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Abstract

This paper studies the implications of imperfect competition in firm-to-firm trade. Using a dataset on all transactions between Belgian firms, we find that firms charge higher markups if they have higher input shares among their buyers. We interpret this as firms competing as oligopolies to supply inputs to each buyer and build a model in which they charge different markups to different buyers. We use the estimated model to quantify how distortionary firm-to-firm markups are. Reducing all markups in firm-to-firm trade by 20 percent increases welfare by around 7 percent, suggesting large distortions due to double marginalization. We then investigate how endogenous markups in firm-to-firm trade alter predictions of the transmission of shocks. In the counterfactual where we take a fall in import prices as the shock, we show that allowing for oligopolistic competition generates larger cost reductions for some firms, and attenuates these for others relative to a case with constant markups. We demonstrate that a measure capturing firms’ positions in the production chain is a key metric in explaining this heterogeneity.

JEL codes: F12, L13, L14
Keywords: Firm-to-firm networks, imperfect competition

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1 Introduction

Firms largely operate and compete in relationships with other firms. Firms often deliver their output to multiple firms, and they often purchase inputs from multiple firms. These buyer-supplier relationships create a complex network of firm-to-firm transactions. One such complexity is that the set of firms that a firm competes against when supplying to a certain buyer may be different from those when supplying to a different buyer. This paper studies the nature of this competition in firm-to-firm trade and analyzes its implications.

We examine detailed administrative data on all domestic firm-to-firm transactions in Belgium. We explore and quantify to what extent firms engage in imperfect competition when they sell their outputs to each individual firm. In Belgium, the firm-to-firm network is extremely sparse, and firms’ outputs pass through many other firms until they reach final demand. To the extent that firms are in non-competitive environments when they engage in firm-to-firm trade, the distortions they face due to firm-to-firm markups are likely heterogeneous depending on how downstream they are. Quantification of these distortions requires accounting for the underlying structure of the firm-to-firm network.

We also explore the implications that markups in firm-to-firm trade have on counterfactual predictions. The Belgian data reveals large skewness in the input shares firms have across suppliers. If these input shares reflect suppliers’ abilities to charge markups, then accounting for the endogeneity of markups for each firm-to-firm pair becomes important when analyzing how shocks transmit through the economy. In response to shocks, firm-to-firm markups may change through changing firm-to-firm input shares, thus attenuating or amplifying both firm-level and aggregate outcomes.

The data points out to the importance of focusing on firm-to-firm relationships when studying firms’ competition. We find that firms charge higher average markups when they have larger input shares amongst their buyers. Firm-level average markups are measured by either computing accounting markups or estimating markups following De Loecker and Warzynski (2012). This positive relationship holds even after controlling for the firms’ sectoral market shares. We interpret this fact as firms competing as oligopolies to supply inputs to each buyer. In addition to the firm-level market share within a sector, the firm’s pairwise input shares for each buyer capture the pair-level pricing power the firm has for each of its buyers.

Motivated by this fact, we build a model of oligopolistic competition in firm-to-firm trade. With a nested CES structure in the production function that builds on Atkeson and Burstein (2008), firms charge different markups to each buyer firm. The more conventional implementation is where a firm’s total sales share among same-sector firms determines its firm-level markup; in our model, the markup a firm charges a buyer depends on the firm’s share in the buyer’s intermediate goods purchases. As firms compete with different sets of firms when selling to each buyer, the shares that firms have in each buyer’s intermediate goods vary across buyers. Therefore, the model puts emphasis on the firms’
pricing powers that vary across buyers.

Mapping the data to our model, we estimate the CES parameters in both preference and production functions. We obtain these estimates so that the firm-level average markups – averages of the model implied markups on sales to other producers and to the final consumer – provide the best fit of those implied by the data. The estimated CES parameters reveal that firms generally charge higher markups in their sales to other firms than in their sales to final demand.

Equipped with these estimates, we conduct two separate counterfactual exercises that explore the implications of oligopolistic competition in firm-to-firm trade. In the first counterfactual exercise, we quantify how distortionary firm-to-firm markups are. With each firm along the production chain charging a different markup in the observed firm-to-firm trade network, the degree of double marginalization can be large. From the estimated model we back out markups for each buyer-supplier pair in the data and consider the reduction in those markups in firm-to-firm trade as the shock. We find that in response to a 20 percent reduction in firm-to-firm markups, aggregate welfare – measured as the level of household consumption – increases by 7 percent.

We contrast these results to those obtained by assuming a sectoral roundabout production economy. In the sectoral roundabout production economy we impose a simple network structure in which there are two sets of common composite goods, one of which is used as intermediate goods and the other as final consumption goods. In this exercise we keep the initial firm-level sales, firm-level inputs, firms’ markups charged on sales to final demand, and firm-level average markups on intermediate goods sales consistent with our model. The impact of the markup reduction on welfare turns out to be smaller under the sectoral roundabout production economy: in response to a 20 percent reduction in markups charged on common composite intermediate goods, aggregate welfare increases by less than 5 percent. Failing to fully account for the observed firm-to-firm trade network leads to a smaller magnitude of distortion because the sectoral roundabout production economy cannot capture the heterogeneity in cost reductions that firms face. Under the observed firm-to-firm trade network, some firms that are downstream experience extreme cost reductions leading to greater movements in the aggregate due to non-linearities in the system.

In the second counterfactual exercise we explore how oligopolistic competition in firm-to-firm trade affects predictions of the transmission of shocks both at the firm-level and at the aggregate level. We shock an exogenous parameter in the model – the price of foreign goods – and see how the model’s predictions differ from those without endogenous markups in firm-to-firm trade. As a benchmark, we consider a special case of the model where we impose markups that are heterogeneous across buyers but constant.

Implementing endogenous markups leads to two counteracting effects on top of the effects predicted under constant markups. First, endogenous markups imply an incomplete pass-through from a change in the supplier’s input price to the change in its output price. When the price of foreign
goods fall, firms may increase their markups in response to reductions in their input costs. We call this the “attenuation effect,” as firms’ cost changes are not fully passed on to their buyers attenuating both firm-level and aggregate responses. Second, when a firm faces a reduction in its input costs, the other suppliers that sell goods to the firm’s buyers may reduce their markups in the face of increased competition. We call this the “pro-competitive effect,” as this amplifies the downstream effects of cost reductions.

We characterize the magnitudes of these two counteracting effects operating within each buyer-supplier pair. We find it important to account for endogenous markups in firm-to-firm trade to understand cost changes at the firm-level. In response to a foreign price change, around half of the firms face higher markups from their suppliers on average while the rest face lower markups. We demonstrate that a measure capturing the firms’ respective positions in the production chain is a key metric in explaining this heterogeneity. The more exposed a firm is to foreign inputs through its domestic suppliers, the higher markups the firm faces from its suppliers on average. And overall, under the uniform foreign price reduction that reduces the costs of all importers directly and of almost all firms indirectly, we find that accounting for endogenous markups in firm-to-firm trade has quantitatively small effects on aggregate welfare. The two counteracting effects largely cancel each other out in the aggregate.

This paper contributes to the literature studying the implications of imperfect competition in intermediate goods markets. Grassi (2018) develops a model in which firms engage in oligopolistic competition in an economy with sectoral input-output linkages and studies the contribution of firm-level shocks on the aggregate dynamics. Effects similar to our attenuation and pro-competitive effects are studied extensively in other contexts. For example, Feenstra, Gagnon, and Knetter (1996) study how the degree of price pass-through varies with the firm’s export market share. Amiti, Itskhoki, and Konings (2017) study how firms’ prices respond to changes in the prices of their competitors. Atkeson and Burstein (2008) focus on incomplete price pass-through to explain deviations of international relative prices from relative PPP. These papers analyze oligopolistic competition where firms compete with others within the same sector, implying that the firm’s market power is captured by its market share in its sector. In contrast, we propose a more granular view on the competition between firms. In addition to the firm-level market share within the sector being the determinant of the firm’s market power, we suggest that the pair-level input shares across its buyers are also relevant metrics.

1As in Grassi (2018), we focus on strategic complementarities across suppliers in the style of Atkeson and Burstein (2008). See Neiman (2011) for a similar model of variable markups that allows for arm’s length and intra-firm transactions. Mongey (2018) studies inflation and output responses to monetary shocks in oligopolistic market structures. Morlacco (2018) and Macedoni and Tyazhelnikov (2018) cast attention to firms’ market power as buyers of goods in international markets. For imperfect competition where complementarities arise from the demand side, see also Krugman (1979), Ottaviano, Tabuchi, and Thissé (2002), Melitz and Ottaviano (2008), and Zhelobodko, Kokovin, Parenti, and Thissé (2012).

2There are also cases in which aggregate volatilities can be captured by the distribution of market shares. See for example Gabaix (2011), where the Herfindahl-Hirschman Index (HHI) is the main metric that captures aggregate volatility.
in capturing the firm’s ability to charge markups.

This paper is also related to the vast literature investigating the implications of distortions. Our approach to assess the quantitative impact of distortions is similar to those in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), in which they compute the aggregate counterfactuals upon hypothetical reductions of wedges.\footnote{Other papers that investigate misallocations arising from imperfect competition, resource misallocations, and financial frictions include Hopenhayn and Rogerson (1993), Chari, Kehoe, and McGrattan (2007), Epifani and Gancia (2011), Fernald and Neiman (2011), Buera, Kaboski, and Shin (2011), Oberfield (2013), Bartelsman, Haltiwanger, and Scarpetta (2013), Midrigan and Xu (2014), Asker, Collard-wexler, and De Loecker (2014), Sandleris and Wright (2014), Hopenhayn (2014), Moll (2014), Edmond, Midrigan, and Xu (2015), Buera and Moll (2015), Peters (2016), Diingra and Morrow (2016), Gopinath, Kaleml-Ozcan, Karabarbounis, and Villegas-Sanchez (2017), Haltiwanger, Kulick, and Syverson (2017), Sraer and Thesmar (2018), Edmond, Midrigan, and Xu (2018), Bilbiie, Ghironi, and Melitz (2018), and Behrens, Mion, Murata, and Suedekum (2018).} We focus on a particular source of distortions, imperfect competition in firm-to-firm trade, and quantify how much distortion it creates in the aggregate. Focusing on imperfect competition in firm-to-firm trade also connects our paper to research on on firm boundaries and vertical relationships, seminally developed by Coase (1937). Our findings that reducing markups in firm-to-firm transactions can substantially lower firms’ costs relate our paper to the literature studying incentives of firms to vertically integrate. The efficiency motive for vertical integration has been intensively studied and empirically investigated for selected sectors (see Lafontaine and Slade, 2007, for a survey on this literature).\footnote{Antras (2003) investigates the relationship between vertical integration and trade, and develops a model with incomplete-contracting and allocation of property rights. For empirical investigations on the efficiency motives of vertical integration, see for example Grimm, Winston, and Evans (1992), Waterman and Weiss (1996), Chipty (2001), Hastings and Gilbert (2005), and Hortaçu and Syverson (2007).}

We also relate this paper to the important work by Baqaee and Farhi (2018), which provides a framework for aggregating micro shocks at the first-order or second-order approximation, using a general model with distortions such as markups.\footnote{An important benchmark in this literature is the work by Hulten (1978), which shows that the information on the structure of the production network is irrelevant in an efficient and closed economy up to a first order approximation. Building on this result, Baqaee and Farhi (2017) analyze the importance of second order effects of firm-level TFP shocks in an efficient economy. For other papers that investigate the effects beyond Hulten (1978)’s network irrelevance result, see Altinoglu (2015), Liu (2016), and Bigio and La’o (2017), which model firms facing financial constraints, and Pasten, Schoenle, and Weber (2017), which constructs a model with price rigidities.} Using U.S. firm-level data, they find that eliminating firm-level markups would increase aggregate TFP by around 20 percent.\footnote{Consistent with the findings from De Loecker and Eeckhout (2017), they find the distortions that firms’ markups create to increase over time.} In this paper we capture the heterogeneous markups firms potentially charge different buyers and investigate the distortions created by markups in firm-to-firm trade. The markups we back out using the structure of the model are generally higher in firm-to-firm trade than in firms’ sales to final demand. This implies that we consider the reductions in markups that are initially at higher levels than the firm-level markups one obtains by imposing them to be the same across destinations. We impose more structure on the production functions and the competition environment, and focus on the global firm-level and aggregate outcomes in response to large shocks. In doing so, we employ the technique developed by Dekle,
Eaton, and Kortum (2007), which enables us to compute the counterfactual outcomes with just the observed input shares and the estimated CES parameters.

Lastly, this paper also contributes to the literature on domestic production networks. The empirical literature has investigated shocks transmission through production networks. By examining firms sourcing from Japanese firms impacted by the 2011 Tohoku earthquake, Carvalho, Nirei, Saito, and Tahbaz-Salehi (2014) and Boehm, Pandalai-Nayar, and Flaaen (2016) have found that shocks to suppliers transmit to buyer firms. Barrot and Sauvagnat (2016) have also found shock transmission through production linkages by looking at firms sourcing from firms located in places hit by natural disasters in the U.S. In the context of sector-to-sector linkages, Acemoglu, Akcigit, and Kerr (2015) study the propagation of demand and supply shocks. Motivated by this evidence, we focus on how shocks transmit through the production network once oligopolistic competition in firm-to-firm trade is accounted for.

This paper proceeds as follows. Section 2 describes the data. This section also shows that suppliers charge higher markups if their input shares to buyers are higher. Section 3 outlines the model of oligopolistic competition in firm-to-firm trade along with several alternative models for comparison to the counterfactual results. In Section 4 we estimate the model’s underlying parameters. With the estimated model we quantify how distortionary markups in firm-to-firm trade are in Section 5. Section 6 investigates accounting for endogenous markups in firm-to-firm trade affects predictions of the transmission of shocks. Finally, Section 7 concludes.

For works studying the structure of domestic production networks, see Atalay, Hortacsu, Roberts, and Syverson (2011). Bernard, Dhyne, Magerman, Manova, and Moxnes (2018) explore the importance of firm-to-firm relationships in generating observed firm-size heterogeneity. For works on production networks in international trade, see handbook chapter of Chaney (2016).

A growing number of papers observe how extensive margins in firm-to-firm linkages play a role in the aggregate. For examples, see Baqee (2014), Lim (2015), Bernard, Moxnes, and Saito (2016), Oberfield (2017), Tintelnot, Kikkawa, Mogstad, and Dhyne (2018), and Taschereau-Dumouchel (2018).

In one of our counterfactual exercises, we consider the change in the foreign price as the shock and look at its firm-level and aggregate consequences. See, for example, Gopinath and Neiman (2014), Halpern, Koren, and Szeidl (2015), Magyari (2016), Antras, Fort, and Tintelnot (2017), Furusawa, Inui, Ito, and Tang (2017), and Tintelnot, Kikkawa, Mogstad, and Dhyne (2018) for papers studying the effects of import shocks on firms. On how such firm-level or other micro shocks lead to aggregate fluctuations, Gabaix (2011) and Carvalho and Gabaix (2013) show that firm-level shocks may not wash out in the aggregate if the firm-size distributions are fat-tailed. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) illustrate that firm-level shocks may lead to aggregate fluctuations if input-output structures are asymmetric. Di Giovanni, Levchenko, and Mejean (2014) and Magerman, De Bruyne, Dhyne, and Van Hove (2016) study the two potential sources of aggregate fluctuations together. Yeh (2016) points out that large firms tend to be less volatile, leading to mitigated effects of fat-tailed firm size distributions in the aggregate. Papers that study the importance of micro shocks on aggregate volatility include Jovanovic (1987), Durlauf (1993), Bak, Chen, Scheinkman, and Woodford (1993), Horvath (1998), Horvath (2000), Carvalho (2010), Foerster, Sarte, and Watson (2011), Di Giovanni, Levchenko, and Mejean (2014), Stella (2015), Atalay (2017), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017).
2 Data and evidence

2.1 Dataset and sample selection

Our main dataset is the National Bank of Belgium (NBB) Business-to-Business (B2B) transactions database, which is a panel of VAT-ID to VAT-ID transactions among the universe of Belgian VAT-IDs from 2002–2014. As explained in detail in Dhyne, Magerman, and Rubinova (2015), all enterprises in Belgium are assigned unique VAT-IDs and are required to report total yearly sales exceeding 250 Euro to other VAT-IDs. We also make use of the VAT declarations where we observe their total sales and total purchases.

We merge the datasets with the annual account filings and the international trade dataset. From the annual accounts we observe the primary sector of each VAT-ID (NACE Rev. 2, 4-digit), total sales, labor cost, ownership relations to other VAT-ID’s, location (ZIP code), and other variables that are standard in the annual accounts. The international trade dataset contains the values of imports and exports of goods at the VAT-country-product (CN 8-digit)-year level.

One firm can have multiple VAT-IDs. We focus on competitions and pricing decisions that occur across firm boundaries. The nature of these may be different from those within firm boundaries. Thus, we aggregate VAT-IDs up to the firm-level using ownership filings in the annual accounts and foreign ownership filings in the Balance of Payments survey. The Balance of Payments survey reports each VAT-ID, the name, and the country of a foreign firm that owns at least 10 percent of the shares, along with the associated ownership share. We group all VAT-IDs into firms if they are linked with more than or equal to 50 percent of ownership, or if they share the same foreign parent firm that holds more than or equal to 50 percent of their shares. See Appendix A.1 for further details.

We select private and non-financial sector Belgian firms that report positive sales, labor cost, and at least one full-time equivalent employee as our sample for analysis. Following De Loecker, Fuss, and Van Biesebroeck (2014), we select firms that report tangible assets of more than 100 Euro and positive total assets for at least one year throughout our sample period. Table 1 describes the coverage of our selected sample compared to the Belgian aggregate statistics.10 The numbers in Table 1 are identical to those in Table 1 in Tintelnot, Kikkawa, Mogstad, and Dhyne (2018), as we follow the same sampling and aggregation procedures. Note that the total sales in our sample turn out to be larger than those in the aggregate statistics. The differences can be explained by the fact that the output values in the aggregate statistics sum up value added for trade intermediaries instead of gross output, hence the smaller numbers in the aggregate statistics.

10In Appendix A.2 we also report the coverage of the full sample constructed in Dhyne, Magerman, and Rubinova (2015). There we also provide the sectoral composition of our sample, the aggregate statistics of the B2B dataset, and some descriptive statistics of the production network.
Table 1: Coverage of selected sample

<table>
<thead>
<tr>
<th>Year</th>
<th>Private, non-financial Imports</th>
<th>Exports</th>
<th>Selected sample Count</th>
<th>V.A. Sales</th>
<th>Imports</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>182</td>
<td>458</td>
<td>178</td>
<td>193</td>
<td>88,301</td>
<td>231</td>
</tr>
<tr>
<td>2007</td>
<td>230</td>
<td>593</td>
<td>254</td>
<td>267</td>
<td>95,941</td>
<td>299</td>
</tr>
<tr>
<td>2012</td>
<td>248</td>
<td>671</td>
<td>317</td>
<td>319</td>
<td>98,745</td>
<td>356</td>
</tr>
</tbody>
</table>

Note: All numbers except for Count are in billions of Euro in current prices. Belgian GDP and output are for all private and non-financial sectors. Data for Belgian aggregate statistics are from Eurostat. Value added is computed as the firms’ sales minus imports and their purchases from other Belgian firms in the selected sample. Total sales in our selected sample are larger than total output in the aggregate statistics because the output values in the aggregate statistics sum up the value added for trade intermediaries instead of their gross output.

All analyses in this paper we focus on firms from the Table 1 sample and on the firm-to-firm network among those in the selected sample. For the transactions between the selected firms and the non-selected firms, we do not consider the sales of selected firms that go to non-selected firms. On the input side, we classify input purchases to selected firms from non-selected firms as labor costs. Thus labor costs can be interpreted as a composite good that come from outside the selected firms. In Appendix A.2 we report the fractions of firms’ inputs that are affected by these classifications.

Table 2 shows the aggregate statistics using our selected sample. The number of firm-to-firm links in the economy is much smaller than the number of all possible links among all firms, indicating that the production network is extremely sparse.

Table 2: Aggregate statistics of the B2B dataset

<table>
<thead>
<tr>
<th>Year</th>
<th>Num. links</th>
<th>Num. links / Possible links</th>
<th>Total B2B sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>4,187</td>
<td>0.05%</td>
<td>199</td>
</tr>
<tr>
<td>2007</td>
<td>4,848</td>
<td>0.05%</td>
<td>206</td>
</tr>
<tr>
<td>2012</td>
<td>5,026</td>
<td>0.05%</td>
<td>225</td>
</tr>
</tbody>
</table>

Note: This table shows aggregate statistics of the firm-to-firm network, among the firms selected from the procedure described in Section 2.1. Number of links are in the thousands and the total B2B sales are in billions of Euro in current prices.

2.2 Skewed input shares across suppliers

With the data sample described, we first point to the fact that the distribution of the shares that suppliers have in a buyer’s input purchases is very skewed. For each buyer-supplier pair, we compute the share of sales from the supplier firm $i$ to the buyer firm $j$ out of $j$’s total input purchases:

$$s_{ij} = \frac{\text{Sales}_{ij}}{\text{InputPurchases}_j}. \quad (1)$$
Input purchases here includes all purchases from other Belgian firms in our sample and imports. Figure 1 displays a histogram of the input shares to the largest suppliers for all buyer firms in 2012. The figure restricts scope to firms with at least 10 suppliers. The input share of the largest supplier for the median firm in this figure is 29 percent.

**Figure 1:** Input shares of the largest suppliers

Note: $s_{ij}^m$ is defined as firm $i$’s goods share among firm $j$’s input purchases from other Belgian firms and abroad. The above histogram shows the distribution of $\max_i (s_{ij}^m)$, which is the maximum value of $s_{ij}^m$ for each buyer firm $j$ in 2012 that has at least 10 suppliers. The median value is 0.29.

Together with the fact that the median firm has 33 suppliers, the figure reveals that suppliers’ input shares are highly skewed throughout the economy. For each buyer, few suppliers tend to account for most input purchases. Appendix A.3 presents a histogram of the Herfindahl-Hirschman Index (HHI) of $s_{ij}^m$ for the same set of firms with at least 10 suppliers. We find that 50 percent of firms have a HHI above 0.15. 26 percent of firms have a HHI above 0.25.\(^{11}\)

### 2.3 Markups and input shares

We then explore if these heterogeneities across buyer-supplier pairs are important when considering firms’ markups. To do this, we determine if firm-level markups and firms’ average buyer input shares are positively associated with each other, even after controlling for firm-level sectoral market shares. A positive relationship suggests that firms’ market power contains pair-level components that come from each individual buyer in addition to firm-level components that are captured by sectoral market shares.

\(^{11}\)In Appendix A.4 we also present the analogous figures for the revenue shares, $r_{ij}$, which is defined as the share of firm $i$’s sales to $j$ out of firm $i$’s total sales.
shares.

Firm-level markups, $\mu_{i,t}$, are measured as the ratio of firms’ total sales over variable costs (the sum of input purchases and labor costs). This measure of firm-level markups is consistent with the model we construct in Section 3, which is static and features CRS production technologies. As firms might use additional factors, such as capital inputs, we consider alternative measures of firm-level markups in Appendix A.7 following De Loecker and Warzynski (2012).12

Firm-level sectoral market shares, $S_{cMktShare_{i,t}}$, are computed at the NACE 4-digit level. This measure captures firms’ market power in models that feature oligopolistic competition where firms’ outputs are aggregated at the sectoral level.

We construct a measure that captures the input shares firms have within their buyers. Using the pairwise input shares defined in equation (1), we compute firm $i$’s weighted average input shares to its buyers at year $t$, $s^m_{i,j,t}$, as

$$s^m_{i,j,t} = \frac{\sum_{j \in W_{i,t}} \text{InputPurchases}_{j,t} s^m_{i,j,t}}{\sum_{j \in W_{i,t}} \text{InputPurchases}_{j,t}} = \frac{\sum_{j \in W_{i,t}} \text{Sales}_{i,j,t}}{\sum_{j \in W_{i,t}} \text{InputPurchases}_{j,t}},$$

where $W_{i,t}$ is the set of $i$’s buyers at year $t$. Total input purchases are assigned as weights for each buyer firm.

With these variables, we run the following regression:

$$\mu_{i,t} = \beta S_{cMktShare_{i,t}} + \gamma s^m_{i,j,t} + \varphi X_{i,t} + \delta_t + \epsilon_{i,t}, \quad (2)$$

where firm-level controls and year fixed effects are included. Table 3 reports the results. The specification of the first column includes sector fixed effects, and the specifications of the second and the third columns include firm fixed effects. First, unsurprisingly, in all specifications we see a positive relationship between markups and firm-level market shares. The result in the third column, for example, indicates that within each firm, an increase of one standard deviation in the firm’s market share is associated with an increase of around 2.2 percentage points in the firm’s markup. More interestingly, even after controlling for these sectoral market shares, the coefficients on the firms’ average input shares to buyers are positive. The third column indicates that within each firm a single standard deviation increase in average input shares to buyers leads to around an increase of 3.9 percentage points in the firm’s markup. Controlling for firms’ size in each sector, firms have greater ability to charge markups if they have higher shares within their buyers’ inputs.

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12We exclude the user cost of capital in the calculation of markups in our baseline case. This is because the firm-to-firm trade data may capture purchases of capital goods. Adding a measure of user cost of capital leads to double counting of capital goods.
Table 3: Firm-level markups and input shares

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<tbody>
<tr>
<td>SctrMktShare(_{i,t}) (4-digit)</td>
<td>0.0219</td>
<td>0.0154</td>
<td>0.0221</td>
</tr>
<tr>
<td></td>
<td>(0.00280)</td>
<td>(0.00174)</td>
<td>(0.00201)</td>
</tr>
<tr>
<td>Average input share (\bar{s}_{i,j}^{\mu})</td>
<td>0.0524</td>
<td>0.0412</td>
<td>0.0391</td>
</tr>
<tr>
<td></td>
<td>(0.00395)</td>
<td>(0.00300)</td>
<td>(0.00290)</td>
</tr>
<tr>
<td>N</td>
<td>809722</td>
<td>781627</td>
<td>781627</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sector FE</td>
<td>4-digit</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.105</td>
<td>0.638</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

The positive correlation between markups, \(\mu_i\), and average input shares, \(\bar{s}_{i,j}^{\mu}\), after controlling for firms’ sectoral market shares indicate that firms’ sectoral market shares are not perfectly collinear with their average input shares to buyers. Appendix A.5 illustrates that this is indeed the case. In particular, we demonstrate that a firm with a high input share in a particular buyer is not necessarily large in terms of total sales. Therefore, in generating the observed distributions of pairwise input shares, \(s_{i,j}^{\mu}\), pairwise match components play a large role in addition to firm-level components. The relative size of the two coefficients is also worth discussing. Across the specifications, we see larger coefficients on the average input shares compared to those on the firm-level market shares. Additionally, we show in Table 11 in Appendix A.6 that the R-squared tends to increase more when adding the average input shares on the RHS, as opposed to adding the firm-level market shares. These results indicate that the variations in the average input shares within buyers’ inputs are more important for firms’ ability to charge markups than the variations in the sectoral market shares.

While our results show that buyer-supplier match specific components play an important role in explaining firm-level markups, there are several forces that drive these results. One can interpret these match specific components as firms customizing deliveries across buyers or selling goods of different qualities. One may also rationalize in line with theories in which these match specific components develop over time, such as relation-specific sunk costs. To partly account for these time-varying components, we control for the firm’s age and also for the average relationship age across its buyers. The positive correlation between markups and average input shares is robust even after these additional controls are added, meaning that there are also time-invariant aspects in the match specific components.

13 This is consistent with the time-invariant firm-country specific factors determining the exporters’ distribution of sales across countries, documented in Bernard, Moxnes, and Ullteit-Moe (2018). Another explanation could be non-homotheticities in production functions, as in Blaum, Lelarge, and Peters (2018). However, the positive correlation
potential sources that drive these components but treat them as pair-specific constant variables in the production functions, that reflect how suitable goods from each supplier are as inputs for the buyer.

The positive correlation is robust under different average measures of $s_{ij}$, such as taking simple averages or median values. Furthermore, it is also robust when using other measures of pairwise input shares. For example, instead of using $s_{ij}^m$ we use $s_{ij}$, which is the firm $i$’s sales share in $j$’s total variable inputs (input purchases plus labor costs). Another alternative share we use is the supplier’s sales share among the buyer’s goods inputs that are classified the same as the supplier’s, either at the 2-digit or 4-digit level. We report these results and those of other robustness checks in Appendix A.6.
3 Model

In the previous section, the Belgian firm-to-firm trade data revealed that firms charge higher markups when they have higher input shares within their buyers. We interpret this fact as firms competing as oligopolies to supply inputs to each buyer. In this section, we set up a model of oligopolistic competition in firm-to-firm trade. Throughout the model we assume a small, open economy, where we take the foreign price and the foreign demand shifter as given.

3.1 Preference

There is a representative household providing $L$ units of labor. Households have a CES preference over all firms’ goods with the substitution parameter $\sigma$. We assume that firms’ goods are substitutes, thus $\sigma > 1$. We also assume that households do not directly consume foreign goods in the heterogeneous goods sector. The household’s preference is denoted as

$$U = \left( \sum_{i \in \Omega} \beta_{iH} q_{iH}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}},$$  

(3)

where $\Omega$ denotes the set of domestic firms. $q_{iH}$ denotes the quantity of goods that firm $i$ sells to the household. Given the price that $i$ charges to the household, $p_{iH}$, $q_{iH}$ can be written as

$$q_{iH} = \beta_{iH}^{\sigma} p_{iH}^{\frac{1-\sigma}{\sigma}} E,$$

(4)

where $E$ denotes the aggregate expenditure. $P$ denotes the aggregate price index:

$$P = \left( \sum_{i \in \Omega} \beta_{iH}^{\sigma} p_{iH}^{\frac{1-\sigma}{\sigma}} \right)^{\frac{1}{1-\sigma}}.$$

(5)

Demand from abroad is modeled with the same structure as the domestic household. Let $I_{iF}$ be an indicator of whether firm $i$ is an exporter or not. Given a price that $i$ charges on exported goods, $p_{iF}$, export quantity, $q_{iF}$, can be written as

$$q_{iF} = I_{iF} p_{iF}^{\sigma} D^*,$$

(6)

where $D^*$ is the exogenous demand shifter from abroad.
3.2 Technology and market structure

Each firm produces a single differentiated good. In addition to labor inputs, they purchase goods from other firms and/or purchase imported goods as intermediate goods. On the output side, they sell goods directly to final demand, to other domestic firms, and/or export. We treat firms to be infinitesimal in the final demand market and assume Dixit and Stiglitz (1977) monopolistic competition. Thus firms charge constant markups on their goods when selling to the final consumer. We also assume that firms apply the same markups when exporting.

When considering firm-to-firm trade markets, the assumption of infinitesimal suppliers for each buyer is not consistent with the data. In Section 2.2, we showed that firms tend to have highly concentrated input share distributions. A handful of top supplier firms account for the majority of firms’ goods purchases. Moreover, in Section 2.3, we found that firms charge higher markups when they have higher input shares to buyers. Therefore, we assume oligopolistic competition in firm-to-firm trade, where firms charge different markups to different buyers depending on the shares they have in each buyer’s goods purchases. In doing so, we apply the framework of Atkeson and Burstein (2008) to firms’ pricing decisions in the relationships with each buyer.

Let $Z_i$ be firm $i$’s set of domestic suppliers and let $I_{Fi}$ be the indicator for the importing status of firm $i$. We denote $i$’s sector as $u$ and $j$’s sector as $v$. We assume nested CES structures in firms’ production functions. A firm first combines domestically supplied goods into sector-level intermediate goods bundles. Then it combines these sectoral goods and imported goods into a different intermediate goods bundle. Finally, the firm combines labor inputs and the intermediate goods bundle to produce its output. We denote the elasticity of substitution across firms’ goods in sector $u$ as $\sigma_u$. The substitution parameter across sectoral goods and imported goods is $\rho$, and the substitution parameter across labor inputs and the intermediate goods bundle is $\eta$. We assume all substitution parameters are above one.\(^{14}\)

The implied unit cost of firm $i$ is

$$c_i = \phi_i^{-1} \left( \omega_l^\eta w^{1-\eta} + \omega_m^\eta p_{mi}^{1-\eta} \right)^{1/\eta},$$

(7)

where $\phi_i$ is $i$’s core productivity. $\omega_l$ and $\omega_m$ denote CES weights in the production function on labor and intermediate goods. $w$ denotes wage, and $p_{mi}$ is the firm-specific price index of intermediate goods. $p_{mi}$ is another aggregate of firm $i$’s sector-level domestic intermediate price indices, $p_{vi}^m$, and the foreign price, $p_F$. $p_{mi}$ and $p_{vi}^m$ vary with firms’ sourcing strategy, $Z_i$ and $I_{Fi}$, along with the saliency

\(^{14}\)We do not impose any restrictions concerning the relative magnitudes among $\{\sigma_u\}$, $\rho$, and $\eta$ when we estimate them in Section 4.
parameters, $\alpha_{ji}$ and $\alpha_{Fi}$:

$$p_{mi} = \left( \sum_v \alpha_v^p (p_{mi}^v)^{1-p} + I_F \alpha_{Fi}^p p_{Fi}^{1-p} \right)^{1/p},$$

$$p_{vi}^m = \left( \sum_{j \in \mathcal{V} \cap \mathcal{V}_i} \alpha_{ji}^{\sigma_{vi}(j)} p_{ji}^{1-\sigma_{vi}(j)} \right)^{1-\sigma_{vi}(j)}.$$

(8)

$\mathcal{V}$ denotes the set of firms in sector $v$. The term $p_{ji}$ denotes the price that firm $j$ charges for its goods when selling to firm $i$. $p_F$ is the exogenous price of the foreign good. The terms $\alpha_{ji}$ and $\alpha_{Fi}$ reflect how suitable goods from firm $j$ and foreign imports are as inputs for firm $i$.

Before discussing the market structures of the final demand market and of the firm-to-firm markets, we derive the firms’ shares on inputs implied by the above CES structures. The share of firm $i$’s variable costs spent on labor, $s_{li}$, is

$$s_{li} = \omega_i^{\eta} w^{1-\eta} \phi_i^{1-\eta}.$$

(9)

The intermediate goods’ share, $s_{mi}$, becomes

$$s_{mi} = 1 - s_{li} = \frac{\omega_i^p p_{mi}^{1-\eta}}{c_i^{1-\eta} \phi_i^{1-\eta}}.$$  

(10)

Among $i$’s variable costs spent on intermediate goods, the share of sector $v$ goods, $s_{vi}^m$, and the share of foreign goods, $s_{Fi}^m$, are, respectively,

$$s_{vi}^m = \alpha_v^p \frac{(p_{vi}^m)^{1-p}}{p_{mi}^{1-p}},$$

$$s_{Fi}^m = I_F \alpha_{Fi}^p \frac{p_{Fi}^{1-p}}{p_{mi}^{1-p}}.$$  

(11)

Among $i$’s variable costs spent on sector $v$ goods, the share of firm $j$’s goods, $s_{ji}^{(j)}$, is

$$s_{ji}^{(j)} = \alpha_{ji}^{\sigma_{vi}(j)} \frac{p_{ji}^{1-\sigma_{vi}(j)}}{(p_{vi}^m)^{1-\sigma_{vi}(j)}}.$$  

(12)

Analogously, we can write $s_{ji}$ and $s_{Fi}$ respectively as the shares of $j$’s goods and foreign goods out of $i$’s total variable costs, $s_{ji} = s_{ji}^{(j)} s_{vi}^m s_{mi}$ and $s_{Fi} = s_{Fi}^m s_{mi}$.

Finally, we turn to the market structures. We assume monopolistic competition when firms sell to final demand. Firms charge a constant markup over marginal cost, and we assume the same when
firms export:

\[ p_{ij} = p_i = \frac{\sigma}{\sigma - 1} c_i. \]  

(13)

We introduce oligopolistic competition in firm-to-firm trade in the following way. When selling to firm \( i \), firm \( j \) sets price \( p_{ji} \) that maximizes variable profits by taking as given prices of \( i \)'s other suppliers and \( i \)'s unit cost and output, \( c_i \) and \( q_i \). Solving the firm’s profit maximization problem yields the following price:

\[ p_{ji} = \frac{\mu_{ji} c_i}{\varepsilon_{ji} - 1} \]

\[ \varepsilon_{ji} = \sigma_{v(j)} \left(1 - s_{v(j)}^v\right) + \rho s_{v(j)}^v \left(1 - s_{v(j)}^m\right) + \eta s_{v(j)}^m s_{v(j)}^m. \]  

(14)

The price implies that the markup firm \( j \) charges on firm \( i \), \( \mu_{ji} \), depends on the input shares that \( j \)'s goods have in \( i \)'s intermediate goods, \( s_{v(j)}^v \) and \( s_{v(j)}^m \). If the supplier \( j \) in sector \( v \) has an infinitesimally small share in buyer \( i \)'s intermediate goods bundle \( (s_{v(j)}^v \to 0) \), then all the competition the supplier \( j \) engages in are with the other suppliers in sector \( v \) sharing the same buyer \( i \). The price converges to the value obtained assuming monopolistic competition, a constant markup of \( \sigma_{v(j)} \sigma_{v(j)} - 1 \). As the supplier’s input share on the buyer increases, then not only does the supplier compete with the other suppliers, but also with other suppliers in sectors other than \( v \) and the labor input that buyer firm \( i \) employs. Thus, the demand elasticity the supplier faces, \( \varepsilon_{ji} \), is a weighted average of \( \sigma_{v(j)}, \rho, \) and \( \eta \). These weights are constructed from the shares \( s_{v(j)}^v \) and \( s_{v(j)}^m \). When the supplier \( j \) is the only firm supplying the buyer \( (s_{v(j)}^v, s_{v(j)}^m \to 1) \), the markup converges to \( \eta \). The intuition of how pairwise markups depend on pairwise shares are identical to what is described in Atkeson and Burstein (2008). The key difference is that here the relevant shares and markups are defined for each buyer-supplier pair.

As aforementioned, we assume that the supplier takes as given the buyer’s unit cost and output. This is consistent with the assumption of Bertrand competition, where firms take as given all others’ prices, including the prices of their buyers. A plausible alternative would be to assume that the supplier firm internalizes the change in demand for the buyer’s good when determining price. In this case, the supplier needs to know the output composition of the buyer firm to infer the elasticity of demand the buyer is facing. As firms are unlikely to observe the flow of goods distant in the production chain, we find our assumption to be reasonable.\(^{16}\)

\(^{15}\)See Appendix B.1 for firm \( j \)'s maximization problem.

\(^{16}\)The assumption that firms have incomplete information about firms that are distant in the production chain is similar to that considered by Antrás and de Gortari (2017). In Appendix B.2 we discuss in detail the optimal prices that firms charge their buyers under alternative assumptions. When a firm internalizes the effect of its price on the demand for the buyer’s goods, the markup it charges not only depends on \( s_{v(j)}^v \) and \( s_{v(j)}^m \), but also on quantities that the buyer sells to other firms and the quantities that it sells to final demand. One can also assume that firms take as given a constant demand elasticity buyers are presumed to face. In this case, if one assumes the value of the demand elasticity is \( \eta \), the pricing equation collapses to that of equation (14). In Appendix B.2 we also discuss optimal prices when firms engage in Cournot competition instead of Bertrand competition.
This assumption is also consistent with the empirical evidence. Section 2.3 confirmed that firms’ markups are correlated with the firms average input shares within their buyers. We further investigate if firms’ markups are correlated with the average input shares their buyers have within those buyers’ buyers. We find that the coefficient on these second-degree average input shares is not significant and close to zero. These results indicate that although firms charge higher markups when possessing have higher input shares in their buyers, this is not necessarily the case when their buyers have higher input shares. See Table 19 in Appendix A.6 for details.

Finally, we describe firms’ output and profits. A firm sells goods to households, abroad (if the firm is an exporter), and also to other domestic firms. Therefore we have

\[ q_i = q_{iH} + q_{iF} + \sum_{j \in W_i} \alpha_{ij}^{\sigma_{a(i)}} \frac{p_{ij}^{1-\sigma_{a(i)}}}{p_{m}^{1-\sigma_{a(i)}}} s_{a(i)}^m s_{m} c_j q_j, \]  

(15)

where we \( W_i \) is the set of \( i \)'s buyers. Firm \( i \)'s profits come from three sources: sales to households, exports, and sales to other domestic firms. So the variable profit of firm \( i \) can thus be described as

\[
\pi_i = \frac{1}{\sigma} \beta_{ih} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{\sigma}{\sigma - 1} \right) E_i^{\frac{1-\sigma}{\sigma}} + I_{iF} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{\sigma}{\sigma - 1} \right) E_i^{\frac{1-\sigma}{\sigma}} \]

Exports to HH

\[
+ \sum_{j \in W_i} \frac{1}{\sigma_{ij}} \frac{p_{ij}^{1-\sigma_{a(i)}}}{p_{m}^{1-\sigma_{a(i)}}} s_{a(i)}^m s_{m} c_j q_j. \]

(16)

3.3 Equilibrium

We close the model by assuming that firms’ profits are distributed back to the household. We also assume balanced trade. The household’s budget constraint becomes

\[ E = wL + \sum_{i \in \Omega} \pi_i. \]  

(17)

The trade balance and labor market clearing conditions are the following:

\[ [\text{TB}] : 0 = \sum_{i \in \Omega} I_{iE} p_{iH}^{1-\sigma} D_i - \sum_{i \in \Omega} I_{iF} S_{F_i} c_i q_i, \]

(18)

\[ [\text{LMC}] : wL = \sum_{i \in \Omega} S_{hi} c_i q_i. \]

(19)

We then characterize the equilibrium.
Definition (Equilibrium). Take as given the foreign demand shifter, $D^*$, and the foreign price, $p_F$. An equilibrium is a set of variables, \( \{w, P, E, q_i\} \), that satisfy equations (4)–(19).

Using the system of equations above that defines the equilibrium, in Sections 5 and 6 we solve for the equilibrium changes in firm-level costs and aggregate welfare, taking the changes in firm-to-firm markups or the changes in the foreign price as the shock. We implement the technique developed by Dekle, Eaton, and Kortum (2007), which enables us to compute the counterfactual outcomes with only shares directly observed in the data, \( \{s_{li}, s_{mi}, s_{ij}, s_{F,i}, s_{iH}\} \), and the estimated CES parameters.\(^{17}\)

### 3.4 Alternative models as benchmarks

Before estimating the CES parameters and conducting counterfactual exercises, we provide with variations of alternative modeling assumptions useful in benchmarking the counterfactual results of Sections 5 and 6.

#### Sectoral roundabout production economy

In Section 5 we consider reductions in firm-to-firm markups and quantify the distortions arising from double marginalization in firm-to-firm trade. To evaluate the results, we compare them with those from a sectoral roundabout production economy, in the spirit of Eaton and Kortum (2002). In this economy there are two sets of sector-level composite goods; one that is used as intermediate goods, the other as final consumption goods. We specify the firms’ cost function as the following:

\[
\begin{align*}
    c_i &= \phi_i^{-1} \left( \omega_i^j w^{1-\eta} + \omega_{m_{i}p_{m_i}}^{1-\eta} \right)^{\frac{1}{1-\eta}} \\
    p_{m_i} &= \left( \alpha_{iB_i}^{\rho} \left( \prod_v P_{vB}^{\gamma(u,v)} \right)^{1-\rho} + \alpha_{F,F}^{\rho} p_{F}^{1-\rho} \right)^{\frac{1}{1-\rho}} \\
    P_{vB} &= \left( \sum_{j \in V} \alpha_{jv}^{\sigma} p_{jB_k}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.
\end{align*}
\]

\( P_{vB} \) is the price index of the sector-\( v \)-specific composite good that is used as an intermediate good. It is a CES aggregate of prices firms charge in the intermediate goods market, \( p_{jB_k} \). \( \gamma_{uv} \) is the Cobb-Douglas share of sector \( v \) inputs in the production of sector \( u \)’s intermediate goods. Analogously, the final consumption good combines all sector-level composite goods. The aggregate price index \( P \) can be expressed as \( P = \prod_v P_{vH}^{\gamma_{vH}} \) where \( \gamma_{vH} \) is the Cobb-Douglas share of sector \( v \) goods among household’s consumption. The sector-\( v \)-specific price index in final consumption, \( P_{vH} \), can be expressed as \( P_{vH} = \left( \sum_{j \in V} \beta_{jH}^{v} p_{jH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \). We assume that firms charge constant markups, \( \mu_{iB_k} \).

\(^{17}\)See Appendices B.3 and B.5 for the system of equilibrium changes.
and $\mu_{iH_R}$, to both composite goods for intermediate goods and final consumption, $p_{iB_R} = \mu_{iB_R}c_i$ and $p_{iH_R} = \mu_{iH_R}c_i$. We assume these constant markups to be consistent with the average markups firms charge on intermediate goods sales and on their sales to final demand in our baseline model.\textsuperscript{18}

This sectoral roundabout production economy is useful as a benchmark because it assumes a simple network structure while keeping firm-level variables (such as firms’ sales, firms’ inputs, firms’ markups on final demand) and firm-level average markups on intermediate goods sales still consistent with the data. This roundabout economy has few production layers of intermediate goods, whereas the real firm-to-firm network features a much more complex production network structure. See Appendix B.4 for the system of equations solving for the changes in equilibrium variables under this sectoral roundabout production economy.

**Economy with constant markups**

In Section 6 we consider changes in the price of foreign goods as the shock and analyze whether accounting for oligopolistic competition in firm-to-firm trade alters predictions of the transmission of shocks. To this end, we consider as a benchmark an economy where firms charge constant markups in firm-to-firm trade. To make the comparison as consistent as possible, we assume firms charge the same heterogeneous markups that are implied by the baseline economy. However, here we assume markups are constant and do not change in response to shocks. This alternative model with constant markups is close to what is considered in Tintelnot, Kikkawa, Mogstad, and Dhyne (2018), but with sectoral layers in the production functions. We present the system of equations solving for the changes in equilibrium variables in Appendix B.6.\textsuperscript{19}

\textsuperscript{18}Specifically, we assume $\mu_{iH_R} = \sigma - 1$ and $\mu_{iB_R} = \frac{\sum_{j} p_{ij}q_{ij}c_iq_i - p_{iH_R}q_{iH_R} + p_{iF_R}q_{iF_R}}{c_{ij}q_{ij}p_{ij}q_{ij}}$.

\textsuperscript{19}To benchmark the results from Section 6, we additionally consider a back-of-the-envelope calculation of the economy’s aggregate response under the assumption of perfect competition. Analogous to the argument made by Hulten (1978), with perfect competition and other assumptions, one can solve for the aggregate counterfactual changes using firm-level information alone. We outline this approach in Appendix B.7.
4 Estimation

The counterfactual exercises using the model setup in the previous section require estimates of the CES parameters in the preference and production functions, \{\{\sigma_u, \rho, \eta, \sigma\}\}, and observables from the Belgian firm-to-firm trade data. In this section we describe the estimation procedures for the CES parameters.

We estimate the CES parameters, \{\{\sigma_u, \rho, \eta, \sigma\}\}, by exploiting the variations of sales and input shares at the firm-to-firm level in the data. Recall that in equation (14), pairwise markups, \(\mu_{ij} = \frac{\epsilon_{ij}}{\epsilon_{ij-1}}\), are functions of parameters, \{\{\sigma_u(i), \rho, \eta\}\}, and observable input shares, \(s_{ij}^{u(i)}\) and \(s_{ij}^{m(i)}\). We have also assumed markups firms charge on goods sold to domestic households and on exports, \(\mu_{ih}\), to be \(\sigma/\sigma - 1\).

In our static model, a firm’s total variable input cost — sum of labor costs, purchases from other firms, and imports, \(c_i q_i\) — has to equal the firm’s total sales, each deflated by destination-specific markups, \(\sum_j V_{ij} \mu_{ij} + V_{ih} \mu_{ih} + V_{if} \mu_{if}\). Denote the total variable input costs implied from the model as \(C_i = \sum_j V_{ij} \mu_{ij} + V_{ih} \mu_{ih} + V_{if} \mu_{if}\). Represent the difference between the observed input costs and the model implied input costs, relative to the observed input costs as

\[
\epsilon_i = \frac{c_i q_i - C_i}{c_i q_i}.
\] (21)

We assume that the accounting identity \(c_i q_i = C_i\) holds in the data up to a measurement error, \(\epsilon_i\):

**Assumption 1.** \(\epsilon_i\) are measurement errors and constant variables for each firm.

In the Belgian dataset we observe the input costs, \(c_i q_i\), firm \(i\)'s sales to firm \(j\), \(V_{ij}\), firm \(i\)'s sales to households, \(V_{ih}\), and firm \(i\)'s exports, \(V_{if}\), for all firms and input shares at the buyer-supplier level, \(s_{ij}^{u(i)}\) and \(s_{ij}^{m(i)}\). Using these observables, we estimate the CES parameters, \{\{\sigma_u, \rho, \eta, \sigma\}\}, by minimizing the squared sum of the measurement errors, \(\epsilon_i\):

\[
\min_{\{\sigma_u, \rho, \eta, \sigma\}} \sum_i \left(\frac{c_i q_i - C_i}{c_i q_i}\right)^2.
\] (22)

Since firms’ markups to final demand, \(\mu_{ih}\), are constants of \(\sigma/\sigma - 1\), the variations in the ratio of firms’ sales to final demand and exports \((V_{ih} + V_{if})\) over firms’ total inputs \((c_i q_i)\) pin down the value of \(\sigma\). Firm-to-firm markups, \(\mu_{ij}\), are functions of pair specific shares, \(s_{ij}^{u(i)}\) and \(s_{ij}^{m(i)}\), and parameters, \{\sigma_u, \rho, \eta\}. Thus the ratio of firm-to-firm sales \((V_{ij})\) over suppliers’ input costs \((c_i q_i)\), and the input shares \(s_{ij}^{u(i)}\) and \(s_{ij}^{m(i)}\) jointly determine the value of the two parameters.\(^{20}\)

We use the categorization of “intermediate SNA/ISIC aggregation A*38” in NACE Rev.2 classi-

\(^{20}\)Edmond, Midrigan, and Xu (2015) use a similar procedure with sectoral market shares to infer one of the CES parameters in models with endogenous markups.
fication, which leaves us to estimate 29 sectoral substitution parameters of $\sigma_u$ and three parameters of $\sigma$, $\rho$, and $\eta$.\footnote{See European Commission (2008) for details. We aggregate two A+38 codes, CD and CE, into one sector.} Finally, the model cannot accommodate firms total sales less than their variable input costs. We drop these firms from the estimation sample, losing around 15 percent of firms that account for around 28 percent of output. We report the estimation results in Table 4.\footnote{To evaluate the sensitivity of estimates to firms in the network, for each sector we draw firm-level samples from the data with replacements and compute the standard deviations of the estimates from the re-sampled data. However, as these firm-level observations are interdependent on the activities of their suppliers and buyers, standard asymptotic properties may not hold with the re-sampled data. See Chandrasekhar (2015) for discussions on conducting inference using network data.}
Table 4: Estimated CES parameters

(a) η, ρ, and σ

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>s.e.</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>η</td>
<td>1.92</td>
<td>0.13</td>
<td>1.28</td>
</tr>
<tr>
<td>ρ</td>
<td>2.16</td>
<td>0.22</td>
<td>0.07</td>
</tr>
<tr>
<td>σ−1</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

(Labor and goods) (Sectoral goods and imports in production) (Firms’ goods in consumption)

(b) Sectoral estimates of σₜₓ

<table>
<thead>
<tr>
<th>Description of sector</th>
<th>Estimate</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, forestry, and fishing</td>
<td>2.45</td>
<td>0.28</td>
</tr>
<tr>
<td>Mining and quarrying</td>
<td>2.40</td>
<td>0.28</td>
</tr>
<tr>
<td>Manufacture of food products, beverages, and tobacco products</td>
<td>3.37</td>
<td>0.46</td>
</tr>
<tr>
<td>Manufacture of textiles, apparel, leather, and related products</td>
<td>2.27</td>
<td>0.24</td>
</tr>
<tr>
<td>Manufacture of wood and paper products, and printing</td>
<td>3.08</td>
<td>0.40</td>
</tr>
<tr>
<td>Manufacture of coke, refined petroleum products, chemicals, and chemical products</td>
<td>2.69</td>
<td>0.32</td>
</tr>
<tr>
<td>Manufacture of pharmaceuticals, medicinal chemical, and botanical products</td>
<td>5.11</td>
<td>2.80</td>
</tr>
<tr>
<td>Manufacture of rubber and plastics products, and other non-metallic mineral products</td>
<td>3.53</td>
<td>0.48</td>
</tr>
<tr>
<td>Manufacture of basic metals and fabricated metal products, except machinery and equipment</td>
<td>2.98</td>
<td>0.38</td>
</tr>
<tr>
<td>Manufacture of computer, electronic, and optical products</td>
<td>2.33</td>
<td>0.25</td>
</tr>
<tr>
<td>Manufacture of electrical equipment</td>
<td>3.58</td>
<td>0.49</td>
</tr>
<tr>
<td>Manufacture of machinery and equipment n.e.c.</td>
<td>2.93</td>
<td>0.37</td>
</tr>
<tr>
<td>Manufacture of transport equipment</td>
<td>2.44</td>
<td>0.74</td>
</tr>
<tr>
<td>Other manufacturing, and repair and installation of machinery and equipment</td>
<td>2.41</td>
<td>0.27</td>
</tr>
<tr>
<td>Electricity, gas, steam and air-conditioning supply</td>
<td>2.05</td>
<td>0.70</td>
</tr>
<tr>
<td>Water supply, sewerage, waste management, and remediation</td>
<td>2.30</td>
<td>0.25</td>
</tr>
<tr>
<td>Construction</td>
<td>3.59</td>
<td>0.50</td>
</tr>
<tr>
<td>Wholesale and retail trade, repair of motor vehicles and motorcycles</td>
<td>2.27</td>
<td>0.25</td>
</tr>
<tr>
<td>Transportation and storage</td>
<td>3.00</td>
<td>0.39</td>
</tr>
<tr>
<td>Accommodation and food service activities</td>
<td>3.48</td>
<td>0.48</td>
</tr>
<tr>
<td>Publishing, audiovisual and broadcasting activities</td>
<td>2.50</td>
<td>0.29</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>2.07</td>
<td>0.40</td>
</tr>
<tr>
<td>IT and other information services</td>
<td>2.24</td>
<td>0.24</td>
</tr>
<tr>
<td>Real estate activities</td>
<td>1.76</td>
<td>0.15</td>
</tr>
<tr>
<td>Legal, accounting, management, architecture, engineering, technical testing, and analysis activities</td>
<td>1.70</td>
<td>0.13</td>
</tr>
<tr>
<td>Scientific research and development</td>
<td>4.76</td>
<td>2.94</td>
</tr>
<tr>
<td>Other professional, scientific and technical activities</td>
<td>2.60</td>
<td>0.31</td>
</tr>
<tr>
<td>Administrative and support service activities</td>
<td>2.41</td>
<td>0.27</td>
</tr>
<tr>
<td>Other services</td>
<td>2.63</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Note: Standard errors are based on 25 bootstrap samples drawn with replacements. The samples are drawn at the firm-level for each sector.

In the production function the substitution parameter across labor and goods is 1.92. Within intermediate goods, the substitution parameter across sectoral goods and imported inputs is 2.16.
the preference function, the substitution parameter across goods is 4.55. The estimated values fall in ranges not far from the findings of different approaches. Chan (2017) finds labor and intermediates to be gross substitutes. The survey of Anderson and van Wincoop (2004) finds that, within sectors, the elasticity of substitution across goods in the production function ranges from around 5 to 10 depending on the aggregation. Our estimates of $\sigma_u$ are slightly lower than this because our estimates pick up the substitutability of firms goods among the small set of suppliers that firms source from in each sector instead of the substitutability of goods among all firms in each sector.\footnote{Our approach of estimating CES parameters is different from that of other papers that estimate substitution parameters at higher frequencies. For example Boehm, Pandalai-Nayar, and Flaen (2016), Barrot and Sauvagnat (2016), and Atalay (2017) find much lower estimates in the production function parameters.}

We turn to the estimates under alternative setups. In our model, firms engage in Bertrand competition in firm-to-firm trade. As an alternate specification one can assume that firms engage in Cournot competition, which leads to a different formula for pairwise markups $\mu_{ij}$:

$$p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1} c_j$$

$$\varepsilon_{ji} = \left( \frac{1}{\sigma_{v(j)}} \left( 1 - s^v_{ji} \right) + \frac{1}{\rho} s^{s(j)} \left( 1 - s^m_{ji} \right) + \frac{1}{\eta} s^{s(j)} s^{m(j)} \right)^{-1}.$$

We estimate the three parameters in this setup and report the results in Appendix C.2.

Our estimates for the parameters are not affected when assuming oligopolistic competition in the final goods market. This is because for most firms, shares in the final goods consumption are very small, which validates our assumption of monopolistic competition.

Finally, it is worth highlighting the lack of capital goods in our model. We sum firms’ total labor costs, purchases from other domestic firms, and imported goods in our measurement of firms’ total inputs, $c_i q_i$. Missing capital inputs will lower our measurement of $c_i q_i$. If the degree of capital intensity is correlated with the firm’s sales, it violates our assumption of uncorrelated errors. To accommodate this potential issue, we account for firms’ capital inputs in two alternative ways: scaling up labor costs of firms uniformly by assuming a common labor-to-capital share; computing firm-level capital costs from the annual accounts data. We report the results in Appendix C.3.
5 How distortionary are markups in firm-to-firm trade?

With the estimated parameters, in this section we explore how distortionary markups in firm-to-firm trade are. The observed input shares at the buyer-supplier level, $s_{ij}^{(m)}$ and $s_{ij}^{(n)}$, and the CES parameters enable us to back-out the pair-specific markups implied by the model (see equation (14)). We consider a reduction in those markups in firm-to-firm trade, $\mu_{ij}$, as the shock.

Because firms’ outputs pass through many other firms until reaching final demand, the effect of a reduction in a markup that firm $i$ charges to firm $j$, $\hat{\mu}_{ij}$, will be amplified when firm $j$ reduces markups to its buyers. We feed in the shock of $\hat{\mu}_{ij}$, where $\mu_{ij}$ is the markup backed-out from the data, and consider the following system of counterfactual changes:

$$\hat{c}_i^{1-\eta} = s_i \hat{w}^{1-\eta} + s_m \hat{p}_m^{1-\eta}$$

$$\hat{p}_m^{1-\rho} = \sum_v s_v^m (\hat{p}_v^m)^{1-\rho} + \hat{s}_v^m$$

$$(\hat{p}_v^{m})^{1-\sigma_v} = \sum_{j \in Z \cup \gamma} s_j^{(v)} \hat{p}_j^{1-\sigma_v(i)}$$

$$\hat{C}_i = \frac{V_{ih}}{C_i \mu_{ih}} \hat{s}_{ih} \hat{E} + \frac{V_{if}}{C_i} \hat{V}_{if} \hat{q} + \frac{1}{C_i} \sum_j \frac{V_{ij} \hat{s}_{ij}}{\mu_{ij}} \hat{C}_j$$

$$\hat{E} = \frac{1}{1 - \sum_i \frac{1}{\hat{p}_m^{1-\rho} V_{ih} \hat{s}_{ih}}} \left( \frac{wL}{E} \hat{w} + \sum_i \frac{\pi_i}{E} \left( \sum_k \frac{1}{\pi_i} V_{ik} \hat{\mu}_{ik} \hat{\mu}_{ik} - \frac{1}{\mu_{ih}} \hat{s}_{ih} \hat{E}_k \frac{1}{\mu_{ih}} V_{if} \hat{V}_{if} \right) \right)$$

$$\hat{w} = \frac{1}{wL} \sum_i \frac{V_{ij} \hat{s}_{ij}}{\mu_{ij} \mu_{ih}}$$

(C_i denotes the total input values of firm $i$ implied by the model: $C_i = \sum_j \frac{V_{ij}}{\mu_{ij}} + \frac{V_{if}}{\mu_{ih}} + \frac{V_{ih}}{\mu_{ih}}$. Furthermore, $s_j^{(v)} = \hat{p}_j^{1-\sigma_v(i)} \hat{c}_j^{1-\sigma_v(i)} (\hat{p}_v^{m})^{(1-\sigma_v(i)-1)}$, $s_v^{m} = (\hat{p}_v^m)^{1-\rho} \hat{s}_v^m$, $\hat{s}_m = \hat{p}_m^{1-\eta} \hat{c}_i^{1-\eta}$, $\hat{s}_{ij} = s_j^{(v)} \hat{s}_m \hat{s}_{ij}$, $\hat{s}_{ih} = \hat{w}^{1-\eta} \hat{c}_i^{1-\eta} \hat{p}_v^{1-\sigma}$, $\hat{V}_{if} = \hat{c}_i^{1-\sigma}$.)

Taking the data into the system above reveals that firms’ total input values implied by the model, $C_i$, do not necessary match the observed input values, $c_i q_i$. While we minimized the difference between the two when estimating the CES parameters, the model under the estimated parameters is still not entirely consistent with the data. For some firms the observed inputs, $c_i q_i$, are larger than the model implied values, $C_i$. For other firms, the observed input values seem lower than is necessary to produce what is sold. To be consistent with the estimation strategy, in the counterfactual analyses we take the error term in equation (21), $\epsilon_i = \frac{c_i q_i - C_i}{c_i q_i}$, as constants. We designate $\xi_i$ as the difference between the observed input values and model implied input values, $\xi_i = c_i q_i - C_i$. With this assumption, the changes in the observed inputs, $c_i q_i$, are equal to the changes in the model implied inputs, $\hat{C}_i$, and are also equal to the changes in the difference between the two, $\hat{\xi}_i$. One may alternatively take the values of $\xi_i$ as constant numbers, and solve for both $c_i q_i$ and $\hat{C}_i$ using the
relationship \( \bar{c}_i \bar{q}_i = c_i \hat{C}_i + \xi_i \). However, for firms with negative values of \( \xi_i \) and under extreme cases where the values of \( \hat{C}_i \) are low, \( \bar{c}_i \bar{q}_i \) can become negative and not well defined. We treat the observed trade balance as fixed in the counterfactual analyses. We outline the detailed steps solving the system of counterfactual changes in Appendix B.3.

To evaluate the results, we contrast them with the results from the sectoral roundabout production economy described in Section 3.4. The sectoral roundabout production economy imposes a particular structure on the production network. There are two distinct composite goods, one of which is used as intermediate goods and the other is used as a final consumption good. The sectoral roundabout production economy does not match the observed firm-to-firm transactions but matches the firm-level exports, imports, domestic sales, labor costs, domestic purchases, value added, markups charged to final demand sales, and firm-level average markups charged to sales to intermediate goods. Therefore, this comparison with the roundabout production economy is useful for evaluating the implications of markup distortions that account for the real firm-to-firm network. We consider the reduction in the markups firms charge to the composite good used as intermediate goods as the shock. We outline the system of counterfactual changes under the sectoral roundabout production economy in Appendix B.4.

We focus on a 20 percent reduction in markups under the two models. That is, we feed in the shock of \( \hat{\mu}_{ij} = \frac{(\mu_{ij} - 1) \times 0.8 + 1}{\mu_{ij}} \) for the baseline economy and \( \hat{\mu}_{ijB} = \frac{(\mu_{ijB} - 1) \times 0.8 + 1}{\mu_{ijB}} \) for the sectoral roundabout production economy. We present the results of firm-level cost changes under the two economies in Figure 2. In both economies the cost changes are bounded from above by the increases in the nominal wage, \( \hat{\psi} \), which are 4.2 percent in the baseline economy and 4.4 percent in the sectoral roundabout production economy. While there is a large heterogeneity in the cost changes under the baseline economy, the cost changes under the sectoral roundabout production economy are more compressed. In the baseline economy some firms’ costs decrease by up to 37 percent, the largest decline in firm-level costs in the sectoral roundabout production economy is only by 14 percent. This is because the sectoral roundabout production economy cannot capture the within sector heterogeneities in firms’ exposure to other firms’ goods.

\[^{24}\text{We present results for the other magnitudes of markup reductions in Appendix D.3.}\]
We find that the magnitudes of firm-level cost reductions in the baseline economy are correlated with measures that capture how downstream or upstream firms are positioned. One way to measure the downstreamness of firms is to compute firms’ revenue share of sales to domestic final demand, $r_{iH} = \frac{V_{iH}}{V_i}$. Another approach is to compute the upstreamness measure by Antràs, Chor, Fally, and Hillberry (2012). Firm-level cost changes, $\hat{c}_i$, are negatively correlated with $r_{iH}$ with correlation of $-0.22$, and are positively correlated with the upstreamness measure with correlation of $0.14$. See Appendix D.1 for details.

We next turn to the effects on the aggregate welfare. Table 5 reports the aggregate welfare effects of a 20 percent reduction in firm-to-firm markups in the baseline economy using the observed firm-to-firm transaction data, and of a 20 percent reduction in markups on the composite good used as intermediate goods in the sectoral roundabout production economy. We also report the changes in the aggregate expenditures, $\hat{E}$, price indices relative to nominal wage, $\hat{P}/\hat{w}$, and aggregate profits, $\hat{\Pi}$. The aggregate welfare goes up by 7.3 percent in the baseline case; in the sectoral roundabout production economy, the increase in welfare is less than 5 percent. We also see that the baseline economy predicts larger magnitudes in the movements of all other aggregate variables.

We highlight that the aggregate expenditures, $E = wL + \Pi - \sum_i \xi_i - TB$, are not only affected by the change in the nominal wage, $w$, but also by the change in aggregate profits, $\Pi$, and the change in the sum of firm-level differences in observed input costs and model implied input costs, $\sum_i \xi_i$.

The baseline economy exhibits a larger increase in the aggregate profits, contributing to the larger increase in the aggregate expenditure. When firms set prices to buyers, firms do so by maximizing profits taking as given the demand shifters they face. But here we consider the case where all firms reduce their markups at the same time, leading to changes in the demand shifters firms face through
the general equilibrium. In the observed firm-to-firm network, firms’ outputs go through multiple firms until final demand, resulting in larger magnitudes of these general equilibrium effects. On the other hand, in the sectoral roundabout production economy all firms’ output reach final demand through a layer of the composite goods, resulting in smaller magnitudes of changes in the demand shifters.

In addition to these changes in profits, as markups go down and firms use greater amount of inputs, the differences in observed input costs and model implied input costs, $\sum_i \xi_i$, become larger as well. This stems from the assumption that we keep the firm-level ratio of $\epsilon_i = \xi_i / c_i q_i$ fixed instead of the values of the differences, $\xi_i$. In both the baseline economy and in the sectoral roundabout economy, the total differences, $\sum_i \xi_i$, increase by 31 percent. These increases in $\sum_i \xi_i$ move in the direction that will decrease the aggregate expenditures, $E$. Therefore we interpret the 2.1 percent increase in the aggregate expenditure as a conservative estimate. In order to only take into account the effects of the changes in the nominal wage and the aggregate profits on the changes in aggregate welfare, in Appendix D.2 we present results on the aggregate changes from three different approaches. First, we compute the changes in aggregate welfare without considering the change in $\sum_i \xi_i$. We define $\hat{E}$ as $wL + \Pi - TB$ and present the aggregate changes that come from the changes in $\hat{E}$. Second, we present counterfactual results in which we treat $\xi_i$ as fixed instead of treating $\epsilon_i$ as fixed.\footnote{As mentioned on page 24, this approach cannot accommodate extreme cases.} Third, we follow the approach by Ossa (2014) and first eliminate the differences between the observed and model implied input values, $\xi_i$. We solve for the counterfactual changes by forcing the observed differences, $\xi_i$, to zero. The resulting economy becomes fully consistent with the model, with which we can solve for the counterfactual changes under $\hat{\mu}_{ij}$. In all three approaches aggregate expenditure, $E$, increases by larger amounts.

Table 5: Aggregate effects of a 20 percent reduction in markups

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Sectoral roundabout</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U} = \hat{E}/\hat{P}$</td>
<td>1.073</td>
<td>1.049</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>1.021</td>
<td>1.010</td>
</tr>
<tr>
<td>$\hat{P}/\hat{w}$</td>
<td>0.913</td>
<td>0.920</td>
</tr>
<tr>
<td>$\hat{\Pi}$</td>
<td>1.100</td>
<td>1.079</td>
</tr>
</tbody>
</table>

Note: In the baseline case we take the baseline model using the observed firm-to-firm trade network in 2012. We feed a 20 percent reduction in all markups in firm-to-firm trade as the shock, $\hat{\mu}_{ij} = (\mu_{ij} - 1) \times 0.8 + 1$. In the roundabout production case we take the roundabout production economy using the observed firm-level sales and inputs in 2012. We feed a 20 percent reduction in all markups charged to the composite good used as intermediate goods, $\hat{\mu}_{iB} = (\mu_{iB} - 1) \times 0.8 + 1$. It is worthwhile to put these numbers in context with other papers in the literature. Baqae and Farhi (2018) use firm-level data with sectoral Input-Output data from the U.S. and find that eliminating firm-level markups would lead to an increase in the TFP by around 20 percent at the
second-order approximation. Instead of taking first-order or second-order approximations, we impose a particular structure on how markups are determined at the firm-to-firm level, and compute the welfare benefits of reducing those markups. Although these numbers are not directly comparable, one reason we predict large aggregate effects is that the markups we back out are generally higher in firm-to-firm trade than in firms’ sales to final demand. With our estimates of the CES parameters, our model indicates that while firms charge markups of 1.28 in their sales to final demand, firms on average charge markups of around 1.62 to other firms.\(^{26}\) Instead of assuming firm-level markups that are common across destinations, we incorporate these differences and consider reductions in markups that are initially at higher levels.

Another reason relates to the greater effects of our baseline economy compared to the sectoral roundabout production economy. Considering the observed firm-to-firm network generates substantial heterogeneity in firm-level cost changes, and there are a few firms that experience extreme cost reductions.\(^{27}\) Due to non-linearities in the system of equilibrium changes (23), firms with large cost reductions obtain larger shares in both the final demand market and among their buyers’ inputs, which leads to greater movements in the aggregate. As we saw in Figure 2, the sectoral roundabout production economy fails to capture large heterogeneity in firm-level cost changes. In Appendix D.3 we also consider the system of first-order approximated equilibrium changes and illustrate the non-linearities of the system by analyzing different magnitudes of shocks. We find that under first-order approximation, eliminating all firm-to-firm markups would lead to a 30 percent approximate increase in the aggregate welfare.

\(^{26}\) The level of these accounting markups are generally higher than other estimates of markups for Belgium (see for example De Loecker et al., 2018). This is primarily because our measure of accounting markups only take into account imports, labor costs, and inputs from other firms as production costs, and exclude other costs such as capital usage costs.

\(^{27}\) Because of this subset of firms that experience extreme cost reductions, we cannot compute counterfactual changes upon shocks of magnitudes larger than 50 percent reduction in markups. Beyond this point, there are firms with \(\hat{c}_i\) so close to zero that a numerical solution is not obtained.
6 How do markups in firm-to-firm trade alter predictions of the transmission of shocks?

In the previous section, we backed out markups at the buyer-supplier level and took the reduction of these markups as the exogenous shock. The exercise informed us of the magnitude of distortions that are present due to double marginalization in the observed firm-to-firm network. Another point of interest is the treatment of these markups as endogenous variables and the investigation of how these endogenous markups in firm-to-firm trade alter predictions of transmission shocks.

In this section we take the exogenous parameter of the model, the price of foreign goods, \( p_F \), and use its changes as the shock. Similar to the approaches taken in the previous section, we take as given the change in foreign price, \( \hat{p}_F \), and consider the system of counterfactual changes presented below:

\[
\begin{align*}
\hat{c}_i^{1-\eta} &= s_{ii} \hat{\omega}^{1-\eta} + s_{mi} \hat{p}_m^{1-\eta} \\
\hat{p}_m^{1-\rho} &= \sum_s s_{vi}^m (\hat{p}_m^s)^{1-\rho} + s_{Fi}^m \hat{p}_F^{1-\rho} \\
(\hat{p}_m^s)^{1-\sigma_j} &= \sum_{ji \in E} s_{ji}^{(j)} \rho_j^{1-\sigma_j} \hat{c}_j^{1-\sigma_j} \\
\hat{\mu}_{ji} &= \hat{\epsilon}_{ji} - 1 \\
\hat{\hat{C}}_i &= \frac{1}{C_i \mu_i} \hat{\hat{s}}_{ii} \hat{\hat{E}} + \frac{1}{C_i \mu_i} \hat{V}_{iF} \hat{V}_{iF} + \frac{1}{C_i} \sum_j \hat{V}_{ij} \hat{\hat{C}}_j \\
\hat{\hat{E}} &= \frac{1}{1 - \sum_i \frac{1}{\mu_i} \hat{\hat{V}}_{iH} \hat{\hat{s}}_{ii}} \left( \frac{wL}{E} \hat{\hat{w}} + \sum_i \frac{\pi_i}{E} \left( \sum_k \frac{1}{\pi_k} \hat{V}_{ik} \hat{\hat{\mu}}_{ik} \hat{\hat{\mu}}_{ik} - \frac{1}{\mu_i} \hat{\hat{s}}_{iH} \hat{\hat{C}}_i + \frac{1}{\mu_i} \hat{\hat{V}}_{iF} \hat{V}_{iF} \right) \right) \\
\hat{\hat{w}} &= \frac{1}{wL} \sum_i s_{ii} \hat{c}_{ii} q_{ii} \hat{\hat{C}}_{ii}.
\end{align*}
\]

Furthermore, \( \epsilon_{ji} \hat{\epsilon}_{ji} = \sigma_i \hat{\sigma}_{ji} (1 - \hat{s}_{ji}^{(j)} \hat{s}_{ji}^{(j)}) + \rho_i \hat{\rho}_{ji} \hat{s}_{ji}^{(j)} (1 - \hat{s}_{vi}^{(j)} \hat{s}_{vi}^{(j)}) + \eta_i \hat{\eta}_{ji} \hat{s}_{ji}^{(j)} \hat{s}_{vi}^{(j)} \hat{s}_{vi}^{(j)} \hat{s}_{ji}^{(j)}, \) \( \hat{s}_{ji}^{(j)} = \hat{\mu}_{ji}^{1-\sigma_j} \hat{\hat{c}}_j^{1-\sigma_j} (\hat{p}_m^{s_{ji}})^{\sigma_j(1-\sigma_j)}, \) \( \hat{s}_{vi}^{(j)} = (\hat{p}_m^{s_{ji}})^{1-\rho} \hat{\hat{p}}_{mi}^{1-\rho}, \) \( \hat{s}_{mi} = (\hat{p}_m^{s_{ji}})^{1-\eta} \hat{\hat{p}}_{mi}^{1-\eta}, \) \( \hat{s}_{iH} = \hat{\hat{c}}_i^{1-\sigma_j} \hat{\hat{p}}_{iH}^{1-\sigma_j}, \) and \( \hat{\hat{V}}_{iF} = \hat{\hat{c}}_i^{1-\sigma_j}. \) The above system is different from the system presented in (23) in that we now have the shock \( \hat{p}_F \) in the second equation and that the changes in firm-to-firm markups, \( \hat{\mu}_{ij} \), are additional endogenous variables to solve for. As in the previous section, we treat the error terms in equation (21), \( e_i = \frac{\hat{e}_i}{\hat{q}_{ii}} \), and observed trade balance as constants.

We focus on a 20 percent reduction in the foreign price as the shock, \( \hat{p}_F = 0.8 \), and analyze its firm-level and aggregate consequences.\(^{28}\) We compare these results with those from the alternative model of constant markups laid out in Section 3.4.

We plot the histogram of firm-level cost changes under the above system in Figure 3a. Nominal

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\(^{28}\)We present results for the other magnitudes of foreign price reductions in Appendix E.5.
wage goes up by 3.22 percent to ensure trade balance. The firm-level cost changes are bounded from above by this wage change. We compare these firm-level cost changes to those obtained from a model with constant markups. In this model firms charge constant markups, $\mu_i$, which is the average of the markups firms charge to different destinations in the baseline economy. The system of counterfactual changes is presented in Appendix B.6. Since the nominal wage increases by 3.18 percent in the constant markups economy, we work with cost changes normalized for the wage changes, $\hat{c}_i/\hat{w}$, to make comparisons. Figure 3b plots the differences in these normalized firm-level cost changes, relative to the normalized cost change in the constant markups economy, $\frac{\hat{c}_{\text{endog}}}{\hat{w}_{\text{endog}} - \frac{\hat{c}_{\text{const}}}{\hat{w}_{\text{const}}}}$. For example, if a firm has $\frac{\hat{c}_{\text{endog}}}{\hat{w}_{\text{endog}} - \frac{\hat{c}_{\text{const}}}{\hat{w}_{\text{const}}}} = 0.05$, then the firm experiences smaller cost reduction relative to the nominal wage by 5 percent, compared to the cost reduction relative to the nominal wage under constant markups. Incorporating endogenous markups has very different implications to cost changes across firms in the economy. Around 50 percent of firms experience smaller cost reductions in endogenous markups compared to constant markups, while the rest experience larger cost reductions. In terms of magnitudes, 10 percent of firms experience cost reductions that are smaller than under constant markups by 1 percent or more; 12 percent of firms experience cost reductions that are greater than under constant markups by 1 percent or more.

**Figure 3:** Histograms of cost changes, $\hat{c}_i$, endogenous markups and constant markups

![Histograms of cost changes](image)

Note: The left figure plots the distribution of firm-level cost changes using the baseline model under a 20 percent reduction in the foreign price. The right figure plots the distribution of the differences in normalized cost changes under the baseline economy and the constant markups economy. The right figure is truncated from below, and the minimum value of $\frac{\hat{c}_{\text{endog}}}{\hat{w}_{\text{endog}} - \frac{\hat{c}_{\text{const}}}{\hat{w}_{\text{const}}}}$ is -0.45.

We then characterize these heterogeneous implications on firm-level costs. Which firms experience larger cost reductions and which firms experience smaller cost reductions when incorporating endogenous markups? For the sake of characterization, we focus on the first-order approximated
system of equilibrium changes. Equation (25) shows the first-order approximated change of the cost of firm $i$, $\frac{dc_i}{c_i}$. When one accounts for endogenous markups at the buyer-supplier level, the changes in markups charged by its suppliers, $\frac{d\mu_j}{\mu_j}$, enters as additional variables that affect firm $i$’s cost. The changes in firms’ unit costs are affected by the changes in the unit costs of their suppliers, the changes in the markups these suppliers charge, change in the nominal wage, and change in the foreign price, each weighted by the firms’ exposure to these inputs.

$$\frac{dc_i}{c_i} = \frac{dw}{w} + \sum_{j \in Z_i} s_{ji} \left( \frac{d\mu_{ji}}{\mu_{ji}} + \frac{dc_j}{c_j} \right) + s_{Fi} \frac{dp_F}{p_F}.$$  \hspace{1cm} (25)

The changes in markups that $i$’s suppliers charge, $\frac{d\mu_j}{\mu_j}$, can be decomposed into two counteracting forces that push $i$’s cost in opposite directions. Equation (26) shows the decomposition. The first term captures what we call the attenuation effect, as the reduction in supplier $j$’s cost leads to an increase in the markup $j$ charges $i$, $\mu_{ji}$. On the other hand, the second term in equation (26) shows that the markup $\mu_{ji}$ is also affected by $i$’s other suppliers besides $j$. If the prices of other suppliers and imported goods decline on average, then the supplier $j$ reduces its markup to $i$ in face of increased competition. We call this second effect the pro-competitive effect.\footnote{See Feenstra, Gagnon, and Knetter (1996), Atkeson and Burstein (2008), Amiti, Itskhoki, and Konings (2017) for similar strategic complementarities that operate at the firm-level within each sector.}

$$\frac{d\mu_{ji}}{\mu_{ji}} = -\Gamma_{ji} \frac{dc_j}{c_j} + \Gamma_{ji} \frac{dp_{ji}}{p_{ji}}.$$  \hspace{1cm} (26)

The term $\Gamma_{ji}$ represents the elasticity of the markup $\mu_{ji}$ with respect to the supplier’s cost $c_j$:

$$\Gamma_{ji} = \frac{\partial \mu_{ji}}{\partial c_j} \frac{c_j}{\mu_{ji}} = \frac{\Upsilon_{ji}}{1 + \Upsilon_{ji}},$$  \hspace{1cm} (27)

and

$$\Upsilon_{ji} = \left( \varepsilon_{ji} - \sigma_{s(j)i} \right) \left( 1 - \sigma_{s(j)i} \right) \left( 1 - s_{v(j)i}^{(j)} \right) + s_{v(j)i}^{(j)} \left( \eta - \rho \right) \left( 1 - \rho \right) s_{v(j)i}^{(j)} \left( 1 - s_{m(j)i}^{(j)} \right) s_{m(j)i}^{(j)} \varepsilon_{ji} \left( \varepsilon_{ji} - 1 \right).$$  \hspace{1cm} (28)

The term $\frac{dp_{ji}}{p_{ji}}$ represents the average change in input prices of $i$ excluding the price of $j$’s goods sold to $i$.\footnote{See Appendix B.5.2 for details.}

As one can see from equation (26), the magnitudes of both the attenuation and pro-competitive effects are governed by two components. The term $\Gamma_{ji}$, which is the elasticity of markup with respect to supplier $j$’s cost, governs the maximum possible magnitudes of the two. If the supplier firm $j$ has infinitesimal share in buyer $i$’s inputs, $s_{v(j)i}^{(j)} \rightarrow 0$, then the elasticity term converges to 0. The elasticity
term also converges to 0 when the supplier firm \( j \) is the only supplier of buyer \( i \), \( s_{ji}^{x(j)}, s_{vi(j)i}^{m} \to 1 \). Hence both attenuation and pro-competitive effects can have large magnitudes when the pair-specific input shares are in the intermediate range. This point can be conveyed visually when we collapse the model to a single-sector model. Once we assume \( \sigma_u = \rho \), then the relevant input share that determines the markup becomes \( s_{ji}^m = s_{ji}^{x(j)} s_{vi(j)i}^{m} \) from equation (14), and equation (28) collapses to \( \Gamma_{ji} = \frac{(\epsilon_{ji} - \rho)(1 - \rho)(1 - s_{ji}^m)}{\epsilon_{ji}(\epsilon_{ji} - 1)} \). Figure 4 plots markups, \( \mu_{ji} \), and the elasticity of markups, \( \Gamma_{ji} \), with respect to the input share, \( s_{ji}^m \). One can see that \( \Gamma_{ji} \) displays a hump shape with respect to the input share, \( s_{ji}^m \), and when the input share is around 0.7, both attenuation and pro-competitive effects can have large magnitudes.

**Figure 4:** Markup \( \mu_{ji} \) and elasticity \( \Gamma_{ji} \) with respect to input share \( s_{ji}^m \), single-sector model

In addition to the elasticity term \( \Gamma_{ji} \), the magnitudes of the attenuation and pro-competitive effects are each influenced by how much shock the supplier or other suppliers received, \( \frac{\delta \epsilon_{ji}}{c_j} \) and \( \frac{\delta \rho_{p_j}}{\rho_p} \), respectively. For example, even if the input share for a specific pair is in the region where the elasticity \( \Gamma_{ji} \) is large, if the supplier’s cost did not decrease at all, there will be no attenuation effect. The magnitudes of cost reductions by the suppliers govern the magnitudes of attenuation effects within the same values of input shares. Likewise, the average magnitudes of price changes by other suppliers determine the magnitudes of pro-competitive effects within the same value of input shares. Furthermore, the markup that the supplier \( j \) charges \( i \) would see net decrease if supplier \( j \)’s cost reduction is greater than the average cost reductions of other suppliers. It would see net increase if other suppliers received greater cost reductions than what \( j \) received.

Having characterized the changes in markups at the buyer-supplier level, we move our attention to firm-level changes in average markups. As seen in equation (25), the additional force that affects firm \( i \)’s costs by taking into account endogenous markups can be summarized by the average change
in markups that suppliers charge to firm $i$, $\sum_{j \in Z_i} \frac{d\mu_j}{\mu_j}$. As discussed above, these changes in markups are governed by the input shares suppliers have and the relative magnitudes of the suppliers’ cost changes to the other suppliers’ cost changes. Suppose that firm $i$ has multiple suppliers. The largest supplier, $j$, with input shares $s^{v(j)}_{ji}$ and $s^{m(j)}_{vi(j)i}$ which predict a large markup elasticity, $\Gamma_{ji}$, receives the largest cost reduction among $i$’s suppliers. Then the supplier $j$ will increase its markup, and $i$ will experience higher markups from its suppliers on average. If, on the other hand, $j$ does not experience any cost reduction while other suppliers do, then $j$ will reduce its markup and $i$ will experience lower markups from its suppliers on average.

To approximate these firm-level changes in average markups, we consider a measure that captures firms’ indirect exposure to foreign inputs, $s^{Indirect}_{Fi}$. We first construct the measure of “total foreign input share,” $s^{Total}_{Fi}$, that captures firm $i$’s exposure to foreign inputs by summing its direct exposure, its suppliers’ exposure, and so on:

$$s^{Total}_{Fi} = s^{Fi}_{Fi} + \sum_{k \in Z_i} s^{s_{ki}}_{Fi} s^{Total}_{Fk}.$$  

We then subtract firms’ direct exposure to foreign inputs: $s^{Indirect}_{Fi} = s^{Total}_{Fi} - s^{Fi}_{Fi}$. Figure 5 plots these average changes in suppliers’ markups, $\sum_{j \in Z_i} \frac{d\mu_j}{\mu_j}$, against firms’ indirect exposure to foreign inputs, $s^{Indirect}_{Fi}$. There is a positive correlation between the two measures. Consider a firm with high value of $s^{Indirect}_{Fi}$, meaning that the supplier with a high input share is highly exposed to foreign imports. In this case the supplier with the high input share increases its markup and the firm will experience higher markups on average. This positive correlation informs us that the firm’s position in its production network is an important determinant of whether the firm experiences greater or smaller cost changes than those implied from models with constant markups.

To confirm this point, we present the correlations of firm-level cost changes and firm-level variables in Appendix E.1. The measure of the total foreign input share, $s^{Total}_{Fi}$, has the largest correlation with the cost changes under endogenous markups among other firm-level variables such as total sales, import share, and the number of suppliers. Even so, the measure of indirect foreign input share, $s^{Indirect}_{Fi}$, predicts best the differences in cost changes under endogenous markups and constant markups, $\frac{c^{endog}_{i} - c^{const}_{i}}{1 - c^{const}_{i} / c^{const}_{i}}$.

---

31 We follow the definition of $s^{Total}_{Fi}$ by Tintelnot, Kikkawa, Mogstad, and Dhyne (2018).
Figure 5: Average change in markups, $\sum_{j \in Z_i} s_{ji} \frac{d\mu_j}{\mu_j}$, and indirect exposure to foreign inputs, $s_{Fi}^{Indirect}$, on a 20 percent reduction in the price of foreign goods, against firms’ indirect exposure to foreign goods, $s_{Fi}^{Indirect}$. The least-squares line has a y-intercept of -0.0002 and slope coefficient of 0.0009. The R-squared is 0.15. The correlation between the two variables is 0.39.

In addition to the firms’ position in the production network, the nature of the shock itself also affects how firms’ cost changes behave differently from those under constant markups. The shock we focus on here affects all importers (accounting for around 20 percent of all firms) directly, and many other firms indirectly at the same time. The median value of the total foreign input share, $s_{Fi}^{Total}$, is around 39 percent. As the shock affects many firms in the economy at the same time, many firms have multiple suppliers which experience roughly the same degree of cost reductions. In these cases, both the attenuation effects and the pro-competitive effects tend to cancel each other out. To illustrate this point, in Appendix E.2 we study an alternative shock where we hit only one importer with the foreign price reduction.\textsuperscript{32} We demonstrate that the positive correlation between the average changes in markups, $\sum_{j \in Z_i} s_{ji} \frac{d\mu_j}{\mu_j}$, and firms’ exposure to the firm’s goods is much stronger. Moreover, we show that the differences in firm-level cost changes between the baseline model and under constant markups become greater. In this case, 71 percent of firms experience cost reductions that are smaller than under constant markups by 1 percent or more, and 9 percent of firms experience cost reductions that are greater than under constant markups by 1 percent or more.

Finally, we turn to the aggregate counterfactual changes. We report in Table 6 the counterfactual changes of aggregate welfare, expenditure, price index relative to nominal wage, and profits across the

\textsuperscript{32}We choose the importer with the largest number of domestic buyer as the firm receiving the shock.
two models. The baseline economy predicts quantitatively similar changes in the aggregate variables to those predicted by the economy with constant markups. At the firm-level, allowing endogenous markups attenuated cost reductions for around half of the firms and amplified cost reductions for the rest. In the aggregate, these cost changes at the firm-level almost cancel out and produce a small net-amplification effect, resulting in a slightly larger reduction in the aggregate price index normalized for the change in nominal wage.\footnote{In Appendix E.2 we also present results on the change in aggregate welfare under a shock to one importer. In this case, the net effect of the two counteracting forces goes in the opposite direction in the aggregate. The aggregate effects are attenuated once incorporating endogenous markups, and the magnitude of the net effects becomes much larger than it would be considering uniform foreign price reduction. In Appendix E.3 we compare the results with those from a back-of-the-envelope calculation of the economy’s aggregate response under the assumption of perfect competition. Under perfect competition and other assumptions, we can solve for the aggregate counterfactual changes using firm-level information alone (See the approach outline in Appendix B.7). Finally, we show in Appendix E.5 results analogous to those in Table 6 under different magnitudes of shocks.}

**Table 6: Aggregate effects of a 20 percent reduction in the foreign price**

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Constant markups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U = \frac{\hat{E}}{\hat{P}}$</td>
<td>1.2726</td>
<td>1.2721</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>1.1501</td>
<td>1.1493</td>
</tr>
<tr>
<td>$\hat{P}/\hat{\psi}$</td>
<td>0.8756</td>
<td>0.8757</td>
</tr>
<tr>
<td>$\hat{\Pi}$</td>
<td>1.3286</td>
<td>1.3285</td>
</tr>
</tbody>
</table>
7 Conclusion

In this paper we studied the implications of imperfect competition in firm-to-firm trade. We proposed a novel view on competition between firms. In addition to the market shares within sectors determining firms’ market power, we suggest that the relative size of the firm in the total input sourcing of its buyers is also a relevant metric. The data on firm-to-firm transactions supports this view; firms charge higher markups if they have higher average input shares within their buyer firms, controlling for their sectoral market shares.

Using a model of oligopolistic competition in firm-to-firm trade where firms charge different markups to different buyers, we offered two counterfactual exercises. We first investigated the amount of distortion caused by variable markups in firm-to-firm trade. We backed out markups for each buyer-supplier pair in the data and found that the magnitudes of distortions coming from markups can be larger than previously suspected. Reducing all markups in firm-to-firm relationships by 20 percent could increase aggregate welfare by around 7 percent.

We also explored the effect of endogenous markups in firm-to-firm trade on the predictions of shock transmissions in both the aggregate and at the firm-level. Compared to a model featuring constant markups in firm-to-firm trade, we found a large heterogeneity at the firm-level. In the counterfactual where we take a fall in import prices as the shock, we found that incorporating endogenous markups in firm-to-firm trade would amplify cost changes for some firms and attenuate cost changes for others. We characterized these differences and demonstrated that a measure of the firm’s exposure to the indirect shock through its domestic suppliers is an important metric in explaining this firm-level heterogeneity.

While we focused on two specific counterfactual exercises, all results and intuitions offer insights into responses to other types of shocks, such as industry-level shocks or firm-level shocks. Moreover, our framework would be useful in analyzing the effects of various policies such as international trade policies and competition policies.
References


Yeh, C. (2016). Are firm-level idiosyncratic shocks important for U.S. aggregate volatility?

A Data and additional empirical results

A.1 Aggregating VAT-IDs into firms

Our datasets are all at the VAT-ID level. Using the same procedure as in Tintelnot, Kikkawa, Mogstad, and Dhyne (2018), we aggregate the VAT-IDs into firms. As mentioned in the main text, we group all VAT-IDs into firms if they are linked with more than or equal to 50 percent of ownership, or if they share the same foreign parent firm that holds more than or equal to 50 percent of their shares. To determine if the two VAT-IDs share the same foreign parent firm, we use a “fuzzy string matching” method and compare the all possible pairs of the foreign parent firms’ names. In order to correct for misreporting, we pair two separate VAT-IDs into one firm if the two were paired as one firm in the year before and the year after.

We then identify one VAT-ID as the “head VAT-ID” for each group of multiple VAT-IDs. This “head VAT-ID” will work as the identifier of the firm. We also make corrections on which VAT-ID becomes the “head VAT-ID” of the firm, so that the identifiers of the firms become consistent over time. For the procedure to choose the “head VAT-ID” and the corrections, see Appendix C.1 of Tintelnot, Kikkawa, Mogstad, and Dhyne (2018).

When converting the VAT-ID level variables into firm level variables, we simply sum up the variables if the variables are numeric. For variables such as total sales and inputs, we correct for double counting that arises from VAT-ID-to-VAT-ID trade that occur within firms. For other variables including the firm’s age and sector, we take the values of the firm’s “head VAT-ID”.

A.2 Coverage and descriptive statistics

Table 7 reports the coverage of the full sample constructed in Dhyne, Magerman, and Rubinova (2015).

<table>
<thead>
<tr>
<th>Year</th>
<th>All Belgian firms</th>
<th>Selected sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Count</td>
<td>V.A.</td>
</tr>
<tr>
<td>2002</td>
<td>714,469</td>
<td>226</td>
</tr>
<tr>
<td>2007</td>
<td>782,006</td>
<td>315</td>
</tr>
<tr>
<td>2012</td>
<td>860,373</td>
<td>322</td>
</tr>
</tbody>
</table>

Note: All numbers except for Count are in billions of Euro in current prices. Data for Belgian aggregate statistics are from Eurostat. Value added is computed as the firms’ sales minus imports and their purchases from other Belgian firms in the selected sample. The sample for “All Belgian firms” cover all firms in the dataset constructed in Dhyne, Magerman, and Rubinova (2015). The “Selected sample” are the sample selected from the procedure described in Section 2.1.

Table 8 shows the sectoral composition of our selected sample.
Table 8: Sectoral composition of the selected sample in 2012

<table>
<thead>
<tr>
<th>Sector</th>
<th>Count</th>
<th>V.A.</th>
<th>Sales</th>
<th>Imports</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture and Mining</td>
<td>2,805</td>
<td>28.5</td>
<td>49.4</td>
<td>16.9</td>
<td>10.9</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>16,577</td>
<td>138</td>
<td>272</td>
<td>146</td>
<td>193</td>
</tr>
<tr>
<td>Utility and Construction</td>
<td>20,421</td>
<td>23.3</td>
<td>77.0</td>
<td>27.8</td>
<td>17.5</td>
</tr>
<tr>
<td>Wholesale and Retail</td>
<td>31,117</td>
<td>87.8</td>
<td>241</td>
<td>84.1</td>
<td>53.4</td>
</tr>
<tr>
<td>Service</td>
<td>27,825</td>
<td>79.1</td>
<td>127</td>
<td>17.6</td>
<td>16.9</td>
</tr>
<tr>
<td>Total</td>
<td>98,745</td>
<td>356</td>
<td>874</td>
<td>292</td>
<td>292</td>
</tr>
</tbody>
</table>

Note: This table shows the sectoral composition of firms selected from the procedure described in Section 2.1. All numbers except for Count are in billions of Euro in current prices. Value added is computed as the firms’ sales minus imports and their purchases from other Belgian firms in the selected sample. Agriculture and Mining corresponds to NACE 2-digit codes 01 to 09, Manufacturing corresponds to NACE 2-digit codes 10 to 33, Utility and Construction corresponds to NACE 2-digit codes 35 to 43, Whole sale and Retail corresponds to NACE 2-digit codes 45 to 47, and Service corresponds to NACE 2-digit codes 49 to 63, 68 to 82, and 94 to 96.

Table 9 shows the distribution of the pairwise input shares, $s_{ij}^m$, in 2012, defined as the share of goods from firm $i$, among $j$’s input purchases. We also report the distributions for the number of suppliers and buyers. Though the median firm has as many as 33 suppliers, the median value of the pairwise input share, $s_{ij}^m$, is very small. In addition, one can see that the distribution of the number of buyers is much more skewed than the number of suppliers.

Table 9: Descriptive statistics of the production network

<table>
<thead>
<tr>
<th>$s_{ij}^m = \frac{\text{Sales}_{ij}}{\text{InputPurchases}_j}$</th>
<th>Mean</th>
<th>Percentiles</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.80%</td>
<td>0.00%</td>
<td>0.03%</td>
<td>0.19%</td>
<td>0.89%</td>
<td>3.56%</td>
<td></td>
</tr>
<tr>
<td>Num. suppliers</td>
<td>51</td>
<td>11</td>
<td>19</td>
<td>33</td>
<td>56</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>Num. buyers</td>
<td>51</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>35</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows statistics of the firm-to-firm network, among the firms selected from the procedure described in Section 2.1.

Finally, Table 10 describes the shares of firms’ inputs affected by the classification described on page 7. After the sample selection process, we classify input purchases to selected firms from non-selected firms as labor costs.

Table 10: Shares of re-classified labor costs

<table>
<thead>
<tr>
<th>Shares of labor cost, from non-selected firms</th>
<th>Median</th>
<th>Mean</th>
<th>Weighted mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.30</td>
<td>0.35</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Note: The table reports the median, mean, and weighted mean fractions of firms’ labor cost that were originally their purchases from non-selected firms. Firms’ labor costs are used as weights.
A.3 HHI of input shares across suppliers

In this section we compute the HHI of the pair-specific input shares for all buyer firms $j$, across suppliers $i$. The input shares, $s^i_{mj} = \frac{\text{Sales}_{ij}}{\text{InputPurchases}_j}$, are defined as firm $i$’s goods share among $j$’s purchases from other Belgian firms and imports. Figure 6 displays the histogram of these firm-level HHI.

**Figure 6: HHI of suppliers’ input shares**

![Histogram of HHI of suppliers’ input shares](image)

Note: $s^i_{mj}$ is defined as firm $i$’s goods share among firm $j$’s input purchases from other Belgian firms and abroad. The above histogram shows the HHI of $s^i_{mj}$ for all buyer firms $j$ in 2012 that have more than 10 suppliers. The median value is 0.15. The two vertical lines indicate HHI being 0.15 and 0.25.

While there is no perfect reference for the HHI for suppliers’ input shares for each buyer firm, the US Department of Justice and FTC consider markets in which the HHI is between 0.15 and 0.25 to be moderately concentrated. Markets in which the HHI is above 0.25 are considered highly concentrated (U.S. Department of Justice and Federal Trade Commission, 2010). We find here that 50 percent of firms have a HHI above 0.15. 26 percent of firms have a HHI above 0.25.

A.4 Distribution of firms’ output shares

Figure 7 plots a histogram for the output shares of the largest buyers for all supplier firms in 2012 that have at least 10 buyers. The output share, $r_{ij} = \frac{\text{Sales}_{ij}}{\text{Sales}_i}$, is defined as the sales share of firm $i$’s output that were sold to firm $j$. The output share of the largest buyer for the median firm in this figure is 7 percent.
**A.5 Disconnect between pairwise input shares and sectoral market shares**

We showed in Section 2.2 that firms have skewed input shares across their suppliers. However, high skewness in input shares may simply be caused by firm-level components. For example, one may argue that the skewness of input shares across suppliers is coming from the skewness in the suppliers’ productivity distribution. If that is indeed the case, one would expect that a firm with a high input share on a particular buyer would also be one with high total sales. Nevertheless, the results in Section 2.3 suggested otherwise. Firm-level markups, which have total sales on the numerator, were not perfectly collinear with firms’ average input shares to their buyers. To investigate this further, we compute for each firm the rank correlation between its suppliers’ input shares and their total sales.

Consider the firm on the left of Figure 8. This firm is purchasing goods worth 10, 5, and 1 Euro from its three suppliers, $a$, $b$, and $c$, respectively. The three suppliers’ total sales are 100, 50, and 10 Euro. The ordering of the firm’s suppliers according to the input shares aligns with the ordering of their total sales. Thus, the rank correlation for the firm is 1. One the other hand, consider the firm on the right of the figure. The transaction values are identical to the firm on the left, but the three suppliers’ total sales are 10, 50, and 100 Euro, respectively. Here the ordering of the two are opposite, so the rank correlation for the firm is $-1$. 

**Note:** $r_{ij}$ is defined as the share of firm $i$’s goods that were sold to firm $j$, out of firm $i$’s total sales. The above histogram shows the distribution of $\max_j (r_{ij})$, which is the maximum value of $r_{ij}$ for each supplier firm $i$ in 2012 that have more than 10 buyers. The median value is 0.07.
Figure 8: Example for computing rank correlations

Total sales of supplier: €100 \( \rightarrow \) €50 \( \rightarrow \) €10
Transaction value: €10 \( \rightarrow \) €5 \( \rightarrow \) €1

Rank correlation for the buyer: 1 \( \rightarrow \) -1

Figure 9 displays the histogram of the correlation coefficients. The median firm’s coefficient is around 0.12. 30 percent of firms have correlation coefficients that are zero or negative. This result indicates that a firm with high input share on a particular buyer is not necessarily large. It illustrates that pairwise match components play a large role in firm-to-firm trade in addition to firm-level components. Instead of computing the rank correlations, we find that the results when we compute the Pearson correlations also have a mass of firms around zero correlation, and even have a lower median coefficient value. Figure 10 shows the histogram of the Pearson correlation coefficients.

Figure 9: Histogram of rank correlation of suppliers’ input shares and total sales

Note: This figure shows a histogram of Spearman’s rank correlation coefficients between \( s^i_{ij} \) and Sales, for suppliers of \( j \) for all \( j \) with 5 or more suppliers. The vertical line depicts the median correlation coefficient of 0.12.

Indeed, in Figures 9 and 10 we plot the unconditional correlations which do not take into account the difference in the goods produced by suppliers. The low correlations in the figure may come from

\(^{34}\) This becomes the case if the distributions of firms’ output shares to each buyer are skewed. See Appendix A.4 for a figure analogous to Figure 1, but for output shares. The output shares are indeed skewed, where more than 20 percent of the output of a median firm goes to its largest buyer.
A.6 Additional results on markups and input shares

First, we show that the firms’ average input shares on buyers tend to have greater power in explaining the variation of firms’ average markups, compared to firm-level market shares. In Table 11 we report the regression results when we add the two RHS variables one by one, for each of the three specifications in Table 3. The 4th, 8th and 12th columns are identical to the three columns in Table 3. For each specification reported in the main text, we add three additional specifications. One with neither average input shares nor firm-level market shares on the RHS, and ones with each variable without the other. In all three sets of specifications, the increase in R-squared by adding average input shares alone on the RHS is larger than or almost equal to the increase in R-squared by adding sectoral
Figure 11: Median rank correlations

Note: For each buyer firm $j$, we compute the rank correlations of suppliers’ input shares $s_{mj}$ and Sales$_i$, for each sector in which 5 or more of $j$’s suppliers are in. This figure shows a histogram of the median correlation coefficients, across each sector-to-sector pairs. The vertical line depicts the median value of 0.19.

market shares alone.
Figure 12: Median Pearson correlations

Note: For each buyer firm $j$, we compute the Pearson correlations of suppliers' input shares $s_{ij}$ and Sales$_i$, for each sector in which 5 or more of $j$'s suppliers are in. This figure shows a histogram of the median correlation coefficients, across each sector-to-sector pairs. The vertical line depicts the median value of 0.06.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare, t</td>
<td>0.0224</td>
<td>0.0219</td>
<td>0.0153</td>
<td>0.0154</td>
<td>0.0221</td>
<td>0.0221</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00283)</td>
<td>(0.00280)</td>
<td>(0.00174)</td>
<td>(0.00174)</td>
<td>(0.00202)</td>
<td>(0.00201)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average input share $\overline{s^m_{i,j}}$</td>
<td>0.0534</td>
<td>0.0524</td>
<td></td>
<td></td>
<td>0.0412</td>
<td>0.0412</td>
<td>0.0391</td>
<td>0.0391</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00396)</td>
<td>(0.00395)</td>
<td></td>
<td></td>
<td>(0.00299)</td>
<td>(0.00300)</td>
<td>(0.00291)</td>
<td>(0.00290)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>809722</td>
<td>809722</td>
<td>809722</td>
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<td>781627</td>
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</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
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<td>4-digit</td>
<td>4-digit</td>
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</tr>
<tr>
<td>Firm FE</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
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<tr>
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<td>Yes</td>
<td>Yes</td>
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<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
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</tr>
<tr>
<td>R2</td>
<td>0.1033</td>
<td>0.1040</td>
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<td>0.1050</td>
<td>0.6375</td>
<td>0.6376</td>
<td>0.6377</td>
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<td>0.6385</td>
<td>0.6387</td>
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<td>0.6389</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.
We then show here that the positive relationship between markups and firms’ average input shares are robust in other specifications. Table 12 shows additional results when firm-level fixed effects are included, and Table 13 shows additional results when sector-level fixed effects are included. Table 14 shows results when we control for the denominator of $\bar{s}_{i,t}$; buyers’ total input purchases, and Table 15 shows results when we additionally control for the average relationship age with its buyers. Since the data starts from 2002, we count the relationship age of a buyer-supplier pair as the number of years after its first observation from 2002. Then we take the mean of these relationship age across firms’ buyers.

In our main specification, we drop firms that have no sales to other Belgian firms. Table 16 shows the results when we include such firms in the regression, by treating their average input shares to other firms as zero.

Furthermore, as an alternative measure of input shares we use the supplier’s sales share among the buyer’s inputs that are classified as the same goods as the supplier’s, either at the 2-digit or 4-digit level. The results are reported in Tables 17 and 18.

Table 12: Firm-level markups and input shares, with firm fixed effects

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare$_{i,t}$ (4-digit)</td>
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<td>0.0154</td>
<td>0.0221</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00174)</td>
<td>(0.00174)</td>
<td>(0.00201)</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>SctrMktShare$_{i,t}$ (2-digit)</td>
<td></td>
<td></td>
<td>0.0153</td>
<td>0.0153</td>
<td>0.0264</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00256)</td>
<td>(0.00255)</td>
<td>(0.00380)</td>
<td></td>
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<tr>
<td>Average input share $\bar{s}_{i,t}$</td>
<td>0.0412</td>
<td>0.0391</td>
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<td>0.0390</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.00300)</td>
<td>(0.00290)</td>
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<td>(0.00290)</td>
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<tr>
<td>Year FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Controls</td>
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<td>No</td>
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<tr>
<td>R2</td>
<td>0.638</td>
<td>0.638</td>
<td>0.639</td>
<td>0.638</td>
<td>0.638</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.
Table 13: Firm-level markups and input shares, with sector fixed effects

<table>
<thead>
<tr>
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<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare_i,t (4-digit)</td>
<td>0.0224</td>
<td>0.0219</td>
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<td></td>
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<tr>
<td></td>
<td>(0.00283)</td>
<td>(0.00280)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SctrMktShare_i,t (2-digit)</td>
<td></td>
<td></td>
<td>0.0220</td>
<td>0.0215</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00226)</td>
<td>(0.00223)</td>
</tr>
<tr>
<td>Average input share $s^m_{i,t}$</td>
<td>0.0524</td>
<td></td>
<td>0.0540</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00395)</td>
<td></td>
<td>(0.00422)</td>
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<td>809727</td>
<td>809727</td>
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<td>Year FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
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<td>Sector FE</td>
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<td>4-digit</td>
<td>2-digit</td>
<td>2-digit</td>
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<td>Controls</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>R2</td>
<td>0.104</td>
<td>0.105</td>
<td>0.0719</td>
<td>0.0729</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

Table 14: Firm-level markups and input shares, controlling for buyers’ size

<table>
<thead>
<tr>
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<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare_i,t (4-digit)</td>
<td>0.0235</td>
<td>0.0221</td>
</tr>
<tr>
<td></td>
<td>(0.00264)</td>
<td>(0.00201)</td>
</tr>
<tr>
<td>Average input share $s^m_{i,t}$</td>
<td>0.0417</td>
<td>0.0385</td>
</tr>
<tr>
<td></td>
<td>(0.00358)</td>
<td>(0.00291)</td>
</tr>
<tr>
<td>N</td>
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<td>781627</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sector FE</td>
<td>4-digit</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.108</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, age, and buyers’ total input purchases.
Table 15: Firm-level markups and input shares, controlling for average relationship age

<table>
<thead>
<tr>
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<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare&lt;sub&gt;i,t&lt;/sub&gt; (4-digit)</td>
<td>0.0220</td>
<td>0.0221</td>
</tr>
<tr>
<td></td>
<td>(0.00279)</td>
<td>(0.00201)</td>
</tr>
<tr>
<td>Average input share &lt;sup&gt;z&lt;/sup&gt;&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>0.0566</td>
<td>0.0381</td>
</tr>
<tr>
<td></td>
<td>(0.00398)</td>
<td>(0.00289)</td>
</tr>
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<td>781627</td>
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<tr>
<td>Year FE</td>
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<td>Yes</td>
</tr>
<tr>
<td>Sector FE</td>
<td>4-digit</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.106</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, age, and average relationship age.

Table 16: Firm-level markups and input shares, including firms without firm-to-firm sales

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare&lt;sub&gt;i,t&lt;/sub&gt; (4-digit)</td>
<td>0.0231</td>
<td>0.0157</td>
<td>0.0228</td>
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<tr>
<td></td>
<td>(0.00291)</td>
<td>(0.00166)</td>
<td>(0.00197)</td>
</tr>
<tr>
<td>Average input share &lt;sup&gt;z&lt;/sup&gt;&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>0.0383</td>
<td>0.0350</td>
<td>0.0333</td>
</tr>
<tr>
<td></td>
<td>(0.00385)</td>
<td>(0.00272)</td>
<td>(0.00265)</td>
</tr>
<tr>
<td>N</td>
<td>921346</td>
<td>895043</td>
<td>892891</td>
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<td>Year FE</td>
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<td>Yes</td>
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<td>Sector FE</td>
<td>4-digit</td>
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<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
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<td>No</td>
<td>Yes</td>
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<tr>
<td>R2</td>
<td>0.112</td>
<td>0.648</td>
<td>0.642</td>
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</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.
Table 17: Firm-level markups and input shares, input shares taking into account sectors (2-digit)

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>SctrMktShare&lt;sub&gt;,t&lt;/sub&gt; (4-digit)</td>
<td>0.0205</td>
<td>0.0196</td>
<td>0.0266</td>
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<tr>
<td></td>
<td>(0.00265)</td>
<td>(0.00182)</td>
<td>(0.00211)</td>
</tr>
<tr>
<td>Average sectoral input share (\bar{s}_{i,t}^{m}) (4-digit)</td>
<td>0.0245</td>
<td>0.0156</td>
<td>0.0149</td>
</tr>
<tr>
<td></td>
<td>(0.00194)</td>
<td>(0.00119)</td>
<td>(0.00118)</td>
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<tr>
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<td>786575</td>
<td>786575</td>
</tr>
<tr>
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<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sector FE</td>
<td>4-digit</td>
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<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.0830</td>
<td>0.612</td>
<td>0.613</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Average sectoral input share, \(\bar{s}_{i,t}^{m}\), is calculated as the supplier \(i\)'s sales share among its buyers inputs that are classified as the same goods as \(i\)'s, at the 2-digit level. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

Table 18: Firm-level markups and input shares, input shares taking into account sectors (4-digit)

<table>
<thead>
<tr>
<th></th>
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<th>(2)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare&lt;sub&gt;,t&lt;/sub&gt; (2-digit)</td>
<td>0.0205</td>
<td>0.0151</td>
<td>0.0254</td>
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<tr>
<td></td>
<td>(0.00200)</td>
<td>(0.00228)</td>
<td>(0.00354)</td>
</tr>
<tr>
<td>Average sectoral input share (\bar{s}_{i,t}^{m}) (2-digit)</td>
<td>0.0288</td>
<td>0.0203</td>
<td>0.0194</td>
</tr>
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<td>(0.00143)</td>
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<td>840712</td>
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<td>Yes</td>
</tr>
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<td>Sector FE</td>
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<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
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<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Controls</td>
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<td>Yes</td>
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<tr>
<td>R2</td>
<td>0.0820</td>
<td>0.608</td>
<td>0.609</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Average sectoral input share, \(\bar{s}_{i,t}^{m}\), is calculated as the supplier \(i\)'s sales share among its buyers inputs that are classified as the same goods as \(i\)'s, at the 4-digit level. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

Finally, we investigate whether input shares of firms’ buyers in their buyers are correlated with the firms’ markups. Similar to the definition of the weighted average input shares to its buyers, \(\bar{s}_{i,j}^{m}\),
we compute firm $i$’s weighted average input shares of its buyers, $\overline{s}^m_{i,t}$, as

$$\overline{s}^m_{i,t} = \sum_{j \in W_{i,t}} \frac{\text{InputPurchases}_{i,t}}{\sum_{k \in W_{i,t}} \text{InputPurchases}_{k,t}} s^m_{i,j,t}$$

$$\overline{s}^m_{i-1,t} = \sum_{j \in W_{i,t}} \frac{\text{InputPurchases}_{j,t}}{\sum_{k \in W_{i,t}} \text{InputPurchases}_{k,t}} s^m_{i,j,t}$$

Table 19 shows the results when we add this second-degree average input shares as another control variable. While the coefficients on both sectoral market shares and the average input shares are almost unchanged from those in Table 3, the coefficient on the second-degree average input shares are close to zero and not significant in the specifications with firm fixed effects.

Table 19: Firm-level markups and input shares, add second degree average input shares

<table>
<thead>
<tr>
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<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare$_{i,t}$ (4-digit)</td>
<td>0.0219</td>
<td>0.0154</td>
<td>0.0221</td>
</tr>
<tr>
<td></td>
<td>(0.00279)</td>
<td>(0.00174)</td>
<td>(0.00201)</td>
</tr>
<tr>
<td>Average input share $\overline{s}^m_{i,t}$</td>
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<td>(0.00396)</td>
<td>(0.00301)</td>
<td>(0.00291)</td>
</tr>
<tr>
<td>Second degree average input share $\overline{s}^m_{i-1,t}$</td>
<td>-0.00698</td>
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<td>-0.000737</td>
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<td>781627</td>
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<td>Yes</td>
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<tr>
<td>Sector FE</td>
<td>4-digit</td>
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<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.105</td>
<td>0.638</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

### A.7 Alternative markup estimates

In the main text, we recover firm-level average markups, $\mu_i$, using the equation implied from the static model with CRS production function: $\mu_i = \frac{\text{Sales}_i}{\text{InputPurchases}_i + \text{LaborCosts}_i}$. To account for additional heterogeneity such as usage in capital inputs, first we assume that the user cost of capital consists of capital depreciation rate and the interest rate. Following Dhyne, Petrin, Smeets, and Warzynski (2017), we set the yearly depreciation rate as 8 percent and set the interest rate as the long-term interest rate in Belgium. Table 20 reports the results when we add user cost of capital as one of the variable costs of the firm.
Table 20: Firm-level markups and input shares, user cost of capital in markups

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SctrMktShare(_{it}) (4-digit)</td>
<td>0.0414</td>
<td>0.0255</td>
<td>0.0288</td>
</tr>
<tr>
<td></td>
<td>(0.00281)</td>
<td>(0.00188)</td>
<td>(0.00204)</td>
</tr>
<tr>
<td>Average input share (\overline{x}_{it})</td>
<td>0.0214</td>
<td>0.0245</td>
<td>0.0235</td>
</tr>
<tr>
<td></td>
<td>(0.00243)</td>
<td>(0.00197)</td>
<td>(0.00195)</td>
</tr>
<tr>
<td>N</td>
<td>890228</td>
<td>863458</td>
<td>863458</td>
</tr>
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<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sector FE 4-digit</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.115</td>
<td>0.730</td>
<td>0.731</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The coefficients are X-standardized. Standard errors are clustered at the NACE 2-digit-year level. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. Controls include firms’ number of suppliers, number of buyers, employment, total assets, and age.

Another approach that we take to account for firms usage of capital and also in heterogeneity in production technology across sectors is to recover firm-level markups following De Loecker and Warzynski (2012). We show that the positive correlation between firms’ markups and their average input shares are still present even under these alternative markup estimates.

We first briefly describe the estimation procedure. When a firm is engaging in cost minimization under the existence of at least one flexible input \(X\), the markup of firm \(i\) at time \(t\) can be expressed as

\[
\mu_{it} = \theta_X p_{it} q_{it},
\]

where \(\theta_X\) is firm \(i\)’s output elasticity with respect to \(X\), and \(p_{it}^X X_{it}\) is the input value of \(X\). As the input value share of the flexible input \(X\) is directly observed, it remains for us to estimate the value of \(\theta_X\) to recover firm-level markups. In order to estimate the output elasticity, we assume a translog production function. We also assume that the technology parameters do not vary within sectors, thus we estimate the production function sector by sector at the NACE 2-digit level. We also allow for measurement errors in the output. Therefore, the production function to estimate becomes

\[
y_{it} = \alpha l_{it} + \alpha k_{it} + \alpha m_{it} + \alpha l_{it}^2 + \alpha k_{it}^2 + \alpha m_{it}^2 + \alpha l_{it} k_{it} + \alpha k_{it} m_{it} + \alpha l_{it} m_{it} + \omega_{it} + \varepsilon_{it},
\]

where \(y_{it}, l_{it}, k_{it},\) and \(m_{it}\) denote gross output, labor, capital, and material inputs, all in logs. The estimates from a least squares model would be biased as firm productivity \(\omega_{it}\) is unobserved, and is potentially correlated with the inputs of the firm, which results in biased estimates of the technology parameters \(\alpha\). To overcome this issue, we follow Levinsohn and Petrin (2003) and use a “proxy”
method. We assume that the innovation process of the firm-level productivities follow:

$$\omega_{it} = g_t (\omega_{it-1}) + \xi_{it}.$$ 

We identify $\alpha$ via the following moment conditions:

$$E[\xi_{it}(\alpha) z_{it}] = 0,$$

where $z_{it}$ is a vector of lagged input variables:

$$z_{it} = [l_{it-1}, k_{it}, m_{it-1}, l_{it-1}^2, k_{it}^2, m_{it-1}^2, l_{it-1}k_{it}, k_{it}m_{it-1}, l_{it-1}m_{it-1}].$$

The underlying assumption is that capital inputs are chosen a period ahead, and should be orthogonal to the future innovations of productivity. For other inputs, it is assumed that lagged variables are orthogonal to productivity innovations, as they are already chosen by the firm.

We estimate $\alpha$ via GMM, and recover $\theta^X$ by assuming that material inputs are flexible. Once we recover firm-level markups $\mu_{it}$, we run the regression of equation (2) in the main text. Table 21 reports the results. Also in these alternative estimates of firm-level markups, there is a positive relationship between markups and firms’ average input shares within their buyers even after controlling for firm size variables.

**Table 21**: Firm-level markups and input shares, markups from De Loecker and Warzynski (2012)

<table>
<thead>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
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<tr>
<td>SctrMktShare$_{i,t}$ (4-digit)</td>
<td>0.00387</td>
<td>0.000417</td>
<td>0.000595</td>
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<tr>
<td>(0.000997)</td>
<td>(0.000567)</td>
<td>(0.000566)</td>
<td></td>
</tr>
<tr>
<td>Average input share $s_{m,i,t}$</td>
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<td>(0.00167)</td>
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</tr>
<tr>
<td>Sector FE</td>
<td>4-digit</td>
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<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Controls</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>R2</td>
<td>0.752</td>
<td>0.946</td>
<td>0.947</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. We use firm-level markups recovered using methods from De Loecker and Warzynski (2012) as the LHS variables. The regression exclude outliers in the top and bottom 1 percent of the markup distribution. The coefficients are $X$-standardized. Standard errors are clustered at NACE 2-digit-year level.
B Theoretical results

B.1 Derivation of equation (14)

Consider firm $j$ selling its goods to $i$. Firm $j$ chooses $p_{ji}$ to maximize profits, taking into account the effect of $p_{ji}$ on $i$’s price indices for its intermediate goods, $p_{mi}$ and $p_{mv}^{m}(j)i$. It takes as given $i$’s unit cost and production, $c_i$, and $q_i$, as well as $i$’s sourcing sets, $Z_i$ and $I_{F_i}$. The firm’s problem is as follows:

$$\max_{p_{ji}} \left(p_{ji} - c_j\right)q_{ji}$$

subject to:

$$q_{ji} = \alpha_{ji}^{\sigma_{v}(j)}\alpha_{ji}^{\rho}p_{ji}^{-\sigma_{v}(j)}(p_{mji})^{\sigma_{v}(j)-1}(p_{mv}^{m}(j)i)^{1-p}p_{mv}^{m}m_i$$

Solving the above problem while taking into account that $\frac{\partial p_{mi}}{\partial p_{ji}} \neq 0$ and $\frac{\partial p_{mv}^{m}(j)i}{\partial p_{ji}} \neq 0$ yields

$$p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1}c_j$$

$$\varepsilon_{ji} = \sigma_{v}(j) \left(1 - s_{ji}^{v(j)}\right) + \rho s_{ji}^{v}(j) \left(1 - s_{mv}^{m}(j)i\right) + \eta s_{ji}^{v}(j) s_{mv}^{m}(j)i.$$ 

B.2 Alternative market structures

In our model we assume the following when firms participate in firm-to-firm trade. When selling to firm $i$, firm $j$ sets price $p_{ji}$ by internalizing the effect of $p_{ji}$ on $j$’s price indices for its intermediate goods, $p_{mi}$ and $p_{mv}^{m}(j)i$. However, it takes as given $i$’s unit cost and total production, $c_i$ and $q_i$. This yields our pricing equation of

$$p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1}c_j$$

$$\varepsilon_{ji} = \sigma_{v}(j) \left(1 - s_{ji}^{v(j)}\right) + \rho s_{ji}^{v}(j) \left(1 - s_{mv}^{m}(j)i\right) + \eta s_{ji}^{v}(j) s_{mv}^{m}(j)i.$$ 

In this section we discuss alternative market structures in firm-to-firm trade.

B.2.1 Cournot competition

Instead of assuming Bertrand competition one can alternatively assume that firms engage in Cournot competition, where firms set quantity $q_{ji}$ to maximize variable profits. In that case, the demand elasticity that firm $j$ faces, $\varepsilon_{ji}$, becomes a weighted harmonic mean of the CES parameters $\sigma_{v}(j)$, $\rho$, 

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and \( \eta \):

\[
p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1} c_j
\]

\[
\varepsilon_{ji}^{-1} = \frac{1}{\sigma_{\nu(j)}} \left( 1 - s_{ji}^{\nu(j)} \right) + \frac{1}{\rho} s_{ji}^{\nu(j)} \left( 1 - s_{vi(j)i}^m \right) + \frac{1}{\eta} s_{ji}^{\nu(j)} s_{\nu(j)i}^m.
\]

### B.2.2 Fixed demand shifters of buyers

Next we consider a case where firm \( j \) takes as given the two demand shifters that firm \( i \) faces - one from sales to other firms \((D_{iB})\) and another from sales to final demand \((D_{iH})\):

\[
q_i = c_i^{\sigma_{\alpha(i)}} D_{iB} + c_i^{\sigma_{\alpha}} D_{iH}.
\]

When one solves this problem the pricing equation becomes

\[
p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1} c_j
\]

\[
\varepsilon_{ji} = \sigma_{\nu(j)} \left( 1 - s_{ji}^{\nu(j)} \right) + \rho s_{ji}^{\nu(j)} \left( 1 - s_{vi(j)i}^m \right) + s_{ji}^{\nu(j)} s_{\nu(j)i}^m \left( 1 - s_{mi} \right) \eta + s_{mi} \left( s_{iB}^{\theta} \sigma_{\alpha(i)} + s_{iH}^{\theta} \sigma \right)
\]

The term \( s_{iB}^{\theta} \) is the quantity output share of firm \( i \)’s goods that were shipped to other firms, and the term \( s_{iH}^{\theta} \) is the quantity output share of firm \( i \)’s goods that were shipped to final demand:

\[
s_{iB}^{\theta} = \frac{c_i^{\sigma_{\alpha(i)}} D_{iB}}{q_i}
\]

\[
s_{iH}^{\theta} = \frac{c_i^{\sigma_{\alpha}} D_{iH}}{q_i} = 1 - s_{iB}^{\theta}.
\]

This implies that the firm needs to know the quantity output shares of its buyers.

### B.2.3 Constant demand elasticity for buyers’ goods

We also consider a case where firm \( j \) does not know the output compositions of its buyer \( i \), but assumes that \( i \) is facing a common demand elasticity of \( \theta \). In this case \( q_i \) can be written as

\[
q_i = c_i^{-\theta} D_i,
\]
in which firm $j$ takes as given the demand shifter, $D_j$. When one solves the problem of firm $j$ under this setup, the pricing equation becomes

$$ p_{ji} = \frac{\epsilon_{ji}}{\epsilon_{ji} - 1} c_j $$

$$ \epsilon_{ji} = \sigma_{\psi(j)} \left(1 - s_{ji}^{\psi(j)}\right) + \rho s_{ji}^{\psi(j)} \left(1 - s_{\psi(j)}^{m} \right) + s_{ji}^{\psi(j)} s_{\psi(j)}^{m} \left((1 - s_{m}) \eta + s_m \theta \right). $$

Notice that if we additionally assume that $\theta = \eta$, the above equation collapses to equation (14).

### B.3 System of equilibrium changes upon changes in markups

#### B.3.1 System of exact changes

In this section we take the observed firm-to-firm trade network, and present the system of equations that pins down changes in the equilibrium variables upon exogenous changes in $\{\hat{\mu}_{ij}\}$.

The total variable inputs observed in the data is denoted by $c_iq_i = wL_i + \sum_k V_{ki} + V_{Fi}$. Also denote total input cost of firm $i$ implied from the model as $C_i = \sum_j V_{ij} \hat{\mu}_{ij} + \sum_k V_{ik} \hat{\mu}_{ik}$. The difference between the two is denoted as $\xi_i = c_iq_i - C_i$. We take the error term in equation (21), $\epsilon_i = \frac{\xi_i}{c_iq_i}$, as constants. With this assumption, the changes in the observed inputs, $\hat{c_iq_i}$, are equal to the changes in the model implied inputs, $\hat{C}_i$, and also to the changes in the difference between the two, $\hat{\xi}_i$. We also denote trade balance as $TB$ and treat them as fixed.

We now have the following system of equations defining the equilibrium:

$$ E = wL + \sum_i \pi_i - \sum_i \xi_i - TB $$

$$ \pi_i = \sum_{k \in W_i} \frac{\mu_{ik} - 1}{\mu_{ik}} V_{ik} + \frac{\mu_{iH} - 1}{\mu_{iH}} (V_{iH} + V_{iF}) $$

$$ V_{ik} = s_{ki}^{\psi(i)} s_{\psi(i)k}^{m} s_{mk} c_k q_k $$

$$ V_{iH} = s_{iH} E $$

$$ V_{iF} = \mu_{iF}^{1-\sigma} c_i^{1-\sigma} D^* $$

$$ \hat{\xi}_i = c_iq_i - C_i $$

$$ = c_iq_i - \left( \frac{V_{iH}}{\mu_{iH}} + \frac{V_{iF}}{\mu_{iH}} + \sum_j \frac{V_{ij}}{\mu_{ij}} \right). $$

From this system, we then proceed with the following steps and compute the hat-variables upon $\hat{\mu}_{ij}$.

1. Guess the change in nominal wage, $\hat{w}$. 

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2. Compute the changes in prices with
\[
\hat{c}_i^{1-\eta} = s_i \hat{w}^{1-\eta} + s_{mi} \hat{p}_m^{1-\eta}
\]
\[
\hat{p}_m^{1-\rho} = \sum_s s_{vi} (\hat{p}_m^{1-\rho}) + s_{Fi}
\]
\[
(\hat{p}_m^{1-\sigma_v} = \sum_{j \in Z} \sum_{i \in V} s_{vi} (\hat{p}_m^{1-\sigma_v})
\]

3. Compute other variables' changes with
\[
\hat{s}_{vi} = \hat{\mu}_{ji}^{1-\sigma_v} \hat{c}_j^{1-\sigma_v} (\hat{p}_m^{1-\sigma_v})^{-1}
\]
\[
\hat{s}_{mi} = \hat{p}_m^{1-\sigma_v} \hat{c}_i^{1-\sigma_v}
\]
\[
\hat{s}_H = \hat{w}^{1-\eta} \hat{c}_i^{1-\eta}
\]
\[
\hat{s}_{ji} = \hat{s}_{ji} \hat{s}_{mi} \hat{s}_m
\]
\[
\hat{\mu} = \left( \sum_i s_{ii} \hat{c}_i^{1-\sigma} \right)^{-1}
\]
\[
\hat{\nu} = \hat{c}_i^{1-\sigma} \hat{\mu}^{1-\sigma}
\]
\[
\hat{V}_{iF} = \hat{c}_i^{1-\sigma}
\]

4. Compute \( \hat{C}_i \) from
\[
\hat{C}_i = \frac{1}{C_i} \frac{V_{iH}}{\mu_{iH}} \hat{s}_{iH} \hat{E} + \frac{1}{C_i} \frac{V_{iF}}{\mu_{iF}} \hat{V}_{iF} + \frac{1}{C_i} \sum_j \frac{V_{ij} \hat{s}_{ij}}{\mu_{ij}} \hat{C}_j
\]
\[
\hat{E} = \frac{1}{1 - \sum_i \frac{1}{E} \frac{\mu_{iH}}{\mu_{iH}} V_{iH} \hat{s}_{iH}} \times \left\{ \frac{wL}{E} \hat{W} - \frac{T B}{E} - \sum_i \hat{E} \hat{C}_i \right\} + \frac{1}{E} \sum_i \pi_i \left( \sum_k \frac{V_{ik}}{\pi_i} \frac{\hat{\mu}_{ik} \mu_{ik} - 1}{\hat{\mu}_{ik} \mu_{ik}} \hat{s}_{ik} \hat{C}_k + \frac{1}{\pi_i} \frac{\mu_{iH} - 1}{\mu_{iH}} V_{iF} \hat{V}_{iF} \right) \right\}.
\]

5. Update \( \hat{w} \) from
\[
\hat{w} = \frac{1}{wL} \sum_i s_{ii} c_i \hat{s}_{ii} \hat{C}_i,
\]
and iterate from Step 1 until \( \hat{w} \) converges.

**B.3.2 System of first-order approximated changes**

We take the system of equilibrium changes in the previous section and solve for the first-order approximated changes. Now we denote the shock with \( \{ \Delta_{i,j} \} \).
1. Guess the change in nominal wage, \( \frac{d}{d} w \).

2. Compute the changes in prices with

\[
\frac{d}{c_i} = \frac{s_{ii}}{w} + \sum_{j \in Z} s_{ji} \left( \frac{d}{\mu_j} + \frac{d}{c_j} \right).
\]

3. Compute other variables’ changes with

\[
\frac{d}{p_{i}^{m}} = \sum_{j \in Z, j \in V} s_{ij} \left( \frac{d}{\mu_j} + \frac{d}{c_j} \right),
\]

\[
\frac{d}{p_{mi}} = \sum_{v} s_{vi} \frac{d}{p_{vi}^{m}},
\]

\[
\frac{d s_{ji}^{m}}{s_{ji}^{m}} = (1 - \alpha_j) \left( \frac{d}{\mu_j} + \frac{d}{c_j} \right) - \frac{d}{p_{vi}^{m}} \frac{d}{p_{vi}^{m}},
\]

\[
\frac{d s_{i}^{m}}{s_{i}^{m}} = (1 - \rho) \left( \frac{d}{p_{vi}^{m}} - \frac{d}{p_{mi}} \right),
\]

\[
\frac{d s_{i}}{s_{i}} = (1 - \eta) \left( \frac{d}{w} - \frac{d}{c_i} \right),
\]

\[
\frac{d s_{i}}{s_{i}^{m}} = \frac{d s_{i}}{s_{i}^{m}} + \frac{d s_{mi}}{s_{mi}},
\]

\[
\frac{d P}{P} = \sum_{i} s_{iH} \frac{d}{c_i},
\]

\[
\frac{d s_{iH}}{s_{iH}} = (1 - \alpha) \left( \frac{d}{c_i} - \frac{d}{P} \right),
\]

\[
\frac{d V_{EF}}{V_{EF}} = (1 - \alpha) \frac{d}{c_i}.
\]

4. Compute \( \frac{d}{c_i} \) from

\[
\frac{d C_i}{C_i} = \frac{1}{C_i} \left( \frac{d}{s_{iH}^{m}} + \frac{d}{E} \right) + \frac{1}{C_i} \frac{V_{iF} dV_{iF}}{V_{EF}} + \frac{1}{C_i} \sum_{j} V_{ij} \frac{d s_{ij}}{s_{ij}^{m}} \left( \frac{d}{s_{ij}} - \frac{d}{s_{ij}} \right) \frac{d C_j}{C_j} \frac{d}{c_j}.
\]

\[
\frac{d E}{E} = \frac{1}{1 - \sum_{k} \frac{1}{\mu_{ik} - 1} V_{iH}} \times
\]

\[
\left\{ \sum_{i} \mu_{iH} - 1 \frac{d s_{iH}}{s_{iH}} V_{iH} + w L \frac{d}{w} - \sum_{i} \frac{d}{c_i} \right\}.
\]

\[
+ \sum_{i} \left( \sum_{k} V_{ik} \mu_{ik} - 1 \mu_{ik} \frac{d s_{iH}}{s_{iH}} \frac{d C_k}{C_k} + \mu_{iH} - 1 \frac{d V_{iF}}{V_{EF}} \right) \frac{d}{c_i}.
\]
5. Update \( \frac{dw}{w} \) from

\[
\frac{dw}{w} = \frac{1}{w \sum_i w_l} \left( ds_l + \frac{dC}{C} \right),
\]

and iterate from Step 1 until \( \frac{dw}{w} \) converges.

\section*{B.4 System of equilibrium changes upon changes in markups, sectoral roundabout production network}

We also consider a sectoral roundabout production network that is consistent with the observed firm-level data, and present the system of equations that pins down changes in the equilibrium variables upon exogenous changes in markups. We consider a network where there are two distinct composite goods, one of which is used as intermediate goods and another of which is used as a final consumption good. The cost function for firm \( i \) is expressed by equation (20). As in the previous section, the total variable inputs observed in the data is denoted by

\[
c_i q_i = w_l + \sum_k V_{ki} + V_{Fi},
\]

and the total input cost implied from the model is denoted as

\[
C_i = \sum_j V_{ij} \mu_i + V_{iH} \mu_i + V_{iF} \mu_i. 
\]

The difference between the two is denoted as \( \xi_i = c_i q_i - C_i \). We also assume that the error terms in the estimating equation (22), \( \epsilon_i = \xi_i c_i q_i \), are constants. Therefore we have \( \hat{\xi}_i = \hat{c}_i q_i = \hat{C}_i \).

We assume that firms charge constant markups \( \mu_i = \frac{\sum_j V_{ij} + V_{iH} + V_{iF}}{c_i} \) to both composite goods. This assumption makes markups at the firm-level consistent with the baseline model with variable markups. We take the reduction in the markups that firms charge to the composite good used for intermediate inputs, \( \hat{\mu}_{iB_R} \), as the shock.

Before proceeding with the following steps, we first compute the share of sector \( u \) goods in household consumption as \( \gamma_{uH} = \frac{\sum_i V_{iu} V_{H}}{\sum_i V_{iH}} \). Denote the share of sector \( v \) goods in the domestic intermediate input bundle for sector \( u \) as \( \gamma_{vu} = \frac{\sum_i \sum_{sH} V_{i}}{\sum_i \sum_{sH} V_{i}} \). Also denote \( s_{iH}^{H} \) as firm \( i \)'s share of sales to household among other firms in the same sector, thus \( s_{iH} = s_{iH}^{H} \gamma_{uH} \). Analogously, denote \( s_{iB}^{B} \) as firm \( i \)'s share of sales to other firms, among other firms in the same sector.

1. Guess the change in nominal wage, \( \hat{w} \).

2. Compute the changes in prices with

\[
\hat{c}_i^{1-\eta} = s_{hi} \hat{w}_i^{1-\eta} + s_{mi} \hat{P}_m^{1-\eta},
\]

\[
\hat{P}_m^{1-\rho} = s_{Di} \left( \prod_v \hat{P}_{vB}^{\sigma_{\ell}(v)} \right)^{1-\rho} + s_{Fi}^m,
\]

\[
\hat{P}_{v(j)B}^{1-\sigma_{\ell}(j)} = \sum_{j \in Y} s_{j(i)} B_{jB}^{1-\sigma_{\ell}(j)} c_j^{1-\sigma_{\ell}(j)}.
\]
3. Compute other variables’ changes with

\[
\hat{p}_{vH}^{1-\sigma} = \sum_{j \in v} s^H_{v(j)} \hat{s}^{1-\sigma}
\]

\[
\hat{p} = \prod_v \hat{p}_{vH}^{1-\sigma}
\]

\[
\hat{s}_{m}^{s_{Di}} = \left(\prod_v \hat{p}_{vH}^{1-\sigma}\right)^{1-\rho} \hat{p}_{mi}^{1-\rho}
\]

\[
\hat{s}_{mi} = \hat{p}_{mi}^{1-\eta} \hat{c}_i^{1-\eta}
\]

\[
\hat{s}_{fi} = \hat{w}^{1-\eta} \hat{c}_i^{1-\eta}
\]

\[
\hat{s}_{mi}^{B} = \mu_{iB}^{1-\sigma(u_i)} \hat{c}_i^{1-\sigma(u_i)} \hat{p}_{u(i)B}^{1-\sigma(u_i)-1}
\]

\[
\hat{s}_{mi}^{H} = \hat{c}_i^{1-\sigma} \hat{p}_{u(i)H}^{1-\sigma(u_i)}
\]

\[
\hat{V}_{iF} = \hat{c}_i^{1-\sigma}
\]

4. Compute \(\hat{C}_i\) from

\[
\hat{C}_i = \frac{1}{C_i} \sum_{v} s^H_{v(i)} \hat{C}_i + \frac{1}{C_i} \sum \frac{s_{mi}^{s_{Di}}}{\mu_i^{B} s_{mi}^{s_{Bi}}} \left( \sum_{v} \sum_{j \in v} s_{mi}^{s_{Di}} s_{mi}^{s_{Bi}} \hat{C}_j \right) + \frac{1}{C_i} V_{iF} \hat{V}_{iF}
\]

\[
E = \frac{1}{1 - \sum_{i} \frac{\mu_i - 1}{\mu_i} s_{Hi}^{s_{Hi}}} \times \left\{ \frac{wL}{E} \hat{w} + \frac{1}{E} \sum_i \frac{\mu_i \hat{p}_{u(i)B}}{\mu_i^{B} s_{mi}^{s_{Bi}}} \left( \sum_{v} \sum_{j \in v} s_{mi}^{s_{Di}} s_{mi}^{s_{Di}} \hat{C}_j \right) \right. \\
\left. + \frac{1}{E} \sum_i \frac{\mu_i - 1}{\mu_i} V_{iF} \hat{V}_{iF} - \frac{TB}{E} \frac{\sum_i \xi_i \hat{C}_i}{E} \right\}
\]

5. Update \(\hat{w}\) from

\[
\hat{w} = \frac{1}{wL} \sum_i wL \hat{s}_i \hat{C}_i
\]

and iterate from Step 1 until \(\hat{w}\) converges.

**B.5 System of equilibrium changes upon changes in the foreign price**

Here we present the system of equations that pins down changes in the equilibrium variables upon an exogenous change in the price of foreign goods, \(\hat{p}_F\).
B.5.1 System of exact changes

First we present the system under endogenous markups. Unlike the system in Appendix B.3, markups \( \{\mu_{ij}\} \) are endogenous variables. In this section, we assume that the error terms in the estimating equation (22), \( \epsilon_i = \frac{\xi_i c_i q_i}{c_{ij} q_{ij}} \), are constants. Therefore we have \( \hat{\epsilon}_i = c_{ij} q_{ij} = \hat{C}_i \).

We first focus on the exact changes, and follow the steps below.

1. Guess the change in nominal wage, \( \hat{w}_1 \).

2. Compute the changes in prices and shares with

\[
\hat{c}_i^{1-\eta} = s_i \hat{w}_1^{1-\eta} + s_{mi} \hat{p}_m^{1-\eta} \\
\hat{p}_m^{1-\rho} = \sum_{v} s_{vi} (\hat{p}_m^{v})^{1-\rho} + s_{vi} \hat{p}_m^{1-\rho} \\
(\hat{p}_m^v)^{1-\sigma_v} = \sum_{j \in Z_i, j \in V} s_{v}^{(j)} \hat{p}_j^{1-\sigma_v} \hat{c}_j^{1-\sigma_v} \\
\hat{p}_j = \hat{\epsilon}_j \frac{\hat{e}_j - 1}{\hat{e}_j \hat{e}_j - 1} \\
\hat{e}_j = \sigma_v (1 - s_{v}^{(j)} \hat{p}_j) + \rho s_{v}^{(j)} (1 - s_{v}^{(j)} \hat{p}_j) + \eta s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)} \\
\hat{\epsilon}_j = 1 \frac{\hat{e}_j}{\hat{e}_j} (\sigma_v (1 - s_{v}^{(j)} \hat{p}_j) + \rho s_{v}^{(j)} (1 - s_{v}^{(j)} \hat{p}_j) + \eta s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)}) \\
\hat{s}_{v}^{(j)} = \hat{\mu}_{ji}^{1-\sigma_v (j)} s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)} s_{v}^{(j)} \\
\hat{s}_{v}^{(j)} = (\hat{p}_v^m)^{1-\rho} \hat{p}_m^{1-\rho} \\
\hat{P} = \left( \sum_{i} s_{ih} \hat{c}_i^{1-\sigma} \right)^{1/\alpha} \\
\hat{V}_h = \hat{\epsilon}_i^{1-\sigma} \hat{P}^{\sigma-1} \\
\hat{V}_v = \hat{\epsilon}_i^{1-\sigma}.
4. Compute $\hat{C}_i$ from

$$\hat{C}_i = \frac{V_{iH}}{C_i \mu_{ii}} \hat{s}_{ii} \hat{E} + \frac{V_{iF}}{C_i \mu_{ii}} \hat{V}_{iF} + \frac{1}{C_i} \sum_j V_{ij} \hat{s}_{ij} \hat{C}_j$$

$$\hat{E} = \frac{1}{1 - \sum_i \frac{1}{E} \hat{s}_{ii} \hat{E}}$$

$$\times \left\{ \frac{wL}{E} \hat{\nu} - \sum_i \frac{\xi_i}{E} \hat{C}_i - \frac{T B}{E} + \sum_i \frac{\pi_i}{E} \left( \sum_k \frac{1}{\pi_i} V_{ik} \hat{\mu}_{ik} - \frac{1}{\pi_i} \hat{s}_{ik} \hat{C}_k + \frac{1}{\pi_i} \mu_{ii} - \frac{1}{\pi_i} V_{iF} \hat{V}_{iF} \right) \right\}.$$  

5. Update $\hat{w}$ from

$$\hat{w} = \frac{1}{wL} \sum_i s_i c_i \hat{s}_i \hat{C}_i,$$

and iterate from Step 1 until $\hat{w}$ converges.

### B.5.2 System of first-order approximated changes

We then turn to the system of first-order approximated changes. Taking first order approximations of the system presented in the previous section, we obtain the following steps.

1. Guess the change in nominal wage, $\frac{dw}{w}$.

2. Compute the changes in prices and markups with

$$\frac{dc_i}{c_i} = \frac{\hat{s}_i}{w} + \sum_{j \in Z} s_{ji} \left( \frac{\hat{\mu}_{ji}}{\mu_{ji}} \frac{\hat{c}_j}{c_j} + S_{F_i} \frac{\hat{p}_F}{p_F} \right)$$

$$\frac{d\mu_{ji}}{\mu_{ji}} = -\Gamma_{ji} \frac{dc_j}{c_j} + \Gamma_{ji} \frac{\hat{p}_j}{p_j},$$

where $\Gamma_{ji}$ equals the elasticity of markup $\mu_{ji}$ with respect to the supplier’s cost $c_j$:

$$\Gamma_{ji} = -\frac{\partial \mu_{ji}}{\partial c_j} \frac{c_j}{\mu_{ji}} = \frac{1}{1 + \Gamma_{ji}}$$

$$\Gamma_{ji} = \left( \sigma_{v(j)} - \sigma_{v(j)} (1 - s_{v(j)}(1 - \sigma_{v(j)})) + s_{v(j)} (1 - \rho) s_{v(j)} (1 - \rho) \right) \hat{e}_{ji}(\hat{e}_{ji} - 1).$$

The term $\frac{dp_{ji}}{p_{ji}}$ is expressed as:

$$\frac{dp_{ji}}{p_{ji}} = \frac{\gamma^{\nu_{i}(j)}}{\Gamma_{ji}} \sum_{k \in V_{i}(j), k \neq j} s_{i(k)}^{v(j)} \left( \frac{d\mu_{ki}}{\mu_{ki}} + \frac{dc_k}{c_k} \right) + \frac{\gamma^{p_{i}(j)}}{\Gamma_{ji}} \left( \sum_{k \in V_{i}(j), k \neq j} s_{i(k)}^{m} \left( \frac{d\mu_{ki}}{\mu_{ki}} + \frac{dc_k}{c_k} \right) + S_{F_i} \frac{\hat{p}_F}{p_F} \right)$$
where

\[
\tau^{e(\mathcal{V})}_{ij} = \frac{(\varepsilon_{ji} - \sigma_{v(j)}) (1 - \sigma_{v(j)}) - (\eta - \rho) (1 - \rho) s^{v(j)}_{ji} (1 - s^{m}_{vji}) s^{m}_{vji}}{\varepsilon_{ji} (\varepsilon_{ji} - 1)}
\]

\[
\tau^{e(\mathcal{U})}(\mathcal{V})_{ij} = \frac{s^{v(j)}_{ji} (\eta - \rho) (1 - \rho) s^{m}_{vji}}{\varepsilon_{ji} (\varepsilon_{ji} - 1)}
\]

\(\frac{dp_{ji}}{pp}\) is the weighted average of the price changes of \(i\)'s suppliers other than \(j\), with suppliers in the same sector as \(j\)'s sector \((k \in \mathcal{V}(j), k \neq j)\) and suppliers in different sectors as \(j\)'s sector \((k \in \mathcal{U}, u \neq v(j))\) having different elasticity weights.\(^{35}\)

3. Compute other changes in prices and shares with

\[
\frac{dp_{vi}^m}{p_{vi}^m} = \sum_{j \in \mathcal{Z}_i, v \in \mathcal{V}} s^{v(j)}_{ji} \left( \frac{dp_{ji}}{\mu_{ji}} + \frac{dc_{ji}}{c_i} \right)
\]

\[
\frac{dp_{mi}}{p_{mi}} = \sum_{v} s^{v}_{vi} \frac{dp_{vi}^m}{p_{vi}^m} + s^{m}_{F_i} \frac{dp_{F}}{p_{F}}
\]

\[
\frac{ds^{v(j)}_{ji}}{s^{v(j)}_{ji}} = (1 - \sigma_{v}) \left( \frac{dp_{ji}}{\mu_{ji}} + \frac{dc_{ji}}{c_i} - \frac{dp_{mi}^m}{p_{mi}^m} \right)
\]

\[
\frac{ds^{m}_{vi}}{s^{m}_{vi}} = (1 - \rho) \left( \frac{dp_{vi}^m}{p_{vi}^m} - \frac{dp_{mi}}{p_{mi}} \right)
\]

\[
\frac{ds_{mi}}{s_{mi}} = (1 - \eta) \left( \frac{dp_{mi}}{p_{mi}} - \frac{dc_{i}}{c_i} \right)
\]

\[
\frac{ds_{ji}}{s_{ji}} = (1 - \eta) \left( \frac{dw}{w} - \frac{dc_{i}}{c_i} \right)
\]

\[
\frac{dP}{P} = \sum_{i} s_{iH} \frac{dc_{i}}{c_i}
\]

\[
\frac{ds_{iH}}{s_{iH}} = (1 - \sigma) \left( \frac{dc_{i}}{c_i} - \frac{dP}{P} \right)
\]

\[
\frac{dV_{iF}}{V_{iF}} = (1 - \sigma) \frac{dc_{i}}{c_i}
\]

\(^{35}\)For illustration, consider a single-sector model where we impose \(\sigma_{u} = \rho\). Then we obtain

\[
\tau^{e}_{ji} = \frac{(\varepsilon_{ji} - \rho) (1 - \rho) (1 - s^{m}_{vji}) s^{m}_{vji}}{\varepsilon_{ji} (\varepsilon_{ji} - 1)}
\]

and

\[
\frac{dp_{ji}}{pp} = \frac{1}{1 - \rho} \left( \sum_{k \neq j} s^{m}_{ki} \left( \frac{dp_{ki}}{p_{ki}} + \frac{dc_{ki}}{c_i} \right) + s^{m}_{F_i} \frac{dp_{F}}{p_{F}} \right).
\]
4. Compute \( \frac{dC_i}{C_i} \) from
\[
\frac{dC_i}{C_i} = \frac{1}{C_i} \frac{V_{iH}}{\mu_{iH}} \left( \frac{ds_{iH}}{s_{iH}} + \frac{dE}{E} \right) + \frac{1}{C_i} \frac{V_{iF}}{\mu_{iH}} \left( \frac{ds_{iF}}{s_{iF}} + \frac{dC_i}{C_i} \right)
\]
\[
\frac{dE}{E} = \frac{1}{E} \left( 1 - \sum_k \frac{E_{kH}}{E_{kH}} \right) \times \left\{ \sum_i \frac{\mu_{iH} - 1}{\mu_{iH}} \frac{dC_i}{C_i} \right\}
\]
\[
+ \sum_i \left\{ \sum_k \frac{\mu_{ik} - 1}{\mu_{ik}} \left( \frac{1}{\mu_{ik} - 1} \frac{dC_k}{C_k} + \frac{dC_i}{C_i} \right) + \frac{\mu_{iH} - 1}{\mu_{iH}} \frac{dV_{iF}}{V_{iF}} \right\}
\]

5. Update \( \frac{dw}{w} \) from
\[
\frac{dw}{w} = \frac{1}{wL} \sum_i w_i \left( \frac{ds_{il}}{s_{il}} + \frac{dC_i}{C_i} \right).
\]

**B.6 System of equilibrium changes upon changes in the foreign price, constant markups**

Now we consider the system under constant markups, where firms charge heterogeneous markups of \( \mu_{ij} \) and \( \mu_{iH} \) as in the baseline economy. We back-out these markups as we do for the baseline economy, but treat them as fixed in the counterfactual exercise. As in the previous section, we assume that the error terms in the estimating equation (22), \( \epsilon_i = \frac{\xi_i}{\bar{e}_{iH}} \), are constants.

1. Guess the change in nominal wage, \( \hat{w} \).

2. Compute the changes in prices and shares with
\[
\begin{align*}
\hat{c}_i^{1-\eta} &= s_{il} \hat{w}^{1-\eta} + s_{mP_{mi}}^{1-\eta} \\
\hat{p}_{mi}^{1-p} &= \sum_v s_{v1}^{m} (\hat{p}_{v1}^{m})^{1-p} + s_{vF}^{m} \hat{p}_{F}^{1-p} \\
(\hat{p}_{v1}^{m})^{1-c_v} &= \sum_{j \in Z, j \in V} s_{ji}^{m} \hat{c}_j^{1-c_v}.
\end{align*}
\]
3. Compute other changes in prices and shares with

\[ \hat{s}_{\mu}^{(j)} = \hat{c}_{j}^{1-\sigma \epsilon_{j}} \left( \hat{p}_{vi(j)}^{m} \right)^{\sigma \epsilon_{j}} \]

\[ \hat{s}_{vi} = (\hat{p}_{vi})^{1-p} \hat{p}_{mi}^{1-p}. \]

\[ \hat{s}_{mi} = \hat{p}_{mi}^{1-\eta \epsilon_{i}}. \]

\[ \hat{s}_{li} = \hat{w}^{1-\eta \epsilon_{i}}. \]

\[ \hat{s}_{ji} = \hat{s}_{vi}^{(j)} \hat{s}_{mi}. \]

\[ \hat{P} = \left( \sum_{i} s_{iH} \hat{c}_{i}^{1-\sigma} \right)^{1/\sigma} \]

\[ \hat{s}_{iH} = \hat{c}_{i}^{1-\sigma} \hat{p}_{i}^{1-\sigma} \]

\[ \hat{V}_{iF} = \hat{c}_{i}^{1-\sigma}. \]

4. Compute \( \hat{C}_{i} \) from

\[ \hat{C}_{i} = \frac{1}{C_{i} \mu_{iH}} \hat{s}_{iH} \hat{E} + \frac{1}{C_{i} \mu_{iH}} \hat{v}_{iF} \hat{V}_{iF} + \frac{1}{C_{i}} \sum_{j} \frac{V_{ij}}{\mu_{ij}} \hat{s}_{ij} \hat{C}_{j} \]

\[ \hat{E} = \frac{1}{1 - \sum_{i} \frac{1}{E} \mu_{iH} \hat{V}_{iH} \hat{s}_{iH}} \times \left\{ \frac{wL}{E} \hat{w} - \sum_{i} \frac{E}{E} \hat{C}_{i} - \frac{TB}{E} + \sum_{i} \frac{\pi_{i}}{E} \left( \sum_{k} \frac{1}{\pi_{k}} - \frac{1}{\mu_{ik}} - \frac{1}{\mu_{ik}} \hat{V}_{iH} \hat{s}_{ik} \hat{C}_{k} + \frac{1}{\pi_{i}} \frac{1}{\mu_{iH}} \frac{1}{\mu_{iH}} \hat{V}_{iF} \hat{V}_{iF} \right) \right\}. \]

5. Update \( \hat{w} \) from

\[ \hat{w} = \frac{1}{wL} \sum_{i} s_{iH} \hat{q}_{i} \hat{s}_{iH} \hat{C}_{i} \]

and iterate from Step 1 until \( \hat{w} \) converges.

B.7 Alternative economy with perfect competition

To benchmark the results from Section 6 we additionally consider a back-of-the-envelope calculation of the economy’s aggregate response under the assumption of perfect competition. With perfect competition and other assumptions, one can solve for the aggregate counterfactual changes using firm-level information alone, making the information on firm-to-firm relationships irrelevant in aggregate counterfactual analyses. This result resembles that of Hulten (1978) and of Baqaee and Farhi (2017), but focus on global changes in response to large shocks, in a setup of an open economy. We demonstrate this with the following proposition and lemma. Consider the change in aggregate welfare, given an exogenous change in the foreign price, and denote the change in variable \( x \) from the pre-shock equilibrium \( x \) to the post-shock equilibrium \( x' \) be \( \hat{x} = x'/x \).
Assumption 2. Only composite final consumption goods are exported.

Assumption 3. Preferences and technologies have common CES parameters, \( \sigma = \eta = \rho = \sigma_u \).

Assumption 4. Goods are competitively priced, \( p_i = c_i \forall i \in \Omega \).

Assumption 5. There is no change in nominal wage level, \( \hat{w} = 1 \).

Proposition 1 (Network irrelevance with a common CES parameter). Suppose that Assumptions 2-5 hold. Denote \( \tilde{\sigma} \) as the common CES parameter in Assumption 3. Then the change in aggregate welfare, \( \hat{U} \), can be expressed as:

\[
\hat{U} = \left( \sum_{i \in \Omega} \frac{p_i q_i}{E + \text{Exports}} (s_{iH} + s_{Fi} \hat{p}_F^{1-\sigma}) \right)^{\frac{1}{\tilde{\sigma}}}. \tag{29}
\]

Proof. From Assumption 4, no firm generates profits. Hence, the change in welfare, \( \hat{U} \), is the inverse of the change in the aggregate price index:

\[
\hat{U} = \hat{P}^{-1}. \tag{30}
\]

From Assumptions 3, 4, and equation (5), we have

\[
\hat{p}^{1-\sigma} = \sum_{i \in \Omega} s_{iH} \hat{c}_i^{1-\sigma}, \tag{31}
\]

where \( \tilde{\sigma} \) is the common CES parameter and \( s_{iH} \) is firm \( i \)'s share in the final demand market for the heterogeneous goods sector: \( s_{iH} = \frac{p_i^{1-\sigma}}{p_{iH}^{1-\sigma}} \). From Assumptions 3, 4, and equation (7), we obtain the change in unit costs: \( \hat{c}_i^{1-\sigma} = \sum_k s_{iH} \hat{c}_k^{1-\sigma} + s_{iH} + s_{Fi} \hat{p}_F^{1-\sigma} \). Rearranging this into matrix form yields \( \square \)

\[
\hat{c}^{1-\sigma} = (I - S)'^{-1} (s_F + s_{Fi} \hat{p}_F^{1-\sigma}), \tag{32}
\]

where the \( (i, j) \) element of matrix \( S \) is \( s_{ij} \), and \( s_{Fi} \) and \( s_{iH} \) are vectors where their \( i \)'th elements are \( s_{Fi} \) and \( s_{iH} \).

On the output side, the revenue of firm \( i, p_i q_i \), is the sum of sales to households, exports, and sales to other firms. From Assumption 2, the share of each firm among exports are equal to that among sales to households, \( s_{iH} \). Thus from Assumptions 2 and 4, we obtain

\[
p_i q_i = s_{iH} E + s_{iH} \text{Exports} + \sum_j s_{ij} p_j q_j. \tag{33}
\]
Rearrange this into matrix form and obtain

\[
\frac{p \circ q}{E + \text{Exports}} = (I - S)^{-1} s_H, \tag{34}
\]

where \( s_H \) is a vector where its \( i \)'th element is \( s_{iH} \). Equation (34) implies that the firm-level measure \( \frac{p \circ q}{E + \text{Exports}} \) captures the centrality of each firm as a supplier of goods to final demand (including exports). The assumption of perfect competition makes the two matrices identical to each other in equations (32) and (34). Finally, combine equations (31), (32), and (34) to yield

\[
\hat{P}^{1-\sigma} = \sum_{i \in \Omega} \frac{p_i q_i}{E + \text{Exports}} \left( s_{ii} + s_{Fi} \hat{P}_F^{1-\sigma} \right). 
\]

Then from equation (30), we have

\[
\hat{U} = \left( \sum_{i \in \Omega} \frac{p_i q_i}{E + \text{Exports}} \left( s_{ii} + s_{Fi} \hat{P}_F^{1-\sigma} \right) \right)^{\frac{1}{1-\sigma}}. \tag{35}
\]

This result shows that under these assumptions, one does not need any information on how firms are linked with other firms in evaluating aggregate welfare changes. Firms’ direct exposure to the shock are captured by firms’ foreign input shares, \( s_{Fi} \). The importance of each firm in the production network is captured by the Domar (1961) weight adjusted for aggregate exports, \( \frac{p_i q_i}{E + \text{Exports}} \).

However, in order to compute the changes in price index and welfare, one needs to know the value of \( \tilde{\sigma} \) in addition to the firm-level observables. In the following lemma, we impose a stronger assumption in the preference and technologies and obtain a network irrelevance result where the aggregate welfare changes can be computed solely by firm-level observables.

**Assumption 6.** Preferences and technologies have Cobb-Douglas form, \( \sigma = \eta = \rho = \sigma_u = 1 \).

**Lemma 1** (Network irrelevance with Cobb-Douglas). Suppose that Assumptions 2, 4, and 6 hold. Then the change in aggregate welfare, \( \hat{U} \), can be expressed as:

\[
\ln \hat{U} = - \sum_{i \in \Omega} \frac{p_i q_i}{E + \text{Exports}} s_{Fi} \ln \hat{P}_F. \tag{35}
\]

Under the Cobb-Douglas assumption in both the preference and technologies, one obtains a log-linear expression where the aggregate welfare change is essentially a weighted sum of shocks that hit each firm. This case is useful as a benchmark since the necessary variables to compute this counterfactual change in the aggregate welfare are all observables in standard datasets.

We now discuss the assumptions. First, it is worth noting that the four assumptions in both Proposition 1 and Lemma 1 work as sufficient conditions in making the firm-to-firm information
irrelevant in the aggregate. In both Proposition 1 and Lemma 1, instead of having firms export their differentiated goods separately abroad, we assume that goods from all firms are bundled up to a composite final good, and that they are either consumed by the domestic household or exported abroad (Assumption 2). By treating the exports of firms in the same way as their sales to final demand, we can use aggregate final consumption and aggregate exports as the denominator of the Domar weights. Assumptions 3 and 4 in Proposition 1, and Assumptions 4 and 6 in Lemma 1 are also important. As we show in detail in the proof, we obtain equations (29) and (35) because firms’ Domar weights adjusted for aggregate exports, \( \frac{p_{\text{dom}}}{E + \text{Exports}} \), which capture the importance of firms as suppliers of goods, coincides with a measure of firms’ importance as consumers of primary goods. When either of these assumptions is violated, it creates a wedge between the two.\(^{36}\) Lastly, Assumption 5 is also needed in Proposition 1 because in order to compute the change in nominal wage, \( \hat{w} \), firm-level variables are not sufficient and one needs information on firm-to-firm sales.

C Additional estimation results

C.1 Distribution of errors

Figure 13 plots distribution of firm-level errors, \( \epsilon_i \), from equation (21) under the estimated CES parameters. The density plot is truncated at \( \epsilon_i = -1 \). The errors are concentrated around \( \epsilon_i = 0.2 \), and have a thick left tail. Overall this indicates that under the estimated parameters, firms overall purchase more inputs than they need in order to produce their observed outputs.

Figure 13: Distribution of \( \epsilon_i \)

Note: The figure displays the distribution of firm-level errors, \( \epsilon_i \), from equation (21).

\(^{36}\)See Baqae (2014) for a similar argument.
C.2 Assuming Cournot competition

When assuming Cournot competition in firm-to-firm trade instead, equation (14) becomes

\[ p_{ji} = \frac{\varepsilon_{ji}}{\varepsilon_{ji} - 1} c_j \]

\[ \varepsilon_{ji} = \left( \frac{1}{\sigma_{v(j)}} (1 - s_{v(j)}) + \frac{1}{\rho} s_{v(j)} (1 - s_{v(j)i}) + \frac{1}{\eta} s_{v(j)} s_{v(j)i} \right)^{-1}. \]

We follow the same procedure described in Section 4 and obtain the estimates shown in Table 22.
Table 22: Estimated CES parameters under Cournot competition

(a) $\eta$, $\rho$, and $\sigma$

<table>
<thead>
<tr>
<th>Description of sector</th>
<th>Estimate</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\eta}$</td>
<td>0.47</td>
<td>0.11</td>
</tr>
<tr>
<td>$\frac{1}{\rho}$</td>
<td>0.45</td>
<td>0.10</td>
</tr>
<tr>
<td>$\frac{\sigma}{\sigma-1}$</td>
<td>1.28</td>
<td>0.05</td>
</tr>
</tbody>
</table>

(b) Sectoral estimates of $\sigma_u$

<table>
<thead>
<tr>
<th>Description of sector</th>
<th>Estimate</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, forestry, and fishing</td>
<td>2.44</td>
<td>0.29</td>
</tr>
<tr>
<td>Mining and quarrying</td>
<td>2.33</td>
<td>0.30</td>
</tr>
<tr>
<td>Manufacture of food products, beverages, and tobacco products</td>
<td>3.41</td>
<td>0.48</td>
</tr>
<tr>
<td>Manufacture of textiles, apparel, leather, and related products</td>
<td>2.22</td>
<td>0.27</td>
</tr>
<tr>
<td>Manufacture of wood and paper products, and printing</td>
<td>3.05</td>
<td>0.41</td>
</tr>
<tr>
<td>Manufacture of coke, refined petroleum products, chemicals, and chemical products</td>
<td>2.69</td>
<td>0.35</td>
</tr>
<tr>
<td>Manufacture of pharmaceuticals, medicinal chemical, and botanical products</td>
<td>4.25</td>
<td>0.70</td>
</tr>
<tr>
<td>Manufacture of rubber and plastics products, and other non-metallic mineral products</td>
<td>3.48</td>
<td>0.50</td>
</tr>
<tr>
<td>Manufacture of basic metals and fabricated metal products, except machinery and equipment</td>
<td>2.97</td>
<td>0.39</td>
</tr>
<tr>
<td>Manufacture of computer, electronic, and optical products</td>
<td>2.31</td>
<td>0.28</td>
</tr>
<tr>
<td>Manufacture of electrical equipment</td>
<td>3.53</td>
<td>0.52</td>
</tr>
<tr>
<td>Manufacture of machinery and equipment n.e.c.</td>
<td>2.92</td>
<td>0.39</td>
</tr>
<tr>
<td>Manufacture of transport equipment</td>
<td>2.36</td>
<td>0.33</td>
</tr>
<tr>
<td>Other manufacturing, and repair and installation of machinery and equipment</td>
<td>2.36</td>
<td>0.27</td>
</tr>
<tr>
<td>Electricity, gas, steam and air-conditioning supply</td>
<td>2.08</td>
<td>0.47</td>
</tr>
<tr>
<td>Water supply, sewerage, waste management, and remediation</td>
<td>2.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Construction</td>
<td>3.65</td>
<td>0.52</td>
</tr>
<tr>
<td>Wholesale and retail trade, repair of motor vehicles and motorcycles</td>
<td>2.27</td>
<td>0.25</td>
</tr>
<tr>
<td>Transportation and storage</td>
<td>2.98</td>
<td>0.39</td>
</tr>
<tr>
<td>Accommodation and food service activities</td>
<td>3.67</td>
<td>0.53</td>
</tr>
<tr>
<td>Publishing, audiovisual and broadcasting activities</td>
<td>2.48</td>
<td>0.30</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>2.04</td>
<td>0.22</td>
</tr>
<tr>
<td>IT and other information services</td>
<td>2.21</td>
<td>0.24</td>
</tr>
<tr>
<td>Real estate activities</td>
<td>1.69</td>
<td>0.15</td>
</tr>
<tr>
<td>Legal, accounting, management, architecture, engineering, technical testing, and analysis activities</td>
<td>1.68</td>
<td>0.14</td>
</tr>
<tr>
<td>Scientific research and development</td>
<td>4.90</td>
<td>0.83</td>
</tr>
<tr>
<td>Other professional, scientific and technical activities</td>
<td>2.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Administrative and support service activities</td>
<td>2.38</td>
<td>0.28</td>
</tr>
<tr>
<td>Other services</td>
<td>2.53</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note: Standard errors are based on 25 bootstrap samples drawn with replacements. The samples are drawn at the firm-level for each sector.
C.3 Accounting for capital inputs

In the model, total input $c_i q_i$ is an aggregate of labor costs and goods purchases. Here we account for capital inputs by interpreting labor as the composite input of labor and capital. As we do not directly observe capital rental costs for each firm, we take two alternate approaches.

First, we assume that firms have common labor shares, and uniformly scale up labor cost. We use the aggregate labor share of $2/3$ that we compute as the total labor cost divided by the total value added. Second, we assume that the user cost of capital consists of capital depreciation rate and the interest rate. Following Dhyne, Petrin, Smeets, and Warzynski (2017), we set the yearly depreciation rate as 8 percent and set the interest rate as the long-term interest rate in Belgium. We compute the capital rental costs using fixed tangible assets reported in the annual accounts. We report the estimation results in Table 23.
Table 23: Estimated CES parameters accounting for capital

(a) $\eta$, $\rho$, and $\sigma$

<table>
<thead>
<tr>
<th>Description of sector</th>
<th>$\eta$</th>
<th>$\rho$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Labor and goods)</td>
<td>2.05</td>
<td>3.03</td>
<td>5.29</td>
</tr>
<tr>
<td>(Sectoral goods and imports in production)</td>
<td>1.63</td>
<td>2.55</td>
<td>4.79</td>
</tr>
<tr>
<td>(Firms’ goods in consumption)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common labor share</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual accounts</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Sectoral estimates of $\sigma_u$

<table>
<thead>
<tr>
<th>Description of sector</th>
<th>Common labor share</th>
<th>Annual accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, forestry, and fishing</td>
<td>3.07</td>
<td>3.57</td>
</tr>
<tr>
<td>Mining and quarrying</td>
<td>2.66</td>
<td>2.59</td>
</tr>
<tr>
<td>Manufacture of food products, beverages, and tobacco products</td>
<td>4.97</td>
<td>3.73</td>
</tr>
<tr>
<td>Manufacture of textiles, apparel, leather, and related products</td>
<td>3.23</td>
<td>2.27</td>
</tr>
<tr>
<td>Manufacture of wood and paper products, and printing</td>
<td>4.79</td>
<td>3.48</td>
</tr>
<tr>
<td>Manufacture of coke, refined petroleum products, chemicals, and chemical products</td>
<td>3.51</td>
<td>2.90</td>
</tr>
<tr>
<td>Manufacture of pharmaceuticals, medicinal chemical, and botanical products</td>
<td>10.26</td>
<td>5.73</td>
</tr>
<tr>
<td>Manufacture of rubber and plastics products, and other non-metallic mineral products</td>
<td>5.33</td>
<td>4.23</td>
</tr>
<tr>
<td>Manufacture of basic metals and fabricated metal products, except machinery and equipment</td>
<td>4.69</td>
<td>3.36</td>
</tr>
<tr>
<td>Manufacture of computer, electronic, and optical products</td>
<td>3.16</td>
<td>2.73</td>
</tr>
<tr>
<td>Manufacture of electrical equipment</td>
<td>5.40</td>
<td>3.27</td>
</tr>
<tr>
<td>Manufacture of machinery and equipment n.e.c.</td>
<td>3.86</td>
<td>3.09</td>
</tr>
<tr>
<td>Manufacture of transport equipment</td>
<td>3.41</td>
<td>4.42</td>
</tr>
<tr>
<td>Other manufacturing, and repair and installation of machinery and equipment</td>
<td>3.03</td>
<td>2.38</td>
</tr>
<tr>
<td>Electricity, gas, steam and air-conditioning supply</td>
<td>2.13</td>
<td>2.16</td>
</tr>
<tr>
<td>Water supply, sewerage, waste management, and remediation</td>
<td>2.52</td>
<td>2.34</td>
</tr>
<tr>
<td>Construction</td>
<td>6.43</td>
<td>3.83</td>
</tr>
<tr>
<td>Wholesale and retail trade, repair of motor vehicles and motorcycles</td>
<td>2.94</td>
<td>2.39</td>
</tr>
<tr>
<td>Transportation and storage</td>
<td>4.48</td>
<td>3.22</td>
</tr>
<tr>
<td>Accommodation and food service activities</td>
<td>7.87</td>
<td>4.47</td>
</tr>
<tr>
<td>Publishing, audiovisual and broadcasting activities</td>
<td>3.18</td>
<td>2.48</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>2.96</td>
<td>2.51</td>
</tr>
<tr>
<td>IT and other information services</td>
<td>3.54</td>
<td>2.30</td>
</tr>
<tr>
<td>Real estate activities</td>
<td>2.03</td>
<td>2.28</td>
</tr>
<tr>
<td>Legal, accounting, management, architecture, engineering, technical testing, and analysis activities</td>
<td>2.38</td>
<td>1.79</td>
</tr>
<tr>
<td>Scientific research and development</td>
<td>11.52</td>
<td>3.99</td>
</tr>
<tr>
<td>Other professional, scientific and technical activities</td>
<td>3.77</td>
<td>2.62</td>
</tr>
<tr>
<td>Administrative and support service activities</td>
<td>3.50</td>
<td>2.39</td>
</tr>
<tr>
<td>Other services</td>
<td>8.07</td>
<td>2.80</td>
</tr>
</tbody>
</table>

In the two cases above, the estimates of most parameters are larger than those without taking account capital inputs. Inflating firms’ labor costs by adding capital usage costs lead to smaller firm-level markups, and these lower accounting markups are accommodated by the larger CES parameters.
D Additional results from Section 5

D.1 Correlation between \( \hat{c}_i \) and downstreamness measures

We present the correlations between firm-level cost changes, \( \hat{c}_i \), and several measures that capture how downstream firms are in the production network. We consider log-total sales, \( \log(V_i) \), log-sales to domestic final demand, \( \log(V_{iH}) \), and revenue share of sales to domestic final demand, \( r_{iH} = \frac{V_{iH}}{V_i} \). In addition, we also consider the total revenue share of sales to domestic final demand. This measure takes into account not only the firm’s own sales to domestic final demand but also its buyers’ sales and their buyers’ sales and so on: \( r_{iH}^{total} = r_{iH} + \sum_j r_{ij} r_{jH}^{total} \), where \( r_{ij} \) is the share of sales to firm \( j \), out of firm \( i \)’s total sales. Finally, we also consider the upstreamness measure, \( Upstreamness_i \), defined by Antrás, Chor, Fally, and Hillberry (2012). Table 24 reports the results.

<table>
<thead>
<tr>
<th></th>
<th>( \log(V_i) )</th>
<th>( \log(V_{iH}) )</th>
<th>( r_{iH} )</th>
<th>( r_{iH}^{total} )</th>
<th>( Upstreamness_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation with ( \hat{c}_i )</td>
<td>-0.08</td>
<td>-0.18</td>
<td>-0.22</td>
<td>-0.17</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Note: This table shows the correlations between firm-level cost changes, \( \hat{c}_i \), upon a 20 percent reduction in firm-to-firm markups in the baseline economy. \( Upstreamness_i \) is a firm-level upstreamness measure defined by Antrás, Chor, Fally, and Hillberry (2012).

D.2 Taking out the effects of the changes in \( \sum_i \xi_i \)

We first present results where we consider the change in aggregate expenditure without considering the changes in \( \sum_i \xi_i \). We define the aggregate expenditure without taking into account \( \sum_i \xi_i \), \( \hat{E} \), as \( wL + \Pi - TB \) and consider \( \hat{E} \) as \( \frac{wL}{\hat{E}} \hat{\hat{w}} + \frac{\Pi}{\hat{E}} \hat{\hat{I}} - \frac{TB}{\hat{E}} \). We report the results on \( \hat{E} \) and the analogous change in aggregate welfare, \( \hat{\hat{u}} \), in Table 25.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Sectoral roundabout</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{u} = \hat{E}/\hat{\hat{P}} )</td>
<td>1.129</td>
<td>1.108</td>
</tr>
<tr>
<td>( \hat{E} )</td>
<td>1.074</td>
<td>1.063</td>
</tr>
<tr>
<td>( \hat{\hat{P}}/\hat{\hat{w}} )</td>
<td>0.913</td>
<td>0.920</td>
</tr>
<tr>
<td>( \hat{\hat{I}} )</td>
<td>1.100</td>
<td>1.079</td>
</tr>
</tbody>
</table>

Note: \( \hat{E} \) is defined as \( wL + \Pi - TB \), and the variables in the table are computed using the equilibrium changes from the system described in Appendix B.4.

Another way to not take into account the changes in \( \sum_i \xi_i \) is to consider the system of counterfactual changes by fixing the values of \( \xi_i \). As noted on page 24, this approach does not allow us to consider extreme cases, but the aggregate welfare is only affected by the change in the nominal wage.
and aggregate profits. We solve the system of equilibrium changes in Appendix B.4, but now solving for both $\hat{c}_i \hat{q}_i$ and $\hat{C}_i$ separately, using the relationship $\hat{c}_i \hat{q}_i = \frac{c_i}{c_i} \hat{C}_i + \frac{\xi_i}{c_i}$. Table 26 presents the results.

Table 26: Aggregate effects when treating $\xi_i$ as fixed

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Sectoral roundabout</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U} = \hat{E}/\hat{P}$</td>
<td>1.173</td>
<td>1.141</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>1.124</td>
<td>1.099</td>
</tr>
<tr>
<td>$\hat{P}/\hat{w}$</td>
<td>0.908</td>
<td>0.917</td>
</tr>
<tr>
<td>$\hat{\Pi}$</td>
<td>1.140</td>
<td>1.106</td>
</tr>
</tbody>
</table>

Note: The table reports the results when we solve for the equilibrium changes in the system described in Appendix B.4, but now solving for both $\hat{c}_i \hat{q}_i$ and $\hat{C}_i$ separately using the relationship $\hat{c}_i \hat{q}_i = \frac{c_i}{c_i} \hat{C}_i + \frac{\xi_i}{c_i}$.

Finally, we follow the approach by Ossa (2014) and first eliminate the differences between the observed and model implied input values, $\xi_i$. We first solve for the counterfactual changes by forcing the observed differences, $\xi_i$, to zero. The resulting economy becomes fully consistent with the model, with which we can solve for the counterfactual changes under $\hat{\mu}_i$. In the estimated model the sum of the differences between the observed and model implied input values, $\Sigma_i \xi_i$, turn out to be positive. This means that under the estimated parameters, firms overall purchase more inputs than they need in order to produce their observed outputs. By eliminating these differences, we obtain an economy where firms produce more, and gain larger profits than the observed economy. Because firms purchase greater amount of inputs, the reduction in firm-to-firm markups results in a greater increase in aggregate expenditure, compared to the case in which we solve for the counterfactual changes using the observed economy.

Table 27: Aggregate effects by first eliminating $\xi_i$

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Sectoral roundabout</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U} = \hat{E}/\hat{P}$</td>
<td>1.156</td>
<td>1.129</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>1.107</td>
<td>1.085</td>
</tr>
<tr>
<td>$\hat{P}$</td>
<td>0.908</td>
<td>0.915</td>
</tr>
<tr>
<td>$\hat{\Pi}$</td>
<td>1.147</td>
<td>1.111</td>
</tr>
</tbody>
</table>

Note: The table reports the results when we solve for the equilibrium changes in the system described in Appendix B.4, but first solving the system taking $\xi_i = 0$ as the shock.

D.3 First-order approximated changes and different magnitudes of shocks

Here we report numbers analogous to those of the baseline model in Table 5, with first-order approximation and under different magnitudes of shocks. See the system of first-order approximated equilibrium changes in Appendix B.3.2.
Table 28: Welfare effects of reduction in firm-to-firm markups

<table>
<thead>
<tr>
<th></th>
<th>10 percent reduction</th>
<th>20 percent reduction</th>
<th>30 percent reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline, ( \hat{x} )</td>
<td>FOA, ( \frac{dx}{x} )</td>
<td>Baseline, ( \hat{x} )</td>
</tr>
<tr>
<td>Agg. Welfare</td>
<td>1.032</td>
<td>0.030</td>
<td>1.073</td>
</tr>
<tr>
<td>Agg. Expenditure</td>
<td>1.010</td>
<td>0.010</td>
<td>1.021</td>
</tr>
<tr>
<td>Agg. price index</td>
<td>0.979</td>
<td>-0.019</td>
<td>0.952</td>
</tr>
<tr>
<td>Agg. profits</td>
<td>1.0437</td>
<td>0.043</td>
<td>1.100</td>
</tr>
<tr>
<td>Nominal wage</td>
<td>1.020</td>
<td>0.018</td>
<td>1.042</td>
</tr>
</tbody>
</table>

Note: This table shows the results from the baseline model using the observed firm-to-firm trade network in 2012 and feed different magnitudes of reduction in all markups in firm-to-firm trade. The baseline results are denoted in terms of equilibrium changes, \( \hat{x} \), and the first-order approximated results are denoted in log-changes, \( \frac{dx}{x} \).

E Additional results from Section 6

E.1 Correlations between cost changes and firm-level variables

Figure 14 plots the correlations between firm-level cost changes under endogenous markups, \( \hat{c}_{\text{endog}}^i \), and firm-level variables. It also plots the correlations between the difference in firm-level cost changes under endogenous and constant markups, \( \frac{\hat{c}_{\text{endog}}^i - \hat{c}_{\text{const}}^i}{\hat{w}_{\text{endog}}} - \frac{\hat{c}_{\text{const}}^i}{\hat{w}_{\text{const}}} \), and firm-level variables.

Figure 14: Correlations between cost changes and firm-level variables

Note: The black bars depict the univariate correlations between firms’ cost changes under endogenous markups, \( \hat{c}_{\text{endog}}^i \), and firm-level variables. The white bars depict the correlations between the differences in firm-level cost changes under endogenous markups and constant markups, \( \frac{\hat{c}_{\text{endog}}^i - \hat{c}_{\text{const}}^i}{\hat{w}_{\text{endog}}} - \frac{\hat{c}_{\text{const}}^i}{\hat{w}_{\text{const}}} \), and firm-level variables.

E.2 Shock to one importer

Here we consider a shock of foreign price reduction that hits a single importer, firm I. In this exercise we pick the importer with the largest number of domestic buyer as firm I. Analogous to Figure 3b,
we first plot the differences in firm-level cost changes normalized for changes in nominal wages, across two economies: the baseline economy and the economy with constant markups. We plot the histogram of these differences in Figure 15. Compared to Figure 3b in which we consider a uniform foreign price change that affected all importers, Figure 15 shows larger differences between cost changes under endogenous markups and cost changes under constant markups. In this case, 71 percent of firms experience cost reductions that are smaller than under constant markups by 1 percent or more, and 9 percent of firms experience cost reductions that are greater than under constant markups by 1 percent or more.

Figure 15: Differences in $\hat{c}_i$, endogenous vs. constant markups

![Histogram of cost differences]

Note: The figure plots the distribution of the differences in normalized cost changes under the baseline economy and the constant markups economy. The right figure is truncated from below, and the minimum value of $\frac{\hat{c}_{\text{endog}}_i - \hat{c}_{\text{const}}_i}{\hat{w}_{\text{endog}}_i - \hat{w}_{\text{const}}_i}$ is -0.47.

We then plot in Figure 16 the firms’ average change in markups, $\sum_{j \in Z_i} s_{ji} \frac{\partial \mu_j}{\partial \mu_j}$, against the measure capturing how close first are to the shock. We construct a measure $s_{iI}^{Total}$ that captures firms’ exposure to firm $I$’s goods:

$$s_{iI}^{Total} = \sum_{k \in Z_i} s_{ki} s_{kI}^{Total} \quad \text{if } i \neq I$$

$$s_{iI}^{Total} = 1 \quad \text{if } i = I.$$

As in Figure 5, one can also see a positive correlation between the two measures. In this case the correlation between the two measures is 0.59, larger than that under the uniform foreign price change.
Figure 16: Average change in markups, $\sum_{j \in Z_i} s_{ji} \frac{d\mu_j}{\mu_j}$, and exposure to firm $I$, $s_{Ii}^{Total}$

Note: The figure plots the first-order approximated changes in average markups charged from suppliers, $\sum_{j \in Z_i} s_{ji} \frac{d\mu_j}{\mu_j}$, upon a 20 percent reduction in the price of foreign goods that firm $I$ imports, against firms’ exposure to firm $I$’s goods, $s_{Ii}^{Total}$. The least-squares line has a y-intercept of -0.00002 and slope coefficient of 0.0011. The R-squared is 0.35. The correlation between the two variables is 0.59.

Finally in Table 29, we report the differences in the aggregate predictions under the baseline economy and under constant markups, across the two shocks. We show the differences in the counterfactual changes of aggregate welfare, normalized by the aggregate changes predicted under constant markups. While endogenous markups amplify the increase in aggregate welfare by around 0.2 percent when considering the uniform price change as the shock, once we consider the price change hitting a single firm, then incorporating endogenous markups attenuate the increase in aggregate welfare by around 3 percent.

Table 29: Differences in aggregate predictions under baseline and constant markups

<table>
<thead>
<tr>
<th>Shock to all importers</th>
<th>Shock to firm $I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(U_{endog} - U_{const})/(U_{const} - 1)$</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

Note: The table shows the differences in aggregate predictions under the two models, normalized by the aggregate changes predicted under constant markups. We consider 20 percent reduction in the price of foreign goods.

E.3 From Lemma 1 to the baseline economy

In this section we characterize the differences between the aggregate predictions under Lemma 1 outlined in Appendix B.7 and under the baseline model. To do so, we start with Lemma 1 and relax
the assumptions one by one and also add assumptions needed to make the underlying model closer to the baseline model.

Table 30 reports the results. In the left column we present the results from Lemma 1, which involved Assumptions 2, 4, and 6. In the next column we first relax Assumption 2, which assumes that exports are done by the composite final good. The third column further relaxes the Cobb-Douglas assumption (Assumption 6), and introduce CES production functions and preference using the estimated parameters. Further, in order to compute for nominal wage changes that ensure trade balance we introduce Assumption 1 of constant measurement errors, $\epsilon_i$. The fourth column further relaxes the perfect competition assumption (Assumption 4). This model is identical to the model under constant markups introduced in Section 3.4. Finally, we introduce oligopolistic competition, which makes the model to our baseline model. Comparing the aggregate predictions across models, we find that relaxing the Cobb-Douglas assumption and using CES structures makes quantitatively the largest change, making the prediction of the model closer to that of the baseline model.

<table>
<thead>
<tr>
<th>Lemma 1</th>
<th>Perfect competition, Cobb-Douglas</th>
<th>Perfect competition, CES</th>
<th>Constant markups, CES</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumptions</td>
<td>2, 4, and 6</td>
<td>4 and 6</td>
<td>1 and 4</td>
<td>1</td>
</tr>
<tr>
<td>$U = E/P$</td>
<td>1.129</td>
<td>1.109</td>
<td>1.284</td>
<td>1.272</td>
</tr>
<tr>
<td>$\hat{E}$</td>
<td>1</td>
<td>1</td>
<td>1.162</td>
<td>1.149</td>
</tr>
<tr>
<td>$\hat{P}/\hat{w}$</td>
<td>0.886</td>
<td>0.902</td>
<td>0.874</td>
<td>0.876</td>
</tr>
<tr>
<td>$\hat{\Pi}$</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>1.329</td>
</tr>
</tbody>
</table>

Note: This table shows aggregate predictions upon a 20 percent reduction in the price of foreign goods.

### E.4 Proposition 1, with different values of $\bar{\sigma}$

Table 31 reports the change in aggregate welfare predicted as in Proposition 1 outlined in Appendix B.7. We show results when we feed in no shock, $\hat{p}_F = 1$, and 20 percent reduction in the foreign price, $\hat{p}_F$. For the common CES parameter $\bar{\sigma}$, we use three different values: the estimates values for $\sigma$, $\rho$, and $\eta$.

Notice that even when we feed in no shock, Proposition 1 predicts declines in welfare when we feed in the data. When $\hat{p}_F = 1$, equation (29) becomes $\sum_i \frac{\hat{p}_F(s_i + \hat{p}_F)}{E + \text{Exports}}$, which should equal to 1 under the assumptions for the Proposition and the trade balance condition. However, since aggregate expenditure is larger than the sum of labor costs, $\sum_i \frac{\hat{p}_F(s_i + \hat{p}_F)}{E + \text{Exports}}$ in the data is around 0.919. Therefore the change in aggregate welfare is biased downwards under Proposition 1 when one feeds in the data.
Table 31: Aggregate effects from Proposition 1

<table>
<thead>
<tr>
<th>Value of $\bar{\sigma}$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$\eta$</th>
<th>$\bar{\sigma}$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U}$</td>
<td>0.977</td>
<td>0.930</td>
<td>0.913</td>
<td>1.137</td>
<td>1.068</td>
<td>1.047</td>
<td></td>
</tr>
</tbody>
</table>

E.5 Different magnitudes of shocks

Table 32 reports the aggregate predictions under different magnitudes of foreign price change, from the baseline model together with the alternative models.

Table 32: Aggregate effects of reductions in the foreign price

<table>
<thead>
<tr>
<th>$\hat{p}_F = 1.3$</th>
<th>$\hat{p}_F = 1.2$</th>
<th>$\hat{p}_F = 0.8$</th>
<th>$\hat{p}_F = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U}$</td>
<td>$\hat{E}/\hat{P}$</td>
<td>$\hat{E}/\hat{\tilde{w}}$</td>
<td>$\hat{U}$</td>
</tr>
<tr>
<td>Baseline</td>
<td>Baseline</td>
<td>Baseline</td>
<td>Baseline</td>
</tr>
<tr>
<td>Constant markups</td>
<td>Lemma 1</td>
<td>Baseline</td>
<td>Lemma 1</td>
</tr>
<tr>
<td>0.826</td>
<td>0.826</td>
<td>0.867</td>
<td>0.867</td>
</tr>
<tr>
<td>0.912</td>
<td>0.913</td>
<td>1</td>
<td>0.932</td>
</tr>
<tr>
<td>1.115</td>
<td>1.115</td>
<td>1.153</td>
<td>1.084</td>
</tr>
<tr>
<td>0.796</td>
<td>0.796</td>
<td>NA</td>
<td>0.844</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\hat{U} = \hat{E}/\hat{P}$</th>
<th>$\hat{E}$</th>
<th>$\hat{P}/\hat{\tilde{w}}$</th>
<th>$\hat{U}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>Lemma 1</td>
<td>Baseline</td>
<td>Lemma 1</td>
</tr>
<tr>
<td>1.273</td>
<td>1.272</td>
<td>1.129</td>
<td>1.536</td>
</tr>
<tr>
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341 “Alexandre Lamfalussy and the monetary policy debates among central bankers during the Great Inflation”, by I. Maes and P. Clement, Research series, April 2018.