DSGE Models in a Data-Rich Environment

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Importance of large data sets: Evidence from factor models

- **Forecasting**
  

- **Monetary policy**
  
  [Bernanke and Boivin (2003), Giannone, Reichlin and Sala (2004)]

- **VAR**
  
  [Bernanke, Boivin and Eliasz (2005), Forni, Giannone, Lippi, Reichlin (2004)]

- All existing evidence based on largely non-structural models. Limited ability to:
  - Determine sources of fluctuations
  - Perform counterfactual experiments
  - Analyze optimal policy
Usefulness of large data sets raises interesting questions

- Why is the additional information useful?

- Are the costs of ignoring this information important?
  - For structural modeling?
  - For optimal conduct of policy?
Estimated DSGE models

- Important developments

- Promising empirical success
  [Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007)]

- Now increasingly taken seriously as empirical models
Estimated DSGE models

- Important developments

- Promising empirical success
  [Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007)]

- Now increasingly taken seriously as empirical models

- Estimated based on a handful of data series
  \[\rightarrow\] at odds with fact that CB and financial market participants monitor large number of data series!
**Goal of this paper**

- To explore role of large data sets for estimated DSGE models
- By product: Provide economic interpretation of latent factors

**Preview of the main results**

- More precise estimation of the state of the economy
- Improvements in “forecasting” with additional information
- Different conclusions about structure of economy and sources of business cycles
  - Different propagation mechanism (e.g. less habit formation and inflation indexing)
  - Fewer and smaller structural shocks
Why more data in a DSGE context?
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- No scope if:
  - Model well specified
  - Theoretical concepts directly observed by agents and econometrician
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Why more data in a DSGE context?

- No scope if:
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  - Theoretical concepts directly observed by agents and econometrician

- Empirical evidence: at least one assumption violated

- We assume theoretical concepts partially observed by econometrician
  - Employment: Discrepancies between household and payroll surveys
  - Inflation: GDP deflator, CPI
  - Productivity shock: oil prices, commodity prices

- If indeed we are missing information in DSGE estimation: all parameter estimates potentially distorted!
Why more data in a DSGE context? What is Employment?
Why more data in a DSGE context? What is Inflation?
Outline of presentation

- Data-rich environment
  - A simple example: RBC model
  - General framework
  - Estimation

  - Results

- Conclusion

- Research in progress: Optimal policy in data-rich envt.
Why more information? A simple RBC model

Households maximize lifetime utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t [\log (c_t) + v \log (1 - l_t)], \quad 0 < \beta < 1, \ v > 0 \]

subject to

\[
\begin{align*}
y_t &= e^{a_t k_t^{1-\alpha} l_t^\alpha}, \quad 0 < \alpha < 1 \\
y_t &= c_t + k_{t+1} - (1 - \delta) k_t, \\
a_t &= \rho a_{t-1} + \varepsilon_t, \quad 0 \leq \rho < 1, \ \varepsilon_t \sim N(0, \sigma).\end{align*}
\]
RBC example continued...

- Linearized solution has the form:

\[
\begin{align*}
\hat{y}_t &= d_1 \hat{k}_t + d_2 a_t \\
\hat{c}_t &= d_3 \hat{k}_t + d_4 a_t \\
\hat{l}_t &= d_5 \hat{k}_t + d_6 a_t \\
\hat{k}_t &= g_1 \hat{k}_{t-1} + g_2 a_{t-1} \\
a_t &= \rho a_{t-1} + \varepsilon_t
\end{align*}
\]

\[
\begin{align*}
z_t &= D S_t, \quad z_t = [\hat{y}_t, \hat{c}_t, \hat{l}_t]'
\end{align*}
\]

where \( D, G \) and \( H \) are functions of model parameters

- Suppose we estimate the model on the basis of only one variable (no stochastic singularity):

\[
F_t = \hat{y}_t = [d_1 \ d_2] S_t
\]
RBC example: How to link model and data?

- One indicator, no measurement error
  \[ X_t = \hat{y}_t = d_1 k_t + d_2 a_t \]
  - e.g. \( X_t \) = real GDP
  - No scope for judgment or soft data

- One indicator, measurement error (Sargent, 1989):
  \[ X_t = \hat{y}_t + e_t = d_1 \hat{k}_t + d_2 a_t + e_t \]
  - 1 shock and 1 measurement error: identification from dynamics
  - Identification problems?
RBC example (cont.): Proposed solution

- Multiple indicators with **known** relationships to theoretical concepts

\[
X_t = \begin{bmatrix}
\text{real GDP} \\
\text{real NI}
\end{bmatrix} = \begin{bmatrix}
1 \\
\lambda_{NI}
\end{bmatrix} \hat{y}_t + e_t \\
= \begin{bmatrix}
d_1 \\
\lambda_{NI} d_1
\end{bmatrix} \begin{bmatrix}
d_2 \\
\lambda_{NI} d_2
\end{bmatrix} \begin{bmatrix}
\hat{k}_t \\
a_t
\end{bmatrix} + e_t
\]

- Helps disentangle meas. error from structural shocks

- Multiple indicators with **unknown** link

- E.g., soft data, oil price (e.g., as indicator of \(a_t\))

\[
X_t = \begin{bmatrix}
\text{real GDP} \\
\text{soft data}
\end{bmatrix} = \begin{bmatrix}
d_1 \\
\lambda_3
\end{bmatrix} \begin{bmatrix}
d_2 \\
\lambda_4
\end{bmatrix} \begin{bmatrix}
\hat{k}_t \\
a_t
\end{bmatrix} + e_t
\]
Benefits of exploiting more information: Intuition

- Measurement error identifiable from cross-section of indicators
  
  Example: \( x_{it} = f_t + e_{it}, i = 1, \ldots, n_X \)
  
  - If \( n_X = 1 \), and both \( f_t \) and \( e_{it} \) are i.i.d. \( \implies \) Not identified
  
  - If \( n_X = 1 \), \( f_t \) is AR(1) and \( e_{it} \) is i.i.d. \( \implies \) Identified (from dynamics)
  
  - If \( n_X > 1 \), and both \( f_t \) and \( e_{it} \) are i.i.d. \( \implies \) Identified (from cross-section)

- Permits the identification of more structural shocks

- Don’t have to take a stand \( a \ priori \) on the relative importance of measurement errors vs structural shocks

- More efficient (consistent) estimate of the latent factors
  
  - \( \text{var}(\hat{f}_t) \) is of order \( 1/n_X \) [Stock Watson (2002), Forni et al. (2000)]
Data-rich environment: General framework

- Linear(ized) DSGE model:

\[
AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} = B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cs_t \\
S_t = Ms_{t-1} + \varepsilon_t
\]

Note: Agents assumed to know the model and model concepts

- Solution (REE):

\[
\begin{align*}
  z_t &= DS_t \\
  S_t &= GS_{t-1} + H\varepsilon_t \\
  S_t &\equiv \begin{bmatrix} Z_t \\ s_t \end{bmatrix}
\end{align*}
\]

Dynamics of any variable entirely determined by vector of state variables \( S_t \)
Variables of interest (e.g. inflation): \( F_t \)

- Defined as
  \[
  F_t \equiv F \begin{bmatrix}
  z_t \\
  S_t
  \end{bmatrix}
  \]
  
  where \( F \) is a selection matrix

- Related to state vector
  \[
  F_t = \Phi S_t,
  \]
  where
  \[
  \Phi \equiv F \begin{bmatrix}
  D \\
  I
  \end{bmatrix}
  \]
Linking theory and data: Known link

\[ X_{F,t} = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t} \]

where \( \Lambda_F \) has only one non-zero element on each row

- Concepts with multiple indicators. Examples:
  - Employment: household vs. establishment surveys
  - Prices: GDP deflator, PCE deflator, CPI, ....
**Linking theory and data: Known link**

\[ X_{F,t} = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t} \]

where \( \Lambda_F \) has only one non-zero element on each row

- Concepts with multiple indicators. Examples:
  - Employment: household vs. establishment surveys
  - Prices: GDP deflator, PCE deflator, CPI,....

- Special cases:
  - No measurement error: \( X_{F,t} = F_t = \Phi S_t \)
  - Standard treatment of measurement error: Sargent (1989)

\[ X_{F,t} = F_t + e_{F,t} = \Phi S_t + e_{F,t} \]

single indicator for each concept

\( \rightarrow \) Maintains that a small number of series contains all relevant information
Linking theory and data: Unknown link

\[ X_{S,t} = \Lambda_S S_t + e_{S,t} \]

where \( \Lambda_S \) is completely unrestricted (e.g. commodity prices)

- \( X_{S,t} \) helps estimate the state vector \( S_t \)

- Partially observed state variables / exogenous shocks
  - E.g. productivity shock: oil or commodity prices may provide information

- More flexible exploitation of information
Empirical model: Summary

- Transition equation:
  \[ S_t = G S_{t-1} + H \varepsilon_t \]

- Observation equation:
  \[ X_t = \Lambda S_t + e_t \]
  where
  \[ X_t \equiv \begin{bmatrix} X_{F,t} \\ X_{S,t} \end{bmatrix}, \quad e_t \equiv \begin{bmatrix} e_{F,t} \\ e_{S,t} \end{bmatrix}, \quad \Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}. \]

- Comments:
  - Related to non-structural factor models, but we impose DSGE model on transition equation of latent factors
  - Factors have economic interpretation: state variables
  - Interpret info. in data set through lenses of DSGE model
  - Can do counterfactual experiments, study optimal policy
Application: Smets and Wouters (2004) [i.e., CEE (2005) with shocks]

- **State-of-the-art DSGE model:**
  - Popular as fits apparently well, good for forecasting
  - Many frictions, many shocks

- **Households**
  - Consume aggregate of all goods, habit formation (external)
  - Supply specialized labor on monopolistically competitive labor mkt
  - Rent capital services to firms
  - Decide how much capital to accumulate

- **Firms:**
  - Choose labor and capital inputs
  - Supply differentiated goods on monopolistically competitive goods mkt

- **Prices and wages reoptimized at random intervals (Calvo)**
  - If not reoptimized: indexed to past inflation and CB’s inflation target
Model summary (Log-linearized)

- **Consumption Euler equation**

\[
C_t = \frac{h}{1 + h} C_{t-1} + \frac{1}{1 + h} E_t C_{t+1} + \frac{\sigma_c - 1}{\sigma_c (1 + \lambda_w) (1 + h)} (L_t - E_t L_{t+1}) - \frac{(1 - h)}{(1 + h) \sigma_c} (i_t - E_t \pi_{t+1}) + \varepsilon^b_t
\]

- **Investment Euler equation**

\[
I_t = \frac{1}{1 + \beta} I_{t-1} + \frac{\beta}{1 + \beta} E_t I_{t+1} + \frac{1/\varphi}{1 + \beta} \left( Q_t + \varepsilon^I_t \right)
\]

- **Real value of capital**

\[
Q_t = - (i_t - E_t \pi_{t+1}) + \frac{1 - \tau}{1 - \tau + \bar{r}^k} E_t Q_{t+1} + \frac{\bar{r}^k}{1 - \tau + \bar{r}^k} E_t \bar{r}^k_{t+1} + \eta^Q_t
\]

- **Capital accumulation**

\[
K_t = (1 - \tau) K_{t-1} + \tau I_{t-1} + \tau \varepsilon^I_{t-1}
\]
Model summary (cont.)

- Optimal real wage set by household

\[ w_t = \frac{\beta}{1 + \beta} E_t w_{t+1} + \frac{1}{1 + \beta} w_{t-1} + \frac{\beta}{1 + \beta} E_t \pi_{t+1} \]
\[ - \frac{1 + \beta \gamma_w}{1 + \beta} \pi_t + \frac{\gamma_w}{1 + \beta} \pi_{t-1} \]
\[ - \frac{\lambda_w (1 - \beta \xi_w) (1 - \xi_w)}{(1 + \beta) (\lambda_w + (1 + \lambda w) \sigma_L) \xi_w} \]
\[ \times \left[ w_t - \sigma_L L_t - \frac{\sigma_c}{1 - \eta} (C_t - hC_{t-1}) + \epsilon^{L}_t \right] + \eta^{w}_t \]

- Optimal price setting by firms

\[ \pi_t = \frac{\beta}{1 + \beta \gamma_p} E_t \pi_{t+1} + \frac{\gamma_p}{1 + \beta \gamma_p} \pi_{t-1} \]
\[ + \frac{(1 - \beta \xi_p) (1 - \xi_p)}{(1 + \beta \gamma_p) \xi_p} \left[ \alpha r^k + (1 - \alpha) w_t - \epsilon^{a}_t \right] + \eta^{p}_t \]
Model summary (cont.)

- **Labor demand**

\[ L_t = -w_t + (1 + \psi) r_t^k + K_{t-1} \]

- **Goods market equilibrium**

\[ Y_t = (1 - \tau k_y - g_y) C_t + \varepsilon_t^G + \tau k_y I_t + \bar{r}^k k_y \psi r_t^k \]
\[ = \phi \left[ \varepsilon_t^a + \alpha K_{t-1} + \alpha \psi r_t^k + (1 - \alpha) L_t \right] \]

- **Monetary policy reaction function**

\[ i_t = (1 - \rho) \left[ r_{\pi 0} \pi_t + r_{\pi 1} \pi_{t-1} + r_{y 0} Y_t + r_{y 1} Y_{t-1} \right] + \rho i_{t-1} + \eta_t^i \]
Smets and Wouters (2004): Model solution

- 7 variables of interest: \( F_t = [i_t, Y_t, C_t, I_t, \pi_t, w_t, L_t]' \)

- 9 shocks: \( s_t = [\epsilon_t^a, \epsilon_t^b, \epsilon_t^G, \epsilon_t^L, \epsilon_t^I, \eta_t^Q, \eta_t^P, \eta_t^w, \eta_t^i]' \)

- State vector

\[
S_t = [i_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, \pi_{t-1}, w_{t-1}, K_{t-1}, \epsilon_{t-1}^I, s_t]' \]

- State-space representation:
  - Transition equation

\[
S_t = GS_{t-1} + H\epsilon_t
\]

  - Observation equation

\[
X_t = \Lambda S_t + e_t
\]
Estimation method

- Difficult problem to estimate (large dimension)

- Standard methods (e.g. MLE) difficult to implement in this case

- MCMC methods:
  - Empirical approximation of the posterior distribution. Does not rely on gradient method
  - Draw iteratively from conditional distributions (solves the high-dimensionality problem)
  - Priors can help make the estimation better behaved
  - Here priors same as in Smets and Wouters (2004)

- 100,000 iterations; check Markov chains are “well behaved”
Goal: Characterize the posterior distribution of $\Theta = \{\Theta_M, \gamma\}$

$$
\pi \left( \Theta | \tilde{X}_T \right) \propto \mathcal{L}(\Theta | \tilde{X}_T) \ p(\Theta)
$$

where

$\tilde{X}_T = (X_1, X_2, ..., X_T)$,

$\Theta_M =$ structural model’s param.

$\gamma =$ observation equ. param.

$p(\Theta)$

Likelihood of $\Theta = \mathcal{L}(\Theta | \tilde{X}_T) \equiv p(\tilde{X}_T | \Theta)$

or

$$
= \int p(\tilde{X}_T | \tilde{S}_T, \Theta) \ p \left( \tilde{S}_T | \Theta \right) d\tilde{S}_T
$$
**MCMC Implementation: How to compute** $\pi(\Theta|\tilde{X}_T)$?

Iterative steps:

1. Given $\Theta^{(g-1)} = \left\{ \Theta^{(g-1)}_M, \gamma^{(g-1)} \right\}$, draw $\tilde{S}^{(g)}_T$ from

   $$p\left(\tilde{S}_T^g|\Theta^{(g-1)}, \tilde{X}_T\right) = p(\tilde{X}_T|\tilde{S}_T, \Theta^{(g-1)})p(\tilde{S}_T|\Theta^{(g-1)})$$

   – use Carter-Kohn (1994)

2. Given $\tilde{S}^{(g)}_T, \Theta^{(g-1)}_M$ draw $\gamma^{(g)}$ from $p\left(\gamma|\Theta^{(g-1)}_M, \tilde{S}^{(g)}_T, \tilde{X}_T\right)$

   – conditional on $S_t$, observation equ. is linear: known dist. (Chib, 1993)

3. Given $\gamma^{(g)}, \tilde{S}^{(g)}_T$, draw $\Theta^{(g)}_M$ from $\pi\left(\Theta^{(g)}_M|\gamma^{(g)}, \tilde{S}^{(g)}_T, \tilde{X}_T\right)$

   – use RW Metroplis accept/reject step; proposal scalar Student-t dist.
• Posterior: Empirical distribution of the draws

\[ \pi \left( \Theta \mid \tilde{X}_T \right) \propto \left[ \int p(\tilde{X}_T \mid \tilde{S}_T, \Theta) \, p(\tilde{S}_T \mid \Theta) \, d\tilde{S}_T \right] p(\Theta) \]
Specifications of observation equation: $X_t = \Lambda S_t + e_t$

- **Case SW**: Standard estimation (as in Smets and Wouters)
  
  $$X_{1,t} = F_t = \Phi S_t$$

  where

  $$X_{1,t} = [\text{Fed funds, GDP, cons., invest., } %\Delta GDP \text{ defl, real wage, hours worked}]'$$

- **Case A** = Case SW + Measurement error (as in Sargent, 1989):

  $$X_{1,t} = F_t + e_t = \Phi S_t + e_t$$

Restrictions of model used to estimate latent variables in $F_t$ (identification problems?)
Specifications of observation equation (cont.)

- **Case B** = Case A + 7 new indicators of $F_t$ (14 series in total)

$$
\begin{align*}
X_{1,t} &= F_t + e_{1,t} \\
X_{2,t} &= \Lambda_2 F_t + e_{2,t}
\end{align*}
\iff
X_t = \Lambda_2 F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t}
$$

$$
X_{2,t} = \left[ \begin{array}{c}
\text{cons. excl. food \& energy, priv. invest.,} \\
\text{CPI, core CPI, PCE defl, empl. (HH and est. surveys)}
\end{array} \right]'
$$

- E.g. for inflation: use GDP defl., PCE defl., CPI

- **Case C** = Same as case B, but add 25 PC if all remaining data series: with unrestricted loading matrix on new indicators

$$
\begin{align*}
X_{F,t} &= \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t} \\
X_{3,t} &= \Lambda_S S_t + e_{S,t}
\end{align*}
\iff
X_t = \Lambda_S S_t + e_t
$$
Evidence of “measurement error”

Distribution of correlations between latent concepts (Case A) and reference indicators

- Real GDP, Y
- Consumption, C
- Investment, I
- GDP deflator growth, π
- Real wage, w
- Hours worked, L
Empirical results: Estimated latent variables

- Interest rate: $i$
- Output: $Y$
- Consumption: $C$
- Investment: $I$
- Real wage: $w$
- Employment: $L$

Data A B C
Empirical results: Estimated inflation
Correlation between observable indicators and corresponding latent concepts

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fed funds rate</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Real Consumption</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Real fixed Investment</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>GDP defl. inflation</td>
<td>0.73</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>PCE ex. food and Energy</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Gross Real Investment</td>
<td>0.94</td>
<td>0.95</td>
<td>0.94</td>
</tr>
<tr>
<td>PCE deflator</td>
<td>0.70</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>core-CPI</td>
<td>0.53</td>
<td>0.82</td>
<td>0.81</td>
</tr>
<tr>
<td>CPI</td>
<td>0.54</td>
<td>0.83</td>
<td>0.82</td>
</tr>
<tr>
<td>Employment HH Survey</td>
<td>0.89</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>Payroll Employment</td>
<td>0.81</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

- PCE deflator: good indicator of inflation
Estimated Inflation: Median, 5th and 95th percentiles

- More data $\implies$ more precise estimate of inflation
More information leads to more precise estimates of the latent variables

<table>
<thead>
<tr>
<th>Concept</th>
<th>Case A st. dev.</th>
<th>Case B Relative to case A</th>
<th>Case C Relative to case A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate $R_t$</td>
<td>0.000</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Output $Y_t$</td>
<td>0.342</td>
<td>0.93</td>
<td>1.01</td>
</tr>
<tr>
<td>Consumption $C_t$</td>
<td>0.450</td>
<td>0.93</td>
<td>1.01</td>
</tr>
<tr>
<td>Investment $I_t$</td>
<td>0.908</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>Inflation (annualized) $\pi_t$</td>
<td>0.500</td>
<td><strong>0.91</strong></td>
<td><strong>0.65</strong></td>
</tr>
<tr>
<td>Real wage $w_t$</td>
<td>0.478</td>
<td>1.04</td>
<td>1.06</td>
</tr>
<tr>
<td>Hours worked $L_t$</td>
<td>0.311</td>
<td>0.76</td>
<td>0.97</td>
</tr>
</tbody>
</table>

- At each date: compute st. dev. of concept, based on draws; Report average over time

- Gains particularly important for inflation as large "measurement error"
In-sample “Forecasting” performance: One-step ahead RMSE’s

- Skeptics: want forecasts of data releases $X_{1,t}$, not latent model concepts
  $\implies$ in-sample forecasting (estimation on entire sample)
  – Compute RMSE wrt to seven primary indicators (GDP, GDP defl., ....)

<table>
<thead>
<tr>
<th>Primary indicator ($X_{1,t}$)</th>
<th>Case A RMSE</th>
<th>Case B Relative to case A</th>
<th>Case C Relative to case A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fed funds rate</td>
<td>0.52</td>
<td>1.08</td>
<td>1.12</td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.55</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Real Consumption</td>
<td>0.59</td>
<td>0.93</td>
<td>0.97</td>
</tr>
<tr>
<td>Real Investment</td>
<td>1.64</td>
<td>0.97</td>
<td>0.88</td>
</tr>
<tr>
<td>GDP defl. inflation</td>
<td>0.20</td>
<td>0.95</td>
<td>0.90</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.75</td>
<td>1.03</td>
<td>0.96</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.49</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>Overall</td>
<td>-9.26</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

- Case A designed to explain fluctuations in those indicators

- Cases B, C: designed to explain fluctuations in larger set of indicators
  $\implies$ surprising that it B, C do better!
  – due to fact that state is estimated more precisely
Benefit of adding more information

- More precise estimates of latent variables, in particular inflation

- Better “forecasts” of key reference series
## Estimated structural parameters

<table>
<thead>
<tr>
<th>Prior Distribution</th>
<th>SW</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Mean</td>
<td>St.Err.</td>
<td>Type</td>
<td>Mean</td>
</tr>
<tr>
<td>φ</td>
<td>Normal</td>
<td>4</td>
<td>1.5</td>
<td>(0.88)</td>
</tr>
<tr>
<td>h</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.71</td>
</tr>
<tr>
<td>φ</td>
<td>Normal</td>
<td>1.25</td>
<td>0.125</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$1/\psi$</td>
<td>Normal</td>
<td>0.2</td>
<td>0.075</td>
<td>0.32</td>
</tr>
<tr>
<td>$\gamma_\omega$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$r_{\pi 0}$</td>
<td>Normal</td>
<td>1.8</td>
<td>0.1</td>
<td>1.78</td>
</tr>
<tr>
<td>$r_{\pi 1}$</td>
<td>Normal</td>
<td>-0.3</td>
<td>0.1</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

### Implied parameters

- **pseudo EIS:** \( \frac{1-h}{(1+h)\sigma_e} \)
  - \( 0.110 \)
  - \( 0.099 \)
  - \( 0.167 \)
  - \( 0.204 \)

- **slope of PC:** \( \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p} \)
  - \( 0.011 \)
  - \( 0.007 \)
  - \( 0.012 \)
  - \( 0.018 \)

- Also, variance of estimated shocks typically lower in cases A, B, C.
Estimated time series of capital and shocks

\[ K \text{: Capital stock} \]

\[ \varepsilon^a \text{: Productivity shock} \]

\[ \varepsilon^b \text{: Preference shock} \]

\[ \varepsilon^L \text{: Labor supply shock} \]

\[ \varepsilon^G \text{: Gov. expenditure shock} \]

\[ \varepsilon^I \text{: Invest.-specific shock} \]
Estimated time series of shocks

- $\eta^Q$: Equity premium shock
- $\eta^p$: Price markup shock
- $\eta^\omega$: Wage markup shock
- $\eta^i$: Monetary policy shock
Variance decompositions
Conclusions

• Proposes a general framework that exploits information from data-rich environment for estimation of DSGE models

• Imperfect measurement provides scope for using additional indicators

• Retain structure of model, so can do counterfactual experiments

• Applies method to state-of-the-art model
Conclusions: Findings

Adding more information leads to:

- Different estimates of the state of the economy (inflation)
- More precise estimates of the state of the economy (inflation)
- In-sample “forecasting” performance: improvements
- Different conclusions about the nature of propagation and sources of business cycle fluctuations
Research in Progress:

“Optimal Monetary Policy in a Data-Rich Environment”

- Empirical evidence: large data sets relevant for
  - for forecasting
  - to explain monetary policy, and macro variables

- Questions:
  - Why? To assess state of the economy...
  - What are welfare benefits of exploiting information from large data sets?
Research in Progress:

“Optimal Monetary Policy in a Data-Rich Environment”

- Welfare benefits of CB filtering from a large data set

- Assume:
  - agents know model, param. and state (i.e., realized shocks)
  - CB knows model, param, but not state: observes large number of signals

  - Here extended to data-rich envt.:
    - more accurate assessment of state by CB
    - should improve performance of policy, hence welfare
Benefits of using large data set: A filtering exercise

- "Estimated" DSGE model (Giannoni Woodford, 2003)
  - historical policy; CB responds to observable indicators
  - estimate "true" parameters, historical states, shocks, meas. errors...using large data set

- Compare welfare:

  1. Historical policy:
     - CB filters states from large data set
     - CB responds to few observable indicators

  2. Optimal policy (in progress):
     - Full information about parameters, shocks, no meas. error
     - CB estimates states using small data set (Svensson and Woodford)
     - CB estimates states using large data set
Preliminary results (based on shortcuts)

- Model: Giannoni-Woodford (2003) w/ only efficient supply shocks

- Data-Rich CB: implements policy on the basis of the “true” inflation (case C)

- Data-Poor CB: responds instead to actual data (GDP deflator)

- Comparing losses: 23% higher for Data-Poor CB

<table>
<thead>
<tr>
<th></th>
<th>var((\pi_t))</th>
<th>var((Y_t))</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data-rich CB</td>
<td>2.0028</td>
<td>11.6477</td>
<td>3.6402</td>
</tr>
<tr>
<td>Data-poor CB</td>
<td>2.6206</td>
<td>5.8146</td>
<td>4.4689</td>
</tr>
</tbody>
</table>
Avenues for future research

- Optimal monetary policy rule in a data-rich environment


\[
i_t = \bar{i}_{t|t} \\
0 = a(L) \bar{i}_t + B(L) E_t \left[ C \left( L^{-1} \right) (\tau_t - \tau_t^*) \right]
\]

Desirable properties: determinacy, robustness to shock processes...

- Real time application with mixed frequencies
MCMC: Implementation

- Goal: Characterize the posterior distribution of $\Theta = \{\Theta_M, \gamma\}$

$$
\pi(\Theta|\tilde{X}_T) \propto \mathcal{L}(\Theta|\tilde{X}_T) \ p(\Theta)
$$

where

$\tilde{X}_T = (X_1, X_2, \ldots, X_T)$,

$\Theta_M =$ structural model’s param.

$\gamma =$ observation equ. param.

prior $= p(\Theta)$

Likelihood of $\Theta = \mathcal{L}(\Theta|\tilde{X}_T) \equiv p(\tilde{X}_T|\Theta)$

or

$$
= \int p(\tilde{X}_T|\tilde{S}_T, \Theta) \ p(\tilde{S}_T|\Theta) \ d\tilde{S}_T
$$
MCMC Implementation: How to compute $\pi \left( \Theta | \tilde{X}_T \right)$?

Iterative steps:

1. Given $\Theta(g-1) = \left\{ \Theta_M^{(g-1)}, \gamma^{(g-1)} \right\}$, draw $\tilde{S}_T^{(g)}$ from

   $$p \left( \tilde{S}_T | \Theta^{(g-1)}, \tilde{X}_T \right) = p(\tilde{X}_T | \tilde{S}_T, \Theta^{(g-1)}) p(\tilde{S}_T | \Theta^{(g-1)})$$

2. Given $\tilde{S}_T^{(g)}, \Theta_M^{(g-1)}$ draw $\gamma^{(g)}$ from $p \left( \gamma | \Theta_M^{(g-1)}, \tilde{S}_T^{(g)}, \tilde{X}_T \right)$
   - conditional on $S_t$, observation equ. is linear

3. Given $\gamma^{(g)}, \tilde{S}_T^{(g)}$, draw $\Theta_M^{(g)}$ from $\pi \left( \Theta_M^{(g)} | \gamma^{(g)}, \tilde{S}_T^{(g)}, \tilde{X}_T \right)$
   - use Metropolis-Hasting accept/reject step

- Posterior: Empirical distribution of the draws

   $$\pi \left( \Theta | \tilde{X}_T \right) \propto \left[ \int p(\tilde{X}_T | \tilde{S}_T, \Theta) p(\tilde{S}_T | \Theta) \, d\tilde{S}_T \right] p(\Theta)$$
Details on Step 1

Drawing from $p\left( \tilde{S}_T | \Theta, \tilde{X}_T \right)$: Carter and Kohn (1994):

$$p\left( \tilde{S}_T | \Theta, \tilde{X}_T \right) = p\left( S_T | \Theta, \tilde{X}_T \right) \prod_{t=1}^{T-1} p\left( S_t | S_{t+1}, \Theta, \tilde{X}_T \right)$$

$$S_T | \Theta, \tilde{X}_T \sim N \left( S_T | T, P_T | T \right)$$

$$S_t | S_{t+1}, \Theta, \tilde{X}_T \sim N \left( S_t | t, S_{t+1}, P_t | t, S_{t+1} \right)$$

where

$$S_T | T = E \left( S_T | \Theta, \tilde{X}_T \right)$$

$$P_T | T = Cov \left( S_T | \Theta, \tilde{X}_T \right)$$

$$S_t | t, S_{t+1} = E \left( S_t | S_{t+1}, \Theta, \tilde{X}_T \right)$$

$$P_t | t, S_{t+1} = Cov \left( S_t | S_{t+1}, \Theta, \tilde{X}_T \right)$$
Details on Step 2: Drawing from $\pi \left( \gamma | \Theta_M^{(g-1)}, \tilde{S}_T^{(g)}, \tilde{X}_T \right)$

- Assuming $R_{kl} = 0$, $k \neq l$, and a proper (conjugate) but diffuse Inverse-Gamma $(3, 0.001)$ prior for $R_{kk}$, the posterior is of the form:

$$R_{kk} | \tilde{X}_T, \tilde{S}_T \sim iG \left( \tilde{R}_{kk}, T + 0.001 \right)$$

where $\tilde{R}_{kk} = 3 + \hat{e}'_k \hat{e}_k + \hat{\Lambda}'_k \left( M_0^{-1} + \tilde{S}'_T^{(k)} \tilde{S}_T^{(k)} \right)^{-1} \hat{\Lambda}_k$.

- We draw values for $\Lambda_k$ from the posterior $N \left( \bar{\Lambda}_k, R_{kk} \bar{M}_k^{-1} \right)$, where $\bar{\Lambda}_k = \bar{M}_k^{-1} \left( \tilde{S}'_T^{(k)} \tilde{S}_T^{(k)} \right) \hat{\Lambda}_k^{(k)}$ and $\bar{M}_k = M_0 + \tilde{S}'_T^{(k)} \tilde{S}_T^{(k)}$. 
Details on Step 3: Drawing from $\pi\left(\Theta_M | \gamma(g), \tilde{S}_T^{(g)}, \tilde{X}_T\right)$

- Elements of $\Theta_M$ drawn from a proposal scalar Student $t$-distribution, with mean centered around $\Theta_M^{(g-1)}$, and variance calibrated to yield appropriate acceptance rates.

- A draw $\Theta_M^*$ is accepted with probability $\min(1, r)$ where

$$r = \frac{\pi\left(\Theta_M^* | \gamma(g), \tilde{S}_T^{(g)}, \tilde{X}_T\right)}{\pi\left(\Theta_M^{(g)} | \gamma(g), \tilde{S}_T^{(g)}, \tilde{X}_T\right)}$$