Unique Monetary Equilibria with Interest Rate Rules*

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Abstract

In contrast to previous literature, we show that there are interest rate rules that implement unique equilibria in standard monetary models. This is a contribution to a literature that either concentrates on conditions for local determinacy, or criticizes that approach showing that local determinacy might be associated with global indeterminacy. We also contribute

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Question

- How can monetary policy implement unique equilibria?

  (Can interest rate policy provide a nominal anchor?)

- Apparent success with inflation targeting.

  - Success is attributed to some form of a Taylor rule.
  - In monetary models, Taylor rules do not pin down unique equilibria

    * Equilibria may be locally unique, but not globally so.

- Massive literature starting with Sargent and Wallace (1975) and McCallum (1981), including recent literature on local and global determinacy in models with nominal rigidities:
  - Conditions for a unique local equilibrium. There is a unique local equilibrium when there is also a continuum of divergent solutions.

    McCallum (1981), Woodford (2003), Clarida, Gali and Gertler (1999, 2000), Carlstrom and Fuerst (2001, 2002), Benhabib, Schmitt-Grohe and Uribe (2001a), among others.
  - Conditions for local determinacy may be conditions for global indeterminacy.

    Benhabib, Schmitt-Grohe and Uribe (2001b, 2002, 2003), Schmitt-Grohe and Uribe (2001).
  - Christiano and Rostagno (2002) and Atkeson, Chari and Kehoe (2007).
  - Loisel (2006). Similar mechanism to ours in a linear dynamic model.

- We show that it is possible to implement a unique equilibrium globally with an interest rate feedback rule.

Outline

- Flexible price economy with a finite horizon: nominal interest rates are not a sufficient policy instrument.
- In an economy with an infinite horizon there is one rule that guarantees global uniqueness.
- Robustness.
  - Richer structures. Capital.
  - Sticky prices.

A model with flexible prices

- Identical households, competitive firms, and a government.
- Preferences over consumption and leisure.
- The production uses labor only with a linear technology.
- There are shocks to productivity and government expenditures $s_t = [A_t, G_t]$. The initial realization s_0 is given. Discrete distribution. The number of states in period t is Φ_t.
- Cash-in-advance constraint with the timing structure as in Lucas (1980).
- Finite horizon economy. The economy lasts for $T+1$ periods, from period 0 to period T. After T, there is an assets market subperiod for the clearing of debts.

Households

- Preferences:

$$U = E_0 \left\{ \sum_{t=0}^{T} \beta^t u\left(C_t, L_t\right) \right\}$$

- Budget constraints:

$$M_t + B_t + E_t Q_{t,t+1} Z_{t+1} \leq \mathbb{W}_t,\ 0 \leq t \leq T$$

$$\mathbb{W}_{t+1} = M_t + R_t B_t + Z_t - P_t C_t + W_t N_t - P_t T_t,\ 0 \leq t \leq T$$

- No-Ponzi:

$$\mathbb{W}_{T+1} \geq 0$$

- Cash-in-advance constraint

$$P_t C_t \leq M_t,\ 0 \leq t \leq T$$

- FOC:

$$\frac{u_L(t)}{u_C\left(t\right)} = \frac{W_t}{R_t P_t}, \; 0 \leq t \leq T$$

$$\frac{u_C\left(t\right)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}}\right], \; 0 \leq t \leq T-1$$

$$Q_{t,t+1} = \beta \frac{u_C(t+1)}{u_C(t)} \frac{P_t}{P_{t+1}}, \; 0 \leq t \leq T-1$$

$$\mathbb{W}_{T+1} = 0$$

Firms

- The firms are competitive and prices are flexible.
- Production function of the representative firm is linear

$$Y_t = A_t N_t,\ 0 \le t \le T$$

- The equilibrium real wage is

$$\frac{W_t}{P_t} = A_t,\ 0 \le t \le T$$

Government

- The policy variables are lump sum taxes T_t, state noncontingent interest rates R_t, state contingent nominal returns $Q_{t,t+1}^{-1}$, money supplies M_t, state noncontingent public debt B_t, state contingent debt Z_{t+1}.
- Policy: Maps sequences of quantities, prices and policy variables into sets of sequences of the policy variables.
- Budget constraints:

$$M_0 + B_0 = \mathbb{W}_0$$

$$\begin{aligned} & M_t + B_t \\ = \; & M_{t-1} + R_{t-1}B_{t-1} + P_{t-1}G_{t-1} - P_{t-1}T_{t-1}, \\ 1 \; \leq \; & t \leq T \end{aligned}$$

$$\mathbb{W}_{T+1} = M_T + R_T B_T + P_T G_T - P_T T_T = 0$$

Market clearing

$$C_t + G_t = A_t N_t,\ 0 \leq t \leq T$$

$$1 - L_t = N_t,\ 0 \leq t \leq T$$

Equilibrium

Equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, P_t\}$ are

$$C_t + G_t = A_t(1 - L_t),\, 0 \leq t \leq T$$

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},\, 0 \leq t \leq T$$

From these get $C_t = C(R_t)$, $L_t = L(R_t)$

$$P_t C_t = M_t,\, 0 \leq t \leq T$$

which gives $P_t = \frac{M_t}{C(R_t)}$

$$\frac{u_C\,(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right],\, 0 \leq t \leq T - 1$$

The equilibrium conditions for the variables $\{R_t, M_t, P_t\}$ are:

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t\left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}}\right],$$
$$0 \leq t \leq T-1$$

$$P_t = \frac{M_t}{C(R_t)}, \; 0 \leq t \leq T$$

Interest Rate Policy.

- Interest rates are set in every date and state.

  There is a unique equilibrium if prices (money supply?) are set in every state at date T

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}}\right],$$

$$0 \leq t \leq T-1$$

$$P_T = \frac{M_T}{C(R_T)}$$

- Deterministic economy.
- Uncertainty. Need a nominal anchor for every history.

- Arbitrarily large time horizon.
- Does it matter whether policy is conducted with an interest rate or a money supply rule?
- Does it matter which particular feedback rule is used?
- Does it matter whether prices are flexible or sticky?
- Preferences and technology?

Finite vs Infinite Horizon

- Need to set prices (money supply) in every state at date T and after that in $\Phi_t - \Phi_{t-1}$ states for every $t \geq T+1$.

$$\frac{u_C(C(R_t), L(R_t))}{P_t} = \beta R_t E_t \left[\frac{u_C(C(R_{t+1}), L(R_{t+1}))}{P_{t+1}} \right], \; t \geq 0$$

$$P_t = \frac{M_t}{C(R_t)}, \; t \geq 0$$

- Standard approach: Local determinacy. A unique local equilibrium and multiple global equilibria.

- In the infinite horizon, preferences are relevant:

- The utility function is additively separable and logarithmic in consumption.

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\right) + v(L_t) \right] \right\}$$

- Equilibrium conditions:

$$\frac{1}{M_t} = \beta R_t E_t \frac{1}{M_{t+1}}$$

$$P_t = \frac{M_t}{C(R_t)}$$

$$C_t = C(R_t)$$

$$L_t = L(R_t)$$

- In the finite horizon economy:

$$\frac{1}{M_t} = \beta R_t E_t \left[\frac{1}{M_{t+1}} \right], \, t = 0, ..., T-1$$

When the money supply is set exogenously in every state the nominal interest rates, R_t, are determined for $t = 0, ..., T-1$. It is still necessary to set exogenously the interest rates, R_T, in every terminal state.

Single Instrument Feedback Rules

- Interest rate rules such that there is a unique equilibrium:

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

ξ_t is an exogenous variable.

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}}$$

- From

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right]$$

get

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0$$

- From
$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t},\ t \geq 0$$
and
$$C_t + G_t = A_t(1 - L_t),\ t \geq 0$$
and the cash in advance conditions, determine uniquely the variables C_t, L_t, and M_t.

- From
$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$
get P_t.

- The policy rule resembles the rules followed by central banks?

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

- Depending on the exogenous process ξ_t, can implement each allocation in a set of implementable allocations, including the (Friedman rule) optimal allocation.

- Define a set of implementable equilibria where the sequences of policy variables can be any sequences that satisfy the government budget constraint.

Definition 1 *The set of implementable allocations, prices and policy variables* $\{C_t, L_t, P_t, R_t \geq 1, M_t, B_t, T_t\}$ *is the set of sequences that satisfies conditions of the problem for the households, the conditions for the firms and the budget constraint of the government.*

- The interest rate rule can be used to implement uniquely each implementable equilibrium in Definition 1.

- With

$$\xi_t = \frac{1}{k\beta^t}, t \geq 0,$$

where k is a positive constant, get $R_t = 1$.

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}$$

- Let $C_t = C^*(A_t, G_t)$, $L_t = L^*(A_t, G_t)$ be the first best allocation. The price level is given by

$$\frac{u_C(C^*(A_t, G_t), L^*(A_t, G_t))}{P_t} = \frac{1}{k\beta^t}, \ t \geq 0.$$

- The equilibrium money stock is obtained using the cash-in-advance constraint if it holds with equality.

- There are other possible equilibrium processes for the path of the price level associated with the Friedman rule. The rule with $\xi_t = \frac{\mu_t}{k(\rho\beta)^t}$,where $\mu_t = \rho\mu_{t-1} + \varepsilon_t$ and ε_t is a white noise, also implies $R_t = 1$ and achieves the first best allocation with different processes for the price level depending on the choice of k, ρ and ε_t.

Money supply rules

Proposition 1 *Suppose the cash-in-advance constraint holds exactly. Every equilibrium in Definition 1 can be implemented (uniquely) with the money supply feedback rule,*

$$M_t = \frac{C_t u_C(t)}{\xi_t},$$

where ξ_t is an exogenous variable.

- Using the cash in advance conditions with equality,

$$\frac{u_C(t)}{P_t} = \xi_t$$

- Using the intertemporal conditions,

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

  This, together with the intratemporal conditions and the resource constraints, determine C_t, L_t, P_t, R_t, all $t \geq 0$ and s^t.

Robustness: Capital

- Intertemporal condition

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}}\right]$$

- Interest rate rule

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

- Get

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0,$$

and

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}.$$

- Once the sequence of nominal interest rates R_t is determined, the allocations in the model with capital are also uniquely determined and then the price level is also determined uniquely.

Sticky prices: Prices set in advance

- Continuum of firms, indexed by $i \in [0,1]$, each producing a differentiated good also indexed by i. The firms are monopolistic competitive and set prices in advance with different lags.

- C_t is now the composite good

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

  $c_t(i)$ is consumption of good i.

- The demand function for each good i is

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t,$$

  $p_t(i)$ is the price of good i and P_t is the price level,

$$P_t = \left[\int p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}.$$

- The households' intertemporal and intratemporal conditions are as before.

- The government must finance an exogenous path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 0.$$

  and minimizes expenditures on G_t, so that

$$\frac{g_t(i)}{G_t} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta}.$$

- Market clearing

$$c_t(i) + g_t(i) = A_t n_t(i),$$

$$\int_0^1 n_t(i) di = N_t.$$

- Can write the resource constraints as

$$(C_t + G_t) \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t.$$

- A fraction α_j firms set prices j periods in advance with $j = 0, ...J - 1$. Firms decide the price for period t with the information up to period $t - j$ to maximize profits:

$$E_{t-j} \left[Q_{t-j,t+1} \left(p_t(i) y_t(i) - W_t n_t(i) \right) \right],$$

subject to

$$y_t(i) \le A_t n_t(i)$$

and

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t,$$

where $y_t(i) = c_t(i) + g_t(i)$ and $Y_t = C_t + G_t$.

- The optimal price for a firm setting the price for period t, j periods in advance, is

$$p_t(i) \equiv p_{t,j} = \frac{\theta}{(\theta-1)} E_{t-j}\left[\eta_{t,j}\frac{W_t}{A_t}\right],$$

where

$$\eta_{t,j} = \frac{Q_{t-j,t+1}P_t^{\theta}Y_t}{E_{t-j}\left[Q_{t-j,t+1}P_t^{\theta}Y_t\right]}.$$

- Substituting the state contingent prices $Q_{t-j,t+1}$ in the price setting conditions, and using the intertemporal condition as well as the households' intratemporal condition, we obtain

$$E_{t-j}\left[\frac{u_C(t)}{R_t}P_t^{\theta-1}A_t\left(1-L_t\right) - \frac{\theta}{(\theta-1)}u_L(t)P_t^{\theta-1}\left(1-L_t\right)\frac{P_t}{p_{t,j}}\right] = 0,\ j = 0, ...J-1.$$

Proposition 2 *When prices are set in advance, if policy is conducted with the interest rate feedback rule*

$$R_t = \frac{\xi_t}{E_t \frac{\beta u_C(t+1)}{P_{t+1}}},$$

where ξ_t is an exogenous variable, there is a unique equilibrium.

$$\frac{u_C(t)}{P_t} = \xi_t,\ t \geq 0$$

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}},\ t \geq 0$$

These conditions together with the resource constraints

$$(C_t + G_t) \sum_{j=0}^{J-1} \alpha_j \left(\frac{p_{t,j}}{P_t} \right)^{-\theta} = A_t N_t,$$

the intratemporal conditions

$$E_{t-j} \left[\frac{u_C(t)}{R_t} P_t^{\theta-1} A_t \left(1 - L_t\right) - \frac{\theta}{(\theta - 1)} u_L(t) P_t^{\theta-1} \left(1 - L_t\right) \frac{P_t}{p_{t,j}} \right] = 0,\ j = 0, ...J{-}1,$$

the conditions for the price level

$$P_t = \left[\sum_{j=0}^{J-1} \alpha_j \left(p_{t,j}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

and the cash in advance constraints with equality, determine uniquely the variables C_t, L_t, P_t, $p_{t,j}$, $j = 0, ...J-1$, and M_t. $p_{0,j}$, $j = 1, ...J-1$ are exogenous.

Calvo (1983) staggered prices

- Standard newkeynesian model.
- Exogenous velocity made arbitrarily large, so that it is a cashless economy.
- Log-linearized model.
- Loisel (2006).

- Cashless economies
$$\frac{P_t C_t}{v_t} \leq M_t,$$
where $v_t \to \infty$.

- In the limit case, the households conditions are
$$\frac{u_C(t)}{u_L(t)} = \frac{P_t}{W_t},\ t \geq 0,$$
and
$$\frac{u_C(t)}{P_t} = E_t \left[R_{t+1} \frac{\beta u_C(t+1)}{P_{t+1}} \right].$$

- Calvo pricing
- In each period, a fraction $1-\alpha$ of firms can choose optimally their prices, p_t^*.
- Optimal price

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{u_c(t+j+1)P_t}{P_{t+j+1}} s_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{1+\theta} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{u_c(t+j+1)P_t}{P_{t+j+1}} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j}}.$$

$s_{t+j} = \frac{W_{t+j}}{A_{t+j} P_{t+j}}$ is the real marginal cost.

- The expression for the price level is

$$P_t^{1-\theta} = (1-\alpha)\, p_t^{*1-\theta} + \alpha \left(P_{t-1}\right)^{1-\theta}$$

- Loglinearize around a steady-state with zero inflation. Let $\pi_t = \frac{P_t}{P_{t-1}}$.
- Price setting condition (Phillips curve):

$$\widehat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\widehat{s}_t + \beta E_t \widehat{\pi}_{t+1}$$

where

$$\widehat{s}_t = \widehat{\varpi}_t - \widehat{A}_t$$

- The loglinearization of the intratemporal and intertemporal conditions gives

$$\phi_c \widehat{C}_t + \phi_L \widehat{L}_t = \widehat{\varpi}_t$$

where $\phi_x = \frac{\partial \frac{u_L(t)}{u_C(t)}}{\partial x} \frac{u_C(t)}{u_L(t)} x$, $x = C, L$, and

$$E_t \widehat{R}_{t+1} - E_t \left(\widehat{\pi}_{t+1} \right) = \widehat{r}_t$$

where

$$\widehat{r}_t \equiv -\left(\frac{u_{cc} C}{u_c} \right) E_t \left(\widehat{C}_{t+1} - \widehat{C}_t \right) - \left(\frac{u_{cl} L}{u_c} \right) E_t \left(\widehat{L}_{t+1} - \widehat{L}_t \right).$$

- The loglinearization of the resource constraints

$$\left(C_t + G_t \right) \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta} di = A_t N_t$$

is

$$\frac{C}{Y} \widehat{C}_t + \frac{G}{Y} \widehat{G}_t = \widehat{A}_t + \widehat{N}_t,$$

with

$$\widehat{L}_t = -\kappa \widehat{N}_t$$

where $\kappa = \frac{N}{L}$.

- Using the households intratemporal conditions and the feasibility conditions, can write the allocations, $\widehat{C}_t$, $\widehat{N}_t$, $\widehat{L}_t$ as functions of the real wage $\widehat{\varpi}_t$ and the shocks.

- The expression for the Phillips curve is

$$\widehat{\pi}_t = \lambda \left(\widehat{\varpi}_t - \widehat{A}_t \right) + \beta E_t \widehat{\pi}_{t+1}$$

- Consider the following interest rate rule

$$\widehat{R}_{t+1} = \widehat{r}_t + \widehat{P}_{t+1} - \xi_t$$

where ξ_t is an exogenous process.

- Together with

$$E_t \widehat{R}_{t+1} - E_t \left(\widehat{\pi}_{t+1} \right) = \widehat{r}_t,$$

get

$$\widehat{P}_t = \xi_t,$$

so that the price level is uniquely pinned down.

- Since $\widehat{\pi}_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t = \xi_{t+1} - \xi_t$, from the Phillips curve, determine $\widehat{\varpi}_t$ uniquely.

- From the intratemporal and resource constraints, get $\widehat{L}_t$, $\widehat{N}_t$ and $\widehat{C}_t$.

- This pins down $\widehat{r}_t$ and therefore $\widehat{R}_{t+1}$.

- In order to implement the equilibrium with zero inflation, need $\xi_t = 0$. Then $\widehat{P}_t = 0$, so that inflation is the steady state zero inflation. From

$$\widehat{\pi}_t = \lambda \left(\widehat{\varpi}_t - \widehat{A}_t \right) + \beta E_t \widehat{\pi}_{t+1},$$

have $\widehat{\varpi}_t = \widehat{A}_t$, as under flexible prices.

- The rule is the same independently of the price setting restrictions.
- $\widehat{P}_t = \xi_t$. Under flexible prices we have, instead of the Phillips curve,

$$\widehat{\varpi}_t = \widehat{A}_t$$

The allocations, $\widehat{L}_t$, $\widehat{N}_t$ and $\widehat{C}_t$, are determined uniquely. With this can determine uniquely $\widehat{r}_t$ and therefore $\widehat{R}_{t+1}$.

- Share of firms $0 < \alpha < 1$ set prices one period in advance while the remaining $1 - \alpha$ firms set flexible prices

$$\widehat{P}_t = \alpha \widehat{P}_t^S + (1-\alpha)\widehat{P}_t^f,\ t \geq 0$$

$$\widehat{P}_t^S = E_{t-1}\left[\widehat{P}_t\right],\ t \geq 1,$$

$$\widehat{P}_t^f = \widehat{W}_t - \widehat{A}_t.$$

- Then

$$\alpha\left[\widehat{P}_t - E_{t-1}\widehat{P}_t\right] = (1-\alpha)\left[\widehat{\omega}_t - \widehat{A}_t\right],\ t \geq 1$$

$$\alpha\left[\widehat{P}_0 - \widehat{P}_0^S\right] = (1-\alpha)\left[\widehat{\omega}_0 - \widehat{A}_0\right]$$

where $\widehat{P}_0^S$ is exogenous. Once $\widehat{P}_t$ is determined uniquely, so is $\widehat{\omega}_t$. The nominal interest rates and the allocations are determined uniquely as above.

A simple endowment economy

- Euler equation for the representative household:

$$\frac{u_c(Y_t)}{P_t} = R_t E_t \frac{\beta u_c(Y_{t+1})}{P_{t+1}}$$

  $\{Y_t\}$ is the endowment process.

- In log deviations from a deterministic steady state with constant inflation π^*:

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

  where $r_t = \frac{u_c(Y_t)}{\beta E_t u_c(Y_{t+1})}$, or

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}.$$

Monetary policy in the endowment economy

- Interest rate target.
  - Unique path for the conditional expectation of inflation $E_t\widehat{\pi}_{t+1}$,
  - but not for the initial price level, nor the distribution of realized inflation across states.
- Current fedback rule:

$$\widehat{R}_t = \widehat{r}_t + \tau\widehat{\pi}_t$$

- Equilibria:

$$\tau\widehat{\pi}_t - E_t\left(\widehat{\pi}_{t+1}\right) = 0$$

  - Equilibrium with $\widehat{\pi}_t = 0$ and $\widehat{R}_t = \widehat{r}_t$.
  - The equilibrium with $\widehat{\pi}_t = 0$ is locally unique if $\tau > 1$ (Taylor principle).
  - Continuum of divergent solutions.
  - In the nonlinear model divergent solutions can converge to another steady state.

- Forward looking rules do not guarantee local determinacy:

$$\widehat{R}_t = \widehat{r}_t + \tau E_t \widehat{\pi}_{t+1}$$

and

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{\pi}_{t+1}$$

implies

$$(\tau - 1) E_t (\widehat{\pi}_{t+1}) = 0$$

- For $\tau \neq 1$, only expected inflation is pinned down, not the distribution of prices across states.

- With a backward rule

$$\widehat{R}_t = \widehat{r}_t + \tau \widehat{\pi}_{t-1},$$

the dynamic equation is

$$\tau \widehat{\pi}_{t-1} - E_t (\widehat{\pi}_{t+1}) = 0.$$

  - If $\widehat{\pi}_{-1} = 0$, there is a solution with $\widehat{\pi}_t = 0$ all t.
  - There are again multiple solutions and a locally determinate solution, $\widehat{\pi}_t = 0$, with $\tau > 1$, provided $\widehat{\pi}_{-1} = 0$.

- Wicksellian interest rate rules (Woodford, 2003) have the interest rate respond to the price level rather than inflation.
- Policy rule
$$\widehat{R}_t = \widehat{r}_t + \phi \widehat{P}_t,$$
where $\phi > 0$.
- Euler equation
$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$
- Equilibria:
$$(1 + \phi)\, \widehat{P}_t - E_t \widehat{P}_{t+1} = 0$$
  - Equilibrium with $\widehat{P}_t = 0$ and $\widehat{R}_t = \widehat{r}_t$.
  - The equilibrium with $\widehat{P}_t = 0$ is locally unique if $\phi > 0$.
  - Continuum of divergent solutions.

Rules that implement unique equilibria

- Price level targeting rule:

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} + \widehat{\xi}_t$$

  where $\widehat{\xi}_t$ is an exogenous random variable.

- Euler equation

$$\widehat{R}_t = \widehat{r}_t + E_t \widehat{P}_{t+1} - \widehat{P}_t,$$

- Equilibria

$$\widehat{P}_t = \widehat{\xi}_t.$$

- Intuition (Cochrane, 2007): Infinite eigenvalue.

Concluding remarks.

- Interest rate rules can implement unique local equilibria with stable prices. These are normally associated with multiple global equilibria.

- One way out is the rule proposed in this paper. Problems of robustness as in all this literature.