Self-Fulfilling Risk Panics

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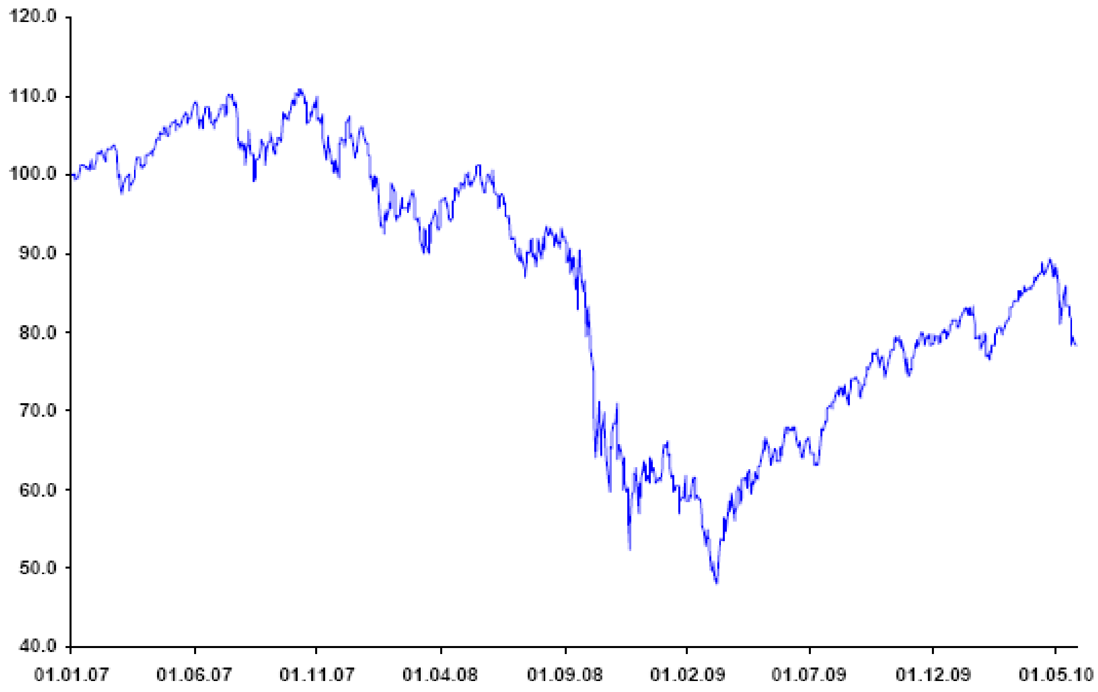
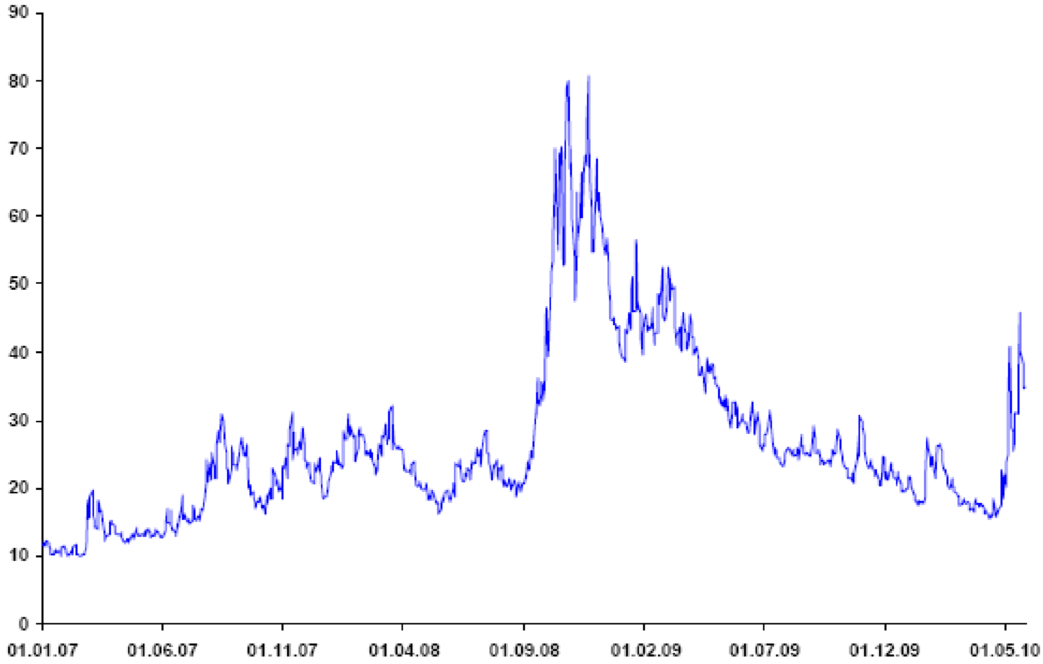
Stock Price



Risk



How can we explain such sharp Breaks?

- Changes in fundamentals and news (Lehman Brothers, Greek crisis) are not large enough.
  - Problems had been brewing for some time and were not suddenly revealed. A full year elapsed between the start of the crisis and the panic of the fall 2008.
- Assuming exogenous movements in risk premium is not satisfactory.
- Our focus: generate endogenous large movements in risk in a standard model.

Our Contribution: a Theory of Risk Panics

- Self-fulfilling shifts in risk, defined as the variance of the future asset price.
- Key ingredient: the asset price depends on risk.
  - Tomorrow's asset price and risk depend on perceived risk on subsequent days.
  - If risk is expected to move over time, prices will also be more volatile, justifying these risk changes.
- Dynamic mapping of risk onto itself, with possible multiple solutions.

Main Findings

- Multiplicity of equilibria: one with high risk and low asset price, one with low risk and high asset price.
- Two sources of multiplicity:
  - "Static": contemporaneous interaction of risk, liquidity and wealth.
  - "Dynamic": new aspect linked to self-fulfilling beliefs about future risk.
- Dual nature of macroeconomic shocks: fundamental and sunspot on which agents coordinate. High volatility equilibria are "sunspot-like" equilibria with this dual role.
- Shifting between equilibria: risk panics with shifts in expectations, risk, asset price, liquidity, and leverage.

Related literature

- Finance literature on market liquidity and leverage.
    - Adrian and Shin (2008), Brunnermeier and Pedersen (2009), Danielsson et al. (2009), Gromb and Vayanos (2002,2008), He and Krishnamurthy (2008a, b), Kyle and Xiong (2001) and Xiong (2001).
- Notice that we do not rely on time-varying volatility of shocks.
- Multiplicity of equilibria in terms of risk, not merely the asset price level.

Outline of the Presentation

- Illustrate multiplicity of equilibria when there is a link between the asset price and risk.
- Include this in a simple mean-variance portfolio choice model, with bonds and risky equity.
- Extend to a general equilibrium setting, requiring approximations.
- Consider redistribution shocks between investors (losses of leveraged institutions).
  - Show how the model generates "risk panics" similar to the crisis.

The Key Mechanism

- Simple linear relation between the current asset price and uncertainty about the future asset price (risk):

$$Q_t = \lambda_0 - \lambda_1 Risk_t = \lambda_0 - \lambda_1 Var_t \left(Q_{t+1}\right)$$

- Take a sunspot variable that is extrinsic to the model. A key assumption is that it is persistent:

$$S_t = \rho S_{t-1} + \varepsilon_t$$

  ε_t has a symmetric distribution with mean 0 and variance σ^2. Also $var\left(\varepsilon_t^2\right) = \omega^2$.

- The asset price is of the form:

$$Q_t = \tilde{Q} - V\left(S_t\right)^2$$

Equilibria

- Fundamental equilibrium. The asset price is constant and there is no risk: $Q_t = \lambda_0$. Notice $V = 0$.
- Sunspot equilibrium. The asset price is affected by the sunspot:

$$Q_t = \tilde{Q} - V\left(S_t\right)^2$$

where:

$$\tilde{Q} = \lambda_0 - \lambda_1 V^2 \omega^2 \quad ; \quad V = \left[4\lambda_1 \rho^2 \sigma^2\right]^{-1} > 0$$

- The persistence of the sunspot variable implies time-varying risk:

$$\begin{aligned} Risk_t &= V^2 Var_t \left[2\rho S_t \varepsilon_{t+1} + \left(\varepsilon_{t+1}\right)^2\right] \\ &= 4V^2 \left(S_t\right)^2 \rho^2 \sigma^2 + V^2 \omega^2 \end{aligned}$$

Current and future risk

- Any non-zero realization of S_t implies that the future asset price is more sensitive to future shocks, i.e. $Risk_t$ is higher.
- At the same time, $Risk_{t+1}$ is higher:

$$Risk_{t+1} = 4V^2\rho^2\sigma^2\left[\rho^2\left(S_t\right)^2 + 2\rho S_t\varepsilon_{t+1} + \left(\varepsilon_{t+1}\right)^2\right] + V^2\omega^2$$

and more volatile:

$$\begin{aligned} Var_t\left(Risk_{t+1}\right) &= 16V^4\rho^4\sigma^4\left[4\rho^2\sigma^2\left(S_t\right)^2 + \omega^2\right] \\ &= 16V^2\rho^4\sigma^4 Risk_t \end{aligned}$$

Intuition

- The future asset price Q_{t+1} is a linear function of future risk $Risk_{t+1}$. This gives a mapping between current and future risk:

$$Risk_t = Var_t\left(Q_{t+1}\right) = \lambda_1^2 Var_t\left(Risk_{t+1}\right)$$

- As $Var_t\left(Risk_{t+1}\right) = \left(4V\rho^2\sigma^2\right)^2 Risk_t$, one solution is zero risk (fundamental, $V = \lambda_1 = 0$).
- The other solution is positive risk, with an impact on the asset price (sunspot).
- The sunspot serves as a focal point for beliefs about risk.

Sunspot-like Equilibrium

- S_t can also have a fundamental role. Consider:

$$Q_t = \lambda_0 - \lambda_1 Risk_t + \lambda_2 S_t$$

- The asset price is of the form:

$$Q_t = \tilde{Q} + vS_t - V(S_t)^2$$

- Fundamental equilibrium. $V = 0$; risk is constant; $Risk_t = \lambda_2^2 \sigma^2$.
- Sunspot like equilibrium. $V > 0$ (identical to previous) and risk is time-varying:

$$Risk_t = (v - 2V\rho S_t)^2 \sigma^2 + V^2 \omega^2$$

Fundamental and Sunspot-like Impact

- Manuelli and Peck (1992):

  > *There are two ways that random fundamentals can influence economic outcomes. First, randomness affects resources which intrinsically affects prices and allocation. Second, the randomness can endogenously affect expectations or market psychology, thereby leading to excessive volatility.*

- Spears, Srivastava and Woodford (1990) also present a model with sunspot-like equilibria:

  > *...a sharp distinction between "sunspot equilibria" and "non-sunspot equilibria" is of little interest in the case of economies subject to stochastic shocks to fundamentals.*

Introducing this in Portfolio Choice

- A standard expected utility framework without portfolio constraint does not lead to a link between the asset price and risk.
  - the only risk that matters then is asset payoff uncertainty.
  - the asset price depends on the covariance of future asset payoff with the stochastic discount factor.
- Realistic constraints on risk-exposure (value-at-risk or margin constraints) make asset demand (and asset prices) depend on uncertainty about the future price.
- We adopt a simple mean-variance portfolio choice model to capture the impact of portfolio constraints while minimizing complexity.
  - Asset demand depends on risk about the future asset price
  - Simple linear relationship between the asset price and risk in equilibrium.

A Simple Mean-Variance Model

- Closed economy with infinite horizon.
- Overlapping generations of agents that are born with wealth W_I, constant across generations.
- Agents purchase a risk-free bond, with constant exogenous return R (we later consider a variable rate) and risky equity, that is a claim on K trees with stochastic productivity $A_t K$. The equity price is Q_t, and the rate of return on equity is:

$$R_{K,t+1} = \left(A_{t+1} + Q_{t+1}\right) / Q_t$$

- Stochastic dividend ($m \geq 0$ reflects the fundamental role of S):

$$A_t = \bar{A} + mS_t \quad ; \quad S_t = \rho S_{t-1} + \varepsilon_t$$

Portfolio Choice

- Denote the portfolio share invested in the risky equity by α_t. Agents maximize a mean-variance utility in terms of the portfolio return:

$$E_t R^p_{t+1} - 0.5\gamma Var_t \left(R^p_{t+1} \right) \quad ; \quad R^p_{t+1} = \alpha_t R_{K,t+1} + (1 - \alpha_t) R$$

- The optimal portfolio reflects the expected excess return on equity scaled by its variance:

$$\alpha_t = \frac{E_t R_{K,t+1} - R}{\gamma Var_t (R_{K,t+1})}$$

- The clearing of the asset market equates the demand and supply for equity, leading to a relation between the equity price and its variance:

$$\alpha_t W_I = Q_t K$$

$$E_t \left(A_{t+1} + Q_{t+1} \right) - RQ_t = \frac{\gamma K}{W_I} Var_t \left(A_{t+1} + Q_{t+1} \right)$$

Equilibria with no Fundamental Role

- Consider that S plays no fundamental role and the dividend is constant ($m = 0$). The asset price is of the form:

$$Q_t = \tilde{Q} - V \left(S_t \right)^2$$

- Fundamental equilibrium implies no risk ($V = 0$):

$$Q_t = \tilde{Q}_{fund} = \bar{A} / \left(R - 1 \right)$$

- The sunspot equilibrium entails time-varying risk and a low average asset price ($V > 0$):

$$Var_t \left(A_{t+1} + Q_{t+1} \right) = 4V^2 \left(S_t \right)^2 \rho^2 \sigma^2 + V^2 \omega^2$$

$$\tilde{Q} = \tilde{Q}_{fund} - \frac{V}{R-1} \left(\sigma^2 + \frac{\gamma K}{W_I} V \omega^2 \right) < \tilde{Q}_{fund}$$

Equilibria with Fundamental Role

- We now introduce a fundamental role for S ($m > 0$). The asset price is of the form:

$$Q_t = \tilde{Q} + vS_t - V\left(S_t\right)^2$$

- The fundamental equilibrium entails constant risk ($V = 0$):

$$Q_t = \tilde{Q} + m\frac{\bar{A}\rho}{R-\rho}S_t \quad ; \quad Var_t\left(A_{t+1} + Q_{t+1}\right) = \left(1 + \frac{\bar{A}\rho}{R-\rho}\right)^2 m^2\sigma^2$$

- The sunspot-like equilibrium entails time-varying risk ($V > 0$, identical to previous):

$$Var_t\left(A_{t+1} + Q_{t+1}\right) = \left(m + v - 2V\rho S_t\right)^2 \sigma^2 + V^2\omega^2$$

General Equilibrium Model

- The interest rate on the bond is endogenously determined by the clearing of the bond market.
- If the bond is in fixed supply, the clearing of both asset markets requires a constant portfolio share, pinning down the expected return and risk.
- Introduce a bond supply that is a function of the interest rate.
- A new group of agents, households, that live for two periods. Endowment W_H that is invested in bonds and the technology a risk-free technology with decreasing returns to scale where I units of investment yield $\left[v I - 0.5 I^2 \right] / \eta$ units of output. The demand for bonds from households is:

$$W_H - v + \eta R_t$$

Asset Market Clearing

- The clearing of the bond market is:

$$(1-\alpha_t)\,W_I + W_H - \upsilon + \eta R_t = 0$$

- Combining with the clearing of the equity market, $\alpha_t W_I = Q_t K$, we get a the bond interest rate as a function of the asset price:

$$Q_t K + \upsilon - \eta R_t = W_I + W_H$$

Solving the Model

- The endogenous interest rate leads to a non-linearity in the asset market clearing. The model is solved using a numerical solution.
- We conjecture a functional form for the log equity price that is quadratic in S:

$$q_t = \tilde{q} + vS_t - V\left(S_t\right)^2$$

- A quadratic approximation of the asset market clearing implies:

$$Z_0 + Z_1 S_t + Z_2 \left(S_t\right)^2 = 0$$

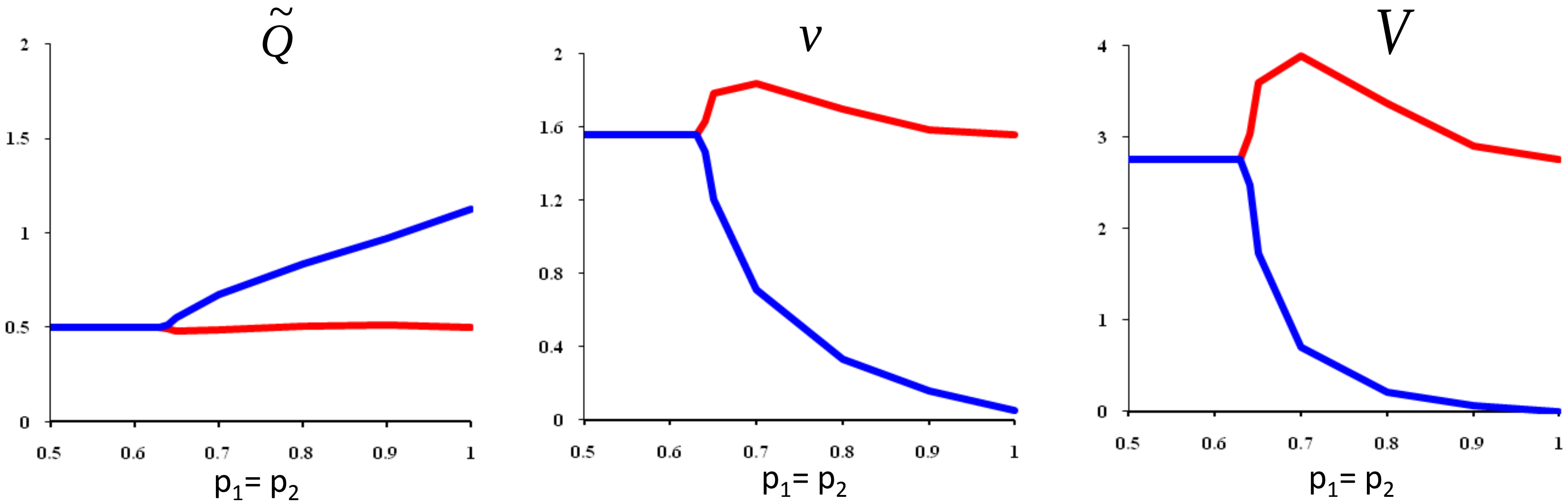
  where the Z's are functions of v, V and $\tilde{q}$. Setting each Z to zero gives the implicit solution of the coefficients.
- There are multiple equilibria: fundamental and sunspot like. Sunspot like equilibria are characterized by time varying risk and a more volatile asset price.
- Limitation: the economy is always in a given equilibrium.

Switching Equilibria

- Introduce an exogenous Markov process that shifts the economy between states. Agents know the probabilities of switching, and expectations are computed taking the possibility of switching into account.
- Two states: a low risk state (denoted by 1) that converges to the fundamental equilibrium when the probability of switching goes to zero, and a high risk state (denoted by 2) that converges to the sunspot-like equilibrium. The probability of remaining in state i is p_i.
- There are now 6 parameters to solve from the asset market clearing conditions in the two states.
- The two states converge if the probability of switching is high enough. The impact of switching on the parameters occurs mostly for the low risk state.

Figure 4 Switching Equilibria*

blue=low risk state; red=high risk state



* This is based on the parameters of Figure 2. When p_1=p_2=1, the high and low risk states correspond exactly to equilibria 1 and 2 in Figure 3.

Risk Panics

- A panic is a switch from the low to the high risk state.
- It is characterized by:
    - increase in risk, and the volatility of risk.
    - decrease in the equity price.
    - decrease in the portfolio share put in equity (lower leverage).
    - decrease in the interest rate.
- Three key points are worth stressing.

A Panic is not a Change in Fundamental

- The state variable S remains unchanged in a panic.
- Its role however changes, as it acquires a sunspot role that dominates its fundamental role. The state variable becomes the focal point for a self-fulfilling change in risk perceptions.
- A panic is distinct from a financial accelerator that amplifies effect of a change in fundamentals through financial constraints (which is a fundamental equilibrium).

The Impact of the Panic depends on the Level of the Fundamental

- The difference of the asset price between the two states is:

$$q_{2,t} - q_{1,t} = \tilde{q}_2 - \tilde{q}_1 + \underbrace{(v_2 - v_1)}_{>0} S_t - (V_2 - V_1)(S_t)^2$$

- $q_{2,t} - q_{1,t} < 0$. The more negative the fundamental, the bigger the asset price change, as well as the other impacts of the panic.
- With $p_1 = p_2 = 0.65$, the asset price falls by 13% when S is at its mean value of 0, but by 65% when S is two standard deviations below the mean.

The Fundamental has a larger Price Impact after the Panic

- After the panic the state variable S has a sunspot role.
- Changes in S now affect the economy primarily through its role as a focal point for expectations.
- An improvement in the fundamental can lead to a quick recovery of asset markets (markets are on edge).

Financial Shocks

- To bring the market closer to the current crisis, we introduce a financial shock, which is a redistribution of endowments between investors and households, leaving the total endowment unchanged.
- Interpreted as losses suffered by investors on risky securities. Allowing for a change in total endowment does not change the results.
- We simplify the model by assuming the dividend shocks are iid, so the relative wealth of investors and households is the only state variable, as financial shocks are persistent.
- The financial shock is denoted by θ_t. A higher θ_t is a lower wealth of investors:

$$W_{I,t} = \exp\left[-m\theta_t - 0.5m^2\theta_t^2\right]\bar{W}_I$$

$$\theta_t = \rho_\theta \theta_{t-1} + \varepsilon_t$$

- This specification ensures that the endowment is linear in the shock in a quadratic approximation:

$$W_{I,t} = \bar{W}_I\left(1 - m\theta_t\right)$$

Solution

- We conjecture a functional form for the log equity price that is quadratic in the state variable:

$$q_t = \tilde{q} - v\theta_t - V\left(\theta_t\right)^2$$

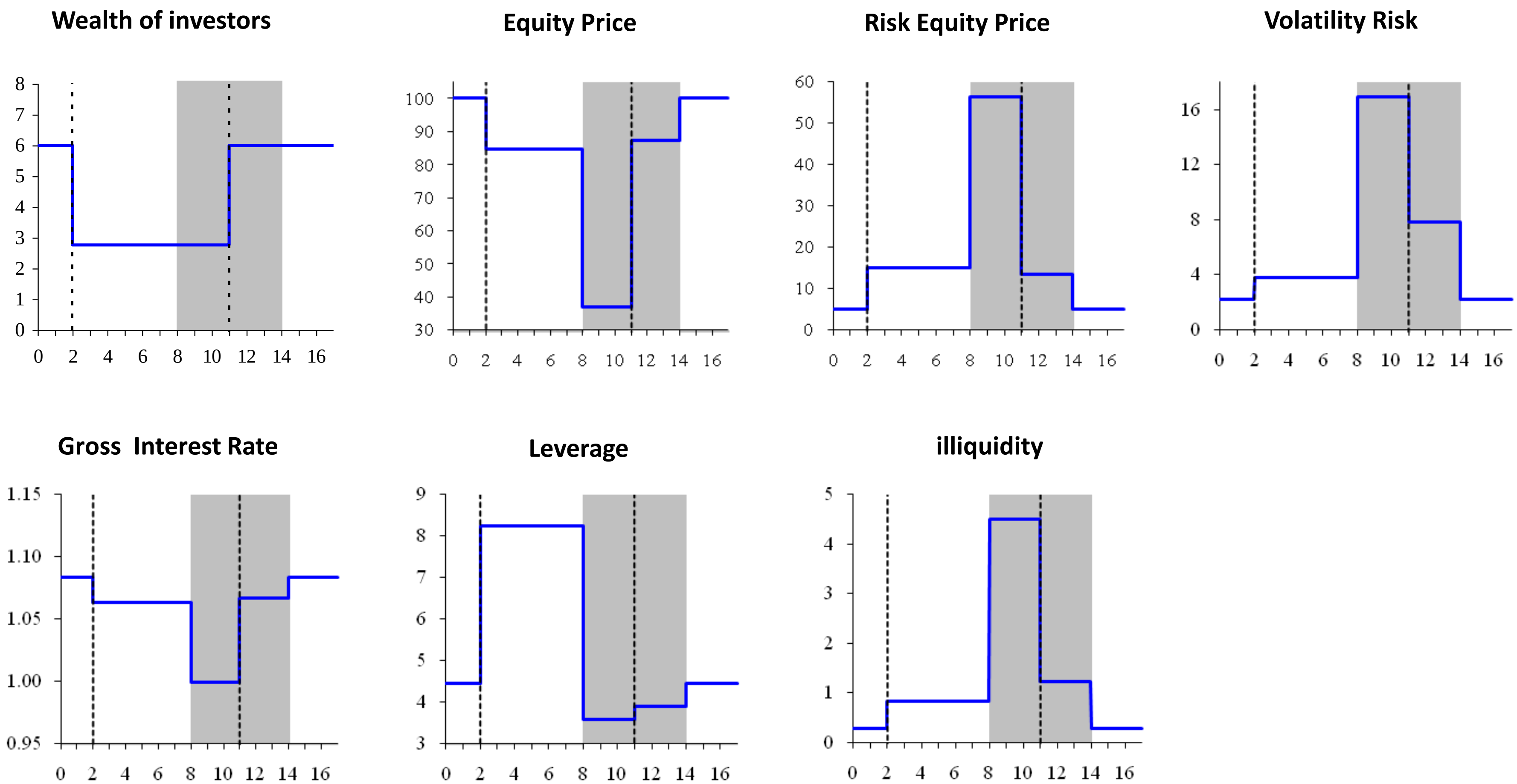
- In addition to the dynamic multiplicity described above (which hinges on persistence of the state variable), there is a static multiplicity.
  - Perceptions of higher risk lead investors to substitute away from equity. Market liquidity is lower, and the asset price is more sensitive to shock, justifying the perception of higher risk.
  - This is most clearly illustrated by setting $\rho_\theta = 0$.
- The dynamic multiplicity is similar to the one presented for dividend shocks. The exact nature of the shock plays little role in the sunspot-like equilibrium, as the variable mostly serves as a focal point.

Application to the Crisis

- The model is too stylized to finely match the crisis, but can it deliver the broad pattern?
- The distinct stages are of particular interest: moderate impact in the first year of the crisis, followed by severe disruptions in the wake of Lehman Brothers.
- We consider four successive situations to illustrate the combinations of the state in which the economy is (low or high risk) and the state variable (high or low wealth of investors).
  - Initial stage: low risk state, high investors' wealth.
  - Second stage: low risk state, low investors' wealth (summer 2007 - September 2008).
  - Third stage: high risk state, low investors' wealth (post Lehman).
  - Fourth stage: high risk state, high investors' wealth (recapitalization of leveraged financial firms).
- Asset prices are lower, and risk is higher when investors wealth is low, or the economy is in the high risk state. Each of these two aspects alone has only a small impact, but their combination is potent.

Figure 8 Model Simulation

shaded area = high risk equilibrium; vertical lines = endowment shock



The economy starts in the low risk equilibrium. At time 2 the share of the endowment of leveraged institutions to total endowment shifts from 0.4 to 0.1. The economy stays in the low risk equilibrium until time 8, at which point is shifts to the high risk equilibrium. At time 11 endowments shift back towards the initial allocation. The economy remains in the high risk equilibrium until time 14, at which points it shifts back to the low risk equilibrium.

$$\overline{A}=0.15; \upsilon - W = 190; \eta = 200; \sigma_a = 0.1; \sigma_\theta = 0.1; \rho_\theta = 0.7; \gamma = 1; \overline{W_I} = 6; m = 2; K = 20; p_1 = 0.95; p_2 = 0.7$$

Extensions

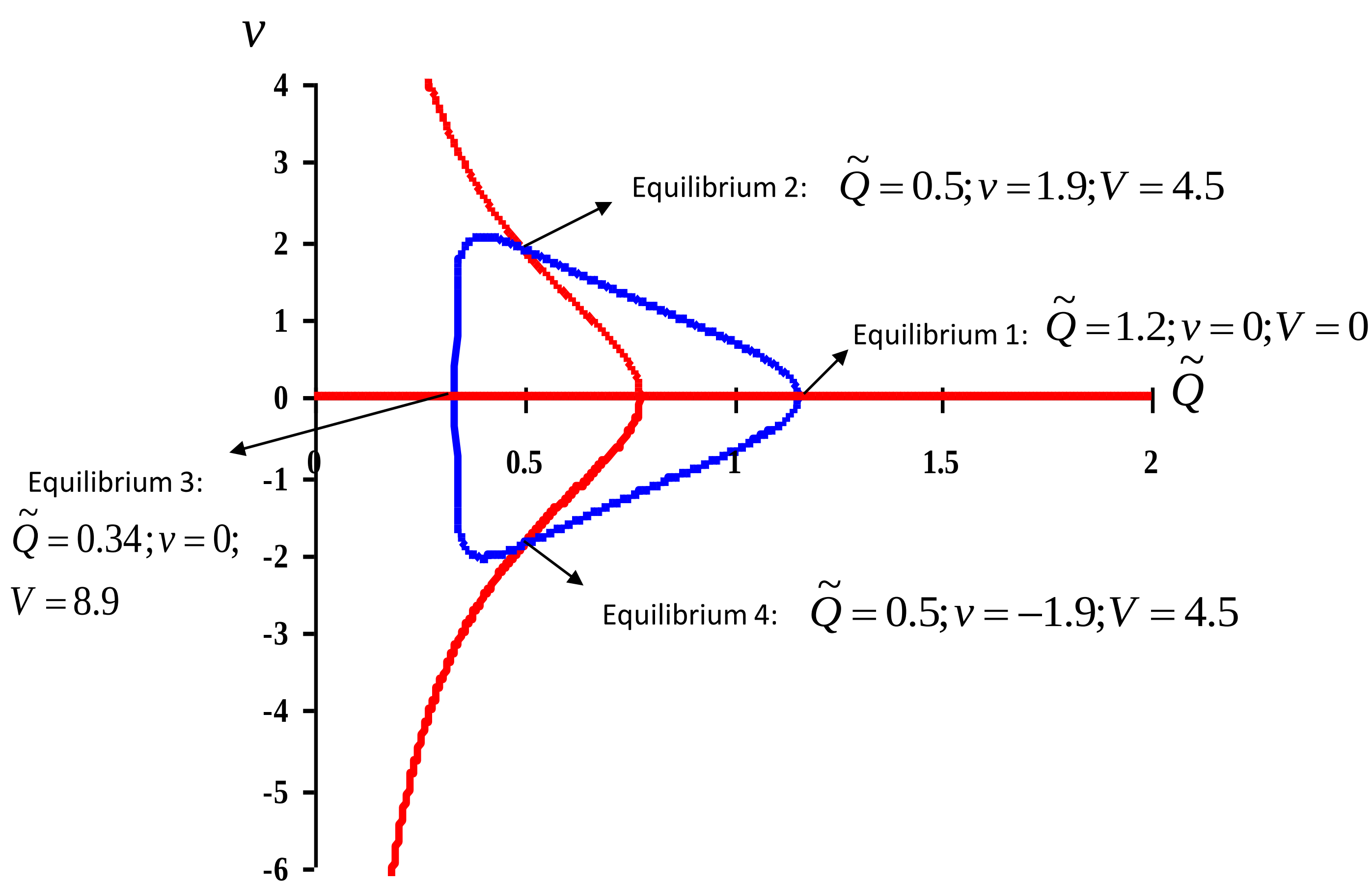
- Allow endowments to also include equity, so asset prices affect wealth directly. High volatility equilibrium is amplified.
- Open economy setting: capital flows recently dominated by (de)leveraging. Preliminary results consistent with recent developments.
- Introducing an impact on the real economy.

Conclusions

- Simple general equilibrium model of portfolio choice.
- Rich set of results regarding liquidity and risk: static and dynamic multiplicity.
- Multiple equilibria and sunspot phenomena that could explain many features of the recent financial crisis.
- Open avenues for much more future work.

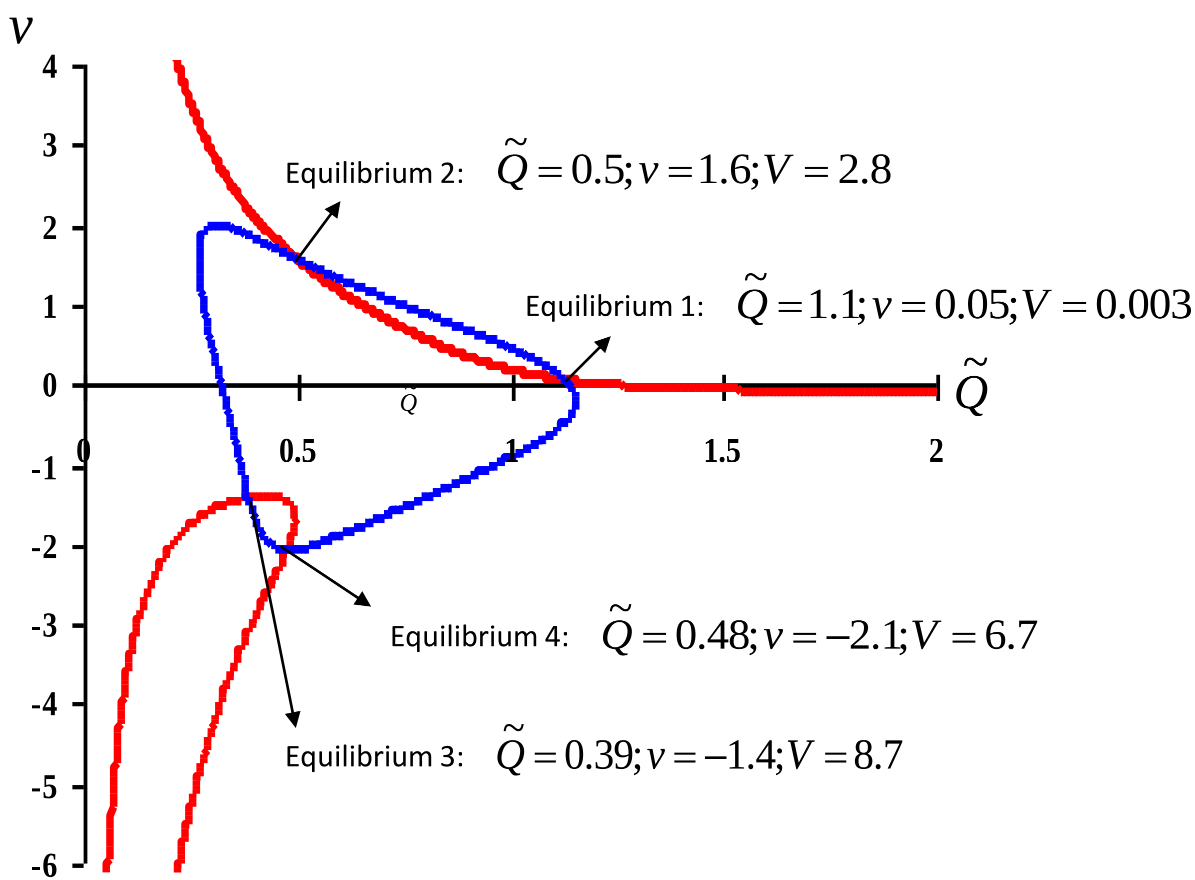
APPENDIX SLIDES

Figure 1 Sunspot Equilibria*



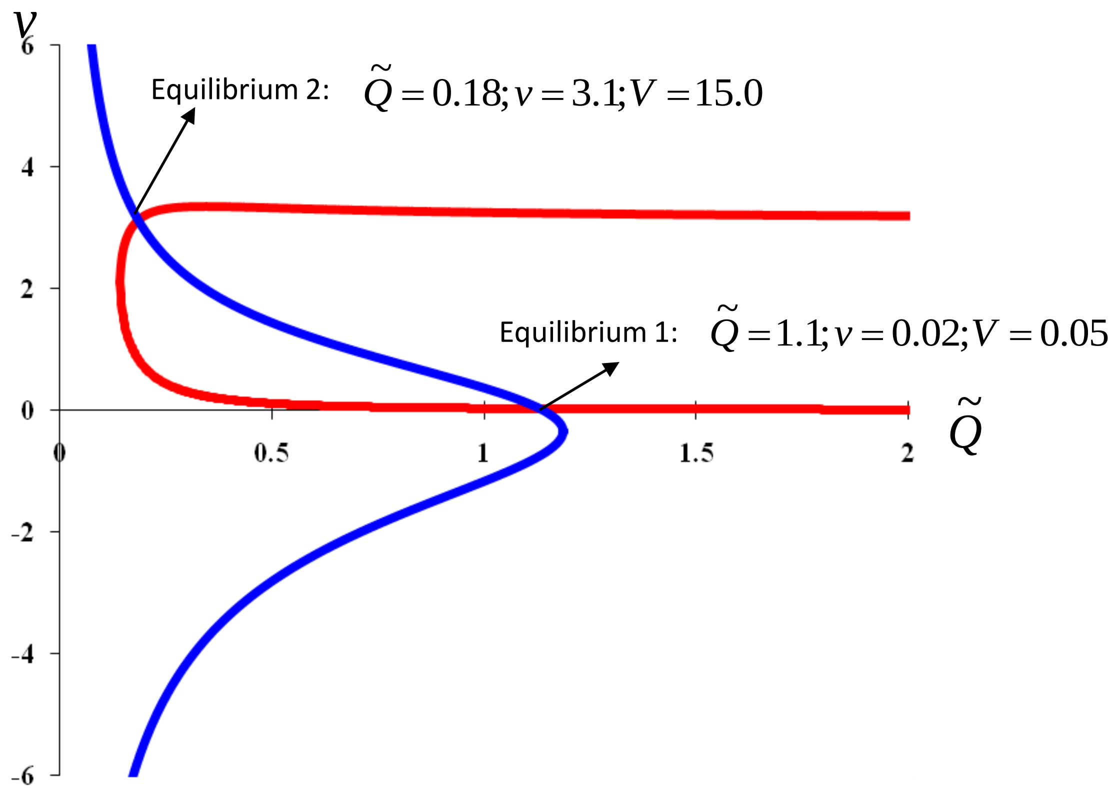
$^*\bar{A}=0.3; \upsilon-W=0.1; \eta=1; \sigma=0.4; \rho=0.4; \gamma=4; W_I=2; m=0; K=1$

Figure 2 Sunspot-Like Equilibria*



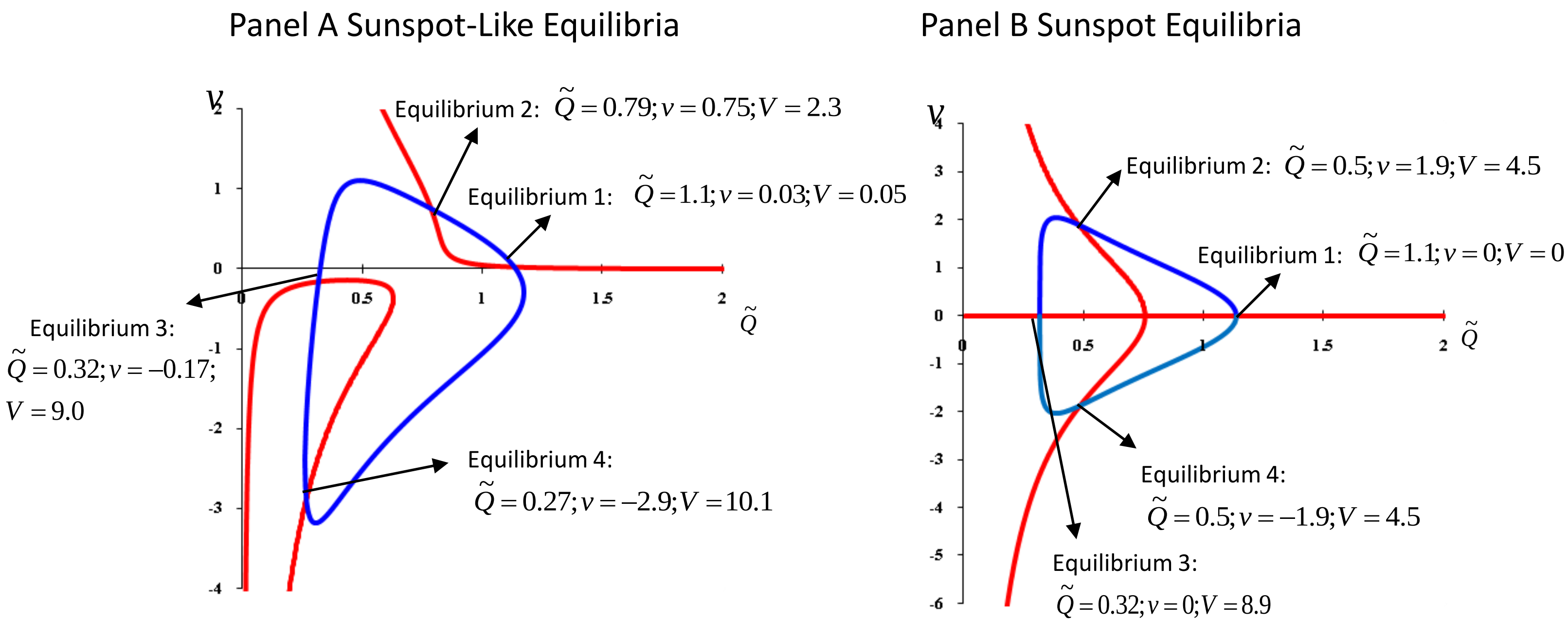
$^*\overline{A} = 0.3; \upsilon - W = 0.1; \eta = 1; \sigma = 0.4; \rho = 0.4; \gamma = 4; W_I = 2; m = 1; K = 1$

Figure 5 Equilibria with Financial Shocks: No Persistence*



* $\overline{A}=0.3; \upsilon-W=0.1; \eta=1; \sigma=0.4; \sigma_\theta=0.4; \rho_\theta=0; \gamma=4; \overline{W_I}=2; m=2; K=1$

Figure 6 Sunspot and Sunspot-Like Equilibria with Financial Shocks*



* Panel A assumes ρ_θ=0.4, m=2. Panel B assumes ρ_θ=0.4, m=0 ; otherwise the parameters are the same as in Figure 5.

Time Line Simulation

period	2 (Q1, 2007)	8 (Q3, 2008)	11 (Q2, 2009)	14 (Q1, 2010)
	drop in wealth of leveraged institutions	switch to high risk state (risk panic)	wealth leveraged institutions restored to pre-crisis level	switch back to low risk state