DSGE Models in a Data-Rich Environment

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Importance of large data sets: Evidence from factor models

- Forecasting

  [Stock and Watson (1999, 2002), Forni, Hallin, Lippi, Reichlin (2000)]

- Monetary policy

  [Bernanke and Boivin (2003), Giannone, Reichlin and Sala (2004)]

- VAR

  [Bernanke, Boivin and Eliasz (2005), Forni, Giannone, Lippi, Reichlin (2004)]

- All existing evidence based on largely non-structural models. Limited ability to:
  - Determine sources of fluctuations
  - Perform counterfactual experiments
  - Analyze optimal policy

Usefulness of large data sets raises interesting questions

- Why is the additional information useful?

- Are the costs of ignoring this information important?
  - For structural modeling?
  - For optimal conduct of policy?

Estimated DSGE models

- Important developments
  [Altug (1989), McGrattan (1994), Leeper and Sims (1994), Rotemberg and Woodford (1997), Ireland (1997, 2001), Kim (2000), Schorfheide (2000), Christiano, Eichenbaum and Evans (2005), Amato and Laubach (2003), Smets and Wouters (2003, 2007), Altig, Christiano, Eichenbaum and Linde (2003), Rabanal and Rubio-Ramírez (2003), Julliard, Karam, Laxton and Pesenti (2004), LOWW (2005), Justiniano and Primiceri (2006), ...]

- Promising empirical success

  [Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007)]

- Now increasingly taken seriously as empirical models

Estimated DSGE models

- Important developments
  [Altug (1989), McGrattan (1994), Leeper and Sims (1994), Rotemberg and Woodford (1997), Ireland (1997, 2001), Kim (2000), Schorfheide (2000), Christiano, Eichenbaum and Evans (2005), Amato and Laubach (2003), Smets and Wouters (2003, 2007), Altig, Christiano, Eichenbaum and Linde (2003), Rabanal and Rubio-Ramírez (2003), Julliard, Karam, Laxton and Pesenti (2004), LOWW (2005), Justiniano and Primiceri (2006), ...]

- Promising empirical success

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- Now increasingly taken seriously as empirical models

- Estimated based on a handful of data series
  $\Longrightarrow$ at odds with fact that CB and financial market participants monitor large number of data series!

Goal of this paper

- To explore role of large data sets for estimated DSGE models
- By product: Provide economic interpretation of latent factors

Preview of the main results

- More precise estimation of the state of the economy
- Improvements in "forecasting" with additional information
- Different conclusions about structure of economy and sources of business cycles
  - Different propagation mechanism (e.g. less habit formation and inflation indexing)
  - Fewer and smaller structural shocks

Why more data in a DSGE context?

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  - Model well specified
  - Theoretical concepts directly observed by agents and econometrician

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- No scope if:
  - Model well specified
  - Theoretical concepts directly observed by agents and econometrician

- Empirical evidence: at least one assumption violated

- We assume theoretical concepts partially observed by econometrician
  - Employment: Discrepancies between household and payroll surveys
  - Inflation: GDP deflator, CPI
  - Productivity shock: oil prices, commodity prices

- If indeed we are missing information in DSGE estimation:
  all parameter estimates potentially distorted!

Why more data in a DSGE context? What is Employment?

Why more data in a DSGE context? What is Inflation?

Outline of presentation

- Data-rich environment
  - A simple example: RBC model
  - General framework
  - Estimation

- Application: Smets and Wouters (2004) model
  - Results

- Conclusion

- Research in progress: Optimal policy in data-rich envt.

Why more information? A simple RBC model

Households maximize lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + v \log(1 - l_t)\right], \qquad 0 < \beta < 1, \ v > 0$$

subject to

$$\begin{aligned} y_t &= e^{a_t} k_t^{1-\alpha} l_t^{\alpha}, \qquad 0 < \alpha < 1 \\ y_t &= c_t + k_{t+1} - (1-\delta) k_t, \\ a_t &= \rho a_{t-1} + \varepsilon_t, \qquad 0 \leq \rho < 1, \ \varepsilon_t \sim N(0, \sigma). \end{aligned}$$

RBC example continued...

- Linearized solution has the form:

$$\left.\begin{aligned}\hat{y}_t &= d_1\hat{k}_t + d_2a_t\\ \hat{c}_t &= d_3\hat{k}_t + d_4a_t\\ \hat{l}_t &= d_5\hat{k}_t + d_6a_t\end{aligned}\right\} \quad z_t = DS_t, \qquad z_t = \left[\hat{y}_t,\ \hat{c}_t,\ \hat{l}_t\right]'$$

$$\left.\begin{aligned}\hat{k}_t &= g_1\hat{k}_{t-1} + g_2a_{t-1}\\ a_t &= \rho a_{t-1} + \varepsilon_t\end{aligned}\right\} \quad S_t = GS_{t-1} + H\varepsilon_t, \qquad S_t = \left[\hat{k}_t, a_t\right]'$$

  where D, G and H are functions of model parameters

- Suppose we estimate the model on the basis of only one variable (no stochastic singularity):

$$F_t = \hat{y}_t = [d_1 \quad d_2]\, S_t$$

RBC example: How to link model and data?

- One indicator, no measurement error

$$X_t = \hat{y}_t = d_1 \hat{k}_t + d_2 a_t$$

  – e.g. $X_t =$ real GDP

  – No scope for judgment or soft data

- One indicator, measurement error (Sargent, 1989):

$$X_t = \hat{y}_t + e_t = d_1 \hat{k}_t + d_2 a_t + e_t$$

  – 1 shock and 1 measurement error: identification from dynamics

  – Identification problems?

RBC example (cont.): Proposed solution

- Multiple indicators with **known** relationships to theoretical concepts

$$X_t = \begin{bmatrix} \text{real GDP} \\ \text{real NI} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_{NI} \end{bmatrix} \hat{y}_t + e_t$$

$$= \begin{bmatrix} d_1 & d_2 \\ \lambda_{NI} d_1 & \lambda_{NI} d_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + e_t$$

  - Helps disentangle meas. error from structural shocks

- Multiple indicators with **unknown** link

- E.g., soft data, oil price (e.g., as indicator of a_t)

$$X_t = \begin{bmatrix} \text{real GDP} \\ \text{soft data} \end{bmatrix} = \begin{bmatrix} d_1 & d_2 \\ \lambda_3 & \lambda_4 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + e_t$$

Benefits of exploiting more information: Intuition

- Measurement error identifiable from cross-section of indicators

  Example: $x_{it} = f_t + e_{it}$, $i = 1, ..., n_X$

  - If $n_X = 1$, and both f_t and e_{it} are i.i.d. $\Longrightarrow$ Not identified
  - If $n_X = 1$, f_t is AR(1) and e_{it} is i.i.d. $\Longrightarrow$ Identified (from dynamics)
  - If $n_X > 1$, and both f_t and e_{it} are i.i.d. $\Longrightarrow$ Identified (from cross-section)

- Permits the identification of more structural shocks

- Don't have to take a stand *a priori* on the relative importance of measurement errors vs structural shocks

- More efficient (consistent) estimate of the latent factors

  - $var(\hat{f}_t)$ is of order $1/n_X$ [Stock Watson (2002), Forni et al. (2000)]

Data-rich environment: General framework

- Linear(ized) DSGE model:

$$\begin{aligned} AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} &= B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cs_t \\ s_t &= Ms_{t-1} + \varepsilon_t \end{aligned}$$

Note: Agents assumed to know the model and model concepts

- Solution (REE):

$$\begin{aligned} z_t &= DS_t \\ S_t &= GS_{t-1} + H\varepsilon_t \end{aligned}$$

$$S_t \equiv \begin{bmatrix} Z_t \\ s_t \end{bmatrix}$$

Dynamics of any variable entirely determined by vector of state variables S_t

Variables of interest (e.g. inflation): F_t

- Defined as

$$F_t \equiv F \begin{bmatrix} z_t \\ S_t \end{bmatrix}$$

where F is a selection matrix

- Related to state vector

$$F_t = \mathbf{\Phi} S_t,$$

where

$$\mathbf{\Phi} \equiv F \begin{bmatrix} D \\ I \end{bmatrix}$$

Linking theory and data: Known link

$$X_{F,t} = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t}$$

where Λ_F has only one non-zero element on each row

- Concepts with multiple indicators. Examples:
  - Employment: household vs. establishment surveys
  - Prices: GDP deflator, PCE deflator, CPI, ...

Linking theory and data: Known link

$$X_{F,t} = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t}$$

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- Concepts with multiple indicators. Examples:
  – Employment: household vs. establishment surveys
  – Prices: GDP deflator, PCE deflator, CPI, ...

- Special cases:
  – No measurement error: $X_{F,t} = F_t = \Phi S_t$
  – Standard treatment of measurement error: Sargent (1989)

$$X_{F,t} = F_t + e_{F,t} = \Phi S_t + e_{F,t}$$

single indicator for each concept
$\Longrightarrow$ Maintains that a small number of series contains all relevant information

Linking theory and data: Unknown link

$$X_{S,t} = \Lambda_S S_t + e_{S,t}$$

where Λ_S is completely unrestricted (e.g. commodity prices)

- $X_{S,t}$ helps estimate the state vector S_t
- Partially observed state variables / exogenous shocks
  - E.g. productivity shock: oil or commodity prices may provide information
- More flexible exploitation of information

Empirical model: Summary

- Transition equation:

$$S_t = GS_{t-1} + H\varepsilon_t$$

- Observation equation:

$$X_t = \Lambda S_t + e_t$$

where

$$X_t \equiv \begin{bmatrix} X_{F,t} \\ X_{S,t} \end{bmatrix}, \qquad e_t \equiv \begin{bmatrix} e_{F,t} \\ e_{S,t} \end{bmatrix}, \qquad \Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}.$$

- Comments:

  – Related to non-structural factor models, but we impose DSGE model on transition equation of latent factors

  – Factors have economic interpretation: state variables

  – Interpret info. in data set through lenses of DSGE model

  – Can do counterfactual experiments, study optimal policy

Application: Smets and Wouters (2004) [i.e., CEE (2005) with shocks]

- State-of-the-art DSGE model:
  - Popular as fits apparently well, good for forecasting
  - Many frictions, many shocks

- Households
  - Consume aggregate of all goods, habit formation (external)
  - Supply specialized labor on monopolistically competitive labor mkt
  - Rent capital services to firms
  - Decide how much capital to accumulate

- Firms:
  - Choose labor and capital inputs
  - Supply differentiated goods on monopolistically competitive goods mkt

- Prices and wages reoptimized at random intervals (Calvo)
  - If not reoptimized: indexed to past inflation and CB's inflation target

Model summary (Log-linearized)

- Consumption Euler equation

$$C_t = \frac{h}{1+h}C_{t-1} + \frac{1}{1+h}E_tC_{t+1} + \frac{\sigma_c - 1}{\sigma_c(1+\lambda_w)(1+h)}(L_t - E_tL_{t+1}) - \frac{(1-h)}{(1+h)\sigma_c}(i_t - E_t\pi_{t+1}) + \varepsilon_t^b$$

- Investment Euler equation

$$I_t = \frac{1}{1+\beta}I_{t-1} + \frac{\beta}{1+\beta}E_tI_{t+1} + \frac{1/\varphi}{1+\beta}\left(Q_t + \varepsilon_t^I\right)$$

- Real value of capital

$$Q_t = -(i_t - E_t\pi_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_tQ_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_tr_{t+1}^k + \eta_t^Q$$

- Capital accumulation

$$K_t = (1-\tau)K_{t-1} + \tau I_{t-1} + \tau\varepsilon_{t-1}^I$$

Model summary (cont.)

- Optimal real wage set by household

$$\begin{aligned} w_t &= \frac{\beta}{1+\beta}E_t w_{t+1} + \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} \\ &\quad -\frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\ &\quad -\frac{\lambda_w(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)(\lambda_w+(1+\lambda_w)\sigma_L)\xi_w} \\ &\quad \times\left[w_t - \sigma_L L_t - \frac{\sigma_c}{1-h}(C_t - hC_{t-1}) + \varepsilon_t^L\right] + \eta_t^w \end{aligned}$$

- Optimal price setting by firms

$$\begin{aligned} \pi_t &= \frac{\beta}{1+\beta\gamma_p}E_t\pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} \\ &\quad +\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\left(1+\beta\gamma_p\right)\xi_p}\left[\alpha r_t^k + (1-\alpha)w_t - \varepsilon_t^a\right] + \eta_t^p \end{aligned}$$

Model summary (cont.)

- Labor demand

$$L_t = -w_t + (1 + \psi) r_t^k + K_{t-1}$$

- Goods market equilibrium

$$\begin{aligned} Y_t &= (1 - \tau k_y - g_y) C_t + \varepsilon_t^G + \tau k_y I_t + \bar{r}^k k_y \psi r_t^k \\ &= \phi \left[\varepsilon_t^a + \alpha K_{t-1} + \alpha \psi r_t^k + (1 - \alpha) L_t \right] \end{aligned}$$

- Monetary policy reaction function

$$i_t = (1 - \rho) \left[r_{\pi 0} \pi_t + r_{\pi 1} \pi_{t-1} + r_{y0} Y_t + r_{y1} Y_{t-1} \right] + \rho i_{t-1} + \eta_t^i$$

Smets and Wouters (2004): Model solution

- 7 variables of interest: $F_t = [i_t, Y_t, C_t, I_t, \pi_t, w_t, L_t]'$

- 9 shocks: $s_t = \left[\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^G, \varepsilon_t^L, \varepsilon_t^I, \eta_t^Q, \eta_t^p, \eta_t^w, \eta_t^i\right]'$

- State vector

$$S_t = \left[i_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, \pi_{t-1}, w_{t-1}, K_{t-1}, \varepsilon_{t-1}^I, s_t'\right]'$$

- State-space representation:
  – Transition equation

$$S_t = GS_{t-1} + H\varepsilon_t$$

  – Observation equation

$$X_t = \Lambda S_t + e_t$$

Estimation method

- Difficult problem to estimate (large dimension)

- Standard methods (e.g. MLE) difficult to implement in this case

- MCMC methods:
  – Empirical approximation of the posterior distribution. Does not rely on gradient method
  – Draw iteratively from conditional distributions (solves the high-dimensionality problem)
  – Priors can help make the estimation better behaved
  – Here priors same as in Smets and Wouters (2004)

- 100,000 iterations; check Markov chains are "well behaved"

MCMC: Implementation

- Goal: Characterize the posterior distribution of $\Theta = \{\Theta_M, \Upsilon\}$

$$\pi\left(\Theta|\tilde{X}_T\right) \propto \mathfrak{L}(\Theta|\tilde{X}_T)\, p(\Theta)$$

where
$\tilde{X}_T = (X_1, X_2, ..., X_T)\,,$
$\Theta_M =$ structural model's param.
$\Upsilon =$ observation equ. param.
prior $= p(\Theta)$

Likelihood of $\Theta = \mathfrak{L}(\Theta|\tilde{X}_T) \equiv p(\tilde{X}_T|\Theta)$
or $= \int p(\tilde{X}_T|\tilde{S}_T, \Theta)\, p\left(\tilde{S}_T|\Theta\right) d\tilde{S}_T$

MCMC Implementation: How to compute $\pi\left(\Theta|\tilde{X}_T\right)$?

Iterative steps:

1. Given $\Theta^{(g-1)} = \left\{\Theta_M^{(g-1)}, \Upsilon^{(g-1)}\right\}$, draw $\tilde{S}_T^{(g)}$ from

$$p\left(\tilde{S}_T|\Theta^{(g-1)}, \tilde{X}_T\right) = p(\tilde{X}_T|\tilde{S}_T, \Theta^{(g-1)})p(\tilde{S}_T|\Theta^{(g-1)})$$

   – use Carter-Kohn (1994)

2. Given $\tilde{S}_T^{(g)}$,$\Theta_M^{(g-1)}$ draw $\Upsilon^{(g)}$ from $p\left(\Upsilon|\Theta_M^{(g-1)}, \tilde{S}_T^{(g)}, \tilde{X}_T\right)$
   – conditional on S_t, observation equ. is linear: known dist. (Chib, 1993)

3. Given $\Upsilon^{(g)}, \tilde{S}_T^{(g)}$, draw $\Theta_M^{(g)}$ from $\pi\left(\Theta_M^{(g)}|\Upsilon^{(g)}, \tilde{S}_T^{(g)}, \tilde{X}_T\right)$
   – use RW Metroplis accept/reject step; proposal scalar Student-t dist.

- Posterior: Empirical distribution of the draws

$$\pi\left(\Theta|\tilde{X}_T\right) \propto \left[\int p(\tilde{X}_T|\tilde{S}_T,\Theta)\ p(\tilde{S}_T|\Theta)\ d\tilde{S}_T\right] p(\Theta)$$

Specifications of observation equation: $X_t = \Lambda S_t + e_t$

- **Case SW**: Standard estimation (as in Smets and Wouters)

$$X_{1,t} = F_t = \Phi S_t$$

where

$$X_{1,t} = [\text{Fed funds, GDP, cons., invest., }\%\Delta GDP\text{ defl, real wage, hours worked}]'$$

- **Case A** $=$ Case SW $+$ Measurement error (as in Sargent, 1989):

$$X_{1,t} = F_t + e_t = \Phi S_t + e_t$$

Restrictions of model used to estimate latent variables in F_t (identification problems?)

Specifications of observation equation (cont.)

- **Case B** = Case A + 7 new indicators of F_t (14 series in total)

$$\left.\begin{array}{l} X_{1,t} = F_t + e_{1,t} \\ X_{2,t} = \Lambda_2 F_t + e_{2,t} \end{array}\right\} \Longleftrightarrow X_t = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t}$$

$$X_{2,t} = \left[\begin{array}{c} \text{cons. excl. food \& energy, priv. invest.,} \\ \text{CPI, core CPI, PCE defl, empl. (HH and est. surveys)} \end{array} \right]'$$

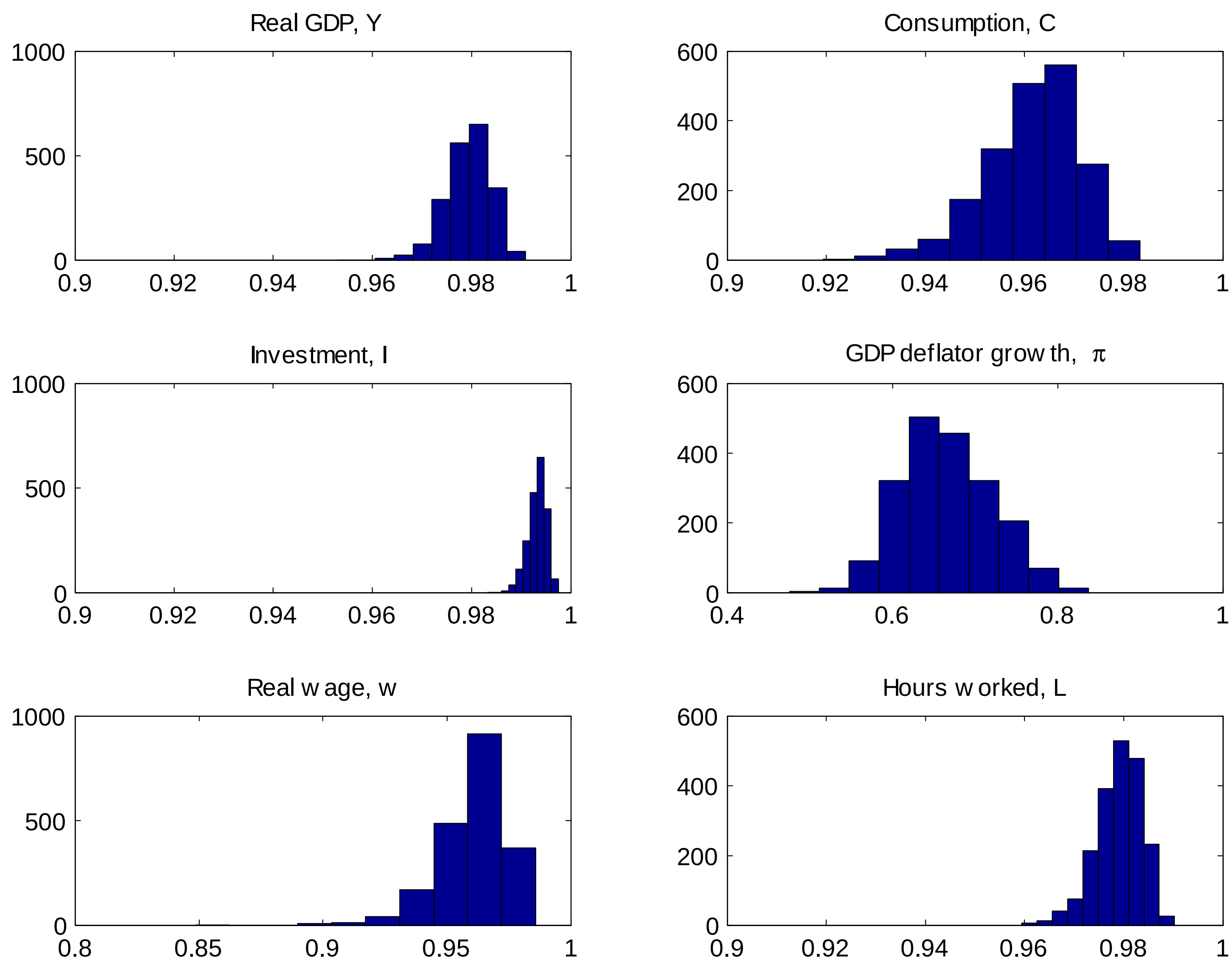
  – E.g. for inflation: use GDP defl., PCE defl., CPI

- **Case C** = Same as case B, but add 25 PC if all remaining data series: with unrestricted loading matrix on new indicators

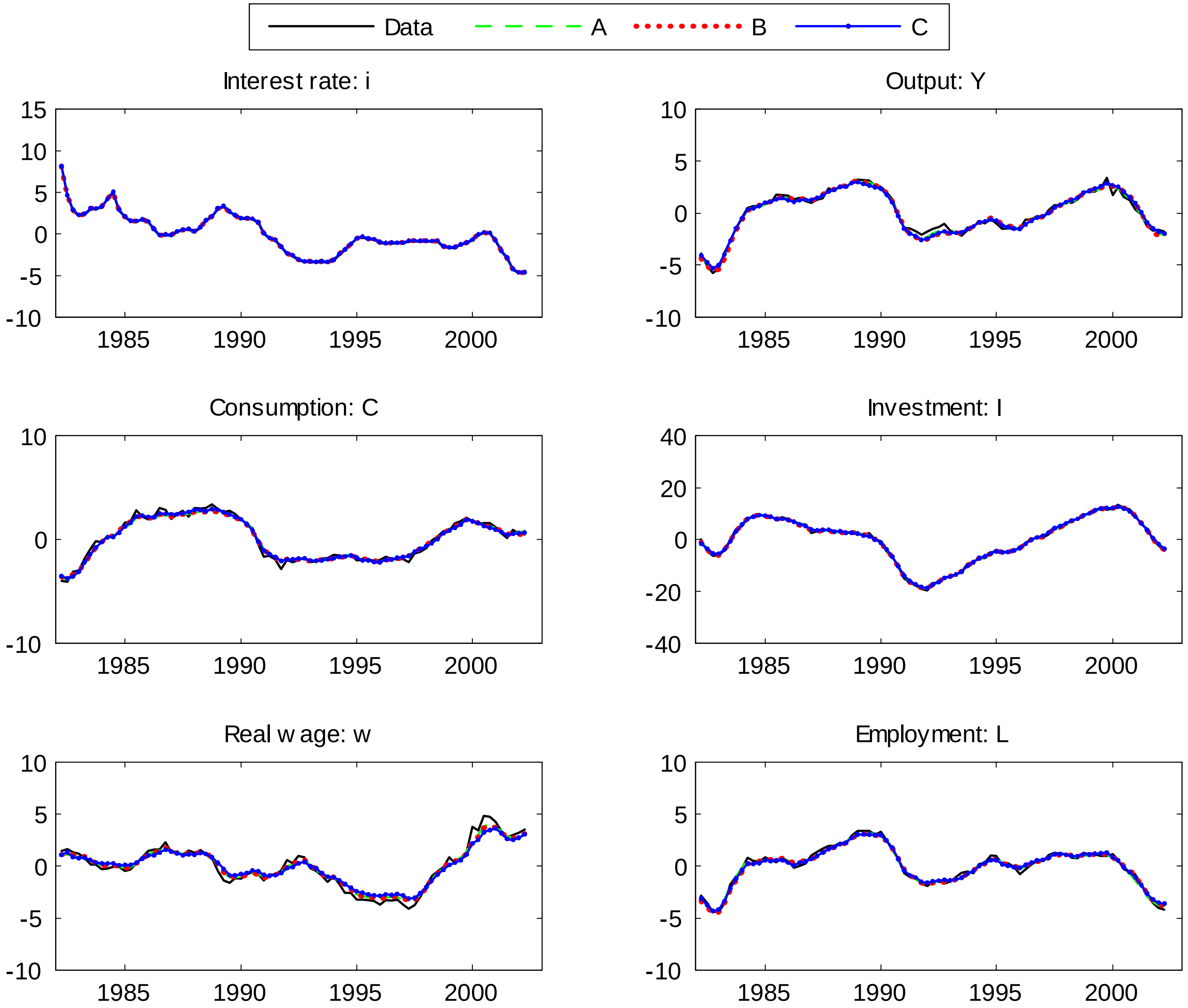
$$\left.\begin{array}{l} X_{F,t} = \Lambda_F F_t + e_{F,t} = \Lambda_F \Phi S_t + e_{F,t} \\ X_{3,t} = \Lambda_S S_t + e_{S,t} \end{array}\right\} \Longleftrightarrow X_t = \Lambda S_t + e_t$$

Evidence of "measurement error"

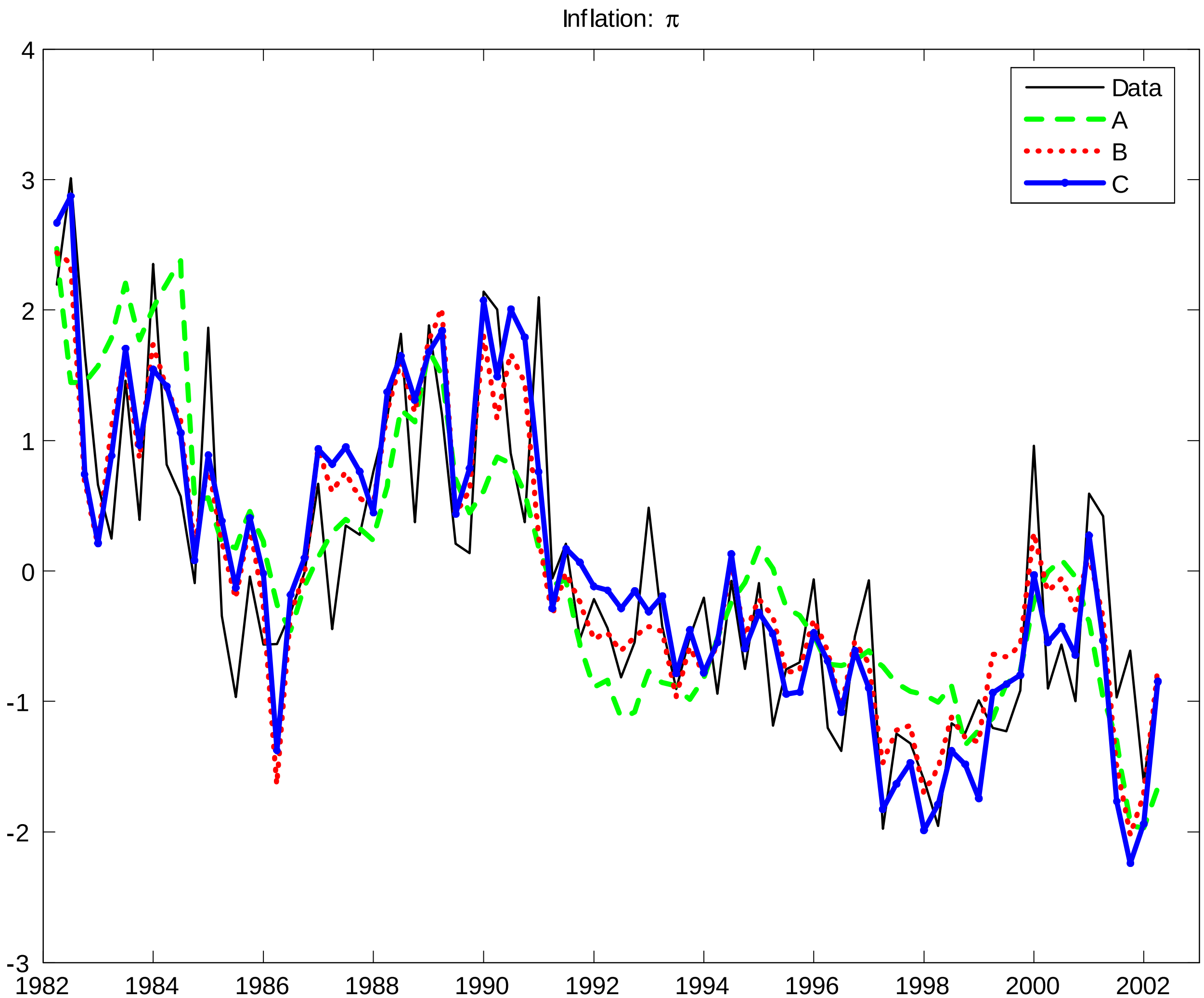
Distribution of correlations between latent concepts (Case A) and reference indicators



Empirical results: Estimated latent variables



Empirical results: Estimated inflation

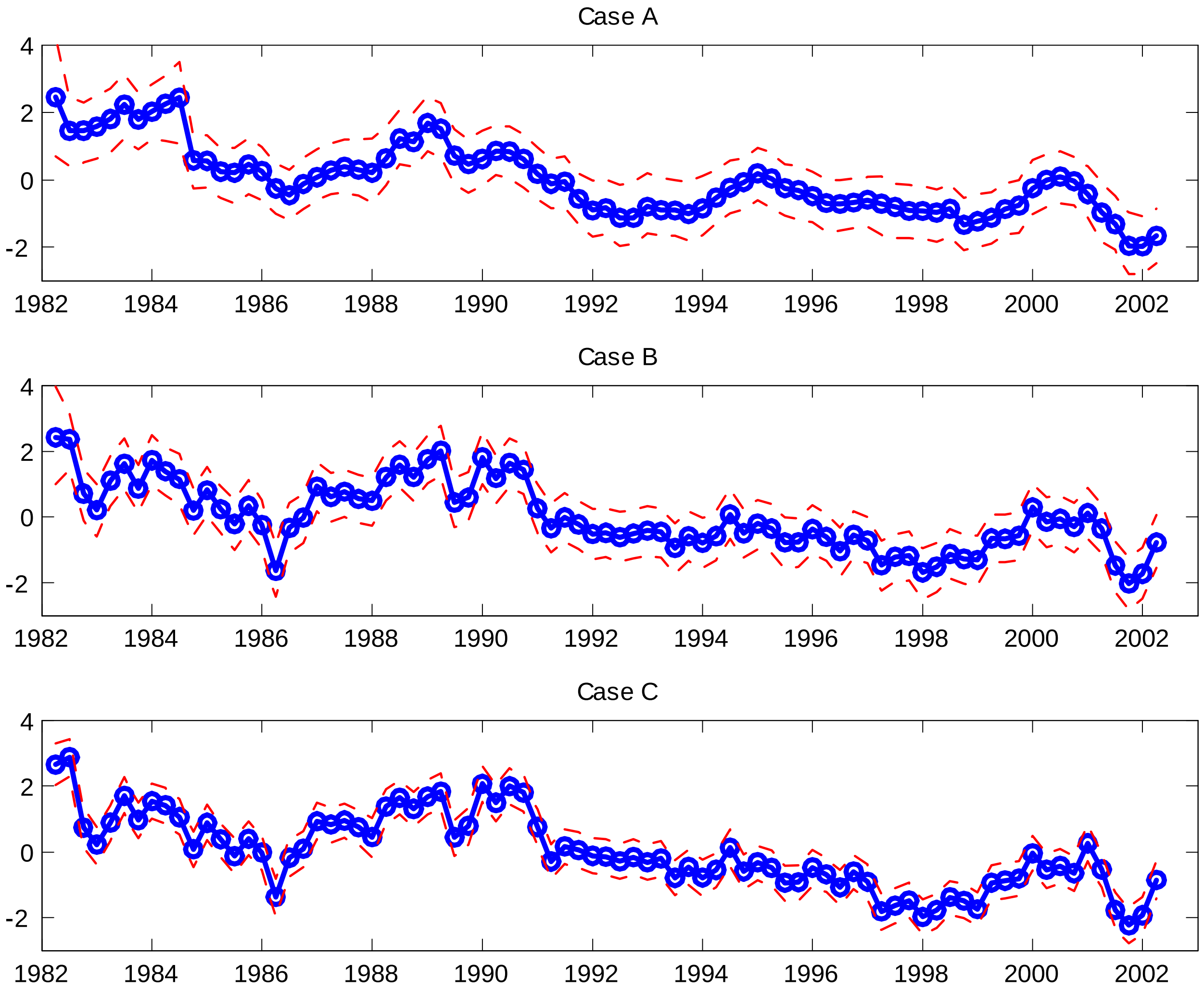


Correlation between observable indicators and corresponding latent concepts

		Case A	Case B	Case C
$X_{1,t}$	Fed funds rate	1.00	1.00	1.00
	Real GDP	0.99	0.98	0.98
	Real Consumption	0.98	0.99	0.98
	Real fixed Investment	0.99	0.99	0.99
	GDP defl. inflation	0.73	0.86	0.86
	Real wage	0.99	0.99	0.98
	Hours worked	0.99	0.98	0.98
$X_{2,t}$	PCE ex. food and Energy	0.98	0.99	0.98
	Gross Real Investment	0.94	0.95	0.94
	PCE deflator	0.70	**0.92**	**0.93**
	core-CPI	0.53	0.82	0.81
	CPI	0.54	0.83	0.82
	Employment HH Survey	0.89	0.92	0.92
	Payroll Employment	0.81	0.85	0.85

- PCE deflator: good indicator of inflation

Estimated Inflation: Median, 5th and 95th percentiles



- More data $\Longrightarrow$ more precise estimate of inflation

More information leads to more precise estimates of the latent variables

Concept		Case A st. dev.	Case B	Case C
			Relative to case A	
Interest rate	R_t	0.000	—	—
Output	Y_t	0.342	0.93	1.01
Consumption	C_t	0.450	0.93	1.01
Investment	I_t	0.908	0.94	0.89
Inflation (annualized)	π_t	0.500	0.91	0.65
Real wage	w_t	0.478	1.04	1.06
Hours worked	L_t	0.311	0.76	0.97

- At each date: compute st. dev. of concept, based on draws; Report average over time

- Gains particularly important for inflation as large "measurement error"

In-sample “Forecasting” performance: One-step ahead RMSE’s

- Skeptics: want forecasts of data releases $X_{1,t}$, *not* latent model concepts
  $\Longrightarrow$ *in-sample* forecasting (estimation on entire sample)
  – Compute RMSE wrt to seven primary indicators (GDP, GDP defl., ...)

Primary indicator ($X_{1,t}$)	Case A RMSE	Case B	Case C
		Relative to case A	
Fed funds rate	0.52	1.08	1.12
Real GDP	0.55	1.00	1.02
Real Consumption	0.59	0.93	0.97
Real Investment	1.64	0.97	0.88
GDP defl. inflation	0.20	0.95	0.90
Real wage	0.75	1.03	0.96
Hours worked	0.49	1.02	1.04
Overall	-9.26	0.98	0.98

- Case A designed to explain fluctuations in those indicators

- Cases B, C: designed to explain fluctuations in larger set of indicators
  $\Longrightarrow$ surprising that it B, C do better!
  – due to fact that state is estimated more precisely

Benefit of adding more information

- More precise estimates of latent variables, in particular inflation

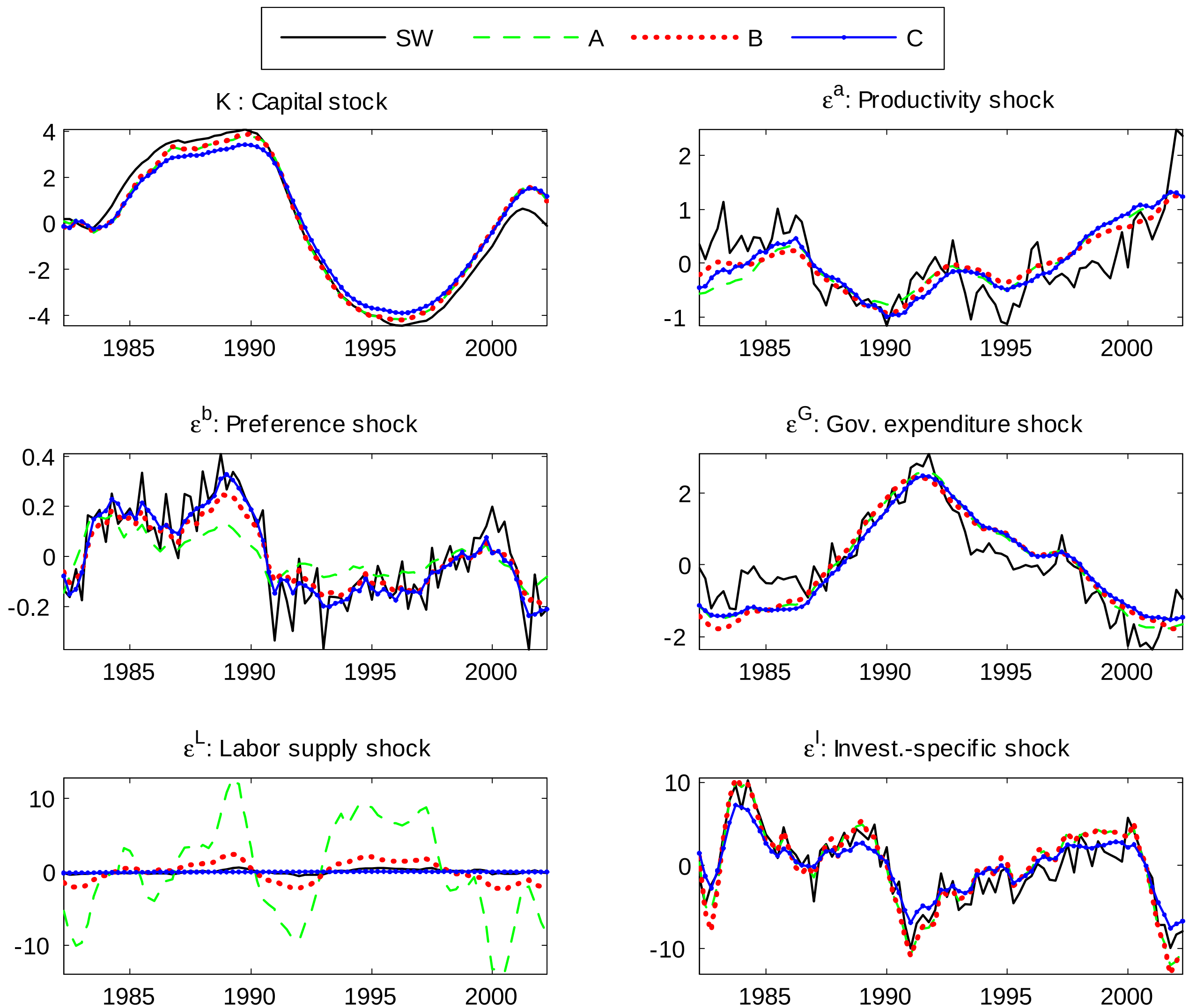
- Better "forecasts" of key reference series

Estimated structural parameters

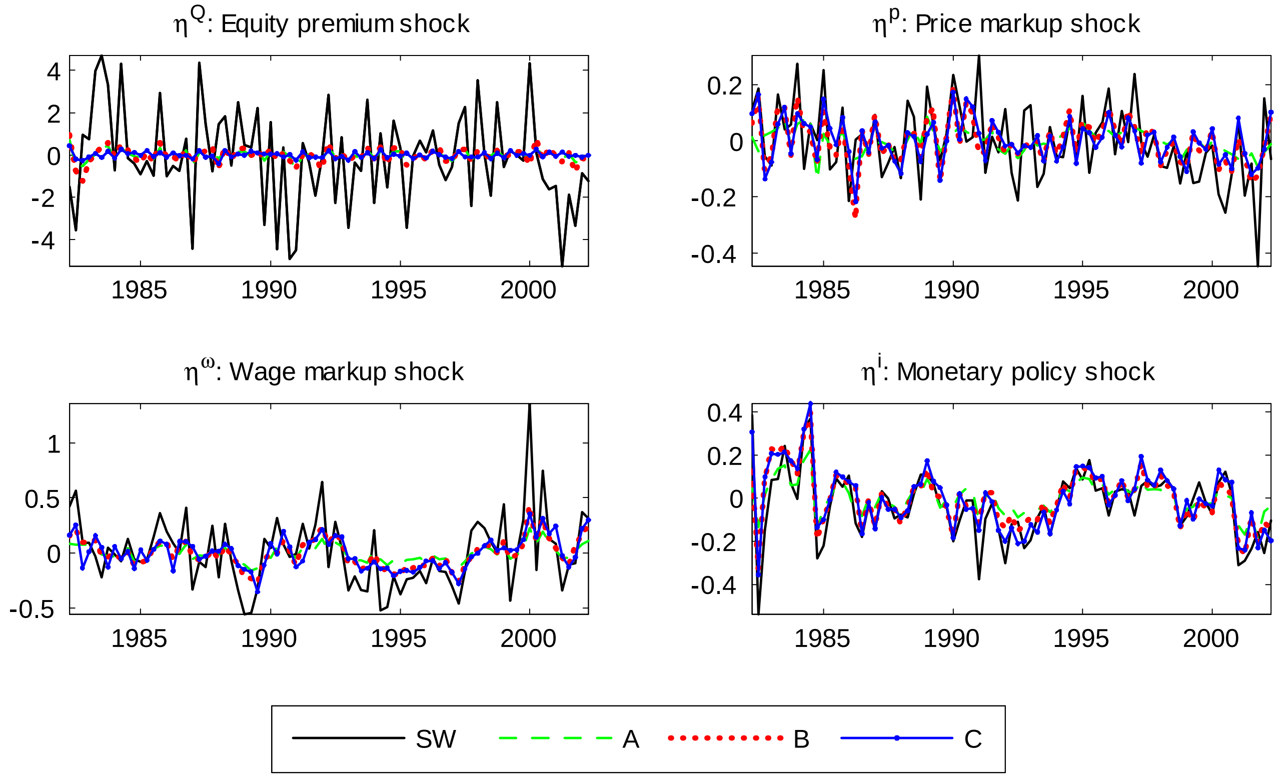
	Prior Distribution			SW	Case A	Case B	Case C
	Type	Mean	St.Err.				
φ	Normal	4	1.5	5.36	5.88	6.17	3.81
				(0.88)	(1.11)	(1.13)	(1.04)
h	Beta	0.7	0.1	0.71	0.75	0.54	0.50
				(0.07)	(0.07)	(0.27)	(0.27)
ϕ	Normal	1.25	0.125	1.42	1.24	1.37	1.26
				(0.08)	(0.07)	(0.07)	(0.07)
$1/\psi$	Normal	0.2	0.075	0.32	0.27	0.26	0.27
				(0.06)	(0.06)	(0.06)	(0.06)
γ_ω	Beta	0.5	0.15	0.39	0.45	0.43	0.48
				(0.12)	(0.14)	(0.14)	(0.14)
γ_p	Beta	0.5	0.15	0.66	0.72	0.50	0.36
				(0.08)	(0.19)	(0.15)	(0.14)
$r_{\pi 0}$	Normal	1.8	0.1	1.78	1.81	1.72	1.66
				(0.08)	(0.10)	(0.10)	(0.09)
$r_{\pi 1}$	Normal	-0.3	0.1	-0.22	-0.22	-0.30	-0.39
				(0.09)	(0.12)	(0.10)	(0.09)
Implied parameters							
pseudo EIS: $\frac{1-h}{(1+h)\sigma_c}$				0.110	0.099	0.167	0.204
slope of PC: $\frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}$				0.011	0.007	0.012	0.018

- Also, variance of estimated shocks typically lower in cases A, B, C.

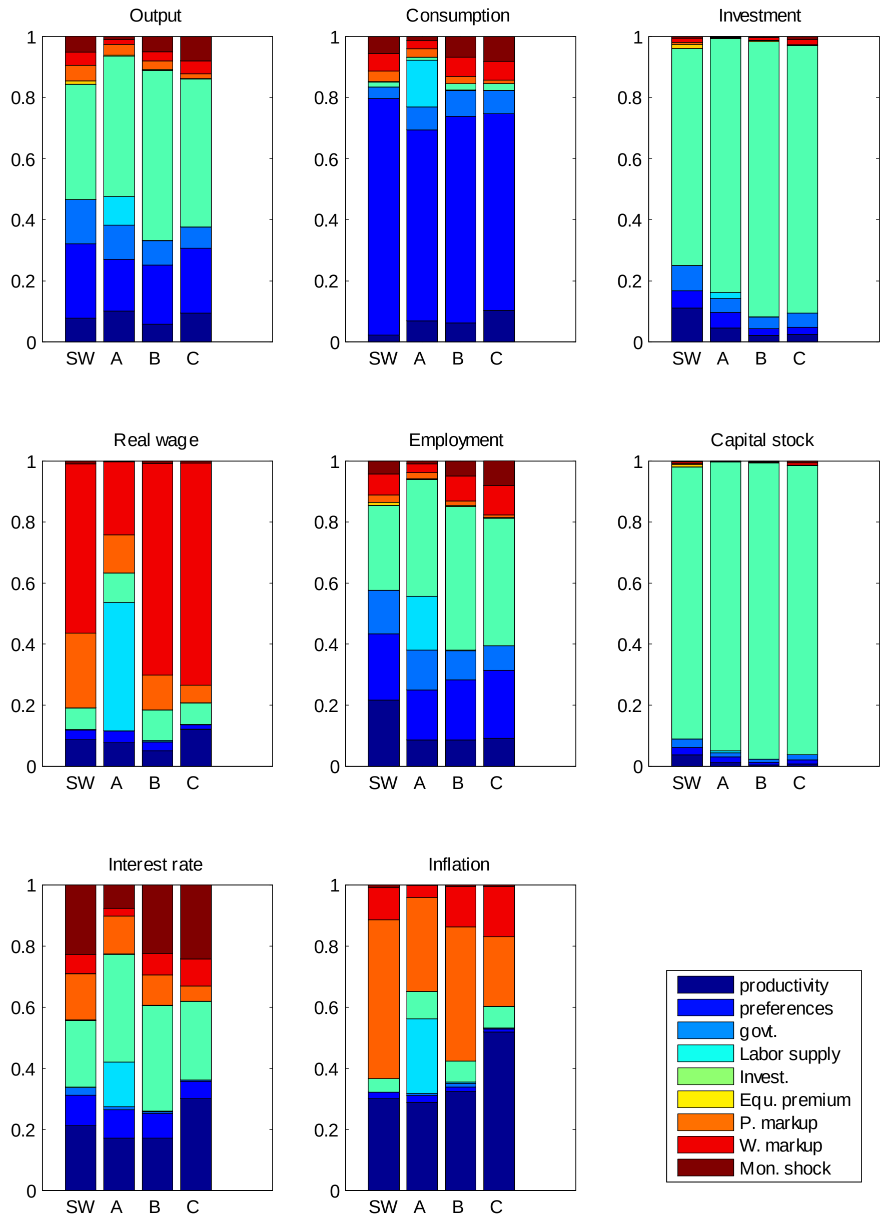
Estimated time series of capital and shocks



Estimated time series of shocks



Variance decompositions



Conclusions

- Proposes a general framework that exploits information from data-rich environment for estimation of DSGE models

- Imperfect measurement provides scope for using additional indicators

- Retain structure of model, so can do counterfactual experiments

- Applies method to state-of-the art model

Conclusions: Findings

Adding more information leads to:

- Different estimates of the state of the economy (inflation)
- More precise estimates of the state of the economy (inflation)
- In-sample "forecasting" performance: improvements
- Different conclusions about the nature of propagation and sources of business cycle fluctuations

Research in Progress:

"Optimal Monetary Policy in a Data-Rich Environment"

- Empirical evidence: large data sets relevant for
  - for forecasting
  - to explain monetary policy, and macro variables

- Questions:
  - Why? To assess state of the economy...
  - What are welfare benefits of exploiting information from large data sets?

Research in Progress:

"Optimal Monetary Policy in a Data-Rich Environment"

- Welfare benefits of CB filtering from a large data set

- Assume:
  - agents know model, param. and state (i.e., realized shocks)
  - CB knows model, param, but not state: observes large number of signals

- Opt. policy with asymmetric info: Svensson-Woodford (2004)
  - Here extended to data-rich envt.:

    $\Longrightarrow$ more accurate assessment of state by CB

    $\Longrightarrow$ should improve performance of policy, hence welfare

Benefits of using large data set: A filtering exercise

- "Estimated" DSGE model (Giannoni Woodford, 2003)
  – historical policy; CB responds to observable indicators
  – estimate "true" parameters, historical states, shocks, meas. errors...using large data set

- Compare welfare:

  1. Historical policy:
     – CB filters states from large data set
     – CB responds to few observable indicators

  2. Optimal policy (in progress):
     – Full information about parameters, shocks, no meas. error
     – CB estimates states using small data set (Svensson and Woodford)
     – CB estimates states using large data set

Preliminary results (based on shortcuts)

- Model: Giannoni-Woodford (2003) w/ only efficient supply shocks
- Data-Rich CB: implements policy on the basis of the "true" inflation (case C)
- Data-Poor CB: responds instead to actual data (GDP deflator)
- Comparing losses: **23% higher for Data-Poor CB**

	var(π_t)	var(Y_t)	Loss
Data-rich CB	2.0028	11.6477	3.6402
Data-poor CB	2.6206	5.8146	4.4689

Avenues for future research

- Optimal monetary policy rule in a data-rich environment

  – GW (2002,2003): General characterization of optimal target criterion

$$
\begin{aligned}
i_t &= \bar{\imath}_{t|t} \\
0 &= a(L)\,\bar{\imath}_t + B(L)\,E_t\left[C\left(L^{-1}\right)(\tau_t - \tau_t^*)\right]
\end{aligned}
$$

  Desirable properties: determinacy, robustness to shock processes...

- Real time application with mixed frequencies

MCMC: Implementation

- Goal: Characterize the posterior distribution of $\Theta = \{\Theta_M, \Upsilon\}$

$$\pi\left(\Theta|\tilde{X}_T\right) \propto \mathfrak{L}(\Theta|\tilde{X}_T)\ p(\Theta)$$

where
$\tilde{X}_T = (X_1, X_2, ..., X_T)$,
Θ_M = structural model's param.
Υ = observation equ. param.
prior = $p(\Theta)$

Likelihood of $\Theta = \mathfrak{L}(\Theta|\tilde{X}_T) \equiv p(\tilde{X}_T|\Theta)$
or $= \int p(\tilde{X}_T|\tilde{S}_T, \Theta)\ p\left(\tilde{S}_T|\Theta\right) d\tilde{S}_T$

MCMC Implementation: How to compute $\pi\left(\Theta|\tilde{X}_T\right)$?

Iterative steps:

1. Given $\Theta^{(g-1)} = \left\{\Theta_M^{(g-1)}, \Upsilon^{(g-1)}\right\}$, draw $\tilde{S}_T^{(g)}$ from

$$p\left(\tilde{S}_T|\Theta^{(g-1)}, \tilde{X}_T\right) = p(\tilde{X}_T|\tilde{S}_T, \Theta^{(g-1)})p(\tilde{S}_T|\Theta^{(g-1)})$$

2. Given $\tilde{S}_T^{(g)}$,$\Theta_M^{(g-1)}$ draw $\Upsilon^{(g)}$ from $p\left(\Upsilon|\Theta_M^{(g-1)}, \tilde{S}_T^{(g)}, \tilde{X}_T\right)$
   – conditional on S_t, observation equ. is linear

3. Given $\Upsilon^{(g)}, \tilde{S}_T^{(g)}$, draw $\Theta_M^{(g)}$ from $\pi\left(\Theta_M^{(g)}|\Upsilon^{(g)}, \tilde{S}_T^{(g)}, \tilde{X}_T\right)$
   – use Metroplis-Hasting accept/reject step

- Posterior: Empirical distribution of the draws

$$\pi\left(\Theta|\tilde{X}_T\right) \propto \left[\int p(\tilde{X}_T|\tilde{S}_T, \Theta)\; p(\tilde{S}_T|\Theta)\; d\tilde{S}_T\right] p(\Theta)$$

Details on Step 1

Drawing from $p\left(\tilde{S}_T|\Theta^{(g-1)}, \tilde{X}_T\right)$: Carter and Kohn (1994):

$$p\left(\tilde{S}_T|\Theta, \tilde{X}_T\right) = p\left(S_T|\Theta, \tilde{X}_T\right) \prod_{t=1}^{T-1} p\left(S_t|S_{t+1}, \Theta, \tilde{X}_T\right)$$

$$\begin{aligned} S_T|\Theta, \tilde{X}_T &\sim N\left(S_{T|T}, P_{T|T}\right) \\ S_t|S_{t+1}, \Theta, \tilde{X}_T &\sim N\left(S_{t|t,S_{t+1}}, P_{t|t,S_{t+1}}\right) \end{aligned}$$

where

$$\begin{aligned} S_{T|T} &= E\left(S_T|\Theta, \tilde{X}_T\right) & P_{T|T} &= Cov(S_T|\Theta, \tilde{X}_T) \\ S_{t|t,S_{t+1}} &= E\left(S_t|S_{t+1}, \Theta, \tilde{X}_T\right) & P_{t|t,S_{t+1}} &= Cov\left(S_t|S_{t+1}, \Theta, \tilde{X}_T\right) \end{aligned}$$

Details on Step 2: Drawing from $\pi\left(\Upsilon | \Theta_M^{(g-1)}, \tilde{S}_T^{(g)}, \tilde{X}_T\right)$

- Assuming $R_{kl} = 0$, $k \neq l$, and a proper (conjugate) but diffuse Inverse-Gamma (3, 0.001) prior for R_{kk}, the posterior is of the form:

$$R_{kk} | \tilde{X}_T, \tilde{S}_T \sim iG\left(\bar{R}_{kk}, T + 0.001\right)$$

where $\bar{R}_{kk} = 3 + \hat{e}_k' \hat{e}_k + \hat{\Lambda}_k' \left(M_0^{-1} + \tilde{S}_T^{(k)\prime} \tilde{S}_T^{(k)}\right)^{-1} \hat{\Lambda}_k$.

- We draw values for Λ_k from the posterior $N\left(\bar{\Lambda}_k, R_{kk} \bar{M}_k^{-1}\right)$, where $\bar{\Lambda}_k = \bar{M}_k^{-1} \left(\tilde{S}_T^{(k)\prime} \tilde{S}_T^{(k)}\right) \hat{\Lambda}_k^{(k)}$and $\bar{M}_k = M_0 + \tilde{S}_T^{(k)\prime} \tilde{S}_T^{(k)}$.

Details on Step 3: Drawing from $\pi\left(\Theta_M|\Upsilon^{(g)},\tilde{S}_T^{(g)},\tilde{X}_T\right)$

- Elements of Θ_M drawn from a proposal scalar Student t-distribution, with mean centered around $\Theta_M^{(g-1)}$, and variance calibrated to yield appropriate acceptance rates

- A draw Θ_M^* is accepted with probability $\min(1, r)$ where

$$r = \frac{\pi\left(\Theta_M^*|\Upsilon^{(g)},\tilde{S}_T^{(g)},\tilde{X}_T\right)}{\pi\left(\Theta_M^{(g)}|\Upsilon^{(g)},\tilde{S}_T^{(g)},\tilde{X}_T\right)}$$