Unique Monetary Equilibria with Interest Rate Rules*

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Abstract

In contrast to previous literature, we show that there are interest rate rules that implement unique equilibria in standard monetary models. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. We also contribute

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Question

• How can monetary policy implement unique equilibria? (Can interest rate policy provide a nominal anchor?)

• Apparent success with inflation targeting.
  
  – Success is attributed to some form of a Taylor rule.
  
  – In monetary models, Taylor rules do not pin down unique equilibria
    
    * Equilibria may be locally unique, but not globally so.
• Massive literature starting with Sargent and Wallace (1975) and McCallum (1981), including recent literature on local and global determinacy in models with nominal rigidities:

  – Conditions for a unique local equilibrium. There is a unique local equilibrium when there is also a continuum of divergent solutions.


  – Conditions for local determinacy may be conditions for global indeterminacy.


• We show that it is possible to implement a unique equilibrium globally with an interest rate feedback rule.

Outline

• Flexible price economy with a finite horizon: nominal interest rates are not a sufficient policy instrument.

• In an economy with an infinite horizon there is one rule that guarantees global uniqueness.

• Robustness.
  – Sticky prices.
A model with flexible prices

- Identical households, competitive firms, and a government.
- Preferences over consumption and leisure.
- The production uses labor only with a linear technology.
- There are shocks to productivity and government expenditures \( s_t = [A_t, G_t] \). The initial realization \( s_0 \) is given. Discrete distribution. The number of states in period \( t \) is \( \Phi_t \).
- Cash-in-advance constraint with the timing structure as in Lucas (1980).
- Finite horizon economy. The economy lasts for \( T + 1 \) periods, from period 0 to period \( T \). After \( T \), there is an assets market subperiod for the clearing of debts.
Households

- Preferences:
  \[ U = E_0 \left\{ \sum_{t=0}^{T} \beta^t u(C_t, L_t) \right\} \]

- Budget constraints:
  \[ M_t + B_t + E_t Q_{t,t+1} Z_{t+1} \leq W_t, \ 0 \leq t \leq T \]
  \[ W_{t+1} = M_t + R_t B_t + Z_t - P_t C_t + W_t N_t - P_t T_t, \ 0 \leq t \leq T \]

- No-Ponzi:
  \[ W_{T+1} \geq 0 \]

- Cash-in-advance constraint
  \[ P_t C_t \leq M_t, \ 0 \leq t \leq T \]
• FOC:

\[ \frac{u_L(t)}{u_C(t)} = \frac{W_t}{R_t P_t}, \quad 0 \leq t \leq T \]

\[ \frac{u_C(t)}{P_t} = R_t E_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right], \quad 0 \leq t \leq T - 1 \]

\[ Q_{t,t+1} = \beta \frac{u_C(t + 1)}{u_C(t)} \frac{P_t}{P_{t+1}}, \quad 0 \leq t \leq T - 1 \]

\[ W_{T+1} = 0 \]
Firms

• The firms are competitive and prices are flexible.

• Production function of the representative firm is linear

\[ Y_t = A_t N_t, \ 0 \leq t \leq T \]

• The equilibrium real wage is

\[ \frac{W_t}{P_t} = A_t, \ 0 \leq t \leq T \]
Government

- The policy variables are lump sum taxes $T_t$, state noncontingent interest rates $R_t$, state contingent nominal returns $Q_{t,t+1}^{-1}$, money supplies $M_t$, state noncontingent public debt $B_t$, state contingent debt $Z_{t+1}$.

- Policy: Maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables.

- Budget constraints:

\[
M_0 + B_0 = \mathbb{W}_0
\]

\[
M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}G_{t-1} - P_{t-1}T_{t-1},
1 \leq t \leq T
\]

\[
\mathbb{W}_{T+1} = M_T + R_TB_T + P_TG_T - P_TT_T = 0
\]
Market clearing

\[ C_t + G_t = A_t N_t, \ 0 \leq t \leq T \]

\[ 1 - L_t = N_t, \ 0 \leq t \leq T \]
Equilibrium

Equilibrium conditions for the variables \( \{C_t, L_t, R_t, M_t, P_t\} \) are

\[
C_t + G_t = A_t(1 - L_t), \quad 0 \leq t \leq T
\]
\[
\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \quad 0 \leq t \leq T
\]

From these get \( C_t = C(R_t), \ L_t = L(R_t) \)

\[
P_tC_t = M_t, \quad 0 \leq t \leq T
\]

which gives \( P_t = \frac{M_t}{C(R_t)} \)

\[
\frac{u_C(t)}{P_t} = R_tE_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right], \quad 0 \leq t \leq T - 1
\]
The equilibrium conditions for the variables \( \{R_t, M_t, P_t\} \) are:

\[
\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right],
\]

\[0 \leq t \leq T - 1\]

\[P_t = \frac{M_t}{C(R_t)}, \quad 0 \leq t \leq T\]
Interest Rate Policy.

• Interest rates are set in every date and state.

There is a unique equilibrium if prices (money supply?) are set in every state at date $T$

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right],$$

$$0 \leq t \leq T - 1$$

$$P_T = \frac{M_T}{C(R_T)}$$

• Deterministic economy.

• Uncertainty. Need a nominal anchor for every history.
• Arbitrarily large time horizon.

• Does it matter whether policy is conducted with an interest rate or a money supply rule?

• Does it matter which particular feedback rule is used?

• Does it matter whether prices are flexible or sticky?

• Preferences and technology?
Finite vs Infinite Horizon

- Need to set prices (money supply) in every state at date $T$ and after that in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T + 1$.

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[ \frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \ t \geq 0$$

$$P_t = \frac{M_t}{C(R_t)}, \ t \geq 0$$

- Standard approach: Local determinacy. A unique local equilibrium and multiple global equilibria.
• In the infinite horizon, preferences are relevant:

• The utility function is additively separable and logarithmic in consumption.

\[ U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) + v(L_t) \right] \right\} \]
• Equilibrium conditions:

\[
\frac{1}{M_t} = \beta R_t E_t \frac{1}{M_{t+1}}
\]

\[
P_t = \frac{M_t}{C(R_t)}
\]

\[
C_t = C(R_t)
\]

\[
L_t = L(R_t)
\]
In the finite horizon economy:

\[ \frac{1}{M_t} = \beta R_t E_t \left[ \frac{1}{M_{t+1}} \right], \quad t = 0, \ldots, T - 1 \]

When the money supply is set exogenously in every state the nominal interest rates, \( R_t \), are determined for \( t = 0, \ldots, T - 1 \). It is still necessary to set exogenously the interest rates, \( R_T \), in every terminal state.
Single Instrument Feedback Rules

- Interest rate rules such that there is a unique equilibrium:

\[ R_t = \frac{\xi_t}{E_t^{\beta u_C(t+1)}}, \]

\( \xi_t \) is an exogenous variable.
\[ R_t = \frac{\xi_t}{E_t \beta u_C(t+1) P_{t+1}} \]

- From

\[ \frac{u_C(t)}{P_t} = R_t E_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right] \]

get

\[ \frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0 \]

and

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \ t \geq 0 \]
• From

\[ \frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \quad t \geq 0 \]

and

\[ C_t + G_t = A_t(1 - L_t), \quad t \geq 0 \]

and the cash in advance conditions, determine uniquely the variables \( C_t, L_t, \) and \( M_t. \)

• From

\[ \frac{u_C(t)}{P_t} = \xi_t, \quad t \geq 0 \]

get \( P_t. \)
• The policy rule resembles the rules followed by central banks?

\[ R_t = \frac{\xi_t}{E_t^{\beta u_C(t+1)}}, \]

• Depending on the exogenous process \( \xi_t \), can implement each allocation in a set of implementable allocations, including the (Friedman rule) optimal allocation.
• Define a set of implementable equilibria where the sequences of policy variables can be any sequences that satisfy the government budget constraint.

**Definition 1** The set of implementable allocations, prices and policy variables \( \{C_t, L_t, P_t, R_t \geq 1, M_t, B_t, T_t\} \) is the set of sequences that satisfies conditions of the problem for the households, the conditions for the firms and the budget constraint of the government.

• The interest rate rule can be used to implement uniquely each implementable equilibrium in Definition 1.
• With
\[ \xi_t = \frac{1}{k \beta^t}, \ t \geq 0, \]
where \( k \) is a positive constant, get \( R_t = 1 \).
\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}} \]

• Let \( C_t = C^*(A_t, G_t) \), \( L_t = L^*(A_t, G_t) \) be the first best allocation. The price level is given by
\[ u_C(C^*(A_t, G_t), L^*(A_t, G_t)) = \frac{1}{k \beta^t}, \ t \geq 0. \]

• The equilibrium money stock is obtained using the cash-in-advance constraint if it holds with equality.

• There are other possible equilibrium processes for the path of the price level associated with the Friedman rule. The rule with \( \xi_t = \frac{\mu_t}{k(\rho \beta)^t} \), where \( \mu_t = \rho \mu_{t-1} + \varepsilon_t \) and \( \varepsilon_t \) is a white noise, also implies \( R_t = 1 \) and achieves the first best allocation with different processes for the price level depending on the choice of \( k, \rho \) and \( \varepsilon_t \).
Money supply rules

**Proposition 1** Suppose the cash-in-advance constraint holds exactly. Every equilibrium in Definition 1 can be implemented (uniquely) with the money supply feedback rule,

\[ M_t = \frac{C_t u_C(t)}{\xi_t}, \]

where \( \xi_t \) is an exogenous variable.

- Using the cash in advance conditions with equality,
  \[ \frac{u_C(t)}{P_t} = \xi_t \]

- Using the intertemporal conditions,
  \[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \]

This, together with the intratemporal conditions and the resource constraints, determine \( C_t, L_t, P_t, R_t \), all \( t \geq 0 \) and \( s^t \).
Robustness: 
Capital

- Intertemporal condition

\[ \frac{u_C(t)}{P_t} = R_tE_t \left[ \frac{\beta u_C(t + 1)}{P_{t+1}} \right] \]

- Interest rate rule

\[ R_t = \frac{\xi_t}{E_t \beta u_C(t+1)} \]

- Get

\[ \frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0, \]

and

\[ R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \]

- Once the sequence of nominal interest rates \( R_t \) is determined, the allocations in the model with capital are also uniquely determined and then the price level is also determined uniquely.
Sticky prices: Prices set in advance

- Continuum of firms, indexed by $i \in [0, 1]$, each producing a differentiated good also indexed by $i$. The firms are monopolistic competitive and set prices in advance with different lags.

- $C_t$ is now the composite good

$$C_t = \left[ \int_0^1 c_t(i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}}, \theta > 1,$$

$c_t(i)$ is consumption of good $i$.

- The demand function for each good $i$ is

$$c_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} C_t,$$

$p_t(i)$ is the price of good $i$ and $P_t$ is the price level,

$$P_t = \left[ \int p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}.$$
• The households’ intertemporal and intratemporal conditions are as before.

• The government must finance an exogenous path of government purchases \( \{G_t\}_{t=0}^{\infty} \), such that

\[
G_t = \left[ \int_0^1 g_t(i)^{\theta-1} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0.
\]

and minimizes expenditures on \( G_t \), so that

\[
\frac{g_t(i)}{G_t} = \left( \frac{p_t(i)}{P_t} \right)^{-\theta}.
\]

• Market clearing

\[
c_t(i) + g_t(i) = A_t n_t(i),
\]

\[
\int_0^1 n_t(i) di = N_t.
\]
• Can write the resource constraints as

\[
(C_t + G_t) \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t.
\]

• A fraction \(\alpha_j\) firms set prices \(j\) periods in advance with \(j = 0, \ldots, J - 1\). Firms decide the price for period \(t\) with the information up to period \(t - j\) to maximize profits:

\[
E_{t-j} \left[ Q_{t-j,t+1} (p_t(i)y_t(i) - W_t n_t(i)) \right],
\]

subject to

\[y_t(i) \leq A_t n_t(i)\]

and

\[y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t,\]

where \(y_t(i) = c_t(i) + g_t(i)\) and \(Y_t = C_t + G_t\).
• The optimal price for a firm setting the price for period $t$, $j$ periods in advance, is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta - 1)} E_{t-j} \left[ \eta_{t,j} \frac{W_t}{A_t} \right],$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1} P_t^\theta Y_t}{E_{t-j} \left[ Q_{t-j,t+1} P_t^\theta Y_t \right]}.$$

• Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions, and using the intertemporal condition as well as the households’ intratemporal condition, we obtain

$$E_{t-j} \left[ \frac{u_C(t)}{R_t} P_t^{\theta-1} A_t (1 - L_t) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} (1 - L_t) \frac{P_t}{p_{t,j}} \right] = 0, \ j = 0, \ldots, J-1.$$
Proposition 2 When prices are set in advance, if policy is conducted with the interest rate feedback rule

\[ R_t = \frac{\xi_t}{E_t^{\beta u_C(t+1)}}, \]

where \( \xi_t \) is an exogenous variable, there is a unique equilibrium.
\[
\frac{u_C(t)}{P_t} = \xi_t, \ t \geq 0
\]

\[
R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}, \ t \geq 0
\]

These conditions together with the resource constraints

\[
(C_t + G_t) \sum_{j=0}^{J-1} \alpha_j \left( \frac{p_{t,j}}{P_t} \right)^{-\theta} = A_t N_t,
\]

the intratemporal conditions

\[
E_{t-j} \left[ \frac{u_C(t)}{R_t} P_t^{\theta-1} A_t (1 - L_t) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} (1 - L_t) \frac{P_t}{p_{t,j}} \right] = 0, \ j = 0, \ldots, J-1,
\]

the conditions for the price level

\[
P_t = \left[ \sum_{j=0}^{J-1} \alpha_j (p_{t,j})^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\]
and the cash in advance constraints with equality, determine uniquely the variables $C_t, L_t, P_t, p_{t,j}, j = 0, \ldots J - 1$, and $M_t$. $p_{0,j}, j = 1, \ldots J - 1$ are exogenous.
Calvo (1983) staggered prices

- Standard newkeynesian model.
- Exogenous velocity made arbitrarily large, so that it is a cashless economy.
- Log-linearized model.
• Cashless economies
\[ \frac{P_t C_t}{v_t} \leq M_t, \]
where \( v_t \to \infty \).

• In the limit case, the households conditions are
\[ \frac{u_C(t)}{u_L(t)} = \frac{P_t}{W_t}, \quad t \geq 0, \]
and
\[ \frac{u_C(t)}{P_t} = E_t \left[ R_{t+1} \frac{\beta u_C(t + 1)}{P_{t+1}} \right]. \]
• Calvo pricing

• In each period, a fraction $1 - \alpha$ of firms can choose optimally their prices, $p_t^*$. 

• Optimal price

$$p_t^* = \frac{\theta}{\theta - 1} \frac{E_{t,j=0}^{\infty} (\alpha/\beta)^j u_c(t+j+1)P_t}{P_{t+j+1}} s_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{1+\theta} Y_{t+j}$$

$$s_{t+j} = \frac{W_{t+j}}{A_{t+j}P_{t+j}}$$

is the real marginal cost.

• The expression for the price level is

$$P_t^{1-\theta} = (1 - \alpha) p_t^{1-\theta} + \alpha (P_{t-1})^{1-\theta}$$
• Loglinearize around a steady-state with zero inflation. Let \( \pi_t = \frac{P_t}{P_{t-1}} \).

• Price setting condition (Phillips curve):

\[
\hat{\pi}_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \hat{s}_t + \beta E_t \hat{\pi}_{t+1}
\]

where

\[
\hat{s}_t = \hat{\omega}_t - \hat{A}_t
\]
• The loglinearization of the intratemporal and intertemporal conditions gives

\[ \phi_c \hat{C}_t + \phi_L \hat{L}_t = \hat{\omega}_t \]

where \( \phi_x = \frac{\partial u_L(t) u_C(t)}{\partial x} x, x = C, L, \) and

\[ E_t \hat{R}_{t+1} - E_t (\hat{\pi}_{t+1}) = \hat{r}_t \]

where

\[ \hat{r}_t \equiv - \left( \frac{u_{cc} C'}{u_c} \right) E_t (\hat{C}_{t+1} - \hat{C}_t) - \left( \frac{u_{cl} L}{u_c} \right) E_t (\hat{L}_{t+1} - \hat{L}_t) . \]

• The loglinearization of the resource constraints

\[ (C_t + G_t) \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t \]

is

\[ \frac{C}{Y} \hat{C}_t + \frac{G}{Y} \hat{G}_t = \hat{A}_t + \hat{N}_t , \]
with

\[ \hat{L}_t = -\kappa \hat{N}_t \]

where \( \kappa = \frac{N}{L} \).

- Using the households intratemporal conditions and the feasibility conditions, can write the allocations, \( \hat{C}_t, \hat{N}_t, \hat{L}_t \) as functions of the real wage \( \hat{\omega}_t \) and the shocks.

- The expression for the Phillips curve is

\[ \hat{\pi}_t = \lambda \left( \hat{\omega}_t - \hat{A}_t \right) + \beta E_t \hat{\pi}_{t+1} \]
• Consider the following interest rate rule

\[ \hat{R}_{t+1} = \hat{r}_t + \hat{P}_{t+1} - \xi_t \]

where \( \xi_t \) is an exogenous process.

• Together with

\[ E_t \hat{R}_{t+1} - E_t (\hat{\pi}_{t+1}) = \hat{r}_t, \]

get

\[ \hat{P}_t = \xi_t, \]

so that the price level is uniquely pinned down.

• Since \( \hat{\pi}_{t+1} = \hat{P}_{t+1} - \hat{P}_t = \xi_{t+1} - \xi_t \), from the Phillips curve, determine \( \hat{\omega}_t \) uniquely.

• From the intratemporal and resource constraints, get \( \hat{L}_t, \hat{N}_t \) and \( \hat{C}_t \).

• This pins down \( \hat{r}_t \) and therefore \( \hat{R}_{t+1} \).
In order to implement the equilibrium with zero inflation, need $\xi_t = 0$. Then $\hat{P}_t = 0$, so that inflation is the steady state zero inflation. From

$$\hat{\pi}_t = \lambda \left( \hat{\omega}_t - \hat{A}_t \right) + \beta E_t \hat{\pi}_{t+1},$$

have $\hat{\omega}_t = \hat{A}_t$, as under flexible prices.
• The rule is the same independently of the price setting restrictions.
• \( \hat{P}_t = \xi_t \). Under flexible prices we have, instead of the Phillips curve,
\[
\hat{\omega}_t = \hat{A}_t
\]
The allocations, \( \hat{L}_t, \hat{N}_t \) and \( \hat{C}_t \), are determined uniquely. With this can determine uniquely \( \hat{r}_t \) and therefore \( \hat{R}_{t+1} \).
• Share of firms $0 < \alpha < 1$ set prices one period in advance while the remaining $1 - \alpha$ firms set flexible prices

$$\hat{P}_t = \alpha \hat{P}_t^S + (1 - \alpha) \hat{P}_t^f, \quad t \geq 0$$

$$\hat{P}_t^S = E_{t-1} \left[ \hat{P}_t \right], \quad t \geq 1,$$

$$\hat{P}_t^f = \hat{W}_t - \hat{A}_t.$$

• Then

$$\alpha \left[ \hat{P}_t - E_{t-1} \hat{P}_t \right] = (1 - \alpha) \left[ \hat{\omega}_t - \hat{A}_t \right], \quad t \geq 1$$

$$\alpha \left[ \hat{P}_0 - \hat{P}_0^S \right] = (1 - \alpha) \left[ \hat{\omega}_0 - \hat{A}_0 \right]$$

where $\hat{P}_0^S$ is exogenous. Once $\hat{P}_t$ is determined uniquely, so is $\hat{\omega}_t$. The nominal interest rates and the allocations are determined uniquely as above.
A simple endowment economy

- Euler equation for the representative household:

\[
\frac{u_c(Y_t)}{P_t} = R_t E_t E_t u_c(Y_{t+1}) \frac{\beta u_c(Y_{t+1})}{P_{t+1}}
\]

\{Y_t\} is the endowment process.

- In log deviations from a deterministic steady state with constant inflation \(\pi^*\):

\[
\hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t,
\]

where \(r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}\), or

\[
\hat{R}_t = \hat{r}_t + E_t \hat{\pi}_{t+1}.
\]
Monetary policy in the endowment economy

- Interest rate target.
  - Unique path for the conditional expectation of inflation $E_t \hat{\pi}_{t+1}$,
  - but not for the initial price level, nor the distribution of realized inflation across states.

- Current feedback rule:
  $$\hat{R}_t = \hat{r}_t + \tau \hat{\pi}_t$$

- Equilibria:
  $$\tau \hat{\pi}_t - E_t (\hat{\pi}_{t+1}) = 0$$
  
  - Equilibrium with $\hat{\pi}_t = 0$ and $\hat{R}_t = \hat{r}_t$.
  - The equilibrium with $\hat{\pi}_t = 0$ is locally unique if $\tau > 1$ (Taylor principle).
  - Continuum of divergent solutions.
  - In the nonlinear model divergent solutions can converge to another steady state.
• Forward looking rules do not guarantee local determinacy:
\[ \hat{R}_t = \hat{r}_t + \tau E_t \hat{\pi}_{t+1} \]
and
\[ \hat{R}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} \]
implies
\[ (\tau - 1) E_t (\hat{\pi}_{t+1}) = 0 \]
• For \( \tau \neq 1 \), only expected inflation is pinned down, not the distribution of prices across states.
• With a backward rule
\[ \hat{R}_t = \hat{r}_t + \tau \hat{\pi}_{t-1}, \]
the dynamic equation is
\[ \tau \hat{\pi}_{t-1} - E_t (\hat{\pi}_{t+1}) = 0. \]
  - If \( \hat{\pi}_{-1} = 0 \), there is a solution with \( \hat{\pi}_t = 0 \) all \( t \).
  - There are again multiple solutions and a locally determinate solution, \( \hat{\pi}_t = 0 \), with \( \tau > 1 \), provided \( \hat{\pi}_{-1} = 0 \).
• Wicksellian interest rate rules (Woodford, 2003) have the interest rate respond to the price level rather than inflation.

• Policy rule

\[ \hat{R}_t = \hat{r}_t + \phi \hat{P}_t, \]

where \( \phi > 0 \).

• Euler equation

\[ \hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t, \]

• Equilibria:

\[ (1 + \phi) \hat{P}_t - E_t \hat{P}_{t+1} = 0 \]

– Equilibrium with \( \hat{P}_t = 0 \) and \( \hat{R}_t = \hat{r}_t \).

– The equilibrium with \( \hat{P}_t = 0 \) is locally unique if \( \phi > 0 \).

– Continuum of divergent solutions.
Rules that implement unique equilibria

- Price level targeting rule:

\[
\hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} + \hat{\xi}_t
\]

where \( \hat{\xi}_t \) is an exogenous random variable.

- Euler equation

\[
\hat{R}_t = \hat{r}_t + E_t \hat{P}_{t+1} - \hat{P}_t,
\]

- Equilibria

\[
\hat{P}_t = \hat{\xi}_t.
\]

Concluding remarks.

- Interest rate rules can implement unique local equilibria with stable prices. These are normally associated with multiple global equilibria.

- One way out is the rule proposed in this paper. Problems of robustness as in all this literature.