

# New-Keynesian Macroeconomics and the Term Structure

Geert Bekaert, Seonghoon Cho and  
Antonio Moreno

April 2005

# Macro 102: Standard New Keynesian Dynamic System

1) AS equation (Phillips curve): Inflation dynamics

- Inflation depends on real variables; unemployment, output
- Concept of natural rate of unemployment or output

2) IS equation: Output dynamics

- Output is linked to past and future output and the real interest rate.

3) Taylor rule: Monetary authority reacts to the output gap and deviations of expected inflation from targeted inflation

Monetary Transmission:

- Effect of real interest rate on output in IS equation ( $\phi$  parameter)
- Effect of output gap on inflation in AS equation ( $\kappa$  parameter)
- Expectational Terms

# Dual Motivation

⌘ Standard New-Keynesian (NK) Macro Model:  
has some clear deficiencies:

☑ Truly Minimalist (e.g. Clarida, Galí and Gertler (1999)):

☒ Only 3 equations: AS, IS, MP rule

☒ Only 3 variables: inflation, output, short-term interest rate  $\Rightarrow$  Very Limited Information Sets

☑ Does not generate enough persistence to match the data

## ⌘ Models of the Term Structure (T-S) of Interest Rates

- ☒ In principle, no link from Factors to Economics (e.g. Dai and Singleton (2000))
- ☒ Lack of Economic Interpretation of the Term Structure
- ☒ Typically do not match the yield data

# Goals in this paper

- ⌘ Enlarge the NK Macro Model with relevant unobservable variables which are filtered out using Term Structure Information
  - ☑ Provide a Better Fit of the data
  - ☑ Get more reliable Estimates of structural parameters
  - ☑ Enrich Agents' Information Sets: Add realism
- ⌘ Nest the NK Macro Model into a standard Affine Term Structure Model  $\Rightarrow$  Explain the Dynamics of the Term Structure in terms of macro variables

# Outline Talk

I. Related Literature/Contributions

II. Model

- Motivation New-Keynesian Model
- Term Structure Model

III. Data and Estimation; Model Selection

IV. Empirical Results

- Macro
- Term Structure

V. Extensions

# Related Literature/Motivation

⌘ NK Macro Model Estimation: Rotemberg, Woodford (1999), Ireland (2000), Cho, Moreno (2004)

⌘ Phillips Curves and IS Curves: Galí, Gertler (1999), Fuhrer, Rudebusch (2004)

⇒ - weak transmission mechanism  
- hard to generate persistence

# Related Literature/Motivation

⌘ Empirical links between spreads and macro variables:

- Estrella, Mishkin (1998); Harvey (1988); Ang, Piazzesi and Wei (2004)

Term structure variables predict real growth

- Diebold, Rudebusch, Aruoba (2003):

Macro variables predict term structure variables

⌘ Persistence Puzzle: Gurkaynak, Sack, Swanson (2003), Ellingsen, Soderstrom (2001)



# Related Literature/Motivation

## ⌘ Macro finance:

- Ang, Piazzesi (2003): Gaussian price of risk model

$$r_t = a + b'X_t$$

$$X_t = \Phi X_{t-1} + \Sigma \varepsilon_t$$

$$X_t = [3 \text{ latent variables}, 2 \text{ macro variables}]$$

$\Phi$  and  $\Sigma$  block diagonal!

Macro factors particularly important for the spread factor

- Competing papers: Rudebusch, Wu (2003), Hördal, Tristani, Vestin (2003)

## Key Features of our NK Model

⌘ DSGE model with

☑ Monopolistic competition with  
Inflation Indexation ( AS )

☑ External Habit Persistence ( IS )

☑ Modified Taylor Rule ( MP )

⌘ Plus Two unobservables

# Unobservables

⌘ Time-Varying Inflation Target:  $\pi_t^*$

☑ Central Bank targets  $\pi_t^*$

⌘ Natural Rate of Output:  $y_t^n$

☑ Output Gap is  $y_t - y_t^n$

☑ Central Bank targets  $y_t^n$

# IS Curve

⌘ Utility function:

$$U(C_t, N_t) = \frac{F_t C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi}, \quad F_t = C_{t-1}^\eta G_t$$

- Exogenous Habit
- Separable Labour Supply

$$\Rightarrow y_t = \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \varepsilon_{IS,t}$$

$$\mu = \frac{\sigma}{\sigma + \eta}; \phi = \frac{1}{\sigma + \eta}$$

# AS Curve

- ⌘ Monopolistically competitive firms produce goods but prices are partially sticky:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + w \hat{s}_t$$

where  $\delta = f$  (the degree of indexation to previous inflation)

$w = f$  (proportion "optimizing" firms)

$\hat{s}_t = \log$  real marginal cost

- ⌘ Natural rate of output ( $Y_t^n$ ): output that would prevail at perfectly flexible prices and shocks at steady state. With simple production function ( $Y_t = \zeta_t \cdot N_t$ ):

$$y_t^n = \lambda y_{t-1}^n + \varepsilon_{y^n, t}$$

- ⌘ Linking marginal cost and output

Model:  $\hat{s}_t = (\chi + \sigma)(y_t - y_t^n - \lambda(y_{t-1} - y_{t-1}^n)) - g_t - (1 + \chi) \ln \zeta_t$

Standard:  $\hat{s}_t = v(y_t - y_t^n)$

# Inflation Target

⌘ Anchored in the expectations of long run inflation, adjusted for lagged inflation target.

$$\pi_t^{LR} = (1 - d) \sum_{j=0}^{\infty} d^j E_t \pi_{t+j}$$

$$\Rightarrow \pi_t^{LR} = d E_t \pi_{t+1}^{LR} + (1 - d) \pi_t$$

$$\pi_t^* = \omega \pi_{t-1}^* + (1 - \omega) \pi_t^{LR} + \varepsilon_{\pi^*, t}$$

$$\Rightarrow \pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \varepsilon_{\pi^*, t}$$

⌘ Also more standard specification:

$$\pi_t^* = \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_{t-1} + \varepsilon_{\pi^*, t}$$

## Enlarged NK Model

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta) \pi_{t-1} + \kappa (y_t - y_t^n) + \varepsilon_{AS,t}$$

$$y_t = \mu E_t y_{t+1} + (1 - \mu) y_{t-1} - \phi (i_t - E_t \pi_{t+1}) + \varepsilon_{IS,t}$$

$$y_t^n = \lambda y_{t-1}^n + \varepsilon_{y^n,t}$$

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \varepsilon_{\pi^*,t}$$

$$i_t = \rho i_{t-1} + (1 - \rho) i_t^* + \varepsilon_{MP,t}$$

$$i_t^* = \bar{i}_t + \left[ \beta_\pi (E_t \pi_{t+1} - \pi_t^*) + \gamma (y_t - y_t^n) \right]$$

# Desired Nominal Interest Rate

$$i_t^* = \bar{i}_t + \left[ \beta(E_t \pi_{t+1} - \pi_t^*) + \gamma(y_t - y_t^n) \right]$$

$$(1) \bar{i}_t = \bar{r} + E_t \pi_{t+1}$$

$$(2) \bar{i}_t = \bar{r}_t + E_t \pi_{t+1}$$

$$(3) \bar{i}_t = \bar{i}$$

$$\bar{r}_t = \frac{1}{\phi} [\mu E_t y_t^n + (1 - \mu) y_{t-1}^n - y_t^n]$$



# Rational Expectations Equilibrium

## ⌘ Structural Model

$$Bx_t = AE_t x_{t+1} + Jx_{t-1} + C\varepsilon_t$$

$$x_t = \left[ \pi_t, y_t, i_t, y_t^n, \pi_t^* \right]$$

## ⌘ Reduced Form Model

$$x_t = \Omega x_{t-1} + \Gamma \varepsilon_t$$

subject to non-linear restrictions

⇒ Serve as State Variable Dynamics

## Term Structure (T-S) Model

⌘ IS Curve Implied by:  $E_t [M_{t+1} (1 + i_t)] = 1$

⌘ Implied Pricing Kernel:

$$M_{t+1} = \psi \frac{Y_{t+1}^{-\sigma} Y_t^\eta G_{t+1}}{Y_t^{-\sigma} Y_{t-1}^\eta G_t} \Pi_{t+1}^{-1}$$

⌘ Bond pricing

$$E_t [M_{t+1} P_{n-1,t+1}] = P_{n,t}$$

$P_{n,t}$  n-period bond price

## Term Structure (T-S) Model

⌘ Implied Pricing Kernel (in logs):

$$m_{t+1} = \ln(\psi) - \sigma y_{t+1} + (\sigma + \eta) y_t - \eta y_{t-1} - \pi_{t+1} + g_{t+1} - g_t$$

⌘ Bond pricing falls into the Affine class:

⊠  $p_{n,t} = a_n + b'_n x_t$

⊠  $p_{n,t}$  n-period bond price

⊠  $a_n$  constant risk premium (EH embedded)

⊠  $b_n$  factor loading

# Term Structure (T-S) Model

⌘ Bond yield: 
$$i_{n,t} = -\frac{a_n}{n} - \frac{b_n}{n} x_t$$

⌘ Yield Spread: 
$$sp_{n,t} = -\frac{a_n}{n} - \left(\frac{b_n}{n} + e_3\right)' x_t$$

## Strategy

⌘ Filter unobservable inflation target and the output gap through the term structure.

⌘ Macro Model Solution expressed as:

$$\boxed{\wedge} \quad x_t = \Omega x_{t-1} + \Gamma \varepsilon_t \quad x_t = \left[ \pi_t, y_t, i_t, y_t^n, \pi_t^* \right]$$

⌘ T-S Model Implied by NK System:

$$\boxed{\wedge} \quad z_t = B_z x_t \quad z_t = \left[ \pi_t, y_t, i_t, sp_{n_1,t}, sp_{n_2,t} \right]$$

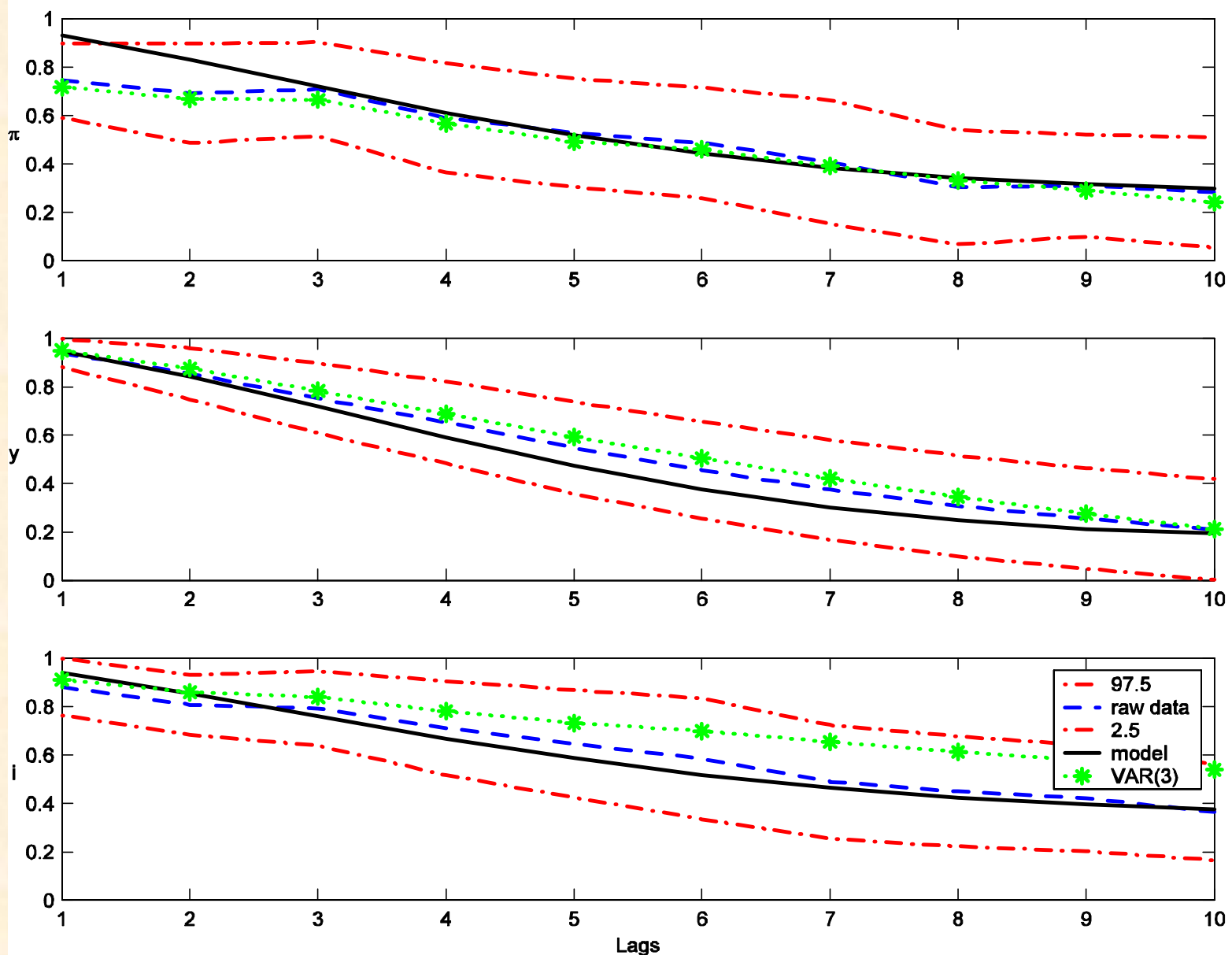
⌘ Full System of Observables can be estimated:

$$\boxed{\wedge} \quad z_t = \Omega_z z_{t-1} + \Gamma_z \varepsilon_t = B_z \Omega B_z^{-1} z_{t-1} + B_z \Gamma \varepsilon_t$$

# Data and Estimation

- ⌘ Model estimated with U.S. data (1961-2003): CPI inflation, Detrended Output, 3 month T-Bill, 3 and 5 year spreads
- ⌘ GMM Estimation Approach with Bootstrapped Standard Errors
  - First stage with “theoretical” weighting matrix
  - Iterate till convergence
  - 7 of 12 specifications converged

# Autocorrelogram: Criterion to Choose among Models



## Estimated NK Model

$$\pi_t = 0.61^* E_t \pi_{t+1} + 0.39^* \pi_{t-1} + 0.06^* (y_t - y_t^n) + \varepsilon_{AS,t}$$

$$y_t = 0.42^* E_t y_{t+1} + 0.58^* y_{t-1} - 0.13^* (i_t - E_t \pi_{t+1}) + \varepsilon_{IS,t}$$

$$i_t = 0.72^* i_{t-1} + 0.28^* i_t^* + \varepsilon_{MP,t}$$

$$i_t^* = E_t \pi_{t+1} + \left[ 0.53^* (E_t \pi_{t+1} - \pi_t^*) + 0.001 (y_t - y_t^n) \right]$$

$$y_t^n = 0.96^* y_{t-1}^n + \varepsilon_{y^n,t}$$

$$\pi_t^* = 0.492^* E_t \pi_{t+1}^* + 0.498^* \pi_{t-1}^* + 0.01 \pi_t + \varepsilon_{\pi^*,t}$$



Parameter	CR,EI,N	Std Error		Cross-Model			
	Estimate	GMM	Bootstrap	Average	Std Error	Min	Max
$\delta$	0.611	( 0.010)	( 0.031)	0.572	( 0.038)	0.525	0.626
$\kappa$	0.064	( 0.007)	( 0.022)	0.075	( 0.044)	0.041	0.174
$\sigma$	3.156	( 0.466)	( 1.632)	3.205	( 0.322)	2.727	4.952
$\eta$	4.294	( 0.470)	( 1.383)	4.780	( 1.024)	3.469	6.796
$\rho$	0.723	( 0.028)	( 0.083)	0.799	( 0.068)	0.705	0.915
$\beta_{\pi}$	1.525	( 0.148)	( 0.251)	2.381	( 0.954)	1.525	2.999
$\gamma$	0.001	( 0.047)	( 0.020)	0.013	( 0.020)	0.001	0.057
$\lambda$	0.958	( 0.006)	( 0.026)	0.945	( 0.020)	0.910	0.975
$\omega$	0.877	( 0.013)	( 0.031)	0.924	( 0.040)	0.810	0.979
d	0.866	( 0.014)	( 0.041)	0.959	( 0.062)	0.757	0.990
$\sigma_{AS}$	1.249	( 0.053)	( 0.123)	1.326	( 0.162)	1.191	1.690
$\sigma_{IS}$	0.671	( 0.033)	( 0.055)	0.682	( 0.027)	0.658	0.739
$\sigma_{MP}$	2.177	( 0.119)	( 0.287)	2.538	( 1.105)	1.222	4.18
$\sigma_{y^n}$	1.380	( 0.115)	( 0.817)	1.691	( 0.878)	0.601	2.947
$\sigma_{\pi^*}$	0.730	( 0.059)	( 0.723)	0.854	( 0.698)	0.370	2.388

Implied Parameter	CR,EI,N Estimate	Std Error			Cross-Model		
		GMM	Bootstrap	Average	Std Error	Min	Max
$\mu$	0.424	( 0.013)	( 0.026)	0.420	( 0.013)	0.401	0.445
$\phi$	0.134	( 0.017)	( 0.029)	0.126	( 0.023)	0.085	0.160
$\varphi_1$	0.500	( 0.003)	( 0.007)	0.395	( 0.229)	0.492	0.533
$\varphi_2$	0.492	( 0.003)	( 0.005)	0.377	( 0.219)	0.466	0.497
$\varphi_3$	0.008	( 0.002)	( 0.003)	0.003	( 0.004)	0.000	0.009
$\bar{\varphi}_2$				0.934	( 0.006)	0.929	0.941
$\bar{\varphi}_3$				0.048	( 0.012)	0.034	0.057
$\hat{\chi}$				2.022			
$\hat{\omega}$				0.025			
$\hat{\tau}$				0.703			
$\hat{\theta}$				0.818			
$\hat{\zeta}$				0.293			
$\hat{\vartheta}$				1.685			

# Macro Findings

- ⌘ Statistically significant Monetary Transmission Mechanism: Phillips curve parameter and elasticity of output to real rate significant: T-S Information is doing the job!
- ⌘ Both Forward and Backward-Looking Components significant. Unobservables quite persistent
- ⌘ What do the unobservables look like?

Figure 2: Output Gap and Natural Rate of Output

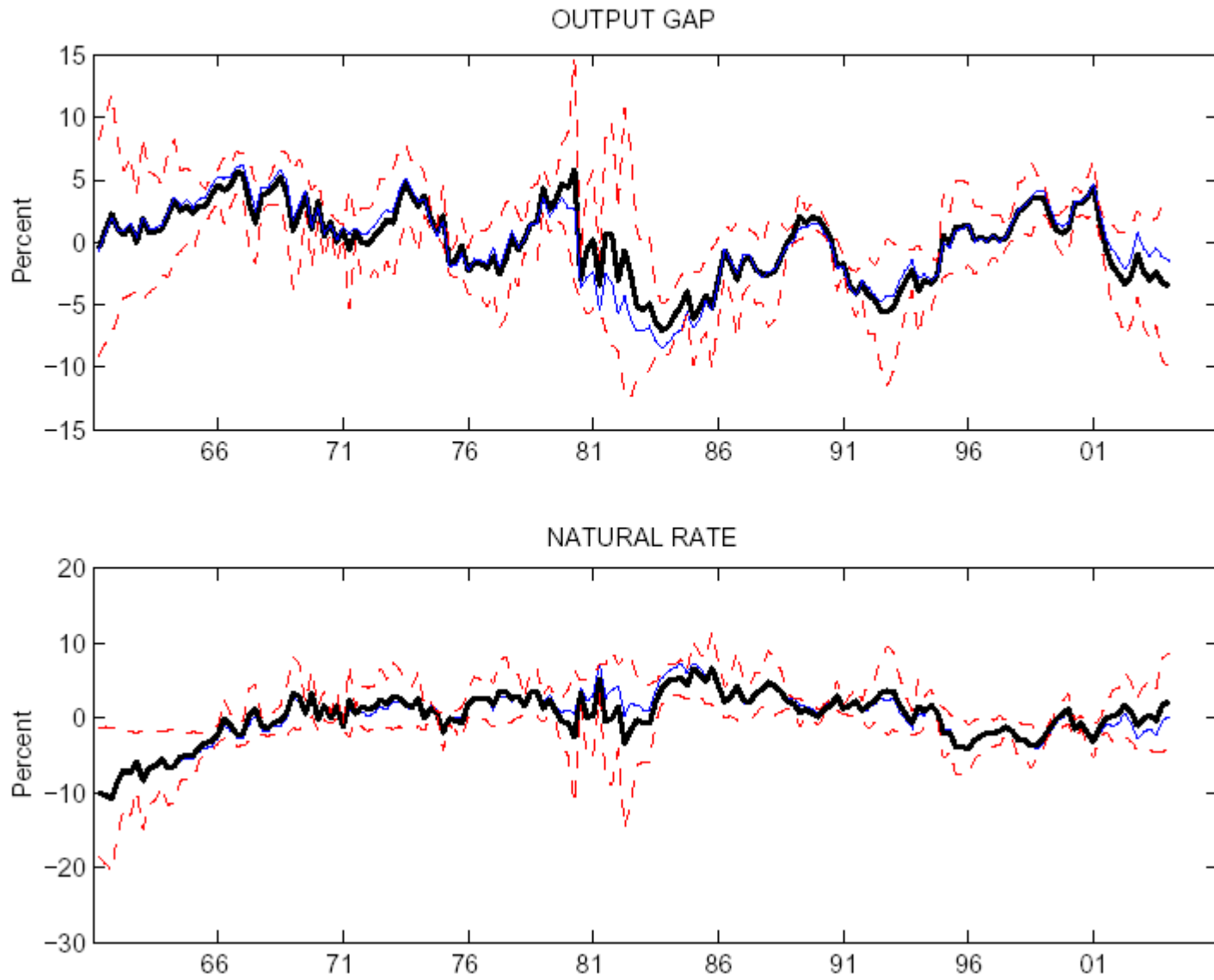


Figure 3: Inflation Target and Inflation

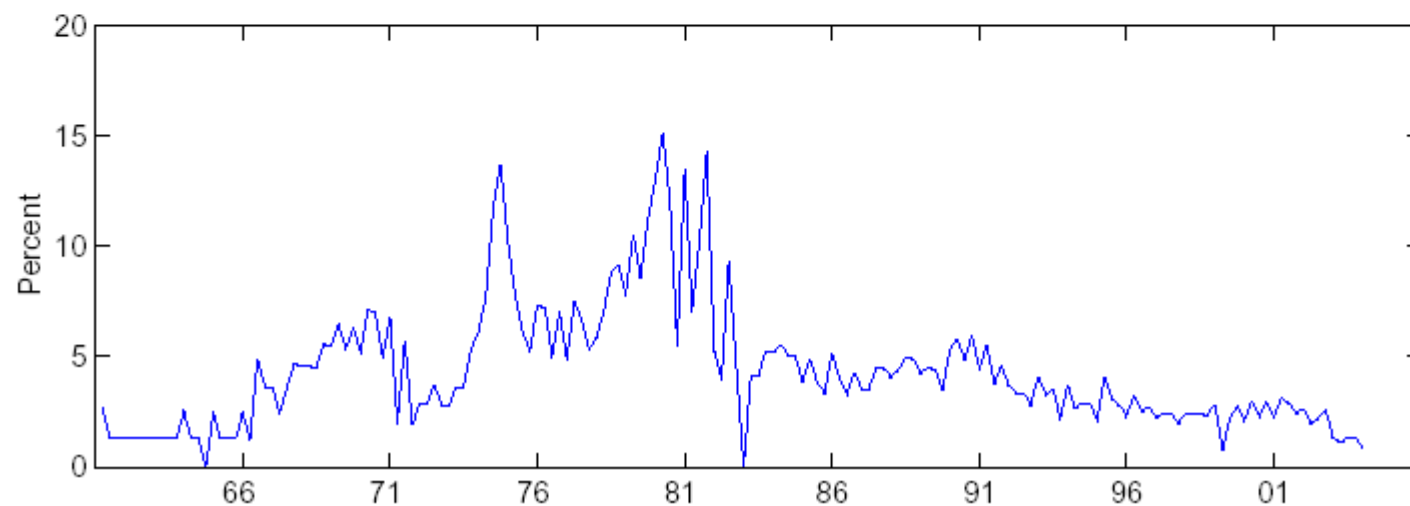
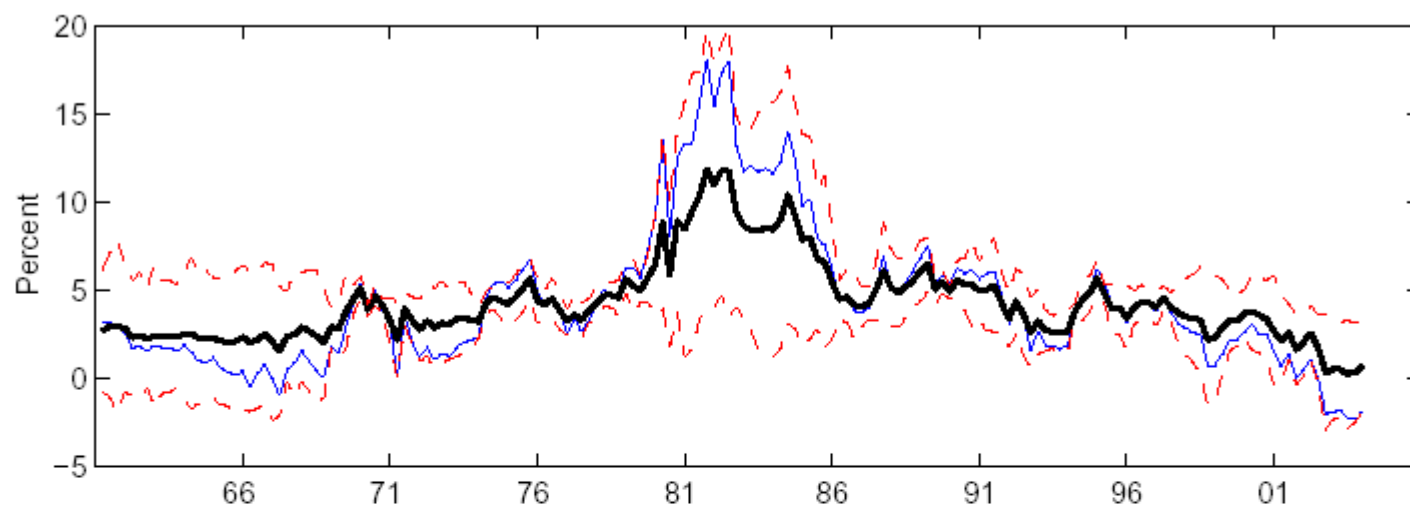


Figure 4: Impulse Response Functions of Macro Variables

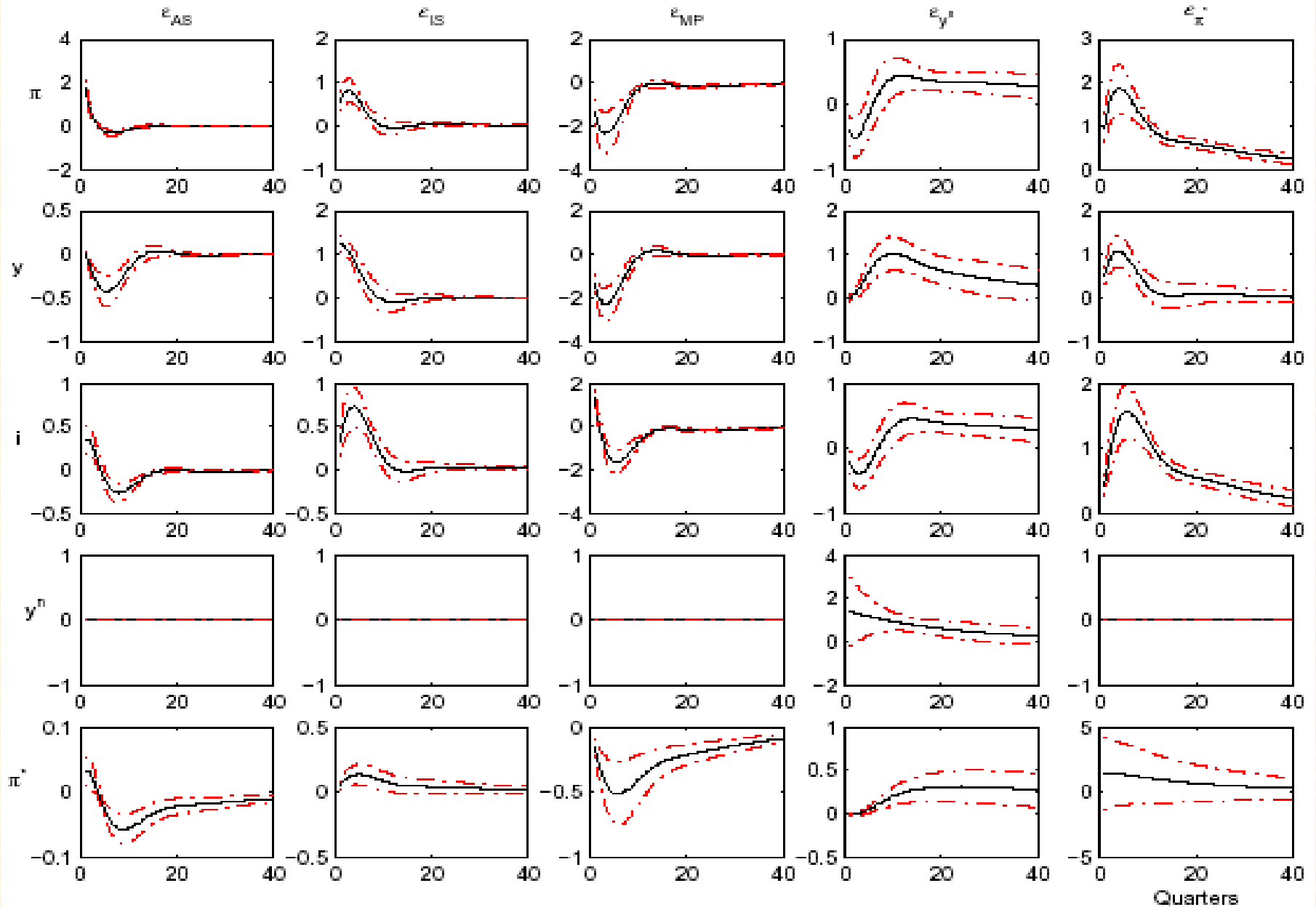
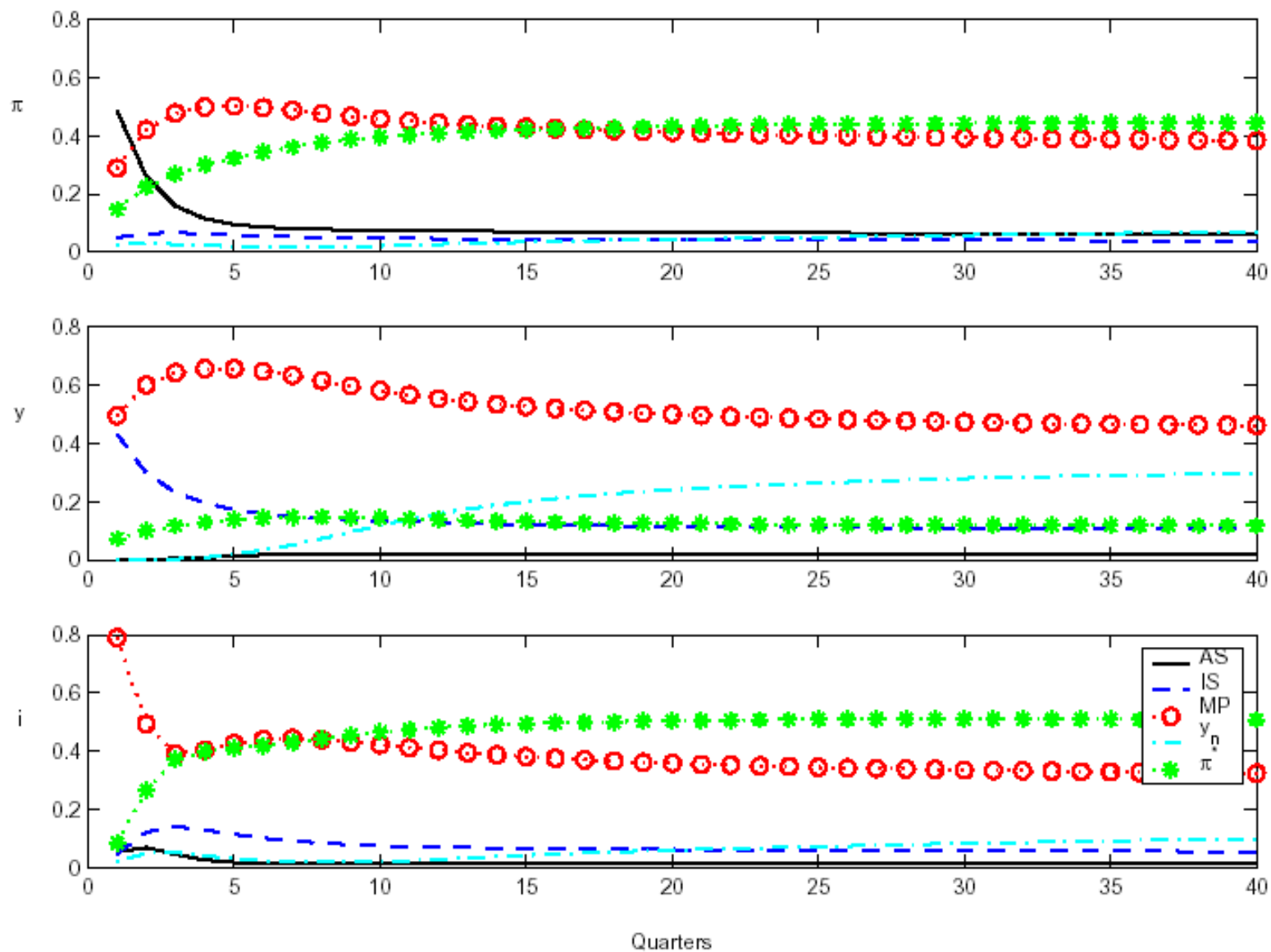


Figure 5: Variance Decompositions for the Macro Variables



## Term Structure Findings

⌘ 5 Macro factors match the Yield Fairly well

⌘ Interpretable Yield Movement

☒ Factors are not orthogonal

☒ Can be interpreted in terms of Structural Shocks

$$i_{n,t} = - \frac{b'_n}{n} x_t = - \frac{b'_n}{n} \Omega x_{t-1} - \frac{b'_n}{n} \Gamma \varepsilon_t$$

⌘ Monetary Policy and Inflation Target shocks are the major driving forces of the T-S.

⌘ Predictability and Excess sensitivity



Figure 6: Fit for Yields

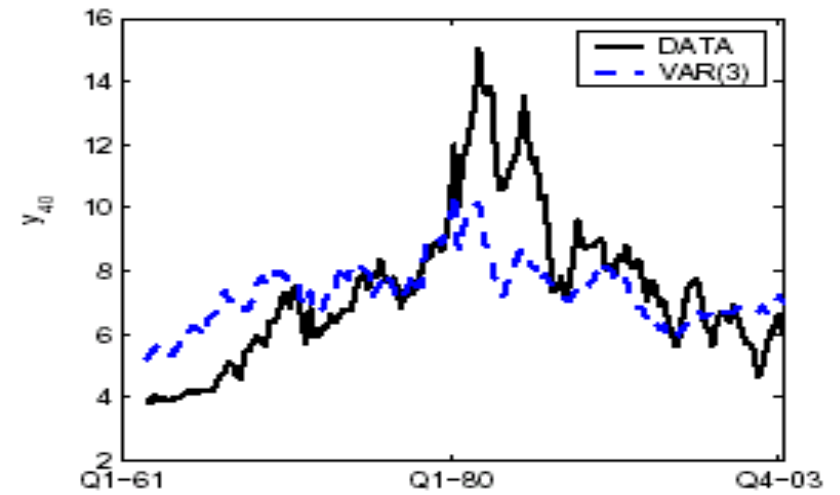
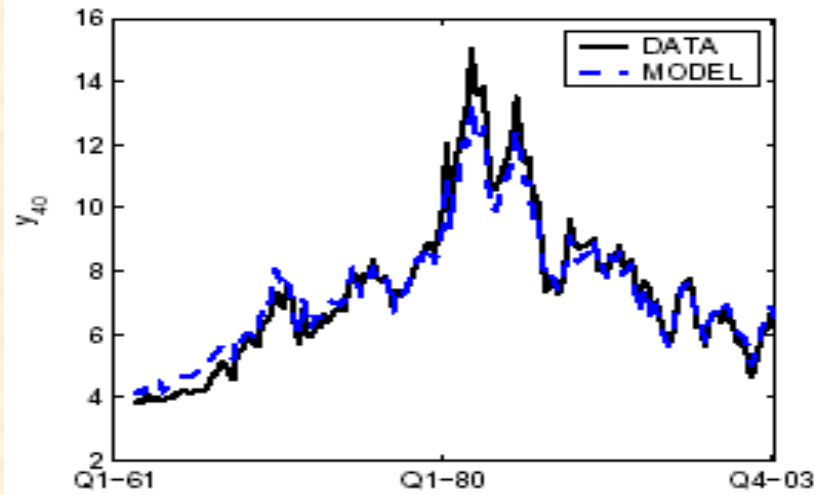
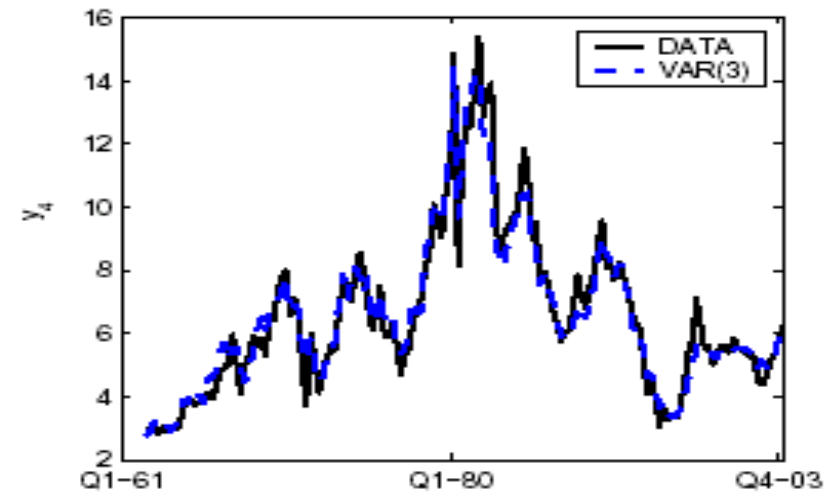
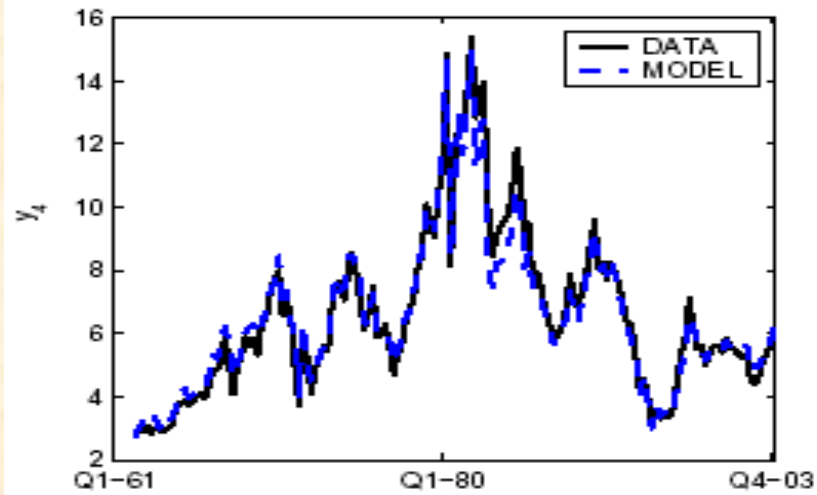


Figure 8: Variance Decompositions for the Term Structure Factors

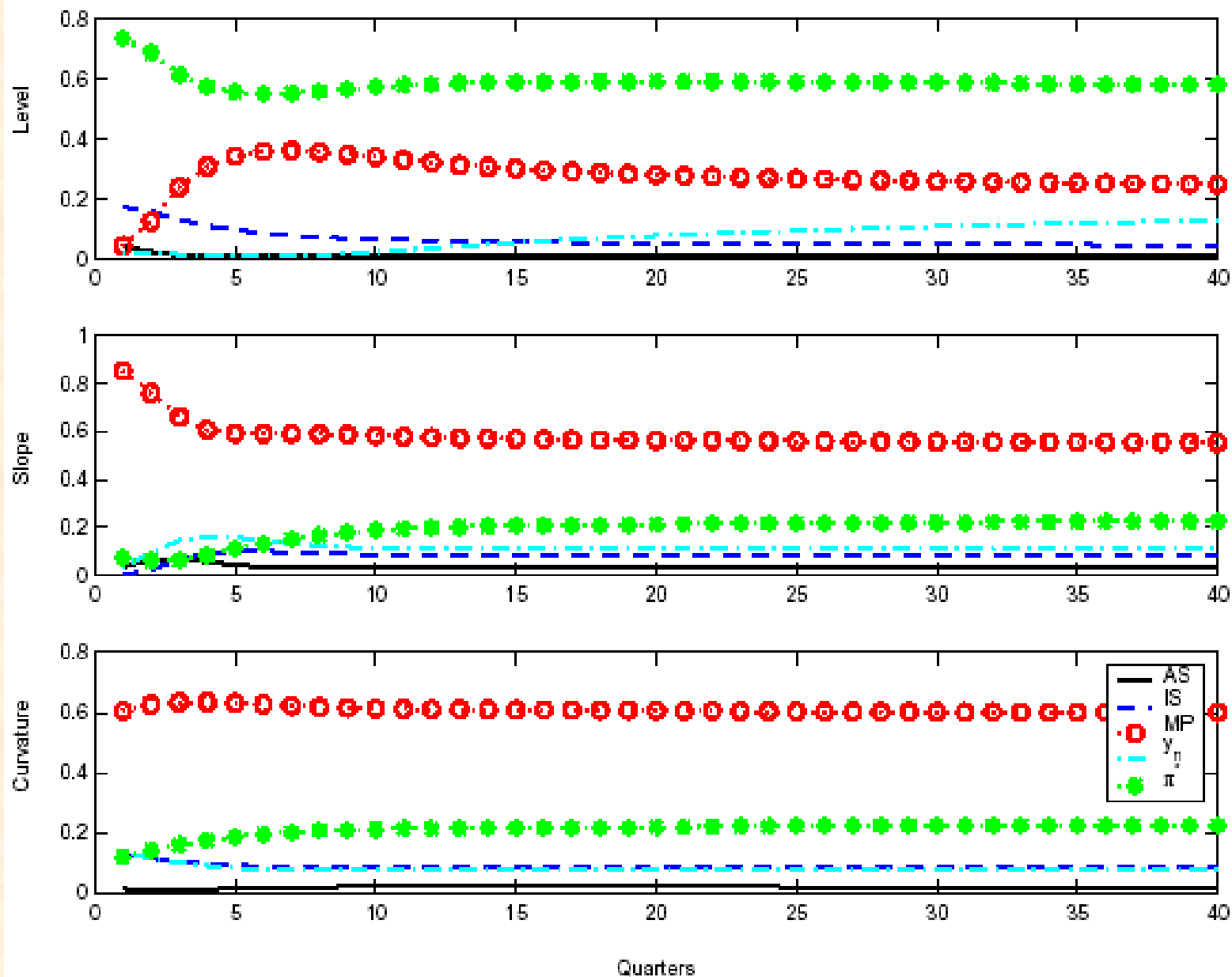
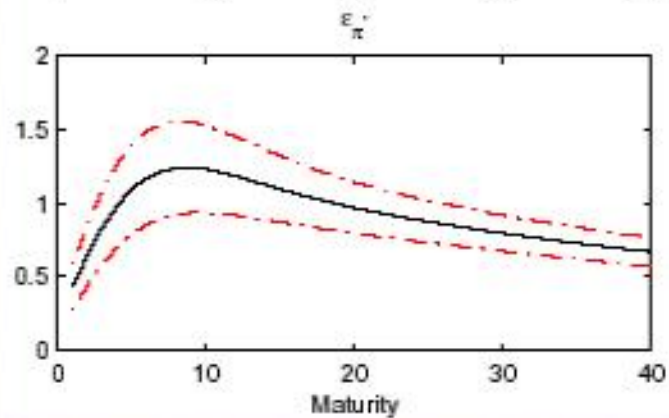
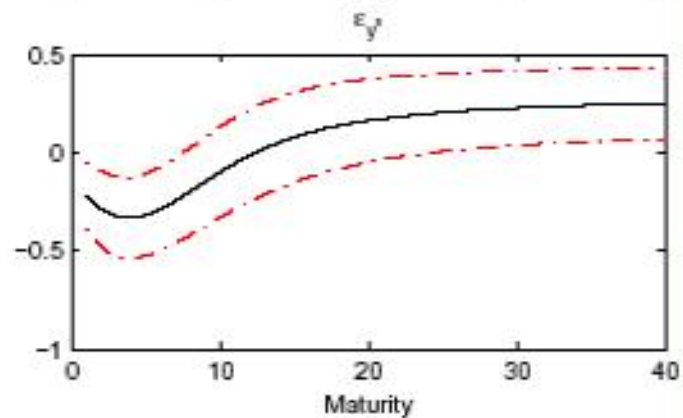
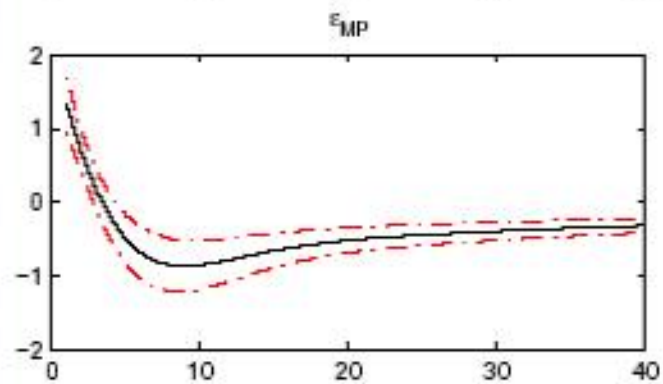
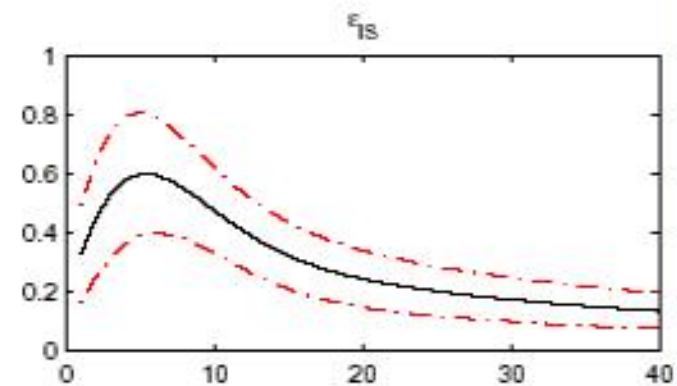
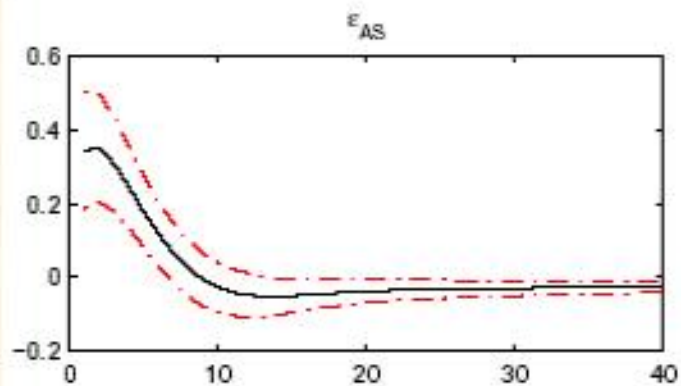


Figure 9: Contemporaneous Responses of Yields to Macro Shocks



# Main Results

- ⌘ Phillips Curve and Interest Rate Demand Channel both Large and Statistically Significant
  - ☑ Strong and Significant Monetary Policy Transmission Mechanism to Output and Inflation
- ⌘ Monetary Factors Drive the Yield Curve
  - ☑ Level is driven by inflation target shocks
  - ☑ Slope and Curvature driven by Monetary Policy Shock

# Extensions

⌘ Include the permanent component of output in the Macro System:

☑ Assume that Output and Natural Rate are Cointegrated

⌘ Generate Time-Varying Risk Premia

☑ Square Root Processes

☑ Devise Theoretical Model with endogenous Risk Premia

⌘ Think about regime shifts/structural breaks