

Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria

S. Boraĝan Aruoba

University of Maryland

Frank Schorfheide

University of Pennsylvania, CEPR, NBER

March, 2013

What We Do

- ZLB on nom. interest rate has become empirically relevant for U.S.
- Once the ZLB is explicitly included in a New Keynesian DSGE model, it has many equilibria. We consider three:
 - “the” equilibrium **near the targeted-inflation steady state**;
 - a minimal state-variable equilibrium **near the deflation steady state**;
 - a **sunspot equilibrium** that switches between a **targeted-inflation** and a **deflation regime**.
- Solve for equilibria using piece-wise smooth, global approximations of decision rules.
- Analyze the dynamics of these three equilibria, especially near ZLB.
- Conditional on a set of model parameters, we
 - extract sequence of exogenous shocks that rationalize U.S. data from 2000:Q1 to 2010Q3
 - conditional on the filtered states, we conduct policy experiments
- Conclusion: Effectiveness of fiscal and monetary policy significantly different under the different equilibria.

Some Related Work

- As in Judd, Maliar, and Maliar (2011), Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez (2012) and in Gust, Lopez-Salido and Smith (2012), we are using projection methods to solve a stochastic model with 2 endogenous and 3 exogenous states:
 - JMM2011, FVGGQRR2012 and GLSS2012 solve for **targeted-inflation equilibrium**; we also solve for two alternative equilibria.
 - Some differences in the implementation of solution method.
- Benhabib, Schmitt-Grohe and Uribe (2001a,b): ZLB causes multiple equilibria. Mertens and Ravn (2012) also consider a sunspot equilibrium, but
 - our model contains a full set of shocks in addition to the sunspot shock;
 - we conduct policy experiments conditional on states extracted from data.
- Braun and Korber (2011), Christiano, Eichenbaum and Rebelo (2011), FVGGQRR2012 and Mertens and Ravn (2012) find fiscal multipliers much larger than one at ZLB. Our results show lower multipliers.

- Multiple equilibria in a two-equation model
- A New Keynesian DSGE model with ZLB constraint
 - Multiple steady states
 - Local dynamics near deflation and targeted-inflation steady states
 - Nonlinear solution
- Quantitative Analysis
 - Extract model state variables from U.S. data
 - Model dynamics
 - Conduct policy experiments

Two-Equation Model

Adapted from Benhabib, Schmitt-Grohe, and Uribe (2001) and Hursey and Wolman (2010).

- Fisher equation:

$$R_t = r\mathbb{E}_t[\pi_{t+1}]$$

- Monetary policy rule

$$R_t = \max \left\{ 1, r\pi_* \left(\frac{\pi_t}{\pi_*} \right)^\psi \exp[\sigma\epsilon_t] \right\}, \quad \epsilon_t \sim iidN(0, 1), \quad \psi > 1$$

- Combine:

$$\mathbb{E}_t[\pi_{t+1}] = \max \left\{ \frac{1}{r}, \pi_* \left(\frac{\pi_t}{\pi_*} \right)^\psi \exp[\sigma\epsilon_t] \right\}$$

- Model has two steady states. ($\sigma = 0$)
 - Targeted-inflation steady state (π_* , $R_* = r\pi_*$)
 - Deflation steady state ($\pi_D = 1/r$, $R_D = 1$)

Local Dynamics Near Steady States

- We use “hat”’s to denote percentage deviations from the respective steady states
- Dynamics near **targeted-inflation steady state** (π_* , $R_* = r\pi_*$):
 - Write $\mathbb{E}_t[\hat{\pi}_{t+1}] = \max \{-\ln(r\pi_*), \psi\hat{\pi}_t + \sigma\epsilon_t\}$
 - For small σ the ZLB is essentially non-binding: $\mathbb{E}_t[\hat{\pi}_{t+1}] = \psi\hat{\pi}_t + \sigma\epsilon_t$
 - The LRE system has a unique stable solution: $\hat{\pi}_t = -\frac{1}{\psi}\sigma\epsilon_t$
- Dynamics near **deflation steady state** ($\pi_D = 1/r$, $R_D = 1$):
 - Write $\mathbb{E}_t[\tilde{\pi}_{t+1}] = \max \{0, -(\psi - 1)\ln(r\pi_*) + \psi\tilde{\pi}_t + \sigma\epsilon_t\}$
 - For small σ the ZLB is essentially binding: $\mathbb{E}_t[\tilde{\pi}_{t+1}] = 0$
 - The LRE system has many stable solutions:
 $\tilde{\pi}_t = -\frac{1}{\psi}(1 + M)\sigma\epsilon_t + \zeta_t$, where M is some constant and ζ_t is a sunspot shock.

Equilibria Considered in this Paper

Returning to the original nonlinear difference equation...

- Mimicking the local dynamics around the **targeted-inflation steady state**

$$\pi_t = \pi_* \gamma_* \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right].$$

- Mimicking similar dynamics around the **deflation steady state**

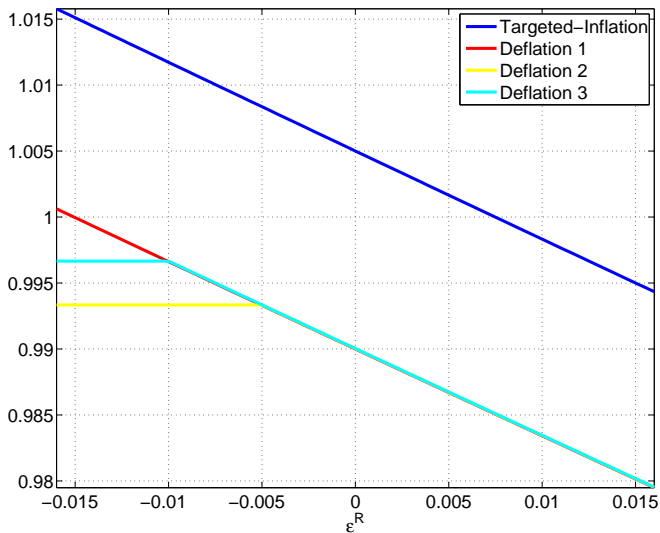
$$\pi_t = \pi_* \gamma_D \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right].$$

- Sunspot equilibria that **alternate** between **targeted-inflation** and **deflation** regimes:

$$\pi_t = \pi_* \gamma(s_t) \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right]$$

where $s_t \in \{0, 1\}$ follows a Markov-switching process.

Inflation Decision Rules in Simple Model



Note: $R_t = 1$ in the deflation equilibria and $R_t = R_*$ in the targeted-inflation equilibrium.

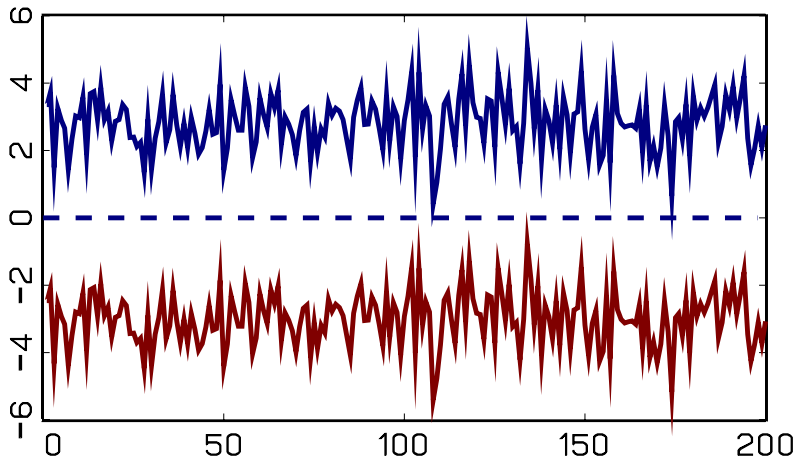
- Benhabib, Schmitt-Grohe, and Uribe (2001) equilibria that move the economy from targeted to deflation, e.g.:

$$\pi_t = \pi_* \max \left\{ \gamma_* \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right] \exp \left[-\psi^{t-t_0} \right], \gamma_D \exp \left[-\frac{1}{\psi} \sigma \epsilon_t \right] \right\}$$

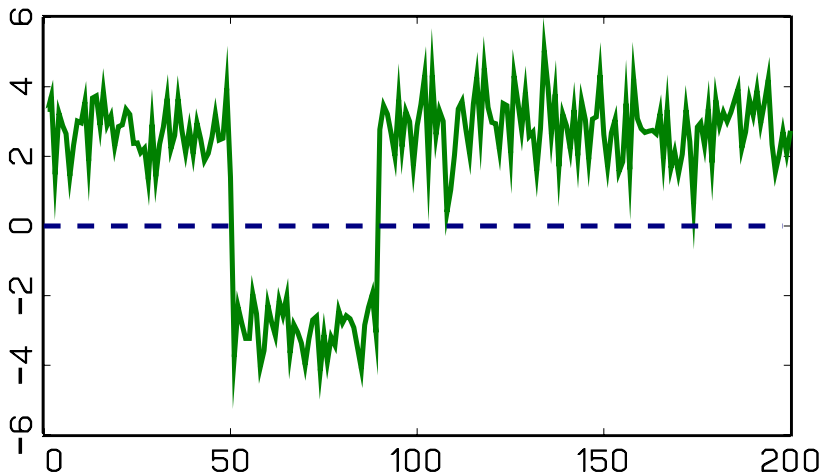
- “Endogenous Sunspot” Equilibrium: Sunspot switches based on fundamental shocks. (with $\bar{\epsilon} > \underline{\epsilon}$)

$$s_t = \begin{cases} 1 & \text{if } s_{t-1} = 1 \text{ and } \epsilon_t > \underline{\epsilon} \\ 0 & \text{if } s_{t-1} = 1 \text{ and } \epsilon_t < \underline{\epsilon} \\ 0 & \text{if } s_{t-1} = 0 \text{ and } \epsilon_t < \bar{\epsilon} \\ 1 & \text{if } s_{t-1} = 0 \text{ and } \epsilon_t > \bar{\epsilon} \end{cases}$$

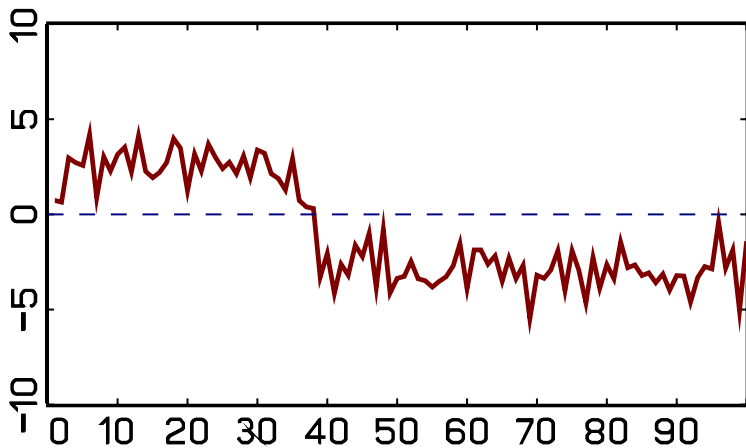
Targeted-Infl. & Deflation Eq.



Exogenous Sunspots

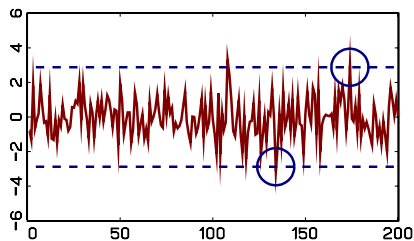


BSGU Dynamics

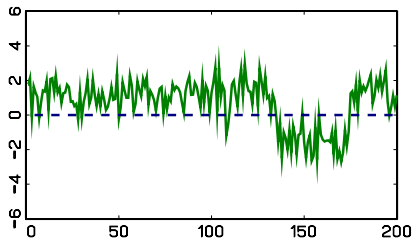


Inflation Dynamics in Simple Model

Standardized Shock



Endogenous Sunspots



The Next Steps

- We now consider a small-scale New Keynesian model...
- and compute three equilibria:
 - a targeted-inflation equilibrium,
 - a deflation equilibrium,
 - a Markov-switching sunspot equilibrium
- We solve for “minimal-state-variable” equilibria by
 - postulating flexible functional forms for agents' decision rules;
 - parameterizing these functions such that the equilibrium conditions are satisfied.

“Standard” Small-Scale New Keynesian DSGE Model

- No discount factor shock. Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - H_{t+s} + \chi V \left(\frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

“Standard” Small-Scale New Keynesian DSGE Model

- No discount factor shock. Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - H_{t+s} + \chi V \left(\frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

- Intermediate good j is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t$$

“Standard” Small-Scale New Keynesian DSGE Model

- **No discount factor shock.** Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - H_{t+s} + \chi V \left(\frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

- Intermediate good j is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t$$

- Intermediate goods producers face **quadratic price adjustment costs:**

$$AC_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi} \right)^2 Y_t(j),$$

“Standard” Small-Scale New Keynesian DSGE Model

- **No discount factor shock.** Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - H_{t+s} + \chi V \left(\frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

- Intermediate good j is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t$$

- Intermediate goods producers face **quadratic price adjustment costs**:

$$AC_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi} \right)^2 Y_t(j),$$

- Monetary policy rule **with ZLB enforced**:

$$R_t = \max \left\{ 1, \left[r \pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}.$$

“Standard” Small-Scale New Keynesian DSGE Model

- No discount factor shock. Households maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} - H_{t+s} + \chi V \left(\frac{M_{t+s}}{P_{t+s}A_{t+s}} \right) \right) \right].$$

- Intermediate good j is produced by a monopolist with technology:

$$Y_t(j) = A_t H_t(j), \text{ where } \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t$$

- Intermediate goods producers face quadratic price adjustment costs:

$$AC_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi} \right)^2 Y_t(j),$$

- Monetary policy rule with ZLB enforced:

$$R_t = \max \left\{ 1, \left[r\pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_{R,t}} \right\}.$$

- Resource constraint (g_t is a generic demand shock):

$$C_t + AC_t = \frac{1}{g_t} Y_t.$$

Equilibrium Conditions

Equilibrium is $\{c_t, \pi_t, y_t, R_t\}$ (in terms of detrended variables, i.e., $c_t = C_t/A_t$ and $y_t = Y_t/A_t$)

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$1 = \frac{1}{\nu} (1 - c_t^\tau) + \phi (\pi_t - \bar{\pi}) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\bar{\pi}}{2\nu} \right] \\ - \phi \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{y_{t+1}}{y_t} (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \right]$$

$$c_t = \left[\frac{1}{g_t} - \frac{\phi}{2} (\pi_t - \bar{\pi})^2 \right] y_t$$

$$R_t = \max \left\{ 1, \left[r \pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{y_t}{y_{t-1}} z_t \right)^{\psi_2} \right]^{1-\rho R} R_{t-1}^{\rho R} e^{\sigma R \epsilon_{R,t}} \right\}$$

and the laws of motion for g_t , z_t and $\epsilon_{R,t}$.

Two Steady States

- Let $\lambda = \nu(1 - \beta)$, assume $\psi_1 > 1$.
- Targeted-Inflation steady state inflation equals π_* and

$$R_* = r\pi_*$$

$$c_* = \left[1 - \nu - \frac{\phi}{2}(1 - 2\lambda) \left(\pi_* - \frac{1 - \lambda}{1 - 2\lambda} \bar{\pi} \right)^2 + \frac{\phi}{2} \frac{\lambda^2}{1 - 2\lambda} \bar{\pi}^2 \right]^{1/\tau}$$

- Deflation steady state nominal interest rate is $R_D = 1$ and:

$$\pi_D = 1/r$$

$$c_D = \left[1 - \nu - \frac{\phi}{2}(1 - 2\lambda) \left(\pi_D - \frac{1 - \lambda}{1 - 2\lambda} \bar{\pi} \right)^2 + \frac{\phi}{2} \frac{\lambda^2}{1 - 2\lambda} \bar{\pi}^2 \right]^{1/\tau}$$

Approximate Dynamics In Targeted-Inflation Equilibrium

- Let $\tau = 1$, $\gamma = 1$, $\bar{\pi} = \pi_*$, $\psi_1 = \psi$, $\psi_2 = 0$, and $\rho_R = \rho_g = \rho_z = 0$.
- Linearizing around targeted-inflation steady state yields the system

$$\hat{R}_t = \max \left\{ -\ln(r\pi_*), \psi \hat{\pi}_t + \sigma_R \epsilon_{R,t} \right\}$$

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - (\hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}])$$

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa_* \hat{c}_t,$$

- Then solution is piece-wise linear

$$\hat{R}_t(\epsilon_{R,t}) = \max \left\{ -\ln(r\pi_*), \frac{1}{1 + \kappa\psi} \left[\psi(\kappa + \beta)\mu_\pi^* + \kappa\psi\mu_c^* + \sigma_R\epsilon_{R,t} \right] \right\}$$

$$\hat{c}_t(\epsilon_{R,t}) = \begin{cases} \frac{1}{1 + \kappa\psi} \left[(1 - \psi\beta)\mu_\pi^* + \mu_c^* - \sigma_R\epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi_*) \\ \ln(r\pi_*) + \mu_c^* + \mu_\pi^* & \text{otherwise} \end{cases}$$

$$\hat{\pi}_t(\epsilon_{R,t}) = \begin{cases} \frac{1}{1 + \kappa\psi} \left[(\kappa + \beta)\mu_\pi^* + \kappa\mu_c^* - \kappa\sigma_R\epsilon_{R,t} \right] & \text{if } \hat{R}_t \geq -\ln(r\pi_*) \\ \kappa \ln(r\pi_*) + (\kappa + \beta)\mu_\pi^* + \kappa\mu_c^* & \text{otherwise} \end{cases}$$

Sketch of Solution Method

- Approximate decision rules for $\pi(\mathcal{S}_t)$ and $\mathcal{E}(\mathcal{S}_t)$.

$$\pi_t = \pi(\mathcal{S}_t; \Theta) = \zeta_t f_{\pi}^1(\mathcal{S}_t; \Theta) + (1 - \zeta_t) f_{\pi}^2(\mathcal{S}_t; \Theta)$$

$$\mathcal{E}_t = \mathcal{E}(\mathcal{S}_t; \Theta) = \zeta_t f_{\mathcal{E}}^1(\mathcal{S}_t; \Theta) + (1 - \zeta_t) f_{\mathcal{E}}^2(\mathcal{S}_t; \Theta)$$

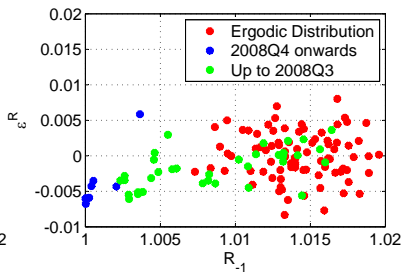
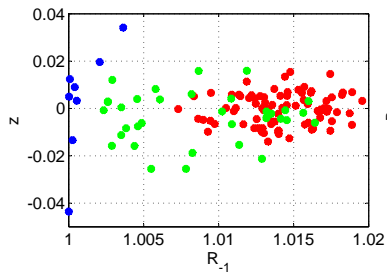
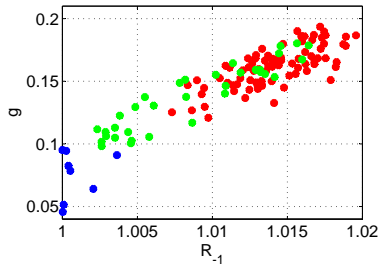
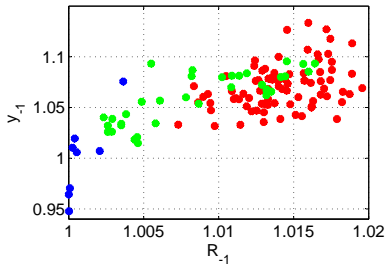
where $\zeta_t = I\{R(\mathcal{S}_t) > 1\}$ and $\mathcal{E}_t \equiv \mathbf{E}_t [c_{t+1}^{-\tau} / \gamma z_{t+1} \pi_{t+1}]$

- f_j^i are linear combinations of a complete set Chebyshev polynomials up to 4th order, parameterized by Θ .
- Two functions are stitched together with endogenous seam.
- Choose Θ to minimize sum squared residuals from the Euler Equations over a grid of points representing the ergodic distribution.
- Details about approximation:
 - 252 unknowns.
 - Running time: Under one minute on a single core with analytical derivatives. (highly parallelizable)
 - Accuracy: Approximation errors in the order of 10^{-4} in consumption units.

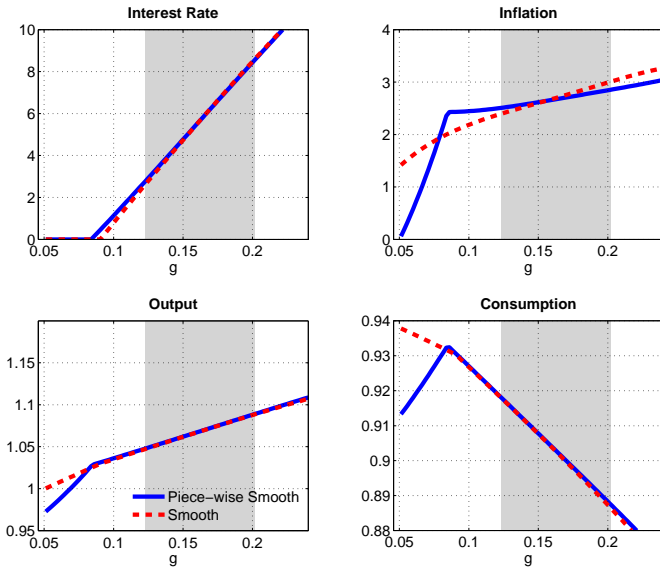
Sketch of Solution Method

- Given a solution, we run an auxiliary particle filter with fixed parameters to obtain filtered states.
- Iterative process (ergodic-set method with a twist):
 - Start with a guess for Θ .
 - Use simulated (and filtered) states to obtain a “representative” grid.
 - Use a time-separated grid algorithm
 - Solve for Θ that minimizes sum of squared residuals.
 - Repeat until convergence.

Solution Grid



Sample Decision Rules - Targeted-Inflation Equilibrium



Gray bands represent 90% of the distribution for g .

- ① Parameter values (from estimating a non-linear version using second-order perturbation, Aruoba, Bocola and Schorfheide, 2011):

$$\nu = 0.1, g_* = \frac{1}{0.85}, \gamma = 1.059, \tau = 1.09, \psi_1 = 1.5, \psi_2 = 0.8,$$

$$\rho_R = 0.55, r = 1.0076, \pi_* = 1.007, \bar{\pi} = 1, \phi = 221.70,$$

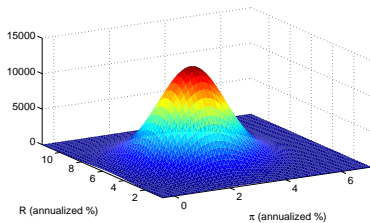
$$\rho_G = 0.89, \rho_Z = 0.26, \sigma_R = 0.0036, \sigma_g = 0.0086, \sigma_z = 0.0072$$

- ② Transition probabilities for sunspot shock

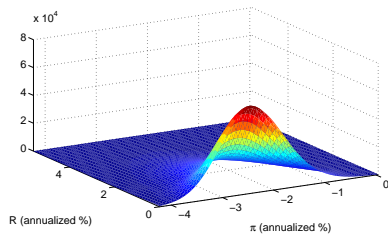
$$p_{**} = 0.99, p_{DD} = 0.99$$

Ergodic Distributions

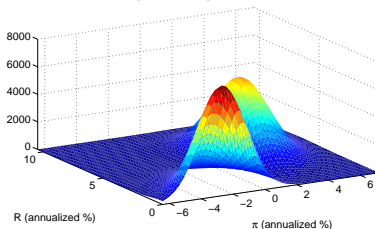
Targeted-Inflation Eq.



Deflation Eq.

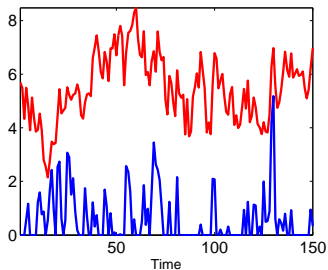


Sunspot Equilibrium

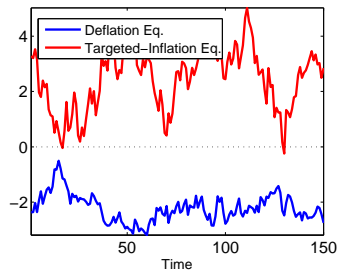


Simulated Paths

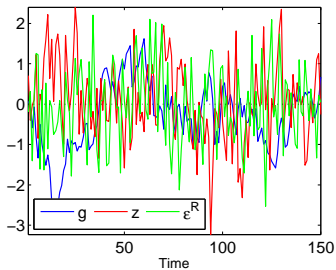
Nominal Interest Rate (annualized %)



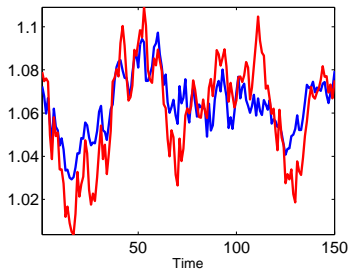
Inflation (annualized %)



Shocks

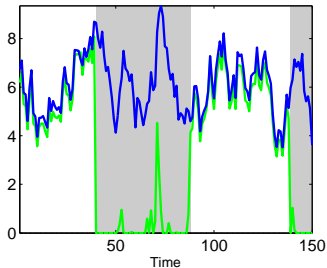


Output

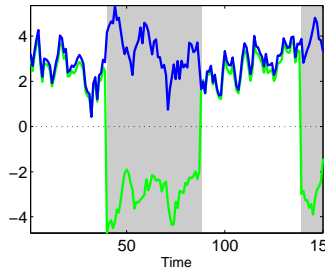


Simulated Paths

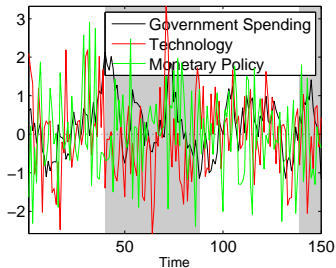
Nominal Interest Rate (annualized %)



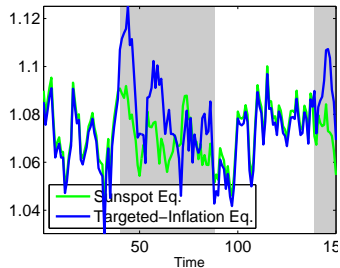
Inflation (annualized %)



Government Spending Shock



Output



Properties of Equilibria

- Inflation responds negatively to a positive demand shock in the **deflation equilibrium** and in the deflation state of the **sunspot equilibrium**.
 - Similar to the findings in Eggertson (2009). AD becomes upward sloping. Log-linearized AD equation.

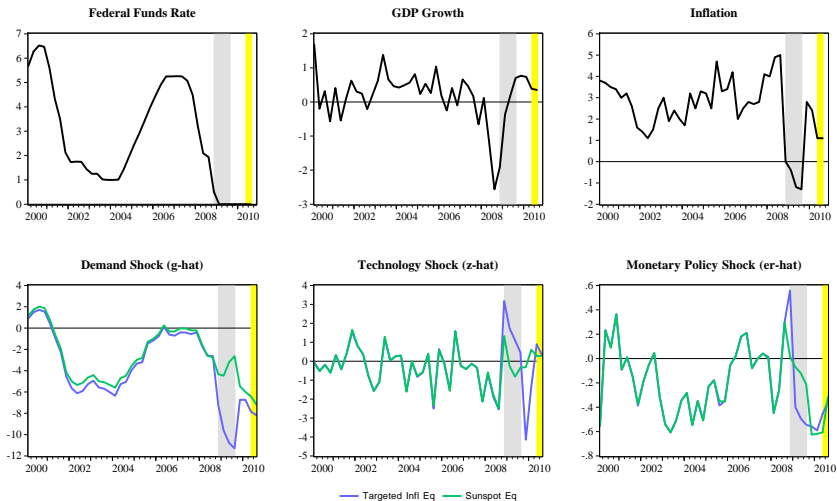
$$\hat{y}_t = E_t \hat{y}_{t+1} - \tau(i_t - E_t \pi_{t+1}) + \hat{g}_t$$

- Expansionary monetary policy less effective in the **deflation equilibrium**.
- ZLB binds 56% of the time in the **deflation equilibrium**. Never in the **targeted-inflation equilibrium**.
- $s_t = 0$ in 47% of the time in the **sunspot equilibrium**. ZLB binds 27% of the time and it is always when $s_t = 0$.
 - May adjust ρ_{**} and ρ_{DD} to obtain numbers that are more empirically relevant.
- Welfare under **deflation equilibrium** comparable to welfare under **targeted-inflation equilibrium**.

Extracting Historical Shocks

- We now use a(n auxiliary) particle filter to extract the latent states for our model.
- Data: output growth, inflation (annualized), and interest rates (annualized) from 2000:Q1 to 2010:Q3
 - Show results for year-on-year CPI inflation.
- We consider the **targeted-inflation equilibrium** and the **sunspot equilibrium**.
- The **deflation equilibrium** is not consistent with the data.
- The **sunspot equilibrium** is the most plausible one given the data.
 - Fit the data with the least extreme shocks.

Data and Historical Shocks



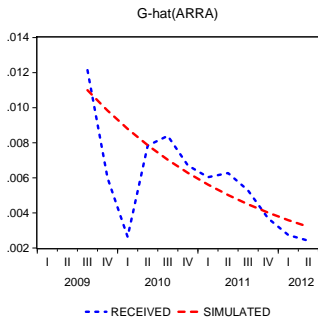
Gray shading indicates deflation regime in sunspot equilibrium.

Yellow shading indicates $0.65 < Pr(s_t = 1) < 1$.

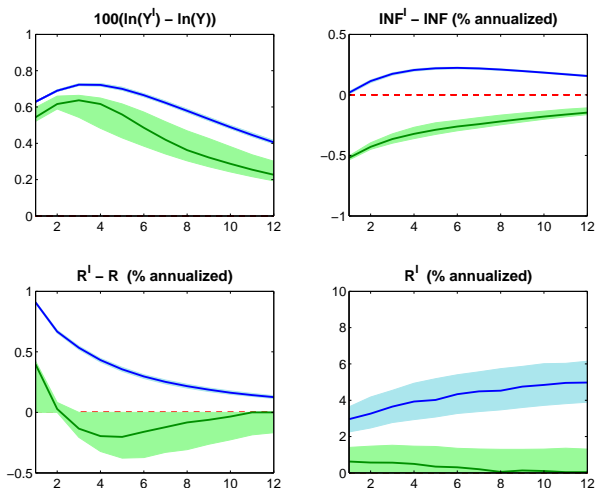
- Standing at the beginning of 2009:Q1 and taking **only** the filtered states in 2009:Q1 as given, we consider
 - ① a fiscal policy intervention calibrated to ARRA;
 - ② a combination of the fiscal policy intervention with an expansionary monetary policy that lasts for one year.
- Mechanics: conditional on time T states we
 - generate draws for future shocks;
 - compute paths $Y_{T+1:T+H}$, $\pi_{T+1:T+H}$, $R_{T+1:T+H}$ without policy intervention;
 - compute paths $Y_{T+1:T+H}^I$, $\pi_{T+1:T+H}^I$, $R_{T+1:T+H}^I$ with policy intervention;
 - inspect the distribution of the intervention effects:
 - $100 \ln(Y_{T+h}^I/Y_{T+h})$;
 - $\pi_{T+h}^I - \pi_{T+h}$ (annualized rates);
 - $R_{T+h}^I - R_{T+h}$ (annualized rates).

Calibration of Fiscal Policy Intervention

- Fiscal policy intervention is calibrated to portion of the American Recovery and Reinvestment Act (ARRA) of February 2009:
 - Tax cuts and benefits;
 - entitlement programs;
 - funding for federal contracts, grants, and loans;
- Convert expenditures into \hat{g}_t^{ARRA} and construct a demand shock that generates a path comparable to \hat{g}_t^{ARRA}



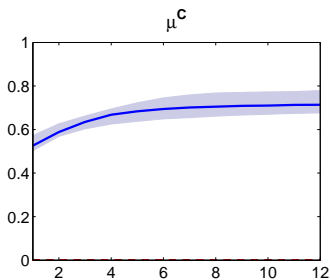
Targeted-Inflation vs. Sunspot Equilibrium: Fiscal Policy



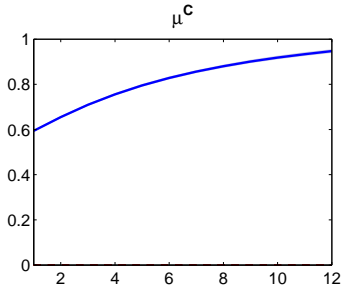
- Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for the **targeted-inflation equilibrium** and the **sunspot equilibrium**.

Government Spending Multipliers

Sunspot Equilibrium



Targeted-Inflation Eq.



$$\text{Multiplier: } \mu_t^c = \frac{\sum_{\tau=1}^t (Y_\tau^I - Y_\tau)}{\sum_{\tau=1}^t (G_\tau^I - G_\tau)}$$

Expansionary Fiscal and Monetary Policy

- At the beginning of 2009:Q1 the Fed contemplates to amplify the effect of the expansionary fiscal policy by an expansionary monetary policy that keeps interest rates at or near zero.
- Recall monetary policy rule:

$$R_t = \max \left\{ 1, \left[r\pi_* \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}} \right\}.$$

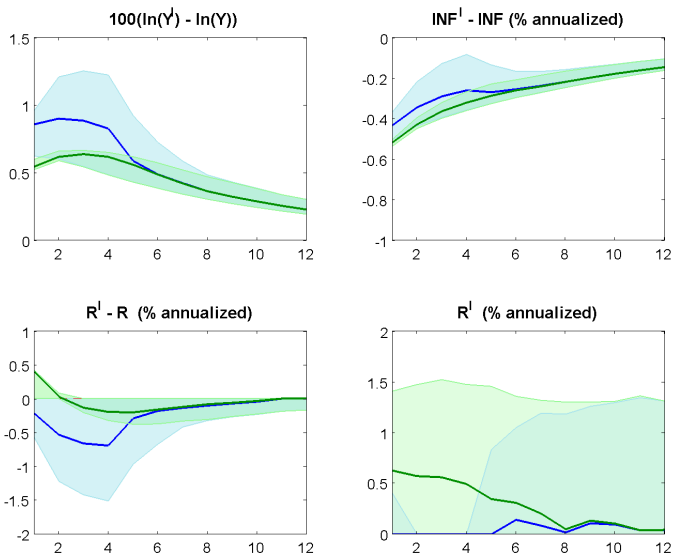
- **Un-intervened paths:**

- all $\epsilon_{g,T+h}$ and $\epsilon_{z,T+h}$ are drawn from $N(0, 1)$;
- $\epsilon_{R,T+h} = 0$.

- **Intervened paths:**

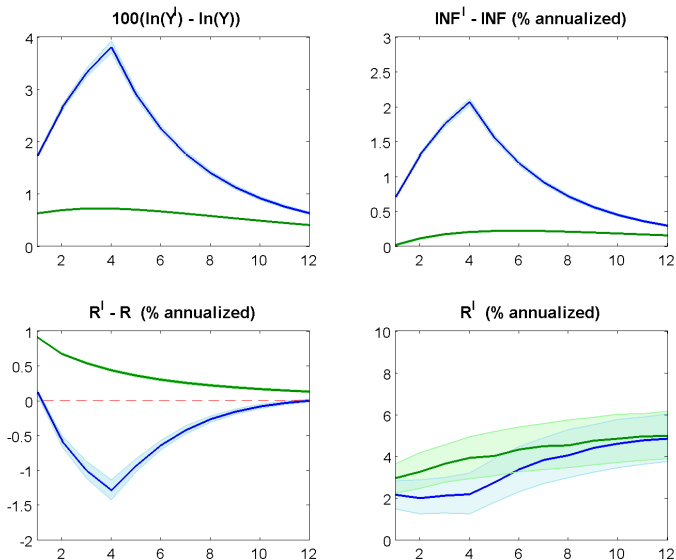
- $\sigma_g \epsilon_{g,T+1} \sim N(0.011, \sigma_g^2)$;
- all other $\epsilon_{g,T+h}$ and $\epsilon_{z,T+h}$ shocks are drawn from $N(0, 1)$;
- solve for the $\tilde{\epsilon}_{R,T+1:T+4} \geq -2\sigma_r$ such that for $h = 1, 2, 3, 4$ $R'_{T+h}(\epsilon_{R,T+h} = 0) - R'_{T+h}(\epsilon_{R,T+h} = \tilde{\epsilon}_{R,T+h})$ is maximized while $\leq 1\%$ (annualized).

Sunspot Equilibrium: Fiscal and Monetary Policy



- Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for **only fiscal intervention** and **both interventions**.

Targeted-Inflation Equilibrium: Fiscal and Monetary Policy

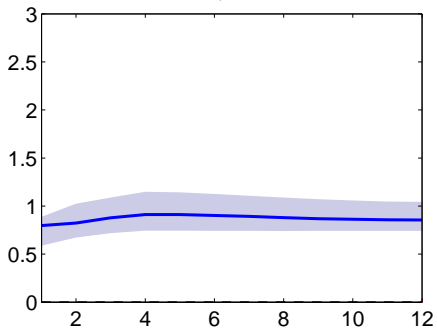


- Figure depicts pointwise medians (solid); 20%-80% percentiles (shaded area) for **only fiscal intervention** and **both interventions**.

Government Spending Multipliers

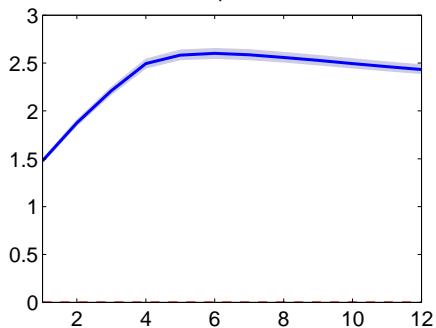
Sunspot Equilibrium

μ^c



Targeted-Inflation Eq.

μ^c



An Alternative Policy Exercise

- Instead of standing at the beginning of 2009Q1, do an ex-post analysis.
- Use the filtered shocks for 2000Q1-2010Q2.
- Two experiments:
 - Fiscal intervention only, calibrated to ARRA.
 - Small issue: Data may already include ARRA.
 - Both fiscal and monetary intervention.
- Intervention dates: 2009Q1 (ZLB) and 2007Q1 (interest rate around 6% p.a.)

Cumulative Government Spending Multipliers

Period	2009Q1		2007Q1	
	Both	Only Fiscal	Both	Only Fiscal
Targeted-Inflation Equilibrium				
1	1.74	1.42	1.44	0.59
2	1.69	1.54	1.85	0.66
3	1.65	1.54	2.21	0.73
4	1.76	1.64	2.46	0.78
5	1.79	1.67	2.56	0.83
6	1.79	1.68	2.62	0.88
Sunspot Equilibrium				
1	0.58	0.58	1.45	0.58
2	0.60	0.60	1.88	0.66
3	0.60	0.61	2.24	0.73
4	0.73	0.70	2.38	0.79
5	0.81	0.77	2.44	0.84
6	0.87	0.82	2.47	0.89

Summary of Empirical Results

- 2008-09 is consistent with a change from the targeted-inflation regime to the deflation regime in the sunspot equilibrium.
 - Data prefers the sunspot model over the targeted-inflation model which needs very large shocks.
- Ex-ante policy exercise in 2009Q1:
 - Cumulative multiplier for targeted-inflation equilibrium just below one versus around 0.7 for the sunspot equilibrium.
 - Very small scope for monetary stimulus for aiding fiscal policy in the sunspot equilibrium, unlike the targeted-inflation equilibrium.
 - Key intuition: The economy escapes / is expected to escape ZLB very quickly in the targeted-inflation equilibrium.
- Ex-post policy exercise:
 - The actual path of the economy in the targeted-inflation economy was “unlikely”: fiscal multiplier larger and monetary policy not effective.
- Since sunspot equilibrium is the more likely explanation of what happened in 2008-2009, policy may not have been very effective.