On the fit and forecasting performance of New-Keynesian models

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Motivation

- DSGE models are not only attractive from a theoretical perspective, but are also emerging as useful tools for forecasting and quantitative analysis;
- Smets and Wouters (SW, 2003, 2005) find that estimated DSGE models a la Christiano et al (JPE, 2005) can compete with Bayesian VARs in terms of the marginal data density.
- This paper revisits some of the evidence on the "good" fit and forecasting performance of New Keynesian DSGE models, using the tools developed in Del Negro and Schorfheide (DS, 2003) for two main reasons.

Motivation (1)

- Sims (2003) criticizes the SW model comparison exercise (based on Bayes factors and posterior odds) on the basis of the argument that the models considered are too sparse.
- In such cases, posterior odds may lead to extreme outcomes and may also be highly dependent on the prior distribution. Sims (2003) proposes to "fill the space of models" to make the model comparison more robust.
- The DSGE-VAR methodology proposed in DS (2003) is one way of doing so.
- In DS (2003), tightly specified DSGE models are used to form a prior for estimating highly parameterised VAR models. The hyper parameter (λ) captures the weight put on the prior. DS (2003) finds that putting a significant weight on the DSGE prior improves the forecasting performance of the VAR

Motivation (1)

- In this paper, the hyper parameter (λ) is interpreted as a factor that scales the inverse of a prior covariance matrix of parameters that capture deviations from the DSGE model restrictions.
  - $\lambda=\infty$ implies that the prior on the misspecification parameters concentrates the mass around zero; DSGE model restrictions on the VAR parameters are dogmatically imposed
  - $\lambda=0$ implies that the prior is flat; no DSGE restrictions imposed
- By considering an entire range of hyperparameter values between the extremes (of the DSGE-VAR and an unrestricted VAR), one allows for varying degrees of deviations from the DSGE model restrictions and the assessment of misspecification becomes more refined and more robust.

Motivation (2)

- An analysis of the estimated stochastic processes in the work of Smets and Wouters indicates the presence of some misspecification:
  - These processes are often estimated to be very persistent and sometime exhibit a clear trend (in spite of being assumed stationary);
  - This is likely to also affect the estimates of the structural parameters (e.g. degree of wage and price stickiness).
- The DSGE-VAR methodology of DS (2003) allows us to relax some of the DSGE restrictions and investigate the effects on the structural model.

Benefits of the methodology

- The hyperparameter that has the highest posterior probability (λ^) can be interpreted as providing an assessment of the degree of misspecification.
- The DSGE-VAR(λ^) provides a natural benchmark for comparing the empirical fit of the DSGE model (Schorfheide, 2001):
  - Out-of-sample forecasting performance
  - Parameter estimates
  - Impulse response functions (identification of VAR)
- This benchmark can be used to analyse the sources of misspecification in the DSGE model

Main findings

- The stationary state-space representation of the DSGE model is well approximated by a VAR with 4 lags, provided the error-correction terms are included;
- The posterior distribution of the hyperparameter λ has an inverse U-shape, indicating that the fit of the autoregressive system can be improved by relaxing the DSGE restrictions. But the DSGE restrictions should not be ignored either.
- This finding is confirmed by out-of-sample forecasting exercises.

Main findings

- Regarding the nature of misspecification of the DSGE model, there are three main findings:
  - There is a shift towards less stickiness in prices and wages, less persistence in the shocks and less habit formation, when misspecification is allowed for;
  - Nevertheless, many the impulse responses in the optimal DSGE-VAR show an increased degree of persistence in particular in response to a government spending and labour supply shock;
  - The real effects of monetary policy are estimated to be larger and more persistent.

Outline of presentation

- Brief overview of the DSGE model:
  - variant of CEE (JPE, 2005) and SW (2003)
- Brief overview of the methodology:
  - similar to Del Negro and Schorfheide (2003), but different interpretation.
- Presentation of main findings;
- Conclusions and future work

The New Keynesian DSGE model

- Model is a variant of Altig et al (2002), Christiano et al (JPE, 2005) and SW (2003);
- Households consume, supply labor, enjoy money, set wages, accumulate capital subject to adjustment costs, choose capital utilisation and have access to a full set of state-contingent assets.

DSGE Model

Households' Problem

Households maximize:

$$\boldsymbol{E}_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\log(C_{t+s}(j) - hC_{t+s-1}(j)) - \frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \frac{\chi}{1-\nu_m} \left(\frac{M_{t+s}(j)}{Z_{t+s} P_{t+s}} \right)^{1-\nu_m} \right]$$

subject to:

$$P_{t+s}C_{t+s}(j) + P_{t+s}I_{t+s}(j) + B_{t+s}(j) + M_{t+s}(j) \leq R_{t+s}B_{t+s-1}(j) + M_{t+s-1}(j) + A_{t+s}(j)$$

$$+ \Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + \left(R^k_{t+s} u_{t+s}(j) \bar{K}_{t+s-1}(j) - P_{t+s} a(u_{t+s}(j)) \bar{K}_{t+s-1}(j) \right),$$

and

$$\bar{K}_t(j) = (1-\delta)\bar{K}_{t-1}(j) + \mu_t \left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right) I_t(j),$$

where firms use the "effective" capital: $K_t(j) = u_t(j)\bar{K}_{t-1}(j)$.

The exogenous shocks φ_t, b_t, and μ_t all follow AR(1) processes (in logs).

Households' Problem – Sticky Wages

Labor packers "pack" $L_t(j)$ labor into $L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_w}} di\right]^{1+\lambda_w}$, and demand:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t,$$

where

$$W_t = \left[\int_0^1 W_t(j)^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}.$$

Households that can reset wages (probability $1-\zeta_w$) solve:

$$\max_{\tilde{W}_t(j)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s b_{t+s} \left[-\frac{\varphi_{t+s}}{\nu_l+1} L_{t+s}(j)^{\nu_l+1}\right]$$

s.t. budget constraint for $s = 0, \ldots, \infty$, demand for $L_{t+s}(j)$, and

$$W_{t+s}(j) = \left(\Pi_{l=1}^{s} (\pi_* e^{\gamma})^{1-\iota_w} (\pi_{t+l-1} e^{z_{t+l-1}})^{\iota_w}\right) \tilde{W}_t(j).$$

The New Keynesian DSGE model

- Intermediate goods producers use capital services and labor; set prices in a monopolistic competitive market and face a random walk productivity process;
- Final goods producers aggregate a continuum of intermediate goods.

Producers' Problem

$$\max P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t(i),$$

subject to the technology:

$$Y_t(i) = \max\left\{K_t(i)^\alpha (Z_t L_t(i))^{1-\alpha} - Z_t \mathcal{F}, 0\right\},$$

where $z_t = \log(Z_t/Z_{t-1})$ follows the process:

$$(z_t - \gamma) = \rho_z (z_{t-1} - \gamma) + \epsilon_{z,t}.$$

Producers' Problem – Sticky Prices

Final goods producers "pack" $Y_t(i)$ into a composite good:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} di\right]^{1+\lambda_{f,t}}, \; (\ln \lambda_{f,t} - \ln \lambda_f) = \rho_{\lambda_f}(\ln \lambda_{f,t-1} - \ln \lambda_f) + \sigma_{\lambda,f}\epsilon_{\lambda,t},$$

and demand:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_t,$$

where

$$P_t = \left[\int_0^1 P_t(i)^{\frac{1}{\lambda_{f,t}}} di\right]^{\lambda_{f,t}}.$$

Producers that can reset prices (probability $1 - \zeta_p$) solve:

$$\max_{\tilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p \left(\tilde{P}_t(i) \left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) - MC_{t+s}\right) Y_{t+s}(i)$$

s.t. demand for $Y_{t+s}(i)$ and , $MC_{t+s} = (1-\alpha)^{-(1-\alpha)} Z_{t+s}^{-(1-\alpha)} \alpha^{-\alpha} W_{t+s}^{1-\alpha} R_{t+s}^{k\;\alpha}$.

Government

$$P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + M_t + B_t$$

where $G_t = (1 - 1/g_t) Y_t$ and:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \epsilon_{g,t}.$$

The central bank follows a nominal interest rate rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}\right]^{1-\rho_R} \sigma_R e^{\epsilon_{R,t}}$$

where Y_t^* can be either the stochastic steady state level of output ($Y_t^* = Y_t^s$) or the level output that would have prevailed in absence of nominal rigidities ($Y_t^* = Y_t^f$).

Detrending and model solution

$$c_t = \frac{C_t}{Z_t},\; y_t = \frac{Y_t}{Z_t},\; i_t = \frac{I_t}{Z_t},\; k_t = \frac{K_t}{Z_t},\; \bar{k}_t = \frac{\bar{K}_t}{Z_t},$$

$$r_t^k = \frac{R_t^k}{P_t},\; w_t = \frac{W_t}{P_t Z_t},\; m_t = \frac{M_t}{P_t Z_t},\; \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\; \tilde{w}_t = \frac{\tilde{W}_t}{W_t},$$

$$\xi_t = \Xi_t Z_t^*,\; \xi_t^k = \Xi_t^k Z_t,$$

Log-linearize + Sims' *gensys* $\rightarrow$ transition equations.

Measurement equations

Output growth (log differences, quarter-to-quarter, in %):

$$100 \times (\ln Y_t - \ln Y_{t-1}) = 100 \times (\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t) + 100\gamma$$

Consumption growth (. . .): $100 \times (\ln C_t - \ln C_{t-1}) = 100 \times (\hat{c}_t - \hat{c}_{t-1} + \hat{z}_t) + 100\gamma$.

Investment growth (. . .): $100 \times (\ln I_t - \ln I_{t-1}) = 100 \times (\hat{i}_t - \hat{i}_{t-1} + \hat{z}_t) + 100\gamma$.

Real wage growth (. . .): $100 \times (\ln W_t - \ln P_t - \ln W_{t-1} + \ln P_{t-1}) = 100 \times (\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t) + 1$

Hours worked (log): $\ln L_t = \hat{L}_t + \ln L^* + \ln L^{adj}$

Inflation (quarter-to-quarter, in %): $100 \times (\ln P_t - \ln P_{t-1}) = 100\hat{\pi}_t + 100 \ln \pi^*$.

Nominal interest rate (annualized, in %): $400 \times (\ln R_t) = 4 \times 100\hat{R}_t + 400 * \ln R^*$.

The DSGE-VAR methodology

- VAR/VECM representation of the DSGE model
- Prior distribution of the VAR parameters given the DSGE parameters
- Posterior distribution of VAR parameters and DSGE parameters
- Identification

VAR/VECM representation

- We approximate DSGE model by VAR

$$\Delta y_t = \Phi_0 + \Phi_1 \Delta y_{t-1} + \ldots + \Phi_p \Delta y_{t-p} + u_t, \quad \boldsymbol{E}[u_t u_t'] = \Sigma \tag{1}$$

- or alternatively by a VECM

$$\Delta y_t = \Phi_0 + \Phi_\beta(\beta' y_{t-1}) + \Phi_1 \Delta y_{t-1} + \ldots + \Phi_p \Delta y_{t-p} + u_t. \tag{2}$$

- Y is $T \times n$ matrix with rows $\Delta y_t'$; X is $T \times k$ matrix with rows $x_t' = [1, \Delta y_{t-1}', \ldots, \Delta y_{t-p}']$; U is $T \times n$ matrix with rows u_t', and $\Phi = [\Phi_0, \Phi_1, \ldots, \Phi_p]'$.

- Write VAR as

$$Y = X\Phi + U; \tag{3}$$

VECM looks similar

VAR/VECM representation

- Log-lin DSGE model has in general no exact finite-order VAR representation, but can often be well approximated with few lags, provided y_t is covariance stationary.

- Define autocovariance matrices

$$\Gamma_{XX}(\theta) = \mathbb{E}_\theta^D[x_t x_t'], \quad \Gamma_{XY}(\theta) = \mathbb{E}_\theta^D[x_t y_t']$$

  $\mathbb{E}_\theta^D[\cdot]$ expectation under DSGE model.

- Let

$$\Phi^*(\theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta). \tag{4}$$

  and

$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta). \tag{5}$$

- Note that $\mathbb{E}^{VAR}_{\Psi^*(\theta),\Sigma^*(\theta)}[x_t x_t'] = \mathbb{E}_\theta^D[x_t x_t'] = \Gamma_{XX}(\theta)$.

Prior distribution of VAR parameters

$$\Sigma_u|\theta \sim \mathcal{IW}\left(\lambda T \Sigma_u^*(\theta), \lambda T - k, n\right) \tag{40}$$

$$\Phi|\Sigma_u, \theta \sim \mathcal{N}\left(\Phi^*(\theta), \frac{1}{\lambda T}\left[\Sigma_u^{-1} \otimes \Gamma_{XX}(\theta)\right]^{-1}\right), \tag{41}$$

where $\mathcal{IW}$ denotes the inverted Wishart distribution. The latter induces a distribution for the discrepancy $\Sigma_u^{\Delta} = \Sigma_u - \Sigma_u^*$.

Posterior distribution of VAR parameters

It is straightforward to show, e.g., Zellner (1971), that the posterior distribution of Φ and Σ is also of the Inverted Wishart – Normal form:

$$\Sigma_u | Y, \theta \sim \mathcal{IW}\Big((\lambda + 1) T \hat{\Sigma}_{u,b}(\theta), (1+\lambda)T - k, n \Big) \tag{43}$$

$$\Phi | Y, \Sigma_u, \theta \sim \mathcal{N}\Big(\hat{\Phi}_b(\theta), \Sigma_u \otimes (\lambda T \Gamma_{XX}(\theta) + X'X)^{-1} \Big), \tag{44}$$

where $\hat{\Phi}_b(\theta)$ and $\hat{\Sigma}_{u,b}(\theta)$ are the given by

$$\begin{aligned} \hat{\Phi}_b(\theta) &= (\lambda T \Gamma_{XX}(\theta) + X'X)^{-1} (\lambda T \Gamma_{XY} + X'Y) \qquad (45) \\ &= (\frac{\lambda}{1+\lambda} \Gamma_{XX}(\theta) + \frac{1}{1+\lambda} \frac{X'X}{T})^{-1} (\frac{\lambda}{1+\lambda} \Gamma_{XY} + \frac{1}{1+\lambda} \frac{X'Y}{T}) \\ \hat{\Sigma}_{u,b}(\theta) &= \frac{1}{(\lambda+1)T} \Big[(\lambda T \Gamma_{YY}(\theta) + Y'Y) - (\lambda T \Gamma_{YX}(\theta) + Y'X) \\ &\quad \times (\lambda T \Gamma_{XX}(\theta) + X'X)^{-1} (\lambda T \Gamma_{XY}(\theta) + X'Y) \Big]. \qquad (46) \end{aligned}$$

Implementation

- Place prior over θ.
- MCMC methods to generate draws from the joint posterior of θ, Φ, and Σ.
- Use marginal data density to form a posterior for λ:

$$p_\lambda(Y) = \int p(Y|\theta, \Sigma, \Phi) p_\lambda(\theta, \Sigma, \Phi) d(\theta, \Sigma, \Phi). \tag{10}$$

Identification

- To assess the DSGE model based on impulse response comparisons between DSGE and VAR/VECM we need a mapping from VAR innovations into structural shocks.

- An (exactly) identified VAR is a triplet: (Φ, Σ, Ω), where Ω is orthonormal:

$$\left(\frac{\partial y_t}{\partial \epsilon_t'}\right)_{VAR} = \Sigma_{tr}\Omega.$$

- The DSGE model is identified: there is a matrix $\Omega^*(\theta)$ that maps the variance-covariance matrix of innovations into the portion attributed to each shock:

$$\left(\frac{\partial y_t}{\partial \epsilon_t'}\right)_{DSGE} = \Sigma_{tr}^*(\theta)\Omega^*(\theta).$$

- Identification of DSGE-VAR (for each draw of Φ, Σ, θ): $\quad (\Phi, \Sigma, \Omega^*(\theta))$

Findings

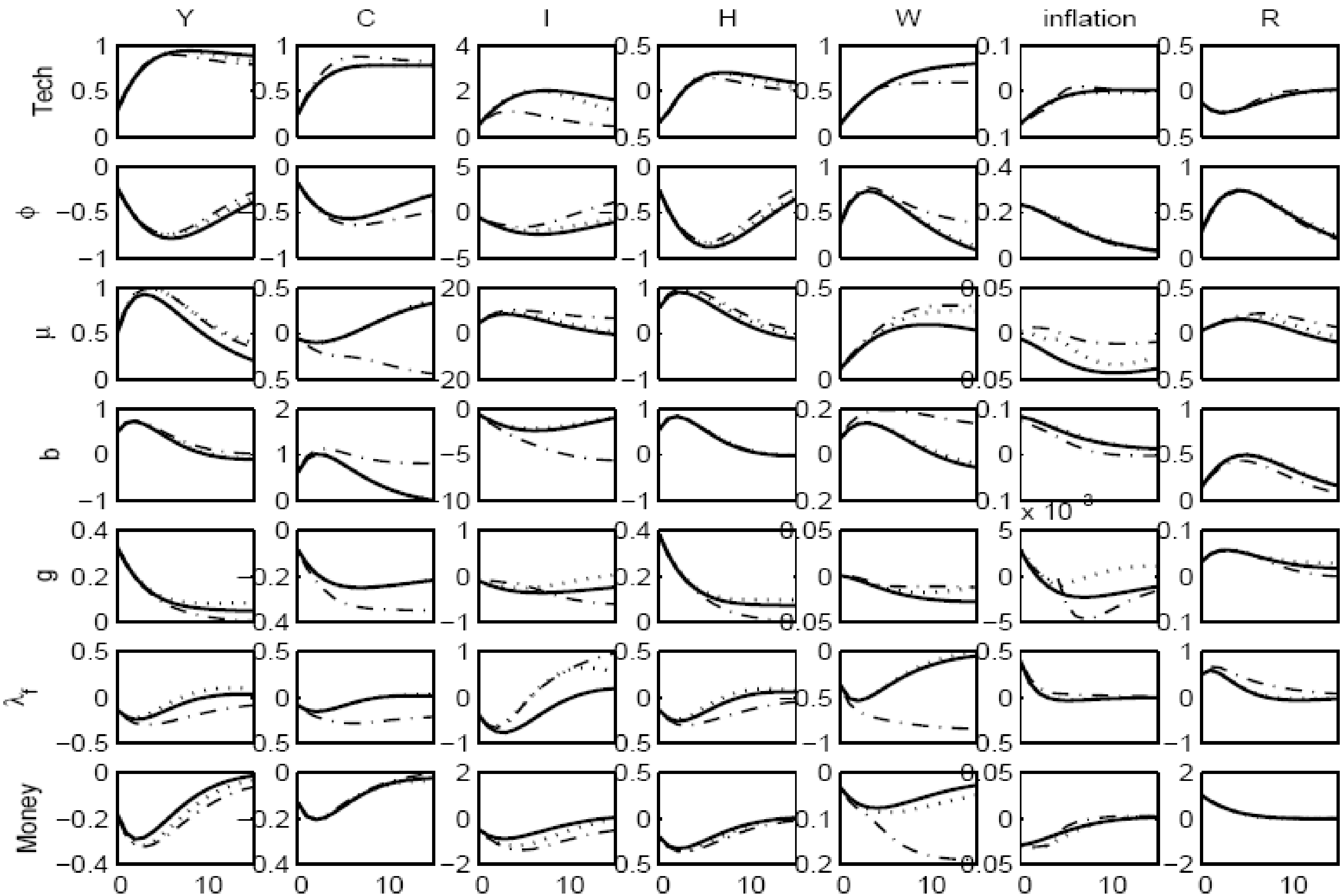
Can DSGE-VAR reproduce the dynamics of the DSGE model when $\lambda \to \infty$? – Dealing with Cointegration

- Y, C, I, and W all grow along a balanced growth path – according to the DSGE model.
- A standard VAR in growth rates (even for $\lambda \to \infty$) does not impose these cointegrating restrictions.
- If we introduce an error correction term into the VAR (we call it VECM):

$$\beta' y_{t-1} = \begin{bmatrix} \ln C_t - \ln Y_t \\ \ln I_t - \ln Y_t \\ \ln(W_t/P_t) - \ln Y_t \end{bmatrix}.$$

  we can reproduce the dynamics of the DSGE model with only a few autoregressive lags.

How Well Do DSGE-VAR/VECM Approximate the DSGE Model?



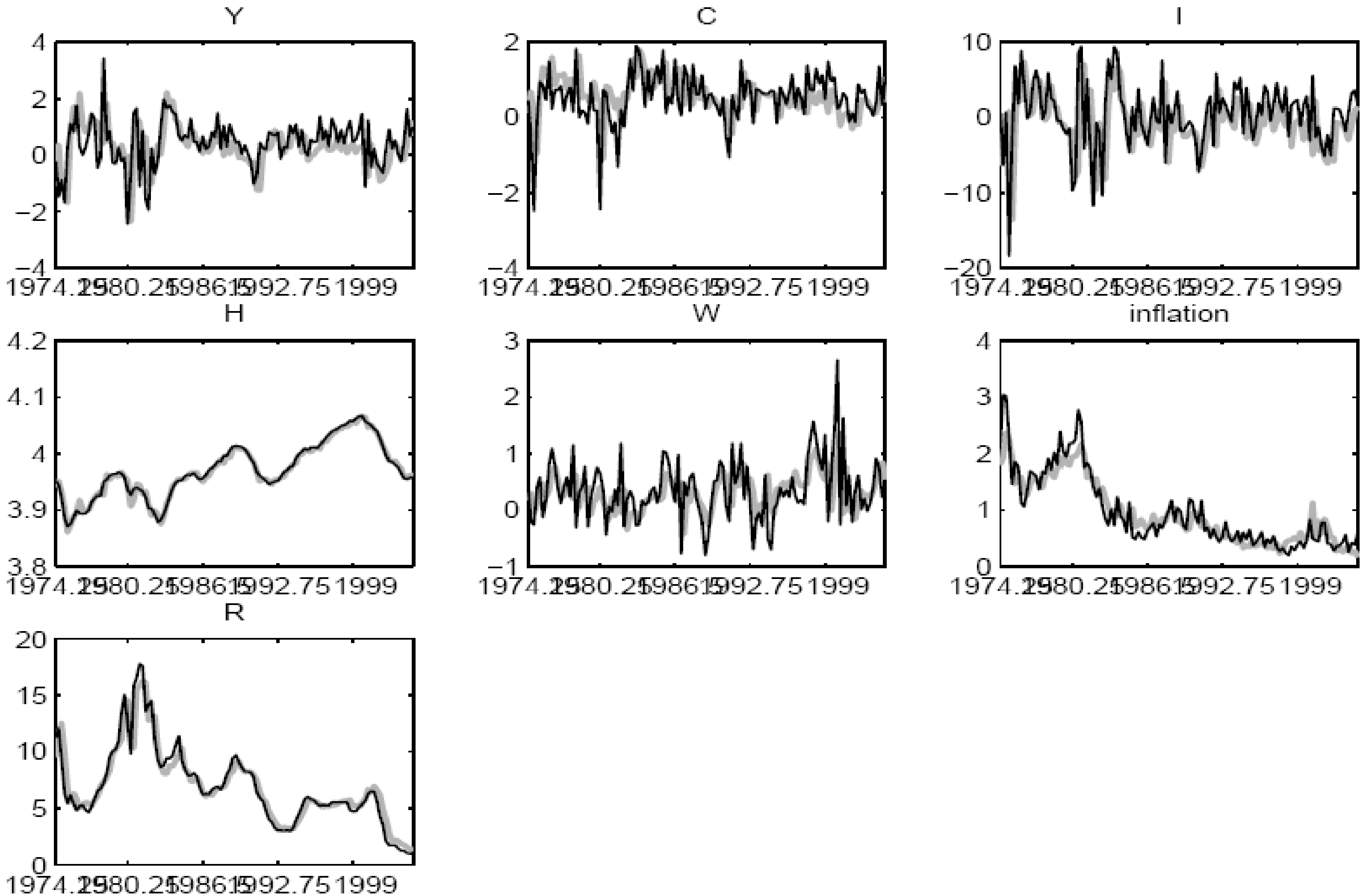
Preliminary: Direct evaluation of the DSGE model

- Choosing the DSGE specification;
- In-sample fit of the DSGE model;
- Parameters of the DSGE model;
- Exogenous processes.

Choosing the DSGE Model Specification

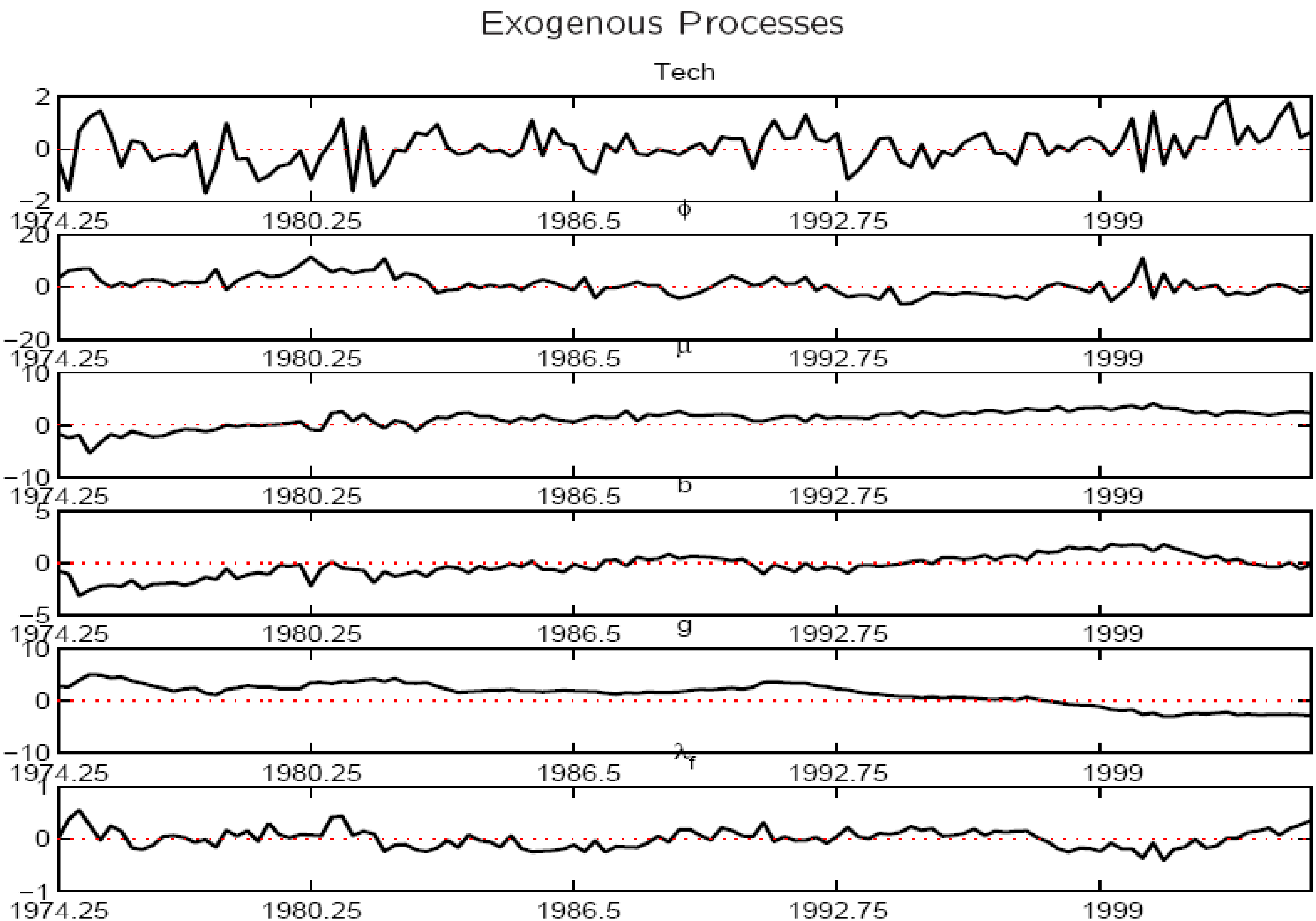
		Log-Differences in Marginal Likelihood wrt Baseline Model	Bayes Factors wrt Baseline Model (in %)
(i)	Dynamic Indexation ($\iota_p = \iota_w = 1$)	-31.49	10^{-12} %
(ii)	Flexible-price Output Target	-2.08	12.49 %
(iii)	$\mathcal{F}$ sets steady-state profits $= 0$	-28.87	10^{-13} %

In-Sample Fit of the DSGE Model



DSGE Model's Parameter Estimates

Parameter	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
α	0.250	0.100	0.172	0.149	0.195
ζ_p	0.750	0.100	0.848	0.814	0.883
s'	4.000	1.500	5.827	3.238	8.115
h	0.800	0.100	0.793	0.725	0.857
$a'\,'$	0.200	0.075	0.167	0.067	0.273
ν_l	2.000	0.750	2.204	1.050	3.271
ζ_w	0.750	0.100	0.936	0.913	0.959
r^*	0.500	0.100	0.389	0.270	0.501
ψ_1	1.700	0.100	1.799	1.640	1.944
ψ_2	0.125	0.100	0.065	0.040	0.090
ρ_r	0.800	0.100	0.815	0.775	0.855
π^*	0.650	0.200	1.026	0.807	1.264
γ	0.500	0.250	0.185	0.085	0.276
χ	0.100	0.100	0.100	0.100	0.100
λ_f	0.300	0.500	0.300	0.300	0.300
g^*	0.150	0.050	0.224	0.199	0.253
ρ_z	0.200	0.050	0.218	0.146	0.294
ρ_ϕ	0.800	0.050	0.705	0.625	0.791
ρ_{λ_f}	0.800	0.050	0.518	0.449	0.589
ρ_μ	0.800	0.050	0.884	0.834	0.937
ρ_b	0.800	0.050	0.811	0.743	0.876
ρ_g	0.800	0.050	0.951	0.928	0.975
σ_z	0.400	2.000	0.702	0.625	0.779
σ_ϕ	1.000	2.000	3.450	1.990	4.886
σ_χ	1.000	2.000	0.000	0.000	0.000
σ_{λ_f}	1.000	2.000	0.192	0.168	0.217
σ_μ	1.000	2.000	0.918	0.742	1.077
σ_b	0.200	2.000	0.538	0.439	0.630
σ_g	0.300	2.000	0.406	0.360	0.454
σ_r	0.200	2.000	0.271	0.242	0.300



Exogenous Processes
Tech
ϕ
μ
b
g
λ_f
2
0
−2
20
0
−20
10
0
−10
5
0
−5
10
0
−10
1
0
−1
1974.25
1980.25
1986.5
1992.75
1999

Evaluation of the DSGE model using the DSGE-VAR toolkit

- In-sample fit: how large a weight should we put on the DSGE model restrictions?
- Out-of-sample forecasting performance (rolling sample 1985QI-2000Q4);
- Comparing parameter estimates;
- Comparing impulse responses;

In-sample fit: How large a weight should we put on the DSGE model prior?

Weight of the DSGE model prior		DSGE-VECM		DSGE-VAR	
(1)	(2)	(3)	(4)	(5)	(6)
λ	$\frac{\text{Artificial}}{\text{Total}}$	Log-Marginal Likelihood	(Difference wrt Best Model)	Log-Marginal Likelihood	(Difference wrt Best Model)
0.5	0.33	-456.41	(-13.03)	-436.07	(-7.29)
0.75	0.43	-443.45	(-0.07)	-428.78	(0)
1	0.50	-443.38	(0)	-434.89	(-6.11)
1.25	0.56	-450.50	(-7.12)	-437.62	(-8.84)
1.5	0.60	-454.14	(-10.76)	-444.20	(-15.42)
2	0.67	-461.13	(-17.75)	-451.23	(-22.45)
5	0.83	-488.65	(-45.27)	-481.04	(-52.26)
∞	1.00	-529.00	(-85.62)	-501.56	(-72.78)
DSGE		-530.48	(-87.10)	-530.48	(-101.70)

Out-of-Sample Root Mean Squared Errors: VECM Specification

		forecast horizon					
		1	2	4	6	8	12
Y	DSGE-VECM($\hat{\lambda}$)	0.577	0.909	1.753	2.505	3.141	3.888
	relative to DSGE	(19.6)	(36.0)	(51.5)	(58.8)	(63.1)	(69.5)
	relative to VECM	(21.3)	(24.4)	(25.0)	(21.5)	(17.0)	(12.7)
C	DSGE-VECM($\hat{\lambda}$)	0.498	0.767	1.375	1.959	2.450	3.226
	relative to DSGE	(22.9)	(33.9)	(42.1)	(46.1)	(49.4)	(54.7)
	relative to VECM	(5.5)	(9.3)	(18.2)	(19.9)	(21.4)	(19.2)
I	DSGE-VECM($\hat{\lambda}$)	3.160	4.955	9.205	13.112	16.520	20.250
	relative to DSGE	(29.3)	(41.0)	(53.1)	(59.7)	(63.7)	(69.9)
	relative to VECM	(13.2)	(12.3)	(9.5)	(4.5)	(-2.1)	(-11.8)
H	DSGE-VECM($\hat{\lambda}$)	0.005	0.009	0.019	0.029	0.038	0.050
	relative to DSGE	(16.3)	(29.9)	(43.7)	(48.5)	(50.2)	(52.5)
	relative to VECM	(19.5)	(21.1)	(16.4)	(13.6)	(9.4)	(-0.2)
W	DSGE-VECM($\hat{\lambda}$)	0.611	1.022	1.875	2.563	3.162	4.256
	relative to DSGE	(1.9)	(-0.3)	(-3.9)	(-8.0)	(-12.0)	(-13.2)
	relative to VECM	(7.5)	(6.4)	(2.5)	(1.5)	(3.6)	(8.8)
Inflation	DSGE-VECM($\hat{\lambda}$)	0.233	0.450	0.833	1.305	1.803	2.820
	relative to DSGE	(2.4)	(8.6)	(15.5)	(14.1)	(14.0)	(14.5)
	relative to VECM	(5.2)	(5.6)	(9.0)	(13.4)	(15.3)	(16.5)
R	DSGE-VECM($\hat{\lambda}$)	0.465	0.780	1.288	1.712	2.180	2.596
	relative to DSGE	(13.1)	(22.4)	(27.4)	(26.5)	(21.2)	(19.4)
	relative to VECM	(28.3)	(28.8)	(29.2)	(31.4)	(30.1)	(29.1)
Multivariate Statistic	DSGE-VECM($\hat{\lambda}$)	1.368	0.939	0.523	0.281	0.117	-0.101
	relative to DSGE	(13.8)	(18.2)	(21.7)	(20.7)	(17.8)	(5.0)
	relative to VECM	(15.0)	(16.3)	(16.0)	(19.7)	(20.4)	(26.5)

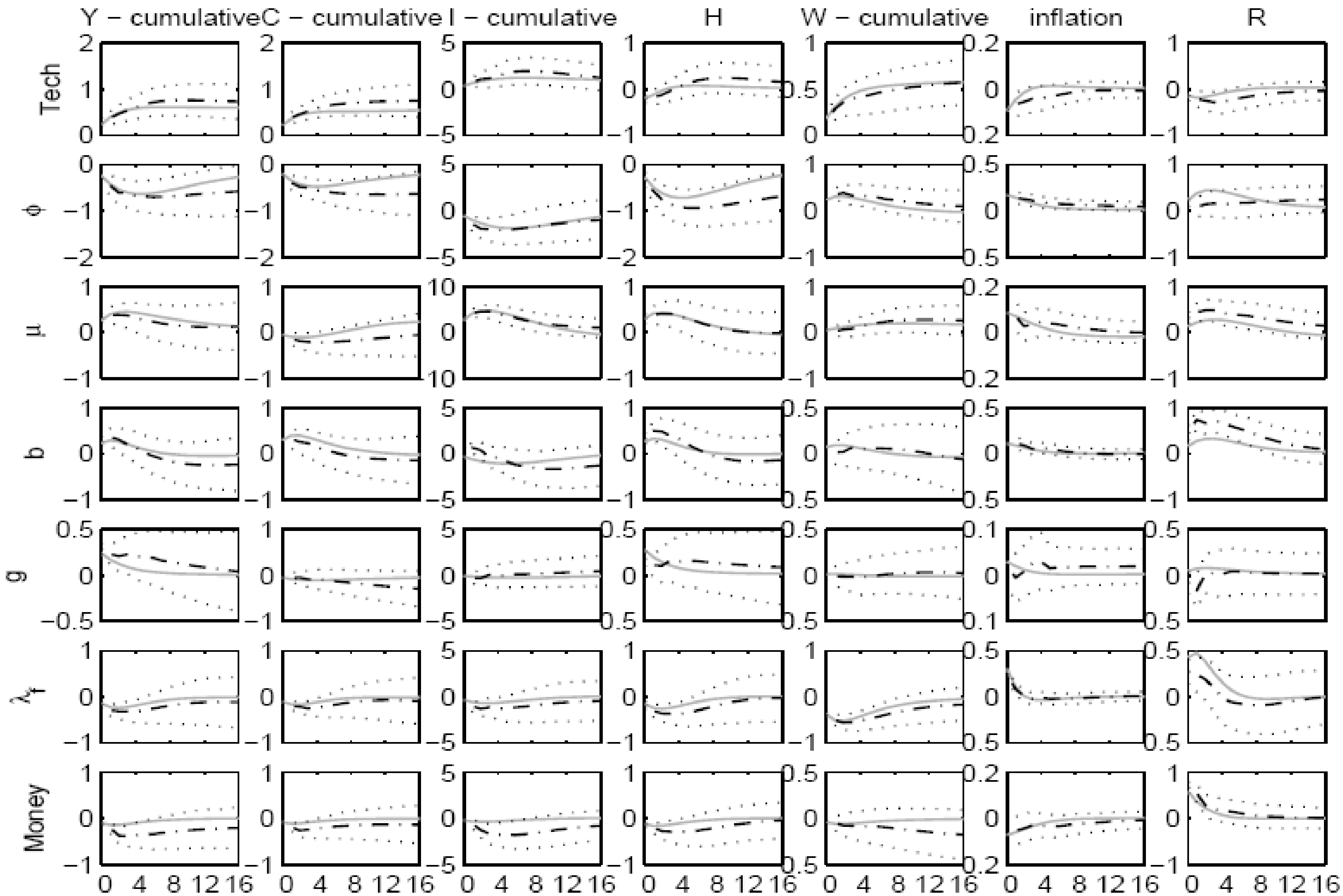
Table 2: DSGE Model's Parameter Estimates (Part I)

	Prior			DSGE Post.			DSGE-VECM($\hat{\lambda}$) Post.		
	Distr.	Mean	Stdd	Mean	Interval		Mean	Interval	
α	$\mathcal{B}$	0.250	0.100	0.172	0.149	0.195	0.153	0.125	0.181
ζ_p	$\mathcal{B}$	0.750	0.100	0.848	0.814	0.883	0.667	0.512	0.834
s'	$\mathcal{N}$	4.000	1.500	5.827	3.238	8.115	4.204	3.999	4.405
h	$\mathcal{B}$	0.800	0.100	0.793	0.725	0.857	0.691	0.585	0.791
a''	$\mathcal{G}$	0.200	0.075	0.167	0.067	0.273	0.243	0.126	0.356
ν_l	$\mathcal{G}$	2.000	0.750	2.204	1.050	3.271	2.285	2.056	2.518
ζ_w	$\mathcal{B}$	0.750	0.100	0.936	0.913	0.959	0.812	0.720	0.905
r^*	$\mathcal{G}$	0.500	0.100	0.389	0.270	0.501	0.496	0.350	0.646
ψ_1	$\mathcal{G}$	1.700	0.100	1.799	1.640	1.944	1.768	1.609	1.993
ψ_2	$\mathcal{G}$	0.125	0.100	0.065	0.040	0.090	0.035	0.000	0.074
ρ_r	$\mathcal{B}$	0.800	0.100	0.815	0.775	0.855	0.799	0.748	0.850
π^*	$\mathcal{N}$	0.650	0.200	1.026	0.807	1.264	0.553	0.296	0.843
γ	$\mathcal{G}$	0.500	0.250	0.185	0.085	0.276	0.466	0.237	0.699
g^*	$\mathcal{B}$	0.150	0.050	0.224	0.199	0.253	0.157	0.095	0.217

Table 3: DSGE Model's Parameter Estimates (Part II)

	Prior			DSGE Post.			DSGE-VECM($\hat{\lambda}$) Post.		
	Distr.	Mean	Stdd	Mean	Interval		Mean	Interval	
ρ_z	$\mathcal{B}$	0.200	0.050	0.218	0.146	0.294	0.210	0.134	0.283
ρ_ϕ	$\mathcal{B}$	0.800	0.050	0.705	0.625	0.791	0.856	0.766	0.951
ρ_{λ_f}	$\mathcal{B}$	0.800	0.050	0.518	0.449	0.589	0.690	0.471	0.895
ρ_μ	$\mathcal{B}$	0.800	0.050	0.884	0.834	0.937	0.706	0.608	0.797
ρ_b	$\mathcal{B}$	0.800	0.050	0.811	0.743	0.876	0.762	0.696	0.841
ρ_g	$\mathcal{B}$	0.800	0.050	0.951	0.928	0.975	0.900	0.846	0.955
σ_z	$\mathcal{IG}$	0.400	2.000	0.702	0.625	0.779	0.475	0.405	0.544
σ_ϕ	$\mathcal{IG}$	1.000	2.000	3.450	1.990	4.886	1.121	0.867	1.369
σ_{λ_f}	$\mathcal{IG}$	1.000	2.000	0.192	0.168	0.217	0.191	0.163	0.219
σ_μ	$\mathcal{IG}$	1.000	2.000	0.918	0.742	1.077	0.725	0.597	0.844
σ_b	$\mathcal{IG}$	0.200	2.000	0.538	0.439	0.630	0.302	0.231	0.370
σ_g	$\mathcal{IG}$	0.300	2.000	0.406	0.360	0.454	0.284	0.241	0.328
σ_r	$\mathcal{IG}$	0.200	2.000	0.271	0.242	0.300	0.169	0.143	0.197

Comparing the Propagation of Shocks



Conclusions

- Application of the DSGE-VAR tool kit to a relatively rich New Keynesian DSGE model suggests that some model features are still at odds with the data:
  - Relaxing the DSGE model restrictions improves the fit in terms of Bayesian marginal likelihood and out-of-sample forecasting performance of the DSGE-VAR.
  - In particular, the co-trending implications of the DSGE model are not fully born out by the data;
  - When this misspecification is not taken into account, the structural parameter estimates are biased towards more persistence (more persistent shocks, higher habit formation and higher degree of nominal frictions)

Work ahead

- Work harder on building models that can be successfully taken to non-detrended data to fulfil Kydland and Prescott (1982)'s original promise of integrating growth and business cycle theory;
- Need to develop approaches that use the DSGE model restrictions, but down-weight those frequencies where the DSGE's model implications are more at odds with the data.
- Monetary policy analysis with misspecified models (Del Negro and Schorfheide, 2004);