Discretionary Monetary Policy in the Calvo Model

Willem Van Zandweghe[1]
Alexander L. Wolman[2]

National Bank of Belgium
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[1] Federal Reserve Bank of Kansas City
[2] Federal Reserve Bank of Richmond, currently on sabbatical at the Bundesbank

What are we doing?

Studying optimal monetary policy without commitment in a model where monop. competitive firms face an exogenous, constant probability of price adjustment

Why are we doing it?

- Calvo model = most popular framework for applied monetary policy analysis, yet studied almost exclusively by approx. around a zero inflation steady state.
- We approx. only by choosing fineness of grid (single state variable, can solve quite accurately), leads to richer trade-offs than in existing literature.
- If want to study discretionary eqlb (Markov) in less ad-hoc sticky price models (state-dep.), this is a natural starting point, for learning about how to approach problem, what kind of problems feasible to study.

Why (details)?

- Papers that take LQ approach (New Keynesian models) never find multiple equilibria.
- In two previous papers (w/King QJE 2004; w/Khan, King FRBWP 2001), we found multiple private-sector equilibria in a Markov-perfect discretionary equilibrium.
  – Taylor style models, with fixed duration prices.
  – Does that result carry over to the Calvo model?

What we find

- Mult.eq. result does not carry over to the Calvo model. Policy-induced complimentarity in pricing behavior weaker here, b/c current price-setters play a smaller role in determining the index of pre-set prices inherited by future monetary authority.
- Steady-state inflation rate can be relatively high under discretion ($>$5% if price adjustment probability$<$0.5)

Plan of talk

1. Related Literature

2. General Model, nesting Calvo and Taylor (2-pd prices)

3. State Variables: Real vs. Nominal

4. Background: Discretionary Equilibrium in Taylor Model

5. Discretionary Equilibrium in Calvo Model
   (a) definition
   (b) algorithm and computational method
   (c) properties (example)

6. Calvo vs. Taylor

7. Conclusion

Related Literature

- Discretionary Policy in Calvo-Style Models- LQ Approach
  - Clarida, Gali, Gertler; Woodford
  - Adam and Billi (zero bound)
  - many others

- Nonlinear Solutions for Discretion with Sticky Prices
  - (Khan), King and Wolman (2001) 2004, Siu (2008)
  - Dotsey and Hornstein (2008)
  - Yun (2005)
  - Anderson, Kim and Yun (2008), closest to this paper

Model: households

- Representative agent, infinitely lived, preferences,

$$\sum_{j=0}^{\infty} \beta^j \left(\ln(c_{t+j}) + \chi\left(1 - n_{t+j}\right)\right), \quad \beta \in (0,1)$$

  – efficiency condition:

$$\chi c_t = w_t,$$

  – consumption Euler equation:

$$1/c_t = \beta\left(1 + R_t\right)\left(1/\left(c_{t+1}\pi_{t+1}\right)\right),$$

- Consumption is Dixit-Stiglitz aggregate:.

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1$$

  – price index

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}.$$

  – individual demands

$$c_t\left(z\right) = \left(\frac{P_t\left(z\right)}{P_t}\right)^{-\varepsilon} c_t,$$

- Exogenous money demand

$$M_t = P_t c_t$$

Model: firms

- Technology:

$$y_t(z) = n_t(z),$$

- Current period profits:

$$d(X_t; P_t, c_t, w_t) = X_t \left(\frac{X_t}{P_t}\right)^{-\varepsilon} c_t - P_t w_t \left(\frac{X_t}{P_t}\right)^{-\varepsilon} c_t.$$

- Value
  - Calvo firms adjust price with probability η. :

$$V_t^{\text{C}} = \max_{X_t} \left\{ \sum_{j=0}^{\infty} Q_{t,t+j} (1-\eta)^j d(X_t; P_{t+j}, c_{t+j}, w_{t+j}) \right\};$$

  - Taylor firms adjust price every two periods:

$$V_t^{\text{JT}} = \max_{X_t} \{ d(X_t; P_t, c_t, w_t) + Q_{t,t+1} d(X_t; P_{t+1}, c_{t+1}, w_{t+1}) \},$$

  where

$$Q_{t,t+j} = \frac{1}{\prod_{k=1}^{j} R_{t-1+k}} = \beta^j \left(\frac{P_t}{P_{t+j}}\right) \left(\frac{c_t}{c_{t+j}}\right).$$

Model: optimal prices and price indices

- Calvo optimal price:

$$\frac{P_{0,t}}{P_t} = \frac{p_{0,t}}{\pi_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\sum_{j=0}^{\infty} (1-\eta)^j \beta^j \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon} w_{t+j}}{\sum_{j=0}^{\infty} (1-\eta)^j \beta^j \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon-1}}, \quad p_{0,t} \equiv \frac{P_{0,t}}{P_{t-1}}$$

- Taylor optimal price:

$$\frac{p_{0,t}^{\mathsf{JT}}}{p_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \cdot \frac{w_t + \beta \left(\frac{p_{t+1}}{p_t} p_{0,t}^{\mathsf{JT}}\right)^{\varepsilon} w_{t+1}}{1 + \beta \left(\frac{p_{t+1}}{p_t} p_{0,t}^{\mathsf{JT}}\right)^{\varepsilon-1}}, \quad p_{0,t}^{\mathsf{JT}} \equiv \frac{P_{0,t}}{P_{0,t-1}}, \quad p_t \equiv \frac{P_t}{P_{0,t-1}}$$

- Calvo price index:

$$P_t = \left(\sum_{j=0}^{\infty} \eta (1-\eta)^j P_{0,t-j}^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \Rightarrow \pi_t = \left(\eta p_{0,t}^{1-\varepsilon} + (1-\eta)\right)^{\frac{1}{1-\varepsilon}}$$

- Taylor price index:

$$P_t = \left(\frac{1}{2} P_{0,t}^{1-\varepsilon} + \frac{1}{2} P_{0,t-1}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \Rightarrow p_t = \left(\frac{1}{2} \left(p_{0,t}^{\mathsf{JT}}\right)^{1-\varepsilon} + \frac{1}{2}\right)^{\frac{1}{1-\varepsilon}}.$$

Model: monetary authority

- Choose nominal money supply, M_t. Without loss of generality, view m.a. as choosing M normalized by P_{t-1} (Calvo) or $P_{0,t-1}$ (Taylor)

- Optimizing m.a. without commitment $\Longleftrightarrow$ different policymaker each period.

Model: labor market clearing

- labor supplied by household = labor demanded by all firms:

$$n_t = \int_0^1 n_t(z)dz.$$

- Calvo model:

$$n_t = \sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j n_{j,t},$$

$n_{j,t}$ = labor input in t by firm that set price in $t-j$. Technology $\Rightarrow$

$$n_t = \sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j c_{j,t}, \text{ now use demand curves:}$$

$$\frac{n_t}{c_t} = \sum_{j=0}^{\infty} \eta \left(1-\eta\right)^j \left(\frac{P_{0,t-j}}{P_t}\right)^{-\varepsilon} \Rightarrow \frac{n_t}{c_t} = \eta \pi_t^{\varepsilon} \left(p_{0,t}^{-\varepsilon} + \left(\frac{1-\eta}{\eta}\right)\frac{n_{t-1}}{c_{t-1}}\right).$$

- Taylor model, analogously:

$$n_t = \frac{1}{2}\sum_{j=0}^{1} n_{j,t}$$

$$\frac{n_t}{c_t} = \frac{1}{2}p_t^{\varepsilon}\left(\left(p_{0,t}^{\text{JT}}\right)^{-\varepsilon} + 1\right)$$

Model: timing

1. Predet. prices ($P_{0,t-j}$, $j > 0$) known at beginning of pd.

2. Monetary authority chooses M (equivalently m).

3. Adjusting firms set prices, all other pd.-t variables determined.

Model: state variables

- Want study Markov Perfect Equilibrium (i.e. no reputational equilibria, no trigger strategies.)

- Need know what are payoff-relevant state variables

- Principle in nominal models: one less p-r.s.v. than # predet variables

- Calvo, 2 predet. ($P_{t-1}, n_{t-1}/c_{t-1}$) one state:
$$\Delta_t \equiv \frac{n_{t-1}}{c_{t-1}} \Rightarrow \Delta_{t+1} = \eta \pi_t^{\varepsilon} \left(p_{0,t}^{-\varepsilon} + \left(\frac{1-\eta}{\eta} \right) \Delta_t \right).$$

- Taylor, 1 predet., no states:
$$\frac{n_t}{c_t} = \frac{1}{2} p_t^{\varepsilon} \left(\left(p_{0,t}^{\mathsf{JT}} \right)^{-\varepsilon} + 1 \right).$$

Discretionary eqlb in Taylor model

(King and Wolman 2004)

- No state vars., so m_t constant in MPE
- Given m, eqlb is fixed point of pricing best response fn:

$$p_{0,t}^{\text{JT}} = \left(\frac{\varepsilon\chi}{\varepsilon - 1}\right) \cdot \left((1 - \theta_{t,t+1}) \cdot m_t + \theta_{t,t+1} \cdot m_{t+1} p_{0,t}^{\text{JT}}\right),$$

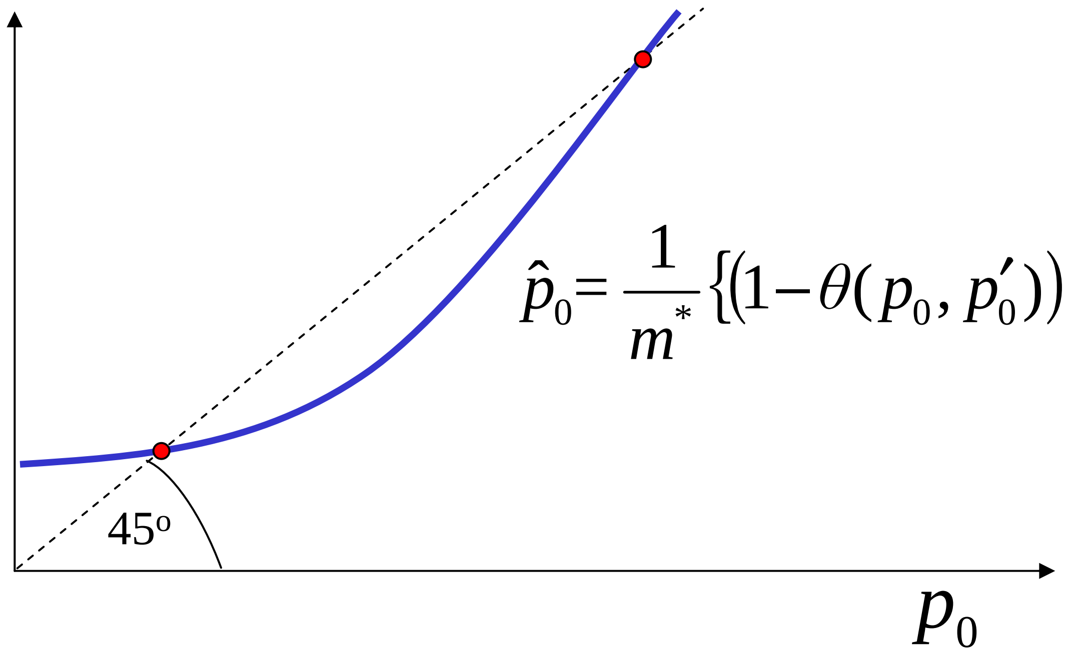
where

$$\theta_{t,t+1} \equiv \frac{\beta \left(\frac{p\left(p_{0,t+1}^{\text{JT}}\right)}{p\left(p_{0,t}^{\text{JT}}\right)} p_{0,t}^{\text{JT}}\right)^{\varepsilon-1}}{1 + \beta \left(\frac{p\left(p_{0,t+1}^{\text{JT}}\right)}{p\left(p_{0,t}^{\text{JT}}\right)} p_{0,t}^{\text{JT}}\right)^{\varepsilon-1}}$$

- Generically, # fixed points = zero or two
- Holds for arbitrary m, so holds in discretionary eqlb:
  – if discretionary eqlb exists, there are endogenous fluctuations (across 2 fixed points)

Two or zero point-in-time equilibria under arbitrary monetary policy

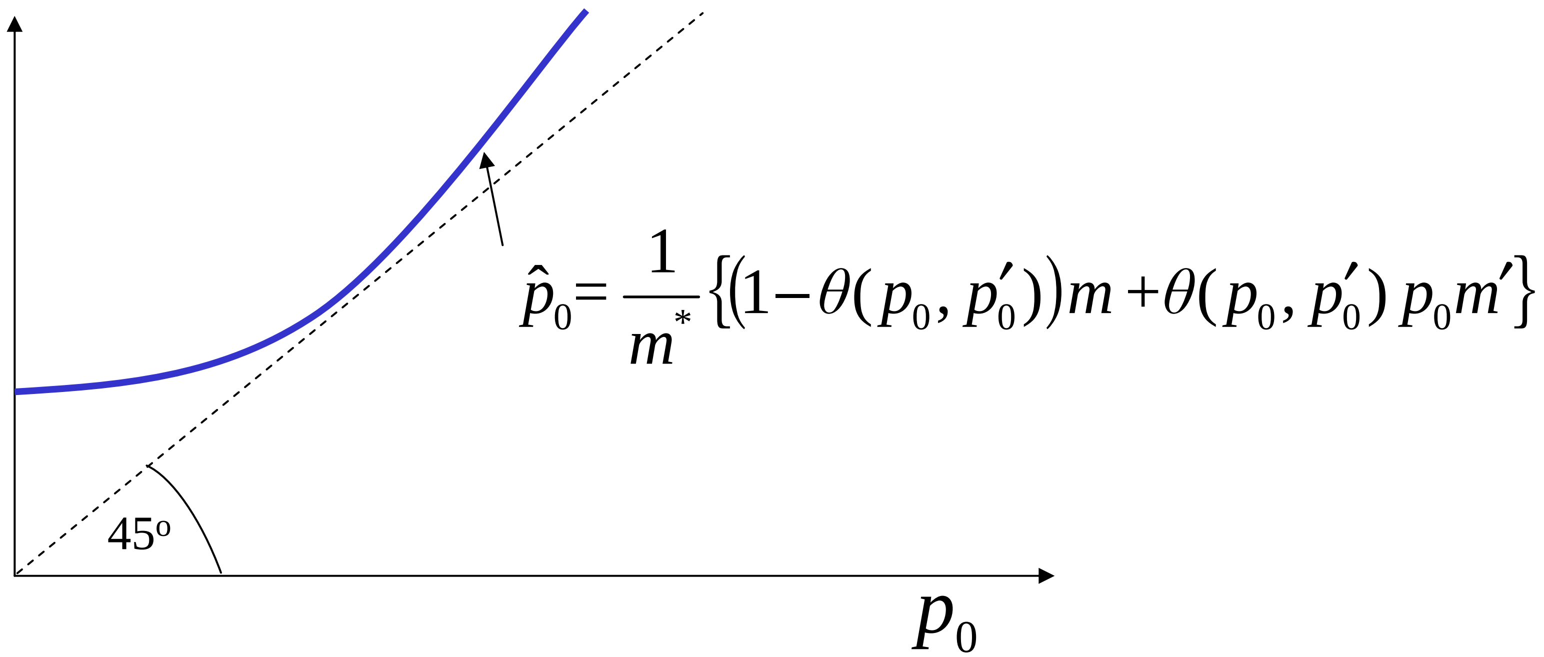
- Strong complementarity generated by combination of
  - any candidate policy makes future nominal M increase with current p_0 (firm's optimal price increasing in future nom. marginal cost, hence M), and
  - positive effect of prices set by other firms today on future inflation, and hence on optimal price for individual firm today.



$$\hat{p}_0 = \frac{1}{m^*}\left\{\left(1-\theta(p_0, p_0')\right)m + \theta(p_0, p_0')\, p_0 m'\right\}$$

Two or zero point-in-time equilibria under arbitrary monetary policy

- Strong complementarity generated by combination of
  - any candidate policy makes future nominal M increase with current p_0 (firm's optimal price is increasing in future nominal M), and
  - positive effect of prices set by other firms today on future inflation, and hence on optimal price for individual firm today.



Background to discretionary eqlb in Calvo model

- State variable. Δ, so $m = \Gamma(\Delta)$ in MPE
- Eqlb for arbitrary $m = \Gamma(\Delta)$, two functions $N(\Delta)$ and $D(\Delta)$:

$$N(\Delta) = \pi^{\varepsilon}\left[w + \beta(1-\eta)N(\Delta')\right],$$

$$D(\Delta) = \pi^{\varepsilon-1}\left[1 + \beta(1-\eta)D(\Delta')\right],$$

where

$$\begin{aligned} p_0 &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\cdot\frac{N(\Delta)}{D(\Delta)}, \\ \pi &= \left(\eta p_0^{1-\varepsilon} + (1-\eta)\right)^{1/(1-\varepsilon)} \\ \Delta' &= \pi^{\varepsilon}\left(\eta p_0^{-\varepsilon} + (1-\eta)\Delta\right), \\ c &= \Gamma(\Delta)/\pi, \\ w &= \chi c. \end{aligned}$$

(form of system suggests fixed point method of computing eqlb. for arbitrary policy $\Gamma(\Delta)$)

Discretionary eqlb defined in Calvo model

A policy function $\Gamma^*(\Delta)$ and a value function $v^*(\Delta)$ that satisfy

$$\begin{aligned} v^*(\Delta) &= \max_m \{\ln c + \chi(1-n) + \beta v(\Delta')\} \\ \Gamma^*(\Delta) &= \arg\max_m \{\ln c + \chi(1-n) + \beta v(\Delta')\} \end{aligned}$$

subject to the price index, market clearing, money demand, labor supply, definition of the state variable, and *optimal pricing*,

$$\begin{aligned} p_0 &= \left(\frac{\varepsilon}{\varepsilon-1}\right) \cdot \frac{\widetilde{N}}{\widetilde{D}} \\ \widetilde{N} &= \pi^{\varepsilon} \left\{\chi \frac{m}{\pi} + \beta(1-\eta) N(\Delta')\right\} \\ \widetilde{D} &= \pi^{\varepsilon-1} \{1 + \beta(1-\eta) D(\Delta')\}, \end{aligned}$$

for $v() = v^*()$, & fns. $N(\Delta)$ and $D(\Delta)$ solve stationary equilibrium for $\Gamma^*(\Delta)$.

Properties of discretionary equilibrium

- “Highest level”: m, welfare etc. as functions of state
  - For given value of state: objective function, policy tradeoffs
    - $*$ For given value of state and given value of m : private sector equilibrium through lens of best response function
- Now particular example: $\varepsilon = 10$, $\eta = 0.5$, $\chi = 4.5$, $\beta = 0.99$
- Caveats:
  - haven’t proved uniqueness
  - results slightly sensitive to computational parameters

Figure 1: Equilibrium as a function of the state

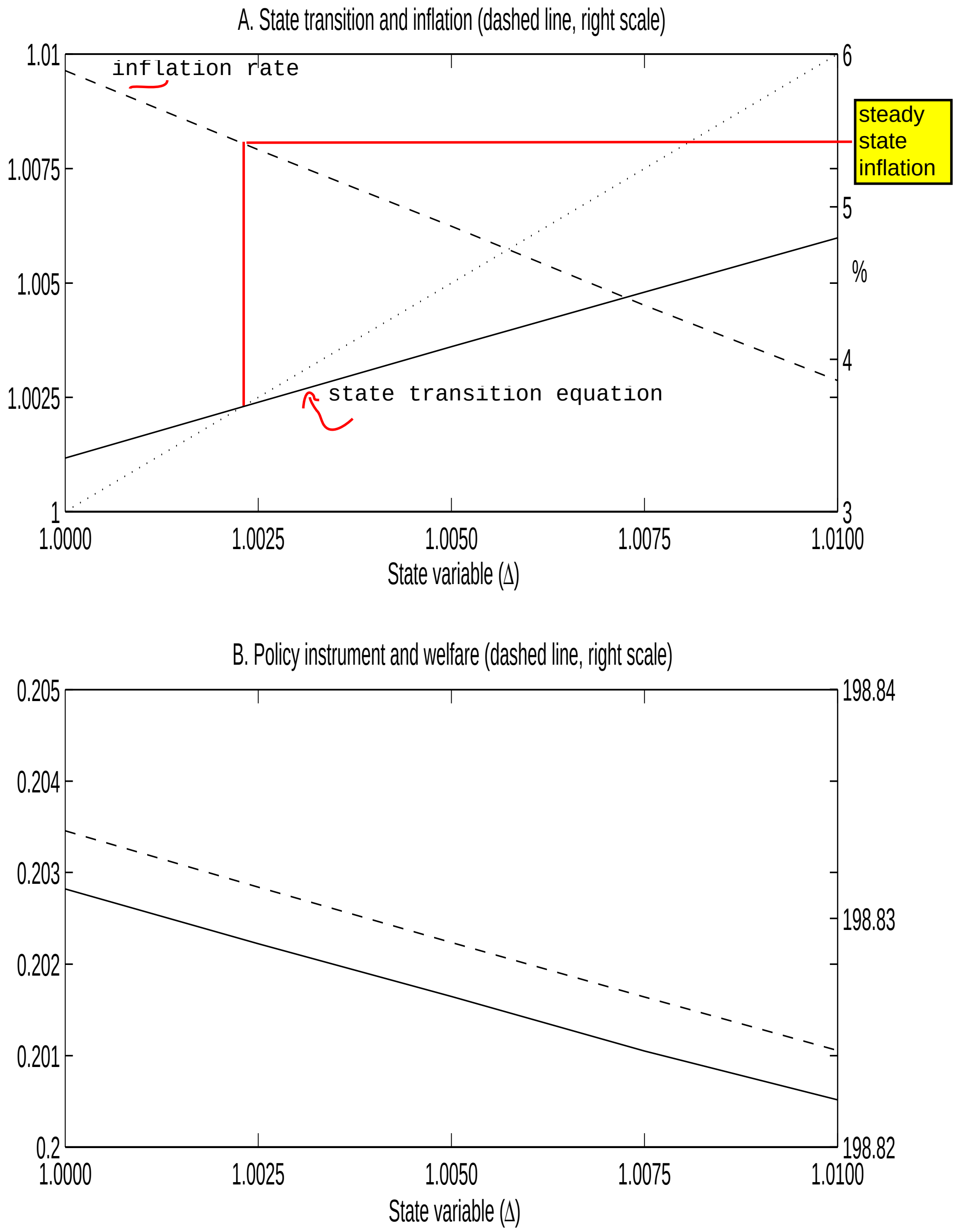


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Figure 3: Policymaker's objective function

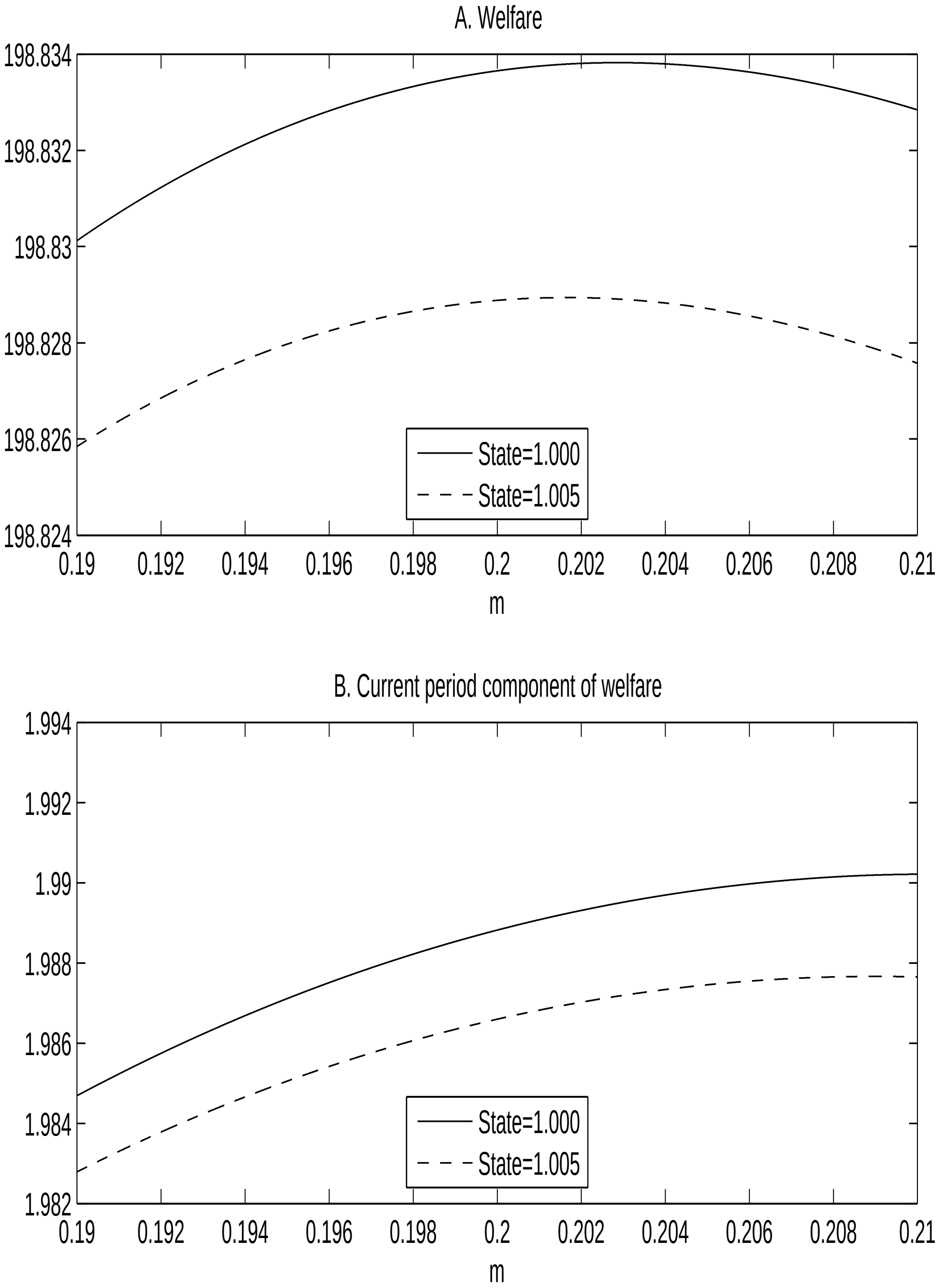


Figure 4: Distortions as functions of m

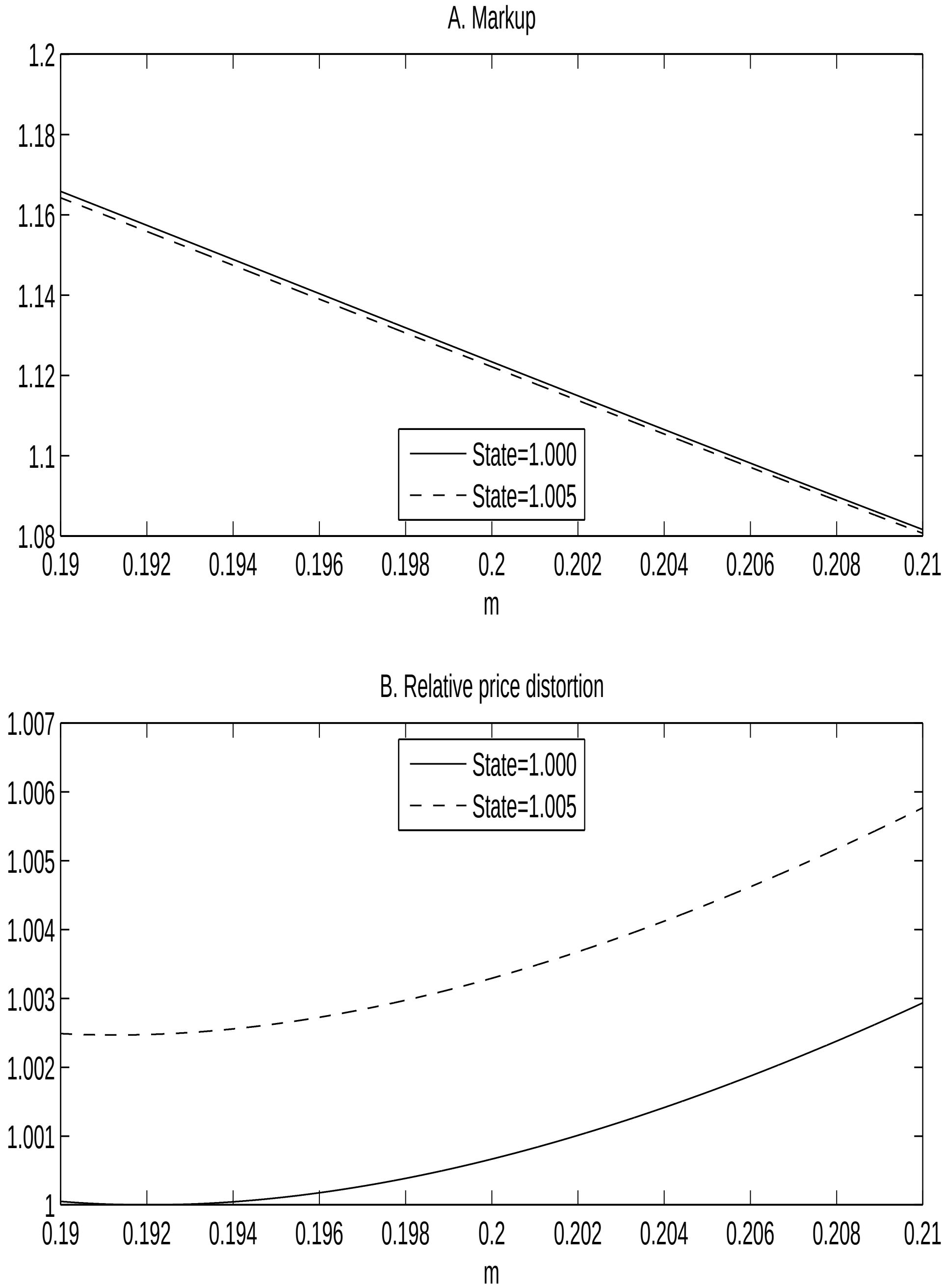
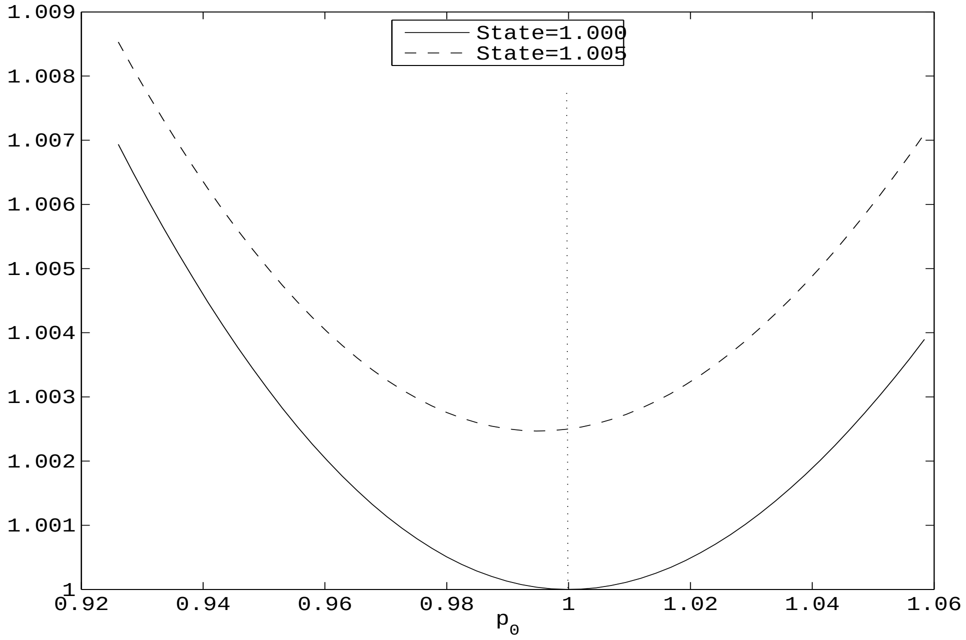


Figure 5: Relative price distortion as function of p_0



Properties of discretionary equilibrium

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Figure 6: Pricing best response function: State = 1.0025, $m = 0.2022$

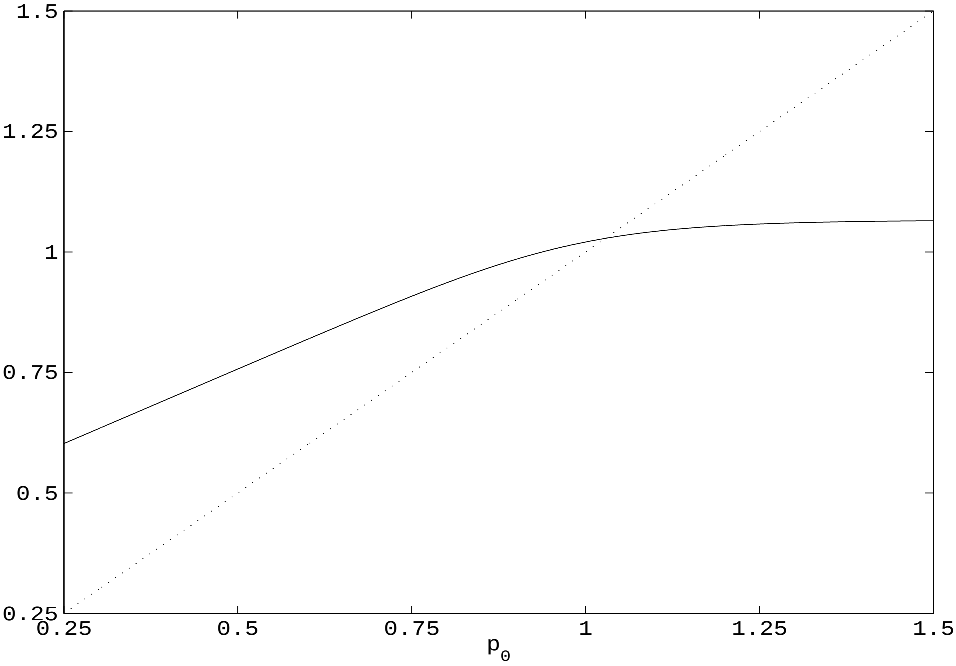
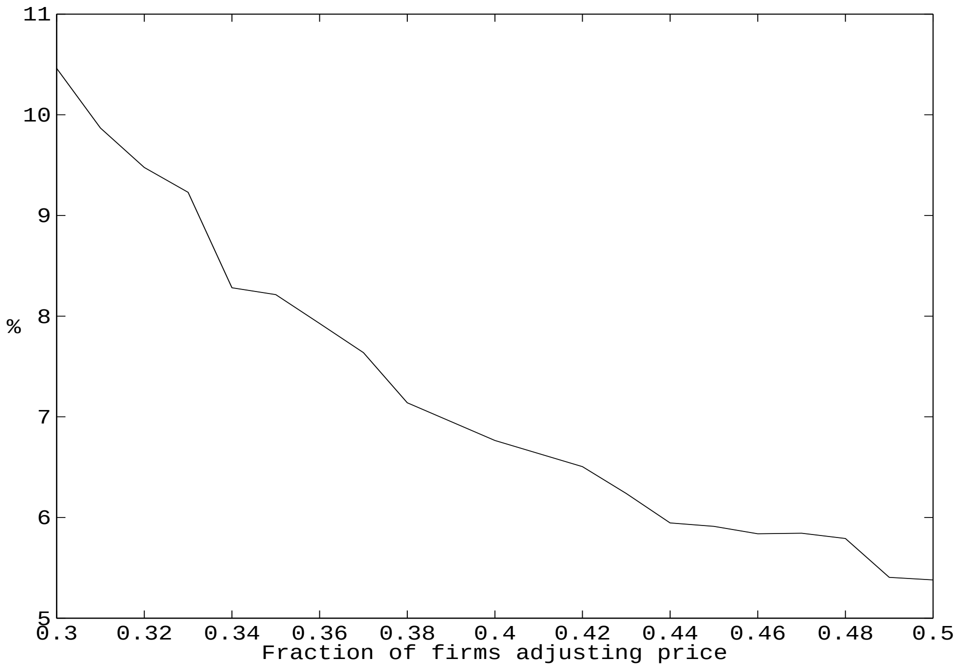


Figure 2: Steady state inflation rate



Comparing Calvo and Taylor

- Taylor: two fixed points, convex best response. If all adjusting firms raise prices very high,

  (a) nominal state responds one-for-one

  (b) individual firm optimally overcompensates, because tomorrow's m.a. raises M one for one. Don't want get stuck with low price and high costs.

- Calvo: unique fixed point to concave pricing best response function. If all adjusting firms raise price very high,

  (a) nominal state barely responds

  (b) individual firm barely responds, because tomorrow's m.a. barely responds.

Price Stickiness and inflation

- In Calvo model, can ask how steady state inflation varies with price stickiness under discretionary policy.
  - stickier prices $\Rightarrow$ higher marginal benefit to surprise inflation (reduce markup), needs to be offset with higher marginal cost (relative price distortion)
  - thus, inflation rate decreasing in fraction of firms adjusting prices

Price Stickiness and inflation

- In Calvo model, can ask how steady state inflation varies with price stickiness under discretionary policy.
  - stickier prices $\Rightarrow$ higher marginal benefit to surprise inflation (reduce markup), needs to be offset with higher marginal cost (relative price distortion)
  - thus, inflation rate decreasing in fraction of firms adjusting prices
- Raises the question of whether the degree of price stickiness should also respond to the inflation rate.

Conclusions

- Discretionary monetary policy in sp models involves interesting ec. mechanisms, incentives for monetary authority that are missing in local approximations around zero inflation:
  - accommodate nominal state in Taylor model $\Rightarrow$ multiple equilibria
  - counteract real state in Calvo model
- High equilibrium inflation rate points to
  - Perils of linearizing around zero inflation
  - Importance of endogenizing price stickiness