - Targeted-inflation steady state (\( \pi_\ast, R_\ast = r\pi_\ast \))
  - Deflation steady state (\( \pi_D = 1/r, R_D = 1 \))
Local Dynamics Near Steady States

- We use “hat”s to denote percentage deviations from the respective steady states.

- Dynamics near targeted-inflation steady state \((\pi_*, R_* = r\pi_*)\):
  - Write \(\mathbb{E}_t[\hat{\pi}_{t+1}] = \max \{ -\ln(r\pi_*), \psi\hat{\pi}_t + \sigma \epsilon_t \} \)
  - For small \(\sigma\) the ZLB is essentially non-binding: \(\mathbb{E}_t[\hat{\pi}_{t+1}] = \psi\hat{\pi}_t + \sigma \epsilon_t \)
  - The LRE system has a unique stable solution: \(\hat{\pi}_t = -\frac{1}{\psi} \sigma \epsilon_t \)

- Dynamics near deflation steady state \((\pi_D = 1/r, R_D = 1)\):
  - Write \(\mathbb{E}_t[\tilde{\pi}_{t+1}] = \max \{ 0, -(\psi - 1) \ln(r\pi_*) + \psi\tilde{\pi}_t + \sigma \epsilon_t \} \)
  - For small \(\sigma\) the ZLB is essentially binding: \(\mathbb{E}_t[\tilde{\pi}_{t+1}] = 0 \)
  - The LRE system has many stable solutions:
    \(\tilde{\pi}_t = -\frac{1}{\psi} (1 + M) \sigma \epsilon_t + \zeta_t\), where \(M\) is some constant and \(\zeta_t\) is a sunspot shock.
Equilibria Considered in this Paper

Returning to the original nonlinear difference equation...

- Mimicking the local dynamics around the targeted-inflation steady state

\[ \pi_t = \pi^* \gamma^* \exp \left[ -\frac{1}{\psi} \sigma \epsilon_t \right]. \]

- Mimicking similar dynamics around the deflation steady state

\[ \pi_t = \pi^* \gamma_D \exp \left[ -\frac{1}{\psi} \sigma \epsilon_t \right]. \]

- Sunspot equilibria that alternate between targeted-inflation and deflation regimes:

\[ \pi_t = \pi^* \gamma(s_t) \exp \left[ -\frac{1}{\psi} \sigma \epsilon_t \right] \]

where \( s_t \in \{0, 1\} \) follows a Markov-switching process.
Note: $R_t = 1$ in the deflation equilibria and $R_t = R_*$ in the targeted-inflation equilibrium.
Benhabib, Schmitt-Grohe, and Uribe (2001) equilibria that move the economy from targeted to deflation, e.g.:

\[ \pi_t = \pi_* \max \left\{ \gamma_* \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right] \exp \left[-\psi^{t-t_0} \right], \ \gamma_D \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right] \right\} \]

“Endogenous Sunspot” Equilibrium: Sunspot switches based on fundamental shocks. (with \( \bar{\epsilon} > \epsilon \))

\[ s_t = \begin{cases} 
1 & \text{if } s_{t-1} = 1 \text{ and } \epsilon_t > \epsilon \\
0 & \text{if } s_{t-1} = 1 \text{ and } \epsilon_t < \epsilon \\
0 & \text{if } s_{t-1} = 0 \text{ and } \epsilon_t < \bar{\epsilon} \\
1 & \text{if } s_{t-1} = 0 \text{ and } \epsilon_t > \bar{\epsilon} 
\end{cases} \]
Inflation Dynamics in Simple Model

Targeted-Infl. & Deflation Eq.
Exogenous Sunspots
Inflation Dynamics in Simple Model

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BSGU Dynamics

ZLB Dynamics
Inflation Dynamics in Simple Model

Standardized Shock

![Graph showing standardized shock]

Endogenous Sunspots

![Graph showing endogenous sunspots]

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ZLB Dynamics
We now consider a small-scale New Keynesian model...

and compute three equilibria:

- a targeted-inflation equilibrium,
- a deflation equilibrium,
- a Markov-switching sunspot equilibrium

We solve for “minimal-state-variable” equilibria by

- postulating flexible functional forms for agents’ decision rules;
- parameterizing these functions such that the equilibrium conditions are satisfied.
No discount factor shock. Households maximize

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1 - \tau} - H_{t+s} + \chi V \left( \frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$
- No discount factor shock. Households maximize
\[ \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1 - \tau} - H_{t+s} + \chi V \left( \frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right]. \]

- Intermediate good \( j \) is produced by a monopolist with technology:
\[ Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t. \]
No discount factor shock. Households maximize

$$E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1 - \tau} - H_{t+s} + \chi V \left( \frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

Intermediate good $j$ is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j),$$

where $lnA_t = ln\gamma + lnA_{t-1} + lnz_t$

Intermediate goods producers face quadratic price adjustment costs:

$$AC_t(j) = \frac{\phi}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi} \right)^2 Y_t(j),$$
No discount factor shock. Households maximize
\[
E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \left( \frac{C_{t+s}}{A_{t+s}} \right)^{1-\tau} - 1 \right) - H_{t+s} + \chi V \left( \frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right].
\]

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\]

Monetary policy rule with ZLB enforced:
\[
R_t = \max \left\{ 1, \left[ r \pi_* \left( \frac{\pi_t}{\pi_*} \right)^\psi_1 \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^\psi_2 \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}.
\]
No discount factor shock. Households maximize

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1 - \tau} - H_{t+s} + \chi V \left( \frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

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Monetary policy rule with ZLB enforced:

$$R_t = \max \left\{ 1, \left[ r \pi_* \left( \frac{\pi_t}{\pi_*} \right)^{\psi_1} \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R e_{R,t}} \right\}.$$

Resource constraint ($g_t$ is a generic demand shock):

$$C_t + AC_t = \frac{1}{g_t} Y_t.$$
Equilibrium Conditions

Equilibrium is \( \{ c_t, \pi_t, y_t, R_t \} \) (in terms of detrended variables, i.e., \( c_t = C_t/A_t \) and \( y_t = Y_t/A_t \))

\[
1 = \mathbb{E}_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right]
\]

\[
1 = \frac{1}{\nu} (1 - c_t^\tau) + \phi (\pi_t - \bar{\pi}) \left[ \left( 1 - \frac{1}{2\nu} \right) \pi_t + \frac{\bar{\pi}}{2\nu} \right]
\]

\[
-\phi \beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \bar{\pi})\pi_{t+1} \right]
\]

\[
c_t = \left[ \frac{1}{g_t} - \frac{\phi}{2} (\pi_t - \bar{\pi})^2 \right] y_t
\]

\[
R_t = \max \left\{ 1, \left[ r \pi_* \left( \frac{\pi_t}{\pi_*} \right)^{\psi_1} \left( \frac{y_t}{y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}
\]

and the laws of motion for \( g_t, z_t \) and \( \epsilon_{R,t} \).
Two Steady States

- Let $\lambda = \nu (1 - \beta)$, assume $\psi_1 > 1$.
- **Targeted-Inflation steady state** inflation equals $\pi^*$ and
  
  $$R^* = r \pi^*$$

  $$c^* = \left[ 1 - \nu - \frac{\phi}{2} (1 - 2\lambda) \left( \pi^* - \frac{1 - \lambda}{1 - 2\lambda} \bar{\pi} \right)^2 + \frac{\phi}{2} \frac{\lambda^2}{1 - 2\lambda} \bar{\pi}^2 \right]^{1/\tau}$$

- **Deflation steady state** nominal interest rate is $R_D = 1$ and:
  
  $$\pi_D = \frac{1}{r}$$

  $$c_D = \left[ 1 - \nu - \frac{\phi}{2} (1 - 2\lambda) \left( \pi_D - \frac{1 - \lambda}{1 - 2\lambda} \bar{\pi} \right)^2 + \frac{\phi}{2} \frac{\lambda^2}{1 - 2\lambda} \bar{\pi}^2 \right]^{1/\tau}$$
Approximate Dynamics In Targeted-Inflation Equilibrium

- Let $\tau = 1$, $\gamma = 1$, $\bar{\pi} = \pi^*$, $\psi_1 = \psi$, $\psi_2 = 0$, and $\rho_R = \rho_g = \rho_z = 0$.
- Linearizing around targeted-inflation steady state yields the system

$$
\hat{R}_t = \max \left\{ -\ln(r\pi^*), \psi \hat{\pi}_t + \sigma_R \epsilon_R,t \right\}
$$

$$
\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - (\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}])
$$

$$
\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \hat{c}_t,
$$

- Then solution is piece-wise linear

$$
\hat{R}_t(\epsilon_{R,t}) = \max \left\{ -\ln(r\pi^*), \frac{1}{1 + \kappa \psi} \left[ \psi (\kappa + \beta) \mu^*_\pi + \kappa \psi \mu^*_c + \sigma_R \epsilon_{R,t} \right] \right\}
$$

$$
\hat{c}_t(\epsilon_{R,t}) = \begin{cases} 
\frac{1}{1 + \kappa \psi} \left[ (1 - \psi \beta) \mu^*_\pi + \mu^*_c - \sigma_R \epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi^*) \\
\ln(r\pi^*) + \mu^*_c + \mu^*_\pi & \text{otherwise}
\end{cases}
$$

$$
\hat{\pi}_t(\epsilon_{R,t}) = \begin{cases} 
\frac{1}{1 + \kappa \psi} \left[ (\kappa + \beta) \mu^*_\pi + \kappa \mu^*_c - \kappa \sigma_R \epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi^*) \\
\kappa \ln(r\pi^*) + (\kappa + \beta) \mu^*_\pi + \kappa \mu^*_c & \text{otherwise}
\end{cases}
$$
Sketch of Solution Method

- Approximate decision rules for \( \pi(S_t) \) and \( \mathcal{E}(S_t) \).

\[
\begin{align*}
\pi_t &= \pi(S_t; \Theta) = \zeta_t f_\pi^1(S_t; \Theta) + (1 - \zeta_t) f_\pi^2(S_t; \Theta) \\
\mathcal{E}_t &= \mathcal{E}(S_t; \Theta) = \zeta_t f_{\mathcal{E}}^1(S_t; \Theta) + (1 - \zeta_t) f_{\mathcal{E}}^1(S_t; \Theta)
\end{align*}
\]

where \( \zeta_t = I\{R(S_t) > 1\} \) and \( \mathcal{E}_t \equiv E_t \left[ \frac{c_{t+1} - \tau_t + 1}{\gamma z_{t+1} \pi_{t+1}} \right] \)

- \( f^i_j \) are linear combinations of a complete set Chebyshev polynomials up to 4th order, parameterized by \( \Theta \).

- Two functions are stitched together with endogenous seam.

- Choose \( \Theta \) to minimize sum squared residuals from the Euler Equations over a grid of points representing the ergodic distribution.

- Details about approximation:
  - 252 unknowns.
  - Running time: Under one minute on a single core with analytical derivatives. (highly parallelizable)
  - Accuracy: Approximation errors in the order of \( 10^{-4} \) in consumption units.
Given a solution, we run an auxiliary particle filter with fixed parameters to obtain filtered states.

Iterative process (ergodic-set method with a twist):

- Start with a guess for $\Theta$.
- Use simulated (and filtered) states to obtain a “representative” grid.
  - Use a time-separated grid algorithm
- Solve for $\Theta$ that minimizes sum of squared residuals.
- Repeat until convergence.
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ZLB Dynamics
Gray bands represent 90% of the distribution for $g$. 

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ZLB Dynamics
Parameter values (from estimating a non-linear version using second-order perturbation, Aruoba, Bocola and Schorfheide, 2011):

\[ \nu = 0.1, \quad g_* = \frac{1}{0.85}, \quad \gamma = 1.059, \quad \tau = 1.09, \quad \psi_1 = 1.5, \quad \psi_2 = 0.8, \]
\[ \rho_R = 0.55, \quad r = 1.0076, \quad \pi_* = 1.007, \quad \bar{\pi} = 1, \quad \phi = 221.70, \]
\[ \rho_G = 0.89, \quad \rho_Z = 0.26, \quad \sigma_R = 0.0036, \quad \sigma_g = 0.0086, \quad \sigma_z = 0.0072 \]

Transition probabilities for sunspot shock

\[ p^{**} = 0.99, \quad p_{DD} = 0.99 \]
Ergodic Distributions

Targeted-Inflation Eq.

Deflation Eq.

Sunspot Equilibrium

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ZLB Dynamics
Properties of Equilibria

- Inflation responds negatively to a positive demand shock in the **deflation equilibrium** and in the deflation state of the **sunspot equilibrium**.
  - Similar to the findings in Eggertson (2009). AD becomes upward sloping. Log-linearized AD equation.
    \[
    \hat{y}_t = E_t \hat{y}_{t+1} - \tau (i_t - E_t \pi_{t+1}) + \hat{g}_t
    \]

- Expansionary monetary policy less effective in the **deflation equilibrium**.

- ZLB binds 56% of the time in the **deflation equilibrium**. Never in the **targeted-inflation equilibrium**.

- \( s_t = 0 \) in 47% of the time in the **sunspot equilibrium**. ZLB binds 27% of the time and it is always when \( s_t = 0 \).
  - May adjust \( \rho_{**} \) and \( \rho_{DD} \) to obtain numbers that are more empirically relevant.
  - Welfare under **deflation equilibrium** comparable to welfare under **targeted-inflation equilibrium**.
We now use a(n auxiliary) particle filter to extract the latent states for our model.

Data: output growth, inflation (annualized), and interest rates (annualized) from 2000:Q1 to 2010:Q3

- Show results for year-on-year CPI inflation.

We consider the targeted-inflation equilibrium and the sunspot equilibrium.

The deflation equilibrium is not consistent with the data.

The sunspot equilibrium is the most plausible one given the data.

- Fit the data with the least extreme shocks.
Gray shading indicates deflation regime in sunspot equilibrium.

Yellow shading indicates $0.65 < Pr(s_t = 1) < 1$. 

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ZLB Dynamics
Standing at the beginning of 2009:Q1 and taking only the filtered states in 2009:Q1 as given, we consider

1. a fiscal policy intervention calibrated to ARRA;

2. a combination of the fiscal policy intervention with an expansionary monetary policy that lasts for one year.

Mechanics: conditional on time $T$ states we

- generate draws for future shocks;
- compute paths $Y_{T+1:T+H}, \pi_{T+1:T+H}, R_{T+1:T+H}$ without policy intervention;
- compute paths $Y^I_{T+1:T+H}, \pi^I_{T+1:T+H}, R^I_{T+1:T+H}$ with policy intervention;
- inspect the distribution of the intervention effects:
  - $100 \ln(Y^I_{T+h}/Y_{T+h})$;
  - $\pi^I_{T+h} - \pi_{T+h}$ (annualized rates);
  - $R^I_{T+h} - R_{T+h}$ (annualized rates).
Fiscal policy intervention is calibrated to portion of the American Recovery and Reinvestment Act (ARRA) of February 2009:
- Tax cuts and benefits;
- entitlement programs;
- funding for federal contracts, grants, and loans;
- Convert expenditures into $\hat{g}_t^{ARRA}$ and construct a demand shock that generates a path comparable to $\hat{g}_t^{ARRA}$
Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for the targeted-inflation equilibrium and the sunspot equilibrium.
Multiplier: $\mu_t^c = \frac{\sum_{\tau=1}^{t}(Y_{\tau}^I - Y_{\tau})}{\sum_{\tau=1}^{t}(G_{\tau}^I - G_{\tau})}$;
At the beginning of 2009:Q1 the Fed contemplates to amplify the effect of the expansionary fiscal policy by an expansionary monetary policy that keeps interest rates at or near zero.

Recall monetary policy rule:

\[
R_t = \max \left\{ 1, \left[ r \pi_* \left( \frac{\pi_t}{\pi_*} \right)^\psi_1 \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^\psi_2 \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}
\]

**Un-intervened paths:**
- all \( \epsilon_{g,T+h} \) and \( \epsilon_{z,T+h} \) are drawn from \( N(0, 1) \);
- \( \epsilon_{R,T+h} = 0 \).

**Intervened paths:**
- \( \sigma_g \epsilon_{g,T+1} \sim N(0.011, \sigma_g^2) \);
- all other \( \epsilon_{g,T+h} \) and \( \epsilon_{z,T+h} \) shocks are drawn from \( N(0, 1) \);
- solve for the \( \tilde{\epsilon}_{R,T+1:T+4} \) such that for \( h = 1, 2, 3, 4 \)
  \[
  R_{T+h}^I(\epsilon_{R,T+h} = 0) - R_{T+h}^I(\epsilon_{R,T+h} = \tilde{\epsilon}_{R,T+h}) \text{ is maximized while } \leq 1\% \text{ (annualized)}.\]
Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for only fiscal intervention and both interventions.
Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for only fiscal intervention and both interventions.
An Alternative Policy Exercise

Instead of standing at the beginning of 2009Q1, do an ex-post analysis.

Use the filtered shocks for 2000Q1-2010Q2.

Two experiments:

- Fiscal intervention only, calibrated to ARRA.
  - Small issue: Data may already include ARRA.

- Both fiscal and monetary intervention.

Intervention dates: 2009Q1 (ZLB) and 2007Q1 (interest rate around 6% p.a.)
### Cumulative Government Spending Multipliers

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<th>Period</th>
<th>2009Q1</th>
<th>2007Q1</th>
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#### Targeted-Inflation Equilibrium

#### Sunspot Equilibrium

<table>
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<th>2007Q1</th>
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</table>
Summary of Empirical Results

• 2008-09 is consistent with a change from the targeted-inflation regime to the deflation regime in the sunspot equilibrium.
  • Data prefers the sunspot model over the targeted-inflation model which needs very large shocks.

• Ex-ante policy exercise in 2009Q1:
  • Cumulative multiplier for targeted-inflation equilibrium just below one versus around 0.7 for the sunspot equilibrium.
  • Very small scope for monetary stimulus for aiding fiscal policy in the sunspot equilibrium, unlike the targeted-inflation equilibrium.
  • Key intuition: The economy escapes / is expected to escape ZLB very quickly in the targeted-inflation equilibrium.

• Ex-post policy exercise:
  • The actual path of the economy in the targeted-inflation economy was “unlikely”: fiscal multiplier larger and monetary policy not effective.

• Since sunspot equilibrium is the more likely explanation of what happened in 2008-2009, policy may not have been very effective.