An Estimated Monetary DSGE Model with Unemployment and Staggered Nash Wage Bargaining

Mark Gertler

New York University

Luca Sala   Antonella Trigari

IGIER, Università Bocconi

March 2008

What we do

1. Estimate variant of a conventional monetary DSGE model as in Christiano et al. (CEE, 2005) and Smets and Wouters (SW, 2007)

2. Allow for unemployment via a variant of the Diamond-Mortensen-Pissarides (DMP) search and matching framework

3. Add wage rigidity via staggered Nash bargaining as in Gertler and Trigari (GT, 2006)

Why we do it

Existing quantitative models

- Employment adjusts along the intensive margin + sticky wages
- Wage rigidity
  - is key in accounting for labor market volatility and, also
  - influences short run output/inflation tradeoff (Blanchard and Galí, 2006)
- Subject to the Barro's (1977) critique: there are unexploited gains from renegotiation

In our model

- As employment adjusts along the extensive margin:
  - address the Barro's critique
  - consistent with evidence
- Address debate over whether the DMP framework can account for labor market volatility absent wage rigidity (Shimer, 2005, Hall, 2005, Hagedorn/Manovskii, 2006)
  - Estimate the key labor market parameters (vs. calibration)
  - Allow for the full range of shocks (vs. technology)
  - Formally test for the importance of wage rigidity (vs. comparing moments)

Model

- As in CEE and SW, basic non-labor market features: price stickiness, habit formation in consumption, investment adjustment costs, variable capital utilization

- Unemployment and staggered wage bargaining as in GT, but:

  wage contracting in nominal terms with indexing for past inflation

- Wholesale (wage setting + hiring) versus retail (price setting) firms

- Growth: technology is non stationary $\longrightarrow$ model-consistent detrending

Main results

- Wage rigidity improves the quantitative performance

- More work needed to ensure robust identification of key labor market parameters

- Similar fit of the data as the SW model

Households

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log \left(c_{t+s} - h c_{t+s-1}\right)$$

- Budget constraint:

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t)\, b_t + r_t^k z_t k_{t-1}^p + \Pi_t - T_t - a(z_t)\, k_{t-1}^p + \frac{B_{t-1}}{p_t}$$

- Capital accumulation: $k_t^p = (1-\delta)\, k_{t-1}^p + \varepsilon_t^i \left(1 - s\left(\frac{i_t}{i_{t-1}}\right)\right) i_t$

- Effective capital: $k_t = z_t k_{t-1}^p$

Unemployment, vacancies and matching

- Each firm i employs $n_t(i)$ workers and post $v_t(i)$ vacancies

- $n_t = \int_0^1 n_t(i)di, \quad v_t = \int_0^1 v_t(i)di$ and $u_t = 1 - n_{t-1}$

- $m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}, \quad q_t = \dfrac{m_t}{v_t}$ and $s_t = \dfrac{m_t}{u_t}$

- Exogenous separation $1 - \rho$

Firms: setup

$$F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa_t}{2} x_t(i)^2 n_{t-1}(i) - r_t^k k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i)$$

- Technology: $y_t(i) = [k_t(i)]^{\alpha} [a_t n_t(i)]^{1-\alpha}$ with $a_t / a_{t-1} = \varepsilon_t^a$
- Workforce dynamics: $n_t(i) = (\rho + x_t(i)) n_{t-1}(i)$
- Hiring rate: $x_t(i) = \dfrac{q_t v_t(i)}{n_{t-1}(i)}$

Firms: rental capital decision

$$r_t^k = p_t^w \alpha \frac{y_t(i)}{k_t(i)} = \alpha \frac{y_t}{k_t}$$

Firms: hiring decision

$$\kappa_t x_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 + \rho \beta E_t \Lambda_{t,t+1} \kappa_{t+1} x_{t+1}(i)$$

with

$$f_{nt}(i) = (1-\alpha) \frac{y_t(i)}{n_t(i)} = (1-\alpha) \frac{y_t}{n_t} = f_{nt}$$

Staggered Nash bargaining: basics

- Each period only a subset of firms/workers negotiate a wage contract

- Each firm negotiates with its existing workforce including new hires

- Workers hired in-between contract settlements receive existing wage

- Form of the contract: fixed nominal wage per period over an exogenous horizon

  $\Rightarrow$ fixed probability $1-\lambda$ to renegotiate the wage

  $\Rightarrow$ λ: average frequency of wage renegotiations

- Tractable generalization of the period-by-period Nash bargaining

- Differences from conventional time-dependent staggered wage setting

  - No unexploited bilateral gains from renegotiating the wage

  - General-equilibrium spillovers of average wages on contract wages

    $\Rightarrow$ extra stickiness in addition to the one generated by λ

Staggered Nash bargaining: the problem

- For the fraction $1 - \lambda$ of firms that are renegotiating at t:

$$w_t^{n*} = \text{arg max}\, H_t(i)^{\eta_t} J_t\,(i)^{1-\eta_t}$$

with $\eta_t = \eta \varepsilon_t^{\eta}$

- For the fraction λ that are not:

$$w_t^n\,(i) = \bar{\gamma} w_{t-1}^n\,(i)\,\pi_{t-1}^{\gamma}$$

Firm's marginal value of a worker (firm's surplus)

$$J_t(i) = p_t^w f_{nt} - w_t(i) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 + E_t \beta \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(i) J_{t+1}(i)$$

Worker's surplus

- Worker's employment value

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1} \left[\rho V_{t+1}(i) + (1-\rho) U_{t+1}\right]$$

- Worker's unemployment value

$$U_t = b_t + \beta E_t \Lambda_{t,t+1} \left[s_t V_{x,t+1} + (1-s_t) U_{t+1}\right]$$

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_{t-1} n_{t-1}} di$$

- Worker surplus

$$H_t(i) = V_t(i) - U_t = w_t(i) - b_t + \beta E_t \Lambda_{t,t+1} \left(\rho H_{t+1}(i) - s_t H_{x,t+1}\right)$$

Staggered Nash bargaining: solution

- The Nash foc is

$$\chi_t(i)\, J_t(i) = (1 - \chi_t(i))\, H_t(i)$$

with the "effective bargaining power" given by

$$\chi_t(i) = \frac{\eta}{\eta + (1-\eta)\, \Sigma_t(i) / \Delta_t}$$

and $\Sigma_t(i), \Delta_t \equiv$ firm and worker cumulative discount factors

- "Horizon effect" $\Rightarrow \chi < \eta$

- As in GT, the solution to the bargaining problem leads to:

$$\Delta_t w_t^* = w_t^o(i) + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$$

as in time-dependent contracting, w_t^* is a weighted sum of a future expected "target"

- Absent the horizon effect, the "target" is the period-by-period Nash wage:

$$\begin{aligned} w_t^o(i) &= \chi\left(p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2\right) \\ &\quad + (1-\chi)\left(b_t + s_{t+1}\beta E_t \Lambda_{t,t+1} H_{x,t+1}\right) + \Phi_t(i) \end{aligned}$$

Retail firms

- Monopolistically competitive, set prices á la Calvo. Price markup shocks ε_t^p.

Monetary and fiscal policy

- Taylor rule: $r_t/r = (r_{t-1}/r)^{\rho^s} \left[(\pi_t/\pi)^{r_\pi} (y_t/y_{nt})^{r_y}\right]^{(1-\rho^s)} \varepsilon_t^r$

- Government balances budget: $g_t + (1 - n_t)\, b_t + B_{t-1}/p_t = T_t + B_t/p_t r_t$

  where $g_t = \left(1 - 1/\varepsilon_t^g\right) y_t$

Aggregate wage dynamics

- The aggregate wage is given by:

$$\begin{aligned}\widehat{w}_t &= \gamma_b \left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a\right) \\ &\quad + \gamma_o \widehat{w}_t^o \\ &\quad + \gamma_f E_t \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a\right)\end{aligned}$$

$\widehat{w}_t^o =$ period-by-period Nash bargained wage economy-wide

$$\gamma_b + \gamma_o + \gamma_f = 1$$

as $\lambda \rightarrow 0$, $\gamma_b, \gamma_f \rightarrow 0$ and $\gamma_o \rightarrow 1$

- Tractable generalization of period-by-period Nash bargaining

Aggregate hiring dynamics

$$\widehat{x}_t = \gamma_{f_n}\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) - \gamma_w \widehat{w}_t + \gamma_\lambda E_t \widehat{\Lambda}_{t,t+1} + \beta E_t \widehat{x}_{t+1}$$

Estimation strategy

- Seven quarterly series as in SW:

  Y_t, C_t, I_t, W_t/P_t, N_t, r_t, π_t

- Sample period: 1960Q1-2005Q1

- One shock for each series

- Log-linearize the model

- Model-consistent detrending for non-stationary variables

- Labor market $\longrightarrow$ 5 new parameters $+ \lambda$
- Calibrate 3 for which there is independent evidence:
  - separation rate: $\rho = 0.895$
  - matches elasticity to unemployment: $\sigma = 0.5$
  - job finding rate: $s = 0.95$
- Estimate
  - relative flow value of unemployment $\bar{b}$
  - bargaining power η
  - wage rigidity λ

- Bayesian methods (see, e.g., An and Schorfheide, 2006)

- Combine the likelihood function of the model with priors for the parameters to be estimated to obtain the posterior distribution

- Let the data speak as much as possible

Table 1: Calibrated parameters

β	δ	α	ξ	g/y	σ	ρ	s
0.99	0.025	0.33	10	0.2	0.5	0.895	0.95

Table 2: Prior and posterior distribution of structural parameters

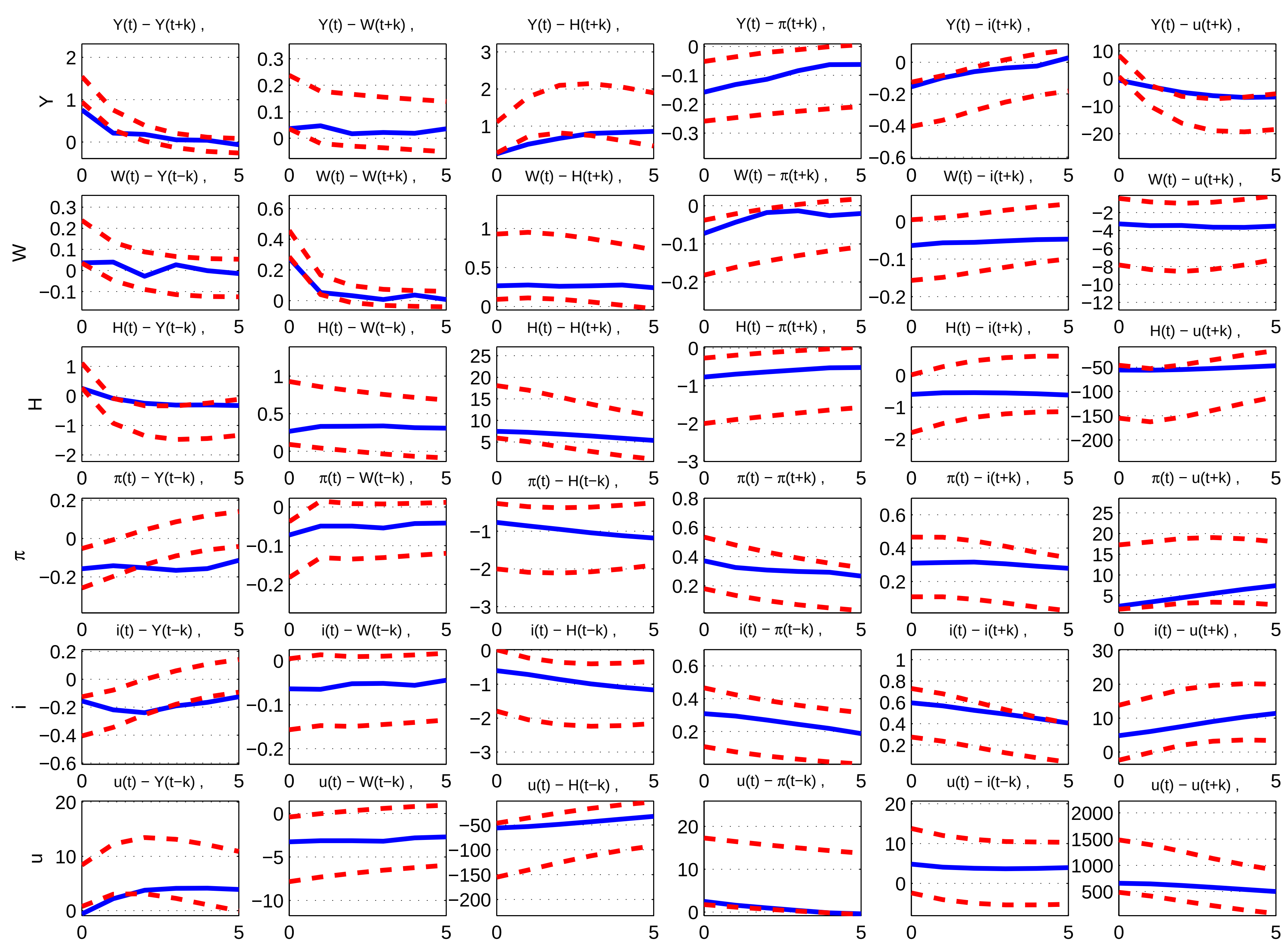
	Parameter	Prior Distribution	Posterior Max	Posterior Mean	Posterior 5%	Posterior 95%
λ	wage stickiness	Beta (0.75,0.1)	0.717	0.725	0.670	0.780
γ	wage indexation	Uniform (0,1)	0.816	0.815	0.713	0.918
$\bar{b}$	unem. flow value	Beta (0.5,0.1)	0.726	0.723	0.664	0.786
η	bargaining power	Beta (0.5,0.1)	0.907	0.907	0.862	0.941
ψ_z	utilization rate elast.	Beta (0.5,0.1)	0.695	0.700	0.623	0.775
η_k	cap. adj. cost elast.	Normal (4,1.5)	2.425	2.375	1.767	3.007
h	habit persistence	Beta (0.5,0.1)	0.727	0.708	0.652	0.757
λ^P	price stickiness	Beta (0.66,0.1)	0.848	0.846	0.803	0.884
γ^P	price indexation	Uniform (0,1)	0.000	0.018	0.001	0.051
ε^P	price markup	Normal (1.15,0.05)	1.405	1.408	1.362	1.454
r_π	inf. coef. in Taylor	Normal (1.7,0.3)	2.015	2.006	1.893	2.125
r_y	gap coef. in Taylor	Gamma (0.125,0.1)	0.333	0.332	0.264	0.405
ρ_s	smoothing in Taylor	Beta (0.75,0.1)	0.773	0.772	0.731	0.811
γ_a	growth rate	Uniform (1,1.5)	1.004	1.004	1.003	1.005

Table 4: Prior and posterior distribution of structural parameters, $\lambda = 0$

	Parameter	Prior Distribution	Posterior Max	Posterior Mean	Posterior 5%	Posterior 95%
$\bar{b}$	unem. flow value	Beta (0.5,0.1)	0.983	0.982	0.975	0.987
η	bargaining power	Beta (0.5,0.1)	0.616	0.589	0.451	0.726
ψ_z	utilization rate elast.	Beta (0.5,0.1)	0.861	0.852	0.783	0.911
η_k	cap. adj. cost elast.	Normal (4,1.5)	1.023	1.179	0.803	1.635
h	habit persistence	Beta (0.5,0.1)	0.801	0.803	0.760	0.840
λ^P	price stickiness	Beta (0.66,0.1)	0.574	0.575	0.512	0.630
γ^P	price indexation	Uniform (0,1)	0.000	0.037	0.003	0.090
ε^P	price markup	Normal (1.15,0.05)	1.347	1.351	1.298	1.407
r_π	inf. coef. in Taylor	Normal (1.7,0.3)	1.927	1.999	1.748	2.297
r_y	gap coef. in Taylor	Gamma (0.125,0.1)	0.013	0.019	0.003	0.043
ρ_s	smoothing in Taylor	Beta (0.75,0.1)	0.685	0.700	0.648	0.746
γ_a	growth rate	Uniform (1,1.5)	1.003	1.003	1.001	1.004

Table 6: Log Marginal Likelihood

Baseline Model	Flex Wage Model
-1191	-1234


Y(t) − Y(t+k) ,
Y(t) − W(t+k) ,
Y(t) − H(t+k) ,
Y(t) − π(t+k) ,
Y(t) − i(t+k) ,
Y(t) − u(t+k) ,
W(t) − Y(t−k) ,
W(t) − W(t+k) ,
W(t) − H(t+k) ,
W(t) − π(t+k) ,
W(t) − i(t+k) ,
W(t) − u(t+k) ,
H(t) − Y(t−k) ,
H(t) − W(t−k) ,
H(t) − H(t+k) ,
H(t) − π(t+k) ,
H(t) − i(t+k) ,
H(t) − u(t+k) ,
π(t) − Y(t−k) ,
π(t) − W(t−k) ,
π(t) − H(t−k) ,
π(t) − π(t+k) ,
π(t) − i(t+k) ,
π(t) − u(t+k) ,
i(t) − Y(t−k) ,
i(t) − W(t−k) ,
i(t) − H(t−k) ,
i(t) − π(t−k) ,
i(t) − i(t+k) ,
i(t) − u(t+k) ,
u(t) − Y(t−k) ,
u(t) − W(t−k) ,
u(t) − H(t−k) ,
u(t) − π(t−k) ,
u(t) − i(t−k) ,
u(t) − u(t+k) ,
Y
W
H
π
i
u

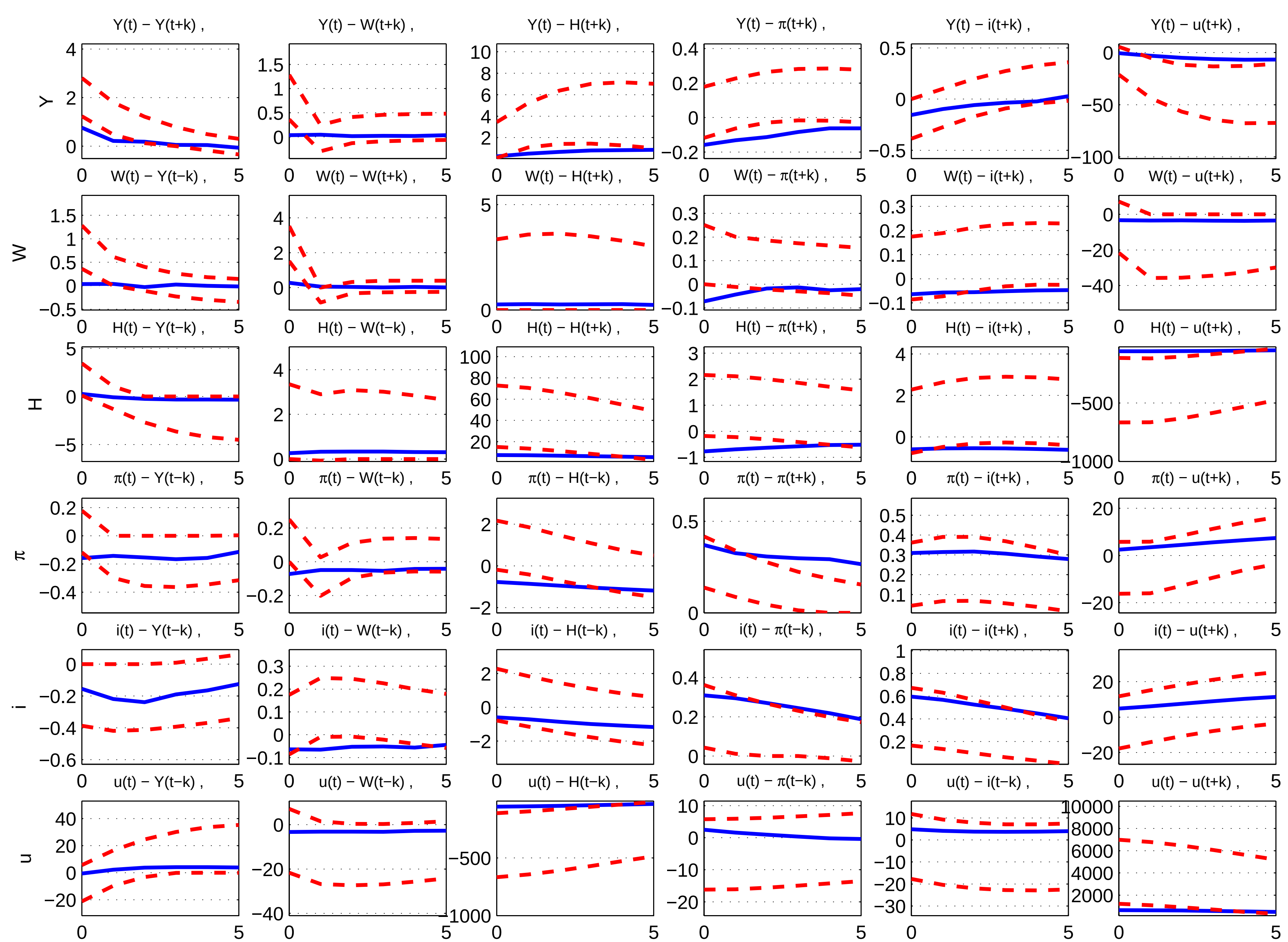
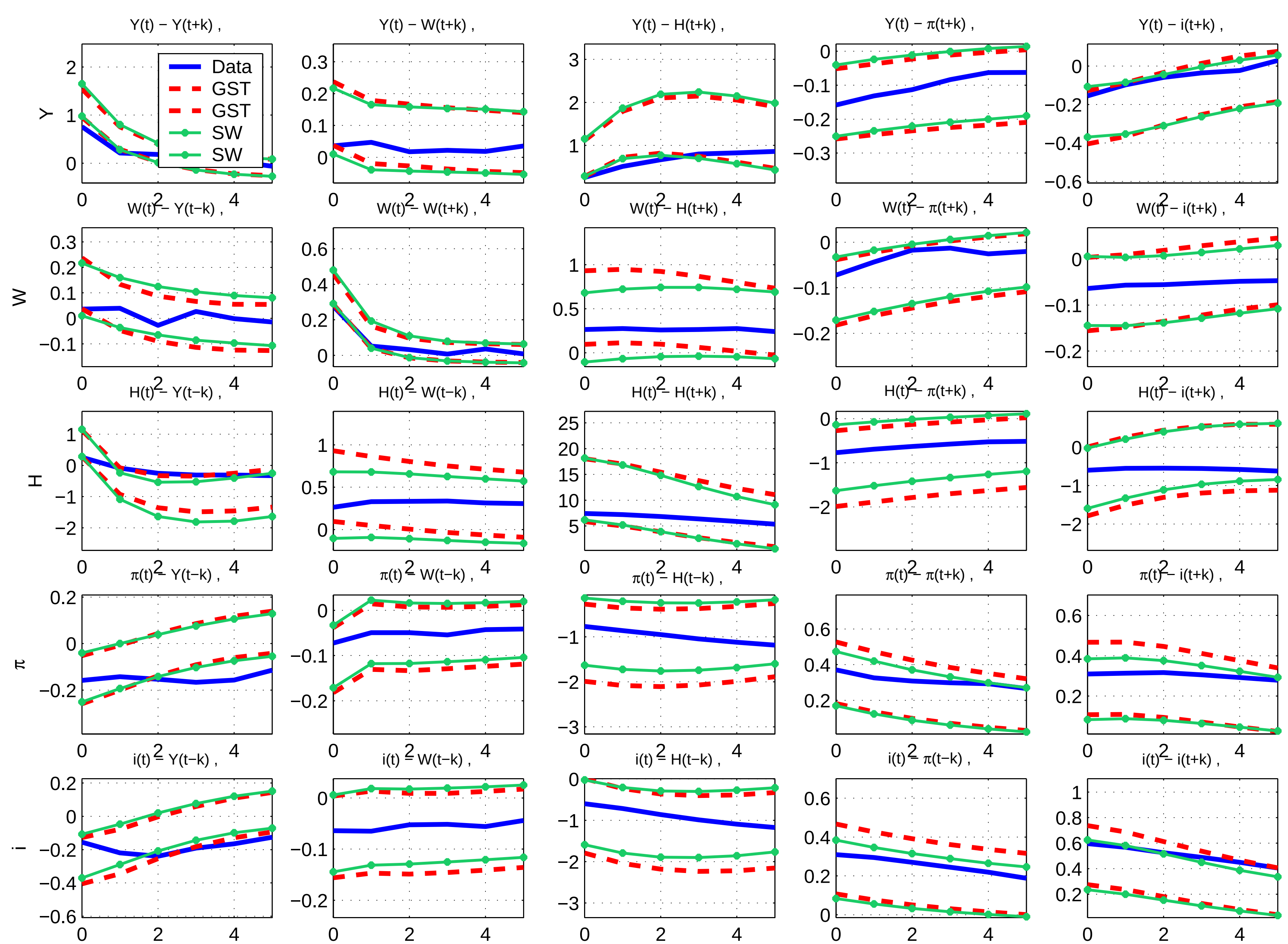

Y(t) − Y(t+k) ,
Y(t) − W(t+k) ,
Y(t) − H(t+k) ,
Y(t) − π(t+k) ,
Y(t) − i(t+k) ,
Y(t) − u(t+k) ,
W(t) − Y(t−k) ,
W(t) − W(t+k) ,
W(t) − H(t+k) ,
W(t) − π(t+k) ,
W(t) − i(t+k) ,
W(t) − u(t+k) ,
H(t) − Y(t−k) ,
H(t) − W(t−k) ,
H(t) − H(t+k) ,
H(t) − π(t+k) ,
H(t) − i(t+k) ,
H(t) − u(t+k) ,
π(t) − Y(t−k) ,
π(t) − W(t−k) ,
π(t) − H(t−k) ,
π(t) − π(t+k) ,
π(t) − i(t+k) ,
π(t) − u(t+k) ,
i(t) − Y(t−k) ,
i(t) − W(t−k) ,
i(t) − H(t−k) ,
i(t) − π(t−k) ,
i(t) − i(t+k) ,
i(t) − u(t+k) ,
u(t) − Y(t−k) ,
u(t) − W(t−k) ,
u(t) − H(t−k) ,
u(t) − π(t−k) ,
u(t) − i(t−k) ,
u(t) − u(t+k) ,
Y
W
H
π
i
u

Table 7: Variance decomposition for output ($\Delta \log y$) at different horizons (in percentage)

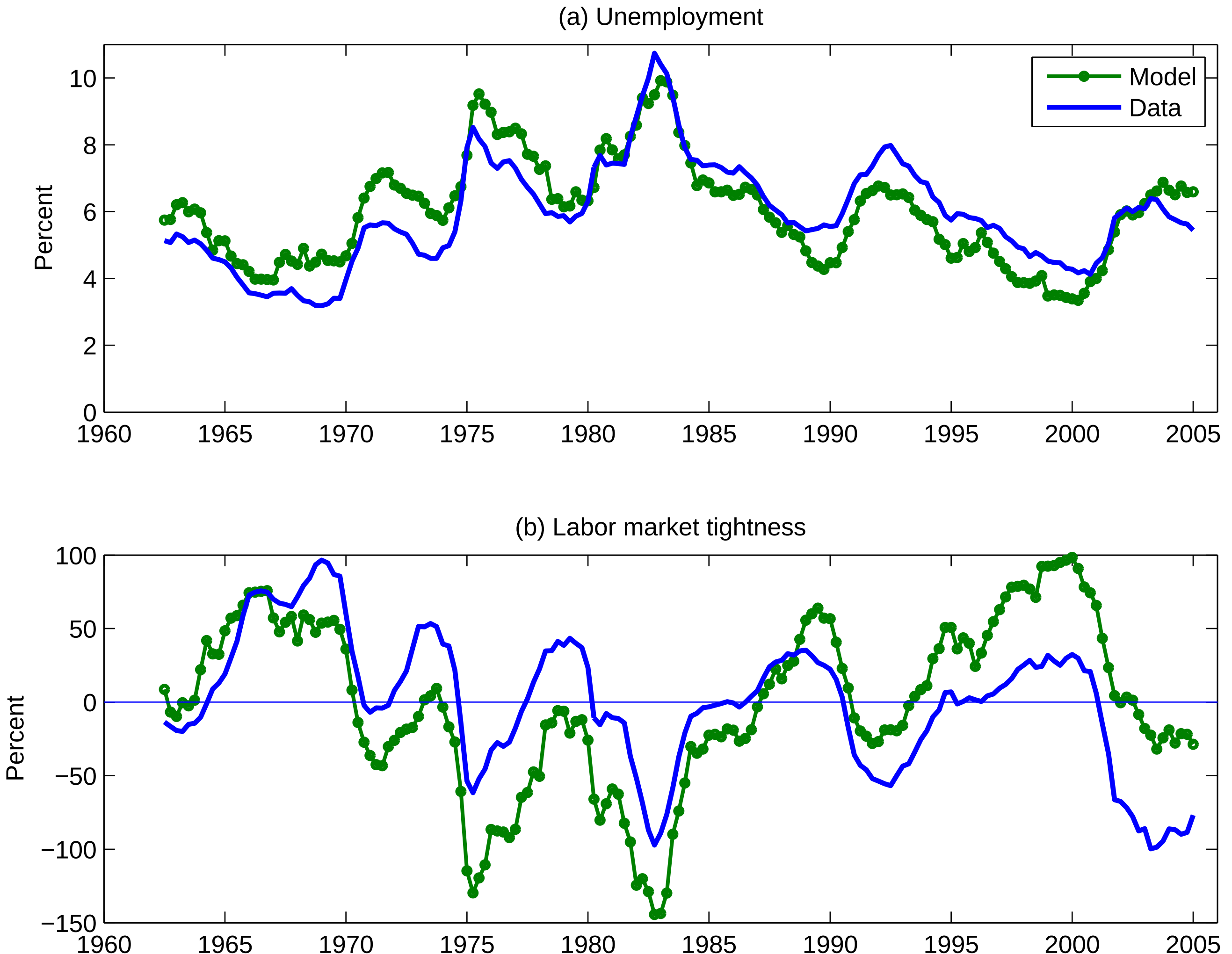
	Horizons			
Shocks	on impact	1 year	4 years	long run
a shock - technology	16.7	32.5	31.0	31.0
r shock - monetary	6.1	5.0	5.4	5.4
b shock - preferences	11.1	9.2	9.5	9.5
i shock - investment	54.8	41.9	42.4	42.4
g shock - government	9.4	8.7	8.2	8.2
p shock - price markup	1.9	2.5	3.2	3.2
w shock - bargaining power	0.0	0.2	0.3	0.3



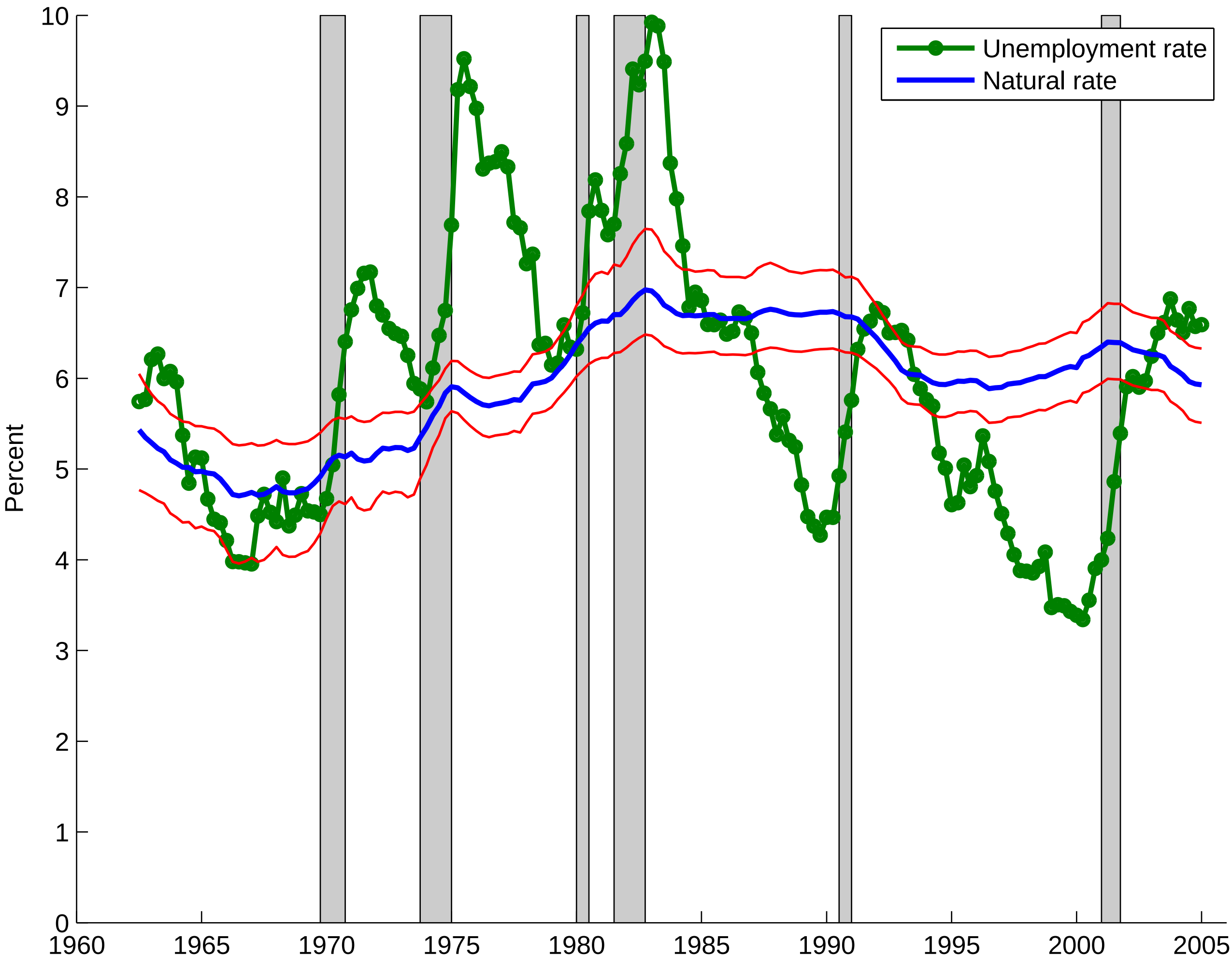
Y(t) − Y(t+k) ,
Y(t) − W(t+k) ,
Y(t) − H(t+k) ,
Y(t) − π(t+k) ,
Y(t) − i(t+k) ,
W(t) − Y(t−k) ,
W(t) − W(t+k) ,
W(t) − H(t+k) ,
W(t) − π(t+k) ,
W(t) − i(t+k) ,
H(t) − Y(t−k) ,
H(t) − W(t−k) ,
H(t) − H(t+k) ,
H(t) − π(t+k) ,
H(t) − i(t+k) ,
π(t) − Y(t−k) ,
π(t) − W(t−k) ,
π(t) − H(t−k) ,
π(t) − π(t+k) ,
π(t) − i(t+k) ,
i(t) − Y(t−k) ,
i(t) − W(t−k) ,
i(t) − H(t−k) ,
i(t) − π(t−k) ,
i(t) − i(t+k) ,
Y
W
H
π
i
Data
GST
GST
SW
SW

Conclusions

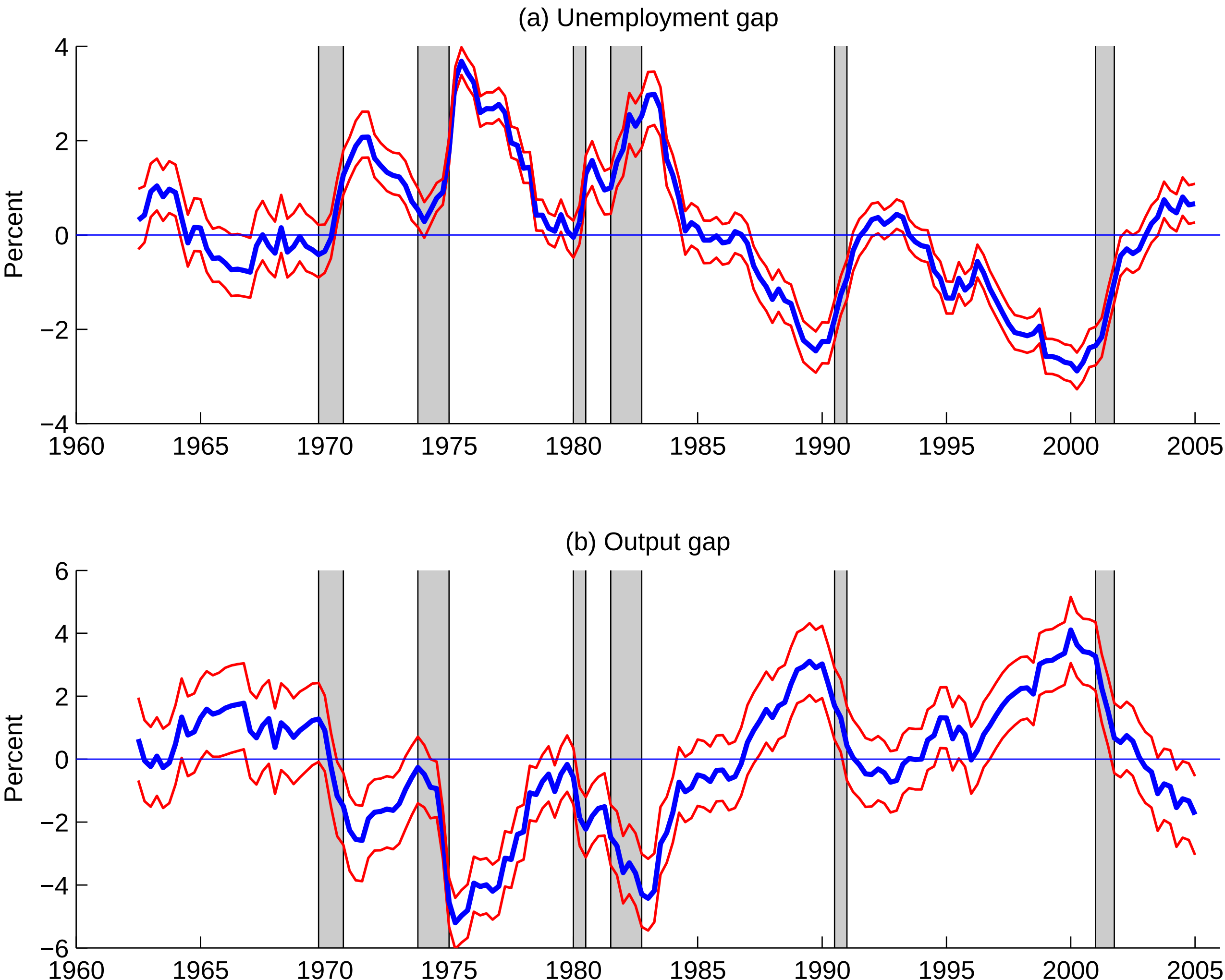
- Develop and estimate a quantitative macro model with
  - wage rigidity
  - efficient bargaining
- Wage rigidity improves the quantitative performance
- More work needed on identification of labor market parameters
  (more variables: unemployment, vacancies, employment?)
- Similar fit as existing quantitative macroeconomic models (CEE, 2005, SW, 2007)

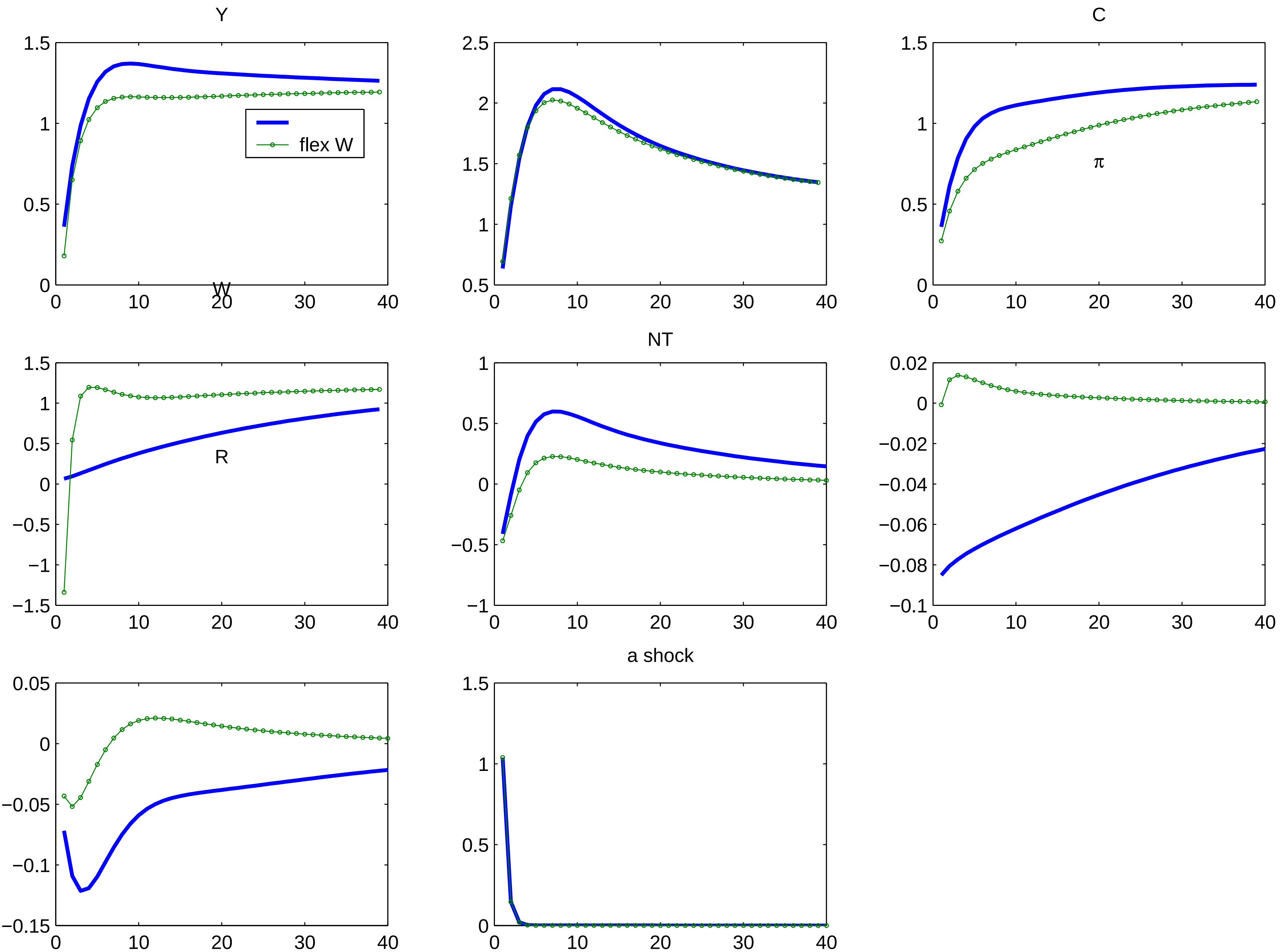


(a) Unemployment
Model
Data
Percent
0
2
4
6
8
10
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
(b) Labor market tightness
Percent
100
50
0
−50
−100
−150
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

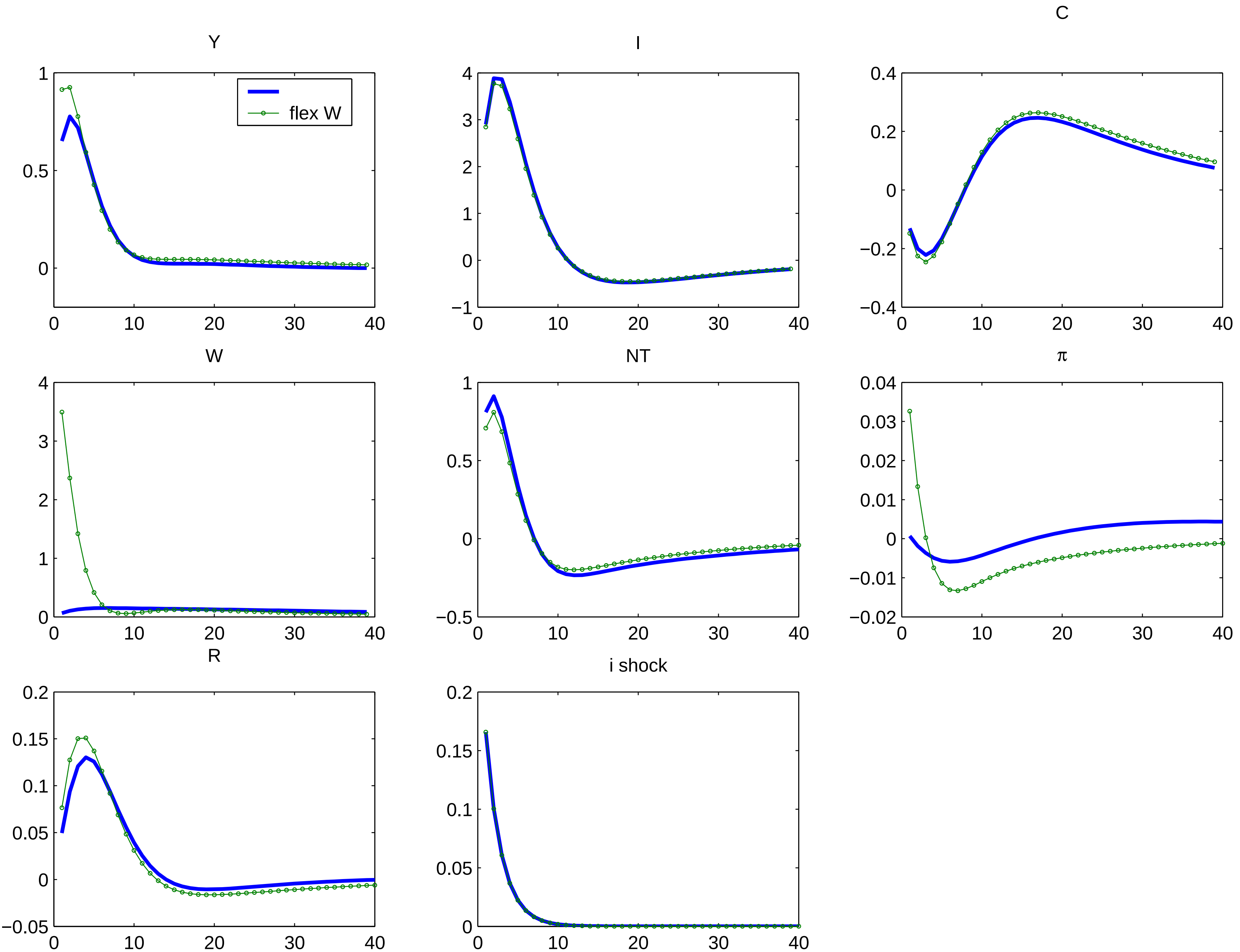


Unemployment rate
Natural rate
Percent
0
1
2
3
4
5
6
7
8
9
10
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005


(a) Unemployment gap
Percent
4
2
0
−2
−4
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
(b) Output gap
Percent
6
4
2
0
−2
−4
−6
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005



Y
flex W
C
π
W
NT
R
a shock



Y
flex W
I
C
W
NT
π
R
i shock

Table 3: Prior and posterior distribution of shock processes

	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ρ_a	Beta (0.5,0.2)	0.140	0.096	0.036	0.167
ρ_r	Beta (0.5,0.2)	0.207	0.179	0.096	0.266
ρ_b	Beta (0.5,0.2)	0.713	0.724	0.660	0.788
ρ_i	Beta (0.5,0.2)	0.605	0.599	0.520	0.675
ρ_p	Beta (0.5,0.2)	0.808	0.814	0.762	0.863
ρ_w	Beta (0.5,0.2)	0.264	0.261	0.199	0.322
ρ_g	Beta (0.5,0.2)	0.991	0.993	0.987	0.998
σ_a	IGamma (0.15,0.15)	1.039	1.025	0.968	1.084
σ_r	IGamma (0.15,0.15)	0.224	0.226	0.206	0.248
σ_b	IGamma (0.15,0.15)	0.362	0.334	0.251	0.436
σ_i	IGamma (0.15,0.15)	0.166	0.165	0.126	0.207
σ_p	IGamma (0.15,0.15)	0.062	0.06	0.046	0.076
σ_w	IGamma (0.15,0.15)	0.578	0.586	0.528	0.650
σ_g	IGamma (0.15,0.15)	0.357	0.358	0.327	0.391

Table 5: Prior and posterior distribution of shock processes, $\lambda = 0$					
	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ρ_a	Beta (0.5,0.2)	0.287	0.282	0.193	0.378
ρ_r	Beta (0.5,0.2)	0.249	0.250	0.162	0.334
ρ_b	Beta (0.5,0.2)	0.351	0.363	0.231	0.507
ρ_i	Beta (0.5,0.2)	0.865	0.852	0.812	0.891
ρ_p	Beta (0.5,0.2)	0.916	0.909	0.866	0.946
ρ_w	Beta (0.5,0.2)	0.984	0.984	0.977	0.990
ρ_g	Beta (0.5,0.2)	0.987	0.987	0.981	0.992
σ_a	IGamma (0.15,0.15)	1.071	1.083	0.992	1.188
σ_r	IGamma (0.15,0.15)	0.237	0.241	0.218	0.267
σ_b	IGamma (0.15,0.15)	0.686	0.738	0.436	1.146
σ_i	IGamma (0.15,0.15)	0.066	0.074	0.059	0.094
σ_p	IGamma (0.15,0.15)	0.093	0.093	0.076	0.115
σ_w	IGamma (0.15,0.15)	0.250	0.274	0.183	0.369
σ_g	IGamma (0.15,0.15)	0.352	0.355	0.324	0.389