An estimated two-country EA-US model with limited exchange rate pass-through
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Abstract

We develop a two-country New Keynesian model with sticky local currency pricing, distribution costs and a demand elasticity increasing with the relative price. These features help to reduce the exchange rate pass-through to import price at the border and down the chain towards consumption price, both in the short and the long run. Oil and imported goods enter at the same time as inputs in the production process and as consumption components. The model is estimated using Bayesian full information maximum likelihood techniques and based on real and nominal macroeconomic series for the euro area and the United States together with the bilateral exchange rate and oil prices. The estimated model is shown to perform well in an out-of-sample forecasting exercise and is able to reproduce most of the cross-series co-variances observed in the data. It is then used for forecast error variance decomposition and historical decomposition exercises.

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# TABLE OF CONTENTS

1. Introduction ....................................................................................................................... 1  
2. The model .......................................................................................................................... 3  
  2.1. Households .................................................................................................................... 4  
  2.1.1. Allocation of demand ................................................................................................. 4  
  2.1.2. International markets and the nominal exchange rate ................................................... 6  
  2.2. Firms ............................................................................................................................. 6  
  2.2.1. Final good firms and the distribution channel ............................................................... 8  
  2.2.2. Homogeneous good assemblers and variable demand elasticity ................................. 9  
  2.2.3. Intermediate good firms ............................................................................................ 10  
  2.3. International trade ......................................................................................................... 15  
  2.4. Resource constraints and monetary policy ...................................................................... 16  
3. Estimation methodology ................................................................................................. 18  
  3.1. Data and shocks ............................................................................................................ 18  
  3.2. Calibration and priors ................................................................................................. 21  
4. Posterior estimates .......................................................................................................... 23  
5. Impulse-response analysis .............................................................................................. 29  
  5.1. A positive productivity shock ........................................................................................ 30  
  5.2. A positive risk premium shock ..................................................................................... 32  
  5.3. A positive UIRP shock .................................................................................................. 34  
  5.4. A negative Rest of the World demand shock ................................................................. 37  
  5.5. Positive oil and import price mark-up shocks ............................................................... 39  
6. Assessing the out-of-sample forecasting performances ................................................... 43  
7. Matching data moments and cross borders spillovers .................................................... 47  
8. Conditional variance decomposition of the forecast errors ............................................. 51  
9. Growth, inflation and exchange rate historical decompositions ...................................... 57  
  9.1. GDPs historical decompositions ................................................................................... 57  
  9.2. Variations in consumption price inflations historical decompositions .......................... 61  
  9.3. Nominal exchange rate historical decomposition ....................................................... 64  
10. Conclusion ...................................................................................................................... 66  
References .................................................................................................................................. 67  
Appendix ..................................................................................................................................... 70  
Figures ........................................................................................................................................ 78  
National Bank of Belgium - Working papers series ............................................................... 89
1 Introduction

Around the turn of the millennium, the “New Open Economy Model” (NOEM) has become a standard tool to analyse the behaviour of the exchange rate and the current account (see Obstfeld and Rogoff, 1995, 1998, 2000). As any New-Keynesian model, it is based on the optimizing behaviour of households and firms, in a monopolistic competitive environment with nominal rigidities in the price and wage setting. The open economy dimension shows up with a foreign good entering in the composition of the final good used for consumption and investment purposes as an imperfect substitute to the domestic good. Their respective share will evolve in function of their relative price, strongly influenced by the real exchange rate. The current account is consistently explained by the intertemporal decisions on the one hand, i.e. the savings minus investment identity, and the intra-temporal decisions, i.e. the allocation of demand between domestic and foreign goods, on the other hand. The exchange rate is determined by the uncovered interest rate parity condition. Starting from these common building blocks, many variants have enriched the literature with differences mainly in the export pricing assumption and the possibility of international risk sharing.

Initially the models developed in this field were theoretical, small and very stylized. Several authors have then started to estimate small scale open economy models, like Ghiaroni (2000), Bergin (2006), Lubik and Schorfheide (2006), Lubik and Schorfheide (2007) or Rabanal and Tuesta (2010). Afterwards, larger and more realistic open and two- or multi-country models have been constructed within central banks,\(^1\) with a continuous trend in complexity. Recently, some authors have put the role of financial frictions under scrutiny (e.g. Kollmann, 2013), while others have gone in the direction of increasing the number of endogenous economies, as e.g. Gomes et al. (2012). These more complex models allow to give a structural shock interpretation of movements observed in exchange rates and trade balances (see Jacob and Peersman, 2013).

The objective of this paper is to construct a medium-sized two-country model for the euro area and the US able to deliver an acceptable empirical fit for a relatively wide set of macro-variables and generate reliable forecasts. We extend the previous works of Smets and Wouters (2003, 2005, 2007) by integrating two closed economy models through international trade in goods and assets. Their initial dataset is extended with information from the net trade flows, import and consumption price inflation, the bilateral nominal exchange rate and oil price fluctuations. For the domestic block, we start from their previous work on the closed economy. For the open economy block, we take inspiration from Jeanfils (2008) and provide the model with different channels that attenuate the price pressures in the wake of exchange rate fluctuations (i.e. the exchange rate pass-through, hereafter ERPT). First, we assume that exporters’ prices are sticky, and set in the currency of the destination market (Betts and Devereux, 1996, 2000). Second, nominal rigidities are combined with a demand elasticity increasing with the relative price (Kimball, 1995).\(^2\) Third, we attribute a role to a distribu-

---

\(^1\)Initial examples of such models are for example Laxton and Pesenti (2003), Benigno and Thoenissen (2003), de Walque et al. (2005), Erceg et al. (2006) or Adolfson et al. (2007).

\(^2\)Note that, from a pure estimation point of view, the Calvo probability and the curvature coefficient of
tion sector in a similar way to Burstein et al. (2003, 2005, 2007) and Corsetti and Dedola (2005). These three layers are likely to affect the ERPT towards import and consumption prices both in the short and in the long run. Staggered price setting and varying demand elasticity reduce the pass-through in the short run, while the presence of a distribution sector allows limiting it in the long run. A limited exchange rate pass through is important for the model to deliver a reasonable fit of international variables such as the exchange rate. If the ERPT is too big, the model requires excessive rigidities in the domestic bloc and unreasonable shocks processes in order to dampen the high observed volatility in the exchange rate. Furthermore, in order to improve the realism and flexibility of the pass-through of import prices to final domestic prices, we differentiate between oil and non-oil imports and assume that they are used both in the final good and the intermediate good production. Changes in the domestic-foreign goods combination are characterized by adjustment costs in order to smooth the impact of relative prices on the allocation of demand. Finally, we consider that only fixed interest rate assets are traded internationally which amounts to assume the absence of risk sharing through portfolio diversification.

After a short discussion on the estimation of the model parameters, we compare the out-of-sample performance of the model with the forecasting power of VAR, BVAR and closed economy DSGE models. The two-country model fares quite well along this dimension, competing with the benchmark models. We check afterwards whether the estimated model is able to reproduce a series of stylized facts of open economy business cycle fluctuations such as the standard deviations, autocorrelations and correlations between the exchange rate, net exports, output and demand components. In particular, the model correctly circumvents the traditional open economy puzzle of the NOEM, i.e. the negative correlation between relative consumption and real exchange rate. As usual in this literature (see Justiniano and Preston, 2010; Adolfson et al., 2007), the model struggles in replicating the observed international synchronisation in output and aggregate demand cycles under the hypothesis that all the estimated shocks are orthogonal. Allowing for a cross-country correlation in risk premium shocks significantly improves this dimension. Such an estimated correlation can be viewed as a short-cut for the representation of an international financial market as in Kollmann (2013) or Dedola and Lombardo (2012). However, no cross-country shock correlation does help to significantly increase the synchronisation of nominal variables like consumption prices or short term interest rates.

Based on the model estimates, we also review the major implications for the domestic and open economy variables of various shocks present in the model (productivity and risk premium, exchange rate risk premium shocks, oil price shocks and price and demand shocks originating in the rest of the world). In this framework, we discuss the role of the elasticity of substitution between foreign and domestic goods. Starting from the impulse-responses for Kimball are not separately identifiable. The introduction of a demand elasticity dependent on relative prices however helps to obtain reasonable values of the Calvo probability.

\(^3\)Noteworthy, with a strictly positive curvature of the demand curve and a strictly positive share of the distribution sector, the steady-state elasticity of demand re-enters the Phillips curve and make import price inflation more sensitive to marginal cost and exchange rate developments.
the different types of shocks, the joint behaviour of the domestic variables and the typical open economy variables (the exchange rate and the net trade balance) are informative to identify the contribution of the major shocks. Conditional variance decomposition of the forecast errors reveals a non-negligible role for open economy shocks. Foreign demand shocks are significant contributors to short-run unexpected output fluctuations in the euro area, though they are less so in the United States. US domestic shocks (mainly risk premium and monetary policy shocks) have some explanatory power for euro area output movements, even though spillovers remain rather limited. In both economies, about a third of the short-term variance of consumption price inflation is explained by foreign shocks (mainly oil price, foreign mark-up and exchange rate shocks) while their importance remains significant at medium and long horizons. Exchange rate risk premium shocks turn out to be very important for explaining the short-run volatility in the exchange rate, but technology, risk premium and monetary policy shocks (especially those originated in the euro area) are of high relevance at longer horizon. While the trade balance is mostly affected by open economy shocks in the short-run, domestic shocks display important influences on long-run swings.

The paper is organised as follows. Section 2 describes the model. The estimation methodology, data and shock processes are discussed in Section 3. Posterior estimation results are presented in Section 4. In Section 5, we analyse the impulse response functions to several shocks. Sections 6 and 7 test the ability of the model in forecasting variables and matching observed co-movements. Finally, Sections 8 and 9 propose conditional variance and historical decompositions, while Section 10 concludes.

2 The model

The model consists of two countries of equal size: the euro area (EA) and the United States (US). Figure 15 in the appendix summarizes the representation of the two-country economy. The two economies share the same structure, with a number of real and nominal frictions that are standard in closed economy DSGE models (see Smets and Wouters, 2007): habit preferences, investment adjustment costs, variable utilization rate of capital, and price and wage stickiness augmented with indexation schemes. In each economy, households consume, invest in physical capital, supply differentiated labour services, set wages, and invest in domestic bonds. In addition, euro area households have access to the US bond market. The production sector, schematically represented on Figure 16 in appendix, is made up of three layers of firms. First, intermediate good firms acting under monopolistic competition produce differentiated types of goods sold domestically and abroad. In their production process intermediate good firms use labour and physical capital services together with foreign goods and oil. A second layer of firms, called homogeneous-good assemblers, buy the differentiated goods from home and foreign intermediate good firms and use a technology à la Kimball (1995) to produce homogeneous domestic ($Y_H$) and foreign goods ($Y_F$ and $Y_F^p$). The third layer of firms consists of final good firms. They use domestic and foreign homogeneous goods as inputs and combine them with a bundle of domestic goods for distributional purposes in order to produce final goods sold to end users, i.e. consumers and investors. This process
is consistent with the distribution channel described in Corsetti and Dedola (2005). In each country, a monetary authority sets the nominal interest rate by following a Taylor-type of interest-rate rule.

As the two countries share the same structure, equations are symmetric for the EA and the US. Therefore, we present the model from the euro area point of view, which we refer to as the home country. A subscript H (respectively F) is attached to variables associated with prices and quantities produced in the home (foreign) country. Whenever variables or parameters are associated with the foreign market, a star exponent \( * \) is used.\(^4\)

### 2.1 Households

There is a continuum of households indexed by \( h \in [0, 1] \). Each household consumes a composite consumption good \( C \), supplies a differentiated labour service \( \ell \) and maximizes the following intertemporal utility:

\[
U_t(h) \equiv E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1 - \sigma_c} (C_{t+j}(h) - H_{t+j})^{1-\sigma_c} \right) \exp \left( \frac{\sigma_c - 1}{1 + \sigma_c} \ell_{t+j}(h)^{1+\sigma_c} \right),
\]

where \( \sigma_c \) is the degree of relative risk aversion (i.e. the inverse of the intertemporal elasticity of substitution for constant labour), \( \sigma_\ell \) is the inverse of the Frisch elasticity, and \( H_t = \lambda_{hab} C_{t-1} \) is the external habit variable, which is proportional to aggregate past consumption.

#### 2.1.1 Allocation of demand

An oil component is included in the households’ consumption composite good \( C \), so that the consumer price is directly impacted by oil prices.\(^5\) Households are assumed to allocate consumption to consumption goods \( C \) and the energy component \( O^D \) using a CES aggregator, with \( \lambda_{oil} \) the price elasticity of demand in global consumption:

\[
C_t(h) = \left( 1 - \phi_{oil} \right)^{\lambda_{oil}} C_t(h)^{\lambda_{oil}} + \phi_{oil} O^D_t(h)^{\lambda_{oil}}.
\]

The corresponding price index is

\[
P_{C,t} = \left( 1 - \phi_{oil} \right)^{1-\lambda_{oil}} P_{c,t}^{1-\lambda_{oil}} + \phi_{oil} P_{oil,t}^{1-\lambda_{oil}} \left( \frac{1}{1 - \lambda_{oil}} \right)^{\frac{1}{\lambda_{oil}}} \exp(\varepsilon^p_t).
\]

\(^4\)For instance, prices of home produced goods sold abroad are denoted by \( P^*_H \). Prices of foreign goods sold in the home (foreign) markets are expressed as \( P^*_F \) (respectively \( P^*_F \)).

\(^5\)Oil prices are modelled as being exogenous and, therefore, by construction are assumed to be entirely supply driven. A more comprehensive oil setup, left for future research, would also consider the possibility of endogenous responses of oil prices to demand shocks. We believe that the work of Stevens (2015) and Forni et al. (2015) can be inspiring avenues for an extension of the model.
where the consumer price shock process $\varepsilon^p_t = \varepsilon^p_{t-1} + \mu^p_t$ is a random walk. We assume that there is a constant wedge between crude oil prices ($P_{oil}$) and the price of oil paid by consumers ($P_{oil}^D$). The wedge arises because crude oil goes through a distribution channel, where it is combined with domestic output, before being distributed to consumers. This wedge is a modelling device aimed at representing refining and distribution margins as well as value added and excise taxes. In the model, it helps us to mitigate the impact of crude oil prices on consumer price inflation. The price of crude oil is expressed in USD, meaning that, in the euro area, $P_{oil,t} = P_{oil,t}^* S_t$, where $S_t$ is the nominal exchange rate computed as the amount of euro per unit of dollar.

The consumption good consists of an index of domestic and foreign consumption goods: 

$$C_t = \left[ \phi^\frac{1}{\lambda} H C^\frac{\lambda-1}{\lambda}_{H,t} + (1 - \phi^H) \frac{1}{\lambda} ((1 - \Omega^c,t) C^\frac{1}{\lambda}_{F,t} - 1)^\frac{1}{\lambda} \right]^{\frac{1}{\lambda-1}} , \quad (4)$$

where $\lambda$ is the Armington elasticity, that is, the degree of intratemporal substitution between domestic and imported goods, and $\phi^H$ determines the demand bias towards the domestic good. A cost $\Omega^c,t$ is associated with adjustment in the use of imported goods in the aggregation process, and has a standard quadratic form:

$$\Omega^c,t = \frac{\Omega^c}{2} \left( \frac{C^\frac{1}{\lambda}_{F,t}}{C^\frac{1}{\lambda}_{F,t-1}} - 1 \right)^2 .$$

This adjustment cost captures the limited ability for household to substitute between home and foreign goods in the short-term. It is also consistent with empirical evidence of slow adjustment of imports to changes in relative prices in the short run (see for instance McDaniel and Balistreri, 2003). The corresponding price index is:

$$P^c,t = \left( \phi^H P^D_{H,t} 1^{1-\lambda} + (1 - \phi^H) P^D_{F,t} 1^{1-\lambda} \right) \frac{1}{1-\lambda} , \quad (5)$$

where the superscript $D$ refers to prices of distributed quantities. Similar to consumption goods, investment goods are a combination of homogeneous domestic and foreign goods. However, they have no oil component. We further assume that the price of the investment good is equal to the price of the non-energy composite good $P^c,t$, and that the same trade elasticity $\lambda$, home bias $\phi^H$ and adjustment cost structure apply to the aggregation of the consumption good and the one of the investment good. Therefore, the composite investment good is obtained as follows:

$$I_t = \left[ \phi^\frac{1}{\lambda} H I^\frac{\lambda-1}{\lambda}_{H,t} + (1 - \phi^H) \frac{1}{\lambda} ((1 - \Omega^i,t) I^\frac{1}{\lambda}_{F,t} - 1)^\frac{1}{\lambda} \right]^{\frac{1}{\lambda-1}} . \quad (6)$$

As explained later, in Footnote 11, this shock helps to cope with constant weights associated with CPI determinants and the fact that actual CPI is affected by some elements (e.g. unprocessed food) that are absent from the model and that bring more volatility to the actual CPI.
2.1.2 International markets and the nominal exchange rate

The budget constraint of domestic households is as follows:

\[
P_{C,t} [C_t(h) + I_t(h)] + \frac{B_{H,t}(h)}{\exp(\varepsilon_t^b)R_t} + \frac{S_t B_{F,t}(h)}{\exp(\varepsilon_t^b)\bar{R}_t} \leq W_t(h)\ell_t(h) + B_{H,t-1}(h) + R^*_t u_t(h)K_{t-1}(h) - \psi(u_t)K_{t-1}(h) + \int Div_t(i,h) \, di.
\]

(7)

Each household receives income from labour services \(W_t(h)\ell_t(h)\), from return on the capital stock diminished by utilization costs \(R^*_t u_t(h) - \psi(u_t(h))\), returns on past position in home \(B_{H,t}(h)\) and international bonds \(B_{F,t}(h)\) and dividend from owning the intermediate good firms in the domestic economy \(\int Div_t(i,h) \, di\). Total income is used to consume goods and oil, to invest in new physical capital and to trade domestic and foreign bonds. It is assumed that an internationally traded bond is issued by the US, and is expressed in dollar currency. Euro area households are consequently able to invest their financial wealth in nominal riskless bonds denominated in home and foreign currency. They receive a return on foreign bonds equal to \(R^*_t = R_t^*\Theta_t\), where \(R_t^*\) is the interest rate bearing on US bonds while \(\Theta_t\) represents some real costs described a few lines below. The optimal positions in bond markets are determined by the first-order conditions from the maximization of households utility (1) subject to the budget constraint (7):

\[
(\partial B_{H,t}(h)) \Xi_t = \exp(\varepsilon_t^b)R_t\beta E_t \left[ \Xi_{t+1} \frac{P_{C,t}}{P_{C,t+1}} \right],
\]

(8)

\[
(\partial B_{F,t}(h)) \Xi_t = \exp(\varepsilon_t^b)\bar{R}_t\beta E_t \left[ \Xi_{t+1} \frac{S_{t+1}}{S_t} \frac{P_{C,t}}{P_{C,t+1}} \right],
\]

(9)

with \(\Xi_t\), the associated Lagrange multiplier, i.e. the marginal utility of consumption. As in Smets and Wouters (2007), the exogenous process \(\varepsilon_t^b\) introduces a wedge between the central bank interest rate and the return on domestic assets held by households. It is assumed to follow an AR(1) process with an i.i.d.-normal error structure, interpreted as a risk premium shock on financial assets.\(^7\) Combining these two Euler equations leads to an arbitrage condition on interest rates, i.e. the uncovered interest-rate parity (or UIRP), which is used to pin down the nominal exchange rate:

\[
E_t \left[ \frac{S_{t+1}}{S_t} \right] = \frac{R_t}{R_t^*} = \frac{R_t}{\bar{R}_t^*\Theta_t}.
\]

(10)

Following Adolfson et al. (2008), positions in the international bond are subject to real costs \(\Theta_t\), modelled as a function of the real holdings of the foreign assets in the entire home economy \(B_{F,t}\), of the expected change in the exchange rate and of an exogenous component:

\(^7\)Note that, as in Smets and Wouters (2007), the risk premium shock also intervenes in the equation for the price of capital, where a positive shock drives down the value of capital and investment.
\[ \Theta_t \equiv \Theta \left( B_{F,t}, E_t(S_{t+1}), S_t, S_{t-1}, \varepsilon_t^s \right) = \exp \left( -\theta_a \frac{S_t B_{F,t}}{P_{C,t} \gamma_t} - \theta_s \left( \frac{E_t(S_{t+1})}{S_t} \frac{S_t}{S_{t-1}} - 1 \right) + \varepsilon_t^s \right), \]

where \( \gamma \) represents the deterministic growth rate of the economy. Individual households take these costs as given in their optimal foreign holdings decisions. If EA households are net lenders, they receive a net return lower than the foreign interest rate, while if they are net debtors, they are charged a premium over the foreign interest rate. This mechanism ensures stationarity of the net foreign asset position. The presence of expected changes in the exchange rate in the cost function (11) is motivated by empirical evidence on the negative correlation between risk premia and expected changes in the exchange rate. If future exchange rates are easier to anticipate (because their fluctuations display some persistence), domestic households will require a lower expected return on their foreign bond holdings. From an empirical point of view, the estimation of \( \theta_s \) helps to capture the persistence in exchange rate data. The autoregressive process \( \varepsilon_t^s \) captures exogenous variations in international financial market conditions, and is often referred to as an international risk premium shock.

Households’ optimal decisions on consumption, investment, capital and utilization rates are standard and replicate closely the structure of Smets and Wouters (2003, 2007). Accordingly, capital adjustment through variations in investment and utilization is costly, and an investment-specific technology shock is introduced in the model. The treatment of the labour market follows closely Erceg et al. (2000) and Smets and Wouters (2007). On the supply side, trade unions bring together many households specializing in the same type of labour (with their joint mass remaining infinitesimal), and decide on wages. A complete set of securities markets and full consumption risk sharing across households is assumed in order to make budget constraints independent from the optimal wage set. As a result, all unions will face an identical problem and select the same optimal wage. In addition, we assume that only a fraction \( 1 - \xi_w \) of unions, drawn randomly from the population, can re-optimize their wage at each period. The non-adjusted wages are indexed by the deterministic growth rate of the economy \( \gamma \) and a weighted average of trend inflation \( \bar{\pi} \) and of previous period inflation. We refer to the log-linear equations presented in Appendix B for the formulation of these standard optimal mechanisms.

### 2.2 Firms

The three layers of firms, briefly described above, articulate as follow. Monopolistic intermediate good firms produce differentiated goods and sell them to home and foreign homogeneous good assemblers. These assemblers act in perfect competition and use a technology à la Kimball (1995) in order to aggregate their inputs into homogeneous domestic and foreign goods. The homogeneous goods are then combined with a home bundle (i.e. a bundle of domestic goods used as complements in a distribution channel) by final good firms in order to distribute goods to final users (i.e. consumers and investors).
2.2.1 Final good firms and the distribution channel

Final good firms are indexed by \( f \). They use Leontieff technology to produce retail goods \( Y_{i,t}^D(f) \) from homogeneous goods, \( Y_{i,t} \), and a home bundle, \( Y_{d,H,t}^D \), with \( i = H, F \). Their production function is therefore:

\[
Y_{i,t}^D(f) = \min \left[ (1 + \delta_f)Y_{i,t}(f); \frac{1+\delta_f}{\delta_f} Y_{d,H,t}^D \right] \quad \text{with} \quad i = H, F , \tag{12}
\]

where the superscript \( D \) is used to denote distributed goods, and \( d \) stands for the home bundle used for distribution. In Corsetti and Dedola (2005) and Jeanfils (2008), a non-tradable sector is introduced and home and foreign goods are combined with a fixed proportion of non-tradable goods in order to be distributed to final users. In order to avoid the complexity of working with two monopolistic sectors, we assume that the additional input used for distribution purposes is a home bundle composed of domestic homogeneous goods instead of non-tradable goods. Therefore, imported goods are combined with a fixed proportion of domestic goods, which represent the distribution services. The final good firms act in perfect competition, taking all prices as given. Cost minimization implies that \((1 + \delta_f)Y_{i,t}(f) = \frac{1+\delta_f}{\delta_f} Y_{d,H,t}^D = Y_{i,t}^D(f)\), as it is optimal to have no unused inputs at equilibrium. Therefore the demand function for inputs is a linear function of the distributed goods:

\[
Y_{i,t}(f) = \frac{1}{1 + \delta_f} Y_{i,t}^D(f) \quad \text{with} \quad i = H, F . \tag{13}
\]

The total demands for distributed goods from final users are

\[
Y_{H,t}^D = C_{H,t} + I_{H,t} + G_t + \psi(u_t)K_{t-1} , \tag{14}
\]

\[
Y_{F,t}^D = C_{F,t} + I_{F,t} , \tag{15}
\]

where it is assumed that government spending and capital utilization costs are in terms of domestic distributed goods only. In aggregate, by the zero-profit condition of final good firms, we obtain

\[
P_{H,t}^D = \frac{1}{1 + \delta_f} P_{H,t} + \frac{\delta_f}{1 + \delta_f} P_{H,t} = P_{H,t} , \tag{16}
\]

\[
P_{F,t}^D = \frac{1}{1 + \delta_f} P_{F,t} + \frac{\delta_f}{1 + \delta_f} P_{H,t} . \tag{17}
\]

Equation (17) underlines that the need to combine foreign goods with a home bundle introduces a local distribution cost. The wholesale price \( P_{F,t} \) is only a part of the final retail price, and its changes are only partially reflected by the latter. As discussed in Burstein et al. (2003), Corsetti and Dedola (2005) and Jeanfils (2008), the distribution mechanism is a friction that affects the degree of transmission of exchange rate movements along the pricing chain, i.e. the exchange rate pass-through (ERPT). Changes in the exchange rates mainly affect the wholesale price, which is only a small part of the final retail price of the distributed good. Consequently, the distribution mechanism leads to an imperfect exchange
rate pass-through to the retail prices.

Similar to consumption goods, imported crude oil for consumption purposes $O^c_t$ goes through a distribution channel before being distributed to consumers. The production function of distributed oil and its corresponding price equation are as follows:

$$O^D_t(f) = \min \left[ (1 + \delta_o)O^c_t(f); \frac{1 + \delta_o}{\delta_o}Y^d_{H,t} \right],$$

$$P^D_{oil,t} = \frac{1}{1 + \delta_o} P_{oil,t} + \frac{\delta_o}{1 + \delta_o} P_{H,t},$$

where $P_{oil,t}$ is the price of crude oil expressed in home currency.

### 2.2.2 Homogeneous good assemblers and variable demand elasticity

There exists a multitude of perfectly competitive assemblers who produce homogeneous goods $Y_{H,t}$, $Y_{F,t}$ and $Y^p_{F,t}$ using respectively a continuum of domestic intermediate inputs - $\{Y_{H,t}(i)\}_{i \in [0,1]}$ - and foreign intermediate inputs - $\{Y_{F,t}(j)\}_{j \in [0,1]}$ and $\{Y^p_{F,t}(k)\}_{k \in [0,1]}$ - produced by intermediate good firms. The index $p$ refers to goods dedicated to be used as inputs in the production of intermediate good firms. Following Kimball (1995), their production functions are:

$$1 = \int_0^1 G \left( \frac{Y_{H,t}(i)}{Y_{H,t}} \right) di,$$

$$1 = \int_0^1 G \left( \frac{Y_{F,t}(j)}{Y_{F,t}} \right) dj,$$

$$1 = \int_0^1 G \left( \frac{Y^p_{F,t}(k)}{Y^p_{F,t}} \right) dk,$$

with $G(1) = 1, G'(x) > 0$ and $G''(x) < 0$ for all $x \geq 0$.\(^8\)

Subject to this technology, and taking prices as given, assemblers decide the optimal inputs and output levels in order to maximize their profits. The first-order conditions imply that the demand of the relative individual input is a function of its relative price. For the demand of domestic intermediate good, we obtain:

$$Y_{H,t}(i) = G'^{-1} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right) Y_{H,t},$$

where $I_t = \int_0^1 G' \left( \frac{Y_{H,t}(i)}{Y_{H,t}} \right) \frac{Y_{H,t}(i)}{Y_{H,t}} di$. Similarly, the demand for domestic goods sold abroad

---

\(^8\)As long as the model is solved using a first-order approximation method, there is no need to specify any functional form for $G$. 
As shown in Kimball (1995), the assumptions on $G(.)$ imply that the demand for input is decreasing in its relative price, while the elasticity of demand is a positive function of the relative price. In a Calvo framework, the Kimball aggregator generates demand functions which are more elastic for firms that increase their prices than for firms whose relative price declines as a result of price stickiness. An elasticity of demand that is increasing in the relative price constitutes another friction that hampers the degree of exchange rate pass-through. It makes the desired mark-up decreasing in the relative price, and results in smaller price movements than with a constant elasticity. Ceteris paribus, a depreciation in the exchange rate raises costs expressed in local currency. Foreign firms that can re-adjust their prices are therefore willing to increase them. However, for any given rise in their price, the sensitivity of the demand elasticity raises the cost of deviating from the average price, and dissuades the adjusting firms from setting prices that deviate too far from their competitors. It is important to note that it is the combination of staggered price-setting and variable demand elasticity that makes the mark-ups variable. In a flexible price environment, mark-ups and market shares would remain constant as a same increase in marginal cost expressed in local currency would lead all firms to increase their prices by the same amount.

When intermediate good firms cannot reset prices, they adjust it with respect to a weighted average of past and trend inflation. By the zero-profit condition of assemblers, the Calvo-Kimball price index for the home economy is as follows (a similar development holds for export prices):

$$P_{H,t} = \xi_p \bar{\pi}_{H,t}^{\pi} \bar{\pi}_{H,t-1}^{1-\pi} P_{H,t-1} G^{-1} \left( \frac{\bar{\pi}_{H,t}^{\pi} \bar{\pi}_{H,t-1}^{1-\pi} P_{H,t-1} I_t}{P_{H,t}} \right) + (1 - \xi_p) \bar{P}_{H,t} G^{-1} \left( \frac{\bar{P}_{H,t} I_t}{P_{H,t}} \right). \quad (26)$$

### 2.2.3 Intermediate good firms

Intermediate good firms operate in monopolistic markets and produce intermediate goods that can be sold either at home or abroad. Intermediate good producer $v$ uses the following technology:

$$J_t(v) = \bar{K}_t^\alpha \left( \exp(\varepsilon_t^\alpha) \gamma^t L_t(v) \right)^{-1-\alpha}, \quad (27)$$

$$Y_t(v) = Y_{H,t}(v) + Y_{H,t}^*(v) + Y_{H,t}^{p*}(v)$$

$$= \min \left\{ \frac{1}{(1-\rho_m-\rho_o)} J_t(v); \frac{1}{\varepsilon_m} Y_{F,t}(v); \frac{1}{\varepsilon_o} O_t(v) \right\} - \gamma^t \Phi, \quad (28)$$

where $\bar{K}_t(v)$ are the effective capital services used in production, $L_t(v)$ is aggregate labor input of different types of labor used by the firm, $\Phi$ is a fixed cost of production, and $\varepsilon_t^\alpha$ is an AR(1)
process with an i.i.d.-normal error structure, interpreted as a transitory neutral productivity shock. We follow Smets and Wouters (2007) and introduce labour-augmenting deterministic growth ($\gamma$), which drives the long-term trend of the economy. Cost minimization implies the following demand relationships for inputs

$$\frac{J_t(v)}{O_t(v)} = \frac{1}{\varrho_m - \varrho_o},$$

$$\frac{J_t(v)}{Y_{F,t}(v)} = \frac{1}{\varrho_m - \varrho_o},$$

$$\frac{W_t L_t(v)}{\tau_t K_t(v)} = \frac{1}{\alpha}.$$

The marginal cost of 1 unit of output depends on wages, the rental rate of capital, foreign prices and crude oil prices expressed in home currency:

$$MC_t = (1 - \varrho_m - \varrho_o) \frac{W_t^{1-\alpha} \tau_k^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha} \tau_k^\alpha} + \varrho_m P_{F,t} + \varrho_o P_{oil,t}.$$  \hfill (32)

Intermediate good firm $v$ sets prices (home and abroad) according to Calvo’s (1983) mechanism in order to maximize its profit:

$$\max_{P_{H,t}(v),P_{H,t}(v),\bar{P}_{H,t}(v)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \mathbb{E}_{t+j} \left[ \bar{P}_{H,t}(v) \chi_{t,j} Y_{H,t,j}(v) - MC_{t+j} Y_{H,t,j}(v) \right]$$

$$+ \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \mathbb{E}_{t+j} \left[ S_{t+j} \bar{P}_{H,t}(v) \chi_{t,j} Y_{H,t,j}(v) - MC_{t+j} Y_{H,t,j}(v) \right] + S_{t+j} \bar{P}_{H,t}(v) \chi_{t,j} Y_{H,t,j}(v) - MC_{t+j} Y_{H,t,j}(v),$$

where

$$\chi_{t,j} = \begin{cases} 1 & \text{if } j = 0 \\ \Pi_{k=1}^j \bar{\pi}_{H,t+k-1} & \text{if } j = 1, \ldots, \infty \end{cases},$$

$$\chi_{t,j}^* = \begin{cases} 1 & \text{if } j = 0 \\ \Pi_{k=1}^j \bar{\pi}_{H,t+k-1} & \text{if } j = 1, \ldots, \infty \end{cases},$$

and

$$\chi_{t,j}^{ps} = \begin{cases} 1 & \text{if } j = 0 \\ \Pi_{k=1}^j \bar{\pi}_{H,t+k-1} & \text{if } j = 1, \ldots, \infty \end{cases}.$$
with a weight \( \iota_p \) and to trend inflation with a weight \( (1 - \iota_p) \). Variables \( \chi \) are introduced to take these indexation mechanisms into account. Different Calvo and indexation parameters \( (\xi_{\text{pF}}^\ast, \iota_{\text{pF}}^\ast) \) apply for exported goods. This distinction adds flexibility in the fit of New Keynesian Phillips Curves (NKPCs) for foreign prices with the data on import prices. In the optimization of their prices, intermediate good firms are assumed to internalize the distribution costs of their goods. Therefore, the demand of final good firms is substituted into the demands of homogeneous assemblers (23), (24) and (25), and intermediate good firms consider the following demand constraints:

\[
Y_{H,t}(v) = G' \left( \frac{P_{H,t}^D(v)}{P_{H,t}^D} \right) \frac{1}{1 + \delta_f Y_{H,t}^D},
\]

(34)

\[
Y_{H,t}^*(v) = G' \left( \frac{P_{H,t}^D(v)}{P_{H,t}^D} \right) \frac{1}{1 + \delta_f Y_{H,t}^D},
\]

(35)

\[
Y_{H,t}^p(v) = G' \left( \frac{P_{H,t}^p(v)}{P_{H,t}^p} \right) \frac{1}{1 + \delta_m Y_t^p},
\]

(36)

The assumption of the internalization of distribution costs makes our price setting structure consistent with the pricing-to-market logic of the literature (see for instance Corsetti and Dedola, 2005). A consequence of this assumption is the direct influence of the distribution parameter \( \delta_f^* \) in the NKPC for foreign prices. Given the assumed complementarity in the distribution channel, the retail price of a domestic good in \( t + j \) for a firm that last resets its price in \( t \) can be decomposed as follows:

\[
P_{H,t+j}(v) = \frac{1}{1 + \delta_f} \tilde{P}_{H,t}(v) \chi_{t,j} + \frac{\delta_f}{1 + \delta_f} P_{H,t+j}
\]

(37)

and a similar expression holds for the retail price of exports for direct consumption:

\[
P_{H,t+j}^*(v) = \frac{1}{1 + \delta_f^*} \tilde{P}_{H,t}^*(v) \chi_{t,j}^* + \frac{\delta_f^*}{1 + \delta_f^*} P_{H,t+j}^*.
\]

(38)

Symmetrically, the retail price of imports from a home economy point of view and for foreign producers that last reset prices in \( t \) is:

\[
P_{F,t+j}(v) = \frac{1}{1 + \delta_f} \tilde{P}_{F,t}(v) \chi_{t,j} + \frac{\delta_f}{1 + \delta_f} P_{H,t+j}.
\]

(39)

Marginal costs for producing differentiated goods are the same, be they sold domestically or exported. However the maximization problem with respect to domestic and export prices requires a separate treatment. Due to local market pricing, monopolistic firms face different currencies, demand characteristics and distribution costs when selling home and abroad, and find it optimal to charge different prices in the home and the foreign country. The log-linearised NKPCs for home and foreign inflation (i.e. imported inflation) depict these optimal decisions, and are presented in Appendix B. We report them here for clarity and discussion.
purposes. Small letters with a hat stands for detrended variables expressed in log-deviation from their steady-state values. The home production price inflation in log-linearized form writes as:

\[
\ddot{\pi}_H,t = \frac{\beta \gamma}{1 + \beta \gamma t_p} \ddot{\pi}_{H,t+1} + \frac{t_p}{1 + \beta \gamma t_p} \ddot{\pi}_{H,t-1} + \frac{(1 - \xi_p)(1 - \beta \gamma \xi_p)}{\xi_p(1 + \beta \gamma t_p)} \frac{\eta - 1 + \delta_f}{\eta + \epsilon - 1} (\dot{m}c_t - \dot{p}_H,t) + \xi^H_t - \vartheta \mu^H_t,
\]

(40)

where \( \eta \) is the steady-state price elasticity of demand,\(^9\) and \( \epsilon \) measures the curvature of the demand while \( \beta = \beta \gamma - \sigma_c \).\(^10\) In the absence of distribution costs, \( \delta_f = 0 \) and we obtain an expression similar to the inflation equation of Smets and Wouters (2007). The presence of distribution costs decreases the sensitivity of home inflation to firms’ marginal costs. Shocks to the domestic price mark-up are introduced in the home NKPC through an ARMA process \( \xi^H_t \). Finally, we let the home price inflation react negatively to i.i.d. innovation in the composite consumption good price inflation (\( \mu^C_t \)).\(^11\) Concerning imported goods for direct consumption, the imported inflation from a home country (EA) point of view is

\[
\ddot{\pi}_{F,t} = \frac{(1 - \beta^* \gamma^* \xi^*_p)(1 - \xi^*_p)}{\xi^*_p(1 + \beta^* \gamma^* t^*_p)} \frac{1}{\eta - 1 + \epsilon} \left[ (\eta - 1 - \delta_f)\dot{m}c_t^\nabla + \delta_f \dot{p}_H,t - (\eta - 1)\dot{p}_{F,t} \right] + \frac{\beta^* \gamma^*}{1 + \beta^* \gamma^* t^*_p} \ddot{\pi}_{F,t+1} + \frac{t^*_p}{1 + \beta^* \gamma^* t^*_p} \ddot{\pi}_{F,t-1} + \xi^F_t.
\]

(41)

Foreign exporters set prices in home currency, and take into account their real marginal costs expressed in the home currency, denoted by \( \dot{m}c_t^\nabla \). As mentioned later in Section 2.3, we assume that a share \( \beta_m \) of one country’s imports comes from the other modelled country. This assumption helps to reconcile the theoretical concepts of the two-country model with values observed for trade variables, which cover multi-country concepts. As a result, the total

---

\(^9\)The price elasticity of demand is \( \eta_l = -\frac{G'(z_l)}{\pi G'(z_l)} \), where \( z_l = G^{-1} \left( \frac{p^j_t}{p^j_t} \right) \) with \( j = H, F \). Parameter \( \eta \) defines the steady-state value of the price elasticity of demand.

\(^10\)As in Eichenbaum and Fisher (2007) and de Walque et al. (2006), the curvature of Kimball’s (1995) aggregator is defined as the elasticity of the price elasticity of demand with respect to relative price at steady-state. For instance, for home prices, we have:

\[
\epsilon = \frac{\ddot{\pi}_H/\ddot{p}_H}{\eta_H(z_{ss})} \frac{\partial \eta_H(z_{ss})}{\partial \ddot{p}_H} |_{z_{ss}=1} = 1 + \eta \left( 1 + \frac{G''(1)}{G'(1)} \right)
\]

Therefore, we obtain

\[
\frac{1 + (1 + \delta_f) \frac{G''(1)}{G'(1)}}{2 + \frac{G''(1)}{G'(1)}} = \frac{\eta - 1 - \delta_f}{\eta - 1 + \epsilon}
\]

This result is used to obtain the coefficient in (40).

\(^11\) The innovation \( \mu^C_t \) associated with the consumption price inflation captures, among others, elements (as e.g. unprocessed food) that are absent from the model and bring some extra volatility to the actual CPI. The last term in equation 40 allows to clear this unmodelled CPI component from the price mark-up shock.
marginal costs of foreign exporters are equal to:

\[ \hat{m}c_t^\nabla = \beta_m (\hat{m}c_t^* + \hat{r}s_t) + (1 - \beta_m)\varepsilon_t^{\text{row}}, \]

where \( \varepsilon_t^{\text{row}} \) is the ARMA(1,1) exogenous process driving the Rest of the World marginal cost expressed in domestic currency.\(^{12}\) Due to the distribution channel, foreign exporters consider the level of home prices as a cost through the term \( \delta_f \hat{p}_{H,t} \) in equation (41), while the weight assigned to real marginal costs is reduced. Moreover, local pricing implies that they use Kimball parameters that characterize the demand of home homogeneous assemblers in their optimal decisions.

The component of imports dedicated to be used as inputs in the production process has its own NKPC, which is expressed as follows:

\[
\hat{\pi}_{F,t} = \frac{(1 - \beta^* \gamma^*) \eta - 1}{\eta - 1 + \epsilon} \left( \hat{m}c_t^\nabla - \hat{p}_{F,t}^p \right) + \frac{\beta^* \gamma^*}{1 + \beta^* \gamma^* \eta_{pF}^*} \hat{\pi}_{F,t+1}^p + \frac{\eta_{pF}^*}{1 + \beta^* \gamma^* \eta_{pF}^*} \hat{\pi}_{F,t-1}^p + \varepsilon_{pF}^t. \tag{42}
\]

Compared to (41), terms implying the distribution costs parameter are absent from (42) as there is no distribution channel for imported goods for production purposes. Due to the similar nature of imported goods dedicated to direct consumption and production, we use the same exogenous ARMA process for foreign price mark-up shocks in (41) and (42).

As underlined in Jeanfils (2008), four different factors are likely to affect the exchange rate pass-through. First, a high Calvo parameter \( \xi_{pF}^* \) decreases the frequency of price revisions, and thus lowers the pass-through in the short term. As time elapses after a given shock, the proportion of firms that have been allowed to adjust prices increases continuously, and the effect of this mechanism shades off. Second, positive curvature in the demand curve (\( \epsilon > 0 \)) reduces the pass-through. As the demand from assemblers becomes more elastic when adjusting firms increase their price, the cost of deviating from competitors is more important the higher the \( \epsilon \). Consequently, adjusting firms rise their prices by less than the rate of increase in their costs. Note that this demand elasticity channel disappears in the case of fully-flexible prices. If prices are flexible, all firms would change their prices by the same amount after a move in the exchange rate. Consequently, market shares and mark-ups would remain constant. Hence, the bulk of the effects of the price-dependent elasticity of demand is on short-term dynamics, and disappears in the long run when all prices adjust. Third, the share of distribution services \( \delta_f \) attenuates the exchange rate pass-through. When setting optimal prices, foreign firms realise that their goods need to be combined with the home good

\(^{12}\)However, this exogenous process cannot be disentangled from the one driving the price mark-up shock in the import price, i.e. \( \varepsilon_{pF}^t \). We therefore have to keep in mind in the subsequent analysis that the exogenous process \( \varepsilon_{pF}^t \) represent non explicitly modelled influences affecting the imported prices inflation coming from shocks either on the competitiveness of the market for imports, or on the RoW marginal cost or on the bilateral exchange rate with the RoW.
in order to reach final users. Moreover, an increase in their price is only partially reflected in the retail price of foreign goods. The share of distribution services is constant in our model, meaning that the difference between retail and wholesale prices persists and affects the ERPT in the long-run. Finally, a higher steady state demand elasticity $\eta$ increases the sensitivity of price with respect to marginal cost and exchange rate. This mechanism plays a role only if $\epsilon$ or $\delta_f$ is strictly positive, as it is clear from equation (41). Intuitively, for positive values of either $\epsilon$ or $\delta_f$ or both parameters, the higher the price elasticity, the lower the mark-up and adjusting firms have a lower margin to absorb marginal costs and exchange rate movements. As a result, optimal prices will follow changes in exchange rate and costs more closely.

2.3 International trade

In this section, we report the equations related to international prices and quantities. The Rest of the World is not formally modelled and appears under the form of exogenous processes. From the home country point of view, the demand for exports is expressed as a share of the (non-oil) import demand of the other country ($M_{F,t}$) and an exogenous shock that captures changes in the import demand from the Rest of the World:

$$X_{H,t} = M_{H,t}^{\beta_m} \exp(\epsilon nt). \quad (43)$$

Parameter $\beta_m$ measures the sensitivity of home country exports to foreign country imports. The non-energy component of imported goods for the home country ($M_{H,t}$) depends on the demand for foreign inputs by final goods firms, as well as on the demand for “transit goods” ($X_{F,t}$). The model takes into account the fact that a part of the imported goods are directly exported. In other words, they only transit in the home economy. The introduction of transit goods helps to capture co-movements between home exports and imports.\(^{13}\) To keep track of the amount of these transit goods, we introduce imported good aggregators that combine foreign produced goods used in the home economy ($Y_{F,t}$ and $Y_{p,F,t}$) and transit goods into an aggregate imported good. We therefore obtain:

$$M_{H,t} = \left[ \phi_H^{\lambda_m} \left( Y_{F,t}^{\lambda_m} \right)^{\lambda_m - 1} \lambda_m + (1 - \phi_H^{\lambda_m}) \frac{1}{\lambda_m} X_{F,t}^{\lambda_m - 1} \right]^{\frac{1}{\lambda_m - 1}}, \quad (44)$$

where $\phi_H^{\lambda_m}$ measures the relative proportion of consumed imported goods in non-energy imports, and $\lambda_m$ is the elasticity of substitution between the two types of imported goods. Variable $Y_{F,t}^{\lambda_m}$ depicts total foreign goods, and consists in a CES aggregate of foreign goods for consumption and production purposes:

$$Y_{F,t}^{\lambda_m} = \left[ \phi_F^{\lambda_F} \left( Y_{F,t}^{\lambda_F} \right)^{\lambda_F - 1} \lambda_F + (1 - \phi_F^{\lambda_F}) \frac{1}{\lambda_F} Y_{p,F,t}^{\lambda_F - 1} \lambda_F \right]^{\frac{1}{\lambda_F - 1}}. \quad (45)$$

Parameter $\phi_F^{\lambda_F}$ accounts for the quasi-share of imported goods dedicated to direct consumption. A similar aggregator holds for exported goods, in order to take the transit goods into

\(^{13}\)In our model, the import content of export includes the transit good as well as the part of foreign quantities used for the production of the export good.
account in total exports:

\[
X_{H,t} = \left[ \frac{1}{\lambda_x} \left( Y_{H,t} \right)^{\frac{\lambda_x - 1}{\lambda_x}} + \left( 1 - \phi_x^H \right) \frac{1}{\lambda_x} X_{F,t}^{\frac{\lambda_x - 1}{\lambda_x}} \right]^{\frac{1}{\lambda_x - 1}},
\]

(46)

where \( \phi_x^H \) is a bias parameter that takes into account the proportion of home-produced relative to transit goods in exported goods, and \( \lambda_x \) is the elasticity of substitution between the two types of exported goods. The demand functions for transit goods \( X_{T,t} \) are derived from (46).

Prices associated with imported and exported goods aggregators are as follows:

\[
P_{M,t} = \left[ \phi_m^H \left( P_{F,t}^{T_o} \right)^{1-\lambda_m} + (1 - \phi_m^H) P_{F,t}^{1-\lambda_m} \right]^{\frac{1}{1-\lambda_m}},
\]

(47)

\[
P_{X,t} = \left[ \phi_x^H \left( P_{H,t}^{T_o} \right)^{1-\lambda_x} + (1 - \phi_x^H) P_{F,t}^{1-\lambda_x} \right]^{\frac{1}{1-\lambda_x}},
\]

(48)

where we assume that transit goods have the same price as foreign goods dedicated to consumption \( (P_F) \).

Total imports are composed of non-energy and oil component of imports:

\[
M_{H,t} = \left[ (1 - \phi_m^{oil}) \frac{1}{\lambda_m^{oil}} M_{H,t}^{\frac{\lambda_m^{oil} - 1}{\lambda_m^{oil}}} + \phi_m^{oil} \frac{1}{\lambda_m^{oil}} OIL_t^{\frac{\lambda_m^{oil} - 1}{\lambda_m^{oil}}} \right]^{\frac{1}{1-\lambda_m^{oil}}}.
\]

(49)

The latter equation is associated with the following price index for total imports:

\[
P_{M,t} = \left[ (1 - \phi_m^{oil}) P_{M,t}^{1-\lambda_m^{oil}} + \phi_m^{oil} P_{oil,t}^{1-\lambda_m^{oil}} \right]^{\frac{1}{1-\lambda_m^{oil}}}. 
\]

(50)

CES cost minimization implies that \( P_{M,t} M_{H,t} = P_{M,t} M_{H,t} + P_{oil,t} OIL_t \). The total of imported oil consists of oil imported for consumption and production purposes: \( OIL_t = O_{it}^c + O_{it}^p \). The terms of trade is defined as the ratio of export prices, expressed in home currency, to total import prices: \( TOT_t = \frac{P_{X,t}}{P_{M,t}} \).

### 2.4 Resource constraints and monetary policy

In our open economy model with two countries, consumption includes both domestic and foreign components, subject to different prices, and residents of the euro area are allowed to hold foreign bonds. These features lead to differences in real and nominal domestic resource constraints. The nominal resource constraint combines the aggregate budget constraints and profits of agents, and leads to a relationship that links the net foreign assets to the trade balance. It is used to pin down net foreign assets in the model:

\[
\frac{S_t B_{F,t}}{R_t^i} = S_t B_{F,t-1} + TB_t,
\]

(51)
where the trade balance is defined as $TB_t \equiv P_{X,t}X_{H,t} - P_{M,t}M_{H,t}$. Details about the derivation can be found in Appendix A.

The real resource constraints ensures that production equals demand. In our setup, it is assumed that governments purchase home produced goods only, and government spending ($G_t$) is treated as an exogenous AR(1) process. At the level of intermediate good firms, total domestic production is

$$Y_t = \int Y_t(v)dv = \int Y_{H,t}(v)dv + \int Y^*_t(v)dv + \int Y^{ps}_{H,t}(v)dv + \int Y^{d,H}_{H,t}(v)dv$$

$$+ \int Y^{d,F}_{H,t}(v)dv + \int Y^{d,O}_{H,t}(v)dv$$

$$= \int G^{-1}\left(\frac{P_{H,t}(v)}{P_{H,t}}\right)T_{H,t}Y^h_{H,t}dv + \int G^{-1}\left(\frac{P^*_{H,t}(v)}{P^*_{H,t}}\right)Y^{h*}_{H,t}dv$$

$$+ \int G^{-1}\left(\frac{P^{ps}_{H,t}(v)}{P^{ps}_{H,t}}\right)T^{ps}_{H,t}Y^{ps}_{H,t}dv + \int G^{-1}\left(\frac{P^{d}_{H,t}(v)}{P^{d}_{H,t}}\right)Y^{d}_{H,t}dv$$

$$+ \int G^{-1}\left(\frac{P^{d,F}_{H,t}(v)}{P^{d,F}_{H,t}}\right)T^{d,F}_{H,t}Y^{d,F}_{H,t}dv + \int G^{-1}\left(\frac{P^{d,O}_{H,t}(v)}{P^{d,O}_{H,t}}\right)Y^{d,O}_{H,t}dv$$

$$= s_{H,t}Y^h_{H,t} + s^*_{H,t}Y^{h*}_{H,t} + s^{ps}_{H,t}Y^{ps}_{H,t} + s^{d}_{H,t}Y^{d}_{H,t} + s^{d,F}_{H,t}Y^{d,F}_{H,t} + s^{d,O}_{H,t}Y^{d,O}_{H,t}$$

$$= s_{H,t}\frac{1}{1+\delta_f}Y^D_{H,t} + s^*_{H,t}\frac{1}{1+\delta_f}Y^{D*}_{H,t} + s^{ps}_{H,t}Y^{ps}_{H,t} + s^{d}_{H,t}\frac{\delta_f}{1+\delta_f}Y^{d}_{H,t} + s^{d,F}_{H,t}\frac{\delta_f}{1+\delta_f}Y^{d,F}_{H,t} + s^{d,O}_{H,t}\frac{\delta_f}{1+\delta_f}Y^{d,O}_{H,t}$$

$$+ s^{d,F}_{H,t}\frac{\delta_o}{1+\delta_o}O^D_{H,t}, \quad (52)$$

where $s_x = \int G^{-1}\left(\frac{P_{x,t}(v)}{P_{x,t}}\right)T_{x,t}$ stands for price dispersion measures. In the development above, we use the exponent $(d, x)$ to differentiate between the uses of the home bundle.

Central banks (Fed and ECB) follow a nominal interest rate rule by adjusting their nominal interest rate in response to (i) deviations of inflation from target and (ii) deviations of output from their fully-flexible levels (denoted in the equations by an exponent $f$):\(^{14}\)

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{q_r} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{q_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{q_y} \right]^{1-\rho_r} \left(\frac{Y_t/Y_t^f}{Y_{t-1}^f/Y_{t-1}^f}\right)^{\phi_y} \varepsilon_t^r, \quad (53)$$

where $\pi_t$ is the inflation rate of the consumption composite price and $\varepsilon_t^r$ is the AR1 process followed by the i.i.d. monetary policy shock $\mu^r_t$.

\(^{14}\)The fully-flexible levels of output are the levels that would be observed in a counterfactual economy without any price and wage stickiness and free of mark-up shocks.
3 Estimation methodology

The model is estimated using a Bayesian full-information maximum likelihood approach along the lines of An and Schorfheide (2007) and Smets and Wouters (2007). The approach has the advantage to alleviate the numerical estimation of the large number of equilibrium relationships implied by our DSGE model. In particular, the inclusion of prior information over parameters adds more curvature to the likelihood function and helps to stabilize the non-linear optimization. The estimation of the model’s parameters proceeds as follows. In a first step, we cast the model into a stationary form. All real variables are detrended with the deterministic trend \( \gamma \), and nominal variables are replaced by their real counterpart by dividing by the appropriate deflator. For instance, capital becomes \( k_t = \frac{K_t}{\gamma_t} \) and nominal wages are transformed into real detrended wages as follows: \( w_t = \frac{w_t}{P_{C,t}\gamma_t} \). Secondly, non-linear equilibrium equations are log-linearised and solved using first-order approximation methods. Appendix B reports the log-linearised version of the model. It is augmented by measurement equations which link observed macroeconomic series with the model’s variables in order to form a state-space system. Observed series and exogenous processes are discussed in the next section. Kalman filtering techniques are used to evaluate the joint distribution of the observables, i.e. the likelihood function. Using Bayes’ rule, the likelihood function is then combined with a prior density for the parameters to obtain a posterior distribution. Markov Chain Monte Carlo methods are implemented in a Metropolis-Hastings algorithm in order to numerically find the parameters that optimize the posterior density function. Posterior estimates of optimal parameters are presented in Section 4 below.

3.1 Data and shocks

The database of observed variables used to estimate the above described model closely mimics those used by Smets and Wouters respectively for their EA (2003) and US (2007) estimated models and extends it to some typical open economy variables. For the United States, we use macroeconomic series from the Bureau of Economic Analysis, and for the euro area, the updated Area Wide Model database as described in Fagan et al. (2001). All real variables are expressed in per capita terms, dividing them by the working age population, and in quarter-on-quarter growth rates. For instance, data on output are expressed as follows:

\[
dGDP_t = 100 \cdot \Delta \ln \left( \frac{GDP_t}{Pop_t} \right), \tag{54}
\]

where \( Pop \) is the working age population. Employment is also divided by the working age population and, for the US, is multiplied by individual hours worked. To overcome the fact that there exists no consistent data on hours worked in the euro area, we follow Smets and Wouters (2003) and add the following equation for log-linear euro area employment to the model:

\[
\hat{e}_t = \hat{e}_{t-1} + \tilde{\beta} \gamma (\hat{e}_{t+1} - \hat{e}_t) + (1 - \xi_e) \frac{1 - \xi_e \tilde{\beta} \gamma}{\xi_e} (\hat{e}_t - \hat{e}_t), \tag{55}
\]
where \( \hat{e}_t \) denotes the number of people employed, and \( \hat{\ell}_t \) is the log-linearised labour. The introduction of a Calvo mechanism, where only a fraction \( \xi_c \) of the firms is able to adjust employment to the desired labour input, helps to take into account the fact that the employment variable responds only progressively to macroeconomic shocks. For the US, we work with hourly wage costs while for the euro area a wage cost per head concept is used.

The ten country-specific time series considered in the estimation procedure of the model are the growth rate in real gross domestic product (GDP), real private consumption (CONS), real investment (INVE), hours worked or employment (LAB), and real wages (WAGE), the inflation rate in consumption deflator (PCD), GDP deflator (YED) and import deflator (MTD), the short term interest rate expressed in annual terms (STI)\(^{15}\) and the real level of net trade expressed in GDP percentage (NT).\(^{16}\) To these 20 variables we add two common series, the euro-dollar exchange rate (EXCRN) and the price of crude oil in USD (POIL). The estimation period starts in 1970 Q2 and ends in 2014 Q4. The observed macroeconomic series are linked to variables of the log-linearised version of the model in the following way

\[
\begin{bmatrix}
  d\text{GDP}_t \\
  d\text{CONS}_t \\
  d\text{INVE}_t \\
  d\text{WAGE}_t \\
  d\text{LAB}_t \\
  \text{STI}_t \\
  d\text{PCD}_t \\
  d\text{YED}_t \\
  \text{NT}_t \\
  d\text{MTD}_t \\
  d\text{EXCRN}_t \\
  d\text{POIL}_t \\
\end{bmatrix}
= \begin{bmatrix}
\hat{\gamma} \\
\hat{\ell} \\
\hat{\pi} \\
\hat{\pi}_H \\
\hat{\pi}_M \\
\hat{\pi}_\text{oil} \\
\end{bmatrix} + \begin{bmatrix}
\hat{\gamma}_t - \hat{\gamma}_{t-1} \\
\hat{\ell}_t - \hat{\ell}_{t-1} \\
\hat{\pi}_t - \hat{\pi}_{t-1} \\
\hat{\pi}_H,t - \hat{\pi}_H,t-1 \\
\hat{\pi}_M,t - \hat{\pi}_M,t-1 \\
\hat{\pi}_\text{oil},t - \hat{\pi}_\text{oil},t-1 + \hat{\pi}_\text{oil},t \\
\end{bmatrix}
\]

(56)

where \( \hat{\gamma} = 100(\gamma - 1) \) is the quarterly trend of the economy, and \( \hat{\gamma}_t, \hat{\gamma}_\text{int}, \hat{\pi}_H, \hat{\pi}_M, \hat{\pi}_\text{oil} \) are constants that are estimated along with the structural parameters of the model. Consistent with the concept of value-added output used in the data, the observable on GDP growth is related to domestically produced intermediate good \( \hat{j}_t \), which corresponds to the log-linearised output of equation (27). Finally, \( \bar{\pi} = 100(\bar{\pi}_C/\bar{\beta} - 1) \) is the steady-state nominal interest rate where \( \bar{\pi}_C = 1 + \bar{\pi}/100 \). The coefficient \( \alpha_m \) is steady-state ratio of total imports on GDP. Variables \( c, \hat{m} \) and \( \hat{r}s_t \) stands for log deviations from steady state of respectively composite consumption, total imports, and real exchange rate.

\(^{15}\)The 3-month Federal Fund rate is used in the US dataset. For the euro area, it is the 3-month EURIBOR interest rate as published in the Monthly Bulletin.

\(^{16}\)In the AWM database, reported imports and exports are computed following a gross concept, i.e. mixing trade flows between countries within and outside the euro area. Observing the net trade instead of both its components allows to cancel out the major part of the trade flows inside the euro area. The remaining short run intra trade movement will feed the RoW demand shock.
A total of 22 structural shocks are introduced in order to estimate the model based on the 22 macroeconomic time series. Table 1 reports all the shocks, their symbol and their associated stochastic process. Most of the shocks are standard in the closed economy DSGE literature: shocks to total factor productivity, investment-specific technology, risk premium, exogenous government spending, home price mark-up, wage mark-up and to monetary policy. The risk premium shock intervenes in both the Euler equation for (composite) consumption and the equation that determines the price of capital. In the former equation, its role is to introduce a wedge between the interest rate determined by monetary authorities and the return on assets held by households. In the latter equation, a positive risk premium shock reduces the price of capital and has a deterring effect on investment. The MA component in ARMA processes allows to capture short term volatility in price and wage mark-up shocks.\(^\text{17}\)

Finally, we recognize that we have a formal modelling of the trade channel only, while the two regions considered are also connected through financial and technological links. In order to somehow circumvent this potential weakness and allow further cross-country feedback effects, we introduce and estimate cross-country correlation terms in TFP, risk premium and interest-rate i.i.d. shocks. International shocks (net trade preferences and foreign price mark-up)\(^\text{18}\) capture shocks originating from the Rest of the World. As the latter is partly common to both modelled economies, it certainly makes sense to let them to be correlated as well.

\(^{17}\)For example, the so-called “wage mark-up” shock is actually a mix between a persistent labour supply shock and a i.i.d. mark-up shock. As such it may be interpreted both as a mark-up or a supply shock.

\(^{18}\)Shocks to the import prices of one country include exogenous changes in the prices of imports to the Rest of the World, as well as exogenous movements to the exchange rate between the country and the Rest of the World.
The international spillovers generated by the model will be discussed in Section 7 below. Finally, following Smets and Wouters (2007), we let the productivity shock to be a potential driver for external demand. In their closed economy set-up, this feature goes through their only exogenous demand process $\varepsilon_t$ while in our case it will mainly go through the external demand process $\varepsilon_{Nt}$. However, we do not close the $\varepsilon_t$ channel as it seems empirically relevant for the US economy.

### 3.2 Calibration and priors

Following common practice, we fix the value of some parameters that are poorly identified by the observed variables. This is equivalent to choosing extremely tight priors for these parameters. Calibrated and implied parameters are reported in Table 2. The demand components in proportion of private GDP are fixed at their historical averages. Most calibrated values are standard in the DSGE literature. The capital depreciation rate is fixed at 0.025, which corresponds to 10% annually. The parameter governing the wage mark-up, $\lambda_w$ is set at 0.25. Following Smets and Wouters (2007), the curvature of the demand from homogeneous assemblers (Kimball curvature) is set at 10.

We further assume that the demand for transit goods evolves one-to-one with the demand for exported goods, which implies a value of 0 for $\lambda_x$. The price elasticity of oil demand for consumption, $\lambda_{oil}$ is set at 0.3, as in Natal (2012) and consistent with estimates found in Kilian and Murphy (2014). For the euro area, the share of oil in consumption, $\phi_{oil}$, is approximated through Eurostat data on energy expenditures which averages around 4% for the period from 2000 to 2013. For the US, as in Natal (2012), and in line with US NIPA data, this share is set at 6%. The oil shares in imported and exported goods for the euro area are matched with Eurostat data on the category of mineral fuels, lubricants and related materials (average on the period 1999-2012). For the US, oil shares in imports and exports are set to be broadly in line with OECD data on an equivalent category (that is, “petroleum, petroleum products and related materials”). The home bias in final demand, $\phi_H$, is set equal to 0.87 and 0.89 respectively. For such values, evaluated at the priors means of the other related parameters, total imports represent a share of 17% of GDP for the euro area, as reported by van der Helm and Hoekstra (2009), and 15% of GDP for the US, consistently with US data dating back to the year 2000.

The priors used in the Bayesian estimation procedure are not very different from those used by predecessors in similar kind of estimation and are fully described in Tables 3 to 7 reporting the posterior estimates. The standard deviations of shocks are assumed to follow an inverted Gamma distribution with mean 0.2 and 2 degrees of freedom, which is a pretty loose prior. The autocorrelation (and moving average) parameters of the AR(MA) exogenous processes are assumed to be distributed along a Beta distribution with a mean of 0.5 and a standard deviation of 0.2. When we estimate shocks correlations, we pose as prior for the correlation parameters a normal distribution centered on zero with a standard deviation of 0.3.
Table 2: Calibrated parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>symbol</th>
<th>EA</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption to gdp</td>
<td>$\bar{c}/\bar{y}$</td>
<td>0.583</td>
<td>0.664</td>
</tr>
<tr>
<td>Investment to gdp</td>
<td>$\bar{i}/\bar{y}$</td>
<td>0.213</td>
<td>0.176</td>
</tr>
<tr>
<td>Government spending to gdp</td>
<td>$\bar{g}/\bar{y}$</td>
<td>0.204</td>
<td>0.160</td>
</tr>
<tr>
<td>Capital depreciation</td>
<td>$\tau$</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Wage elasticity</td>
<td>$\lambda_w$</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td>Kimball curvature</td>
<td>$\epsilon$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Substitution transit goods</td>
<td>$\lambda_x$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oil demand elasticity</td>
<td>$\lambda_{oil}$</td>
<td>0.300</td>
<td>0.300</td>
</tr>
<tr>
<td>Oil share in consumption</td>
<td>$\phi_{oil}$</td>
<td>0.040</td>
<td>0.060</td>
</tr>
<tr>
<td>Oil share in exports</td>
<td>$\phi_{x}$</td>
<td>0.040</td>
<td>0.080</td>
</tr>
<tr>
<td>Home bias in cons.</td>
<td>$\phi_H$</td>
<td>0.87</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The prior distribution chosen for the Calvo and indexation parameters in the various NKPCs of the model is a Beta distribution with mean 0.5 and standard deviation 0.15. This prior is also shared by the capital utilization rate adjustment cost parameter. The parameters of the utility function are assumed to be distributed as follows: the inverse of the intertemporal elasticity of substitution is centered on 1.5 with a standard deviation of 0.25, the habit parameter fluctuates around 0.6 with a standard error of 0.1, the Frisch elasticity is assumed to fluctuate around 2 with a standard deviation of 0.75. For the prior of the investment adjustment cost parameter, we follow Christiano et al. (2005) and center it on 4 with a standard error of 1.5. We assume a prior mean of 0.25 for the share of the fixed costs in the production function, with a standard error of 0.125.

The parameter driving the persistence of the Taylor rules is assumed to be Beta distributed around 0.75 with standard error 0.1, while the long run reactions on inflation and the output gap are assumed to be Normal distributed around respectively 1.5 and 0.1, with standard deviation 0.15 and 0.03. The prior for the short run reaction to changes in the output gap is fixed around 0.1.

For the parameters that are specific to the open economy dimension of the model, we impose the following prior distributions. The smoothing parameter of the UIRP condition, $\theta_s$, is Beta distributed around 0.3 with standard error 0.1. The share of home produced goods in foreign imports, $\beta_m$, is assumed to follow a Beta distribution with mean 0.5 and standard error 0.1. The parameters driving the share of oil products and imported goods in the production process, i.e. $\phi_o$ and $\phi_{m}$, are normally distributed respectively around 0.006 and 0.06. Parameters driving the size of the distribution sector related to the imported goods ($\delta_f$) and the oil products ($\delta_o$) follow both a Gamma distribution, with respective means 0.7 and 3.5, i.e. respective shares of the distribution sectors of about 40% and 80%. Finally, the elasticity of substitution between home produced and foreign goods in consumption and investment bundles, $\lambda$, is Gamma distributed around 3 with standard deviation 1, which
corresponds to a pretty loose prior. The associated adjustment cost $\Omega$ shares the same prior distribution as the one assumed for the investment adjustment cost.

4 Posterior estimates

The shock structure of the two economies is identical but some differences show up in the estimates, reported in Tables 3 and 4. Let us quickly pin down the most salient ones. The standard error of the TFP shock is a bit more than 70% higher in the euro area than in the US, for an identical persistence. In what the monetary policy shock concerns, its standard deviation is higher in the US, but this is compensated by a lower persistence than estimated for the euro area. The same kind of difference is observed for the wage mark-up shock. The standard deviation of the risk premium shock is estimated to be quite different in both economies, but this is also one of the less precisely estimated dispersion parameters. The difference between the posterior mode and mean for the standard deviation of the US shock attests to the difficulty to pin down this parameter when including the Great Recession period. Note also that the estimated processes of the shocks to the investment cost functions differ notably between the two regions, being much more persistent in the US than in the euro area. Finally, the typically open economy shocks (import price shocks, export demand shocks, oil price and UIRP shocks) are all characterized by large standard deviations and pretty strong persistences. The standard deviation of the export demand shock is much bigger in the euro area than in the US. This is explained by (i) the above mentioned remaining intra-euro zone trade and (ii) by the fact that the exogenous external demand is estimated to be much more reactive to TFP innovations on the western shore of the Atlantic.

Looking at the correlations estimated between shocks (bottom of Table 3), it appears that productivity shocks are not significantly correlated, indicating that the co-movements in technology shocks do not help to account for transatlantic spillovers. The risk premium and monetary policy shocks are more strongly correlated, confirming that shocks to the financial sector might offer a better cross-country transmission channel. Import price shocks are the most strongly correlated, with a correlation coefficient of 0.54. Indeed, even though Rests of the World are not identical for both economies, they nevertheless share a large common component. It is noteworthy that the correlation between export demand shocks is negative, although not strongly significantly different from zero: on average, a positive external demand shock oriented towards one economy is made at the expense of the trading partner. Finally, we observe that technology shocks create extra internal and external exogenous demands, especially in the United States.

As already mentioned in previous studies (e.g. Smets and Wouters, 2005; de Walque et al., 2005), the estimated structural parameters of domestic economy blocks, reported in Table 6, are fairly similar (i) to their estimates in closed economy set-ups and (ii) in both the euro area and the United States. The only noticeable differences with respect to the latter dimension consist first in an investment adjustment cost estimated higher in the euro area (7.7) than in the US (4.9) and second, in a lower estimated Frisch elasticity for the euro area.
Table 3: Prior and posterior parameter estimates: shocks standard errors and correlations

<table>
<thead>
<tr>
<th>Std errors of the shocks</th>
<th>symbol</th>
<th>distribution</th>
<th>mean</th>
<th>d.f.</th>
<th>Estimated max. posterior</th>
<th>posterior distribution (MH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>mode</td>
<td>std. error</td>
<td>Mean</td>
<td>5%</td>
</tr>
<tr>
<td>Euro area</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td>(\mu_{a, \text{ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.820</td>
<td>0.104</td>
</tr>
<tr>
<td>risk premium</td>
<td>(\mu_{b, \text{ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.378</td>
<td>0.021</td>
</tr>
<tr>
<td>gov. spending</td>
<td>(\mu_{\text{g, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.438</td>
<td>0.047</td>
</tr>
<tr>
<td>investment</td>
<td>(\mu_{\text{inv, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.141</td>
<td>0.009</td>
</tr>
<tr>
<td>monetary policy</td>
<td>(\mu_{\text{mp, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.136</td>
<td>0.012</td>
</tr>
<tr>
<td>price mark-up</td>
<td>(\mu_{\text{pm, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.158</td>
<td>0.015</td>
</tr>
<tr>
<td>wage mark-up</td>
<td>(\mu_{\text{wm, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.222</td>
<td>0.012</td>
</tr>
<tr>
<td>cons. price</td>
<td>(\mu_{\text{cons, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.857</td>
<td>0.117</td>
</tr>
<tr>
<td>import price</td>
<td>(\mu_{\text{ip, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>2.273</td>
<td>0.204</td>
</tr>
<tr>
<td>RoW demand</td>
<td>(\mu_{\text{nt, ea}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.476</td>
<td>0.030</td>
</tr>
<tr>
<td>United States</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td>(\mu_{a, \text{us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.476</td>
<td>0.030</td>
</tr>
<tr>
<td>risk premium</td>
<td>(\mu_{b, \text{us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.445</td>
<td>0.026</td>
</tr>
<tr>
<td>gov. spending</td>
<td>(\mu_{\text{g, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.277</td>
<td>0.029</td>
</tr>
<tr>
<td>investment</td>
<td>(\mu_{\text{inv, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.233</td>
<td>0.014</td>
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<tr>
<td>monetary policy</td>
<td>(\mu_{\text{mp, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.134</td>
<td>0.011</td>
</tr>
<tr>
<td>price mark-up</td>
<td>(\mu_{\text{pm, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.378</td>
<td>0.026</td>
</tr>
<tr>
<td>wage mark-up</td>
<td>(\mu_{\text{wm, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.177</td>
<td>0.010</td>
</tr>
<tr>
<td>cons. price</td>
<td>(\mu_{\text{cons, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>1.145</td>
<td>0.133</td>
</tr>
<tr>
<td>import price</td>
<td>(\mu_{\text{ip, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>1.341</td>
<td>0.113</td>
</tr>
<tr>
<td>RoW demand</td>
<td>(\mu_{\text{nt, us}, t})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.658</td>
<td>0.158</td>
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<tr>
<td>common</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UIRP</td>
<td>(\mu_{s, \text{t}})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.165</td>
<td>0.009</td>
</tr>
<tr>
<td>oil price</td>
<td>(\mu_{\text{oil, \text{t}}})</td>
<td>inv. gamma</td>
<td>0.200</td>
<td>2.000</td>
<td>0.326</td>
<td>0.070</td>
</tr>
</tbody>
</table>

| cross-country correlations | distribution | mean  | std. dev. |          |          |          |          |          |          |          |
|-----------------------------|--------------|-------|------------|----------|----------|----------|----------|----------|----------|
| TFP                         | normal       | 0.000 | 0.300      | 0.067    | 0.076    | 0.064    | -0.084   | 0.211    |
| risk premium                | normal       | 0.000 | 0.300      | 0.194    | 0.082    | 0.166    | 0.008    | 0.324    |
| monetary policy             | normal       | 0.000 | 0.300      | 0.537    | 0.061    | 0.518    | 0.395    | 0.637    |
| import price                | normal       | 0.000 | 0.300      | -0.144   | 0.095    | -0.156   | -0.336   | 0.024    |
| RoW demand                  | normal       | 0.000 | 0.300      |          |          |          |          |          |
Table 4: Prior and posterior parameter estimates: structure of the stochastic processes

<table>
<thead>
<tr>
<th>shocks</th>
<th>ARMA parameters</th>
<th>Prior distribution</th>
<th>Estimated max. posterior distribution</th>
<th>Posterior distribution (MH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>symbol</td>
<td>mean</td>
<td>std. dev.</td>
<td>mode</td>
</tr>
<tr>
<td>Euro area</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFP</td>
<td>AR ρ_a</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.975</td>
</tr>
<tr>
<td>risk premium</td>
<td>AR ρ_b</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.902</td>
</tr>
<tr>
<td>gov. spending (G)</td>
<td>AR ρ_g</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.912</td>
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<tr>
<td>investment</td>
<td>AR ρ_i</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.446</td>
</tr>
<tr>
<td>monetary policy</td>
<td>AR ρ_r</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.281</td>
</tr>
<tr>
<td>price mark-up (PH)</td>
<td>AR ρ_{PH}</td>
<td>beta</td>
<td>0.500 200</td>
<td>0.889</td>
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25
Table 5: Prior and posterior parameter estimates: constant trends

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<td>norm</td>
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<td>0.300</td>
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<tr>
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<td>norm</td>
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<td>0.500</td>
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<td>labour</td>
<td>\ell</td>
<td>norm</td>
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<td>0.100</td>
</tr>
<tr>
<td>net trade</td>
<td>n_{x}</td>
<td>norm</td>
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<td>0.500</td>
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<td>norm</td>
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<td>0.300</td>
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<tr>
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<td>#_{m}</td>
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<td>0.500</td>
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<tr>
<td>labour</td>
<td>\ell</td>
<td>norm</td>
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For the rest, the nominal stickiness as captured by the price and wage Calvo parameters is marginally smaller on the western side of the Atlantic, but this is partly compensated by higher indexations on past inflation.

The remaining estimated parameters are related to the open economy dimension of the model, and are reported in Table 7. A first set of parameters determines the degree of openness of the considered economies with regard to oil and non-oil foreign goods. The fraction of crude oil that is directly used in consumption is equal to \( \phi_{oil} \), which is estimated to be equal to 0.6% in euro area and 0.8% in the US. Taking into account \( \varrho_{o} \), the share of oil required in the production process, imports of oil amount to 0.9% (resp. 1.1%) of GDP in euro area (resp. the US). The share of non-oil imports in GDP is estimated by \( \frac{1-\phi_{oil}}{1+\delta_{f}} \), \( \gamma \), which is estimated to be equal to 14-15% of GDP in both economies. Finally, the share of imported goods that is immediately re-exported, \( (1-\phi_{x}) \), is evaluated respectively at 23.5% and 14%.\(^{19}\)

A second set of parameters determines the pass-through of the foreign prices and exchange rate fluctuations together with the substitution effects they yield in the trade flows. Among them, let us first address the case of the parameters shaping the NKPC for import prices, i.e. equation (41). The share of imported goods which is coming from the modelled partner economy, \( \beta_{m} \), is estimated respectively around 29% (EA) and 47% (US), while the Calvo probability \( \xi_{F} \) is evaluated at 0.31 and 0.35 in each economy, corresponding to an average duration of export pricing contracts of about two quarters.\(^{20}\)

\(^{19}\)Note that the number estimated for the euro area is fairly in line with the evaluation of van der Helm and Hoekstra (2009).

\(^{20}\)We refer here to the Dixon and Kara (2006) way to compute this statistics: \( \frac{1+\xi_{F}}{1-\xi_{F}} \).
Table 6: Prior and posterior parameter estimates: domestic economy behavioral parameters

<table>
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<tr>
<th>parameters</th>
<th>symbol</th>
<th>Prior distribution</th>
<th>Estimated max. posterior</th>
<th>Posterior distribution</th>
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<td>mode</td>
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<td>inv. adj. cost</td>
<td>$1/\Psi''''$</td>
<td>norm</td>
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<td>1.500</td>
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<td>$\sigma_c$</td>
<td>norm</td>
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<td>0.250</td>
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<td>norm</td>
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<td>0.100</td>
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<td>$\Psi''''/\Phi$</td>
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<td>0.150</td>
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<td>0.150</td>
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<td>inv. adj. cost</td>
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Table 7: Prior and posterior parameter estimates: open economy parameters

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<td>0.150</td>
<td>0.354</td>
<td>0.060</td>
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<tr>
<td>index imp. price ι_pF</td>
<td>beta</td>
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<td>0.150</td>
<td>0.356</td>
<td>0.113</td>
<td>0.361</td>
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<tr>
<td>elast. of subst. λ</td>
<td>gamma</td>
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<td>1.000</td>
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<td>4.242</td>
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</tr>
<tr>
<td>(1-share of transit g.) φ_H</td>
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<td>0.860</td>
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<td>0.835</td>
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</table>
area, at 0.20 versus 0.36. Together, these three parameters produce a pass-through of the bilateral exchange rate that is limited in the long run by distribution costs and the share of bilateral trade but, according to this share, does not take that long to produce its full effect.

Finally, the elasticity of substitution between domestic and foreign goods is a key parameter of the open economy block of the model. As highlighted by Corsetti et al. (2008), it does not only manage trade flows in response to relative prices but it also determines the reaction of the exchange rate to the various exogenous shocks. As this parameter governs the substitutability between domestic goods and imported and distributed goods, its value is furthermore mitigated by the estimate of parameter $\delta_f$, driving the share of the distribution sector required to make imported goods available for final demand purposes. The trade elasticity is estimated fairly above unity for both economies, respectively at 2.8 (EA) and 1.8 (US). Such values are necessary to produce an expenditure switching effect that is large enough to guarantee the long-run stability of foreign asset accumulation and moderate the systematic reaction of the exchange rate to exogenous shocks.

5 Impulse-response analysis

After shortly describing the endogenous reactions to two domestic shocks, productivity and risk premium, we devote the major part of this section to the various open economy shocks in the model, i.e. shocks to the exchange rate, the Rest of the World demand, the foreign and the oil prices. This selection of the two domestic shocks is motivated by the fact that they are important contributors to real and nominal fluctuations as presented in the variance and historical decomposition analyses of Section 8 and Section 9. Moreover, cross-country correlation between risk premium shocks are also shown in Section 7 to generate interesting spillovers across countries. Additional impulse-response functions, related to the interest-rate shock, the investment-technology shock, the government spending shock, and the price and wage markup shocks, are reported in Appendix C.

We discuss the impulse-response functions from the euro area point of view. The posterior estimation of the model reveals that the characterizations of the euro area and the US economies are relatively symmetric. Consequently, the dynamics described in this section generally holds for shocks originated in the US.$^{21}$ Note that in this section we switch off the cross-countries shocks correlation and adopt the more traditional view that shocks are purely orthogonal. This simplifies the reading of the transmission mechanisms and allows to introduce the discussion on the possibility to generate increased spillovers by allowing shocks to be correlated over the Atlantic. Finally, the impulse response functions displayed in this section are computed using the estimated posterior mode of the parameters.

$^{21}$ Differences can nevertheless occur in the amplitude of shocks, due to differences in estimated volatilities and persistence between EA and US shocks. These differences are reported in Section 4 above, where we discuss the posterior estimation of the model.
5.1 A positive productivity shock

Figure 1 shows the dynamics of selected variables after a productivity shock in the EA. Although productivity is modelled as a transitory shock, it is estimated as highly persistent with an AR(1) coefficient of 0.975. Consequently, the transitory productivity shock generates long lasting responses. A positive shock boosts output but to a lower extent than its fully-flexible equivalent, hence generating a negative output gap in the euro area. In order to unload extra production and to reflect lower marginal costs, prices of domestically produced goods decline, and drive consumption inflation down. The ECB reacts to the negative output gap and deflationary pressures by adapting the nominal interest rate downwards. Therefore the domestic economy undergoes a deterioration of its production price relative to the faced import price not only because of the initial deflationary pressure but also as a consequence of the ensuing depreciation of the nominal exchange rate. The shift in relative price shares the wealth effect of the productivity shock between the two economies as foreign households will benefit from reduced import prices.

Finally, the sustained aggregate demand in the euro area pulls up the demand for foreign goods, including US goods, and has thus positive effects on the US production. However, the above mentioned euro depreciation limits the positive spillover effects of euro area increased demand on the demand for US goods. The dollar appreciation increases the wealth of US households who adapt their consumption accordingly. Because of this extra push on demand, US firms hire more labour and capital services. The upwards pressure on the production factors prices more than counterbalance the decrease in import prices, pushing the marginal cost, production and consumption prices up.

Consistent with Smets and Wouters (2003) and Smets and Wouters (2007), employment falls after a positive productivity shock. Indeed, the aggregate demand adjusts slowly in the short term due to real and nominal frictions (price stickiness, consumption habit and investment adjustment costs). As a result, the positive response of output on impact is less important than the increase in the level of productivity, and the resulting labour demand is too weak to stimulate employment. The counter-cyclical nature of employment responses makes the productivity shock a bad candidate to explain recession periods accompanied by a fall in employment, as the ones that took place in the beginning of the 00’s and during the recent financial crisis. Moreover, a negative productivity shock implies positive responses of inflation, which is inconsistent with the post-crisis episodes of low inflation and deflation observed in both economies during the Global financial crisis.
Figure 1: Impulse responses to a positive productivity shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
5.2 A positive risk premium shock

The effects of a positive risk premium shock in the euro area are highlighted in Figure 2. The risk premium shock affects variables through two channels: a private saving channel and a capital price channel. In the first mechanism, a positive risk premium shock results in a positive private saving shock, which lowers euro area consumption, output and inflation, and re-directs resources towards investment. However, an increase in the domestic risk premium drives the market value of capital down, with a strong depressing effect on investment. This negative effect dominates the positive influence of the private saving shock, and the total response of investment to a positive risk premium shock turns negative. The fall in aggregate demand dampens labour demand and employment while real wages decline accordingly.

The ECB reacts to the depressed economic conditions by being more accommodative. European real rates decrease and the EA-US real rate gap expands, which induces a depreciation of the euro. Movements in domestic aggregate demand negatively impact the demand for foreign goods, and euro area imports decline. In the US, the appreciation of the dollar leads to lower import prices that pull down consumption inflation. The resulting stimulus in US internal demand more than compensates the decline in exports due to a weak euro area demand and the appreciation of the dollar. Even though US households substitute cheaper foreign goods in their final demand, the potential negative impact of this trade balance effect on output is partly compensated by a higher demand for the US home bundle used in the distribution of foreign goods. All-in-all, the response of US output to a euro area risk premium shock is slightly positive.

To conclude, a positive risk premium shock generates a persistent fall in domestic GDP, a decline in inflation and interest rates. Moreover, it implies pro-cyclical variations in investment, employment and wages together with a depreciation of the euro. These movements are consistent with the recent recession experiences, and makes the risk premium shock a good candidate for explaining the post-crisis period. As said earlier, economic activity reacts in opposite direction depending whether the shock occurred domestically or in the foreign economy, such that this type of shock hinders the potential of the model to generate positive spillovers between the economies. Allowing for positive correlations between risk premium shocks is certainly a promising way to alleviate somehow this problem and to enable variables to react in the same direction as it will be further discussed in Section 7 below.
Figure 2: Impulse responses to a positive risk premium shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
5.3 A positive UIRP shock

Figure 3 reports the impulse responses associated with an UIRP shock simulated in order to induce a 10% depreciation of the euro with respect to the dollar on impact. In this simulation, the Rest of the World is assumed to behave as the US, and faces the same appreciation of their currency with respect to the euro. This assumption enables us to proxy the effects of a depreciation of the euro with respect to the currencies of all extra-EA countries. The last sub-plot shows dynamics in the euro area import and export prices and in the oil prices expressed in euro. The depreciation of the euro leads to a rise in euro area import prices. However, due to Calvo rigidities and to Kimball demand elasticity, the exchange rate pass-through (hereafter ERPT) at the border is imperfect, and euro area import prices (excl. oil) pick up by only 3.35% (in quarterly terms or 14.32% annually) on impact, which implies a ERPT of 33.5% for import prices.\textsuperscript{22} Moreover, as foreign goods go through a distribution channel, import prices at the border only partially reflects the total retail prices of foreign goods, and the ERPT to retail foreign prices (not reported on the figure) is limited to 2.31% on impact (in quarterly terms). Crude oil prices expressed in euro also increase following the depreciation of the currency. Their total impact on retail energy prices is nevertheless limited due to important estimated distribution costs.

The UIRP shock has several effects on domestic output. First, the resulting depreciation of the euro bolsters euro area exports and dampens imports. Moreover, euro area households react to higher import prices by substituting domestic goods for foreign goods. These effects are positive for domestic production. However, the decrease in the demand for foreign goods also translates into a decrease in the demand for the home bundle, with a negative impact on the euro area production. More importantly, the heightening of import prices generates inflationary pressures in the euro area, which leads to a tightening of monetary policy. The implied contractionary effect drives aggregate demand down. Finally, more expensive imported goods used in the production process prevent euro area firms’ marginal costs to adjust downwards on impact, and domestic price inflation is slightly positive. All-in-all, the negative effects dominate the positive outcomes and euro area output goes down after a UIRP shock that depreciates the euro.

In an exercise not reported here for the sake of clarity, following the intuition behind the (static) Marshall-Lerner condition, immediate positive responses of euro area output and employment can be recovered in the case of a trade elasticity sufficiently high - higher than 5 in both areas. In this case, the fall in the demand for foreign goods is so important in the short term that it positively spills over euro area net trade and production, and leads to a sharp decline in US exports. Furthermore, the role of the monetary policy pointed above is

\textsuperscript{22}This estimate of the ERPT only takes the UIRP shock into account. A cleaner estimate of the ERPT implies the consideration of the complete structure of shocks. Based on simulated data by the model (under the assumption that the Rest of the World behaves as the opposite country), and following the methodology of Choudhri and Hakura (2015), regressions and VAR-based estimates of the ERPT for import prices are around 40% in the EA and 30% in the US. For the US, these estimates are close to what regression and VAR models deliver for our data augmented by an effective exchange rate series, that is an ERPT around 32%. For the EA, the simulated ERPT is somewhat overstated, as it is estimated to be around 26.5% in the data.
also crucial to explain the observed initial negative real effects of the depreciation. Would this channel be closed down, e.g. because the economy is blocked in the liquidity trap, the positive real effects of a depreciation would appear much more rapidly, if not instantaneously.

Responses of US variables to the UIRP shock are the mirror images of those of euro area variables. In the US, the dollar appreciation applies a downward pressure on US exports and on US import prices. The ERPT is comparable with the one observed for the euro area. US import prices at the border fall by 3.30% (while retail foreign prices fall by 2.29%). The decline in US import prices leads to deflationary consumption prices that stimulate aggregate demand. In order to address this stronger demand, US firms increase their demand for inputs, which drives real wages and the rental rate of capital up. However, marginal costs remain relatively stable due to the decline in the price of foreign intermediate goods used in the production process. As a result, US domestic production price inflation does not compensate the influence of import prices on consumption prices. This outcome generates an extra stimulus on aggregate demand compared to a model without imports in production.
Figure 3: Impulse responses to an international risk premium shock (UIRP shock), calibrated to induce a 10% depreciation of the euro with respect to the dollar. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
5.4 A negative Rest of the World demand shock

When facing a negative Rest of the World demand shock (cf. Figure 4), output decreases as net exports dampen. The slump in production kicks off a deflationary spiral. Firms reduce their demand for inputs, which lowers employment, real wages and the rental rate of capital. Firms react to lower marginal costs by decreasing domestic prices. The ECB attempts to counteract the deflationary pressures by setting the nominal interest rate to a lower level. The expansionary monetary policy somehow attenuates the weakening effect of the lower rental rate of capital on investment. As a result, investment is stimulated after a negative RoW demand shock, while consumption moderately expands.

Due to the expansionary monetary policy, the euro depreciates, which causes a rise in import prices. This increase weakens the demand for foreign goods and produces negative spillover effects on US exports. Nevertheless, the deterioration in exports is somehow compensated in the US by the positive influence of movements in the bilateral exchange rate. The appreciation of the dollar triggers a fall in import and consumption prices in the US, pushing demand upwards. This is slightly reinforced by the complementarity between domestic and foreign goods in the distribution channel: improvement in US demand for imports translates into a marginal increase in the production of the US home bundle.

The dynamics implied by a negative Rest of the World demand shock share some features with the fluctuations implied by the 2008 crisis in the euro area: slump in output and employment, and the depreciation of the euro with respect to the dollar. Such a shock fits well in the tale of the weakening of the global economy during the crisis, or of the slow global recovery afterwards.
Figure 4: Impulse responses to a negative Rest of the World demand shock (i.e., shock $\varepsilon^{nt}$ from the EA point of view). The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
5.5 Positive oil and import price mark-up shocks

Oil and import price mark-up shocks share the same complex pass-through as they both affect directly and indirectly the consumption price inflation. Directly and immediately via the respective proportion of oil and foreign goods in consumption; indirectly through their role as inputs in the production process, this influence being furthermore delayed by the intervention of nominal stickiness in the price setting. This combination produces a strong initial increase in consumption price inflation which is made persistent through the firm’s marginal cost and price setting process.23

Let us first consider the consequences of an oil price shock on the euro area economy, depicted in Figure 5. The shock is calibrated such as to imply an 10% increase in crude oil prices on impact. The brutal increase in consumption price yields a decrease in demand that is reinforced by the monetary policy tightening. The oil price influence in the marginal cost is sufficient to counterbalance the decrease in wage and capital return resulting from the depressed demand. This implies a persistent rise of the domestic production price inflation that will feed further the consumption price inflation. Finally, oil prices could also pass on the economy through their impact on the exchange rate, improving or deteriorating the value of the money and affecting the relative prices trade flows. However it does not seem to be the case in this model configuration, irrespective of the trade elasticity. The exchange rate reaction is rather reduced as prices and interest rates reactions are pretty similar in both the endogenous economies. The fact that both economies react identically to such a common shock makes it a good potential candidate to improve co-movements in the two modelled economies as it will be later emphasized.

The bottom-right sub-plot of Figure 6 displays the process profile of the euro area mark-up shock in import prices. Its ARMA(1,1) structure is estimated with an AR(1) coefficient of 0.96 and a MA coefficient close to 0.35. These estimates imply a high-frequency volatility, with a shock that quickly declines in the short run, but with effects that remain persistent in the medium and the long run. A foreign price mark-up shock generates a jump in imported inflation. This drives consumption inflation up and has contractionary effects on domestic aggregate demand, associated with substitution effects between foreign and domestic goods. Though real wages and the capital rental rate fall in response of a weak demand for production inputs, the higher prices of intermediated goods imported for production prevent firms’ marginal costs to decline. Differences in relative prices between the two endogenous countries imply an appreciation of the dollar with respect to the euro. This appreciation dampens US exports (reported in the 8th sub-plot) and this deterioration in turn leads to a small decline in US production.

As for the UIRP shock, for particular combinations associating a sufficiently high trade elasticity with small enough adjustment costs in foreign quantities, substitution effects become significant and can lead to an increase in output and employment after a positive foreign mark-up shock. Finally, as noted in the previous section, the foreign price mark-up shocks

23Together with the intrinsic persistence of the shock processes, modelled as ARMA(1,1) structures.
have been estimated to be strongly positively correlated. Following the above analysis, this should help to increase co-movements in prices, exports and output.
Figure 5: Impulse responses to a positive oil price shock, calibrated such as to imply an 10% increase in crude oil prices on impact. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
Figure 6: Impulse responses to a positive foreign price mark-up shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
6 Assessing the out-of-sample forecasting performances

Forecasting performance is an important criterion in the assessment of a model credibility and usefulness for policy analysis. In this subsection, we analyse the out-of-sample forecast accuracy of the estimated two-country DSGE model. We compare the baseline version to several alternative DSGE specifications and also reduced form models such as VAR and BVAR. By evaluating a wide range of models, we are able to test whether predictions based on the theoretically-grounded DSGE model are competitive with those of reduced form approaches. By comparing the outcomes obtained from the models which utilize the prior beliefs (BVAR and DSGE), we check whether the prior information plays a role in improving the forecast accuracy and which prior, a-theoretical or implied by the DSGE restrictions, has more relevant content for predicting the future dynamics. All models are estimated on the same data set.

We would like to examine whether the additional cross-equation restrictions implied by the open economy set-up penalize or instead improve the predictions of the future economic development. Smets and Wouters (2007) demonstrate that the new generation of closed economy DSGE models compare very well with the VAR and BVAR models in terms of the forecast accuracy. The multi-country specification enables us to obtain the projections of the evolution of open economy variables such as net trade and the nominal exchange rate. Beside this we also consider a naïve open economy model which is simply the juxtaposition of two closed economy models plus exogenous processes for the nominal exchange rate, import and oil prices. The consumption price inflation is then computed as a weighted average of domestic, import and oil prices inflations.

We calculate forecasts for 8 country-specific and one common macroeconomic time series: output, consumption, investment, employment, real wages, nominal interest rate, consumption price inflation, the net trade and the nominal exchange rate. Variables are expressed in growth rates, except for the interest and exchange rates and inflation (in levels), and the net trade is expressed in percentage of GDP. The accuracy of the predictions is assessed through the traditional root mean squared forecast error (RMSE) measure, which is computed for one to eight horizon ahead predictions. We use a standard recursive forecast procedure which implies that the model is estimated up to a certain time period where the forecast distribution is computed and the estimation sample is extended by one more data point afterwards. The forecasts are computed for the period from 1995Q1 to 2014Q4. All the models are re-estimated every quarter. We compare the forecasting performances across the following model specifications:

- unrestricted VAR(2);
- BVAR(2) with Minnesota-style prior;
- Baseline DSGE as presented in section 2 with parameters estimated as in section 4;
- Smets and Wouters (2007) closed economy version, with consumption price inflation observed instead of the inflation in the GDP deflator;
- a naïve open economy version of Smets and Wouters (2007) as shortly described above.

Figures 7 and 8 depict the RMSEs across the different models for the euro area and the US variables respectively. Generally, the BVAR outperforms the VAR, which indicates that restricting a parameter space of the VAR with the prior information enables obtaining more accurate predictions. The forecast comparison exercise demonstrates furthermore that, overall, the baseline DSGE model specification does well compared to the reduced form models that are the VAR(2) and BVAR(2). The baseline DSGE model shows superior forecasting performance for US data, while for the euro area this is true for most variables. In forecasts of the nominal exchange rate, shown in the bottom right plot of Figure 8, the baseline DSGE model outperforms the VAR(2) and BVAR(2) models.

If we now compare the baseline DSGE model with pure closed economies Smets-Wouters-type of models, we observe that for all the domestic variables but the EA real consumption, the open economy model provides root mean square forecast errors that are either fairly equal or smaller than its closed economy equivalent. This shows that adding the open economy dimension mainly helps to improve the forecasting power of the Smets and Wouters (2007) model, or at least does not affect strongly its forecasting accuracy. The comparison with the naïve open economy variant of Smets and Wouters (2007) shows comparable results. The fully-fledged open economy model seems superior in some cases, such as for EA and US output, consumption and US inflation.
Figure 7: Root mean squared forecast errors on EA variables for alternative estimated models. The solid line stands for our baseline two-country model (DSGE). Closed economy Smets-Wouters models are in solid line with asterisk (DSGE SWclosed). The naïve open economy version is represented in the dash-dotted line (DSGE SWopen). VAR and BVAR models are represented in respectively red dashed lines and green solid lines with cross. Posterior modes have been used in the simulation.
Figure 8: Root mean squared forecast errors on US variables for alternative estimated models. The solid line stands for our baseline two-country model (DSGE). Closed economy Smets-Wouters models are in solid line with asterisk (DSGE SWclosed). The naïve open economy version is represented in the dash-dotted line (DSGE SWopen). VAR and BVAR models are represented in respectively red dashed lines and green solid lines with cross. Posterior modes have been used in the simulation.
7 Matching data moments and cross borders spillovers

In this subsection, we validate the model by comparing the stylized facts implied by the actual data to those of the simulated data from the estimated two-country DSGE model. More specifically, we calculate and contrast the model-based and empirical cross-correlations between the main observed macroeconomic data series (see for instance Fuhrer and Moore, 1995; Smets and Wouters, 2003; Gertler et al., 2008). We focus here on the following variables: real output, consumption, investment, the real exchange rate, consumption price inflation and the short term nominal interest rate, all in growth rates but the latter two. The empirical cross-correlations are based on a VAR(1) estimated on the same data sample as the DSGE model and covering the period from 1970:Q2-2014:Q4. The model-based cross-correlations are calculated by estimating a VAR(1) on 6000 random samples of 179 observations generated from the DSGE model (that is 100 runs for a sample of 60 parameter draws from the posterior distribution). The empirical and model-based cross-correlations are reported in Figure 9. The solid asterisk line represents the autocorrelation function (ACF) of the data, the solid line reports the autocorrelation function of the model and the dashed lines delimit the ninety percent posterior interval of the model correlations.

Figure 9 shows that, overall, the model does well in capturing the properties of the data. For the majority of variables, the empirical autocorrelation functions (reported on the first diagonal for the EA and the diagonal just above for the US) lie within the model confidence interval. The model does particularly well in reproducing the real exchange rate (RER) and output (Y) ACFs. One can however note a small difficulty for the model to fit the first order autocorrelation for euro area real consumption (C) and investment (I). The net exports (NT) are the only real series for which the data ACF is hitting the upper bound of the ninety percent confidence interval built around the model simulations. However, the model has more pain with the two nominal data series that are the consumption price inflation (INF) and nominal interest rate (R). Their model simulated variance is too small compared to the data and the model fails to generate a persistence in line with the observations. The difficulties in replicating the dynamics of nominal variables appears to be a general problem of sticky-price DSGE models and has already been described in King and Watson (1996) and Smets and Wouters (2003).

Concerning the within country cross-correlations, the model gives very satisfactory results for real GDP and its major components that are consumption and investments. Coming now to the open economy dimension of the model, we can say that it performs also quite well in reproducing the slightly negative (positive) cross-correlations of the real exchange rate with the EA (US) real GDP and its major components. Noteworthy, as displayed by the two sub-plots at the left end of the last row of Figure 9, the model has also no trouble with the traditional Backus and Smith (1993) puzzle. For memory, these authors remarked that the NOEModels fail to generate the observed correlation between the real exchange rate and the

\footnote{For the VAR model in the following companion form: $y_t = F y_{t-1} + \eta_t$, the vector auto-covariance function is computed recursively according to $\Gamma_j = F \Gamma_{j-1} = F^j \Gamma_0$, for $j = 1, 2, \ldots$ Dividing each row and column of $\Gamma_j$ by the square root of the corresponding diagonal element of $\Gamma_0$, yields the vector autocorrelation function.}
relative consumption (or GDP) between countries (see also Chari et al., 2002, on this topic). The typical issue is that this family of models usually focus on monetary and productivity shocks. For such shocks a raise in consumption is accompanied by a real exchange rate depreciation which is in stark contradiction with the observed feature that countries with high consumption on the cyclical frequency tend to appreciate. The model presented here is equipped with a wider set of shocks, among which risk premium and UIRP shocks that drive movements in consumption and real exchange rate in line with empirical observations.

It is well known from the literature (e.g. Justiniano and Preston, 2010) that pure trade open economy models usually do a pretty poor job in reproducing the transatlantic synchronisation of the real business cycles, as they fail to produce sufficient propagations of domestic shocks to the partner economy. The cross-correlations displayed at the penultimate row of Figure 9 prove that the model does not perform that bad in this respect. This is mainly due to the fact that some shocks have been allowed to be correlated. But we must confess that none of these shocks correlations has proved useful in improving the synchronisation of nominal variables, i.e. the consumption price inflation and the nominal interest rate. Let us examine here which of the shock correlations are important and in which dimension. The upper side of Table 8 displays the shock correlations as they have been estimated. Each column refers to an alternative model specification. The penultimate column reports the numbers for the baseline model \((m-7)\), which are the same as in Table 3 above, while in other columns they can be slightly different as the estimated correlations are influenced by the orthogonality imposed to some pairs of shocks. The bottom part of Table 8 presents the different correlations between euro area and US variables as obtained from the data on the one hand (first column), and from model simulations obtained with the corresponding estimated shocks correlations. The first information provided by the table is that, indeed, in the absence of any cross-countries shocks correlations, the real business cycles of the two economies would appear totally disconnected in the model (see Configuration \(m-1\) in the table), while consumption price inflation and interest rates would be negatively correlated between trading countries instead of being strongly positively correlated as in the data. Second, allowing for correlations of the Rest of the World demand and price shocks \((m-2)\) does not seem to help in any dimension. Correlations between productivity shocks \((m-3)\) helps very timidly to obtain a positive correlation between consumptions. Finally, cross-country correlations between financial shocks \((m-4)\) play the most important role in obtaining cross-country simulated co-movements in output and consumption. The risk premium shocks correlations is clearly predominant in this improvement and this is not astonishing as it may be seen a short-cut for the representation of an international financial system as developed by Dedola and Lombardo (2012), Kollmann (2013) or Kamber and Thoenissen (2013). However, Figure 2 tells us that the consumption price and interest rate reactions to a risk-premium shock remain rather limited, even though in the right direction. This explains that the estimated correlation in risk premium shocks is not sufficient to drive simulated inflation or interest rate synchronisation to the levels observed in the data.

It has been observed in Section 5.5 that both economies react in a similar way to oil price shocks which influence directly consumption prices. Increasing the share of oil in consumption
could then help synchronise the nominal sides of the analysed economies. This potential channel is easily assessed by reducing strongly the share of the distribution costs for oil from the high estimated values to 1 (as implemented in Configuration \( m-8 \)): the correlations between the simulated nominal series are strongly enhanced. However, blowing up the share of oil in consumption is extremely costly in terms of log-likelihood as it increases the volatility of consumption price inflation. This is therefore not directly the way to go, but it could be interesting to investigate further the possibility for other commodities to play the role of a common nominal shock.

Table 8: Cross-correlations between the EA and the US. The columns refer to alternative model specifications, differing from each other in terms of the shock correlations that are allowed for. Note: \( m-8 \) is a model where the share of oil in consumption has been increased strongly by setting parameter \( \delta_o \) to 1, which amounts to triple the share of oil in consumption.

<table>
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<th>( m-2 )</th>
<th>( m-3 )</th>
<th>( m-4 )</th>
<th>( m-5 )</th>
<th>( m-6 )</th>
<th>( m-7 )</th>
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<tr>
<td>( \text{corr}(\mu^{nt}<em>{e,t}, \mu^{nt}</em>{u,t}) )</td>
<td>0</td>
<td>-0.143</td>
<td>-0.144</td>
<td>-0.144</td>
<td>-0.144</td>
<td>-0.145</td>
<td>-0.144</td>
<td>0.001</td>
</tr>
<tr>
<td>( \text{corr}(\mu^{pF}<em>{e,t}, \mu^{pF}</em>{u,t}) )</td>
<td>0</td>
<td>0.540</td>
<td>0.540</td>
<td>0.536</td>
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<td>0.536</td>
<td>0.537</td>
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<table>
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<th>correlations between macroeconomic series as computed from data series simulated with</th>
<th>( m-1 )</th>
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<th>( m-3 )</th>
<th>( m-4 )</th>
<th>( m-5 )</th>
<th>( m-6 )</th>
<th>( m-7 )</th>
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<tr>
<td>( \text{corr}(\Delta Y_{e,t}^{ca}, \Delta Y_{u,t}^{us}) )</td>
<td>0.303</td>
<td>0.077</td>
<td>0.056</td>
<td>0.064</td>
<td>0.100</td>
<td>0.086</td>
<td>0.109</td>
<td>0.137</td>
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<td>( \text{corr}(\Delta C_{e,t}^{ca}, \Delta C_{u,t}^{us}) )</td>
<td>0.262</td>
<td>-0.005</td>
<td>-0.001</td>
<td>0.007</td>
<td>0.089</td>
<td>0.045</td>
<td>0.098</td>
<td>0.144</td>
</tr>
<tr>
<td>( \text{corr}(\Delta I_{e,t}^{ca}, \Delta I_{u,t}^{us}) )</td>
<td>0.278</td>
<td>-0.163</td>
<td>-0.130</td>
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<td>-0.112</td>
<td>-0.108</td>
<td>-0.109</td>
<td>-0.087</td>
</tr>
<tr>
<td>( \text{corr}(r_{e,t}^{fu}, r_{u,t}^{us}) )</td>
<td>0.855</td>
<td>0.323</td>
<td>0.232</td>
<td>0.237</td>
<td>0.291</td>
<td>0.271</td>
<td>0.295</td>
<td>0.332</td>
</tr>
<tr>
<td>( \text{corr}(\pi_{e,t}^{C}, \pi_{u,t}^{C}) )</td>
<td>0.859</td>
<td>0.258</td>
<td>0.223</td>
<td>0.226</td>
<td>0.246</td>
<td>0.242</td>
<td>0.249</td>
<td>0.267</td>
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</table>
Figure 9: Comparison of auto- and cross-correlations of the DSGE model (solid green line) and the data (asterisk blue line). Red dashed lines delimit the ninety percent posterior interval of the model correlations. In the upper bloc delimited by the first five rows, the lower triangular part (highlighted by blue boxes) corresponds to the EA. The upper triangular part (red boxes) is its mirror image adapted to the US. The last two rows (green boxes) are associated with cross-country and open economy auto- and cross-correlations. In these rows, RER and EXCRN are respectively the real and nominal exchange rate, rY, rC and rINF are the relative GDP, consumption and inflation of the two countries.
8 Conditional variance decomposition of the forecast errors

Tables 9 to 12 summarize the results for the variance decomposition of the forecast errors for output, inflation, the nominal exchange rate and the trade balance at different horizons based on the model estimates.

The first observation that follows from the output variance decomposition is the predominant role of the domestic shocks for output: around 72% for the euro area and some 93% for the US one period ahead increasing to 89% for euro area at the longer horizon, while it remains fairly constant for the US. The major difference between the two economies is due to the exports demand shock which is responsible for 23% of the output short run unexpected volatility in the euro area but quickly fades away. Note that the exogenous export demand process plays an important role as well in explaining the US output volatility, but as mentioned earlier, this passes much more via the TFP innovations that are allowed to affect the external demand via the $\vartheta_{nt}$ parameter.

Among domestic shocks, the unexpected short-run output fluctuations are mainly explained by demand shocks, that is the risk premium and government spending shocks. Compared to previous calculations for the closed economy models (e.g. Smets and Wouters, 2005), the contribution of exogenous foreign demand shocks/processes goes at the cost of the domestic demand shocks and especially the public spending shocks. This is not surprising given that, in closed economy, the public spending shock are defined as the residual term in the GDP identity. The role of government spending shocks decreases rapidly in both economies while the influence of the risk premium shocks peaks at 42% and 31% respectively in the euro area and the US, at a 4-quarter horizon. It then decreases slowly by one fourth of this value at a 10-year horizon in euro area, while the drop is more drastic in the US, about two third. The output variance over the business cycles, i.e. over a horizon from 10 to 40 quarters, becomes more and more influenced by labor supply shocks. In the euro area the productivity shock replaces progressively the role played initially by the net export shock, while in the US TFP shocks contribution is relatively constant at all horizons.

Spillover effects from shocks in the US economy towards the euro area or the other way round are too weak to explain a significant proportion of the output fluctuations. Trade shocks or processes have a much more significant impact as it has already been shortly discussed, even though short lived. Oil price shocks and exchange rate shocks do have a very small contribution to the output variance at the business cycle frequency. The influence of monetary policy shocks plays mostly at the short and medium horizons and culminates after one year with a contribution to output volatility around 10% in the EA and 12% in the US. Finally the various price shocks explain never more than 3% (EA) to 5% (US) of the output variance in both economies at the longer horizon.

Domestic shocks are also the major source for the forecast error in consumption price
inflation. Spillover effects between US and the euro area are again negligible, but the foreign shocks make up between one third of the variance in the short run and some 17 to 21% at the 10-year horizon. Among these, in the short run, oil shocks account for respectively 15% (EA) and 21% (US), other import price shocks for 14% (EA) and 13% (US) and UIRP shocks for 6% (EA) and 3% (US) of the inflation variance. Domestic mark-up shocks are by far the major source for unexpected volatility of the inflation process: price mark-up shocks are crucial in the short-run (about 42%) and wage mark-up shocks explain the long-run variance (30 to 40%) and take up most of the long-run trends in inflation. Innovations in the consumption price inflation are responsible for some 12-13% of the unexpected inflation volatility at the one-quarter horizon but their influence halves after 10 quarters in both economies. Remaining domestic shocks do explain at most 4% of inflation in the short run, and raises up to around 12-13% at the 10-quarter horizon. Note that their respective contribution can be pretty different in the two analysed economies, with investment specific shocks much more important in the US while in the euro area it is replaced by the risk premium shock.

The uncovered interest parity shock explains above 74% of the one-period-ahead forecast error variance of the nominal exchange rate. As the shock is fairly persistent, its contribution slowly decreases to 54% at a 10-quarter horizon and 29% at the 10-year horizon. The role of other open economy shocks is marginal. Consequently, domestic shocks explain the remaining share of the exchange rate variance, with a predominant role played by European shocks. Productivity, risk premium and monetary policy shocks have the highest contributions, which increase with the forecast horizon. In the US, investment specific shocks emerge at longer horizons.

The decomposition of the trade balance also provides some useful insights. About 70% of the short-run variance, in the trade balance is explained by open economy shocks, a share that decreases progressively towards 22-24% at the 10-year horizon. The UIRP shock strongly dominates in the trade balance decomposition of both areas, indicating a strong nominal component of trade balance variations. It accounts for about 60% of short-term fluctuations, while its contribution significantly drops towards 16% in the long run. In the euro area, exogenous demand shocks contribute to 10% in the short term, but become negligible as the horizon expands. Domestic shocks happen to play a relatively important role as well at long horizons. This is particularly the case of the productivity, risk premium and wage mark-up shocks.

Together these results for the exchange rate and the trade balance decomposition illustrate that the model is able to explain a significant proportion of the dynamics in these variables by structural shocks that have a clear economic interpretation. What is however less satisfactory is that spillover effects of foreign shocks to the domestic variables remain very small. Bilateral trade flows are not sufficiently important to generate strong spillover effects, consistently with the cross-correlation analysis of Section 7.
Table 9: Variance decomposition of EA and US detrended GDP

<table>
<thead>
<tr>
<th></th>
<th>1 quarter</th>
<th>2 quarters</th>
<th>4 quarters</th>
<th>10 quarters</th>
<th>10 years</th>
<th>1 quarter</th>
<th>2 quarters</th>
<th>4 quarters</th>
<th>10 quarters</th>
<th>10 years</th>
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<td>0.894</td>
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<td>0.023</td>
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<td>0.000</td>
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<tr>
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Table 10: Variance decomposition of EA and US consumption price inflation

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<th>1 quarter</th>
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<th>4 quarters</th>
<th>10 quarters</th>
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<td>0.006</td>
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<td>0.002</td>
<td>0.005</td>
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<td>0.004</td>
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<td>0.005</td>
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<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.013</td>
<td>0.023</td>
<td>0.033</td>
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<td>0.357</td>
<td>0.314</td>
<td>0.240</td>
<td>0.173</td>
<td>0.372</td>
<td>0.348</td>
<td>0.307</td>
<td>0.256</td>
<td>0.231</td>
</tr>
<tr>
<td>UIRP</td>
<td>0.064</td>
<td>0.074</td>
<td>0.065</td>
<td>0.048</td>
<td>0.033</td>
<td>0.034</td>
<td>0.044</td>
<td>0.042</td>
<td>0.033</td>
<td>0.028</td>
</tr>
<tr>
<td>oil price</td>
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<td>0.128</td>
<td>0.102</td>
<td>0.074</td>
<td>0.051</td>
<td>0.211</td>
<td>0.173</td>
<td>0.138</td>
<td>0.107</td>
<td>0.091</td>
</tr>
<tr>
<td>RoW demand (EA)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RoW demand (US)</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
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<tr>
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<td>0.060</td>
<td>0.059</td>
<td>0.048</td>
<td>0.037</td>
<td>0.127</td>
<td>0.130</td>
<td>0.125</td>
<td>0.113</td>
<td>0.109</td>
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Table 11: Variance decomposition of the nominal exchange rate

<table>
<thead>
<tr>
<th>Euro area shocks</th>
<th>1 quarter</th>
<th>2 quarters</th>
<th>4 quarters</th>
<th>10 quarters</th>
<th>10 years</th>
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<tr>
<td>productivity</td>
<td>0.183</td>
<td>0.208</td>
<td>0.255</td>
<td>0.345</td>
<td>0.350</td>
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<tr>
<td>risk premium</td>
<td>0.045</td>
<td>0.053</td>
<td>0.073</td>
<td>0.126</td>
<td>0.136</td>
</tr>
<tr>
<td>investment</td>
<td>0.053</td>
<td>0.062</td>
<td>0.079</td>
<td>0.093</td>
<td>0.094</td>
</tr>
<tr>
<td>gov. spending</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>dom. price m-up</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>cons. price</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>wage m-up</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>monetary policy</td>
<td>0.083</td>
<td>0.089</td>
<td>0.097</td>
<td>0.110</td>
<td>0.099</td>
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<table>
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<tr>
<td>productivity</td>
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<td>0.021</td>
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<td>0.048</td>
<td>0.074</td>
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<td>risk premium</td>
<td>0.006</td>
<td>0.006</td>
<td>0.004</td>
<td>0.004</td>
<td>0.009</td>
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<tr>
<td>investment</td>
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<td>0.002</td>
<td>0.003</td>
<td>0.221</td>
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<tr>
<td>gov. spending</td>
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<td>0.006</td>
<td>0.007</td>
<td>0.009</td>
<td>0.007</td>
</tr>
<tr>
<td>dom. price m-up</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>cons. price</td>
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<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>wage m-up</td>
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<td>0.002</td>
<td>0.006</td>
<td>0.010</td>
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<tr>
<td>monetary policy</td>
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<td>0.047</td>
<td>0.044</td>
<td>0.042</td>
<td>0.038</td>
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</table>

<table>
<thead>
<tr>
<th>open econ. shocks</th>
<th>0.737</th>
<th>0.709</th>
<th>0.655</th>
<th>0.539</th>
<th>0.286</th>
</tr>
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<tbody>
<tr>
<td>UIRP</td>
<td>0.734</td>
<td>0.704</td>
<td>0.649</td>
<td>0.528</td>
<td>0.278</td>
</tr>
<tr>
<td>oil price</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RoW demand (EA)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RoW demand (US)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>imp. price (EA)</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>imp. price (US)</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

| open econ. shocks         | 0.737     | 0.709      | 0.655      | 0.539       | 0.286    |
Table 12: Variance decomposition of EA and US trade balance

<table>
<thead>
<tr>
<th></th>
<th>trade balance (EA)</th>
<th>trade balance (US)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 quarter</td>
<td>2 quarters</td>
</tr>
<tr>
<td>Euro area shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>productivity</td>
<td>0.215</td>
<td>0.250</td>
</tr>
<tr>
<td>risk premium</td>
<td>0.050</td>
<td>0.057</td>
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<td>investment</td>
<td>0.099</td>
<td>0.116</td>
</tr>
<tr>
<td>gov. spending</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>dom. price m-up</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td>cons. price</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>wage m-up</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>monetary policy</td>
<td>0.055</td>
<td>0.061</td>
</tr>
<tr>
<td>US shocks</td>
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<tr>
<td>productivity</td>
<td>0.074</td>
<td>0.083</td>
</tr>
<tr>
<td>risk premium</td>
<td>0.016</td>
<td>0.021</td>
</tr>
<tr>
<td>investment</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>gov. spending</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>dom. price m-up</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>cons. price</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>wage m-up</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>monetary policy</td>
<td>0.034</td>
<td>0.035</td>
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<tr>
<td>open econ. shocks</td>
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<td></td>
</tr>
<tr>
<td>UIRP</td>
<td>0.710</td>
<td>0.667</td>
</tr>
<tr>
<td>oil price</td>
<td>0.574</td>
<td>0.578</td>
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<tr>
<td>RoW demand (EA)</td>
<td>0.000</td>
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</tr>
<tr>
<td>RoW demand (US)</td>
<td>0.103</td>
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</tr>
<tr>
<td>imp. price (EA)</td>
<td>0.028</td>
<td>0.031</td>
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<tr>
<td>imp. price (US)</td>
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<td>0.005</td>
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</table>
9 Growth, inflation and exchange rate historical decompositions

This section provides the historical decompositions of the business cycle and inflation fluctuations in terms of the exogenous shocks of the model. Figures 10 and 11 show the contributions of each shocks to the GDP growth, year per year, respectively in euro area and the United States. Figures 12 and 13 display the historical shock decomposition for the changes in consumption price inflation in both economies. Finally, Figure 14 reports the historical decomposition for the nominal exchange rate growth movements. The decompositions are implemented on the detrended values of observed variables, i.e. in deviation from steady-state values. We take profit from our relatively long sample to analyze the period from 1972 to 2014, which includes the two oil shocks, the early nineties recession together with more recent events such as the dot-com bubble burst in 2001, the subprime crisis of 2007-2008 and its evolution into a global financial turmoil followed afterwards by the euro sovereign debt crisis. For additional information, Appendix C reports the historical decomposition of GDP and inflation growth rates in quarterly terms over the period 1999-2014.

From the previous variance decomposition exercise, we can already infer that the period-by-period shocks contributions to output growth will be mainly driven by domestic shocks and should therefore not be very different from the outcome of a closed economy exercise. This will be less true for changes in inflation, given the larger contribution of typical open economy shocks to the forecast error of consumption price inflation.

9.1 GDPs historical decompositions

The main contributors to GDP growth fluctuations identified by the model are the productivity shock, the risk premium shock, the interest-rate shock, the investment technology shock, the government spending shock and (in the euro area only) the RoW demand shock.25 The productivity shock contributes mainly counter-cyclically to the GDP growth in both economies until roughly 1995, with marked differences in intensity. This remains true in the US for the recent period. However, in the euro area, from the second half of the nineties, productivity shocks contribute systematically negatively to output growth, even though much less during the recent major busts. Countercyclical TFP growth contributions reflect the reaction of labour after such a shock usually obtained in New Keynesian models and discussed in Section 5. This confirms the intuition of the impulse-response analysis, in which the productivity shock turned out to be a bad candidate for explaining growth during the financial crisis, as it implies opposite movements in output and employment. However its contribution to growth is significantly negative in the aftermath of the crisis in both economies and the model identifies productivity shocks as one of the main factors that could explain the slow recovery of the economic activity.

25This difference between the two economies has already been discussed earlier, in the impulse responses and variance decomposition sections.
As discussed in Smets and Wouters (2007), the risk premium shock has some similarities with the effects of an external finance premium in a model with financial frictions. A positive shock increases the required premium in financial assets and has a depressing effect on asset prices. Contributions of the risk premium shock are typically pro-cyclical and particularly strong in the recent double dip recession as well as, in euro area, during the EMS crisis. The recession during Volker’s years at the head of the Fed constitutes an exception where monetary policy shocks dominate. But for this episode, the monetary policy as reflected by the interest rate shocks tends to lean against the wind in the United States, being more (less) accommodative than what the Taylor rule implies in recessions (booms). An expansionary departure from the policy rule is also observed in the beginning of the subprime crisis, but it ended in hitting rapidly the zero lower bound, resulting in a crisis enhancing shock. The brand new ECB clearly followed the same accommodative policy as the Fed during the dot-com crisis but failed to do so when the great recession started.

As pointed out by Justiniano et al. (2010), the investment technology shock can be interpreted as a shock to the marginal efficiency of investment, i.e. the ease with which savings are turned into capital, and therefore can reflect the efficiency of financial intermediation. The investment technology shocks contributed heavily to the output drop of 1974 in the EA but not at all in the US. From the beginning of the eighties, such shocks push continuously output down in Europe and especially during the dot-com and financial crisis episodes. The same marked influence of these shocks during the 2000’s recessions is observed in the US as well. The two shocks that capture imperfections in financial markets, i.e. the risk premium shock and the investment technology shock, are among the main drivers of the subprime downturn.

Wage mark-up shocks are the last major determinant of GDP growth fluctuations around the business cycle. In the euro area, there is a clear break in 1985. Before this date, wage mark-ups put a downward pressure on output growth and the other way round afterwards. In the United States such shocks feed growth in a rather pro-cyclical way until the turn of the century, when their contribution fades away. In both economies, the joint contributions of the export demand and government spending shocks is very close to the contribution of the exogenous spending shocks in a closed economy (see Smets and Wouters, 2005). The model suggests that RoW demand shocks mostly supported growth during and after the financial crisis, with a clear exception in 2008 and 2014 for the euro area. The oil price shocks contribution is globally negligible, but we clearly observe the marked influence of the seventies and mid eighties (in reverse direction) oil shocks.
Figure 10: Historical decomposition of EA GDP growth (YoY) from 1972 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
Figure 11: Historical decomposition of US GDP growth (YoY) from 1972 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
9.2 Variations in consumption price inflation historical decompositions

Let us focus here on the (annual) growth rate of consumption price inflation in order to consider stationary series. Both economies home mark-up shocks account for most of the inflation variations during the seventies. It receives a good help from the exchange rate and foreign price mark-up shocks, especially in the euro area, and wage mark-up shocks offer a non negligible support. The rest is due to monetary policy and risk premium shocks, with a more prominent role for them in the euro area. Oil price shocks explain a surprisingly limited share of the inflation variation during this period of high oil price volatility. The QoQ historical decomposition of consumption price inflation (not reported here for this period) highlights more important contributions of the oil price shock. In particular, the contributions peak at the expected dates (1974:Q1 and 1979:Q1). However, as these contributions are not lasting, this is lost in the yearly aggregation.

As expected, monetary policy shocks contributed to the disinflation of the early eighties, with negative contributions from 1979 to 1984. In the euro area the disinflation process is supported by the risk premium and productivity shock contributions, but this is not at all the case in the US where the contribution of risk premium shocks is replaced by this of investment technology shocks. In both economies a good share of the downward pressure on inflation comes from price and wage mark-up shocks, with a more important role for the latter in the US, reflecting the increased flexibility in the labour market initiated under Reagan. The role of oil price, exchange rate and import price shocks in the disinflation process remain pretty limited in the early eighties, especially in the US, but becomes more important in explaining inflation variations by the end of the eighties and the last decade of the millennium.

The nineties are a period of moderate and less volatile inflation which is nearly not influenced by monetary policy shocks as regards the US. In both economies, wage mark-up shocks become a significant deflationary force. The years 2000s started off under the premises of a technological bubble and continued with the burst of a major financial crisis.\textsuperscript{26} The relatively tight monetary policy of the ECB at the beginning of the financial crisis contributed to decrease inflation, while in the US the Fed policy was perceived as tight only one period later, when hitting the zero lower bound. After 2008, and during the sovereign debt crisis, jumps in uncertainty and the lack of confidence implied important levels for the risk premium shock, which contributed to deflationary pressures. In the two countries, the contribution of wage mark-up shocks to inflation is much reduced after 2000, being less positive (resp. negative) in the EA (resp. US) and reflecting the downward pressure on the remuneration of labour. During the great Recession, the main contributors to the decrease in inflation are the home price mark-up shocks and the composite consumption price shock together with the oil price shock. The drop in oil price accounts for the highest contribution in the deepest trough of the inflation series, the fourth quarter of 2008, with a contribution that amounts to one half (EA) and one third (US) of the smoothed value.

\textsuperscript{26}See Figures 24 and 25 in the appendix for a zoom on the period 1999-2014 for QoQ historical decompositions.
Figure 12: Historical decomposition of the variations of EA consumption price inflation (annual growth rate) from 1972 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
Figure 13: Historical decomposition of the variations of US consumption price inflation (annual growth rate) from 1972 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
9.3 Nominal exchange rate historical decomposition

Figure 14 describes the shocks’ contribution to the major exchange rate movements over the period 1972-2014. The series displayed is the de-trended growth rate of the dollar price in euro. Values above the trend represent a dollar appreciation while below the trend we observe a strengthening of the euro. The first observation might be rather disappointing because most of the fluctuations are explained by the UIRP shocks, coherently with the preceding conditional variance decomposition exercise. As observed then, productivity shocks, together with risk premium and monetary policy shocks play a non negligible role in explaining the relative values of the two currencies.

During the first oil crisis episodes, monetary policy shocks on both sides of the Atlantic tended to influence the euro value of a dollar in opposite directions, with monetary authorities pushing for a depreciation of their respective currency. Central banks decisions put a similar tension on the bilateral exchange rate during the early eighties recession but in reverse direction as they tightened their monetary policy to take control of inflation. The EMS crisis episode appears neatly in the exercise as well, with a euro appreciation encouraged first by the Bundesbank restrictive monetary policy followed by other European central banks pegging with the Deutsche Mark and second by the Fed’s reaction to the savings and loans crisis. Since then the action of central banks seems to globally tend to depreciate their respective currency, with some notable exceptions for the ECB, in 2008 when it reacted with some delay to the Great recession, and in 2010-2011 when it attempted to raise interest rates. Exceptions to this trend for the Fed’s occurred in 1999, when it attempted to end a cycle of loose monetary policy, and in 2009 when it hit the interest rate zero-lower-bound.

An increase in the gap between the financial conditions faced by economic agents and the risk free rate has been shown in Section 5.2 to yield to a strong currency depreciation. This is exactly what we observe in the historical decomposition exercise where recession periods correspond systematically, in each country, to currency-devaluing risk premium shocks contributions. This observation is less true for the contributions of productivity shocks which also correspond with a decreased value of the home currency. During the 1974 and early eighties recession, US productivity shocks contributed to a dollar appreciation, while in 2009 EA productivity shocks sustained weakly the value of the euro.
Figure 14: Historical decomposition of the nominal exchange rate growth (YoY) from 1972 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
10 Conclusion

This paper presents a medium-size two-country model for the euro area and the US. Local currency pricing, Calvo setting and variable demand elasticity are introduced in order to limit the exchange rate pass through to wholesale foreign prices, while distribution costs reduce the pass through to retail foreign prices. The model features a rich structure of domestic and open economy shocks, and its parameters are estimated using Bayesian techniques.

The estimation results are validated through out-of-sample forecasting tests and the investigation of generated co-movements. In terms of forecasting performance, the model is shown to compete with reduced-form models and closed economy DSGEs estimated on the same dataset. The co-movements generated by the model capture fairly well the properties of the data. In particular, simulated data closely reproduce the autocorrelations and correlations between the exchange rate, net exports, output and demand components and are in line with the observed negative correlation between relative consumption and real exchange rate. The model has however more difficulties with the nominal series as the consumption price inflation and the nominal interest rate. Finally, a conditional variance decomposition underlines the non-negligible role played by international shocks in the model. Open economy shocks are important contributors to short-term fluctuations of output (especially in the EA) and consumption price inflation. Moreover, domestic shocks, such as technology, risk premium and monetary policy are relevant factors driving long-run swings in the exchange rate and the trade balance.

As usual in the open-economy DSGE literature, the model struggles to generate significant international spillover effects. Interesting synchronisations can be recovered in real variables when cross-country correlations in the risk premium shock are enabled. This result suggests that internationally-linked financial sectors may be an interesting extension to the model, and is likely to bring additional insights on cross-border propagation mechanisms. Common shocks such as oil price shocks could help to improve the spillovers in nominal variables like consumption prices or short term interest rates. However, such an improvement requires the increase of the share of oil in consumption, and is therefore extremely costly in terms of log-likelihood as it increases the volatility of consumption price inflation. It might nevertheless be interesting to investigate further the possibility for other commodities to play the role of a common nominal shock.

Finally, it should be mentioned that an important objective of the EA-US model developed in this paper is to serve as a reliable prequel to a three-country version that integrates the Belgian economy. The challenge of this extended version will be to deal with countries of different sizes, as Belgium is a small open economy with small or even negligible feedback effects on the other two areas. The extension also requires the modelling of a monetary union, with common nominal exchange rate and monetary policy. Moreover, the modelling of the Belgian economy for policy analysis asks for additional features such as a complete fiscal bloc and detailed housing and financial sectors. From an estimation point of view, the posterior results of the EA-US model can serve as a starting point for the estimation of the
three-country model.

References


Bergin, P.R. (2006), ‘How well can the New Open Economy Macroeconomics explain the exchange rate and current account?’, *Journal of International Money and Finance* 25, 5, 675–701.


A Nominal resource constraint

In this section of the appendix, we detail the steps of the derivation of the nominal resource constraint. After simplification, the result of this derivation leads to an equation that relates changes in net foreign assets positions to the trade balance. The aggregation of home households’ budget constraints and the government (here modelled as a shock), assuming that government bonds are in zero net supply and only held by domestic residents, gives:

\[
\int_0^1 P_{C,t}C_t(h)dh + \int_0^1 P_{I,t}I_t(h)dh + P^D_{H,t}G_t + \int_0^1 \frac{S_tB_{F,t}(h)}{R^t_{K}} - S_tB_{F,t-1}(h)dh + P^D_{H,t}\psi(u_t)K_{t-1} = \int_0^1 W_t(h)\ell_t(h)dh + R^t_{K}\int_0^1 u_t(h)K_{t-1}(h)dh + \int_0^1 div(h)dh
\]  

(57)

The last terms stands for dividends of monopolistic intermediate-good firms, which are equal to
Consumption (public and private), investment and related costs can be further developed:

\[
\int_0^1 dv (h) dh = \int_0^1 P_{H,t}(v) Y_{H,t}(v) dv + \int_0^1 S_t P_{H,t}^v(v) Y_{H,t}^v(v) dv \\
+ \int_0^1 S_t P_{H,t}^p(v) Y_{H,t}^p(v) dv + \int_0^1 P_{H,t}(v) Y_{H,t}^d(v) dv \\
- \int_0^1 W_t(v) L_t(v) dv - R^k_t \int_0^1 w_t(v) K_{t-1}(v) dv \\
- \int_0^1 P_{oil,t} O_t^p(v) dv - \int_0^1 P_{F,t} Y_{F,t}^p(v) dv
\]  

(58)

Substituting dividends in equation (57) and simplifying overlapping terms yields to

\[
\int_0^1 P_{H,t}(v) Y_{H,t}(v) dv + \int_0^1 S_t P_{H,t}^v(v) Y_{H,t}^v(v) dv + \int_0^1 S_t P_{H,t}^p(v) Y_{H,t}^p(v) dv + \int_0^1 P_{H,t}(v) Y_{H,t}^d(v) dv \\
= P_{C,t} C_t + P_{i,t} I_t + P_{H,t}^D G_t + \frac{S_t B_{F,t}}{R^k_t} - S_t B_{F,t-1} - P_{H,t}^D \psi(u_t) K_{t-1} + P_{oil,t} O_t^p + P_{F,t} Y_{F,t}^p \\
\Leftrightarrow \ Y_{H,t} \int_0^1 P_{H,t}(v) G^{-1} \left( \frac{P_{H,t}(v)}{P_{H,t}} \right) dv + S_t Y_{H,t} \int_0^1 P_{H,t}^v(v) G^{-1} \left( \frac{P_{H,t}^v(v)}{P_{H,t}^v} \right) dv + Y_{H,t} \int_0^1 P_{H,t}(v) G^{-1} \left( \frac{P_{H,t}(v)}{P_{H,t}} \right) dv \\
= P_{C,t} C_t + P_{i,t} I_t + P_{H,t}^D G_t + \frac{S_t B_{F,t}}{R^k_t} - S_t B_{F,t-1} - P_{H,t}^D \psi(u_t) K_{t-1} + P_{oil,t} O_t^p + P_{F,t} Y_{F,t}^p \\
\Leftrightarrow \ P_{H,t} Y_{H,t} + S_t P_{H,t}^v Y_{H,t}^v + S_t P_{H,t}^p Y_{H,t}^p + P_{H,t} Y_{H,t}^d = P_{C,t} C_t + P_{H,t}^D O_t^p + P_{oil,t} O_t^p + P_{i,t} I_t \\
+ P_{H,t}^D G_t + \frac{S_t B_{F,t}}{R^k_t} - S_t B_{F,t-1} + P_{H,t}^D \psi(u_t) K_{t-1} + P_{F,t} Y_{F,t}^p
\]

(59)

Consumption (public and private), investment and related costs can be further developed:

\[
P_{C,t} C_t + P_{i,t} I_t + P_{H,t}^D G_t + P_{H,t}^D \psi(u_t) K_{t-1} \\
= P_{H,t} Y_{H,t} + P_{F,t}^D C_{F,t} + P_{H,t}^D I_{F,t} + P_{H,t}^D G_{F,t} + P_{H,t}^D \psi(u_t) K_{t-1} \\
= P_{H,t} Y_{H,t} + P_{F,t} Y_{F,t}^p
\]

These last result further develops into

\[
P_{H,t}^D Y_{H,t}^D + P_{F,t}^D Y_{F,t}^p = \frac{1}{1+\delta_f} P_{H,t}(1+\delta_f) Y_{H,t} + \frac{\delta_f}{1+\delta_f} P_{H,t} \frac{1+\delta_f Y_{H,t}^d}{\delta_f} \\
+ \frac{1}{1+\delta_f} P_{F,t}(1+\delta_f) Y_{F,t} + \frac{\delta_f}{1+\delta_f} P_{H,t} \frac{1+\delta_f Y_{F,t}^d}{\delta_f} \\
= P_{H,t} Y_{H,t} + P_{F,t} Y_{F,t} + P_{H,t} Y_{H,t}^{d,H,F}
\]

(60)

Note that retailers of domestic and foreign goods and oil products use the same home bundle, so only one term in $Y^d_{H,t}$ remains.

Substituting all the relevant terms in (59), the nominal resource constraint boils down to

$$S_t P^*_{H,t} Y^*_{H,t} + S_t P^p Y^p_{F,t} - P_{F,t} Y_{F,t} - P_{oil,t} Oil_t = \frac{S_t B_{F,t}}{R_t} - S_t B_{F,t-1}$$  \hspace{1cm} (64)$$

As $P_{X,t} X_{H,t} = S_t P^*_{H,t} Y^*_{H,t} + S_t P^p Y^p_{F,t} + P_{F,t} X_{F,t}$, and $P_{M,t} M_{H,t} = P_{F,t} Y_{F,t} + P^p Y^p_{F,t} + P_{F,t} X_{F,t} + P_{oil,t} Oil_t$, then we can further simplify into

$$\frac{S_t B_{F,t}}{R_t} = S_t B_{F,t-1} + P_{X,t} X_{H,t} - P_{M,t} M_{H,t}$$  \hspace{1cm} (65)$$

which is the final equation (51) reported in the text.

**B The log-linearised model**

As mentioned in the description of the estimation methodology, we start by detrending real variables and express nominal variables in real terms by dividing them by the composite consumer price. Next, the transformed non-linear equations are log-linearised around the steady-state equilibrium of the model. In the following sub-sections, we report the log-linearised equations of our model. In terms of notations, small letters with a hat (e.g. $\hat{x}_t$) denote a detrended variable expressed in log deviations from its steady-state value, where the latter is expressed with a bar ($\bar{X}$).  

As can be seen from the equations reported in this appendix, equations associated with domestic variables replicate closely the closed economy model of Smets and Wouters (2007).

**B.1 Households**

The log-linearised version of the first order condition of households’ maximization problem with respect to consumption, combined with the Euler equation (8), is

$$\dot{c}_t = \frac{\lambda_{hab}/\gamma}{(1 + \lambda_{hab}/\gamma)} \dot{c}_{t-1} + \frac{1}{(1 + \lambda_{hab}/\gamma)} E_t \ddot{c}_{t+1} + \left[ (\sigma_c - 1) \frac{\bar{w}}{\bar{c}} (\sigma_c (1 + \lambda_{hab}/\gamma))^{-1} \right] (\dot{\bar{r}}_t - E_t \ddot{\bar{r}}_{t+1})$$

$$- (1 - \lambda_{hab}/\gamma) (\sigma_c (1 + \lambda_{hab}/\gamma))^{-1} (\dot{x}_t - E_t \ddot{x}_{t+1} + \varepsilon^b_t)$$  \hspace{1cm} (66)$$

where $\dot{c}_t$ is the log deviation of composite consumption $C_t$ with respect to its steady state value, and $\ddot{x}_t$ stands for the composite consumer price inflation.

27 According to this convention, we have that $\ddot{x}_t = \ln(X_t/\bar{X})$. 

72
First-order conditions with respect to investment and capital provide the following log-linearised equations:

\[
\hat{l}_t = \frac{1}{1+\bar{\beta}} \left( \hat{b}_t - \hat{\beta}\gamma (\hat{q}_t - \hat{\beta}\gamma) + \frac{1}{\Psi^\prime_1\gamma^2} (\hat{q}_t - \hat{\beta}\gamma\hat{p}_c,t) + \frac{1}{\Psi^\prime_1\gamma^2} \hat{\varepsilon}_t \right) \tag{67}
\]

\[
\hat{q}_t = \bar{\beta}(1-\tau) E_t \hat{q}_{t+1} + (1-\bar{\beta}(1-\tau)) E_t \hat{r}_{t+1} - \bar{\beta}(\hat{r}_t - E_t \hat{\pi}_{t+1} + \varepsilon_t) \tag{68}
\]

with \(\Psi^\prime\), the steady-state second derivative of the investment adjustment cost function, \(\bar{\beta} = \beta \gamma - \sigma c\) and \(\bar{\beta}(\bar{\gamma} + (1-\tau)) = 1\).

The capital accumulation equation becomes

\[
\hat{k}_t = \frac{(1-\tau)}{\gamma} \hat{k}_{t-1} + \left( 1 - \frac{(1-\tau)}{\gamma} \right) \hat{l}_t + \left( 1 - \frac{(1-\tau)}{\gamma} \right) \varepsilon_t \tag{69}
\]

and, using the fact that \(\bar{r} = \psi'(1)\), the capacity utilization equation is

\[
\hat{u}_t = \frac{\psi'(1)}{\psi''(1)} \hat{r}_t \tag{70}
\]

Wage setting under Calvo stickiness (\(\xi_w\)) and partial indexation (\(\iota_w\)) generates the following wage equation, which relates real wages to past and expected future real wages, past, current and expected price inflation, and the wage mark-up (last term inside brackets).

\[
\hat{w}_t = \frac{1}{1+\beta} \hat{w}_{t-1} + \frac{\beta\gamma}{1+\beta} E_t \hat{w}_{t+1} + \frac{\iota_w}{1+\beta} \hat{\pi}_{t-1} + \frac{\beta\gamma}{1+\beta} E_t \hat{\pi}_{t+1} - \frac{1}{1+\beta} \hat{\gamma} \hat{w}_{t+1} + \frac{1-\beta\gamma}{1+\beta} \hat{\pi}_t
\]

\[
+ \frac{(1-\beta\gamma\xi_w)(1-\xi_w)}{\xi_w} \left[ \frac{1}{1+\frac{\lambda_{hab}}{\gamma}} \hat{c}_t - \frac{\lambda_{hab}/\gamma}{1-\lambda_{hab}/\gamma} \hat{c}_{t-1} + \frac{\lambda_{hab}/\gamma}{1-\lambda_{hab}/\gamma} \hat{c}_{t+1} + \sigma_t \hat{l}_t - \hat{w}_t \right] + \varepsilon_t \tag{71}
\]

The log-linear version of the UIRP equation (10) is expressed in terms of real exchange rates. We assume the net foreign asset position (\(NFA_t = S_t B_{F,t}\)) to be zero in steady-state, linearise the terms in \(NFA\), and transform the other variables in log deviation from steady-state. We end-up with the following equation:

\[
r_s = (1-\theta_s) E_r r_{s_{t+1}} + \theta_s r_{s_{t-1}} + r_s - r_t - E_r \hat{\pi}_{t+1} + E_r \hat{\pi}_{t+1} - \rho_n f_a n f_a + \varepsilon_t \tag{72}
\]

where \(r_s\) is the real exchange rate, and \(\rho_n f_a = \Theta'(0)\hat{y}^\prime\) is useful to ensure stationarity of the solution, and is set to \(10^{-7}\).
B.2 Firms

We obtain traditional results for log-linearised production, marginal productivity and marginal cost functions:

\[
\begin{align*}
\dot{y}_t &= \Phi_y \left( \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t + \varepsilon^a_t \right) \\
\hat{k}_t &= \hat{w}_t - \hat{r}_k + \hat{\ell}_t \\
\hat{mc}_t &= \Phi_y \left( \frac{1}{\Phi_y} - \varrho_m - \varrho_o \right) \left[ \alpha \hat{r}_k + (1 - \alpha) \hat{w}_t - \varepsilon^a_t \right] \\
&+ \Phi_y \left( \varrho_m \hat{p}^p_{F,t} + \varrho_o \hat{p}^p_{oil,t} \right)
\end{align*}
\]

(73) (74) (75)

where effective capital \( \hat{k}_t = \hat{u}_t \hat{k}_t - 1 \) and \( \Phi_y = \bar{y} + \Phi_{\bar{y}} \).

B.2.1 Relative prices

From the Calvo-Kimball home price index (26), we obtain that \( \hat{p}^H(v) = 1 \), where small letters \( p \) denotes prices relative to the composite consumer price. This result implies that \( \hat{p}^D_H(v) = 1 \). This result follows from detrended versions of equations (3) and (5), and combining their log-linearised versions, we obtain the following equation for the level of composite consumer prices:

\[
0 = (1 - \phi_{oil}) \left[ \Phi_{\bar{c}}^H + \delta_f \hat{p}^H + \frac{\Phi_{\bar{c}}^H}{1 + \delta_f} \hat{p}_{F,t} + 1 - \phi_{oil} \hat{p}^D_{oil,t} \right] + \phi_{oil} \hat{p}^D_{oil,t} + \mu^p_t
\]

(76)

Using detrended versions of equations (3) and (5), and combining their log-linearised versions, we obtain the following equation for the level of composite consumer prices:

\[
\hat{\pi}_{H,t} = \frac{\beta}{1 + t_p \hat{p}_{oil}} \frac{t_p}{1 + \beta t_p} \hat{\pi}_{H,t+1} + \frac{1 - \phi_{oil}^H}{1 + \delta_f} \hat{p}_{F,t} + \phi_{oil} \hat{p}^D_{oil,t} + \mu^p_t
\]

(77)

B.2.2 Home inflation

Using the fact that \( G^{t-1}(...) = z_{ss} = 1 \) in steady-state and \( \bar{I} = G'(1) \), the New Keynesian Phillips Curve associated with home inflation is as follows:

\[
\hat{\pi}_{H,t} = \frac{\beta}{1 + \beta t_p} \hat{\pi}_{H,t+1} + \frac{t_p}{1 + \beta t_p} \hat{\pi}_{H,t-1}
\]

\[
+ \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p(1 + \beta t_p)} \left( \frac{1}{\eta + \epsilon - 1} (\hat{mc}_t - \hat{p}_{H,t}) + \varepsilon^H_t - \varrho^p_{oil} \right)
\]

(77)
B.2.3 Foreign inflation

Imported inflation, from a euro area point of view, is

\[
\hat{\pi}_{F,t} = \frac{(1 - \bar{\beta}^* \gamma^* \xi^*_{pF})(1 - \xi^*_p)}{\xi^*_p(1 + \beta^* \gamma^* \xi^*_p)} \frac{1}{\eta - 1 + \epsilon} \left[ (\eta - 1 - \delta_f)\hat{m}c_t^\nabla + \delta_f \hat{p}_{H,t} - (\eta - 1)\hat{p}_{F,t} \right]
\]

\[
+ \frac{\bar{\beta}^* \gamma^*}{1 + \bar{\beta}^* \gamma^* \xi^*_p} \hat{\pi}_{F,t+1} + \frac{\xi^*_p}{1 + \bar{\beta}^* \gamma^* \xi^*_p} \hat{\pi}_{F,t-1} + \xi^*_p \hat{p}^p
\]

(78)

Foreign exporters set prices in home currency, and take into account their real marginal costs expressed in the home currency, denoted by \(\hat{m}c_t^\nabla\). The total marginal costs of foreign exporters consists of marginal costs of the exporters from the opposite country and from the rest of the world. The latter is accounted for by an exogenous shock. Therefore, we have:

\[
\hat{m}c_t^\nabla = \beta_m (\hat{m}c_t^* + \hat{r} \hat{s}_t) + (1 - \beta_m)\epsilon_t^{row}
\]

The component of imports dedicated to be used as inputs in the production process has its own NKPC, which is expressed as follows:

\[
\hat{p}^p_{F,t} = \frac{(1 - \bar{\beta}^* \gamma^* \xi^*_{pF})(1 - \xi^*_p)}{\xi^*_p(1 + \beta^* \gamma^* \xi^*_p)} \left[ \frac{\eta - 1}{\eta - 1 + \epsilon} \hat{m}c_t^\nabla - \frac{\eta - 1}{\eta - 1 + \epsilon} \hat{p}^p_{F,t} \right]
\]

\[
+ \frac{\bar{\beta}^* \gamma^*}{1 + \bar{\beta}^* \gamma^* \xi^*_p} \hat{p}^p_{F,t+1} + \frac{\xi^*_p}{1 + \bar{\beta}^* \gamma^* \xi^*_p} \hat{p}^p_{F,t-1} + \xi^*_p \hat{p}^p
\]

(79)

B.3 International trade

The optimal demands for foreign consumption and investment goods are derived from the CES aggregators (4) and (6):

\[
\Leftrightarrow \hat{c}_{F,t} = \hat{c}_t - \lambda (\hat{p}^D_{F,t} - \hat{p}_{c,t}) - \Omega_{c}\lambda (\hat{c}_{F,t} - \hat{c}_t - (\hat{c}_{F,t-1} - \hat{c}_{t-1}))
\]

\[
+ \beta \Omega_{c}\lambda (\hat{c}_{F,t+1} - \hat{c}_{t+1} - (\hat{c}_{F,t} - \hat{c}_t))
\]

\[
\Leftrightarrow \hat{i}_{F,t} = \hat{i}_t - \lambda (\hat{p}^D_{F,t} - \hat{p}_{c,t}) - \Omega_{i}\lambda (\hat{i}_{F,t} - \hat{i}_t - (\hat{i}_{F,t-1} - \hat{i}_{t-1}))
\]

\[
+ \beta \Omega_{i}\lambda (\hat{i}_{F,t+1} - \hat{i}_{t+1} - (\hat{i}_{F,t} - \hat{i}_t))
\]

(80)

(81)

where \(\hat{c}_t = \hat{c}_t - \lambda_{oil}\hat{p}_{c,t}\) is the demand for non-oil consumption, which is derived from optimal solutions of (2).

From (46), we obtain the log-linearised demand for transit goods:

\[
\hat{x}_{F,t} = \hat{x}_{H,t} - \lambda_x (\hat{p}_{F,t} - \hat{p}_{x,t})
\]

We further make the assumption that responses of transit goods move one-to-one with fluctuations in exports, that is \(\lambda_x = 0\).
After the log-linearisation of the non-oil imported goods equations (44) and (45), and substitution of foreign consumption and investment goods, we obtain

\[
\dot{m}_{H,t} = \frac{\dot{y}_F}{\ddot{y}_F} \dddot{y}_{F,t} + \frac{x_t}{\dot{m}_H} \dddot{x}_{F,t}
\]

\[
= \phi^H_m \left( \frac{\dddot{y}_F}{\dddot{y}_F} \dot{y}_{F,t} + \frac{\ddot{y}_F}{\dddot{y}_F} \dddot{y}_{F,t} \right) + (1 - \phi^H_m) \dddot{x}_{F,t}
\]

\[
= \phi^H_m \left( \frac{\dddot{c}_F}{\dddot{y}_F} \dot{c}_{F,t} + \frac{\dddot{z}_F}{\dddot{y}_F} \dddot{z}_{F,t} \right) + \frac{\dddot{y}_F}{\dddot{y}_F} \frac{1}{\dddot{y}_F} \dddot{y}_{t} + (1 - \phi^H_m) \dddot{x}_{F,t}
\]

(82)

Coefficients \( \frac{\dddot{c}_F}{\dddot{y}_F} \) and \( \frac{\dddot{z}_F}{\dddot{y}_F} \) can be simplified as follows

\[
\frac{\dddot{c}_F}{\dddot{y}_F} \frac{\dddot{y}_F}{\dddot{y}_F} = \frac{\dddot{y}_F}{\dddot{y}_F} \frac{m_H}{\dddot{y}_F} \frac{\dddot{y}_F}{\dddot{y}_F} \frac{\dddot{c}_F}{\dddot{y}_F} \frac{\dddot{c}_F}{\dddot{y}_F} \frac{\dddot{y}_F}{\dddot{y}_F}
\]

\[
= \alpha^{-1}_m (1 - \phi^o_i)^{-1} \phi^H_{m}^{-1} (1 + \delta)^{-1} (1 - \phi^H_c) (1 - \phi^o_i) \alpha_i
\]

(83)

(84)

where \( \dddot{m} \) is the steady state of total imports (including oil). For investment, the same trick applies, except that the term in \( \phi^o_i \) disappears as no oil is used during the investment process.

As a result, we obtain

\[
\dot{m}_{H,t} = \phi^H_m \left[ \alpha^{-1}_m (1 - \phi^o_i)^{-1} \phi^H_{m}^{-1} (1 + \delta)^{-1} (1 - \phi^H_c) \right] \left( (1 - \phi^o_i) \alpha_i \dddot{c}_{F,t} + \alpha_i \dddot{x}_{F,t} \right) + (1 - \phi^H_m) \dddot{x}_{F,t}
\]

(85)

Therefore,

\[
\dot{m}_{H,t} = \phi^H_m \left[ \alpha^{-1}_m (1 - \phi^o_i)^{-1} \phi^H_{m}^{-1} (1 + \delta)^{-1} (1 - \phi^H_c) \right] \left( (1 - \phi^o_i) \alpha_i \dddot{c}_{F,t} + \alpha_i \dddot{x}_{F,t} \right) + (1 - \phi^H_m) \dddot{x}_{F,t}
\]

(86)

The total import price is a weighted average of oil prices and the price of non-oil imported goods.

\[
\dot{p}_{m,t} = (1 - \phi^o_i) \left[ \phi_F \dot{p}_{F,t} + (1 - \phi_F) \dot{p}_{F,t} \right] + \phi^o_i \dot{p}_{oil,t}
\]

(87)

where \( \dddot{m} \) is associated with the total imports \( \dddot{M} \) expressed in log-deviation around steady-state. The share of imports for consumption purposes in total non-oil imports can be inferred from other parameters and steady-state ratios.

\[
\phi_F = \left[ \frac{1 - \phi^H}{1 + \delta_f} \left( (1 - \phi^o_i) \alpha_c + \alpha_i \right) \right] / \left[ \frac{1 - \phi^H}{1 + \delta_f} \left( (1 - \phi^o_i) \alpha_c + \alpha_i \right) + \theta_m \right]
\]

(88)

Exported goods are driven by the demand for import of the other country and an exogenous shock to account in fluctuations in trade with the rest of the world

\[
\dddot{x}_{H,t} = \beta_m \dddot{m}_{F,t} + \dddot{z}_m
\]
B.4 Resource constraints and net foreign assets

Taking into account the distribution channels, equation (52) becomes

\[ Y_t = \left( s_{H,t} \frac{1}{1+\delta} + s_{H,t}^d \frac{\delta}{1+\delta} \right) (C_{H,t} + I_{H,t} + G_t + \psi(u_t)K_{t-1}) + s_{H,t}^* Y_{H,t}^* + s_{H,t}^{p*} Y_{H,t}^{p*} \]
\[ + s_{H,t}^{d} Y_{F,t} + s_{H,t}^{d} \frac{\delta^{oil}}{1+\delta^{oil}} O_t^{D} \]

(89)

Using the fact that price dispersion terms \( s \) vanish in a first-order log-linear approximation of the model, and after substitution, we obtain

\[ \hat{y}_t = \bar{c} \hat{c}_t + \bar{\gamma} \hat{\pi}_t + \frac{\bar{\gamma} k}{\gamma} \hat{u}_t + \bar{\gamma} H \hat{y}_{H,t} + \bar{\gamma} H \hat{y}_{H,t}^* - \bar{\gamma} F \hat{y}_{F,t} + \frac{\delta^{oil}}{1+\delta^{oil}} \hat{\delta}^D \hat{\delta}^{D} \]

(90)

We then substitute log-linear expressions of equation (46) for exports and equations (44) (49) for imports, and for oil. After simplifying for imports of oil and foreign inputs in the production, we end-up with the following log-linearised real resource constraint

\[ \hat{y}_t = \bar{m} \bar{c}_t + \bar{m} \bar{i}_t + \frac{\bar{m} k}{\gamma} \bar{u}_t + \bar{m} \bar{y}_{H,t} + \bar{m} \bar{y}_{H,t}^* - \bar{m} \bar{y}_{F,t} + \frac{\delta^{oil}}{1+\delta^{oil}} \bar{\delta}^{D} \bar{\delta}^{D} \]

(91)

For the log-linearization of (51), we first note that since the position in foreign bonds is assumed to be zero in steady state, the net foreign assets position is also zero. Therefore, we linearise the terms in \( nfa \) and log-linearise the other terms. After simplification, the net foreign equation has the following log-linear approximation:

\[ \beta^* nfa_t = \gamma^{-1} nfa_{t-1} + \alpha_x (\hat{p}_{x,t} + \hat{x}_F - \hat{p}_{m,t} - \hat{m}_t) \]

Consistently with this last equation, we define the linear version of the trade balance as

\[ \hat{tb}_t = \hat{p}_{x,t} + \hat{x}_{H,t} - \hat{p}_{m,t} - \hat{m}_t \]
\[ = \hat{t} t^2_t - \hat{p}_{m,t} + \hat{p}_{F,t} + \hat{x}_{H,t} - \hat{m}_t \]

(92)

B.5 Monetary policy

The monetary policy rule is

\[ \hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r)(\rho^r \hat{\pi}_t + \rho^\hat{\pi}(\hat{y}_t - \hat{y}_{t}^f)) + \rho^\Delta \gamma (\hat{y}_t - \hat{y}_{t-1} - (\hat{y}_{t}^f - \hat{y}_{t-1}^f)) + \epsilon^r \]

(93)

C Figures
Figure 15: Representation of the two-country economy. Domestic structure is represented in black, and follows closely a standard Smets-Wouters framework. Open economy relationships are depicted in red.
Figure 16: Representation of the production process, from the EA viewpoint. IGF = Intermediate-good firms; HGA = Homogenous-good assemblers; FGF = Final-good firms. A subscript H (respectively F) is attached to variables associated with prices and quantities produced in the home (foreign) country. Whenever variables or parameters are associated with the foreign market, a star exponent (*) is used.
Figure 17: Impulse responses to a positive interest-rate shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
Figure 18: *Impulse responses to a positive investment-technology shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.*
Figure 19: Impulse responses to a positive government spending shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
Figure 20: Impulse responses to a positive home price markup shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
Figure 21: Impulse responses to a positive home wage markup shock in the EA. The posterior mode is used for the simulation. All impulse responses are reported as percentage deviations from the steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations from steady-state. Increase in the real exchange rate corresponds to a depreciation of the euro with respect to the dollar.
Figure 22: Historical decomposition of EA GDP growth (QoQ) from 1999 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
Figure 23: Historical decomposition of US GDP growth (QoQ) from 1999 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
Figure 24: *Historical decomposition of the EA inflation growth rate (QoQ) from 1999 to 2014.* Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.
Figure 25: Historical decomposition of the US inflation growth rate (QoQ) from 1999 to 2014. Detrended values of the observed variable are represented by the black line. The posterior mode is used to implement the decomposition.

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297. “Does one size fit all at all times? The role of country specificities and state dependencies in predicting banking crises” by S. Ferrari and M. Pirovano, Research series, May 2016.


305. “Forward guidance, quantitative easing, or both?”, by F. De Graeve and K. Theodoridis, Research series, October 2016.


