Forward guidance with preferences over safe assets
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Abstract

I develop a New Keynesian model with preferences over safe assets (POSA) calibrated using evidence on the wedge between household discount rates and market interest rates. POSA attenuate intertemporal consumption smoothing and thus the household’s responsiveness to future interest rates, the more so the more distant in time they are located, and imply a consumption wealth effect. Therefore, POSA substantially lower the macroeconomic effect of forward guidance policies. By contrast, POSA does not substantially change the effect of the standard shocks of Smets/Wouters (2007) type DSGE models. The results carry over to a model with Iacoviello (2005,2014)-type collateral constraints. Such constraints in themselves tend to strongly amplify the effect of forward guidance.

JEL codes: E52; E62; E32
Keywords: Forward guidance puzzle; preferences over safe assets; monetary policy; collateral constraints.

Author:
Ansgar Rannenberg, National Bank of Belgium – e-mail: ansgar.rannenberg@nbb.be

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1 Introduction

The goal of this paper is to address the so called “Forward Guidance Puzzle”. This term refers to the finding that in DSGE models with nominal rigidities, the GDP and inflationary effects of central bank announcements regarding the future path of the short-term interest rate tend to be very large and to explode in the length of the announced fixed interest rate period (see Del Negro et al. (2015a) and Carlstrom et al. (2015)). In this paper, I show that Preferences Over Safe Assets (POSA) considerably mitigate the puzzle.

The main motivation for POSA is liquidity preference. Krishnamurthy and Vissing-Jorgenson (2012) provide evidence that liquidity preference may extend to assets with a positive yield if they have money-like qualities. An alternative motivation interprets the saving behavior of rich households as evidence for “Capitalist Spirit” type preferences over all forms of wealth, for instance due to the prestige, power and security associated with wealth (see for instance Carroll (2000), Dynant et al. (2004), Francis (2009) and Kumhof et al. (2015) as well as their literature review). Finally, POSA allow to accommodate the finding that empirical estimates of individual discount rates typically exceed the market interest rates relevant for the inter-temporal choice studied to elicit them. A long literature has confirmed the finding of high individual discount rates (see Frederick et al. (2002)).

I parameterize the POSA by drawing on this empirical evidence. In line with the existing literature, I find that without POSA, GDP and inflation increase exponentially in the length of the announced interest rate peg. However, with POSA, the effect is substantially muted, and for reasonable calibrations becomes linear or even concave in the length of the interest rate peg. The reason for the attenuation is twofold. Firstly, POSA implies that the marginal utility of today’s consumption is governed not merely by the discounted expected marginal utility of future consumption but also by the marginal utility of safe assets. This attenuation of intertemporal consumption smoothing compounds the more distant in time a given future consumption choice is located, and implies a smaller effect of future interest rates on current consumption. Secondly, with declining marginal utility from real safe assets, the household aims to smooth her asset holdings, which implies a “wealth effect” of real
safe assets on consumption. This “wealth effect” further attenuates the expansionary effect of forward guidance, as the policy lowers real government debt and thus the real safe asset holdings of households by reducing the government budget deficit and increasing inflation. The associated increase in the marginal utility of government bonds motivates the household to save more. My results are robust to allowing for long-term government debt as in Krause and Moyen (2016). Furthermore, I show that the impact of POSA on the effect of the standard shocks typically used in the DSGE literature is small.

My findings thus differ from Campbell et al. (2016), who also develop a model with POSA, but report that extending an interest rate peg ultimately leads to an explosive increase of GDP and inflation. I obtain a different result because I parameterize POSA based on empirical evidence on household discount rates and because I drop Campbell et al. (2016)’s assumption of constant government debt, thus allowing for the aforementioned “wealth effect”.

Finally, I examine the effect of forward guidance in a model with collateral-constrained borrower households and firms along the lines of Iacoviello (2005, 2014) and Iacoviello and Neri (2010), which to my knowledge has not been done so far. This environment is relevant because credit market frictions were arguably responsible for the length and depth of the Great Recession and the slow ensuing recovery (see Iacoviello (2014), Del Negro et al. (2015b), Cai et al. (2018)), thus giving rise to the need for forward guidance policies in the first place. I show that the presence of binding collateral constraints considerably amplifies the effect of forward guidance. The reason is that the income share of the constrained agents increases in response to the policy. I find that POSA considerably attenuate the effect of the policy even in this environment.

My paper is -as far I am aware- the first to show how POSA can solve the "Forward Guidance Puzzle". More recently, Michaillat and Saez (2018) show that under certain parameter restrictions -i.e. utility discount rates of 20-40% (annualized

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1Most of the results as well as my approach to parameterize POSA based on evidence on the wedge between individual discount rates and market interest rates were already present in an earlier version of the paper circulated as “The effect of fiscal policy and forward guidance with preferences over wealth”, which was available online as of May 2017, see http://programme.exordo.com/iea2017/delegates/presentation/52/
percentage rate) or higher, and very sticky prices-, a stylized New Keynesian model with POSA has a determinate zero lower bound steady state. Based on a qualitative dynamic analysis, they argue, inter alia, that there is no "Forward Guidance Puzzle" around this steady state if monetary policy announces the accommodation of an expected demand shock. As I discuss in Section 2.6, my findings regarding the attenuative effects of POSA do not depend on the parameter restrictions they assume. Furthermore, I allow for the "wealth effect", which they eliminate by assuming real safe assets are in zero net supply, and investigate the role of long-term government debt and collateral constraints. Finally, their results are difficult to compare to mine or the literature because they do not compute the (numerical) differential effect of the announced monetary accommodation.

Other important contributions to the literature on the "Forward Guidance Puzzle" include Del Negro et al. (2015a), who address the puzzle by assuming a Blanchard-Yaari type perpetual youth structure to limit the horizon of unconstrained households. However, in order to achieve a quantitatively important attenuation of the effect of forward guidance, they also have to assume counterfactually high death rates (see McKay et al. 2017).

Finally, there is an ongoing debate on whether heterogenous agents incomplete market models are subject to the "Forward Guidance Puzzle". McKay et al. (2016) develop a New Keynesian incomplete markets model where agents respond less to future real interest rates for fear of running up against their borrowing constraint in the future. However, Werning (2015) argues that this lower sensitivity to future interest rate changes is not a consequence of the incomplete market assumption per se, but relates to other, auxiliary assumptions of McKay et al. (2016), which render idiosyncratic income risk procyclical and liquidity countercyclical\footnote{Werning (2015) argues that with either countercyclical income risk or procyclical liquidity, consumption becomes even more sensitive to current and future interest rates than in a representative agent model.} Werning (2015) argues that with either countercyclical income risk or procyclical liquidity, consumption becomes even more sensitive to current and future interest rates than in a representative agent model.

The remainder of the paper is organized as follows. In Section 2 I develop a
simple New Keynesian model where households have POSA. Section 3 describes how I simulate the forward guidance policy. I maintain this setup throughout the paper. Section 4 discusses the effect of forward guidance in the simple model and a number of variants thereof. It also examines the impact of POSA on the effect of the standard shocks typically assumed in estimated DSGE models. Section 5 summarizes the model with Iacoviello (2005, 2014) type collateral constrained households and firms and the effect of forward guidance in this framework, while details are relegated to Appendix E.

2 The simple model

The model consists of households who have preferences over safe assets (POSA), retailers operating under monopolistic competition and sticky prices, a government and a central bank.

2.1 Households

The representative household derives utility from consumption $C_t$ and her holdings of real government bonds $b_{G,t}$, and disutility from supplying labor $N_t$. Her objective is given by

$$
\sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi_N}{1+\eta} N_{t+i}^{1+\eta} + \frac{\chi_b}{1-\sigma_b} (b_{G,t+i})^{1-\sigma_b} \right]
$$

with $\chi_N, \sigma, \eta > 0$ and $\chi_b, \sigma_b \geq 0$. One motivation for POSA is liquidity preference. Krishnamurthy and Vissing-Jorgenson (2012) argue that liquidity preference may extend to assets with a positive yield if they have money-like qualities, and provide supporting evidence in the form of a positive relationship between the supply of US government debt and the differential between its yields and the yield of other debt-securities. Fisher (2015) also adopts this argument. Another motivation pertains to rich households, who may have “Capitalist Spirit” type preferences over all forms of wealth, meaning that they derive utility from the prestige, power and security associated with wealth. Several authors have argued that such preferences are necessary
to replicate the saving behavior of rich households in US data, namely the positive marginal propensity to save out permanent income changes (see Dynant et al. 2004 and Kumhof et al. 2014), and the level of wealth held by rich households relative to their disposable income (Carroll 2000 and Francis 2009).

The household derives income from supplying labor and her ownership of government bonds and physical capital $K_t$. Her budget constraint is thus given by

$$b_{G,t} + (1 + \tau_C) C_t + I_t = \frac{R_{t-1}}{\Pi_t} b_{G,t-1} + (1 - \tau_w) w_t N_t - T_t + \Xi_t + (1 - \tau_K) r_{K,t} + \tau_K \delta) K_{t-1}$$

(2)

where $I_t$, $\Pi_t$, $R_t$, $w_t$, $\Xi_t$, $r_{K,t}$, $\delta$, $T_t$, $\tau_C$, $\tau_w$ and $\tau_K$ denote investment, the inflation rate, the nominal interest rate on government bonds (which is also the policy rate set by the central bank), the real wage, real profits of firms, the real capital rental, the rate of depreciation, and lump sum, consumption, labor and profit taxes, respectively. I assume that physical capital depreciation can be deducted from taxable profit income. Throughout the paper I adopt the convention that only period $t$ decision variables are indexed with $t$, implying that $b_{G,t}$ denotes the stock of safe bonds at the end of period $t$. Physical capital accumulation is subject to quadratic investment adjustment costs and is given by

$$K_t = (1 - \delta) K_{t-1} + I_t \left(1 - \frac{\xi_t}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$$

(3)

with $\xi_t > 0$. The first order conditions (FOCs) of the household with respect to government bonds, consumption, labor, physical capital and investment are given by
\[
\Lambda_t = \beta E \left\{ \frac{R_t}{\Pi_{t+1}} \Lambda_{t+1} \right\} + \chi_b (b_{G,t})^{-\sigma_b} \tag{4}
\]

\[
\Lambda_t (1 + \tau_C) = C_t^{1-\sigma} \tag{5}
\]

\[
(1 - \tau_w) w_t \Lambda_t = \chi_N N_t^\eta \tag{6}
\]

\[
Q_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left( (1 - \tau_K) r_{K,t+1} + \tau_K \delta + (1 - \delta) Q_{t+1} \right) \right\} \tag{7}
\]

\[
1 = Q_t \left[ 1 - \xi_I \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \xi_I \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{I_{t+1}}{I_t} \right)^2 \xi_I \left( \frac{I_{t+1}}{I_t} - 1 \right) Q_{t+1} \right\} \tag{8}
\]

where \( \Lambda_t \) and denotes the marginal utility of consumption. If \( \chi_b > 0 \), \( \chi_b (b_{G,t})^{-\sigma_b} \) represents an extra marginal benefit from saving over and above the utility associated with the future consumption possibility saving entails (represented by \( \beta E_t \left\{ \frac{R_t}{\Pi_{t+1}} \Lambda_{t+1} \right\} \)). This extra benefit has three (related) consequences. Firstly, \( \Lambda_t \) is now less than proportional to the marginal utility of \( t + 1 \) consumption \( \Lambda_{t+1} \), since it also depends on marginal utility of holding government bonds. Hence there will be less intertemporal consumption smoothing. Furthermore, as \( \Lambda_{t+1} \) is no longer proportional to \( \Lambda_{t+2} \) either, and so on and so forth, the attenuation of intertemporal consumption smoothing compounds as the more distant in time the respective future consumption choice is located. Secondly, the extra benefit of holding government bonds implies that

\[
R_t \leq \frac{1}{E_t \left\{ \frac{\beta \Lambda_{t+1}}{\Lambda_{t+1} \Pi_{t+1}} \right\}} \equiv DIS_t \tag{9}
\]

i.e. the nominal interest rate may be smaller than the households individual discount rate \( DIS_t \). A third consequence of POSA arises under the assumption of declining marginal utility from government bonds (\( \sigma_b > 0 \)), as the household in that case aims to smooth her asset holdings.

The consequences of POSA for intertemporal choice may be further illustrated by linearizing equation (4), which yields
 where a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with the exception of $\hat{b}_{G,t}$, which is expressed as a percentage of steady state GDP. $\theta = \beta R$ for each household discount factor and the real interest rate. Below I will refer to $\theta$ as the discounting wedge. $\theta$ represents the net weight the household attaches to the $t + 1$ marginal utility of consumption. Assuming POSA (i.e. $\chi_b > 0$) implies that $\theta < 1$, and thus less consumption smoothing (as mentioned above). The potential consequence of this attenuation of consumption smoothing for forward guidance can be sensed by iterating equation (10) ad infinitum:

$$\hat{\Lambda}_t = E_t \left\{ \sum_{i=0}^{\infty} \theta^i \left[ \theta \left( \hat{R}_{t+i} - \hat{\Pi}_{t+1+i} \right) - (1 - \theta)\sigma_b Y b_{G,t} \right] \right\}$$

In the absence of POSA ($\theta = 1 \iff \chi_b = 0$), a cut of the nominal interest rate taking place in some future quarter $t + i$, and lasting for one quarter (i.e. a cut of $\hat{R}_{t+i}$), has the same effect on the period $t$ marginal utility of consumption (and thus consumption itself) as a cut of $\hat{R}_t$. By contrast, with POSA ($\theta < 1 \iff \chi_b > 0$), the effect of an interest rate cut in period $t + i$ equals $\theta^{i+1}$, which declines in $i$ and converges to zero as $i$ approaches infinity.

Finally, with declining marginal utility from safe assets ($\theta < 1$ and $\sigma_b > 0$), a decline in the household’s government bond holdings will tend to lower her consumption, as the household attempts to smooth not just consumption but also her real safe asset holdings.

### 2.2 Retailers

There is a continuum of monopolistically competitive firms owned by households, each producing a variety $j$ from a CES basket of goods. They set prices subject to Rotemberg (1983) type quadratic price adjustment costs, which are given by
\[ AC(j)_{P,t} = Y_t(j) \frac{\xi_P}{2} \left( \frac{P(j)_t}{P(j)_{t-1}} \frac{1}{\Pi} - 1 \right)^2 \] (12)

where \( \xi_P > 0 \) denotes the adjustment cost curvature. Retailers employ labor as well as capital using a Cobb Douglas technology:

\[ Y(j)_t = A_t N(j)^{1-\alpha_K} K(j)^{\alpha_K} \] (13)

I assume that physical capital is freely tradeable across firms, implying that each firm faces identical marginal costs. The first order conditions with respect to labor and physical capital thus imply

\[ w_t = m_{c_t} (1 - \alpha_K) \frac{Y_t}{N_t} \] (14)

\[ r_{K,t} = m_{c_t} \alpha_K \frac{Y_t}{K_{t-1}} \] (15)

where \( m_{c_t} \) denote real marginal costs of production.

Finally, optimal price setting implies that up to first order, inflation evolves according to the familiar New Keynesian Phillips curve

\[ \hat{\Pi}_t = \kappa_\pi m_{c_t} + \beta E_t \hat{\Pi}_{t+1} \] (16)

where \( \kappa_\pi \) is a constant depending inversely on the degree of adjustment cost curvature \( \xi_P \).

### 2.3 Government

The government levies taxes on households and buys goods from retailers. Its budget constraint is given by

\[ b_{G,t} = R_{t-1} b_{G,t-1} + G - (T_t + \tau_w w_t N_t + \tau_C C_t + \tau_K (r_{K,t} - \delta) K_{t-1})) \] (17)
where $G$ denotes government expenditure on goods and services. Lump sum taxes $T_t$ follow the fiscal rule

$$
\hat{T}_t = \tau_h \hat{b}_{G,t-1}
$$

(18)

where the hat above lump sum taxes and debt denotes the respective deviation from its steady state as a percentage of steady state GDP, with $\tau_h > 0$. Government expenditure remains constant in all simulations. The attenuation of the macroeconomic effect of forward guidance associated with POSA to be discussed in Section 4 is robust to specifying the fiscal rule in terms of distortionary labor or consumption taxes instead of lump sum taxes (results are available upon request).

Monetary policy is described by the following interest feedback rule:

$$
\hat{R}_t = (1 - d_{P,t}) \left( (1 - \phi_i) \left( \phi_x \hat{\Pi}_t + \frac{\phi_y}{4} \left( \hat{Y}_t - \hat{Y}^p_t \right) \right) + \phi_i \hat{R}_{t-1} \right) + d_{P,t} \hat{R}_{p,t}
$$

(19)

where $\hat{Y}^p_t$ denotes flexible price output, and $d_{P,t}$ denotes a dummy variable which takes a value of one if the Central Bank decides to switch off the interest feedback rule and instead peg the interest rate to $\hat{R}_{p,t}$, and equals 0 otherwise.

### 2.4 Equilibrium

The labor market clears. GDP is the sum of household and government consumption, investment and the cost of price adjustment:

$$
Y_t = C_t + G_t + I_t + Y_t \frac{\xi_t}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2
$$

(20)
2.5 Linearized equations

Linearizing equations (4) to (7) and (13) to (20) yields

\[ \hat{Y}_t = C \hat{Y}_t + I \hat{Y}_t \]
\[ \hat{C}_t = -\theta \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{H}_{t+1} \right] + \theta E_t \hat{C}_{t+1} + (1 - \theta) \frac{\sigma_h}{\sigma_b} \hat{b}_{G,t} \]
\[ \hat{Q}_t = \sigma \left( \hat{C}_t - E_t \hat{C}_{t+1} \right) + \beta (1 - \tau_K) r_K E_t \hat{r}_{K,t+1} + \beta (1 - \delta) E_t \hat{Q}_{t+1} \]
\[ \hat{I}_t = \frac{1}{1 + \beta} \left( \hat{I}_{t-1} + \beta E_t \hat{I}_{t+1} + \hat{Q}_t \right) \]
\[ \hat{\Pi}_t = \kappa \pi \hat{m}_c + \beta E_t \hat{\Pi}_{t+1} \]
\[ \hat{w}_t = \sigma \left( \hat{C}_t + \eta \hat{N}_t \right) \]
\[ \hat{Y}_t = (1 - \alpha_K) \hat{N}_t + \alpha_K \hat{K}_{t-1} \]
\[ \hat{K}_t = \delta \hat{I}_t + (1 - \delta) \hat{K}_{t-1} \]
\[ \hat{b}_{G,t} = \frac{R}{\Pi} \hat{G}_{t-1} + \frac{b_{G}}{\Pi} \left( \hat{R}_{t-1} - \Pi_t \right) - \left( \hat{T}_t + \tau_c \frac{w}{\Pi} \left( \hat{w}_t + \hat{N}_t \right) \right) + \tau_c C \frac{\hat{C}_t}{\Pi} + \tau_k \left( \frac{K}{\Pi} \left( (r_K - \delta) \hat{K}_{t-1} + r_K \hat{r}_{K,t} \right) \right) \]
\[ \hat{R}_t = \left( 1 - \phi_\pi \right) \left( 1 - \phi_\gamma \right) \left( \phi_\pi \hat{H}_t + \frac{\phi_\gamma}{4} \left( \hat{Y}_t - \hat{Y}_t^p \right) \right) + \phi_i \hat{R}_{t-1} + d_{p,t} \hat{R}_{p,t} \]
\[ \hat{T}_t = \tau_c \hat{b}_{G,t-1} \]

where a hat above a variable denotes the deviation of that variable from its own steady state value, with the exception of \( \hat{T}_t, \hat{G}_t \) and \( \hat{b}_{G,t} \), which are expressed as a percentage of steady state GDP. Note that without POSA (\( \theta = 1 \)), the model reduces to the familiar New Keynesian model.

2.6 Calibration

Table 1 displays the calibration. I assume log preferences over consumption (\( \sigma = 1 \)), and a Frisch elasticity of labor supply of 4 (\( \eta = 0.25 \)), as estimated by Smets and Wouters (2007) for a variant of their model without nominal wage stickiness. I assume a steady state price markup \( \mu_p \) of 1.1. I set the tax rates on labor, consumption and capital to the estimates of Trabandt and Uhlig (2011). I set the investment adjustment cost curvature \( \xi_I \), the price markup coefficient \( \kappa_\pi \) and the monetary policy rule parameters \( \phi_\pi, \phi_\gamma \) and \( \phi_i \) to the estimates of Linde et al. (2016)\(^3\). I set

\(^3\)See their Table 3.2.
τ_\text{b} to a small value sufficient to guarantee debt stationarity.

Given these choices, I calibrate some parameters in order to set the steady state values of important model variables close to long run averages of their counterparts in the data, which are reported in Table 1. Given the assumed empirical target for \( \theta := \frac{\beta R}{\Pi} \), to be discussed below, the target for the steady state real interest rate determines the household discount factor \( \beta \). The empirical targets for the labor share, the government debt to GDP ratio and the share of government expenditures on goods and services in GDP determine the elasticity of output w.r.t. physical capital \( \alpha_K \), the debt target implicit in the fiscal rule and \( \frac{G}{Y} \), respectively.

Regarding the household’s POSA, I use the weight on safe assets in the household utility function \( \chi_\text{b} \) to set the discounting wedge \( \theta \) to a target value. For instance, the case without POSA corresponds to \( \theta = 1/\chi_\text{b} = 0 \). Campbell et al. (2016) assume \( \theta \approx 0.99 \). My preferred value is \( \theta = 0.96 \). To obtain evidence on \( \theta \), I draw on estimates of the (time-varying) nominal individual discount rate which the household applies to future nominal income streams, \( DIS_t = \frac{1}{E_t \left\{ \frac{\beta \Lambda_{t+1}}{\Lambda_t} + \Lambda_t \Pi_{t+1} \right\} } \). Given estimates of \( DIS_t \), I exploit that for small weights on safe assets in the utility function (i.e. \( \theta \) smaller than but close to one), \( \theta_t = \frac{R_t}{DIS_t} \) is approximately constant across time in the model. This property is a consequence of intertemporal substitution by the household: An increase in \( R_t \) shifts consumption from \( t \) to \( t+1 \), thus reducing the marginal utility of future consumption and increasing \( DIS_t \). Hence \( \theta \approx \frac{R_t}{DIS_t} \), which given the assumed steady state value of the real interest rate \( \frac{R}{\Pi} \) then allows to pin down \( \beta \). This indirect way of calibrating \( \beta \) avoids two problems pointed out by for instance Frederick et al. (2002) which arise if one interprets the empirical estimates of individual discount rates as measuring \( \beta \) itself. Firstly, calibrating \( \theta \) does not require the assumption that utility is linear in money. Secondly, it accounts for the possibility that estimates of the household discount rate based on choices between nominal amounts reflect inflation expectations (at least under the assumption that the nominal interest rate \( R_t \) reflects the same inflation expectations as DIS).

\[ H \]

More formally, rearranging equation (4) as
\[ 1 - \frac{\chi_\text{b}(b_G,t)^{-\sigma_b}}{A_t} = \beta R_t E_t \frac{\Lambda_{t+1}}{\Pi_{t+1} A_t}, \]
defining \( \theta_t = \frac{R_t}{DIS_t} = 1 - \frac{\chi_\text{b}(b_G,t)^{-\sigma_b}}{A_t} \) and linearizing yields
\[ d\theta_t = \frac{\chi_\text{b}(b_G,t)^{-\sigma_b}}{A_t} \left( A_t + \sigma_b \frac{Y}{Y} b_{G,t} \right) = (1 - \theta) \left( \sigma_b \frac{Y}{Y} b_{G,t} - \frac{1}{\sigma} \hat{C}_t \right). \]

Hence for \( 1 - \theta \) close to zero and reasonable calibrations of \( \sigma_H \) and \( \sigma_b \) even large deviations of \( \hat{C}_t \) and \( b_{G,t} \) would lead to tiny movements in \( \theta_t \), implying that \( \theta \approx \frac{R_t}{DIS_t} \) is a good approximation.
Table 1: Parameters and Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter name</th>
<th>Value</th>
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<tbody>
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<td>$\beta$</td>
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<td>0.9955; 0.9856; 0.9557*</td>
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<td>$\sigma$</td>
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<td>Curvature labor disutility</td>
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<td>$\sigma_b$</td>
<td>Wealth curvature</td>
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<tr>
<td>$\tau_b$</td>
<td>Fiscal rule, response of lump sum taxes to debt level</td>
<td>$\frac{R}{\Pi} - 1 + 0.05$</td>
</tr>
<tr>
<td>$\beta_{\Pi^T}$</td>
<td>Fiscal rule, target debt-to-annual GDP ratio</td>
<td>0.615*</td>
</tr>
<tr>
<td>$G^T$</td>
<td>Steady state government expenditure share</td>
<td>0.2*</td>
</tr>
<tr>
<td>$\phi_I$</td>
<td>Taylor rule inflation</td>
<td>2.0</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Taylor rule output gap</td>
<td>0.4/4</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Taylor rule interest rate smoothing</td>
<td>0.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Empirical target</th>
<th>Model counterpart</th>
<th>Value model</th>
<th>Value data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real short-term interest rate</td>
<td>$\left( \frac{R}{\Pi} \right)^4 - 1$</td>
<td>1.8%</td>
<td>1.8%</td>
<td>Federal Funds rate-CPI inflation</td>
</tr>
<tr>
<td>Government expenditure share</td>
<td>$\frac{G}{\Pi}$</td>
<td>0.2</td>
<td>0.2</td>
<td>BEA</td>
</tr>
<tr>
<td>Government debt-to-GDP ratio</td>
<td>$\frac{b_{G}}{\Pi}$</td>
<td>0.615</td>
<td>0.615</td>
<td>FRED</td>
</tr>
<tr>
<td>Non-farm business labor share</td>
<td>$\frac{N_w}{\Pi}$</td>
<td>0.61</td>
<td>0.61</td>
<td>Bureau of Labor Statistics</td>
</tr>
<tr>
<td>Discounting wedge</td>
<td>$\theta = \beta \frac{R}{\Pi}$</td>
<td>1.0; 0.99; 0.96</td>
<td>0.96</td>
<td>See note below.</td>
</tr>
<tr>
<td>Semi-elasticity demand safe assets w.r.t. (4$\Pi_t$)</td>
<td>$\frac{\theta}{1 - \phi \Pi} \frac{1}{\theta}$</td>
<td>6.0</td>
<td>5.0</td>
<td>Ball (2001)</td>
</tr>
</tbody>
</table>

Note:

- Parameter values labeled with * in the first table were calibrated such that the steady state values of the variables listed in the second table correspond to their empirical counterparts.
- The three values of $\beta$ and $\theta$ correspond to the cases of “No POSA, $\theta = 1$”; “POSA, $\theta = 0.99$, as assumed by Campbell et al. (2017)”; “POSA, $\theta = 0.96$”, based on the evidence on $\theta$ reported in Table 2.
- Given the target for $\theta$ and the calibration of the other parameters, the bond utility weight $\chi_b$ does not matter for the linearized model dynamics and is therefore not reported.
- All empirical targets are averages over the years 1981-2014 or the longest available subsample. For more details on the data sources see Appendix A.
- I computed the price markup coefficient $\kappa$ based on the estimates reported in Table 3.2 of Linde et al. (2016) using the following expression (using their notation) $\kappa = \frac{(1-\xi_p)(1-\epsilon_C)}{\epsilon_p(1+\phi_p-1)\epsilon_p}$. 


Table 2: Empirical evidence on $\theta$

<table>
<thead>
<tr>
<th>Sample period</th>
<th>DIS$_t - 1$(APR)</th>
<th>$R_t - 1$(APR)</th>
<th>Implied $\theta$</th>
<th>Source of DIS$_t$; $R_t$ used for comparison</th>
<th>Estimate of DIS$_t$ based on...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929-1948</td>
<td>33.0*</td>
<td>0.8*</td>
<td>0.82</td>
<td>Friedman (1962,1957); real treasury maturity $\geq$ 10 years</td>
<td>Tests of permanent income hypothesis</td>
</tr>
<tr>
<td>1960</td>
<td>19.6*</td>
<td>2.0*</td>
<td>0.96</td>
<td>Heckman (1976); real 10 year treasury</td>
<td>Estimated life cycle earnings model</td>
</tr>
<tr>
<td>1976</td>
<td>24.1</td>
<td>2.3*</td>
<td>0.95</td>
<td>Hausman (1979); real 10 year treasury</td>
<td>Energy efficiency and price of air conditioners</td>
</tr>
<tr>
<td>1979</td>
<td>122.5</td>
<td>3.0*</td>
<td>0.82</td>
<td>Gately (1980), Median estimate; 10 year treasury</td>
<td>Energy efficiency and price of refrigerators</td>
</tr>
<tr>
<td>1979</td>
<td>27.4</td>
<td>9.5</td>
<td>0.96</td>
<td>Cylke et al. (1982); 5 year treasury</td>
<td>US Military reenlistment decisions</td>
</tr>
<tr>
<td>1972; 1978; 1980</td>
<td>54.7; 64.0; 72.1*</td>
<td>3.2; 2.4; 4.4*</td>
<td>0.9; 0.89; 0.88</td>
<td>Ruderman et al. (1984), Median; 10 year real treasury</td>
<td>Price of household appliances</td>
</tr>
<tr>
<td>1982-1989</td>
<td>18.3</td>
<td>8.6</td>
<td>0.98</td>
<td>Ausubel (1991); one month certificate of deposit</td>
<td>US credit card interest rates</td>
</tr>
<tr>
<td>1992-1993</td>
<td>18.7/ 53.6</td>
<td>6.3</td>
<td>0.97 /0.91</td>
<td>Warner and Pleter (2001); 20 year treasury</td>
<td>US soldiers severance package choices</td>
</tr>
<tr>
<td>1996</td>
<td>22.5/ 28.1</td>
<td>4.2</td>
<td>0.96/ 0.95</td>
<td>Harrison et al. (2002); 1 year money market rate</td>
<td>Experiment, Danish households</td>
</tr>
<tr>
<td>2008</td>
<td>28.2/19.0</td>
<td>1.82/3.7</td>
<td>0.9/0.97</td>
<td>Wang et al. (2016); see note below.</td>
<td>Experiment, US econ. students.</td>
</tr>
</tbody>
</table>

Note:

- If information on the horizon of the choice of the agent under observation was available, $R_t - 1$ is the the safe (e.g. government) interest rate during the year the decision was made with a maturity as close as possible to this horizon. In most other cases, I use the 10 year government bond yield. Numbers marked with a * are estimates of the real personal discount rate. The corresponding $R_t$ I use to compute $\theta$ is therefore a measure of the real interest rate, where expected inflation is assumed to equal the average PCE inflation rate over the current and the preceding 9 years. In case of Friedman (1962, 1957), I calculated the relevant interest rate as the difference between the average interest rate on long-term government bonds (maturity 10 years or more, the only long-term government bond series for this period I am aware of) over the 1929-1948 period, and the average PCE deflator inflation rate.

- Warner and Pleter (2001): The first (second) reported value of $D_t - 1$ is the estimate for officers (enlisted personnel), and analogously for $\theta$.

- Harrison et al. (2002): The first (second) reported value of $D_t - 1$ is the estimate for income rich households (the sample mean), and analogously for $\theta$.

- Wang et al. (2016) allow for hyperbolical discounting and therefore allow the discount rate applied to a payment received one year ahead (the first value) to exceed the discount rate between any future period (the second value). The interest rates used to compute the corresponding two values of $\theta$ are the one year treasury bond rate, and the 9 year forward rate one year hence implied by the one and 10 year treasury bond rate.

- Ausubel's (1991) investigation of the US market for credit cards is frequently cited as evidence in favor of high personal discount rates. In his sample, more than three quarters of customers holding credit credit cards incur finance charges on substantial outstanding balances in spite of credit card interest rates ranging between 18 and 19%, and he cites industry publications saying that about 90% of an issuers outstanding balance accrue interest.
Regarding evidence on $DIS_t$, economists have attempted to estimate the personal discount rate at least since Friedman’s (1957) seminal tests of the permanent income hypotheses by studying economic agents’ behavior when faced with a variety of intertemporal trade-offs (see Table 2). These range from trading off the energy efficiency and price of household appliances (e.g., Hausman 1979, Gately 1979, Ruderman et al. 1984) to the choice between different types of severance packages (Warner and Pleeter 2001), as well as field experiments where probants choose between a payment today and a larger deferred payment (Harrison et al. 2002). As can be obtained from Table 2, the elicited discount rates are quite high, though below the median value obtainable from the comprehensive literature survey of Frederick et al. (2002), which equals (approximately) 35%. What is more, the discount rate estimates also typically exceed safe interest rates with a comparable maturity observed at the time the discount rates were elicited, resulting in an implied value of $\theta$ smaller than one, sometimes substantially so.

Since one may interpret the representative household with unconstrained access to financial markets as representing rich and educated households, the contributions of Harrison et al. (2002) and Warner and Pleeter (2001) are of particular relevance. Harrison et al. (2002) report estimates for (income-) rich households, while Warner and Pleeter (2002) elicit discount rate of officers and enlisted men of the United States armed forces choosing between two severance packages during the 1992-1995 military draw-down. My calibration of $\theta$ is thus at the upper end of what is implied by the evidence listed in Table 2.

I assume log-preferences over safe assets ($\sigma_b = 1$), which is consistent with a balanced growth path. Furthermore, this calibration is also consistent with Krishna-murthy and Vissing-Jorgenson (2012)’s argument that treasury bonds have money like qualities, because $\theta = 0.96$ and $\sigma_b = 1$ imply a semi-elasticity of the demand for bonds with respect to the (annualized) safe interest rate of 6.0 in the model. This

5See his Table 1. For each study reported by Frederick et al. (2002), I calculated the mean of the reported range of the discount factor, and then the median over all midpoints, and finally converted the discount factor into a discount rate.

6The authors report that virtually all of the officers in their sample have a college degree, while according to the Current Population Survey the same was true for only 24.5% of all individuals in the same age group.

7The bond demand semi-elasticity with respect to the annualized interest rate is given by
value is close to estimates of the semi-elasticity of money demand with respect to the opportunity costs of holding money by Ball (2001). However, below I also consider smaller values of $\sigma_b$ to illustrate the role of the “wealth effect” implied by POSA.

Finally, as mentioned in the introduction, my calibration would violate the parameter restrictions which according to Michaillat and Saez (2018) are required to solve the "Forward Guidance Puzzle" by way of creating a determinate zero lower bound steady state. Specifically, their assumption (4) requires $\left(\frac{1}{\beta} - \frac{\bar{R}}{\Pi}\right) \left(\frac{1}{\beta}\right) > \kappa_Y$, where $\kappa_Y = \kappa \left(\sigma_Y C + \frac{\alpha_K + \eta}{1 - \alpha_K}\right)$ denotes the reduced form output coefficient relating in the New Keynesian Phillips Curve (see the discussion in their Section 4.3 for this form of their assumption (4)). Nevertheless, as I show below, POSA still delivers a substantial mitigation of the "Forward Guidance Puzzle”.

3 Simulation setup

The simulation design I adopt is similar to Carlstrom et al. (2015). Specifically, let us denote the path of the policy interest rate expected by economic agents prior to the central bank’s announcement for quarter one and the following quarters as $\hat{R}^{pre}_1$, $\hat{R}^{pre}_2$, $\hat{R}^{pre}_3$, ..., where the superscript “pre” indicates “pre-announcement”. I then assume that in quarter one, the central bank announces to “switch off” its interest feedback rule (31) for a total of $D_L + D_p$ quarters, by setting $d_{p,t} = 1$ during those quarters, and to peg the interest rate at values $\hat{R}_{p,t}$. Furthermore, I assume for quarters 1 to $D_L$, the announced trajectory of the policy rate equals its path expected prior to the central bank’s announcement (i.e. $\hat{R}_{p,t} = \hat{R}^{pre}_{t}$). By contrast, for quarters $D_L + 1$ until $D_L + D_p$, the announced trajectory is located $\Delta$ % below its expected pre-announcement path (i.e. $\hat{R}_{p,t} = \hat{R}^{pre}_{t} - \Delta$). Finally, the central bank promises to return to its standard interest feedback rule (i.e. $d_{p,t} = 0$) in quarter $D_L + D_p + 1$.

To summarize, I assume $\frac{\theta}{1 - \sigma_Y \sigma_C}$, which can be seen by re-arranging equation (10) such that the percentage deviation of real bonds from its steady state $\frac{db_{G,t}}{b_{G,t}} (= \frac{b_{G,t} - b_{G}}{b_{G}})$ is located on the left hand side.
\[
\begin{align*}
    d_{p,t} &= \begin{cases} 
        1 & \text{for } t = 1, 2, \ldots, D_L + D_p \\
        0 & \text{for } t > D_L + D_p 
    \end{cases} \\
    \hat{R}_{p,t} &= \begin{cases} 
        \hat{R}_{t}^{\text{pre}} & \text{for } t = 1, 2, \ldots, D_L \\
        \hat{R}_{t}^{\text{pre}} - \Delta & \text{for } t = D_L + 1, D_L + 2, \ldots, D_L + D_p 
    \end{cases}
\end{align*}
\] (33)

I set \( D_L = 6 \), following Carlstrom et al. (2015). Furthermore, prior to the forward guidance announcements of the US Federal Reserve in September 2011, January 2012 and September 2013, financial markets expected the Federal Funds rate to remain at the zero lower bound for approximately 6 quarters, according to the evidence reported in del Negro et al. (2015a). I set \( \Delta \) to an annualized value of 0.2\% (i.e. \( \Delta = 0.24 \)), which is in line with the effect of the announcement on private sector forecasts of three month treasury bills estimated by del Negro et al. (2015a). Based on these assumptions, I will below investigate the macroeconomic effect of varying \( D_p \), i.e. the length of the period during which the interest rate peg shifts interest rate expectations downwards by \( \Delta \) percent.

4 Results in the simple model

I first discuss results from a model where the aggregate economy wide capital stock is constant and does not depreciate, i.e. \( \dot{K}_i = \frac{I}{Y} = \delta = 0 \). The reason is that the key attenuation mechanism associated with POSA operates via consumption, and abstracting from investment allows to represent aggregate demand and supply as two equations. However, results are very similar with a variable economy wide capital stock, as shown in Section 4.2. Section 4.3 investigates the effect of assuming a more realistic maturity structure of government debt instead of the one quarter maturity assumed so far, following Krause and Moyen (2016). Finally, Section 4.4 compares the response of the model with and without POSA to a series of standard shocks used in the DSGE literature.

\[\text{See their Figures 1 and 3, respectively.}\]
4.1 Results with a constant capital stock

As can be obtained from Figure 1 without POSA (i.e. $\theta = 1$) the impact effect of the policy on GDP and inflation are large and increase exponentially in $D_P$ (see the solid black line). The result mirrors the finding of Carlstrom et al. (2015). However, the increase in GDP and inflation is much lower with POSA and $\theta = 0.96$. For instance, for $D_p = 8$, with linear POSA (i.e. $\sigma_b = 0$, and thus no wealth effect) and $\theta = 0.96$, the increase in GDP and inflation is roughly half as big as without POSA (see the green squares line). With declining marginal utility from safe assets (i.e. $\sigma_b > 0$), and thus a wealth effect, the attenuation effect of POSA is even bigger, and the marginal effect of increasing $D_P$ by one quarter is no longer exponentially increasing in the length of the interest rate peg, but instead decreases in $D_P$. Considering first year averages instead of impact effects yields a (qualitatively) very similar picture (results are available upon request). For the case of $D_p = 8$, Figure 2 illustrates that the attenuation effect of POSA for $\theta = 0.96$ is quite persistent.

To gain intuition for the effects of forward guidance in the model variants considered, it is useful to derive aggregate demand and supply relationships for the first quarter from equations (21), (22), (25), (26), (27) and (28):

\[
\hat{Y}_1 = \frac{1}{\sigma^C} E_t \left\{ \sum_{i=0}^{D_L+D_P-1} \theta^i \left[ \theta \hat{\Pi}_{2+i} + (1 - \theta) \sigma_b \hat{b}_{G,1+i} \right] - \theta^{D_L} \sum_{i=1}^{D_P} \theta^i \hat{R}_f \right\} \tag{34}
\]

\[
\hat{\Pi}_1 = E_t \left\{ \sum_{i=0}^{D_L+D_P-1} \beta^i \kappa_Y \hat{Y}_{1+i} + \beta^{D_L+D_P} \hat{\Pi}_{D_L+D_P+1} \right\} \tag{35}
\]

with $\kappa_Y = \kappa \left( \sigma^C Y + \frac{\alpha + \eta}{\alpha - \kappa} \right)$. Consider first the case of no POSA, i.e. $\theta = 1$. In this case aggregate demand becomes a function of future expected real interest rates alone (the term $(1 - \theta)\sigma_b \hat{b}_{G,1+i}$ vanishes from equation (34)). Assume also that there is no interest rate smoothing ($\phi_i = 0$), implying that there are no endogenous state variables and thus that (in the absence of other shocks) GDP and inflation return to their steady state values immediately after the voluntary pegging of the interest rate at $\hat{R}_p$ ends (i.e. $E_t \hat{Y}_{D_L+D_P+1} = E_t \hat{\Pi}_{D_L+D_P+1} = 0$). Equation (34) then
says that increasing $D_P$ by one quarter has a partial equilibrium effect on GDP (i.e. abstracting from the change in inflation associated with the policy) of $\frac{1-C}{\tau_\sigma}$ in quarter 1, as well as in each of the following quarters leading up to and including $D_L + D_P$. The expected increase in $\hat{Y}_{1+i}$ increases the expectation of $\hat{\Pi}_{2+i}$ as well, with the biggest increase observed in quarter 2 and successively smaller increases in the following quarters. The increase in inflation causes a further decline (increase) in the real interest rate (aggregate demand), which feeds back into inflation. The result of this interaction of aggregate demand and supply is the explosive increase of the two variables in the length of the interest rate peg $D_p$.

Moving from the no POSA case ($\theta = 1$) to the case of linear POSA ($\theta = 0.96$, but $\sigma_b = 0$, implying that the $(1-\theta)\sigma_b\hat{Y}_{bG}\hat{b}_{G,1+i}$ term is still absent from (34)) lowers the partial equilibrium effect of increasing $D_P$ on quarter one GDP $\hat{Y}_1$ to $\frac{1}{\tau_\theta}b^{D_L + D_P}$. Clearly, with POSA ($\theta < 1$), the marginal effect of increasing $D_p$ on $\hat{Y}_1$ declines in $D_p$. Loosely speaking, the future becomes less and less important for today’s consumption choice the more distant in time it is located. As discussed in Section 2.1, the reason is that POSA attenuates consumption smoothing by breaking the proportionality between the current and the discounted expected future marginal utility of consumption.

Finally, allowing for declining marginal utility from safe assets ($\theta = 0.96$, $\sigma_b > 0$) implies that forward guidance also increases the marginal utility of safe assets, thus further attenuating the consumption increase. The increased marginal utility from safe assets results from the persistent decline in the real value of outstanding government bonds, which can be obtained from Figure 2 for the case of $D_p = 8$. This persistent decline is caused by the increase in tax revenues due to the improvement in economic activity and the decline in the real interest rate caused by the increase in inflation. Correspondingly, if the government would adjust lump sum taxes in order to keep real government debt constant (i.e. set $\hat{b}_{G,t} = 0$) instead of the gradual tax adjustment prescribed by (18), results would be identical to the case of linear POSA (i.e. $\sigma_b = 0$).

Since part of the decline in government debt driving the “wealth effect” observed with declining marginal utility from real safe assets (i.e. for $\sigma_b > 0$) is due to the improvement in the government’s budget balance, it is of interest to check whether...
the feedback from economic activity to the primary budget balance observed in the simulation is reasonable. As can be obtained by comparing the first with the final panel of Figure 2, the implied (on impact) semi-elasticity of the primary balance with respect to output is about 0.91. By contrast, empirical estimates for the US are closer to 0.5 (e.g. Botev et al. 2015). However, if I drop the assumption of flexible wages and assume nominally sticky wages instead, the primary budget balance semi-elasticity implied by the simulation drops to about 0.5, while the wealth effect continues to contribute strongly to the overall attenuation delivered by POSA. Results for the sticky wage case are reported in Appendix B.

As mentioned in the introduction, Campbell et al. (2016) also consider a model with POSA but report that nevertheless “in our model, extending a near zero interest rate peg for additional periods leads to initial responses of output and inflation that grow with the length of the extension and eventually become explosive.” Figure 2 confirms that under the assumptions of Campbell et al. (2016), namely $\theta = 0.99$ and no wealth effect (captured here as $\sigma_b = 0$), the attenuation effect of POSA becomes very small (compare the cyan diamond line to the black solid line), and the effects of forward guidance continue to increase exponentially in $D_p$. Hence my calibration of $\theta$, which I based on microeconometric evidence on individual discount rates, is crucial.

For the case of $\theta = 0.99$, allowing for a wealth effect would also attenuate the effect of forward guidance even (compare the cyan diamond to the cyan diamond-dotted line). However, $\theta = 0.99$ and $\sigma_b = 1$ would jointly imply a semi-elasticity of the demand for bonds with respect to its own (annualized) interest rate of 25. This value strongly exceeds values which are typically estimated for the elasticity of money demand (e.g. Ball 2001) and would thus be at odds with the interpretation of POSA as reflecting the money like qualities of safe assets.

Note that in principle, POSA could also attenuate the effect of forward guidance by making the Phillips Curve less forward looking as it implies a lower $\beta$ (see equation

---

9For instance, for $D_p = 10$ and “No POSA”, the government primary balance increases by 2.9 percent and 3.2 percentage points, respectively, and $\frac{2.9}{3.2} \approx 0.91$. Based on first year averages instead of impact effects, the semi-elasticity is 0.82.

10See their table 10.

11See Campbell et al. (2016), p. 72
However, if I keep $\beta$ in equation (35) at its value in the absence of POSA, results are virtually identical (not shown), implying that POSA operates mainly via the aggregate demand side of the economy.

### 4.2 Results with variable physical capital

As can be obtained from comparing the black solid lines in Figures 3 and 1, allowing for variable economy-wide physical capital and thus a role for investment spending slightly reduces the effect of forward guidance because consumption increases by more than investment, but the share of consumption in GDP is now lower. However, this prediction is sensitive to the curvature of the investment adjustment costs $\xi_I$. Importantly, POSA continues to strongly attenuate the effect of forward guidance for my preferred calibration of $\theta = 0.96$.

In Appendix C I briefly discuss results under the assumption that the household derives utility both from safe assets and from physical capital. As I mentioned in Section 2.1, an alternative motivation for POSA is the so called “Capitalist Spirit” type motive, which would suggest that the household might derive utility from all her assets. Furthermore, the model with POSA and capital accumulation by the household implies a higher steady state capital rental $r_K$ and thus lower capital- and investment-to-GDP ratios than the model without POSA due to the lower value of the utility discount factor $\beta$ implied by values of $\theta < 1$. This prediction can be avoided by assuming preferences over physical capital on top of POSA. As can be obtained from Appendix C at the same time, results with both POSA and preferences over capital are very close to those with POSA only.

### 4.3 Long-term government bonds

I now examine the effect of adopting a more realistic maturity structure for government debt, following the assumptions of Krause and Moyen (2016). Specifically, I assume that public debt consists of stochastic long-term bonds. In each period such a bond pays the interest rate determined when the bond was issued, and, with a fixed probability $\omega_{LTD}$, matures and in that case pays back the principal. Since the government issues a large number of these bonds each period, the probability that
an individual bond matures equals the fraction of all bonds maturing each quarter in total outstanding bonds. The total real amount of outstanding stochastic bonds \( b_{G,L,t} \) is thus determined by

\[
    b_{G,L,t} = (1 - \omega_{LTD}) \frac{b_{G,L,t-1}}{\Pi_t} + b_{G,L,n,t}
\]

where \( b_{G,L,n,t} \) denotes total newly issued stochastic bonds. The nominal average interest rate on the total amount of outstanding stochastic bonds \( R_{G,L,t} \) is determined by

\[
    (R_{G,L,t} - 1) b_{G,L,t} = (1 - \omega_{LTD}) \frac{(R_{G,L,t-1} - 1)}{\Pi_t} b_{G,L,t-1} + (R_{G,L,n,t} - 1) b_{L,n,t}
\]

where \( R_{G,L,n,t} \) denotes the market interest rate on stochastic bonds issued in period \( t \) (see Krause and Moyen (2016) for details). The household’s objective is now given by

\[
    \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi N_{t+i}^{1+\eta}}{1+\eta} + \frac{\chi b}{1-\sigma_b} (b_{G,t+i} + b_{G,L,t+i})^{1-\sigma_b} \right]
\]

Hence following Krishnamurthy and Vissing-Jorgenson (2012), I assume that households derive utility from their total holdings of government debt, including short and long-term bonds. Households maximize (38) subject to (2), (36) and (37), by choosing \( C_t, N_t, b_{G,t}, b_{G,L,t}, b_{G,L,n,t} \) and \( R_{G,L,t} \)\(^{12}\). The FOCs with respect to \( b_{G,t}, b_{G,L,t}, b_{G,L,n,t} \) and \( R_{G,L,t} \) imply:

\[
    \Lambda_t = \beta E_t \left\{ \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right\} + \chi_b (b_{G,t} + b_{G,L,t})^{-\sigma_b}
\]

\[
    \Lambda_t = \beta E_t \left\{ \Lambda_{t+1} \frac{R_{L,n,t} - \mu_{t+1} (1 - \omega_{LTD}) (R_{L,n,t+1} - R_{L,n,t})}{\Pi_{t+1}} \right\} + \chi_b (b_{G,t} + b_{G,L,t})^{-\sigma_b}
\]

\(^{12}\)The reason that the average interest rate on the households bond portfolio \( R_{G,L,t} \) is a choice variable is that it is affected by the households purchases of newly issued bonds \( b_{L,n,t} \). By contrast, the market interest rate on newly issued bonds \( R_{G,L,n,t} \) is taken as given by the household (see Krause and Moyen (2016)).
\[
\mu_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\Pi_{t+1}} \left[ 1 + \mu_{t+1} (1 - \omega_{LTD}) \right] \right\}
\]  

(41)

where \(\mu_t\) denotes the Lagrange multiplier on the law of motion of the average interest rate (37). These equations are identical to Krause and Moyen except for the term reflecting the marginal utility of government bonds \(\chi_b (b_{G,t} + b_{G,L,t})^{-\sigma_b}\) in equations (39) and (40). The FOC with respect to short-term bonds (39) is identical to equation (4) above, except for the marginal utility of short-term bonds, which now depends on the sum of short-term and long-term bonds \(b_{G,t} + b_{G,L,t}\). Equations (39) to (41) determine private consumption and the interest rate on newly issued long-term bonds \(R_{L,n,t}\) given the expected paths of the short-term nominal interest rate, inflation, and the supply of total real government bonds \(b_{G,t} + b_{G,L,t}\).

Following Krause and Moyen (2016), I assume that one quarter government debt is now in zero net supply, i.e. \(b_{G,t} = 0\), implying that the government budget constraint becomes

\[
b_{G,L,t} = \frac{R_{G,L,t-1}}{\Pi_t} b_{G,L,t-1} + G_t - (T_t + \tau_w w_t N_t + \tau_C C_t + \tau_K ((r_{K,t} - \delta) K_{t-1}))
\]  

(42)

which differs from its counterpart in the model with one quarter debt (17) only in that for \(\omega_{LTD} < 1\), the (average) interest rate on t-1 government debt \(R_{G,L,t-1}\) no longer equals the \(t - 1\) policy rate, but is instead a weighted average of interest rates on bonds issued in all past periods (i.e. \(R_{G,L,n,t-1}, R_{G,L,n,t-2}\) etc., see equation (37)).

The full (linearized) model then consists of the same equations as those listed in Section 2.5, except that (the linearized versions of) equations (39) and (42) replace (22) and (42), while (the linear counterparts of) equations (36), (37), (40) and (41) are added to the model. Following Krause and Moyen (2016), I set the fraction of bonds maturing each quarter to \(\omega_{LTD} = 0.0472\), which matches the average maturity

\[13\text{This equation can be obtained by combining the government budget constraint expressed in terms of newly issued debt, given by } b_{G,n,t} = \frac{(R_{L,n,t-1} - 1 + \alpha) b_{G,L,t-1} + G_t - (T_t + \tau_w w_t N_t + \tau_C C_t + \tau_K ((r_{K,t} - \delta) K_{t-1}))}{\Pi_t} \text{, which corresponds to equation (13) in Krause and Moyen (2016), with equation (36).}\]
of US debt held by the public of 5.3 years. All other parameters remain as in Table 1.

The effect of forward guidance with long-term government bonds is displayed in Figure 4. Note that without POSA ($\theta = 1$) and linear POSA and thus no wealth effect ($\sigma_b = 0$), assuming long-term instead of one quarter debt affects only the path of government debt and no other variable. The reason is the assumption that lump sum taxes are used to ensure long-run debt stationarity (see equation 18). However, even with declining marginal utility from government bonds ($\sigma_b > 0$), the macroeconomic effect of forward guidance and thus the attenuation caused by POSA are almost identical to the results with one quarter debt (compare Figures 4 and 1), as the path of real government debt is very similar. To put it differently, the decline in real government debt observed with debt of a one quarter maturity is almost exclusively driven by higher inflation and the improvement in the government’s primary balance, both of which carry over to the model with long-term government debt, rather than the decline in the nominal interest rate starting in quarter $D_L + 1$.

In Appendix D, I show that these results are broadly robust against assuming that the household derives utility from the market value of her government bond portfolio rather than the face value as assumed above (see equation (38)). For concave POSA ($\sigma_b > 0$), the attenuation becomes a little smaller than if the face value of government bonds enters the utility function. However, the effect of forward guidance on GDP and inflation continues to be much lower than in the absence of POSA.

### 4.4 POSA and standard shocks

The fact that POSA alleviate the "Forward Guidance Puzzle" raises the question whether it changes the response to any of the more standard shocks found in the DSGE literature. In this section, I therefore subject the model with POSA to shocks typically used in estimated Smets and Wouters (2007) type DSGE models: a standard monetary policy shock, a risk premium shock, a shock to the marginal efficiency of investment (i.e. a shock to the investment adjustment cost technology), a productivity shock, a shock to the price markup of retailers and the wage markup of wage-setting households. The parameterization of the shock processes follows the
estimates of Linde et al. (2016). I use the model with variable physical capital of Section 4.2 but assume that both wages and prices are nominally sticky (for details see the note below Figure 5).

Figure 5 reports the results for the case of no POSA, POSA with short-term and POSA with long-term government debt (with $\theta = 0.96$ and $\sigma_b = 1$), with all shocks signed to trigger a GDP increase. The responses of the variables in the models with POSA is generally very similar to the models without. An exception is the inflation response to the monetary policy shock for the case of a one quarter maturity of government debt, where model with POSA predicts an inflation decline starting in quarter 2. The reason is that with short-term debt only, the average interest rate on government debt equals the declining policy rate. Hence the monetary expansion has an immediate direct negative effect on the path of government debt, which accumulates over time and thus persistently lowers the consumption and the real wage response if households have POSA. Both variables fall slightly but very persistently below their respective steady state values after two years, which lowers firms expected marginal costs and thus causes the observed decline in inflation. More generally, with one quarter debt, the model with POSA tends to predict a (somewhat) smaller increase of GDP and inflation to any expansionary shock which also triggers a decline of the policy interest rate (i.e. the monetary policy shock, the productivity shock, the decline in the wage and price markups and the investment efficiency shock). However, assuming the more realistic maturity structure of government debt discussed in Section 4.3 renders the inflation response to the expansionary monetary policy shock positive even with POSA ("POSA, Long-term government debt", the magenta dashed dotted line) because the average interest rate on government debt $R_{G,L,t}$ barely moves in response to the policy rate cut, implying that government debt declines by much less than with one quarter debt. The decline in $R_{G,L,t}$ is tiny because, firstly, any decline of the interest rate on newly issued bonds $R_{G,L,n,t}$ affects only the small maturing fraction of government bonds $\omega_{LTD} = 0.0472$. Secondly, the decline of $R_{G,L,n,t}$ is far smaller than the short-term policy rate cut that triggered it since it mainly depends on a weighted average of expected future short-term rates.

\[^{14}\text{Up to first order, the first order conditions (39) and (40) imply } \hat{R}_{L,n,t} = \hat{R}_t \left[ R - \theta \frac{(1-\alpha)}{R} \right] + \frac{\theta (1-\alpha)}{R} E_t \hat{R}_{L,n,t+1}. \text{ Hence under reasonable calibrations, the effect of a temporary increase of } \hat{R}_t \text{ on...} \]
In Appendix D.2 I show that if I assume that if the market value of government debt enters the household utility function, the IRFs with and without POSA become almost identical.

Hence POSA does not seem to substantially alter the predictions of a New Keynesian DSGE model as far as the standard shocks considered in the literature are concerned while at the same time considerably alleviating the "Forward Guidance Puzzle".

5 Forward guidance with collateral constraints and POSA

In Appendix E I examine the effect of forward guidance in a model with collateral-constrained borrower households, collateral constrained-borrowing firms, and saver households, along the lines of Iacoviello (2005, 2014) and Iacoviello and Neri (2010). Examining the effect of forward guidance in the presence of credit market frictions is of interest because such frictions arguably played an important role for the length of the Great Recession and the slow ensuing recovery (see Iacoviello 2014, del Negro et al. 2015b, Cai et al. 2018), which gave rise to the need for forward guidance policies in the first place.

The model features three types of households, namely saver households, borrower households and entrepreneurs, as well as monopolistically competitive goods producers owned by saver households. Saver and borrower households derive utility from consumption and housing, disutility from labor, and supply labor to goods producers in monopolistically competitive labor markets. Entrepreneurs accumulate a capital stock consisting of physical capital and commercial real estate, and rent to goods producers. In equilibrium, borrower households and entrepreneurs borrow from financial intermediaries who collect deposits from saver households. Borrowers face binding collateral constraints, where the collateral consists of their respective non-financial assets. Price and wage setting is subject to nominal rigidities. In the version of the model with POSA, I assume that saver households have preferences $\hat{R}_{L,n,t}$ is small.
over both government bonds and their deposits with financial intermediaries.

It turns out that the presence of the binding collateral constraints considerably amplifies the effect of forward guidance, thus worsening the "Forward Guidance Puzzle". The amplification arises because the income share of the constrained agents increases in response to the forward guidance announcement, for two reasons. Firstly, the markup of monopolistically competitive firms (owned by saver households) declines as real marginal costs in the form of real wages and the rental rate on physical capital and commercial real estate rise. Hence the real wage income of borrower households and the total capital rental income earned by entrepreneurs increases more than GDP. Secondly, the decline in the real interest rate caused by the increase in inflation reduces the debt service of borrowers during each quarter in which the real interest rate decline persists. Since borrower households and entrepreneurs have a high marginal propensity to spend, this redistribution amplifies the effect of forward guidance. However, assuming that the saver households have POSA again strongly attenuates the effects of forward guidance on GDP and inflation.

Regarding the response to the more standard shocks investigated in Section 4.4, the IRFs are almost identical in the model with collateral constraints regardless of whether POSA is assumed or not, even for one quarter government debt. 

6 Conclusion

This paper examines how the macroeconomic effects of forward guidance are shaped by preferences over safe assets (POSA) calibrated based on evidence on household individual discount rates. POSA attenuates intertemporal consumption smoothing since the marginal utility of consuming today is no longer exclusively governed by the expected discounted utility of consumption tomorrow, but also by the marginal utility of accumulating the safe asset. Furthermore, the attenuation compounds the more distant the respective future consumption choice is located away from

\[ \text{In the model with collateral constraints, a decline in the policy rate reduces the loan rate of borrower households and firms and thus boosts their disposable income, and also relaxes their borrowing constraints by increasing the house price. These effects are absent from the representative agent model and are sufficiently strong to overcompensate the negative wealth effect on saver household consumption arising in the model with POSA.} \]
today. As a result, POSA lowers the effect of future interest rate changes on current consumption, the more so the more distant they are located in the future. Moreover, with POSA a “wealth effect” further attenuates the effect of forward guidance, as real government debt declines in response to the increase in economic activity and the decline in the real interest rate caused by the policy. This decline increases the marginal utility from safe assets and thus dampens the consumption increase. As a result, POSA strongly attenuates the effect of forward guidance, thus alleviating the so-called “Forward Guidance Puzzle”. At the same time, the impact of POSA on the effect of shocks typically found in Smets-Wouters (2007) type DSGE models is small. My results are robust to assuming that some households and firms face Iacoviello (2005, 2014)-type collateral constraints.

References


[10] Del Negro, Marco/ Giannoni, Marc/ Patterson, Christina (2015a), The forward guidance puzzle, in: FRBNY Staff Reports No. 574.


Note: The graph displays the impact effect of an announcement by the central bank which pegs the future path of the policy interest rate, formally described in equation (33), with $D_L = 6$ and $\Delta = \frac{\pi}{4}$. Horizontal axis: length of the interest rate peg $D_p$. Vertical axis: Deviation of the respective variable from its value in the absence of the policy in quarter 1 (i.e., the impact effect). Government debt is expressed as a percentage of steady state GDP. Inflation is expressed as an annualized percentage rate (APR). The government primary balances is expressed as a percentage of actual GDP. All other variables are expressed as percentage of their respective steady state values.

The results are based on the model of Section 2 and the parameters displayed in Table 1, except that the aggregate economy wide capital stock is assumed constant and does not appreciate, i.e. $\frac{d}{dt} K_t = 0$. 
Figure 2: Effect of forward guidance for $D_p = 8$, constant physical capital

Note: The graph displays the dynamic effect of the simulation described in the note below Figure 1 for the case of $D_p = 8$. Horizontal axis: Quarters. Vertical axis: Deviation of the path of the respective variable from its path in the absence of the policy (in particular, starting in quarter 7, the policy interest rate is fixed at 0.2 percent below its path expected in the absence of the forward guidance policy, and starts converging towards its non-policy path in quarter 15). The policy interest rate is expressed as an annualized percentage rate (APR). All other units are as described in the note below Figure 1.
Figure 3: Impact effect of forward guidance with variable physical capital

Note: See the explanation below graph but note that physical capital is now assumed to be variable.
Figure 4: Impact effect of forward guidance, constant physical capital, long-term gov. debt

Note: See the note below Figure [1] but note that government debt now consists of stochastic long-term bonds as discussed in Section [4.3]
Figure 5: Response of the model with and without POSA to standard shocks

Note: The results are based on model of Section 2 and the calibration reported in Table 1, except for the labor market. “POSA” refers to $\theta = 0.96/\sigma_b = 1$, while “No POSA” refers to $\theta = 1$. “Long-term gov. debt” refers to the model of Section 4.3. The labor market now operates under monopolistic competition and convex wage adjustment costs. Hence the labor supply equation (27) is replaced by the wage setting equation $\hat{w}_t = \frac{1}{1 + \beta} \left( \kappa_w \left( \eta \hat{N}_t + \sigma \hat{C}_t - \hat{w}_t \right) + \beta \hat{E}_t \hat{w}_{t+1} + \beta \hat{E}_t \hat{\Pi}_{t+1} + \hat{w}_{t-1} - \hat{\Pi}_t \right) + \mu_{w,t}$ with $\kappa_w = 0.0094$ and $\eta = 2.2$, as estimated by Linde et al. (2016). Price and wage markup shocks follow ARMA(1,1) processes of the form $\mu_{p,t} = \rho_p \mu_{p,t-1} - \nu_{p,t-1} + \epsilon_{p,t}$, with $\epsilon_{p,t}$ being i.i.d. The remaining exogenous variables follow AR(1) processes. I calibrate the exogenous driving processes according to the estimates of Linde et al. (2016):

<table>
<thead>
<tr>
<th>Shock type</th>
<th>AR(1)/ MA(1) coefficient</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary policy</td>
<td>- / -</td>
<td>0.24</td>
</tr>
<tr>
<td>Risk premium</td>
<td>0.18/ -</td>
<td>0.24</td>
</tr>
<tr>
<td>Investment efficiency</td>
<td>0.71/ -</td>
<td>0.41</td>
</tr>
<tr>
<td>Technology</td>
<td>0.96/ -</td>
<td>0.45</td>
</tr>
<tr>
<td>Price markup</td>
<td>0.9/ 0.72</td>
<td>0.13</td>
</tr>
<tr>
<td>Wage markup</td>
<td>0.98/ 0.92</td>
<td>0.30</td>
</tr>
</tbody>
</table>
I computed the targets as averages over the 1981-2014 period or the longest available subperiod.

- Real Federal Funds rate: “Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted”, and “Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100, Monthly, Seasonally Adjusted”, both obtained from FRED.

- Government debt-to-GDP ratio: “Gross Federal Debt as Percent of Gross Domestic Product, Percent of GDP, Annual, Not Seasonally Adjusted”, obtained from FRED.

- Labor share: Giandrea and Sprague (2017), Figure 1.

- Top 40% income share: “Quintiles of income before taxes: Shares of annual aggregate expenditures and sources of income” tables, Consumer Expenditure Survey (CEX), average over the 1989-2015 period.

The empirical targets relating to household and entrepreneurial balance sheet variables were computed using Flow of Funds data:

- Commercial real estate to GDP ratio: Nonfinancial corporate business, real estate at market value + Nonfinancial noncorporate business, nonresidential real estate at market value.

- Residential real estate to GDP ratio: Households and nonprofit organizations, real estate at market value + Nonfinancial noncorporate business, residential real estate at market value.

B Results for the model of Section 2 for the sticky-wage case (for online publication only)

Figure 6: Impact effect of forward guidance, sticky wages, constant physical capital

Note: See the explanation below graph [1], but note that the labor market now operates under monopolistic competition and convex wage adjustment costs. Hence the labor supply equation (27) is replaced by the wage setting equation
\[
\hat{w}_t = \frac{1}{1+\beta} \left( \kappa_w \left( \eta \hat{N}_t + \sigma \hat{C}_t - \hat{w}_t \right) + \beta \hat{P}_t \hat{w}_{t+1} + \beta \hat{E}_t \hat{P}_{t+1} + \hat{w}_{t-1} - \hat{P}_t \right) + \mu_{w,t}
\]
with \( \kappa_w = 0.0094 \) and \( \eta = 2.2 \), as estimated by Linde et al. (2016).
Figure 7: Impact effect of forward guidance, sticky wages, variable physical capital

Note: See the explanation below graph 1 but note that physical capital is now assumed to be variable, and that the labor market now operates under monopolistic competition and convex wage adjustment costs. Hence the labor supply equation (27) is replaced by the wage setting equation

$$\hat{w}_t = \frac{1}{1+\beta} \left( \kappa_w \left( \eta \hat{N}_t + \sigma \hat{C}_t - \hat{w}_t \right) + \beta \hat{E}_t \hat{w}_{t+1} + \beta E_t \hat{\Pi}_{t+1} + \hat{w}_{t-1} - \hat{\Pi}_t \right) + \mu_{w,t}$$

with $\kappa_w = 0.0094$ and $\eta = 2.2$, as estimated by Linde et al. (2016).
C Results for the model with both POSA and utility from physical capital (for online publication only)

With both POSA and utility from physical capital, the household’s objective becomes

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi N_{t+i}}{1+\eta} + \frac{\chi_b}{1-\sigma_b} (b_{G,t+i})^{1-\sigma_b} + \frac{\chi_K}{1-\sigma_K} (K_{t+i})^{1-\sigma_K} \right] \right\}$$ (43)

with the first order condition with respect to capital given by

$$Q_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} [(1 - \tau_K) r_{K,t+1} + \tau_K \delta + (1 - \delta) Q_{t+1}] \right\} + \frac{\chi_K K_t^{-\sigma_K}}{\Lambda_t}$$ (44)

or, in linearized form

$$\hat{Q}_t = -\left[ \frac{\Lambda_t - \theta_K E_t \hat{\Lambda}_{t+1}}{\Lambda_t} \right] + \beta (1 - \tau_K) r_K E_t \hat{r}_{K,t+1} + \beta (1 - \delta) E_t \hat{Q}_{t+1} - (1 - \theta_K) \sigma_K \hat{K}_t$$ (45)

where $\theta_K \equiv 1 - \frac{\chi_K K_t^{-\sigma_K}}{\Lambda_t}$ and I have used $\theta_K = \beta (K_r - \delta) (1 - \tau_K) + 1$. All other model equations remain unchanged. I calibrate $\chi_K$ by assuming $\theta_K = \theta$, implying that the steady state return on capital $r_K$, the capital-output ratio and the share of investment in GDP are the same as without POSA. I set $\sigma_K = \sigma_b$, and consider the same values of $\sigma_b$ as in Table 1 (i.e. 1, 1/3 and 0).

As can be obtained from by comparing Figures 3 and 3, results with preferences over both safe assets and physical capital are very close to the case of POSA only. Note that adding preferences over physical capital does not render the equation for Tobin’s Q (45) less forward looking, as the coefficient on the expected future price of capital $\beta (1 - \delta)$ and thus the decay of the importance of future events for current investment as their distance from the present increases is the same as with POSA only. Furthermore, the additional wealth effect arising from changes in the physical capital stock $- (1 - \theta_K) \sigma_K \hat{K}_t$ plays virtually no role since the (percentage) increase in the physical capital stock in response to the policy is small. Therefore varying $\sigma_K$
independently from $\sigma_b$ has only a small impact on the simulated effects of forward guidance (results are available upon request).

Finally, I also check the prediction of the model for the marginal propensity to save out of an increase in the household’s permanent income by performing a microsimulation, following the strategy of Kumhof et al. (2015), and compare the prediction to the micro-evidence of Dynant et al. (2015). Details on the microsimulation are provided in Appendix F.1 which explains the exercise for the saver household in the model with collateral constraints. The adaption to a household with objective (43) and budget constraint (2) is straightforward. For $\sigma_b = \sigma_K = 1$, the predicted MPS equals 0.43, with higher (lower) predictions for lower (higher) values of $\sigma_K$ and $\sigma_b$. 0.43 is identical to the value very close which Dynant et al. (2015) estimate for the top 40% of households.
Figure 8: Impact effect of forward guidance with POSA and preferences over physical capital

Note: See the explanation below graph 1 but note that physical capital is now assumed to be variable and the household derives utility from physical capital (see equation (43)). Her FOC with respect to physical capital is thus given by equation (45), all other equations remain the same. I assume \( \theta_K = \theta \) and \( \sigma_K = \sigma_b \).
D  The model with long-term government debt and preferences over the market value of safe assets
(for online publication only)

This Appendix shows the effect of assuming that in the framework with stochastic government bonds discussed in Section 4.3, the household has preferences over the market rather than the face value of her safe assets. In this case her objective becomes

$$\sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi_N}{1+\eta} N_{t+i}^{1+\eta} + \frac{\chi_b}{1-\sigma_b} (b_{G,t+i} + Q_{b,G,L,t+i}b_{G,L,t+i})^{1-\sigma_b} \right]$$

(46)

where $Q_{b,G,L,t}$ denotes the market price of the household’s stochastic bond portfolio which is taken as given by the household. The FOCs with respect to $b_{G,t}$, $b_{G,L,t}$, $b_{G,n,t}$ and $R_{G,L,t}$ imply:

$$\Lambda_t = \beta E_t \left\{ \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right\} + \chi_b (b_{G,t} + Q_{b,G,L,t}b_{G,L,t})^{-\sigma_b}$$

(47)

$$\Lambda_t = \beta E_t \left\{ \Lambda_{t+1} \frac{R_{L,n,t} - \mu_{t+1} (1 - \omega_{LTD}) (R_{L,n,t+1} - R_{L,n,t})}{\Pi_{t+1}} \right\} + Q_{b,G,L,t} \chi_b (b_{G,t} + Q_{b,G,L,t}b_{G,L,t})^{-\sigma_b}$$

(48)

$$\mu_t = \beta E_t \left\{ \Lambda_{t+1} \frac{1}{\Pi_{t+1}} \left[ 1 + \mu_{t+1} (1 - \omega_{LTD}) \right] \right\}$$

(49)

$$\mu_{2,t} = \beta E_t \left\{ \Lambda_{t+1} \frac{1}{\Lambda_t} \left[ \alpha + (1 - \alpha) \mu_{2,t+1} \right] \right\} + \frac{Q_{b,G,L,t} \chi_b (b_{G,t} + Q_{b,G,L,t}b_{G,L,t})^{-\sigma_b}}{\Lambda_t}$$

(50)

Equations (47) and (48) differ from their counterparts in Section 4.3 only regarding the presence of $Q_{b,G,L,t}$ (compare equations (39) and (40) ), while the equation for the Lagrange multiplier on (37) $\mu_t$ is unchanged. Finally, I also report the equation determining $\mu_{2,t}$, which denotes the the Lagrange multiplier on the law of motion.
of long-term bonds \(36\). Since it represents the value of an additional unit of the portfolio of long-term bonds to the household, it follows that

\[
\mu_{2,t} = Q_{b,G,L,t}
\]  

(51)

The full (linearized) model then consists of the same equations as those listed in Section 2.5 except that (the linearized versions of) equations \((47)\) and \((42)\) replace \((22)\) and \((42)\), while (the linear counterparts of) equations \((36)\), \((37)\), \((48)\), \((41)\) and \((50)\) are added to the model.

Note that the market value of outstanding stochastic bonds matters for the dynamics over other variables in the model only if (1) the household has preferences over the market value of her safe assets \(b_{G,t} + Q_{LTD,t}b_{G,L,t}\) (as opposed to the face value as in Section 4.3) and (2) if utility from safe assets is concave \((\sigma_b > 0)\), i.e. if there is a “wealth effect”.

D.1 Effect of forward guidance with preferences over the market value of safe assets

As can be obtained from Figure 9, POSA continues to strongly attenuate the effect of forward guidance even if households derive utility from the market value of their long-term bonds. For concave utility from safe assets \((\sigma_b > 0)\), the attenuation is slightly smaller than if the face value of long-term bonds enters the utility function (compare the blue dashed and the magenta dashed dotted lines in Figures 9 and 4). The reason is that the announced decline in the nominal short-term interest rate \(R_t\) persistently increases the market price of the portfolio of outstanding government bonds \(Q_{b,G,L,t}\) (see also Figure 10). Therefore the market value of government debt \(Q_{b,G,L,t}b_{G,L,t}\) declines somewhat less than the face value (compare the government debt response in Figures 9 and 4). However, for a peg of 10 quarters \((D_p = 10)\), the impact effect of forward guidance on GDP and inflation is still only between one tenth and one fifth of its value in the absence of POSA, depending on the value of \(\sigma_b\)

Regarding the dynamics of the market price of the bond portfolio, Figure 10 shows that \(Q_{b,G,L,t}\) increases on impact and increases further until the quarter when
the short-term interest rate is reduced below its pre announcement value. Linearizing equations (48), (50) and (51), it can be shown that

\[ \hat{Q}_{b,G,L,t} = -\hat{R}_t + (1 - \alpha) E_t \hat{Q}_{b,G,L,t+1} \] (52)

which confirms that the bond price increases as the date of the the short-term interest rate cut approaches.
Figure 9: Impact effect of forward guidance, constant physical capital, long-term gov. debt, preferences over market value

Note: See the note below Figure [1] but note that government debt now consists of stochastic long-term bonds as discussed in Section [D] Furthermore, the panel “Gov. debt” now denotes the deviation of the market value of government debt $Q_{b,G,L,t}b_{G,L,t}$ from its steady state, expressed as a percentage of steady state GDP.
Figure 10: Effect of forward guidance for $D_p = 8$, constant physical capital, long-term gov. debt, preferences over market value

Note: The graph displays the dynamic effect of the simulation described in the note below Figure 9 for the case of $D_p = 8$. Horizontal axis: Quarters. Vertical axis: Deviation of the path of the respective variable from its path in the absence of the policy (in particular, starting in quarter 7, the policy interest rate is fixed at 0.2 percent below its path expected in the absence of the forward guidance policy, and starts converging towards its non-policy path in quarter 15). The policy interest rate is expressed as an annualized percentage rate (APR). All other units are as described in the note below Figure 9.
D.2 Effect of standard shocks with preferences over the market value of safe assets

Figure 11: Response of model with and without POSA to standard shocks

Note: See the note below Figure 5, but note that the results denoted as “POSA, Long-term gov. debt” are now based on the assumption that the household derives utility from the market value of her safe assets, as detailed in Section D.

E The model with collateral constrained households and firms (for online publication only)

The model features three types of households, namely savers, borrower households and entrepreneurs, who are denoted with subscripts $S$, $CC$ and $E$, respectively. I
assume here that only saver households have POSA. However, the attenuation effect of POSA becomes even bigger if borrower households have POSA as well.

In this section I discuss only the optimization problems and collateral constraints, as the model is largely a standard Iacoviello (2005)-type framework. The complete set of model equations is located in Appendix G.1. Note that the price setting equation of retailers (10), the capital accumulation equation (3), the investment first order condition (8), the monetary policy rule (19) and the fiscal rule (18) remain unchanged.

### E.1 Saver households

Saver households are indexed with \( j \) and have preferences over consumption \( C_{S,j,t} \), labor \( N_{S,j,t} \), housing \( H_{S,j,t} \) and their holdings of real safe assets \( b_{S,j,t} \). Real safe assets now include both government bonds and deposits with financial intermediaries, both with a one quarter maturity. The intertemporal utility function of saver households is given by

\[
E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ C_{S,j,t+i}^{1-\sigma_S} \frac{1-\sigma_S}{1+\eta} N_{S,j,t+i}^{1+\eta} + H_{S,j,t+i}^{1-\sigma_{H,S}} + b_{S,j,t+i}^{1-\sigma_{b,S}} \right] \right\}
\]

with \( \chi_{N,S}, \chi_{H,S}, \sigma_{H,S}, \sigma_{b,S} > 0 \) and \( \chi_{b,S} \geq 0 \). Each saver household supplies a labor variety \( j \) which forms part of of a CES basket of saver household labor. They operate in a monopolistically competitive labor market and face nominal wage adjustment costs. Their budget constraint is given by

\[
Q_{H,t} H_{S,j,t} + b_{S,j,t} + (1 + \tau_C) C_{S,j,t} = \frac{R_{t+1}}{H_t} b_{S,j,t-1} + Q_{H,t} H_{S,j,t-1} + (1 - \tau_w) \frac{\xi_w}{2} \left( \frac{W_{S,j,t}}{W_{S,j,t-1}} - 1 \right) + \frac{W_{S,j,t}}{P_t} N_{S,j,t} + \Xi_t - T_{S,t}
\]

where \( Q_{H,t}, H_{S,j,t}, \) and \( \xi_w \) denote the real house price, the household’s ownership of residential real estate, and the curvature of wage adjustment costs, respectively. I abstract from financial intermediary default, implying that bank deposits and gov-
ernment bonds are perfect substitutes and thus both earn the safe interest rate. Since in equilibrium, all saver households set the same wage, the \( j \) subscript can be dropped from now.

The saver households first order conditions are standard with the exception of the FOC with respect to real financial assets for \( \chi_{b,S} > 0 \), which however is identical to equation (4):

\[
\Lambda_{S,t} = \beta_S \left\{ E_t \frac{R_t}{\Pi_{t+1}} \Lambda_{S,t+1} \right\} + \chi_{b,S} b_{S,t}^{-\sigma_{b,S}}
\]  

(55)

Furthermore, note that up to first order, the assumptions regarding wage setting give rise to the standard New Keynesian wage Phillips Curve. The same is true for borrower households.

### E.2 Borrower households

Borrower households are indexed with \( j \) and have preferences over consumption \( C_{CC,j,t} \), labor \( N_{CC,j,t} \) and housing \( H_{CC,j,t} \) of the same form as saver households. Their intertemporal utility function is given by

\[
E_t \left\{ \sum_{i=0}^{\infty} \beta_{CC}^i \left[ \frac{C_{CC,j,t+i}^{1-\sigma_{CC}}}{1 - \sigma_{CC}} - \frac{\chi_{N,CC} N_{CC,j,t+i}^{1+\eta}}{1 + \eta} + \frac{\chi_{H,CC} H_{CC,j,t+i}^{1-\sigma_{H,CC}}}{1 - \sigma_{H,CC}} \right] \right\}
\]  

(56)

Their budget constrained is given by

\[
(1 + \tau_C) C_{CC,j,t} + T_{CC,j,t} + Q_{H,t} H_{CC,j,t} + \frac{R_{CC,t-1} b_{CC,j,t-1}}{\Pi_t} = b_{CC,j,t} + (1 - \tau_w) N_{CC,j,t} \frac{\xi_w}{2} \left( \frac{W_{CC,j,t}}{W_{CC,j,t-1}} \frac{1}{\Pi_t} - 1 \right)^2 \frac{W_{CC,j,t}}{P_t} + Q_{H,t} H_{CC,j,t-1}
\]  

(57)

where \( b_{CC,j,t} \) denotes the real debt of impatient households, on which they pay an interest rate \( R_{CC,t} \). Their borrowing is constrained by the expected present value of their house.
\[ b_{CC,j,t} \leq \frac{k_{CC}E_t\left[\Pi_{t+1}Q_{H,t+1}H_{CC,j,t}\right]}{R_{CC,t}} \] (58)

where \( 0 \leq k_{CC} \leq 1 \). I assume that \( \beta_{CC} < \frac{\Pi}{R_{CC}} \), implying that for equilibria sufficiently close to the steady state, the constraint holds with equality.

### E.3 Entrepreneurs

Entrepreneurial households have preferences over consumption \( C_{E,t} \). Their intertemporal utility function is given by

\[ \sum_{i=0}^{\infty} \beta_E^i \frac{C_{E,t+i}^{1-\sigma_S}}{1-\sigma_E} \] (59)

Entrepreneurial households derive rental income from their ownership of commercial real estate \( H_{E,t} \) and the physical capital stock \( K_{E,t} \). Their budget constraint is given by

\[ \frac{R_{E,t}}{\Pi_t} b_{E,t-1} + (1 + \tau_C) C_{E,t} + T_{E,t} + Q_{H,t} H_{E,t} + Q_t K_{E,t} = b_{E,t} + (Q_{H,t} + (1 - \tau_K) r_{H,t}) H_{E,t-1} + ((1 - \tau_K) r_{K,t}) K_{E,t-1} + \tau_K \delta K_{E,t-1} \] (60)

where \( R_{E,t}, Q_t, r_{H,t}, r_{K,t}, \tau_K \) denote the interest rate on commercial loans, the price of the physical capital stock, the real rental rate on commercial real estate and the physical capital stock and the tax rate on profits, respectively. Following Iacoviello (2014), entrepreneurial borrowing is constrained by the expected present value of the total entrepreneurial capital stock

\[ b_{E,t} \leq \frac{k_E E_t \left[\Pi_{t+1}Q_{H,t+1}H_t + \Pi_{t+1}K_{E,t}\right]}{R_{E,t}} \] (61)

where \( 0 \leq k_E \leq 1 \). I assume that \( \beta_E < \frac{\Pi}{R_E} \), implying that for equilibria sufficiently close to the steady state, the constraint holds with equality.\(^{16}\)

\(^{16}\)Following Iacoviello (2014), I assume that non-real estate capital \( K_t \) enters total collateral at its (real) purchase price, rather than its market price \( Q_t \). My results are however robust to assuming
E.4 Financial intermediaries

Financial intermediaries collect deposits from saver households and lend to borrower households and entrepreneurs. They operate under perfect competition both in the deposit and credit markets, and there is no default risk, implying that the interest rates on loans equal the risk free rate $R_t$:

\begin{align*}
R_{E,t} &= R_t \quad (62) \\
R_{CC,t} &= R_t \quad (63)
\end{align*}

E.5 Retailers

The production function of retailers becomes

\[ Y_{j,t} = A_t N_j^{(1-\omega)(1-\alpha_K-\alpha_H)} N_j^{\omega(1-\alpha_K-\alpha_H)} K_j^{\alpha_K} H_j^{\alpha_H} \]

where $\omega$ denotes the share of impatient households in total labor income. As before, retailers face convex price adjustment costs (equation 12) and there are economy wide markets for all factors of production, implying that marginal costs are identical across firms.

E.6 Equilibrium

Following Iacoviello (2005,2014), I assume that the economy wide housing stock is constant. Market clearing in the goods, housing, capital and credit market then requires

\[ \text{total collateral is computed using } Q_t K_{E,t} \]
\[ C_t = C_{S,t} + C_{CC,t} + C_{E,t} \]  
(64)

\[ Y_t = C_t + I_t + G_t + Y_t \frac{\xi_P}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 \]  
(65)

\[ + \frac{\xi_W}{2} \left( \frac{\Pi_{W,S,t}}{\Pi} - 1 \right)^2 w_{S,t} N_{S,t} + \frac{\xi_W}{2} \left( \frac{\Pi_{W,CC,t}}{\Pi} - 1 \right)^2 w_{CC,t} N_{CC,t} \]  
(66)

\[ K_t = K_{E,t} \]  
(67)

\[ H = H_{S,t} + H_{CC,t} + H_{E,t} \]  
(68)

\[ b_{S,t} = b_{G,t} + b_{CC,t} + b_{E,t} \]  
(69)

### E.7 Calibration

Table 3 reports the calibration. If not otherwise mentioned below, the parameter choices are as in Table 1. I assume that all agents have log preferences over consumption and, where relevant, housing \((\sigma_S = \sigma_{CC} = \sigma_E = \sigma_{H,S} = \sigma_{H,CC} = 1)\). The discount factors of entrepreneurs and impatient households are as in Iacoviello (2005). For entrepreneurs, this assumption implies an average annual internal real rate of return of 8.4%. I assume a maximum loan to value ratio of credit constrained households \(LTV_{CC}\) close to the values in Iacoviello (2014) and Iacoviello and Neri (2010). I set the labor income share of credit constrained households \(\omega_{CC}\) to 0.37, as estimated by Iacoviello (2005) and Iacoviello (2014).

Given these choices, I set the elasticity of output w. r. t. physical capital and commercial real estate \(\alpha_K\) and \(\alpha_H\), the two household type’s weight on utility from housing \((\chi_{H,CC}, \chi_{H,S})\) and the entrepreneurial maximum LTV, respectively, such that the steady state labor share, the ratios of the values of commercial and residential real estate relative to GDP and the steady debt-to-asset ratio of non-financial firms correspond to their empirical counterparts reported in Table 3, respectively. Following Iacoviello (2005), I assume that, in the absence of POSA \((\theta_S = 1)\), the utility weight on housing is identical across household types \((\chi_{H,CC} = \chi_{H,S})\). For \(\theta_S < 1\), I set \(\chi_{H,S}\) such that the share of saver households in the total housing stock is the
same as in the absence of POSA.\footnote{The shares of saver households, impatient households and entrepreneurs in the total housing stock equal 52\%, 14\% and 34\%, respectively.}

Regarding saver households POSA, I keep the values of the household discount factor and the associated value of the discount wedge (now denoted as $\theta_S$ and $\beta_S$, respectively) reported in Table I and, in simulations where $\theta_S < 1$, set the utility curvature with respect to safe assets $\sigma_{b,S}$ to one, in line with the previously cited evidence on the elasticity of money demand.

Since the saver household now derives utility from both safe financial assets and housing, one may interpret her objective as reflecting not just POSA, but also the aforementioned “Capitalist Spirit” type motive, which argues that rich households derive utility from all types of assets. In such a setting, Kumhof et al. (2015) set the wealth curvature parameter(s) (here: $\sigma_{b,S}$ and $\sigma_{H,S}$) such that in a microsimulation of an increase in the rich household’s permanent income, the household’s marginal propensity to save matches empirical microeconometric estimates for her income group. I assume here that entrepreneurs and saver households correspond to the rich. Under my calibration, their share in total household income equals about 76\%, which according to the consumer expenditure survey (CEX) would suggest that they jointly represent approximately the top 40\% of the income distribution. For this group, Dynant et al. (2004) report an MPS of 0.429. For $\sigma_{b,S} = 1$, the income weighted average MPS is with 0.37 close to this value. Details on the microsimulation used to compute the MPS of the top 40\% of households in the model are provided in Appendix F.1.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_S)</td>
<td>Discount factor saver household for (\theta_S = 1.0; \theta_S = 0.96)</td>
<td>0.995; 0.9557*</td>
</tr>
<tr>
<td>(\beta_{CC})</td>
<td>Discount factor borrower households</td>
<td>0.95</td>
</tr>
<tr>
<td>(\beta_E)</td>
<td>Discount factor entrepreneurs</td>
<td>0.98</td>
</tr>
<tr>
<td>(\sigma_{S,\sigma_{CC},\sigma_{E}})</td>
<td>Curvature consumption</td>
<td>1</td>
</tr>
<tr>
<td>(\sigma_{H,CC,\sigma_{H,S}})</td>
<td>Curvature housing</td>
<td>1</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Curvature labor disutility</td>
<td>2.2</td>
</tr>
<tr>
<td>(\sigma_{b,S})</td>
<td>Curvature bonds savers</td>
<td>1*</td>
</tr>
<tr>
<td>(\chi_{H,S})</td>
<td>Saver utility weight on housing for (\theta_S = 1/\theta_S = 0.96)</td>
<td>0.0551/0.58*</td>
</tr>
<tr>
<td>(\chi_{H,CC})</td>
<td>Borrower household utility weight on housing</td>
<td>0.0551*</td>
</tr>
<tr>
<td>(\kappa_\pi)</td>
<td>Price markup coefficient</td>
<td>0.02</td>
</tr>
<tr>
<td>(\kappa_w)</td>
<td>Wage markup coefficient</td>
<td>0.0094</td>
</tr>
<tr>
<td>(\mu_P)</td>
<td>Steady state price markup</td>
<td>1.1</td>
</tr>
<tr>
<td>(\omega_{CC})</td>
<td>Impatient household labor share</td>
<td>0.35</td>
</tr>
<tr>
<td>(\sigma_K)</td>
<td>Output elasticity w.r.t. physical capital</td>
<td>0.26*</td>
</tr>
<tr>
<td>(\sigma_H)</td>
<td>Output elasticity w.r.t. real estate</td>
<td>0.0780*</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Physical capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>(\xi_I)</td>
<td>Investment adjustment cost</td>
<td>5.5</td>
</tr>
<tr>
<td>(LTV_{CC})</td>
<td>LTV impatient Households</td>
<td>0.9</td>
</tr>
<tr>
<td>(LTV_{H},LTV_{K})</td>
<td>LTV entrepreneurs</td>
<td>0.4*</td>
</tr>
<tr>
<td>(\tau_C)</td>
<td>Consumption tax rate</td>
<td>0.05</td>
</tr>
<tr>
<td>(\tau_w)</td>
<td>Labor tax rate</td>
<td>0.28</td>
</tr>
<tr>
<td>(\tau_K)</td>
<td>Capital tax rate</td>
<td>0.3600</td>
</tr>
<tr>
<td>(\tau_b)</td>
<td>Fiscal rule debt</td>
<td>(\frac{R}{\Pi} - 1 + 0.02)</td>
</tr>
<tr>
<td>(\frac{\delta_0}{\delta Y})</td>
<td>Fiscal rule, target debt-to-annual GDP ratio</td>
<td>0.615*</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Steady state government expenditure share</td>
<td>0.2*</td>
</tr>
<tr>
<td>(\phi_{\pi})</td>
<td>Taylor rule: Inflation</td>
<td>2.0</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>Taylor rule: Output gap</td>
<td>0.4</td>
</tr>
<tr>
<td>(\phi_i)</td>
<td>Taylor rule interest rate smoothing</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Note:

- Parameter values labeled with * in Table 3 were calibrated such that the steady state values of the variables listed in Table 4 correspond to their empirical counterparts.
- The two values of \(\beta\) and \(\theta\) correspond to the cases of “No POSA, \(\theta = 1\)” and “POSA, \(\theta = 0.96\),” based on the evidence reported in Table 2.
- Given the target for \(\theta\) and the calibration of the other parameters, the bond utility weight \(\chi_b\) does not matter for the model dynamics and is therefore not reported.
- I computed the price and wage markup coefficients \(\kappa_\pi\) and \(\kappa_w\) based on the estimates reported in Table 3.2 in Linde et al. (2016) using the expressions (using their notation) \(\kappa_\pi = \frac{1-\xi_\pi\beta_{\pi}^{1-\sigma_C}}{\xi_\pi(1+(\phi_\pi-1)\kappa_\pi)} (1-\xi_\pi)\) and \(\kappa_w = \frac{(1-\xi_w\beta_{\pi}^{1-\sigma_C})}{\xi_\pi(1+(\phi_\pi-1)\kappa_\pi)} (1-\xi_\pi)\).
E.8 Results with collateral constrained households and firms

I now perform exactly the same policy simulation as described in Section 3. As can be obtained from Figure 12, the presence of collateral constrained households and entrepreneurs substantially amplifies the impact effect of forward guidance in the absence of POSA (compare the black solid and the black dotted line). This amplification is mainly due to a large increase in consumption by the -newly added-borrowing households and entrepreneurs, as well as a much larger increase in investment spending than in the absence of collateral constraints. For the case of an 8 quarter fixation of the nominal interest rate \( \Delta p = 8 \), Figure 13 shows that this amplification persists in later quarters as well. Assuming POSA strongly attenuates the affect of the policy for values of \( \Delta p > 4 \) (compare the magenta dashed-dotted line and the black solid line).

The reason for the amplification of the effect of forward guidance compared to a model without collateral constraints is that the policy disproportionally benefits households and entrepreneurs, who have a high marginal propensity to spend due to their binding borrowing constraints. Firstly, the markup of monopolistically competitive firms (owned by saver households) declines, implying that the the real wage income of borrower households and the total capital rental income earned by entrepreneurs increase more than GDP, as can be obtained from comparing the first and the final panel of Figure 12.\(^{18}\) Secondly, the decline in the real interest rate caused by the increase in inflation reduces the debt service of borrowers during each quarter which the real interest rate decline persists, which is eventually reflected in higher consumption as well. By contrast, and perhaps surprisingly, the increase in the amount of borrower household and entrepreneurial borrowing allowed by the increase in the real house price seems to mainly affect their housing demand and amount of borrowing, but not their spending on goods and services. Fixing \( Q_{H,t} \) to its steady state value in the collateral constraint (equations 58 and 61) yields results very similar to those of Figure 12 for all variables except the house price and

\(^{18}\)I have also checked the effect of assuming that the policy induced change in monopolistic profits is distributed to all three agents according to their respective steady state pre-tax income share. Under this assumption, the responses of GDP and its components are close to the “No collateral, no POSA” case in Figure 12 (results are available upon request).
<table>
<thead>
<tr>
<th>Empirical target (unless otherwise mentioned)</th>
<th>Model counterpart</th>
<th>Value model</th>
<th>Value data</th>
<th>Source (Details in Appendix A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real short-term interest rate</td>
<td>$\left( \frac{1}{\Pi} \right)^4 - 1$</td>
<td>1.8%</td>
<td>1.8%</td>
<td>Federal Funds rate-CPI inflation rate</td>
</tr>
<tr>
<td>Government expenditure share</td>
<td>$\frac{\theta}{\hat{y}}$</td>
<td>0.2</td>
<td>0.2</td>
<td>BEA</td>
</tr>
<tr>
<td>Government debt-to-GDP ratio</td>
<td>$\frac{b_{G}}{4T^{1/4}}$</td>
<td>0.615</td>
<td>0.615</td>
<td>FRED</td>
</tr>
<tr>
<td>Non-farm business labor share</td>
<td>$\frac{N_{CC}w_{CC} + N_{S}w_{S}}{3}$</td>
<td>0.60</td>
<td>0.61</td>
<td>Bureau of Labor Statistics</td>
</tr>
<tr>
<td>Commercial real estate to GDP ratio</td>
<td>$\frac{Q_{H}H_{E}}{4T}$</td>
<td>0.83</td>
<td>0.83</td>
<td>Flow of Funds</td>
</tr>
<tr>
<td>Residential real estate to GDP ratio</td>
<td>$\frac{Q_{H}(H_{CC} + H_{S})}{4T}$</td>
<td>1.61</td>
<td>1.65</td>
<td>Flow of Funds</td>
</tr>
<tr>
<td>Debt-to-asset ratio non-financial firms</td>
<td>$\frac{Q_{H}H_{E} + Q_{K}K}{4T}$</td>
<td>0.42</td>
<td>0.42</td>
<td>Flow of Funds</td>
</tr>
<tr>
<td>Saver household discounting wedge $\theta_S$</td>
<td>$\theta = \beta \frac{R}{\Pi}$</td>
<td>1.0/0.96</td>
<td>0.96</td>
<td>See Table 2 Section 2.6 for details</td>
</tr>
<tr>
<td>Semi-elasticity demand for safe assets w.r.t. $\left( 4\hat{R}_t \right)$</td>
<td>$\theta_{S} = \frac{1}{1-\delta_{S}} \frac{1}{1+\delta_{S}}$</td>
<td>6.0</td>
<td>5.0</td>
<td>Ball (2001)</td>
</tr>
<tr>
<td>Top 40% income share (not targeted)</td>
<td>$\frac{w_{S}N_{S}(b_{CC}+b_{G})(R_{\Pi}^{1/4}+r_{K}-\delta_{K})K+r_{H}H_{E}}{1-\delta_{K}}$</td>
<td>0.75</td>
<td>0.74</td>
<td>Consumer Expenditure Survey</td>
</tr>
<tr>
<td>MPS top 40% (not targeted)</td>
<td>See appendix F.1</td>
<td>0.37</td>
<td>0.43</td>
<td>Dynant et al. (2004), Figure 3</td>
</tr>
</tbody>
</table>

Note: All empirical targets are averages over the years 1981-2014 or the longest available subperiod.
borrowing.

However, the presence of collateral constraints does weaken the intertemporal substitution channel of the policy, in that there is no direct impact effect of the announced decline of the policy rate on the consumption of borrowers in the presence of binding collateral constraints. The reason is that the borrowing constraint implies that the borrowers total debt is limited by her collateral. Figure 14 plots the partial equilibrium impact effect of the announced change in the policy rate, i.e. the effect if I hold all variables the agent takes as given (as well as for simplicity household’s labor income) constant, except for the interest rate. In partial equilibrium the consumption expenditure of borrower households and entrepreneurs actually declines slightly in response to the expected cut in the policy rate as it increases the benefit of relaxing the constraint by accumulating more collateral, which requires a decline in consumption (as the house price remains unchanged). Hence the “original” expansionary impulse caused by the forward guidance policy arises due to the consumption increase of saver households, which in the NOPOSA cases increases linearly in $D_P$ (Left panel of Figure 14, black solid line).

POSA thus attenuates the effect of forward guidance by lowering the responsiveness of saver household consumption to future real interest rates as well as reducing her real financial wealth, since government debt declines again quite persistently due to the increase in economic activity and inflation. The magnitude of the feedback from economic activity to the governments primary budget balance, which partly drives this wealth effect, appears to be in line with empirical estimates. The semi-elasticity of the first year average of the governments primary balance with respect to the (first year average of) GDP implied by the simulation equals 0.45, somewhat below the aforementioned estimate of Botev et al. (2015).
Figure 12: Impact effect of forward guidance with collateral constraints

Note: The graph displays the impact effect of an announcement by the central bank which pegs the future path of the policy interest rate, formally described in equation (33), with $D_L = 6$ and $\Delta = 0.24$.

- Horizontal axis: length of the interest rate peg $DP$.
- Vertical axis: Deviation of the respective variable from its value in the absence of the policy in quarter 1 (i.e. the impact effect). Government and private sector debt are expressed as a percentage of steady state GDP. The government primary balances is expressed as a percentage of actual GDP. Inflation is expressed as an annualized percentage rate (APR). All other variables are expressed as percentage of their respective steady state values.
- The results depicted as a black solid and a magenta dashed-dotted line are based on the model developed in Appendix E and the parameters displayed in Table 3. “POSA” refers to $\theta_S = 0.96/\sigma_{b, S} = 1$ while “NOPOSA (=No Preferences Over Safe Assets)” refers to $\theta_S = 1$.
- The results depicted as a black dotted line are based on the model of Section 2 except that the labor market now operates under monopolistic competition and convex wage adjustment costs as in the model with collateral constraints, i.e. $\frac{\kappa_w}{2} \left( \frac{W_t}{W_{t-1}} \frac{1}{P_t} - 1 \right)^2 \frac{W_t}{P_t} N_t$, with $\eta = 2.2$ and $\kappa_w = 0.0094$. 
Figure 13: Effect of forward guidance with collateral constraints for $D_p = 8$.

Note: The graph displays the dynamic effect of the simulation described in the note below Figure 12 for the case of $D_p = 8$.

- Horizontal axis: Quarters.

- Vertical axis: Deviation of the path of the respective variable from its path in the absence of the policy (in particular, starting in quarter 7, the policy interest rate is fixed at 0.2 percent below its path in the absence of the forward guidance policy, and starts converging towards its non-policy path in quarter 15). The policy interest rate is expressed as an annualized percentage rate (APR). All other units are as described in the note below Figure 12.
Note: The graph displays the partial equilibrium impact effect of an announcement by the central bank which pegs the future path of the policy interest rate, formally described in equation (33), with $D_L = 6$ and $\Delta = \frac{0.2}{4}$. The horizontal axis depicts the length of the fixed interest rate policy $D_P$. The graph reports the deviation of the respective variable from its value in the absence of the policy in quarter 1 (i.e. the impact effect), expressed as a percentage of its respective steady state. The calibration is displayed in Table 3. Note that “partial equilibrium” means that I hold all variables the respective agents takes as given in her optimization problem as well as labor income constant during the simulation (i.e. $\Pi_t, w_{CC,t}, N_{CC,t}, w_{S,t}, N_{S,t}, r_{K,t}, r_{H,t}, Q_{H,t}, Q_I$), except for the policy interest rate during quarters $D_L + 1$ to $D_P$.

F Model with collateral constraints - micro simulations (for online publication only)

F.1 Microsimulation used to compute saver households MPS

Following Kunhof et al. (2015), I compute the marginal propensity to save out of a permanent income increase of saver and entrepreneurial households in a partial
equilibrium simulation. For the purpose of this exercise, I assume that income arising from sources other than the ownership of safe financial assets is exogenous and denoted as $Y_{S,t}$, implying that the budget constraint of saver households is given by

$$Q_{H,t}H_{S,t} + b_{S,t} + (1 + \tau_C)C_{S,t} = \frac{R_{t-1}}{\Pi_t}b_{S,t-1} + Y_{S,t} - T_{S,t} + Q_{H,t}H_{S,t-1} \quad (70)$$

The linearized partial equilibrium is then described by (for the underlying nonlinear first order conditions see Section G.1):

$$\hat{C}_{S,t} = \theta_S \left( \hat{C}_{S,t} + \frac{1}{-\sigma_S} \left( \hat{R}_t - \hat{\Pi}_{t+1} \right) \right) + (1 - \theta_S) \frac{\sigma_{b,S} Y}{\sigma_S b_S} \hat{b}_{S,t} \quad (71)$$

$$\hat{Q}_{H,t} = (1 - \beta_S) \left( -\sigma_{H,S} \hat{H}_{S,t} + \sigma_S \hat{C}_{S,t} \right) + \beta_S \left[ \sigma_S \left( \hat{C}_{S,t} - \hat{C}_{S,t+1} \right) + \hat{Q}_{H,t+1} \right] \quad (72)$$

$$\hat{b}_{S,t} = \frac{R}{\Pi} \left( \hat{b}_{S,t-1} + \frac{b_S}{Y} \left( \hat{R}_t - \hat{\Pi}_t \right) \right) - (1 + \tau_C) \frac{C_S}{Y} \hat{C}_{S,t} \quad (73)$$

$$\hat{Y}_{S,t} = Y_{S,t-1} + \epsilon_{Y,S,t} \quad (74)$$

$$0 = \hat{\Pi}_t = \hat{r}_{K,t} = \hat{r}_{H,t} = \hat{Q}_t = \hat{Q}_{H,t} = \hat{R}_t \quad (75)$$

where equation (72) is the savers first order condition with respect to housing, equation (74) denotes the income process and (75) reflect the fact that the income increase of the individual saver household does not have an effect on equilibrium prices and aggregate quantities. Note that deviations of $Y_{S,t}$ and $b_{S,t}$ from their respective steady state values are expressed as a percentage of steady state GDP.

In the simulation, I assume a one quarter one-off increase of $\epsilon_{Y,S,t}$, and then compute the MPS over a six year horizon, holding inflation and the nominal interest rate constant. The reason for the six year horizon is that the empirical estimates of the MPS Dynant et al. (2004) uses data on saving rates which is six years apart (see Kumhof et al. (2015) for further details on how to compute the MPS in a way consistent with the empirical estimates). Hence the model counterpart of the Dynant et al. (2004) empirical estimate of the MPS of saver households is given by
The linearized entrepreneurial first order conditions are given by

\[
\begin{align*}
\dot{C}_{E,t} &= E_t \dot{C}_{E,t+1} - \frac{1}{\sigma_E} \left( \dot{R}_{E,t} - E_t \dot{\Pi}_{t+1} \right) - \frac{1}{\sigma_E} \frac{1 - \beta_E R}{\beta_E E} \dot{\Lambda}_{E2,t} \\
\dot{Q}_t &= \beta_E \left[ \sigma_E \left( \dot{C}_{E,t} - E_t \dot{C}_{E,t+1} \right) \left( (1 - \tau_K) r_K + 1 - \delta + \tau_K \delta \right) + (1 - \tau_K) \frac{R}{Q} E_t \dot{r}_{K,t+1} \right] \\
&\quad + \beta_E (1 - \delta) E_t \dot{Q}_{t+1} + k_E \left( 1 - \beta_E \frac{R}{\Pi} \right) \frac{\Pi}{R} \left[ \dot{\Lambda}_{E2,t} + \dot{\Pi}_{t+1} - \dot{R}_{L,E,t} \right] \\
\dot{Q}_{H,t} &= \beta_E \left[ \sigma_E \left( \dot{C}_{E,t} - E_t \dot{C}_{E,t+1} \right) \left( (1 - \tau_K) \frac{r_K}{Q} + 1 \right) + (1 - \tau_K) \frac{r_H}{Q} E_t \dot{r}_{H,t+1} + \dot{Q}_{H,t+1} \right] \\
&\quad + k_E \left( 1 - \beta_E \frac{R}{\Pi} \right) \frac{\Pi}{R} \left[ \dot{\Lambda}_{E2,t} + \dot{\Pi}_{t+1} + \dot{Q}_{H,t+1} - \dot{R}_{L,E,t} \right] \\
\dot{b}_{E,t} &= \frac{R}{\Pi} \left( \frac{b_E}{Y} \left( \dot{R}_{E,t-1} - \dot{\Pi}_t \right) + \dot{b}_{E,t-1} \right) + (1 + \tau_C) \frac{C_E}{Y} \dot{C}_{E,t} + \frac{Q_H H_E}{Y} \left( \dot{H}_{E,t} - \dot{H}_{E,t-1} \right) \\
&\quad - \frac{Q_H H_E}{Y} \frac{r_H}{Q} (1 - \tau_K) \left( \dot{H}_{E,t-1} + \dot{r}_{H,t} \right) + \frac{K_E}{Y} \left( \dot{K}_t - \dot{K}_{t-1} \right) \\
&\quad - \frac{K_E}{Y} (1 - \tau_K) \left( \dot{K}_{t-1} \left( r_K - \delta \right) + r_K \dot{r}_{K,t} \right) + \frac{K_E}{Y} \delta \dot{Q}_t - \dot{Y}_{E,t} \\
\dot{b}_{E,t} &= \frac{k_E Q_H H_E \Pi}{Y R} \left( E_t \dot{\Pi}_{t+1} + E_t \dot{Q}_{H,t+1} + \dot{H}_{E,t} - \dot{R}_{E,t} \right) + \frac{k_E K \Pi}{Y R} \left( E_t \dot{\Pi}_{t+1} + \dot{K}_{E,t} - \dot{R}_{S,t} \right)
\end{align*}
\]

where \( \dot{\Lambda}_{E2,t} = \left( \frac{\Lambda_{E2,t}}{\Lambda_{E,t}} \right) \) and \( \Lambda_{E,2,t} \) denote the Lagrange multiplier on the entrepreneurial borrowing constraint and the marginal utility of real income, respectively. The exogenous income process is the same as for saver households:

\[
\dot{Y}_{E,t} = Y_{E,t-1} + e_{Y,E,t}
\]

From these equations, it is clear that a stable solution to the system described by equations (76) and equations (77) to (82) implies that a one quarter one-off increase of \( e_{Y,E,t} \) and thus a permanent increase in \( \dot{Y}_{E,t} \) causes a permanent increase of entrepreneurial consumption expenditure \( (1 + \tau_C) \frac{C_E}{Y} \dot{C}_{E,t} \) of the same magnitude. \( \dot{H}_{E,t} \),
\( \hat{K}_{E,t}, \hat{b}_{E,t} \) and \( \hat{\Lambda}_{E2,t} \) remain unchanged. Hence \( MPS_E = 0 \), and thus the average MPS of savers and entrepreneurs is given by

\[
MPS = \frac{w_S N_S + \left( \frac{R}{\Pi} - 1 \right) b_S + \Xi}{w_S N_S + \left( \frac{R}{\Pi} - 1 \right) b_S + \Xi + K (r_K - \delta) + H r_H - b_E \left( \frac{R}{\Pi} - 1 \right) MPS_S}
\]

where the fraction denotes the share of saver household pre-tax income in the sum of saver household and entrepreneurial household pre-tax income.

F.2 Microsimulation (partial equilibrium effect) of forward guidance

As in Section F.1, the partial equilibrium simulation holds all variables the individual takes as given as well as labor income constant, except for the interest rates faced by households and entrepreneurs:

\[
\hat{\Pi}_t = \hat{r}_{K,t} = \hat{r}_{H,t} = \hat{\Pi}_t = \hat{w}_{CC,t} + \hat{N}_{CC,t} = 0
\]  
(83)

For the interest rates, I assume

\[
\hat{R}_t = \epsilon_{i,t}
\]  
(84)
\[
\hat{R}_{S,t} = \hat{R}_t - 10^{-11} \hat{b}_{S,t}
\]  
(85)
\[
\hat{R}_{CC,t} = \hat{R}_t
\]  
(86)
\[
\hat{R}_{E,t} = \hat{R}_t + 10^{-11} \hat{b}_{E,t}
\]  
(87)

where \( \epsilon_{i,t} \) denotes an exogenous shock which is used to implement the path of \( \hat{R}_t \) described in the note below Figure 14. Equations (85) and (87) imply the assumption of a negative effect of saver household and entrepreneurial financial wealth on their respective interest rates and thus a positive effect on their consumption. In partial equilibrium, this assumption is necessary to induce long-run stationarity of their respective net financial asset position and thus determinacy, as \( \hat{R}_t \) is now ex-
ogenous and cannot play this role (unlike in general equilibrium). The purpose of the assumption is thus similar to the cost of holding net foreign assets regularly assumed in open economy DSGE models. For the borrower household, the assumption is not necessary because the utility the borrower household derives from housing and the collateral constraint jointly imply a negative effect of borrower household debt on her consumption.

For the borrower household, the linearized first order conditions with respect to consumption and housing, the budget constraint and the borrowing constraint (for the underlying nonlinear first order conditions see Section G.1) are given by

\[
\hat{C}_{CC,t} = -\frac{1}{\sigma_{CC}} \left(\hat{R}_{CC,t} - \hat{\Pi}_{t+1}\right) + E_t \hat{C}_{CC,t+1} - \frac{1}{\sigma_{CC}} \frac{1 - \beta_{CC}}{R \beta_{CC}} \hat{\Lambda}_{CC2,t} + \frac{1 - \beta_{CC}}{R \beta_{CC}} \hat{\Lambda}_{CC2,t} \hat{\Pi}_{t+1} + \frac{1}{\sigma_{CC}} \left(\hat{R}_{CC,t} - \hat{\Pi}_{t+1}\right) \hat{\Lambda}_{CC2,t}
\]

(88)

\[
\hat{Q}_{H,t} = \left(1 - \beta_{CC} - \left(1 - \beta_{CC} \frac{R}{\Pi} \frac{k_{CC} \Pi}{R}\right) \hat{\Lambda}_{CC2,t} + \hat{\Lambda}_{CC2,t} \hat{\Pi}_{t+1}\right) \left(-\sigma_{H,CC} \hat{H}_{CC,t} + \sigma_{CC} \hat{C}_{CC,t}\right)
\]

(89)

\[
\hat{b}_{CC,t} = R \left(\frac{b_{CC}}{Y} \left(\hat{R}_{t-1} - \hat{\Pi}_t\right) + b_{CC,t-1}\right) + (1 + \tau_C) \frac{C_{CC}}{Y} \hat{C}_{CC,t}
\]

(90)

\[
\hat{b}_{CC,t} = k_{CC} \frac{Q_{H,CC}}{Y} E_t \left(\hat{\Pi}_{t+1} + \hat{\Pi}_{t+1} + \hat{H}_{CC,t} - \hat{R}_t\right)
\]

(91)

where \(\hat{\Lambda}_{CC2,t}\) is defined analogously to \(\hat{\Lambda}_{E2,t}\). From these equations, one may describe the effect of the forward guidance policy on the consumption of the credit constrained household displayed in Figure 14 as follows. The borrowing constraint (91) implies that consumption cannot increase on impact to the response to the anticipated decline in \(\hat{R}_{CC,t}\). Instead, the anticipated decline in the interest rate increases current and future expected values of \(\hat{\Lambda}_{CC2,t}\), i.e. the benefit of saving one more unit of income due to the associated future relaxation of the borrowing constraint. Hence the demand for housing (see equation (89)) increases, while consumption declines.

For the entrepreneur, the relevant equations are (77) to (81). Note that un-
der the assumption of partial equilibrium (see equation (83)), (78) and (79) are identical since expected net-returns are equalized across asset classes, and hence 
\[
(1 - \tau_K)^\frac{r_H}{\delta} + 1 = ((1 - \tau_K) r_K + 1 - \delta + \tau_K \delta).
\]
Hence the entrepreneur is indifferent between adjusting its holdings of commercial real estate or physical capital in response to the policy. In the partial equilibrium simulation, and without any loss of generality, I therefore replace (89) with
\[
\hat{H}_{E,t} = 0
\]

G Model with collateral constraints- complete set of equations (for online publication only)

G.1 Nonlinear model

Saver households first order conditions are given by

\[
\Lambda_{S,t} = \frac{C_{S,t}}{1 + \tau_C} \tag{93}
\]

\[
\Lambda_{S,t} = E_t \left\{ \beta_S \frac{R_t}{\Pi_{t+1}} \Lambda_{S,t+1} \right\} + \chi_b S \Lambda_{S,t} \tag{94}
\]

\[
Q_{H,t} = \chi_H \frac{H_{S,t}}{\Lambda_t} + \beta_S E_t \left\{ \frac{\Lambda_{S,t+1}}{\Lambda_{S,t}} Q_{H,t+1} \right\} \tag{95}
\]

\[
\left( \frac{\Pi_{W,S,t}}{\Pi - 1} \right) \frac{\Pi_{W,S,t}}{\Pi} = \frac{(1 - \epsilon_W)}{\xi_W} \left( 1 - \frac{\xi_W}{2} \left( \frac{\Pi_{W,S,t}}{\Pi - 1} \right)^2 \right) + \frac{\epsilon_W}{\xi_W} \Lambda_{S,t} \left( 1 - \tau_W \right) w_{S,t} \tag{96}
\]

\[
+ \beta_S E_t \left\{ \frac{\Lambda_{S,t+1}}{\Lambda_{S,t}} \left( \frac{\Pi_{W,S,t+1}}{\Pi - 1} \right) \left( \frac{\Pi_{W,S,t+1}}{\Pi} \right)^2 N_{S,t+1} \right\}
\]

\[
w_{S,t} = w_{S,t-1} \frac{\Pi_{W,S,t}}{\Pi_t} \tag{97}
\]

Borrower households first order conditions are given by
\[ \Lambda_{CC,t} = \frac{c^{CC}_t}{(1 + \tau_C)} \]  
\[ \Lambda_{CC,t} = \frac{\beta_{CC} E_t \left\{ R_{CC,t+1} \Lambda_{CC,t+1} \right\}}{1 - \Lambda_{CC2,t}} \]  
\[ Q_{H,t} = x_{H,CC} \frac{H^{CC}_t}{\Lambda_{CC,t}} + \beta_{CC} E_t \left\{ \frac{\Lambda_{CC,t+1} Q^{H}_{t+1}}{\Lambda_{CC,t}} \right\} \]  
\[ + \beta_{CC} \Lambda_{CC2,t} \frac{LT_{VCC,C} E_t \left[ \Pi_{t+1} Q^{H}_{t+1} \right]}{R_t} \]  
\[ b_{CC,t} = (1 + \tau_C) \left( C_{CC,t} + T_{CC,t} + Q_{H,t} (H_{CC,t} - H_{CC,t-1}) \right) \]  
\[ + \frac{R_{CC,t-1} b_{CC,t-1}}{H_t} - (1 - \tau_w) N_{CC,t} \left( \frac{W_{CC,t}}{W_{CC,t-1}} \right) \left( \frac{1}{\Pi - 1} \right)^2 W_{CC,t} \]  
\[ b_{CC,t} = k_{CC} E_t \left[ \Pi_{t+1} Q^{H}_{t+1} H_{CC,t} \right] \]  
\[ \left( \frac{\Pi_{W,CC,t}}{\Pi} - 1 \right) \frac{\Pi_{W,CC,t}}{\Pi} = \frac{(1 - \epsilon_W)}{\xi_W} \left( 1 - \frac{\xi_W}{2} \left( \frac{\Pi_{W,CC,t}}{\Pi} - 1 \right) \right)^2 + \frac{\epsilon_W}{\xi_W} \frac{\chi_{CC} N^{\Pi}_{CC,t}}{\Lambda_{CC,t} (1 - \tau_w) W_{CC,t}} \]  
\[ + \beta_{CC} E_t \left\{ \frac{\Lambda_{CC,t+1} \left( \frac{\Pi_{W,CC,t+1}}{\Pi} - 1 \right)}{\Lambda CC,t} \right\} \left( \frac{\Pi_{W,CC,t+1}}{\Pi_{t+1} N_{CC,t}} \right) \]  
\[ w_{CC,t} = w_{CC,t-1} \frac{\Pi_{W,CC,t}}{H_t} \]  

where \( \Lambda_{CC2,t} \) denotes the Lagrange multiplier on the borrower household collateral constraint.

The first order conditions of the entrepreneur are given by
\[ \Lambda_{E,t} = \frac{C_{E,t}^{1-\sigma}}{1 + \tau_C} \]

\[ \Lambda_{E,t} = \frac{\beta E_t \{ \frac{R_{E,t}}{\Pi_{E,t} + 1} \Lambda_{E,t+1} \} }{1 - \frac{\Lambda_{E,t}}{\Pi_{E,t}}} \]

\[ Q_t = \beta E_t \left\{ \frac{\Lambda_{E,t+1}}{\Lambda_{E,t}} \left[ (1 - \tau_K) r_{K,t+1} + \tau_K \delta + (1 - \delta) Q_{t+1} \right] \right\} + \frac{\Lambda_{E,t}}{\lambda_{E,t}} k E_t \left[ \Pi_{E,t+1} \right] \]

\[ Q_{H,t} = \beta E_t \left\{ \frac{\Lambda_{E,t+1}}{\Lambda_{E,t}} \left[ \tau_{H,t+1} \left( 1 - \tau_{K,t+1} \right) + Q_{H,t+1} \right] \right\} + \frac{\Lambda_{E,t}}{\lambda_{E,t}} k E_t \left[ \Pi_{E,t+1} \right] \]

\[ b_{E,t} = \left( 1 + \tau_C \right) C_{E,t} + Q_{H,t} H_{E,t} + Q_t \left( K_{E,t} - (1 - \delta) K_{E,t-1} \right) \]

\[ b_{E,t} = \frac{1}{\Pi_t} \left[ \frac{\Lambda_{E,t+1}}{\lambda_{E,t}} \left[ \Pi_{E,t+1} \right] \right] \]

where \( \Lambda_{E2,t} \) denotes the Lagrange multiplier on the entrepreneurial household collateral constraint.

Financial intermediaries:

\[ R_{E,t} = R_t \]

\[ R_{CC,t} = R_t \]

Retailers first order conditions are given by

\[ \left( \frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} + \epsilon - 1 = \frac{\epsilon}{\xi_p} \left( m \xi + \frac{\xi p}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 \right) \]

\[ + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\lambda_t} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \left[ \Pi_{t+1} \right] \left[ \frac{Y_{t+1}}{Y_t} \right] \right\} \]

\[ w_{S,t} = m \xi (1 - \omega) (1 - \alpha_K - \alpha_H) \frac{Y_t}{N_{S,t}} \]

\[ w_{CC,t} = m \xi \omega (1 - \alpha_K - \alpha_H) \frac{Y_t}{N_{CC,t}} \]

\[ r_{K,t} = m \xi \alpha_K \frac{Y_t}{K_{E,t-1}} \]

\[ r_{H,t} = m \xi \alpha_H \frac{Y_t}{H_{E,t-1}} \]

\[ Y_t = \Lambda_t N_{S,t}^{1-\omega} (1 - \alpha_K - \alpha_H) N_{CC,t}^{1-\alpha_K - \alpha_H} K_{E,t-1}^{\alpha_K} H_{E,t-1}^{\alpha_H} \]
The investment and capital accumulation equations remains unchanged:

\[ 1 = Q_t \left[ 1 - \xi I_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] - \xi I_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \]

\[ + \beta_s E_t \left\{ \frac{\Lambda_{S,t+1}}{\Lambda_{S,t}} \left( \frac{I_{t+1}}{I_t} \right)^2 \xi I_t \left( \frac{I_{t+1}}{I_t} - 1 \right) Q_{t+1} \right\} \]

\[ K_t = (1 - \delta) K_{t-1} + I_t \left( 1 - \xi I_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 \]

Market clearing requires

\[ C_t = C_{S,t} + C_{CC,t} + C_{E,t} \]

\[ Y_t = C_t + I_t + G_t + Y_t \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 \]

\[ K_t = K_{E,t} \]

\[ H = H_{S,t} + H_{CC,t} + H_{E,t} \]

\[ b_{S,t} = b_{G,t} + b_{CC,t} + b_{E,t} \]

The government budget constraint, fiscal rule and monetary policy rule are given by

\[ b_{G,t} = \frac{R_{t-1}}{H_t} b_{G,t-1} + G_t \]

\[ \frac{dT_t}{T_{S,t}} = \tau_b b_{G,t-1} \]

\[ \dot{R}_t = (1 - d_p,t) \left( (1 - \phi_t) \left( \theta_s \bar{Y}_t + \frac{\phi_p}{4} (\bar{Y}_t - \bar{Y}_t^p) \right) + \phi_t \dot{R}_{t-1} \right) + d_p,t \dot{R}_{p,t} \]

G.2 Linearized equations (purely for the benefit of the referee)

The list of equations starts on the next page.
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www.nbb.be

Editor
Pierre Wunsch
Governor of the National Bank of Belgium

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\[ \begin{align*}
C_{S,t} &= -\theta_S \frac{1}{\sigma_S} \left[ R_t - \beta_S E_t \tilde{N}_{t+1} + i_{t,t} + \theta_S E_t C_{S,t+1} + (1 - \theta_S) \frac{\sigma_{S,t+1} Y}{\sigma_S} \left( b_{G,t} + b_{CC,t} + b_{E,t} \right) \right] \\
Q_{H,t} &= (1 - \beta_S) \left( -\sigma_{H,t} \beta_{S,t} + \beta_{CC} E_t \left( C_{S,t} - C_{S,t+1} \right) + \hat{Q}_{H,t+1} \right) \\
\hat{w}_{S,t} &= \frac{1}{1 + \beta_S} \left[ \hat{w} \left( \hat{N}_{S,t} + \beta_{C,t} - \hat{w}_{S,t} \right) + \beta_S E_t \hat{w}_{S,t+1} + \beta_S E_t \tilde{N}_{t+1} + \hat{w}_{S,t-1} - \hat{w}_{t} \right] + \epsilon_{w,t} \\
C_{CC,t} &= -\frac{1}{\sigma_{CC}} \left( R_{CC,t} - E_t \tilde{N}_{t+1} \right) + E_t C_{CC,t+1} - \frac{1}{\sigma_{CC}} \left( 1 - \beta_{CC} \right) \frac{R}{\Pi} + \frac{1}{\sigma_{CC}} \delta_{CC,t} \\
Q_{H,t} &= \left( 1 - \beta_{CC} \right) \left( R_{CC,t-1} - \hat{w}_{t} \right) + \hat{b}_{CC,t-1} \right) + \left( 1 + \tau_{C} \right) \frac{R_{CC,t}}{\Pi} \hat{C}_{CC,t} \\
\hat{b}_{CC,t} &= \frac{R}{\Pi} \left( \frac{b_{CC}}{R} \left( R_{CC,t-1} - \hat{w}_{t} \right) + b_{CC,t-1} \right) + \left( 1 + \tau_{C} \right) \frac{R_{CC,t}}{\Pi} \left( \hat{b}_{CC,t+1} + \hat{b}_{CC,t} \right) \\
\hat{C}_{E,t} &= E_t C_{E,t+1} - \frac{1}{\sigma_E} \left( \hat{R}_{E,t} - \hat{E}_{t+1} \right) - \frac{1}{\sigma_E} \left( 1 - \beta_{E} \right) \frac{R}{\Pi} + \frac{1}{\sigma_E} \lambda_{E2,t} \\
\hat{Q}_{t} &= \beta_{E} \left[ E_t \left( C_{E,t} - C_{E,t+1} \right) \left( (1 - \tau_{E}) k_T + 1 + \hat{w}_{E,t} \right) + \left( 1 - \tau_{E} \right) k_T E_t \hat{Q}_{t+1} + (1 - \delta) E_t \hat{Q}_{t} \right] \\
+ \frac{\hat{k}_{E} \left( 1 - \beta_{E} \right) \frac{R}{\Pi} + \frac{1}{\sigma_E} \left( \lambda_{E2,t} + \hat{E}_{t+1} - \hat{R}_{E,t} \right)}{R} \\
\hat{Q}_{H,t} &= \beta_{E} \left[ \frac{E_t}{\Pi} \left( C_{E,t} - C_{E,t+1} \right) \left( (1 - \tau_{E}) k_T + 1 + \hat{w}_{E,t} \right) + \left( 1 - \tau_{E} \right) k_T E_t \hat{Q}_{t+1} + (1 - \delta) E_t \hat{Q}_{t} \right] \\
+ \frac{\hat{k}_{E} \left( 1 - \beta_{E} \right) \frac{R}{\Pi} + \frac{1}{\sigma_E} \left( \lambda_{E2,t} + \hat{E}_{t+1} - \hat{R}_{E,t} \right)}{R} \\
\hat{b}_{E,t} &= \frac{R}{\Pi} \left( \frac{b_{E,t}}{R} \left( R_{E,t-1} - \hat{w}_{t} \right) + b_{E,t-1} \right) + \left( 1 + \tau_{C} \right) \frac{R_{E,t}}{\Pi} \hat{C}_{E,t} + \left( \hat{R}_{E,t} - \hat{R}_{E,t-1} \right) \frac{Q_{H,E,t}}{\Pi} \\
\hat{R}_{E,t} &= \left( 1 - \beta_{E} \right) \frac{R}{\Pi} + \frac{1}{\sigma_E} \left( \lambda_{E2,t} + \hat{E}_{t+1} - \hat{R}_{E,t} \right) \\
\hat{R}_{CC,t} &= \frac{1}{1 + \beta_{S}} \left[ \hat{I}_{t-1} + \beta_{S} E_t \tilde{N}_{t+1} + \epsilon_{t,t} \right] \\
\hat{K}_{t} &= (1 - \delta) \hat{K}_{t-1} + \delta {\hat{I}}_{t} + (1 + \beta_S) \hat{K}_{t+1} \\
\hat{Y}_{t} &= \frac{C}{Y} \hat{C}_{t} + \frac{G}{Y} \hat{G}_{t} + \frac{I}{Y} \hat{I}_{t} \\
C_{Y} \hat{C}_{t} &= \frac{C_{S}}{Y} \hat{C}_{S,t} + \frac{C_{CC}}{Y} \hat{C}_{CC,t} + \frac{C_{E}}{Y} \hat{C}_{E,t} \\
\hat{R}_{t} &= \frac{H_{O,t}}{H} \hat{R}_{O,t} + \frac{H_{E}}{H} \hat{R}_{E,t} + \frac{H_{CC}}{H} \hat{R}_{CC,t} \\
\hat{Y}_{t} &= \left( 1 - \alpha_{K} - \alpha_{H} \right) \hat{N}_{S,t} + \left( 1 - \alpha_{K} - \alpha_{H} \right) \hat{N}_{CC,t} + \alpha_{K} \hat{K}_{t-1} + \alpha_{H} \hat{R}_{t-1} + \epsilon_{t} \\
\hat{b}_{G,t} &= \frac{R}{\Pi} \hat{b}_{G,t-1} + \frac{b_{G}}{R} \frac{R}{\Pi} \left( \tilde{R}_{t-1} - \tilde{N}_{t} \right) + \frac{G}{Y} \hat{G}_{t} + \frac{T_{t} - \tau_{w}}{Y} \left( \frac{\sigma_{CC}}{Y} \left( \hat{w}_{CC,t} + \hat{N}_{CC,t} \right) + \frac{\sigma_{O,t}}{Y} \left( \hat{w}_{O,t} + \hat{N}_{O,t} \right) \right) \\
\hat{R}_{t} &= (1 - d_{p,t}) \left( 1 - \phi_{t} \right) \left( \phi_{t} \hat{N}_{t} + \frac{d_{p}}{d_{p,t}} \left( \hat{Y}_{t} - \hat{Y}_{t} \right) \right) + d_{p,t} \hat{R}_{p,t} \\
\end{align*} \]


341 “Alexandre Lamfalussy and the monetary policy debates among central bankers during the Great Inflation”, by I. Maes and P. Clement, Research series, April 2018.

