The Magic of the Exchange Rate: Optimal Escape from a Liquidity Trap in Small and Large Open Economies

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Plan

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(Appendix)

6. Conclusions

- Optimal escape from a liquidity trap involves private-sector expectations of a higher future price level (Krugman, Eggertsson-Woodford)
- Credibility problem, difficult to make higher future price level credible (Krugman)
- The magic of the exchange rate (Svensson 01, 03):
  - Current exchange rate indicates private-sector expectations of the future price level
  - Intentional depreciation and crawling peg can induce correct private-sector expectations and implement optimal escape; solves credibility problem (Jeanne and Svensson 03)
  - Foolproof Way: (1) price-level target path, (2) depreciation and peg, (3) exit strategy
  - [Optimal Foolproof Way ($i_t = 0$), slightly different from original FPW ($i_t \geq 0$)]

- Magnitude and direction of int'l impact of optimal escape from liquidity trap depends
  - Case of *negative* int'l output externalities (Complete int'l risksharing, $\sigma < \eta$) (Worst-case scenario)
    Noncooperation ⇒ Lower foreign natural interest rate ⇒ Foreign recession, if foreign liquidity trap
    Cooperation ⇒ Foreign recession optimal (output-gap smoothing across countries)
  - Case of *positive* int'l output externalities (Incomplete int'l risksharing) (Good-case scenario)
    Reduces foreign recession and/or eliminates foreign liquidity trap, if initial foreign liquidity trap
- Optimal or original FPW is good policy
  (Also: Simulations by Coenen-Wieland 03, Meredith 03, IMF Multimod, Fed,...)

2. A model of a two-country world

- Variant of open-economy macro model (Benigno-Benigno 03, Clarida-Galí-Gertler 03, Corsetti-Pesenti, Obstfeld-Rogoff, IMF GEM)
- Home country, foreign country, size: $1-\alpha$, α. Quantities per household
- Home representative household

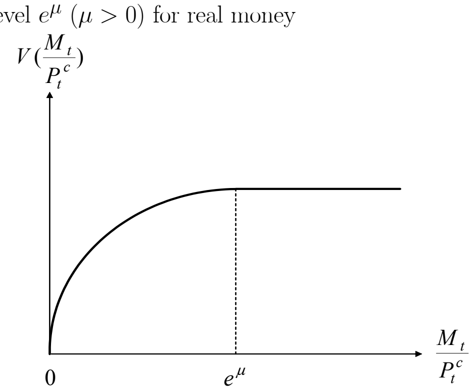
$$\mathrm{E}_t \sum_{j=0}^{\infty} \Delta_{t+j,t} \left[\frac{C_t^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + V(\frac{M_t}{P_t^c}) - \frac{N_t^{1+\varphi}}{1+\varphi} \right]. \qquad (1)$$

Stochastic discount factor

$$\Delta_{t+j,t} \equiv \begin{cases} 1 & \text{for } j = 1, \\ \prod_{l=t}^{t+j-1} \delta_l & \text{for } j > 1, \end{cases} \qquad (2)$$

δ_t ($0 < \mathrm{E}[\delta_t] = \delta < 1$) subjective discount factor between period t and $t+1$, known in period t, exogenous stochastic process; $\rho_t \equiv -\ln \delta_t$ ($\mathrm{E}[\rho_t] = \rho > 0$) rate of time preference;

C_t (aggregate) consumption; σ intertemporal elasticity of substitution; M_t home nominal (base) money; P_t^c CPI; N_t labor supply; Satiation level e^{μ} ($\mu > 0$) for real money



C_{ht}, C_{ft} consumption of home and foreign final goods; η intratemporal elasticity of substitution;

$$C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} C_{ht}^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{ft}^{1-\frac{1}{\eta}}]^{\frac{1}{1-1/\eta}}, \tag{3}$$

$$P_t^c = \left[(1-\alpha)P_t^{1-\eta} + \alpha P_t^{f\ 1-\eta}\right]^{\frac{1}{1-\eta}} \tag{4}$$

$$\equiv P_t\left[(1-\alpha) + \alpha T_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{5}$$

P_t and P_t^f home-currency prices of home and foreign final goods;

$T_t \equiv \frac{P_t^f}{P_t}$ terms of trade.

Log-linear approximation around a steady state (to be determined)

$$p_t^c = (1-\alpha)p_t + \alpha p_t^f = p_t + \alpha\tau_t, \tag{6}$$

$$\tau_t \equiv p_t - p_t^f \tag{7}$$

Producer-currency pricing, perfect exchange-rate pass-through, Law of One Price,

$$p_t^f = p_t^* + s_t, \tag{8}$$

p_t^* (log) foreign-currency price of foreign final goods;

s_t (log) exchange rate

- Production of home and foreign final goods, Y_t and Y_t^*, perfect competition, inputs of nontraded differentiated home and foreign goods, $Y_t(\iota)$ and $Y_t^*(\iota^*)$, $0 \le \iota \le 1$, $0 \le \iota^* \le 1$

$$Y_t \equiv [\int_0^1 Y_t(\iota)^{1-\frac{1}{\xi}} d\iota]^{\frac{1}{1-1/\xi}}, \tag{9}$$

$$Y_t^* \equiv [\int_0^1 Y_t^*(\iota^*)^{1-\frac{1}{\xi}} d\iota^*]^{\frac{1}{1-1/\xi}}, \tag{10}$$

$\xi > 1$ elasticity of substitution between differentiated goods

Price indices

$$P_t = [\int_0^1 P_t(\iota)^{1-\xi} d\iota]^{\frac{1}{1-\xi}},$$

$$P_t^f = [\int_0^1 P_t^f(\iota^*)^{1-\xi} d\iota^*]^{\frac{1}{1-\xi}},$$

- Demand for home and foreign differentiated goods

$$Y_t(\iota) = Y_t\left(\frac{P_t(\iota)}{P_t}\right)^{-\xi}$$

$$Y_t^*(\iota^*) = Y_t^*\left(\frac{P_t^f(\iota^*)}{P_t^f}\right)^{-\xi}$$

Production, exogenous stochastic productivity A_t and A_t^*, monopolistic competition, gross markup $\xi/(\xi-1)$

$$Y_t(\iota) = A_t N_t(\iota),$$

$$Y_t^*(\iota^*) = A_t^* N_t^*(\iota^*),$$

$N_t(\iota)$ and $N_t^*(\iota^*)$ home and foreign households' input of labor

$$N_t = \int_0^1 N_t(\iota) d\iota$$

$$N_t^* = \int_0^1 N_t^*(\iota^*) d\iota^*$$

- Foreign representative household: Same σ, φ, η and average ρ_t^* (ρ); C_t^*, M_t^*/P_t^{c*}, N_t^*

$$p_t^{c*} = \alpha p_t^* + (1-\alpha)(p_t - s_t) = p_t^* - (1-\alpha)\tau_t \tag{11}$$

PPP holds

$$p_t^c = p_t^{c*} + s_t. \tag{12}$$

- Complete international risk-sharing, suitable initial conditions

$$\text{MUC}_t = \text{MUC}_t^*, \qquad c_t = c_t^*; \tag{13}$$

Zero steady-state trade balance

$$p^c + c = p + y, \tag{14}$$

$$p^{c*} + c^* = p^* + y^*, \tag{15}$$

Market equilibrium implies

$$c_t = y_t - \alpha\eta\tau_t, \tag{16}$$

$$c_t^* = y_t^* + (1-\alpha)\eta\tau_t; \tag{17}$$

Terms of trade

$$\tau_t = \frac{1}{\eta}(y_t - y_t^*). \tag{18}$$

Combination of (13) and (16)–(18) gives

$$c_t = c_t^* = (1-\alpha)y_t + \alpha y_t^*. \tag{19}$$

Normalize steady state

$$c = c^* = y = y^* = \tau = 0.$$

2.1 Price setting

- Differentiated goods, monopolistic competition, prices set in advance

$$P_{t+1}(\iota) = \frac{\xi}{\xi-1}\mathrm{E}_t\mathrm{MC}_{t+1}(\iota) = \frac{\xi}{\xi-1}\mathrm{E}_t\frac{W_{t+1}}{A_{t+1}}.$$

$$p_{t+1}(\iota) = \ln\frac{\xi}{\xi-1} + w_{t+1|t} - \dot{a}_{t+1|t}, \tag{20}$$

$p_t(\iota) = p_t$, $y_t(\iota) = y_t$, $n_t(\iota) = n_t$, $y_t = \dot{a}_t + n_t$

$$w_t = p_t^c + (w_t - p_t^c) = p_t^c + \varphi n_t - \frac{1}{\sigma}c_t, \tag{21}$$

$$p_{t+1} = \ln\frac{\xi}{\xi-1} + p_{t+1|t} + \frac{1+\sigma\varphi}{\sigma}y_{t+1|t} + \alpha(1-\frac{\eta}{\sigma})\tau_{\tau+1|t} - (1+\varphi)\dot{a}_{t+1|t}.$$

$$\dot{a} \equiv \frac{1}{1+\varphi}\ln\frac{\xi}{\xi-1}, \qquad a_t \equiv \dot{a}_t - \dot{a}$$

$$p_{t+1} = p_{t+1|t}$$
$$p_{t+1}^* = p_{t+1|t}^*$$

2.2 Potential output

- Home flexprice equilibrium, given foreign output level

$$1 = \frac{\xi}{\xi-1}\frac{1}{A_t}\frac{W_t}{P_t} = \frac{\xi}{\xi-1}\frac{1}{A_t}\frac{P_t^c}{P_t}\frac{W_t}{P_t^c}, \tag{22}$$

$$\begin{aligned} 0 &= \ln\frac{\xi}{\xi-1} - \dot{a}_t + (p_t^c - p_t) + (w_t - p_t^c) \\ &= \ln\frac{\xi}{\xi-1} - \dot{a}_t + \alpha\frac{1}{\eta}(\bar{y}_t - y_t^*) + \varphi(\bar{y}_t - \dot{a}_t) + \frac{1}{\sigma}c_t \\ &= \alpha\frac{1}{\eta}(\bar{y}_t - y_t^*) + \varphi\bar{y}_t + \frac{1}{\sigma}[(1-\alpha)\bar{y}_t + \alpha y_t^*] - (1+\varphi)a_t, \end{aligned}$$

$$\bar{y}_t \equiv b_1 a_t - b_2 y_t^*, \tag{23}$$

$$b_1 \equiv \frac{\tilde{\sigma}(1+\varphi)}{1+\tilde{\sigma}\varphi} > 0, \tag{24}$$

$$b_2 \equiv \frac{\tilde{\sigma}}{1+\tilde{\sigma}\varphi}\alpha(\frac{1}{\sigma} - \frac{1}{\eta}) > 0 \qquad (\sigma < \eta) \tag{25}$$

$$\frac{1}{\tilde{\sigma}} \equiv (1-\alpha)\frac{1}{\sigma} + \alpha\frac{1}{\eta}, \tag{26}$$

Terms of trade effect:

$y_t^* \uparrow \Rightarrow \tau_t \downarrow \Rightarrow (p_t^c - p_t) \downarrow \Rightarrow \frac{P_t^c}{P_t}\frac{W_t}{P_t^c} \downarrow \Rightarrow \bar{y}_t \uparrow \sim \alpha\frac{1}{\eta}$

Consumption effect (perfect international risksharing):

$y_t^* \uparrow \Rightarrow c_t \uparrow \Rightarrow \mathrm{MUC}_t \downarrow \Rightarrow \frac{W_t}{P_t^c} \uparrow \Rightarrow \frac{P_t^c}{P_t}\frac{W_t}{P_t^c} \uparrow \Rightarrow \bar{y}_t \downarrow \sim \alpha\frac{1}{\sigma}$

- Negative international output externality ($\sigma < \eta$): $b_2 > 0$)

- Foreign potential output, given home output

$$\bar{y}_t^* = b_1^* a_t^* - b_2^* y_t, \tag{27}$$

$$b_1^* \equiv \frac{\tilde{\sigma}^*(1+\varphi)}{1+\tilde{\sigma}^*\varphi} > 0,$$

$$b_2^* \equiv \frac{\tilde{\sigma}^*}{1+\tilde{\sigma}^*\varphi}(1-\alpha)(\frac{1}{\sigma} - \frac{1}{\eta}) > 0 \qquad (\sigma < \eta),$$

$$\frac{1}{\tilde{\sigma}^*} \equiv \alpha\frac{1}{\sigma} + (1-\alpha)\frac{1}{\eta},$$

- Negative international output externality ($\sigma < \eta$): $b_2^* > 0$

2.3 Real interest rates, natural interest rates, output gaps and the trade balance

- First-order conditions for intertemporal consumption ($z_{t+j|t} \equiv \mathrm{E}_t z_{t+j}$)

$$c_t = c_{t+1|t} - \sigma(r_t^c - \rho_t), \tag{28}$$

CPI real interest rate

$$r_t^c \equiv i_t - (p_{t+1|t}^c - p_t^c)$$

i_t home nominal interest rate

Home (own-good) real interest rate

$$r_t \equiv i_t - (p_{t+1|t} - p_t)$$

Relation CPI and own-good real interest rate

$$r_t = r_t^c + \alpha(\tau_{t+1|t} - \tau_t)$$

- Home natural interest rates (home flexprice equilibrium, given foreign output)

$$\bar{r}_t \equiv \bar{r}_t^c + \alpha(\bar{\tau}_{t+\tau|t} - \bar{\tau}_t) \equiv \bar{r}_t^c + \alpha\frac{1}{\eta}[(\bar{y}_{t+1|t} - \bar{y}_t) - (y_{t+1|t}^* - y_t^*)]$$

$$\bar{r}_t^c \equiv \rho_t + \frac{1}{\sigma}(\bar{c}_{t+1|t} - \bar{c}_t) \equiv \rho_t + \frac{1}{\sigma}[(1-\alpha)(\bar{y}_{t+1} - \bar{y}_t) + \alpha(y_{t+1|t}^* - y_t^*)]$$

$$\bar{r}_t = \rho_t + d_1(a_{t+1|t} - a_t) + d_2(y_{t+1|t}^* - y_t^*) \tag{29}$$

$$d_1 \equiv \frac{b_1}{\tilde{\sigma}} \equiv \frac{1+\varphi}{1+\tilde{\sigma}\varphi} > 0,$$

$$d_2 \equiv b_2\varphi \equiv \frac{\tilde{\sigma}\varphi}{1+\tilde{\sigma}\varphi}\alpha(\frac{1}{\sigma} - \frac{1}{\eta}) > 0 \qquad (\sigma < \eta),$$

d_2: Effect of unit increase in $y^*_{t+1|t} - y^*_t$

Terms-of-trade change: $\alpha(\bar{\tau}_{t+\tau|t} - \bar{\tau}_t) \downarrow$: $\alpha\frac{1}{\eta}$

Consumption growth (perfect int'l risksharing): $\bar{r}^c_t \uparrow$: $\alpha\frac{1}{\sigma}$

Net effect, for given $\bar{y}_{t+1|t} - \bar{y}_t$: $\alpha(\frac{1}{\sigma} - \frac{1}{\eta})$

Effect on $\bar{y}_{t+1|t} - \bar{y}_t$: $-b_2$

Total effect: d_2, fraction of $\alpha(\frac{1}{\sigma} - \frac{1}{\eta})$

- Aggregate-demand relation

$$y_t = y_{t+1|t} - \tilde{\sigma}[r_t - \rho_t - \alpha(\frac{1}{\sigma} - \frac{1}{\eta})(y^*_{t+1|t} - y^*_t)], \quad (30)$$

$$\bar{y}_t \equiv \bar{y}_{t+1|t} - \tilde{\sigma}[\bar{r}_t - \rho_t - \alpha(\frac{1}{\sigma} - \frac{1}{\eta})(y^*_{t+1|t} - y^*_t)], \quad (31)$$

Output gap

$$x_t \equiv y_t - \bar{y}_t, \quad (32)$$

$$x_t = x_{t+1|t} - \tilde{\sigma}(r_t - \bar{r}_t) \quad (33)$$

- Real interest-rate parity

$$\tau_t = \tau_{t+1|t} - (r_t - r^*_t), \quad (34)$$

Nominal interest-rate parity,

$$s_t = s_{t+1|t} - (i_t - i^*_t). \quad (35)$$

CPI real interest rates are equalized

$$r^c_t = r^{c*}_t. \quad (36)$$

- Home trade balance (net export share in GDP)

$$\text{nx}_t \equiv \frac{P_t Y_t - P^c_t C_t}{P_t Y}$$

Linear approximation is

$$\text{nx}_t = y_t - c_t - (p^c_t - p_t) = \alpha(y_t - y^*_t) - \alpha\tau_t = \alpha(\eta - 1)\tau_t$$
$$= \alpha(1 - \frac{1}{\eta})(y_t - y^*_t).$$

Marshall-Lerner condition: $\eta > 1$.

2.4 Money and the zero lower bound

- The zero lower bound,

$$i_t \geq 0. \quad (37)$$

Log of first-order condition for optimal money holdings

$$m_t - p^c_t = \ln F(e^{c_t}, i_t) < \mu \qquad (i_t > 0), \quad (38)$$
$$m_t - p^c_t \geq \ln F(e^{c_t}, 0) = \mu \qquad (i_t = 0), \quad (39)$$

m_t (log) nominal money supply, $\frac{\partial F(C_t,i_t)}{\partial C_t} > 0$, $\frac{\partial F(C_t,i_t)}{\partial i_t} < 0$ $(i_t > 0)$

Satiation level of money demand, μ

Use (6), (18) and (19):

$$m_t - p_t = l(y_t, y^*_t, i_t) \qquad (i_t > 0), \quad (40)$$
$$m_t - p_t \geq l(y_t, y^*_t, 0) \qquad (i_t = 0), \quad (41)$$

$$l(y_t, y^*_t, i_t) \equiv \frac{\alpha}{\eta}(y_t - y^*_t) + \ln F(e^{(1-\alpha)y_t + \alpha y^*_t}, i_t) \geq l(y_t, y^*_t, i_t)$$
$$l(y_t, y^*_t, 0) \equiv \frac{\alpha}{\eta}(y_t - y^*_t) + \mu$$

Home central bank controls i_t by setting m_t

Summary of model

$$p^c_t = p_t + \alpha\tau_t,$$
$$\tau_t \equiv p_t - p^f_t = \frac{1}{\eta}(y_t - y^*_t),$$
$$p^f_t = p^*_t + s_t$$
$$c_t = c^*_t = (1-\alpha)y_t + \alpha y^*_t,$$
$$c = c^* = y = y^* = \tau = 0,$$
$$p_{t+1} = p_{t+1|t},$$
$$\bar{y}_t = b_1 a_t - b_2 y^*_t \qquad (b_2 \sim \frac{1}{\sigma} - \frac{1}{\eta})$$
$$r_t \equiv i_t - (p_{t+1|t} - p_t) \geq -(p_{t+1|t} - p_t)$$
$$s_t = s_{t+1|t} - (i_t - i^*_t)$$
$$\bar{r}_t = \rho_t + d_1(a_{t+1|t} - a_t) + d_2(y^*_{t+1|t} - y^*_t) \qquad (d_2 \sim \frac{1}{\sigma} - \frac{1}{\eta})$$
$$x_t \equiv y_t - \bar{y}_t = x_{t+1|t} - \tilde{\sigma}(r_t - \bar{r}_t)$$
$$m_t - p_t = l(y_t, y^*_t, i_t) \qquad (i_t > 0)$$
$$m_t - p_t \geq l(y_t, y^*_t, 0) \qquad (i_t = 0)$$

2.5 Monetary-policy objectives

Flexible own-inflation targeting
Home inflation target $\pi \geq 0$:

$$\mathrm{E}_t \sum_{j=0}^{\infty} (1-\delta)\delta^j \frac{1}{2}[(p_{t+j} - p_{t+j-1} - \pi)^2 + \lambda x_{t+j}^2]$$

Foreign inflation target $\pi^* \geq 0$:

$$\mathrm{E}_t \sum_{j=0}^{\infty} (1-\delta)\delta^j \frac{1}{2}[(p_{t+j}^* - p_{t+j-1}^* - \pi^*)^2 + \lambda^* x_{t+j}^{*2}]$$

3. A liquidity trap and optimal escape in a simple case of a small open economy

- Assume foreign country exogenous (home country small open economy)
- Assume foreign country in steady state ($\pi^* \geq 0$)

$$\begin{aligned} y_t^* &= y^* = 0, \\ r_t^* &= \bar{r}_t^* = \rho > 0, \\ p_t^* &= \pi^* + p_{t-1}^* \equiv \hat{p}_t^*, \\ i_t^* &= i^* \equiv \rho + \pi^* > 0, \qquad \text{for all } t. \end{aligned}$$

- Assume a_t, ρ_t iid

$$\begin{aligned} a_{t+1|t} &= 0 \\ \rho_{t+1|t} &= \rho \end{aligned}$$

$\bar{y}_t$ and $\bar{r}_t$ also iid:

$$\begin{aligned} \bar{y}_t &= -b_1 a_t \\ \bar{r}_t &= \rho_t - d_1 a_t \end{aligned}$$

- Prob$\{\bar{r}_t + \pi < 0\}$ small; Prob$\{\bar{r}_t + \pi \geq 0\}$ large, normal case
- Ideal equilibrium

$$\begin{aligned} p_{t+1} &= p_{t+1|t} = p_t + \pi, && (42) \\ x_t &= 0, && (43) \\ i_t &= \bar{r}_t + \pi \geq 0, && (44) \\ r_t &= \bar{r}_t, && (45) \\ m_t &= p_t + l(\bar{y}_t, 0, \bar{r}_t + \pi). && (46) \end{aligned}$$

- Consider home country in period 1 ("present"). Assume expected ideal equilibrium from period 2 on

$$\begin{aligned} p_{3|1} &= p_{2|1} + \pi, && (47) \\ x_{2|1} &= 0, \\ y_{2|1} &= \bar{y}_{2|1} = y = 0, && (48) \\ r_{2|1} &= \bar{r}_{2|1} = \rho > 0, \\ i_{2|1} &= \rho + \pi > 0. \end{aligned}$$

- From (40): $p_{2|1}$ and $m_{2|1}$ linked by

$$p_{2|1} = m_{2|1} + l(0, 0, \rho + \pi). \qquad (49)$$

- Period-2 price level

$$p_2 = p_{2|1}$$

- Period-1 price level predetermined,

$$p_1 = p_{1|0}$$

- Aggregate-demand relation (since $x_{2|1} = 0$)

$$\begin{aligned} x_1 &= -\tilde{\sigma}[i_1 - (p_{2|1} - p_1) - \bar{r}_1] \leq \tilde{\sigma}(\bar{r}_1 + p_{2|1} - p_1), && (50) \\ i_1 &\geq 0, && (51) \end{aligned}$$

By above assumptions

$$\bar{r}_1 = \rho_1 - d_1 a_1.$$

- Relevant loss function

$$\begin{aligned} L_1 &= \frac{1}{2}[\lambda x_1^2 + \delta(p_{2|1} - \hat{p}_2)^2], && (52) \\ \hat{p}_2 &\equiv p_1 + \pi && (53) \end{aligned}$$

- More formal version of Krugman 98. Simplified version of Eggertsson-Woodford 03.

3.1 Optimal escape from a liquidity trap

- Optimal escape from a liquidity trap: Optimal policy under commitment with a binding ZLB
- Combine constraints (50) and (51):

$$x_1 \leq \tilde{\sigma}(\bar{r}_1 + p_{2|1} - p_1) = \tilde{\sigma}(\bar{r}_1 + \pi + p_{2|1} - \hat{p}_2), \qquad (54)$$

Infer i_1 from (50).

- Lagrangian
$$\mathcal{L}_1 = \frac{1}{2}[\lambda x_1^2 + \delta(p_{2|1} - \hat{p}_2)^2] - \phi_1[\tilde{\sigma}(\bar{r}_1 + \pi + p_{2|1} - \hat{p}_2) - x_1],$$
Lagrange multiplier $\phi_1 \geq 0$, complementarity slackness conditions
$$\phi_1[\tilde{\sigma}(\bar{r}_1 + p_{2|1} - p_1) - x_1] = 0.$$
- First-order condition with respect to $p_{2|1}$
$$\delta(p_{2|1} - \hat{p}_2) - \phi_1\tilde{\sigma} = 0.$$
First-order condition with respect to x_1
$$\lambda x_1 + \phi_1 = 0.$$

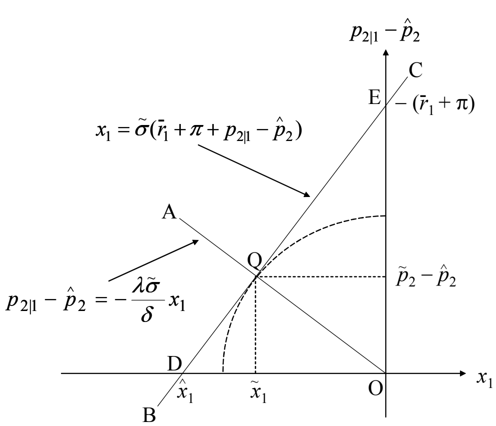
- *Optimal targeting rule* (Svensson 03 JEL)

(N) No liquidity trap: If possible, set $p_{2|1} = \hat{p}_2$ and choose $i_1 \geq 0$ so as to fulfill the target criterion
$$x_1 = 0.$$
(Choose $i_1 = \bar{r}_1 + \pi \geq 0$; then $r_1 = \bar{r}_1$ and $x_1 = 0$.)

(L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $p_{2|1} > \hat{p}_2$ so as to fulfill the target criterion
$$p_{2|1} - \hat{p}_2 = -\frac{\lambda\tilde{\sigma}}{\delta}x_1 > 0. \qquad (55)$$

- Case N: No liquidity trap if and only if
$$\bar{r}_1 + \pi \geq 0.$$
Then $\phi_1 = 0$, ideal equilibrium
$$\begin{aligned} p_{2|1} &= \hat{p}_2, \\ x_1 &= 0, \\ r_1 &= \bar{r}_1, && (56) \\ i_1 &= \bar{r}_1 + \pi \;\geq\; 0, \\ m_1 &= p_1 + l(\bar{y}_1, 0, \bar{r}_1 + \pi) \\ L_1 &= 0. && (57) \end{aligned}$$

- Case L: Liquidity trap if and only if
$$\bar{r}_1 + \pi < 0, \qquad (58)$$
Then $\phi_1 > 0$ and
$$\begin{aligned} i_1 &= 0, && (59) \\ m_1 &\geq p_1 + l(\bar{y}_1 + \tilde{x}_1, 0, 0), && (60) \\ p_{2|1} &= \hat{p}_2 - \frac{\lambda\tilde{\sigma}^2}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) \;\equiv\; \tilde{p}_2 \;>\; \hat{p}_2, && (61) \\ r_1 &= -(\pi + p_{2|1} - \hat{p}_2) \;= -(\pi + \tilde{p}_2 - \hat{p}_2) \;\equiv\; \tilde{r}_1 \;>\; \bar{r}_1 && (62) \\ x_1 &= \tilde{\sigma}(\bar{r}_1 + \pi + p_{2|1} - \hat{p}_2) \;=\; \frac{\delta\tilde{\sigma}}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) \;\equiv\; \tilde{x}_1 \;<\; 0 \\ \tilde{L}_1 &> 0. && (63) \end{aligned}$$



- The price level in period 2 overshoots the price-level target. Inflation expectations exceed the inflation target,
$$p_{2|1} - p_1 = \tilde{p}_2 - p_1 = \pi - \frac{\lambda\tilde{\sigma}^2}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) > \pi. \qquad (64)$$
The optimal policy under commitment hence trades off the right amount of overshooting the future price-level/inflation target for the appropriate reduction in the magnitude of the output gap.

Main insight: Krugman 98.

Precise derivation: Jung-Teranishi-Watanabe 01, Eggertsson-Woodford 03

"Good equilibrium," point Q

3.2 The credibility problem

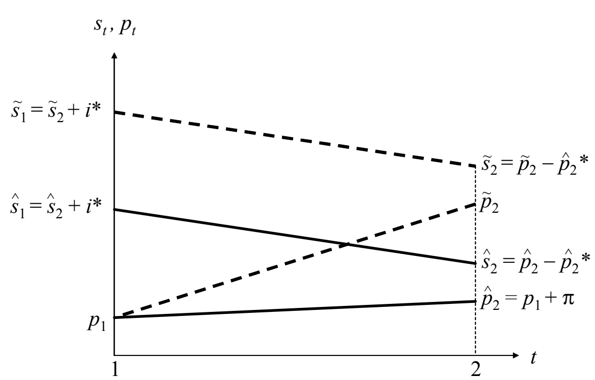
- Krugman 98: Why optimal policy credible? Why not $p_{2|1} = \hat{p}_2$?
  "Commitment to being irresponsible in the future":
  $$m_2 = \tilde{m}_2 \equiv \tilde{p}_2 + l(\bar{y}_2, 0, \bar{r}_2 + \pi)$$
  No commitment mechanism
  Auerbach-Obstfeld 03: Permanent expansion of money supply
  But current $\tilde{m}_1 \geq p_1 + l(\tilde{y}_1, 0, 0)$ may very well be larger than $\tilde{m}_2$
  Japan: "Quantitative easing," mon. base up 50+% since Mar 01.
- Assume optimal policy *not* credible: "Bad equilibrium," higher loss
  $$\begin{aligned} i_1 &= 0, \\ m_1 &\geq p_1 + l(\bar{y}_1 + \hat{x}_1, 0, 0), \\ p_{2|1} &= \hat{p}_2 < \tilde{p}_2, \\ r_1 &= -\pi \equiv \hat{r}_1 > \tilde{r}_1, \\ x_1 &= \tilde{\sigma}(\bar{r}_1 + \pi) \equiv \hat{x}_1 < \tilde{x}_1 \\ \hat{L}_1 &> \tilde{L}_1. \end{aligned}$$

4. The magic of the exchange rate

- How to get from the bad equilibrium to the good equilibrium?
- No zero bound for the exchange rate
- Currency depreciation can stimulate the economy (net export) (Bernanke, McCallum, Meltzer, Orphanides-Wieland)
- Currency depreciation and peg serves as a conspicuous commitment to a higher future price level, induces higher private-sector inflation expectations, reduces the real interest rate (Svensson 01, FPW)
- The right peg can implement the optimal escape from a liquidity trap (Svensson 03 JEP)

- Find exchange-rate path for bad and good equilibrium, respectively
- By (7) and (8), private-sector expectations fulfill
  $$p_{2|1} \equiv p^*_{2|1} + s_{2|1} + \tau_{2|1}.$$
  By (18) and the above assumptions, we have
  $$\tau_{2|1} = \tau = 0.$$
  It follows that
  $$s_{2|1} = p_{2|1} - p^*_{2|1} \quad (65)$$
  $$s_1 = s_{2|1} - (i_1 - i^*). \quad (66)$$

- Bad equilibrium: From (65) and (66)
  $$\begin{aligned} s_{2|1} &= \hat{p}_2 - p^*_2 \equiv \hat{s}_2, \\ s_1 &= \hat{s}_2 + i^* \equiv \hat{s}_1. \end{aligned}$$
- Optimal escape:
  $$\begin{aligned} s_{2|1} &= \tilde{p}_2 - p^*_2 \equiv \tilde{s}_2 > \hat{s}_2, \\ s_1 &= \tilde{s}_2 + i^* \equiv \tilde{s}_1 > \hat{s}_1. \end{aligned} \quad (67)$$



- **Result 1**: s_1 varies one-to-one with $p_{2|1}$.
  Exchange rate is indicator of expectations of future price level
  Japan: Quantitative easing, no depreciation, policy failed
- **Result 2**: Depreciate currency to $s_1 = \tilde{s}_1$ and implement crawling peg with rate of crawl $= -\, i^*$. Induces $s_{2|1} = \tilde{s}_2$ and $p_{2|1} = \tilde{p}_2$!
  Intentional currency depreciation and crawling peg can implement optimal escape from liquidity trap
- Pegging strong currency always feasible; pegging weak currency difficult. Does not require portfolio-balance effects. Just commit to $s_1 = \tilde{s}_1$ and rate of crawl.
- During the initial defense of the peg, the CB may end up accumulating substantial foreign-exchange reserves. Balance-sheet incentive to maintain peg! (Jeanne-Svensson 2003, in progress)
- Once the peg has become credible and private-sector expectations have adjusted, the peg is no longer necessary and binding.

4.1 The Foolproof Way

- Original FPW (Svensson 01)

  (1) Upward-sloping price-level target path, "price gap" to be undone (Bernanke 00, 03)

  (2) Depreciation and (approximately) fixed peg,

$$s_t = \bar{s} \qquad (t \geq 1),$$
$$i_1 = i^* > 0.$$

  Unnecessarily high $p_{2|1}$, not quite optimal (Figure)

  (3) Exit strategy: Floating and flexible inflation/price level targeting once price-level target met

- Optimal FPW

  (1) Price-level target path

$$\hat{p}_t^{\text{o}} = \tilde{p}_2 + (t-2)\pi \qquad (t \geq 1),$$

  Price gap

$$\hat{p}_1^{\text{o}} - p_1 = \tilde{p}_2 - \hat{p}_2 = -\frac{\lambda\tilde{\sigma}^2}{\delta + \lambda\tilde{\sigma}^2}(\bar{r}_1 + \pi) > 0.$$

  (2) Depreciation and crawling peg,

$$s_t = \tilde{s}_1 - (t-1)i^* \qquad (t \geq 1),$$

  (3) Exit strategy as above

- Japan: $\pi = 1\%$/yr. US: $\pi^* = 2\%$/yr, $i^* = 1\%$. Original FPW: $i_1 = i^* + \pi - \pi^* = 0$. Optimal FPW: $i_1 = 0$. In this case, no difference.

5. The international impact

- International impact: Foreign country no longer exogenous

  Simple case: Foreign ideal equilibrium expected from period 2

$$\begin{aligned} p_{3|1}^* &= p_{2|1}^* + \pi^*, \\ x_{2|1}^* &= 0, \\ y_{2|1}^* &= \bar{y}_{2|1}^* = y = 0, \\ r_{2|1}^* &= \bar{r}_{2|1}^* = \rho > 0, \\ i_{2|1}^* &= \rho + \pi^* > 0. \end{aligned} \tag{68}$$

  Period-1 foreign price level predetermined: $p_1^* = p_{1|0}^*$.

  Foreign output gap, potential output and the natural interest rate

$$x_1^* \equiv -\tilde{\sigma}^*[i_1^* - (p_{2|1}^* - p_1^*) - \bar{r}_1^*] \leq \tilde{\sigma}^*(\bar{r}_1^* + p_{2|1}^* - p_1^*), \tag{69}$$
$$i_1^* \geq 0 \tag{70}$$

  $\bar{y}_1^*$ and $\bar{r}_1^*$ both depend on y_1

Foreign flexible own-inflation targeting

$$L_1^* = \frac{1}{2}[\lambda^* x_1^{*2} + \delta(p_{2|1}^* - \hat{p}_2^*)^2]$$
$$\hat{p}_2^* \equiv p_1^* + \pi^*$$

- Discuss international impact in (x_1, x_1^*)-space

  Potential outputs

$$\bar{y}_1 = b_1 a_1 - b_2 y_1^*$$
$$\bar{y}_1^* = b_1^* a_1^* - b_2^* y_1$$

  Output gaps

$$x_1 = y_1 - \bar{y}_1$$
$$x_1^* = y_1^* - \bar{y}_1^*$$

  Output in terms of output gaps

$$y_1 = \bar{\bar{y}}_1 + f_1 x_1 - f_2 x_1^*$$
$$y_1^* = \bar{\bar{y}}_1^* + f_1 x_1^* - f_2^* x_1$$

  World (flexprice) home and foreign potential outputs ($0 \leq b_2 b_2^* < 1$)

$$\bar{\bar{y}}_1 \equiv \frac{b_1 a_1 - b_2 b_1^* a_1^*}{1 - b_2 b_2^*}$$
$$\bar{\bar{y}}_1^* \equiv \frac{b_1^* a_1^* - b_2^* b_1 a_1}{1 - b_2 b_2^*}$$

$$f_1 \equiv \frac{1}{1 - b_2 b_2^*} > 0$$
$$f_2 \equiv \frac{b_2}{1 - b_2 b_2^*} > 0 \qquad (\sigma < \eta)$$
$$f_2^* \equiv \frac{b_2^*}{1 - b_2 b_2^*} > 0 \qquad (\sigma < \eta).$$

Home output decreasing in foreign output gap ($\sigma < \eta$)

f_2: $x_1^* \uparrow \Rightarrow y_1^* \uparrow \Rightarrow y_1 \downarrow$

- Natural interest rates in terms of output gaps

$$\begin{aligned}\bar r_1 &= \rho_1 - d_1 a_1 - d_2 y_1^* \\ &\equiv \bar{\bar r}_1 + g_1 x_1 - g_2 x_1^* \equiv \bar r_1(x_1, x_1^*) \\ \bar r_1^* &= \rho_1^* - d_1^* a_1^* - d_2^* y_1 \\ &\equiv \bar{\bar r}_1^* + g_1^* x_1^* - g_2^* x_1 \equiv \bar r_1^*(x_1^*, x_1)\end{aligned}$$

World home and foreign natural interest rates

$$\begin{aligned}\bar{\bar r}_1 &\equiv \rho_1 - d_1 a_1 - d_2 \bar{\bar y}_1^* \\ \bar{\bar r}_1^* &\equiv \rho_1^* - d_1^* a_1^* - d_2^* \bar{\bar y}_1\end{aligned}$$

$$\begin{aligned}g_1 &\equiv d_2 f_2^* > 0, \\ g_2 &\equiv d_2 f_1 > 0 \qquad (\sigma < \eta), \\ g_1^* &\equiv d_2^* f_2 > 0, \\ g_2^* &\equiv d_2^* f_1 > 0 \qquad (\sigma < \eta),\end{aligned}$$

$\bar r_1$ increasing in x_1, decreasing in x_1^* $(\sigma < \eta)$
g_2: $x_1^* \uparrow \Rightarrow y_1^* \uparrow \Rightarrow \bar r_1 \downarrow$

5.1 Noncooperative commitment equilibrium

- Rewrite constraints

$$x_1 \le \tilde\sigma[\bar r_1(x_1, x_1^*) + \pi + p_{2|1} - \hat p_2], \qquad (71)$$

$$x_1^* \le \tilde\sigma^*[\bar r_1^*(x_1^*, x_1) + \pi^* + p_{2|1}^* - \hat p_2^*] \qquad (72)$$

Home chooses $p_{2|1}$ and x_1 so as to minimize L_1 under commitment, subject to (71), taking y_1^* and $\bar r_1$ as given
Foreign chooses $p_{2|1}^*$ and x_1^* so as to minimize L_1^* under commitment, subject to (72), taking y_1 and $\bar r_1^*$ as given

- Home targeting rule

(N) No liquidity trap: If possible, set $p_{2|1} = \hat p_2$ and choose $i_1 \ge 0$ so as to fulfill the target criterion

$$x_1 = 0.$$

(L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $p_{2|1} > \hat p_2$ so as to fulfill the target criterion

$$p_{2|1} - \hat p_2 = -\frac{\lambda\tilde\sigma}{\delta} x_1 > 0. \qquad (73)$$

- Foreign targeting rule

(N*) No liquidity trap: If possible, set $p_{2|1}^* = \hat p_2^*$ and choose $i_1^* \ge 0$ so as to fulfill the target criterion

$$x_1^* = 0.$$

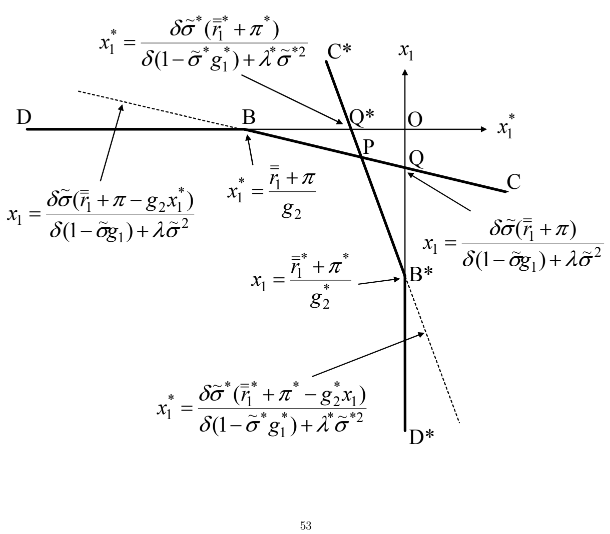
(L*) Liquidity trap: If this is not possible, set $i_1^* = 0$ and choose $p_{2|1}^* > \hat p_2^*$ so as to fulfill the target criterion

$$p_{2|1}^* - \hat p_2^* = -\frac{\lambda^*\tilde\sigma^*}{\delta} x_1^* > 0. \qquad (74)$$

- Combining this with the constraints

$$x_1 = \min\{\tilde\sigma[\bar{\bar r}_1 + g_1 x_1 - g_2 x_1^* + \pi - \frac{\lambda\tilde\sigma}{\delta} x_1], 0\}, \qquad (75)$$

$$x_1^* = \min\{\tilde\sigma^*[\bar{\bar r}_1^* + g_1^* x_1^* - g_2^* x_1 + \pi^* - \frac{\lambda^*\tilde\sigma^*}{\delta} x_1^*], 0\}. \qquad (76)$$



5.1.1 The special case (L, N*)

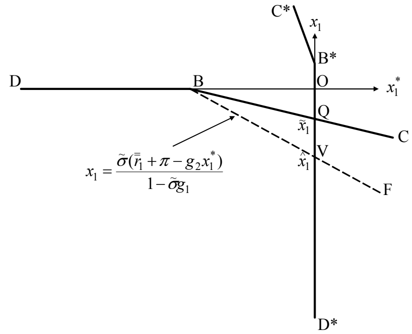
- Point B to the left of O, point B* above O

$$\begin{aligned}\bar{\bar r}_1 + \pi &< 0 \\ \bar{\bar r}_1^* + \pi^* &> 0\end{aligned}$$

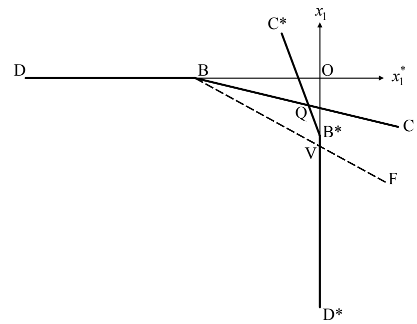
- (N*) Foreign not in liquidity trap, ideal equilibrium

$$\begin{aligned}p_{2|1}^* &= \hat p_2^* \\ x_1^* &= 0 \\ i_1^* &= \bar r_1^*(0, x_1) + \pi^* \equiv \bar{\bar r}_1^* - g_2^* x_1 + \pi^* \ge 0\end{aligned}$$

- (L) Home in liquidity trap, bad equilibrium V, good equilibrium Q
  Home moves from V to Q
  No impact on x_1^*, lower $\bar{r}_1^*$, i_1^*



- Suppose B* between V and Q ($\bar{\bar{r}}_1^* + \pi^* < 0$)



Home moves from bad to good equilibrium, move from V to Q, foreign liquidity trap, $x_1^* < 0$.
Problem? Cf. optimal int'l cooperation!

5.2 Cooperative commitment equilibrium

Constraints

$$x_1 \leq \tilde{\sigma}(\bar{\bar{r}}_1 + \pi + g_1 x_1 - g_2 x_1^* + p_{2|1} - \hat{p}_2),$$
$$x_1^* \leq \tilde{\sigma}^*(\bar{\bar{r}}_1^* + \pi^* + g_1^* x_1^* - g_2^* x_1 + p_{2|1}^* - \hat{p}_2^*)$$

World loss

$$(1-\alpha)L_1 + \alpha L_1^* = (1-\alpha)\frac{1}{2}[\lambda x_1^2 + \delta(p_{2|1} - \hat{p}_2)^2] + \alpha\frac{1}{2}[\lambda^* x_1^{*2} + \delta(p_{2|1}^* - \hat{p}_2^*)^2]$$

Lagrangian

$$\begin{aligned}\mathcal{L}_1 = &(1-\alpha)L_1 + \alpha L_1^* \\ &- (1-\alpha)\phi_1\{\tilde{\sigma}[\bar{\bar{r}}_1 + \pi + g_1 x_1 - g_2 x_1^* + p_{2|1} - \hat{p}_2] - x_1\} \\ &- \alpha\phi_1^*\{\tilde{\sigma}^*[\bar{\bar{r}}_1^* + \pi^* + g_1^* x_1^* - g_2^* x_1 + p_{2|1}^* - \hat{p}_2^*] - x_1^*\}\end{aligned}$$

First-order conditions (plus complementary slackness)

$$\begin{aligned}\delta(p_{2|1} - \hat{p}_2) - \phi_1\tilde{\sigma} &= 0 \\ \delta(p_{2|1}^* - \hat{p}_2^*) - \phi_1^*\tilde{\sigma}^* &= 0 \\ \lambda x_1 - \phi_1\tilde{\sigma}g_1 + \phi_1 + \frac{\alpha}{1-\alpha}\phi_1^*\tilde{\sigma}^* g_2^* &= 0 \\ \lambda^* x_1^* - \phi_1^*\tilde{\sigma}^* g_1^* + \phi_1^* + \frac{1-\alpha}{\alpha}\phi_1\tilde{\sigma}g_2 &= 0\end{aligned}$$

- Optimal targeting rule: Home
  (N) No liquidity trap: If possible, set $p_{2|1} - \hat{p}_2 = 0$ and choose $i_1 \geq 0$ so as to fulfill the target criterion

$$x_1 = -\frac{\alpha}{1-\alpha}\frac{\delta g_2^*}{\lambda}(p_{2|1}^* - \hat{p}_2^*) \leq 0.$$

  (L) Liquidity trap: If this is not possible, set $i_1 = 0$ and choose $p_{2|1} > \hat{p}_2$ so as to fulfill the target criterion

$$p_{2|1} - \hat{p}_2 = -\frac{\lambda\tilde{\sigma}}{\delta(1-\tilde{\sigma}g_1)}x_1 - \frac{\alpha}{1-\alpha}\frac{\tilde{\sigma}g_2^*}{1-\tilde{\sigma}g_1}(p_{2|1}^* - \hat{p}_2^*) > 0.$$

Foreign:

(N*) No liquidity trap: If possible, set $p_{2|1}^* = \hat{p}_2^*$ and choose $i_1^* \geq 0$ so as to fulfill the target criterion

$$x_1^* = -\frac{1-\alpha}{\alpha}\frac{\delta g_2}{\lambda^*}(p_{2|1} - \hat{p}_2) \leq 0.$$

(L*) Liquidity trap: If this is not possible, set $i_1^* = 0$ and choose $p_{2|1}^* > \hat{p}_2^*$ so as to fulfill the target criterion

$$p_{2|1}^* - \hat{p}_2^* = -\frac{\lambda^*\tilde{\sigma}^*}{\delta(1-\tilde{\sigma}^* g_1^*)}x_1^* - \frac{1-\alpha}{\alpha}\frac{\tilde{\sigma}^* g_2}{1-\tilde{\sigma}^* g_1^*}(p_{2|1} - \hat{p}_2) \geq 0.$$

- Consider case (L, N*)

$$p_2^* = \hat{p}_2^*$$

Targeting rule home:

$$p_{2|1} - \hat{p}_2 = -\frac{\lambda\tilde{\sigma}}{\delta(1-\tilde{\sigma}g_1)}x_1 > 0$$

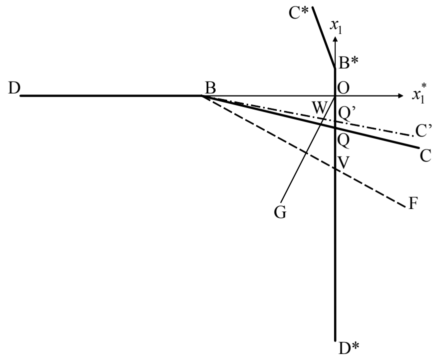
Targeting rule foreign

$$x_1^* = -\frac{1-\alpha}{\alpha}\frac{\delta g_2}{\lambda^*}(p_{2|1} - \hat{p}_2) = \frac{1-\alpha}{\alpha}\frac{g_2}{\lambda^*}\frac{\lambda\tilde{\sigma}}{1-\tilde{\sigma}g_1}x_1 < 0$$

Result:

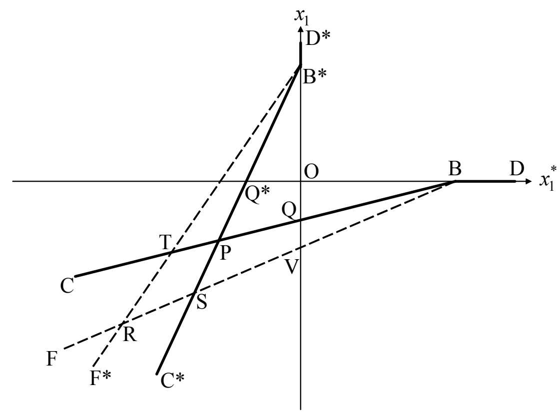
$$\begin{aligned}\tilde{x}_1 < \tilde{\tilde{x}}_1 &< 0, \\ \tilde{\tilde{x}}_1^* < \tilde{x}_1^* &= 0.\end{aligned}$$

- Output-gap smoothing across countries (ray OG)

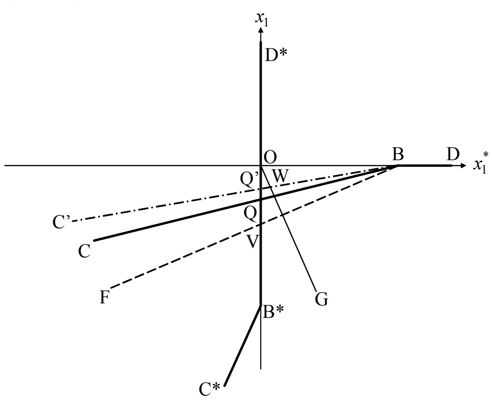


Similar to the case when optimal escape under noncooperation leads to a foreign liquidity trap

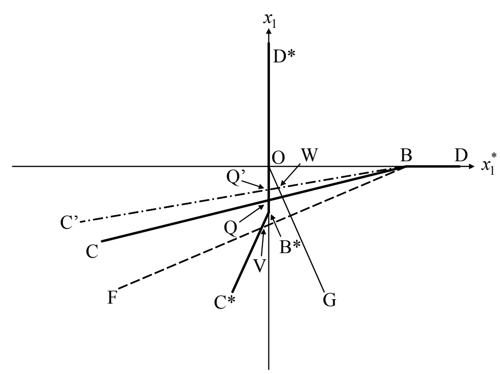
5.3 Positive international output externalities



The case (L, N*)



B* between V and Q



6. Conclusions

- Optimal escape from a liquidity trap involves private-sector expectations of a higher future price level (Krugman, Eggertsson-Woodford)
- Credibility problem, difficult to make higher future price level credible (Krugman)
- The magic of the exchange rate (Svensson 01, 03):
  - Current exchange rate indicates private-sector expectations of the future price level
  - Intentional depreciation and crawling peg can induce correct private-sector expectations and implement optimal escape; solves credibility problem (Jeanne and Svensson 03)
  - Foolproof Way: (1) price-level target path, (2) depreciation and peg, (3) exit strategy
  - [Optimal Foolproof Way ($i_1 = 0$), slightly different from original FPW ($i_1 \geq 0$)]

- Magnitude and direction of int'l impact of optimal escape from liquidity trap depends
  - Case of *negative* int'l output externalities (Complete int'l risksharing, $\sigma < \eta$) (Worst-case scenario)
    Noncooperation ⇒ Lower foreign natural interest rate ⇒ Foreign recession, if foreign liquidity trap
    Cooperation ⇒ Foreign recession optimal (output-gap smoothing across countries)
  - Case of *positive* int'l output externalities (Incomplete int'l risksharing) (Good-case scenario)
    Reduces foreign recession and/or eliminates foreign liquidity trap, if initial foreign liquidity trap
- Optimal or original FPW is good policy
  (Also: Simulations by Coenen-Wieland 03, Meredith 03, IMF Multimod, Fed,...)