The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium, nor of the European Central Bank.

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Abstract

This paper gives an overview of some issues related to market valuation, focusing on the developments on the New York equity markets.

The 42.4 p.c. fall in the S&P 500 price index between 24 March 2000 - when it reached its all-time high - and 31 December 2002 is situated in a very long term perspective. It then appears that some bear markets were more pronounced in the past but that the bull market preceding the 2000-2002 bear market had been particularly long and impressive in extent.

Given this sharp correction, we will discuss whether the S&P 500 was correctly valued at the end of 2002. To this end, we make use of valuation indicators defined as the ratio of the price to a fundamental. The fundamentals considered here are, according to the discount dividend model, annual earnings and, according to Q-theory, net worth.

In December 2002, price-earnings (P/E) still showed a significant overvaluation of equity prices when compared to the historical average over the 1871-2002 period but, since July 2002, the overvaluation has not been significant in the case of Q. The evidence is even more mixed when the comparison is made, for each valuation indicator, with their average over the last 10 years.

Simulations based on VAR models for P/E and Q were carried out to check whether, on two occasions, the S&P 500 in real terms climbed to a level perceived as irrational given past experience, implying that a correction had to be expected. These occasions were the so-called 1929 and 2000 bubbles.

The models showed that, at some point in time before the peak in (real) stock prices was reached, the real S&P 500 exceeded the upper band of the 95 p.c. confidence intervals during both periods. For each of them, the Q model showed earlier and more persistent signals of significant overvaluation of stock prices than for the P/E model. Finally, in December 2002, both models indicated that the stock price had come back largely within the confidence interval.
# TABLE OF CONTENTS

INTRODUCTION .................................................................................................................. 1

1. RECENT AND HISTORICAL EVOLUTION OF THE S&P 500 ......................... 3

2. ARE STOCK PRICES PREDICTABLE? .............................................................. 14

3. VALUATION INDICATORS ............................................................................... 16
   3.1 Theories underlying the fundamental to be compared with stock prices ......... 16
      3.1.1 Discount Dividend Model .................................................................... 16
      3.1.2 Q-theory ......................................................................................... 19
   3.2 Data ............................................................................................................. 20
      3.2.1 Earnings ......................................................................................... 20
      3.2.2 Q .................................................................................................... 22
   3.3 Descriptive analysis .................................................................................... 23
      3.3.1 An initial look at the valuation of stock prices ................................ 23
      3.3.2 Do the valuation indicators send the right signal about stock market prices? 27
      3.3.3 Are misalignments in the valuation indicators corrected by stock price changes? 32
   3.4 Empirical analysis ....................................................................................... 38
      3.4.1 Univariate and multivariate analyses .................................................. 38
      3.4.2 The models .................................................................................... 43
      3.4.3 Simulation exercises ........................................................................ 43

CONCLUSION ................................................................................................................. 50
   Annex 1 Testing the random walk hypothesis ..................................................... 52
   Annex 2 Derivation of the Discount Dividend Model ....................................... 54
   Annex 4 Might an increase in earnings continue to contribute, from now on, to a downward adjustment in P/E? ......................................................... 58
   Annex 5 The VECM Model .............................................................................. 62
   References .............................................................................................................. 65
INTRODUCTION

We take an interest in developments in the American stock markets for three reasons. First, in the United States, stock market movements influence both household and business consumption and investment to a greater extent than in other countries. Second, the New York Stock Exchange (NYSE) is the world’s leading stock market. The importance of the New York market is even greater if we also take account of the National Association of Securities Dealers Automated Quotation (NASDAQ) and the American Stock Exchange (AMEX). Third, movements in stock market prices in the euro area are correlated with those observed in the United States.

The S&P 500 reached an all-time high on 24 March 2000 of 1,527.45 points before plummeting by 42.4 p.c. to 879.82 points on the evening of 31 December 20021.

The purpose of this paper is twofold: to place this event in a historical perspective and to provide some answers to the question whether, despite the sharp correction which has already occurred, shares are still overvalued and prices could therefore fall even further. This paper aims to be accessible to a broad public. It is divided into three sections.

Section 1 comments on the recent development of the S&P 500 stock price index. The main features are a strong and more or less continuous rise between 1995 and 2000, and a sharp fall since the beginning of 2000, which allows the 1995-2000 period as a whole to be considered as a financial bubble; hereafter called the "2000 bubble".

The upward phase ("bull market") had already started in August 1982. In this paper, this movement and the subsequent downward phase ("bear market") are compared with similar phases in the past, especially in real terms. Finally, the parallel has been drawn between the 2000 bubble and the one which occurred in 1929.

Although some conclusions could be drawn from these historical comparisons, one has to take a more in depth look to fully understand the current valuation of the stock market.

Section 2 raises the question of the "random walk" hypothesis. According to the theory of efficient markets, stock market prices are supposed to follow a random walk, and thus it is

1 Unless otherwise specified, all time series data presented in this article have 31 December 2002 as the cut-off date on the basis of information available on 31 January 2003.
impossible to predict their future movements. This hypothesis has been tested and is strongly rejected.

The rejection of the random walk hypothesis implies that stock prices can be forecasted to a certain extent. To this end, it is necessary to use instruments which link the price of shares to a "fundamental". Section 3 is devoted to analysing these valuation indicators. Two fundamentals are mainly examined: earnings and net worth. The first finds its justification from the discount dividend model, while the second comes from the Tobin's Q theory of investment. After a brief review of the key aspects of these theories, section 3 contains a presentation of the data and a descriptive analysis. This starts with an initial outline of the current valuation of stock prices by comparing the latest observed data for each of the valuation indicators with their historical average. We have also checked whether, in the past, the two valuation indicators under review gave the correct signals regarding future movement in stock prices, especially before the major turning points. Finally, we examine whether the extreme variations of the valuation indicators were caused by changes in real stock prices or by changes in real fundamentals.

Section 3 ends with a simulation exercise based on VAR models. We have particularly analysed, for the 1929 and the 2000 bubbles, whether the sharp fluctuations in the stock price were compatible with the evolution of company earnings and net worth.
1. RECENT AND HISTORICAL EVOLUTION OF THE S&P 500

There are several stock markets operating in the United States. The best-known among them are situated in New York; these are, in decreasing order of size according to the stock market capitalisation of the shares quoted on them, the NYSE, the NASDAQ and the AMEX. The comments made in this article relate to the above-mentioned markets and are confined to share transactions.

Over time, some private companies have developed indices enabling the general development of American stock markets to be assessed. These indices may differ from one to another according to criteria such as the method of weighting the index, the markets covered, the type of companies, the number of companies, the breakdown between markets and the relative weight given to the economic sectors. In this paper, we focus on the S&P 500 index.

Since 1995, the S&P 500 has experienced two phases: an almost constant rise between January 1995 and 24 March 2000, and a plunge down to a low point reached on 9 October 2002. A rebound followed until 28 November but the index declined again somewhat in December.

---

2 There are other stock price indices which cover the American markets. In decreasing order of market capitalisation (as at the end of June 2002), these indices are the Wilshire 5000, the Thomson Financial Datastream total market, the S&P 500, the Dow Jones Industrial Average (DJIA) and the Nasdaq Composite. We give preference to the S&P 500 index mainly because it is the reference index for portfolio management and is the only one of these indices for which there are monthly historical series, both for prices and for other series which are useful for our research (dividends, earnings, ...), going back as far as 1871. The availability of such long series is a considerable asset for empirical, statistical and above all econometric work. Other reasons, not specific to the S&P 500, justify that we have recourse to it: the price evolution of this index, together with that of the Thomson Financial Datastream's total market index, is the most closely correlated to that of the broadest stock price index, the Wilshire 5000. The S&P 500 is also broadly representative of the American stock markets: at the end of June 2002, its market value represented 78 p.c. of that of the Wilshire 5000. Finally, unlike the DJIA, it is weighted by stock market capitalisation.

3 Throughout the paper, we shall focus on the price variations of the indices as our main concern is stock market valuation. Nevertheless, an investor also pays attention to dividends as they are part of the return.
Between January 1995 and March 2000, the S&P 500 had just over tripled in value. This important progression of the index led many analysts to qualify it, in retrospect, as a bubble. Consequently, the index showed a sharp correction. It went down 49.1 p.c. to the low reached on 9 October 2002. At the end of the period, the price index still showed a substantial increase since January 1995.

This recent evolution of the S&P 500 can be placed in a long-term perspective as, whereas the S&P 500 was not actually introduced until 1928, in the 1930s the Cowles Commission made it possible to reconstitute a historical monthly series going back to the year 1871.

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4 Cowles and Associates (1939).
5 The data is made available by Professor Robert J. Shiller on the University of Yale website: http://aida.econ.yale.edu/~shiller/data.htm. Whereas data has existed since 1871, it has been shown since 1872 for graphical reasons.
A graph of this index since 1872 on a logarithmic scale (chart 2) shows that:

- over a very long period, the index of stock market prices has risen substantially;

- stock prices declined on several occasions. The sharpest correction was observed between September 1929 and June 1932.

The foregoing analysis suffers from the disadvantage of not taking account of the movement in consumer prices. The shareholder is keen to protect himself against the erosion of the purchasing power of his portfolio caused by the general rise in prices. Some stylised facts can be established on the basis of a real terms series (chart 3):

- stock prices show a steep upward trend;

---

6 In this paper, real terms series always refer to nominal series deflated by the consumer price index (indices 1982-1984 = 100).
after World War I, stock prices significantly exceeded the straight trend line during three sub-periods: around 1929, in the 1960s and from the mid-1980s onwards;

the upward deviation compared to the trend in 1929 was followed by the sharpest correction ever observed; prices having subsequently remained well below the trend until the end of the 1950s;

the upward deviation compared to the trend observed during the 1960s led to a correction in the 1970s, especially in 1973 and 1974;

finally, despite the correction in progress since 2000, prices were still, in December 2002, appreciably above the trend. Even more serious: never, during previous periods characterised by a positive deviation, had this been so great.

Chart 3 - S&P 500: movement since 1872
(in real terms\(^1\), log scale)

\(^1\) Deflated by the consumer price index (indices 1982-1984 = 100).

In the rest of the section an attempt has been made to synthesise the main upward and downward periods of stock market prices, referred to by financial specialists as "bull" and "bear" markets respectively. As our analysis relates to the long term, attention will be paid
to the variations calculated on the basis of monthly averages rather than daily data. Table 1 refers to stock prices in nominal terms.

Table 1 - “Bear markets”\(^1\) at least as significant as the 1987 crash

<table>
<thead>
<tr>
<th>Start of the period (peak)</th>
<th>Date of the lowest point</th>
<th>Value at the lowest point (indices peak = 100)</th>
<th>Number of months between peak and lowest point</th>
<th>End of period (date of return to original value)</th>
<th>Number of months between the lowest point and the return to original value</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 1872</td>
<td>June 1877</td>
<td>52.7</td>
<td>62</td>
<td>February 1880</td>
<td>32</td>
</tr>
<tr>
<td>June 1881</td>
<td>August 1896</td>
<td>57.9</td>
<td>182</td>
<td>December 1900</td>
<td>52</td>
</tr>
<tr>
<td>September 1902</td>
<td>October 1903</td>
<td>70.7</td>
<td>13</td>
<td>March 1905</td>
<td>17</td>
</tr>
<tr>
<td>September 1906</td>
<td>November 1907</td>
<td>62.3</td>
<td>14</td>
<td>August 1909</td>
<td>21</td>
</tr>
<tr>
<td>December 1909</td>
<td>August 1921</td>
<td>62.6</td>
<td>140</td>
<td>January 1925</td>
<td>41</td>
</tr>
<tr>
<td>September 1929</td>
<td>June 1932</td>
<td>15.2</td>
<td>33</td>
<td>November 1958</td>
<td>267</td>
</tr>
<tr>
<td>December 1968</td>
<td>June 1970</td>
<td>71.0</td>
<td>18</td>
<td>March 1972</td>
<td>21</td>
</tr>
<tr>
<td>August 1987</td>
<td>December 1987</td>
<td>73.2</td>
<td>4</td>
<td>July 1989</td>
<td>19</td>
</tr>
<tr>
<td>August 2000</td>
<td>December 2002</td>
<td>60.5</td>
<td>(28)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Sources: Shiller's website (see footnote 5), Thomson Financial Datastream; own calculations.

\(^1\) Based on the S&P 500; stock prices in nominal terms.

The most recent of the significant stock market corrections which preceded latest developments is that of the so-called crash of October 1987. In August 1987 the S&P 500 had peaked. Following a pronounced decline, although it lasted only four months, the S&P had lost (in December 1987) 26.8 p.c. of its August value, and it took altogether 19 months before the index recovered its initial value.

Since 1871, the S&P 500 has shown nine other falls as large as that of the autumn of 1987\(^7\). The occurrence of a stock market crash or "bear market" is therefore not at all exceptional in itself, despite the above-mentioned rising trend. Table 1 also shows other

\(^7\) The stock market crashes taken into account here correspond to the following criteria: we have recourse constantly to the S&P 500 index (previously named Cowles index); the start of the period is a historical peak in the series of the S&P 500 in nominal terms; we consider stock price declines as large as the 1987 crash; we impose no time restriction in our definition of a stock market crash. Mishkin and White (2002) also used what they qualify as the "universally agreed" stock market crashes of October 1929 and October 1987 as benchmarks for their procedure to identify stock market crashes. Their procedure differs from ours on what they refer to as key factors, namely the choice of stock market index, the size of the collapse and the timeframe of the decline. More explicitly, they used the DJIA for 1903 to 1940 and they shifted to the S&P 500 in 1946; they considered stock price declines of over 20 p.c.; and they looked at declines over five different time windows (one day, five days, one month, three months and one year).
lessons:

• the latest crash is characterised by a fall of around 40 p.c. in the S&P 500 between August 2000 and December 2002. The correction is therefore already greater than that of the autumn of 1987 and than several other periods (notably December 1968-June 1970);

• more significant declines than the current one have been observed only in four previous periods. In three of these (1872-1877, 1881-1886 and 1973-1974), the maximum fall was however not more than 50 p.c. Following the October 1929 crash, on the other hand, the fall of S&P 500 down to its lowest point (in June 1932) reached 84.8 p.c. and it took a little over 22 years for the index to return to its original value. This was in November 1958;

• the downward phase of the nine previous bear market periods (since 1871) had lasted on average 54 months (about four and a half years). During these same periods it took on average 60 months (five years) for prices to return to their initial level. In December 2002, the present period began only 28 months ago.

In chart 4, comparison has been made between the present correction and certain “super bear market” periods, i.e. cases of “bear markets” entailing a correction of a size at least equivalent in real terms to that of the present one.
Since 1871, only three other super bear market periods have been identified: September 1906-September 1928, September 1929-November 1958 and December 1968-January 1992.

These periods are longer than those considered in the analysis carried out in nominal terms: as consumer prices tend to increase over time, it takes longer for the S&P 500 to return to the initial level. As a result, some super bear market periods in real terms cover several bear market periods in nominal terms, so that the latter are more numerous than the former\(^8\).

The fall in the S&P 500 observed since August 2000 follows a course close to that of the three preceding super bear markets in real terms. If the current period continues in a similar manner, it may be feared that the trough has not yet been reached, and the periods required to enable the index to reach its lowest point and then return to its preceding peak might be very long. First, at the low points of December 1920, June 1932 and July 1982,

\(^8\) By way of example, the strong inflation of the 1970s did not allow the S&P 500 to retain its December 1968 level in real terms until January 1992, whereas it had reached this level again in nominal terms by March 1972. Also, the super bear market in real terms of the period 1968-1992 comprises three bear markets displayed by the analysis in nominal terms (1968-1970, 1973-1974 and 1987).
the S&P 500 had lost, respectively, 70, 80.6 and 63.6 p.c. of the real value which it had reached when it was at its previous peak, against barely 42.2 p.c. between August 2000 and December 2002. Second, the minimum had been reached after over 14 years, nearly 3 years and over 13 years respectively in the preceding periods whereas in December 2002 a little over 2 years passed since August 2000. Third, in the three preceding periods, a long further period - respectively a little over 5 years, a little over 26 years and nearly 8 years - had elapsed before the S&P 500 returned to its initial volume. Consequently, the total duration of the three above mentioned super bear markets had been respectively 22 years, a little over 29 years and a little over 23 years.

Chart 5 - "Bull markets"

(in real terms, indices low = 100)

We also need to take a look at significant rises in the S&P 500, the so called bull markets. Chart 5 is devoted to a comparison, also in real terms, between the last bull market and the previous ones. No criterion of the extent of the rise in the stock price index is used to define the bull markets. We merely take the trough\(^9\) of the super bear markets as their starting date and their subsequent peaks as the end date.

\(^9\) The trough for the first bull market corresponds to the minimum of the real S&P 500 reached in June 1877.
The cumulative rise of the S&P 500 between the last low point in real terms, observed in July 1982, and the peak reached in August 2000 had been considerable as the index was multiplied by nearly eight. The bull market of the 1920s had been twice as short, and the real stock market price had been multiplied by only five. Other bull markets, on the other hand, were longer. This was the case with the periods extending, on the one hand, between June 1877 and September 1906 and, on the other hand, between June 1932 and December 1968. During the latter period, the real value of the S&P 500 had been multiplied by more than eight.

The bull markets considered in chart 5 ended, more often than not, in a more or less long phase of stock market euphoria, generally coinciding with strong expectations of high future profits thanks to the spread of new technologies: railway and telegraph companies in 1877-1906, automobile, electrical, radio and cinema industries in 1920-1929, "nifty-fifty"\textsuperscript{10} companies such as the high-tech firms IBM and Xerox, as well as high-profile consumer products firms such as Coca-Cola and McDonald's in the 1950s and 1960s and, finally, the companies emblematic of the information and communication technologies (ICT) during the second half of the 1990s. The stock market corrections which followed these phases of euphoria can be explained by a downward revision of future profits. For example, during the first of these super bear markets, the appearance on the scene of two technologies competing with railways, namely mass production of motor cars and trucks, led to the bankruptcy of the main American railway companies by 1917.

To conclude, the stock market correction observed between August 2000 and December 2002 is by no means exceptional either in its length or in its extent. However, the preceding upward movement of prices had been particularly long and impressive in extent.

It is very tempting today to compare the "bubble" of 2000 to that of 1929. As the choice of starting date might have an effect on the conclusions which might be drawn from this comparison, we consider two different criteria to set it: the previous extreme low point\textsuperscript{11} for the real price (chart 6) and the moment when the stock valuation deviates from its historical average, which is estimated at around 15 for the price-earnings ratio\textsuperscript{12} (chart 7).

\textsuperscript{10} This is a list of 50 nifty companies for which the market had high expectations at that time and that traded at very high price-earnings ratios (Shiller, 2000). Siegel (1998) reported average price-earnings (P/E) of 41.9 for these 50 stocks at their market peak in 1972 that he compared with an average P/E of 18.9 for the S&P 500 index.

\textsuperscript{11} This corresponds to the starting point of the bull markets represented in chart 5.

\textsuperscript{12} We shall review this concept in more depth in section 3 and show that the historical geometrical average of the P/E proved to be slightly below 15, i.e. 13.8.
Chart 6 - Comparison between the 1929 and 2000 bubbles
(in real terms, indices low\(^1\) = 100)

Sources: BLS, Shiller's website (see footnote 5), Thomson Financial Datastream; own calculations.

1 The starting date is determined as the moment when the previous extreme low point for the real price has been reached.

Chart 7 - Comparison between the 1929 and 2000 bubbles
(in real terms, indices low\(^1\) = 100)

Sources: BLS, Shiller's website (see footnote 5), Thomson Financial Datastream; own calculations.

1 Defined here by the last month when the price-earnings ratio was lower than 15.
From charts 6 and 7, we conclude that:

- while the rise in real prices was faster at the time of the 1929 bubble, it was also far less impressive;
- in December 2002, real prices were still far above the initial level, whatever criteria are used for choosing the starting date. In June 1932, the correction brought real prices back to their initial level should the starting date be assumed to be the previous extreme low point reached in December 1920 and even barely half of their initial level if the last month during which the price-earnings ratio was under 15, that is October 1927, is referred to.
2. ARE STOCK PRICES PREDICTABLE?

According to the Efficient Markets Hypothesis (EMH) in finance, future stock prices cannot be predicted. As Samuelson (1965) states it, "in an informationally efficient market, asset price changes must be unforecastable if they are properly anticipated, i.e., if they fully incorporate the expectations and information of all market participants". In other words, "a market in which prices always fully reflect available information is called efficient" (Fama, 1970).

The question of the (un)predictability of stock prices in finance is closely related to the random walk hypothesis. Indeed, if stock prices follow a random walk, then they are unforecastable by definition. To illustrate this purpose, take for instance the case where stock prices, $P_t$, are explained by the following equation:

$$P_t = \mu + \rho P_{t-1} + \varepsilon_t$$  \hspace{1cm} (1)

where $\mu$ is a constant, called drift parameter, $\rho$ the estimated autoregressive coefficient and $\varepsilon_t$ is the innovation term or shock\(^{13}\). If stock prices follow a random walk, it involves that $\rho = 1$ and the increments, $\varepsilon_t$, must be white noise. That means that they must respect two conditions: absence of autocorrelation and absence of heteroskedasticity\(^{14}\).

The most important condition requires that increments must be uncorrelated. Simply stated, if stock prices follow a random walk, this means that equation (1) is sufficient to explain the dynamics of $P_t$. Therefore, one of the most direct and easiest way to

---

\(^{13}\) A shock symbolises new information available within the market.

\(^{14}\) Ideally, the increments, $\varepsilon_t$, should be independently and identically distributed with a zero-mean and a constant variance (see Annex 1). In other words, the increments, $\varepsilon_t$, do not have to be autocorrelated and heteroskedastic. In the literature, these conditions are often represented by the following expression: $\varepsilon_t \sim \text{IID} (0, \sigma^2)$. However, it is often recognized that the assumption of identically distributed increments may not be plausible for financial asset prices, especially over long time spans. For example, over a two-hundred-year sample period, many developments (technological progress, market structure, financial innovation, etc.) may explain time-variation in volatility of many financial prices.
test the random walk hypothesis for the stock price is to check for autocorrelation\textsuperscript{15}. Indeed, the presence of autocorrelation in the increments, $\varepsilon_t$, is mostly the sign of an omitted explanatory variable and thus that stock prices might be predictable to some degree.

The homoskedasticity condition, or non-heteroskedasticity in the increments, requires that the variance of increments is constant over time. Since this variance is used for determining the significance level of the explanatory variables, i.e. $P_{r-1}$ in equation (1), this condition warrants an unbiased analysis over the significant variables whatever the sub-sample considered. Actually, with unstable variance, the explanatory variable $\Delta p_{r-1}$, for example, may be significantly different from zero during some sub-periods but not during other sub-periods.

We tested these conditions for the monthly series of the S&P 500 over the sample period January 1871 to December 2002. The results we obtained, detailed in Annex 1, confirm those found recently in empirical literature (Malkiel, 2003 and Shiller, 2003), i.e. a strong rejection of the null hypothesis of no autocorrelation and no heteroskedasticity. Both standard autocorrelation and heteroskedasticity tests clearly suggest the problems of an omitted variable and instability of the variance in the increments. Furthermore, according to the stability of variance condition, if the random walk hypothesis holds for stocks returns, their variance should decrease linearly over time, which also seems to be slightly rejected.

The rejection of the random walk hypothesis, and hence the EMH, in practice explains the emergence of theories, notably in macro-finance, relating the stock price to a fundamental.

\textsuperscript{15} The unit root tests, which explicitly check if the coefficient $p$ is significantly equal to one, are often confused with tests of the random walk hypothesis. Of course, both elements are indirectly related as we will see in section 3.4, but, by construction, the unit root tests are not designed to detect predictability (Campbell et al., 1997, Chapter 2).
3. VALUATION INDICATORS

Given the rejection of the random walk hypothesis, several studies have thus emerged in literature in order to detect what kind of variables may explain the fluctuations of the stock price, i.e. what is (are) the fundamental variable(s) of the stock price. Among potential candidates, the dividends or the earnings, relying on a first theory (discount dividend model) and the net worth, relying on a second theory (Q-theory) are in general considered.

3.1 Theories underlying the fundamental to be compared with stock prices

The discount dividend model (DDM) relies on the idea that a company has to generate earnings in order to expand in the future. As a result, it postulates that the stock price is equal to the sum of all discounted dividends generated by the stock. The Q-theory argues that the equity price of a company should reflect its balance sheet value.

The observed price may then differ, sometimes significantly, from the equilibrium price constructed by applying the hypotheses of the underlying theory. In that case, we can conclude that either the observed price is not justified in the light of the theory, and that a correction is inevitable, or that the hypotheses used for the valuation are inappropriate (Wibaut, 2000). Since we consider that stock prices are mainly driven by the fluctuations of fundamentals, this implies that any significant gap between the current stock prices and its theoretical value (considered as the equilibrium value of equity) has to be viewed as a signal of a future correction in prices.

3.1.1 Discount Dividend Model

A framework for predicting the stock prices is given by the DDM, which relates the current stock prices to the expected discounted dividend in the long run. According to this, the theoretical value of a stock should therefore reflect the discounted sum of the future cash flows associated with the investment. These cash flows consist of dividends (i.e. the distributed part of earnings) and capital gains.

Algebraically, the present value of the stock price may thus be represented by the following equation:
where \( P \) is the current stock price, \( D^e \) is the expected dividend (per share), \( P^e \) is the expected stock price and \( h \) is the discount rate that corresponds here to a constant return for an investment in stocks covering the period from \( i \) to \( N \). However, expecting stocks returns for long time spans is not an easy task, which is the biggest disadvantage of this framework.

Nevertheless, in the late 1950s, this framework has received renewed interest thanks to the restrictions proposed by two economists. By postulating perfect competition and perfect substitutability between the various means of holding wealth, Gordon and Shapiro (1956) show that theoretical stock prices may be reduced to the following expression, which is called the "Gordon-Shapiro formula":

\[
P_t = \frac{D_t(1 + g)}{h - g}
\]  

(3)

where \( D \) is the current dividend (per share) and \( g \) is the assumed constant rate of growth of expected dividends.

This model makes the stock price extremely sensitive to a permanent change in \( h \). The discount factor \( h \) represents the opportunity cost of the investment under consideration. Moreover, the stock return, \( h \), can be broken down as the return on a risk-free asset, \( r \) (which is often represented by the return yielded on a bond issued by a State with a good credit rating), and an equity risk premium related to the features of this type of asset, \( \sigma \).

Therefore, equation 3 becomes:

\[
P_t = \frac{D_t(1 + g)}{r + \sigma - g}
\]  

(4)

By rearranging the model, it is possible to obtain another expression of equation (4): the dividend yield, \( DY \) defined as the ratio of the dividend per share and the stock price:

\[
DY_t = \frac{D_t}{P_t} = \frac{r + \sigma - g}{1 + g}
\]  

(4')

---

16 A detailed description of the links between the simple net return on a stock and equation (3) is given in Annex 2.

17 Note that equation (3) holds if, and only if, \( h > g \).
Moreover, when assuming that a constant fraction $\delta$ of earnings (per share) $E_t$ is paid out as dividends, i.e. $D_t = \delta E_t$, we can obtain from equation (4) the price-earnings ratio (or P/E); the ratio of the stock price and the earnings per share:

$$\frac{P}{E_t} = \frac{\delta (1 + g)}{r + \sigma - g}$$

where the term $\delta$ is known as the payout ratio. Since dividends are the distributed part of the earnings, Gordon-Shapiro's theory amounts to stating that, in the long term, stock prices must move in line with future earnings anticipated by the agents.$^{18}$

In the remainder of the paper, we shall focus on the P/E as the DY proved to be a poor valuation indicator for the following reasons.

The DY can give correct signals of over- or under-valuation of stock prices when comparing its current value to its historical average if and only if the pay-out ratio is constant over time. If this is not the case, in practice, investors can still get decent returns with a low DY, as long as they get higher capital appreciation. Therefore, the DY indicator will be biased. As shown in Chart 15, the pay-out ratio clearly points out a decreasing trend over time. In the latter half of the twentieth century, companies paid out lower dividends in relation to their earnings than in the past (Smithers and Wright, 2001, Brav et al., 2003); undistributed earnings increase companies’ net worth and, hence, their ability to pay additional dividends in the future. Since value is made up of both current dividends and the capacity for future growth, low DYs are not necessarily a sign of an overvalued stock market.

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$^{18}$ Using this theoretical framework, many other specifications (or simplifications) have been proposed in empirical literature. For example, one model postulates that stock price fluctuations are mainly explained by movements in the difference between the earnings yield and the long-run interest rate. This model described in Lander et al. (1997) is often called the Fed model. The authors use a simple error correction model that predicts the return of the S&P based on the deviations from a presumed equilibrium between forecasted earnings yield and yields on bonds. Wetherilt and Weeken (2002) show how this alternative model is actually an extension of Gordon-Shapiro's theory.
Another disadvantage of the DY stems from the fact that dividends are not the only future cash flows\textsuperscript{19}. For example, these cash flows have increasingly been distributed in the form of share buy-backs, especially in recent years\textsuperscript{20} (Brav et al., 2003). If a large part of cash flows is distributed to shareholders otherwise than via dividends, this makes the DY less useful as a summary statistic for valuations.

### 3.1.2 Q-theory

Average Q, formalised by Tobin (1969), is intended to capture the entire market value of the company (including the market value of debt) and compare it with total assets. It can be expressed as follows:

\textsuperscript{19} This point had already been made by Miller and Modigliani (1961) in their seminal paper on stock valuation, which states that the crucial series that markets should be valuing is the total flow of cash between companies and shareholders, not simply dividends.

\textsuperscript{20} The most striking shift happened in the 1980s and 1990s, when significant rates of share buy-backs implied that net new issues were negative (Wright, 2002).
\[
\text{Average } Q = \frac{\text{Market value of Equities + Liabilities}}{\text{Total assets}}
\]  
(5)

which is basically\(^{21}\) the ratio of the discounted value of a company's future earnings to the replacement cost of its \textit{total} capital stock\(^{22}\).

By expressing average \(Q\) as the sum of the expectations on future changes in the capital stock, the rate of profitability (as captured by the ratio between dividends and capital), and future returns, Robertson and Wright (2002) showed that there is barely any evidence that average \(Q\) predicts investment, some evidence that it predicts future profitability, but strong evidence that it predicts future returns.

3.2 Data

The subsection is devoted to the description of the dataset used throughout the rest of the paper. In particular, we look at data on earnings and on \(Q\).

3.2.1 Earnings

There are various ways of measuring earnings and most of them are subject to biases. Two major sources of differences in estimating earnings can be considered: differences in accounting concepts and differences in the estimation period.

As regard differences in the accounting concepts, as reported earnings are officially communicated to the Securities and Exchange Commission and audited in accordance with the Generally Accepted Accounting Principles. They are more reliable than unaudited earnings such as operating earnings and pro forma earnings (S&P, 2002). It is no surprise that operating earnings constantly exceeded as reported earnings, so that P/E based on operating earnings appeared lower and thus less worrying in recent years than P/E based on as reported earnings. As with as reported earnings, some differences appear, depending on the data providers.

As regards differences in the estimation period, some authors resort to anticipated earnings while others use observed earnings.

\(^{21}\) If average \(Q\) exceeds one, then the company can increase earnings by investing.

\(^{22}\) Fixed capital is generally evaluated at the market price (financial concept) which can differ from the accounting concept.
Among the latter category of authors, some make their analysis on the basis of earnings for the last year, others on the basis of smoothed earnings, i.e. earnings calculated on the basis of an average of observations over the last ten years. This version has the advantage of smoothing out fluctuations in earnings due to temporary events and business cycles (Shen, 2000). Earnings tend to be cyclical, but a ten-year period does not necessarily coincide with the fluctuating duration of cycles. Moreover, as real earnings tend to increase over time, average earnings over ten years tend to be lower than the earnings of the most recent year, so that P/Es calculated on an average of earnings are usually higher than P/Es calculated on last year’s earnings.

Authors of the former category (Lander et al., 1997) suggest adopting forward earnings. The advantage of forward earnings is that these series remain closest to the behaviour of investors who work out their investment strategy on the basis of expectations concerning the future behaviour of the markets. However, these forward earnings are supposed to anticipate operating earnings (which, themselves, are not fully reliable; see supra) and they tend to be upward biased$^{23}$ (even when compared to last year's operating earnings). Forward P/Es therefore tend to give the false impression that stock prices are cheaper than they really are.

The earnings per share ($E^{24}$) used in the paper are, according to Shiller (2000, op. cit.), the observed four-quarters trailing$^{25}$ as reported earnings published by S&P. This data is available on Shiller's website$^{26}$.

$^{23}$ Wadhwani (1999) and Panigirtzoglou and Scammel (2002).
$^{24}$ In the rest of the paper, E shall stand for earnings per share.
$^{25}$ For the months of March, June, September and December, E is the sum of the earnings per share of the last four quarters. The E data for the other months is obtained by linear interpolation.
$^{26}$ See footnote 5.
3.2.2 Q

Recourse to average Q as a predictor of future stock price movements has been recently underlined by Smithers and Wright (2001). These authors propose another measure of Q which they labelled as "equity Q", defined as:

\[
\text{Equity Q} = \frac{\text{Market Value of Equities}}{\text{Net Worth}}
\]  

(6)

This definition is in line with the ratio published regularly in the company balance sheets produced alongside the Federal Reserve's flow of funds accounts. These figures can be found on an annual and a quarterly basis for all quoted and unquoted non-financial companies.

Equity Q has the same properties, with respect to the impact of changes in each of its components, as average Q. Thus, a rise in the value of debt increases both ratios, other things being equal. A value of 1 for Equity Q will also imply a value of 1 in average Q definition. In our further analysis, Equity Q is preferred over average Q because this ratio relaxes the strong assumption according to which the market value of a company is unaffected by its method of financing (Smithers and Wright, 2001). Another advantage of Equity Q is that it can be constructed without recourse to capital stock data at market prices, for which the availability is far from easy (Wright, 2002).

We can also express Equity Q per share, so that equation (6) becomes:

\[
\text{Equity Q} = \frac{P_t}{NW_t}
\]  

(7)

where NW stands for Net Worth per share.

According to the theory, the mean value of Q, if we could measure it properly, should be precisely parity (in the case of perfect competition) or somewhat above (in the case of imperfect competition). As Smithers and Wright (2001) maintain, in a world of imperfect statistics, Equity Q has an average below parity. Lower mean values are explained by Wright (2002) by a systematic tendency to over-estimate the replacement value of the

---

27 This assumption holds only under Miller-Modigliani conditions, in particular the absence of company failures (Wright, 2002).
physical capital stock, on account of the fact that economic depreciation may occur at a more rapid rate than recorded depreciation.

Pickford et al. (2002) constructed a monthly proxy for Q using S&P 500 data. In this paper, this series will be referred to as Q. Contrary to Equity Q, Q series therefore includes financial companies and is limited to the 500 companies comprised in the S&P 500 index. It makes it easy to compare Q with P/E. In building this Q series, Pickford et al. (2002, op. cit.) assume that the mis-measurement of depreciation discussed above has an important impact on attempts to derive the estimated net worth of US quoted companies from their published retained earnings, but that the impact is relatively stable over time. This method accordingly measures net worth taking into account an "average" degree of mis-statement of earnings. Hence, recorded earnings were scaled down by an amount that makes Q equal to 1 on average.

Besides the difficulty of measuring the rate of capital depreciation, Q faces another well-known measurement problem. Indeed, capital stock is not measured directly: only the change in the capital stock can be measured.

In the rest of the paper, we have made use of the Q series available on the Smithers and Co website as of January 2003. The series ends on July 2002 (latest available data) as we were not able to update the dataset built by Smithers’ team.

3.3. Descriptive analysis

3.3.1 An initial look at the valuation of stock prices

We can gain an initial idea of the current (and past) valuation of stock prices by considering the ratios between the current stock price and the value of the corresponding fundamental at a given moment (E or NW, depending on the case), and comparing them with their historical average.

---

28 Pickford et al. (2002, op. cit.) further claim that it will not therefore make any allowance for the exceptional mis-statement of earnings that has occurred in recent years. Details of the method used for building the Q series are given in Pickford et al. (2002).

29 These problems apply for Q data in all countries, even in the United States. Outside the United States, high quality net worth statistics over a long period simply do not yet exist. For the United Kingdom, see Smithers and Wright (2001) and MacGrain and Thompson (2002). For Japan, see Ogawa and Kitasaka (1995) and Smithers and Wright (2001).


31 Another way of approaching stock valuation, in the case of the fundamentals used in Gordon-Shapiro’s formula, is to compare the stock price with the theoretical price resulting from equation 4. This method, presented by Wetherilt and Weeken (2002), among others, requires estimating the theoretical price based on assumptions regarding earnings growth (g) and the equity risk premium (\sigma).
This approach has its disadvantages: the choice of the historical benchmark is actually not uncontroversial. Some commentators have argued that historical relationships may have broken down and that long-term averages - such as those calculated by Shiller since 1871 - may therefore no longer be appropriate benchmarks (Wetherilt and Weeken, 2002, op. cit.).

For practical reasons, the aforementioned approach has been used in the current section. We consider either upward or downward deviations from the average as significant if they exceed 25 p.c. in absolute value.

a. Price-earnings

Over the period 1871–2002 as a whole, the P/E calculated for stocks in the S&P 500 averaged 13.8.

Chart 9 - Price-earnings ratio on the S&P 500: historical view

Sources: Shiller’s website (see footnote 5), S&P, Thomson Financial Datastream; own calculations. The black thick horizontal lines represent a fluctuation margin of ± 25 p.c.

Between 1871 and 1995, whenever the P/E significantly exceeded its historical average (e.g., when it was higher than 20), corrections occurred sooner or later; on each occasion,
not only did the P/E revert to its mean, but an over-reaction ensued, with the P/E then remaining below its historical value. Similarly, when the P/E fell substantially short of its historical average (e.g. by dropping below 10), it returned sooner or later to its long-term average, then over-reacted, sometimes overshooting the average to a significant degree.

Since 1995, the P/E has reached historical peaks, with prices sometimes reaching 47 times E in March 2002. The P/E subsequently dropped back to around 29 in December so that the extent of the overvaluation, as measured on the basis of a historical P/E, scaled back to 105 p.c. This value nevertheless surpasses anything seen in the 1871-1995 period. When Alan Greenspan\(^{32}\) issued his warning about the "irrational exuberance" of the stock markets in December 1996, prices stood at just 19 times E.

b. \(Q\)

\(Q\) is shown for the period January 1872 - July 2002 period in chart 10. The historical average is slightly\(^{33}\) above 1. \(Q\) reached a peak in December 1999 which is far beyond the 25 p.c. upward bound, and a sharp correction followed. In July 2002, the extent of overvaluation as measured on the basis of \(Q\), came back to around 25 p.c. above its mean.

**Chart 10 - \(Q\): historical view**

![Chart 10](chart10.png)

Source: Smithers and Co website (see footnote 30); own calculations.
The black thick horizontal lines represent a fluctuation margin of ± 25 p.c.

\(^{32}\) Greenspan (1996).
\(^{33}\) The average is above 1, probably because the scaling down process took place over a slightly shorter period.
c. First conclusions on stock market valuation

In the 1990s, whatever the fundamental considered, both valuation ratios moved substantially away from their long-term averages, and such deviations had never been that large in the past. This prompted some commentators to suggest that equity prices could not depart for much longer from their historical relationships with the fundamentals and therefore needed to fall.

In December 2002, the P/E still showed a significant overvaluation of equity prices when compared to their historical average, but since July 2002 the overvaluation was not that important in the case of Q.

The evidence is even more mixed if, for each valuation indicator, we compare the latest data (those from December 2002) with their average over the last ten years instead of their historical average. We might then conclude either that the stocks are no longer overvalued (on the basis of Q) or that, though still overvalued, they are much less overvalued (on the basis of P/E).

Table 2 - Sensitivity of the valuation indicators to the time horizon chosen for the average

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Value in December 2002</th>
<th>Average 1871-2002</th>
<th>Average 1993-2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/E</td>
<td>28.5</td>
<td>13.8</td>
<td>24.5</td>
</tr>
<tr>
<td>Q</td>
<td>1.32(^1)</td>
<td>1.05</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Sources: Shiller’s website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datastream; own calculations.

\(^1\) 24 July 2002 (latest data available).

In the Gordon-Shapiro framework, a higher value for the average over the last ten years than for the historical average can be justified on the basis of equations (4\(^"\)) (section 3.1.1) by a lower equity risk premium, a lower real risk-free interest rate or a higher growth rate. This third factor relates to New Economy arguments that expectations of higher productivity growth and output growth resulting from the increased usage of ICT supported rapidly rising stock prices in the late 1990s.

With this reservation in mind, we shall keep the 1871-2002 average as the benchmark for the rest of the article.
3.3.2 Do the valuation indicators send the right signal about stock market prices?

We can expect a good valuation indicator to give the right signals about future price movements. In particular, we can hope that it will identify all the major turning points (maxima and minima), but also that it will not give out incorrect signals.

a. Identification of major turning points

Focusing the analysis on major turning points, we investigate whether valuation indicators would have enabled us to spot the "super bear" and bull markets described in section 1.

In order to answer this question, we compare the turning points in the valuation indicators and in the real stock prices. These are shown in chart 11.

The turning points in the real S&P 500 correspond to the starting dates of the bull and super bear markets considered in section 1. They therefore present an alternation of troughs and peaks, delimiting four long cycles.

The turning points in the valuation indicators are defined in relation to the turning points in the real stock price. They are given by the moment when the valuation indicators reach a maximum (or a minimum) in the period comprising the turning point in the real P, during which the valuation indicators constantly deviate from their historical average by more than 25 p.c.\footnote{For example, the turning point for P/E relative to the August 2000 maximum in the real S&P 500 has been defined as follows. First, we select the period during which the P/E exceeded constantly its historical average by 25 p.c. or more. This has been the case between November 1995 and December 2002. Over this period, we then pick up the maximum value for P/E. This has been reached in March 2002, moment when it was worth 3.4 times its historical average. Thus, this turning point for P/E lagged 19 months behind the turning point for the real P. That also means that the signal of overvaluation was not yet fully given at the time the correction of the real S&P 500 started.} This procedure identifies a turning point for each valuation indicator next to a turning point in the real P, except for the P/E which could not signal the super bear market starting in 1906\footnote{In September of that year, even the P/E did not show any overvaluation as it was below its historical average. See table 3 below.}. Hence, we could identify eight turning points for the real P and for Q and seven for P/E.
Chart 11 - Real S&P 500 and ratio between the valuation indicators and their historical average

Sources: BLS, Shiller’s website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datasheet; own calculations.

^Ratio between the valuation indicator and its historical average.
### Table 3 - Signals given by valuation indicators around the major turning points in the real S&P 500

<table>
<thead>
<tr>
<th>Stage</th>
<th>Moment</th>
<th>Value of the valuation indicators</th>
<th>Stage</th>
<th>Moment</th>
<th>Value of the valuation indicators</th>
<th>Lead (+) / Lag (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P/E</td>
<td>Q</td>
<td></td>
<td>P/E</td>
<td>Q</td>
</tr>
<tr>
<td>Trough 1</td>
<td>June 1877</td>
<td>0.68</td>
<td>0.48</td>
<td>June 1877</td>
<td>0.68</td>
<td>0.48</td>
</tr>
<tr>
<td>Peak 1</td>
<td>September 1906</td>
<td>0.98</td>
<td>1.65</td>
<td>January 1906</td>
<td>-</td>
<td>1.70</td>
</tr>
<tr>
<td>Trough 2</td>
<td>December 1920</td>
<td>0.62</td>
<td>0.39</td>
<td>December 1920</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Peak 2</td>
<td>September 1929</td>
<td>1.46</td>
<td>1.87</td>
<td>September 1929</td>
<td>1.46</td>
<td>1.87</td>
</tr>
<tr>
<td>Trough 3</td>
<td>June 1932</td>
<td>0.68</td>
<td>0.36</td>
<td>June 1932</td>
<td>0.68</td>
<td>0.36</td>
</tr>
<tr>
<td>Peak 3</td>
<td>December 1968</td>
<td>1.34</td>
<td>1.84</td>
<td>December 1968</td>
<td>1.34</td>
<td>1.93</td>
</tr>
<tr>
<td>Trough 4</td>
<td>July 1982</td>
<td>0.57</td>
<td>0.48</td>
<td>April 1980</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td>Peak 4</td>
<td>August 2000</td>
<td>2.02</td>
<td>2.51</td>
<td>March 2002</td>
<td>3.38</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller's website (see footnote 5), Smithers and Co website (see footnote 30), S&P, own calculations.

1. Ratio between the indicator and its historical average.
2. Lead (+) or lag (-) of the valuation indicator turning points, in months.
Table 3 shows the moment of the turning point in the real S&P 500, the moment of the associated turning point in the valuation indicators, the value of each indicator at their turning point as well as at the time of the turning points in the real S&P 500 and, finally, the leads or lags of the valuation indicator turning points.

Comparing the timing of the turning points between the valuation indicators and the stock prices in real terms, the P/E leads in 2 out of 7 cases and lags in 1 and the differences range from -19 to 36 months, while Q leads in 3 out of 8 cases and lags in 1 and the differences range from -1 to 35 months.

A non-parametric test performed by Banerji, called Randomization test, can be used to check whether turning points in the valuation indicators lead turning points in the real P and, more importantly, whether these leads are statistically significant. Because of the missing data for P/E in relation to the September 1906 peak in stock price, the test has been applied only for Q.

In the Randomization test, the null hypothesis that the leads of Q over the real P are not statistically significant is tested against the alternative hypothesis that the leads in Q are significantly greater than zero month. The results display a confidence level of 87.5 p.c. for the rejection of the null hypothesis for the whole 1871-2002 period which comprises the eight turning points.

On the basis of this result, if we accept that the leads of Q over the real P are significantly greater than zero month, it might be interesting to find out how much greater than zero month the leads are likely to be. The Randomization test can therefore be conducted for increasing lead span, giving the confidence level at which the null hypothesis is rejected in favour of the alternative hypothesis that the leads at turns significantly exceed one, two, .... months. The full set of confidence levels results in a "lead profile", to use Banerji's terminology.

---

36 Whereas the super bear market starting in September 1906 was signaled by Q with a lead of eight months, P/E did not show any sign of a major overvaluation in September 1906 or even before. Some years beforehand, we could have detected overvaluation of the real S&P 500 on the basis of P/E but this signal should have disappeared as this valuation indicator was below even its historical average when the real P reached their high.

37 See Banerji (2000) for a full description of the test and for a first application to evaluate survey series as leading indicators at business cycle troughs. Also see Van Haelen et al. (2000) for a first application of the test inside the National Bank of Belgium and an extension to peaks and troughs.
The lead profile of Q on real P is depicted in chart 12. The main results are that the null hypothesis of a lead of at most 1 month can be rejected at a confidence level of 87.5 p.c. and that the confidence level decreases with the duration of the leads.

**Chart 12 - Lead profile of Q in relation to the real stock price**

(per cent¹)

![Chart showing lead profile](image)

Sources: BLS, Shiller's website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datastream; own calculations.

¹ The chart shows the confidence level for different durations of leads, i.e. the probability that the null hypothesis of no lead can be rejected for the indicated number of months.

Another key feature expected from a useful valuation indicator is that it should at least point out that the stocks are overvalued at the point when they attained their maximum value. Except for the above mentioned 1906 period for P/E, in all other cases the valuation indicators were more than 25 p.c. above their historical average at the time when P reached its turning point in real terms. Thus, without giving an exact indication of the moment when the bubbles would reach their maximum, the indicators usually signalled that stocks were clearly overvalued, so that the ensuing severe corrections appeared justified in terms of stock market valuation. All the indicators show that the 2000 bubble is considerably larger than the ones which caused the preceding super bear markets.

b. False signals

Some valuation indicators not only failed to warn the markets in time, but also alerted them unnecessarily. On a number of occasions, P/E has reached extremely high values (more
than 25 p.c. above the historical average) without any subsequent significant market correction. This was the case in December 1921, in July 1933 and, to some extent, in January 1992 (see chart 11). On each occasion, these movements can be explained by the extreme volatility of E. The primary, but by no means the only, reason why E are volatile is that they are highly dependent on the state of the economy. When the economy goes into recession, E tend to fall more than proportionally, so that P/E might rise rather than fall, whereas the other valuation indicators either fall or rise at a slower pace. In December 1921 and in January 1992, investors who refrained from buying a basket of American stocks would have missed most of the 1920s and 1990s bull markets respectively.

Generally speaking, Q appears to have given the right signals. However, contrary to P/E, it failed to anticipate the 1987 crash. Since that crash was short-lived, some people might regard this as a sign of reliability rather than weakness.

3.3.3 Are misalignments in the valuation indicators corrected by stock price changes?

Another descriptive analysis can be performed at this stage. It aims to check whether any misalignment of a valuation indicator in relation to its historical average tends to be corrected via changes in price (P) or via changes in fundamentals\(^{38}\) (F). These changes are considered in real terms. We therefore examine the respective contributions of the real P and the real F to the extreme variations in the valuation indicators. An extreme variation means a variation over a period which may vary in length, linking a low point to a high point, and vice versa; in addition, these high and low points have to deviate by more than 25 p.c. from the historical average.

First, we shall identify the extremes (turning points) and hence the extreme variations to be taken into consideration for both indicators (P/E and Q). Next, we shall use charts, in particular, to check whether the upward or downward movements in these indicators can be explained by either variations in real P or variations in real F or both.

Using the ratios between each valuation indicator under review and its corresponding historical average, illustrated in chart 11, we can identify some extremes and link them together (chart 13). The selection of the valuation indicators turning points differs from that

\(^{38}\) Fundamentals are E and NW. They are expressed per share.
of the preceding section. Here, criteria are applied to define the best turning points\textsuperscript{39} for the valuation indicators whereas, in the preceding section, we tried to find turning points in relation to turning points in the real $P$ identified in a first step.

**Chart 13 - Extreme values of the ratios between the valuation indicators\textsuperscript{1} and their historical average**

![Chart 13](chart13.png)

Sources: Shiller's website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datastream; own calculations.

Chart 13 teaches us a number of things.

- We find more extremes for $P/E$ (17) than for $Q$ (11). This difference reflects the greater volatility of $P/E$.
- This alternation of more frequent extremes in the case of $P/E$ is valid only for the period from December 1917 to June 1946; period which also featured a greater frequency of extremes for $Q$.
- With the exception of September 1929 and June 1932 when both indicators recorded an extreme high and an extreme low point respectively, the timing of the extremes differs from one indicator to another. However, except for the period from 1917 to 1946, the general trends were the same for both indicators.

\textsuperscript{39} We impose the alternation of extremes marking overvaluation and undervaluation. As a first step in the selection of extremes, we did not impose such alternation. In this case, it appeared that for each valuation indicator, there were several successive extreme high points causing the valuation indicators to deviate upwards by more than 25 p.c. When the alternation of extremes is imposed, we take only the one with the highest value.
By pinpointing the extremes in the chart, we can define some episodes. We shall consider an episode as comprising successive extreme upward and downward movements in the indicators, thus linking one low point with the next. For example, in the case of the P/E, the first episode covers the period from November 1873 to December 1917, during which – starting from a low point – the P/E reached a peak (in December 1894) before dropping back to another low point. We thus identify eight episodes in the case of P/E and five for Q.

The episodes, explained in the tables in Annex 3, are represented in chart 14 by an arrow linking two points. They consist of upward movements (grey dots) followed by downward movements in the valuation indicators (black dots), a small black circle symbolising the latest downward movement. For a given extreme rise or fall in a valuation indicator, each dot gives the variation in the real P (vertical axis) and the variation in the real F (horizontal axis).

The intersection of the axes enables us to define four quadrants: in the North-West, P is rising in real terms and the real value of F is falling; in the North-East, both the value of P and F are rising in real terms; conversely, both are falling in the South-West; and finally, in the South-East, real P is falling while F is rising in real terms.

The slope of the arrows is a crucial element for the analysis. The more vertical the arrows, the greater the extent to which the extreme variations in the valuation indicators can be attributed to variations in prices (in real terms). Conversely, the more horizontal the arrows, the greater the extent to which the extreme variations in these indicators can be explained by variations in F (in real terms). Finally, an oblique arrow tends to indicate that the variable causing the extreme rise in a valuation indicator contributes only partially to its subsequent correction.

Whatever the valuation indicator considered, there are wide variations in the duration of the episodes and the upward and downward movements which they comprise. For example, in the case of the P/E, disregarding the latest correction, the shortest movement barely lasted 11 months (upward movement in episode 5) and the longest lasted for 23 years (downward movement in episode 1).

The latest episode ended in July 2002 for the Q series.
Comparison of both sides of the chart 14 reveals the following:

- For Q, a rise in the S&P 500 of more than 200 p.c. in real terms has always contributed to extreme rises in this valuation indicator. Similarly, extreme falls in this indicator have always involved a price correction of more than 40 p.c. in real terms.

- Upward movements in the P/E have not always been accompanied by rising P in real terms. Thus, the real value of the stocks increased by less than 100 p.c. during episodes 1, 5 and 6, covering periods from November 1873 to December 1894, and from November 1937 to December 1941, and the period starting on this last date and ending in June 1946. Moreover, stocks actually lost almost 15 p.c. of their real value during the second upward movement in the P/E (South-East quadrant); this happened between December 1917 and December 1921.
Similarly, downward movements in the P/E have not always been accompanied by a price correction. Thus, between December 1921 and October 1923, the drop in the P/E occurred even though P rose by almost 10 p.c. in real terms (North-East quadrant).

Variations in real E have often been much greater than the variations in the real value of NW\textsuperscript{42}.

Falls in NW have rarely contributed to the extreme rises in Q. In fact, during four out of five episodes, Q showed a steep rise despite a rise in the real value of NW. In contrast, in the case of the P/E, a sometimes substantial fall in real E contributed to extreme upward movements in this indicator in six of the eight episodes considered, and E rose by only 4.3 p.c. in real terms during the seventh episode (between June 1949 and December 1961).

In general, a rise in the real value of F contributed to the extreme falls in the valuation indicators. The exceptions comprise a fall of 0.3 p.c. in NW and of 58.2 p.c. in E between September 1929 and June 1932 (respectively, second episode for Q and third episode for P/E).

If we combine the previous two findings, the real value of NW tended to increase steadily, regardless of the movement in the real value of P\textsuperscript{43}. The extreme variations in Q were therefore caused more by extreme variations in P.

The extreme variations in P/E were also influenced by extreme variations in E.

Whatever the valuation indicator considered, the variables behind the rise were usually those which brought about the correction. In the case of Q, this variable was P. In the case of the P/E, E also played a part. When the fall in E contributed to the rise in the P/E, they subsequently always increased, and sometimes to a considerable extent, thus making a sometimes significant contribution to the fall in the P/E. Symmetrically, after the sharp rise in real E between October 1923 and September 1929 (episode 3), E collapsed between September 1929 and June 1932.

The latest extreme episode features the steepest stock price rise in real terms, whatever the valuation indicator considered. That rise was accompanied by a rise in the real value of NW, but a fall in real E. The most recent and most dramatic sub-period of the rise in the P/E, namely 1995-2002 (table 4), comprised two successive moments where the characteristics were very different. Between January 1995 and August 2000, the rise in the P/E (from 15 to 28) was due solely to soaring P, which rose by around 177.7 p.c., while during the same period E increased by 47.8 p.c. Real P and E peaked in August 2000. Between August 2000 and March 2002, a slump in

\textsuperscript{42} The chart on the left is in fact wider than the one on the right.
\textsuperscript{43} Grey and black dots located to the right of the vertical axis.
real E\textsuperscript{44} (-55.1 p.c.) was the only factor contributing to the rise in the P/E (pushing it from 28 to 47). P had in fact already started to fall (-24.9 p.c.).

Table 4 - Changes in P/E over the January 1995-December 2002 period: breakdown according to changes in real P and real E

<table>
<thead>
<tr>
<th>Start</th>
<th>Initial P/E</th>
<th>End</th>
<th>Final P/E</th>
<th>Change in P/E</th>
<th>Change in real P</th>
<th>Change in real E</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1995</td>
<td>14.9</td>
<td>August 2000</td>
<td>28.0</td>
<td>87.9</td>
<td>177.7</td>
<td>47.8</td>
</tr>
<tr>
<td>August 2000</td>
<td>28.0</td>
<td>March 2002</td>
<td>46.7</td>
<td>67.0</td>
<td>-24.9</td>
<td>-55.1</td>
</tr>
<tr>
<td>March 2002</td>
<td>46.7</td>
<td>December 2002</td>
<td>28.5</td>
<td>-39.3</td>
<td>-23.0</td>
<td>26.3</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller's website (see footnote 5), S&P, Thomson Financial Datastream; own calculations.

The latest extreme downward movement in the valuation indicators is due primarily to a correction in real P, but also – in the case of P/E and Q – to a rise in the real value of F. Annex 4 suggests that E could continue to recover, and hence contribute to the still necessary downward adjustment in the P/E.

\textsuperscript{44} In real terms, as reported E showed, between September and March 2002, the third largest fall since 1871 (-55.3 p.c.).
3.4 Empirical analysis

The predictability of stock prices poses the challenge of finding fundamentals for forecasting them. This sub-section shall examine the respective qualities of the two fundamentals considered – earnings and net worth - as suggested by the theories presented in section 3.1.

3.4.1 Univariate and multivariate analyses

The fundamental used for forecasting stock prices must fulfil two criteria. The first is that the fundamental and the stock price must hold a durable relationship over time (mean-reversion criterion). The second criterion requires that the valuation indicator can predict the future movements in stock prices (causality criterion).

a. Mean-reversion

An important criterion for assessing the predictive power of a valuation indicator is that the ratio of price to fundamental (P/F) must mean-revert. This justifies the importance of the historical average and the significance of any misalignment between this ratio and its historical average. If an indicator does not tend to revert to its mean, it is useless to observe that it is deviating from its mean because we shall not be able to state that the ratio shows an over- or undervaluation.

An initial way of looking at the mean-reverting properties of any valuation indicator is to closely track its movement over time and to compare it to its average. A mean-reverting indicator regularly gets pulled back from extreme values - whether high or low - towards its mean. Its average value is an attractor. This means that when it takes a very high value, there is a high probability that it will fall back significantly.

From econometric point of view, the mean-reversion criterion implicitly involves two conditions. First, each variable, i.e. P and F, must be proved to be integrated of the same order, which means they have to display the same degree of non stationarity. Second, a cointegrated relationship must exist between them. This means that whatever the instability of P and F separately, the link between them should be stable. Simply stated, if these variables are cointegrated, then they move together over time. If both tests hold, then any ratio relating both variables will be stationary and considered as a useful indicator.
for predicting future movements in P or in F. Traditionally, these conditions are tested in literature through the unit root tests and the cointegration tests.

For testing unit root process in each variable, Augmented Dickey-Fuller tests (ADF test) were performed whose results are reported in Table 5. The sample period, here as in the following of the sections 3.4.1. and 3.4.2, is from February 1872 to July 2002.

Table 5 - Unit Root Tests based on Augmented Dickey-Fuller Statistics (February 1872 - July 2002)

<table>
<thead>
<tr>
<th>Variables in log</th>
<th>Level</th>
<th>First-difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real P</td>
<td>-1.373</td>
<td>-10.55**</td>
</tr>
<tr>
<td>Real E</td>
<td>-1.972</td>
<td>-9.790**</td>
</tr>
<tr>
<td>Real NW</td>
<td>3.610</td>
<td>-10.69**</td>
</tr>
<tr>
<td>Ratios</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>-3.539*</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>-3.696*</td>
<td>-</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller's website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datastream; own calculations.

Note: Tests performed using ADF tests with a maximum of 12 lags. Critical values are -2.86 and -3.44 at respectively 5 p.c. and 1 p.c. of significance level. * and ** denote rejection of the null hypothesis of a unit root at respectively 5 p.c. and 1 p.c. of significance level.

As suggested by these results, the stock price, and each potential fundamental taken separately, in real terms, contain a unit root in level, i.e. $\rho=1$ in equation (1) in section 2, but are first-difference stationary. From Table 5, we can indeed note that the null hypothesis of unit root is accepted for all the variables in level but strongly rejected when these variables are defined in first-difference. In other words, all these variables taken separately display a $I(1)$ process. Moreover, the existence of a cointegrated relationship between the stock price and each fundamental was tested by using standard Johansen's cointegration tests (Juselius and Johansen, 1990, Johansen, 1992).

The cointegration analysis (not reported here) suggests that there exists one cointegrated vector between the real P and the real E on the one hand and between the real P and the real NW on the other hand. Thus, the relationship between the stock price and each potential fundamental is stable over time, and hence cointegrated. These results are confirmed by

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45 Our dataset covers the period from January 1871 to December 2002 (for the earnings) and from January 1871 to July 2002 (for the net worth). Given that and the number of 12 lags used for the tests, we determined a common sample, i.e. February 1872 to July 2002.

46 Contrary to the unit root tests which use the single equation framework, the Johansen's tests use the vector error correction model (VECM).

47 Standard lag order testing procedures suggest a VAR representation with two lags, i.e. VAR(2), for the model using as fundamental E in real terms, but a VAR representation with five lags, i.e. VAR(5), for the model based on NW in real terms, see Annex 5.
the results of the unit root tests for each valuation indicator (Table 5), which suggest stationarity.

Therefore, at this stage both potential models - P/E and Q - could be considered as equivalent from the statistical point of view in the sense that they both respect the mean-reversion criterion.

b. Causality

Given this mean-reversion criterion, economic theory could indeed predict that a valuation indicator will mean-revert at some point, but not the path or the channel through which the correction will appear. Therefore, to be useful, a valuation indicator should be able to say something about the future movements in stock prices. In that sense, mean reversion is a necessary but not sufficient condition. Additionally, this indicator must be a leading indicator of changes in the level of the stock market. A high value of P/F should indicate not just that the ratio is likely to fall, but that this will come about via stock price changes.

In order to check this assumption, we follow a simple methodology\textsuperscript{48}. Given the existence of the cointegration relationship, a variable of each cointegrated vector at least has to adjust to go back to the long term equilibrium following a specific shock, as suggested by the Granger theorem\textsuperscript{49}.

According to this, if the long term equilibrium is restored through a stock price correction, this implies that the cointegration residual has forecasting power for the short-term variation of P. The results for both models are presented in Tables 6 and 7.

\textsuperscript{48} A similar exercise is applied to the estimation of American and Belgian private consumption functions in respectively Lettau et al. (2001) and Eugène et al. (2003).
\textsuperscript{49} See Granger et al. (1974) and Enders (1995), chapter 6.
### Table 6 - Estimation of the impact of the cointegration residuals on short term variations of \( P \) and \( E \) for several horizons

<table>
<thead>
<tr>
<th>Number of months ((T))</th>
<th>( \sum_{t=1}^{T} \Delta \ln (P_{t,T}) )</th>
<th>Estimated coefficient ((t-Stat))</th>
<th>( R^2 )</th>
<th>( \sum_{t=1}^{T} \Delta \ln (E_{t,T}) )</th>
<th>Estimated coefficient ((t-Stat))</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.001 (-0.47)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.005 (1.15)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>-0.010 (-0.98)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.023 (1.78)</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>-0.018 (-0.94)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.062 (2.42)</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>-0.053 (-1.47)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.150 (3.25)</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>24</td>
<td>-0.101 (-1.83)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.261 (3.79)</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>36</td>
<td>-0.113 (-1.56)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.363 (4.13)</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller’s website (see footnote 5), S&P, Thomson Financial Datastream; own calculations.

Note: Ordinary least squares estimation. The t-Stat between parentheses is based on corrected standard deviations following Newey and West (1987) methodology. Significant coefficients of 5 p.c. are in bold.

Concerning the \( P/E \) model, it appears that the cointegration residual has no forecasting power for the \( P \) growth rate since the estimated coefficient is not significantly different from zero and the adjusted \( R^2 \) statistics is close to zero. At most, the results from Table 6 suggest that the cointegration residual of the \( P/E \) model can predict the growth rate of \( E \) from a 3-month horizon onwards. This means that the value of \( P/E \) is also influenced by variations of \( E \), which confirms the results presented in chart 14.
Table 7 - Estimation of the impact of the cointegration residuals on short term variations of P and NW for several horizons

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\sum_{t=1}^{T} \Delta \ln (P_{t,T})$</th>
<th>$\sum_{t=1}^{T} \Delta \ln (NW_{t,T})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of months (T)</td>
<td>Estimated coefficient (t-Stat)</td>
<td>$R^2$</td>
</tr>
<tr>
<td>1</td>
<td>-0.005 (-1.60)</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>-0.022 (-2.29)</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>-0.047 (-2.72)</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>-0.112 (-3.82)</td>
<td>0.04</td>
</tr>
<tr>
<td>24</td>
<td>-0.214 (-5.04)</td>
<td>0.09</td>
</tr>
<tr>
<td>36</td>
<td>-0.291 (-5.64)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller's website (see footnote 5), Smithers and Co website (see footnote 30), S&P, Thomson Financial Datastream; own calculations.

Note: Ordinary least squares estimation. The t-Stat between parentheses is based on corrected standard deviations following Newey and West (1987) methodology. Significant coefficients of 5 p.c. are in bold.

On the other hand, the results from Table 7 suggest a better performance for the Q model. The cointegration residual does indeed have forecasting power for the growth rate of P from a 3-month horizon onwards, which increases over the time span. The cointegration residual has no forecasting power for the NW growth rate, except maybe for the 2- and 3-year horizons.

It thus seems that the relationship between stock prices and each kind of indicator is not straightforward and clear-cut since the stock price and the fundamental influence each other. That is especially true for the P/E model. Indeed, the expected causality relationship is only checked for the Q model while, for the P/E model, the stock price and the earnings influence each other. This points out, for example, that the sole use of the P/E indicator for forecasting the stock price may be questionable, especially in a univariate framework. Therefore, we estimate the stock price in a multivariate model\(^{50}\).

\(^{50}\) The weak exogeneity test suggests similar conclusions.
3.4.2 The models

Two distinct models are considered: the P/E model and the Q model.

For the estimation exercise, we choose a vector error correction model (VECM) for estimating and simulating the stock price for two specific periods. The intuition behind - and the general features of - the VECM framework are discussed in more details in Annex 5.

Each model consists of a system of two equations where each dependent variable is a function of its past own values, of the past values of the other variable and of the corresponding cointegrating vector. Both equations have the same structure of lags. In other words:

- In the P/E model, the dependent variable for the first equation is the real P, which is a function of its past values and of the past values of E, plus a constant term and a shock. The dependent variable for the second equation is E in real terms, which is a function of its past values and of the past values of P plus a constant term and a shock. For the P/E model, the number of lags chosen is 12 months;
- In the Q model, the dependent variable for the first equation is the real P, which is a function of its past values and of the past values of NW in real terms, plus a constant term and a shock. The dependent variable for the second equation is the NW, which is a function of the past values of NW and of the past values of P, plus a constant term and a shock. For the Q model, the number of lags chosen is 5 months.

3.4.3 Simulation exercises

It must be understand that the simulation exercise presented here is an illustration and, given the simplified version of each model, does not have to be considered as a precise forecasting exercise.

According to the specification of the bubbles discussed in section 1 (see chart 7), two periods have been chosen for comparing the behaviour of the stock price expected by each model and the realised one. The first period starts from November 1927 to December

51 The number of lags was chosen in order to ensure that each VECM model is not misspecified. In this section, a minimum number of lags in the VECM was first chosen to avoid autocorrelation and afterwards to minimise the AIC and HQ information criteria.
1932, so that it contains the "1929 bubble". The second period starts from January 1995 to December 2002, so that it contains the "2000 bubble". The simulation exercise is structured as follows: first, each model is estimated up to the period prior to the sharp fluctuations in the stock price and, second, based on this, dynamic forecasts are generated.

The charts below thus display the forecasts of each model from the beginning of the projection. For the "1929 bubble", the end of the sample period for the estimation is October 1927, while for the "2000 bubble" it is December 1994. The black thick line corresponds to the simulated values and the thin grey curve corresponds to the observed data. Finally, the dotted lines, called the confidence intervals, capture the uncertainty of the forecasts.

At this stage, it appears quite important to clarify how well to interpret the charts below. As previously mentioned, the forecasts are based on VECM estimations for the P/E and Q models. The black thick line therefore reflects both the level of the equilibrium value of the stock price (taking into account the long-term relationship between the stock price and the fundamental) and the short-run dynamics of the model that may temporarily affect this long-run relationship.

The central projection indicates the expected movement in the stock prices if and only if these prices behaved as suggested by each theory. As a result, the values of the variation are in no case a guarantee of the future movement in stock prices in the reality.

Furthermore, the confidence intervals represent the possible extreme values of forecasts given their standard deviation, also depending on the volatility of each model during the estimation period.

Given the speculative element of the sharp movements of the stock price in both periods, it appears normal that the observed stock price diverges durably from the forecasted stock price. In this case, the observed stock price will be considered as over- or undervalued if it diverges from its simulated value. If the observed stock price differs from the simulated value but still lies within the confidence intervals, this could be explained by the short-term behaviour of investors which was already observed in the past. However, if the stock price

---

52 They are based on the variance of the residuals of each model. It is also important to note that, given the high volatility of the observed data during the forecasting period, the black thick line, representing the forecast, may appear horizontal, which does not mean a constant value for the forecasts.
crosses the intervals of confidence, this then means it not only appears completely different from what may be justified by the fundamentals, but it also cannot be explained anymore given all the movements observed in the past\textsuperscript{53}. On this point, the confidence intervals play an interesting role since, by reflecting the uncertainty of the forecasts, they give an appreciation of what appears possible in the movement of stock prices given their whole history, i.e. during the sample period of the estimation. It is worth noting that the value of the confidence intervals are strongly dependent on the time span of the simulation, so that their respective values in the 1929 and 2000 bubbles are not comparable. Indeed, to be consistent with the discussion in section 1, the simulation period counts 6 years for the 1929 bubble and 8 years in the 2000 bubble.

\begin{enumerate}
\item The 1929 bubble
\end{enumerate}

As for the simulated value, both models expect a decrease of the stock price around the beginning of 1928, followed thereafter by a smooth recovery (charts 15 and 16). The average expected increase in the stock price for the period from October 1927 to June 1932 as a whole is, in real terms, 1 p.c. a year for the Q model and 2 p.c. for the P/E model. For the whole period, this represents an increase of between 5 p.c. (for Q) and 10 p.c. (for P/E).

In June 1932, the upper band and the lower band of the forecasts for the stock price lie for example between 0.8 and -0.6 for the P/E model (for a forecast of 0.1 on average) and between 0.5 and -0.5 for the Q model (for a forecast of 0.5 on average). In other words, between October 1927 and June 1932, the confidence intervals for the P/E model point out that the maximum highest increase of the stock price could have been 60 p.c. or the whole period, i.e. more than 10 p.c. a year, while the maximum decrease could be 55 p.c., i.e. 10 p.c. on average. For the Q model, the confidence intervals suggest a maximum increase of 50 p.c. and a maximum decrease of 45 p.c. for the whole period\textsuperscript{54}. The wider confidence intervals for the P/E model confirm the results of the causality test in section 3.4.1, which are explained by a higher volatility of earnings.

\textsuperscript{53} In this sense, the evolution of the observed stock price outside the confidence intervals reflects more a signal of expected correction than a forecast error.

\textsuperscript{54} These confidence intervals may appear very large but the point here is not to precisely forecast the stock price. Indeed, the point in this section is more to analyse the behaviour of the observed data with respect to the central projection, which represents the equilibrium value of the stock price compatible with the fundamental, and the confidence intervals, which give what are the possible values for stock prices given their volatility during the estimation period.
Chart 15 - P/E Model: Simulation of the real stock price from October 1927 to June 1932 (logarithms)

Source: Shiller’s website (see footnote 5); own calculations.
Note: The Y-axis of the chart represents the logarithms of the S&P 500 index in level in real terms (base December 1927).

Chart 16 - Q Model: Simulation of the real stock price from October 1927 to June 1932 (logarithms)

Sources: Shiller’s website (see footnote 5), Smithers and Co website (see footnote 30); own calculations.
Note: The Y-axis of the chart represents the logarithms of the S&P 500 index in level in real terms (base December 1927).
However, as suggested by charts 15-16, the reality has been quite different. Indeed, the stock price was rising rapidly for exceeding the upper band of the confidence intervals. The Q model was also more leading than the P/E in terms of signalling. As shown by Chart 16, the observed stock price exceeded the confidence intervals of Q model in December 1928, i.e. eight months earlier than P/E model (August 1929). On the other hand, the sharp undervaluation was signalled by both models at around the same date in early 1932.

b. The 2000 bubble

As regards the simulated value, both models suggest an average increase around 1.5 p.c. a year in real terms for the S&P 500, i.e. about 10 p.c. for the whole simulation period. Here again, the confidence intervals are larger for the P/E model than for the Q model. Indeed, for the P/E model, the upper band points out a maximum increase of 14 p.c. a year while the lower band suggests a maximum decrease of 9 p.c. a year. For the Q model, the uncertainty is more limited since the upper and lower bands suggest respectively a maximum increase of 8 p.c. a year and a maximum decrease of 6 p.c. for the whole projection period.

Our previous warning concerning the meaning of the central projection is particularly true for the 2000 bubble. Indeed, the current simulation exercise assumes that the P/E and Q models are the right models for explaining the behaviour of stock prices up to 1995. If that were the case, the investor would have been aware of the overvaluation earlier than 2000. Obviously, it has not been the case.

As reported by charts 17 and 18, the significant crossing of the confidence intervals by the observed stock price took place in May 1997 for Q model, i.e. two months earlier than for P/E model (July 1997). Furthermore, the time span for the stock price outside the confidence intervals is much longer for the Q model than for the P/E model. The decreasing trend in stock prices since 2000 puts them within the confidence intervals from February 2001 while the observed stock price came within the confidence intervals of the Q model a year later in March 2002.

55 Starting the simulation exercise four years later, i.e. in 1999, the confidence intervals become wider so that the stock price still stand in between. Nevertheless, the standard misspecification tests deteriorate strongly and the estimated coefficients become unstable.
Chart 17 - P/E Model: Simulation of the real stock price from January 1995 to December 2002 (logarithms)

Sources: BLS, Shiller’s website (see footnote 5), S&P, Thomson Financial Datastream; own calculations.
Note: The Y-axis of the chart represents the logarithms of the S&P 500 index in level in real terms (base December 1927).

Chart 18 - Q Model: Simulation of the real stock price from January 1995 to December 2002 (logarithms)

Sources: BLS, Shiller’s website (see footnote 5), Smithers and Co website (see footnote 30), Thomson Financial Datastream; own calculations.
Note: The Y-axis of the chart represents the logarithms of the S&P 500 index in level in real terms (base December 1927).
The temporary recovery of the stock price after the terrorist attacks in 2001 still appear as "deeply irrational" for Q model, i.e. still outside the intervals. However, for the P/E model, this recovery appeared within the confidence intervals, so that it could be explained by the volatility of the stock price observed previously during the estimation period. Simply stated, this recovery could be considered as possible for the P/E model while it was not for the Q model\textsuperscript{56}.

\textsuperscript{56} Note that the correction observed in stock markets is not straightforward in general. During a correction episode, the stock price fluctuates around a decreasing trend given short-term variations.
CONCLUSION

This paper gives an overview of some issues related to stock market valuation in the United States, focusing on the developments on the New York equity markets.

The impetus for writing the paper was given by the sharp correction of the main New York stock price indices. Thus, S&P 500 lost 42.2 p.c. of its real value between August 2000 and December 2002 (cut-off date). The aim of the paper is twofold: placing this movement in a very long term perspective and, based on history, discussing whether, despite this sharp correction already observed, the S&P 500 is still overvalued.

This correction followed a particularly long and impressive in extent bull market. Up until now, it has also been less pronounced and less long-lasting than other episodes of super bear markets, the most important of which had been the one that began in September 1929 and ended in November 1958, moment when the real S&P 500 reattained its previous peak value.

A valuation of stock markets can usually be expressed in terms of a ratio of the (stock) price to a fundamental. The fundamentals considered here are, according to the discount dividend model, annual earnings and, according to Q-theory, net worth. Our analysis has therefore been focused on two valuation indicators, namely Price-earnings (P/E) and Q.

Deviations of the valuation indicators from their historical averages show overvaluation of the stock index when positive and vice versa. In the 1990s, both valuation indicators moved substantially away from their long-term averages, and such upward deviations had never been that large in the past. This led to the bear market which started in September 2000. In December 2002, the P/E still showed a significant overvaluation of equity prices when compared to their historical average but, since July 2002, the overvaluation was not significant in the case of Q. The evidence is even more mixed when the comparison is made, for each valuation indicator, with their average over the last 10 years, raising the issue of their “fair value”.

On average, over the whole period, Q led the real stock price by one month at major turns and, in most cases, both valuation indicators deviated by more than 25 p.c. from their historical average at the time when stock prices reached their major turning points.
However, some valuation indicators, most frequently the P/E due to the volatility of earnings, alerted the markets unnecessarily.

Whatever the valuation indicator considered, the variables behind the extreme rises in the indicator were usually those which brought about their correction. In the case of Q, this variable was the stock price. In the case of the P/E, earnings also played a part. The latest extreme downward movement in the valuation indicators is due primarily to a correction in real stock prices but also to a rise in the real value of the fundamental. At the end of 2002, the earnings were expected to keep on rising and, hence, to contribute to the still necessary downward adjustment of the P/E.

Simulations based on VAR models for P/E and Q were carried out to check whether, on two occasions, the real S&P 500 climbed to a level perceived as irrational given past experience and theoretical models considered, implying that a correction had to be expected. These occasions were the so-called 1929 and 2000 bubbles.

The models showed that, at some point in time before the peak in (real) stock prices was reached, the real S&P 500 exceeded the upper band of the 95 p.c. confidence intervals during both periods. For each period, Q model showed earlier and more persistent signals of significant overvaluation of stock prices than the P/E model. Finally, in December 2002, both models indicated that the stock price had come back largely within the confidence interval.
Annex 1  Testing the random walk hypothesis

As discussed in section 2, a variable has to fulfil two conditions to be considered as following a random walk. The first and most important condition, called non-autocorrelation, requires that the residuals or increments of equation (1), $\varepsilon_t$, do not depend on their own past values. If this is the case, we shall say the increments are not autocorrelated. The second condition, called homoskedasticity or non-heteroskedasticity, requires a constant variance for the increments, $\varepsilon_t$.

1. Test for autocorrelation

The simplest way to check the first condition for the existence of a random walk process is to estimate equation (1) in first difference and to implement an autocorrelation test on the residuals or increments, $\varepsilon_t$.

In this case, the dependent variable becomes the first-difference of the stock price, $P_t$, which is regressed on a constant and its first-lag. The test consists of measuring the significance of the explanatory variable in the following estimation:

$$\Delta P_t = 0.001 + 0.27 \Delta P_{t-1} + \varepsilon_t$$  \hspace{1cm} (A1)

If $\Delta P_{t-1}$ is significantly different from zero, then the null hypothesis of autocorrelation cannot be rejected. In this case, the estimated coefficient for $\Delta P_{t-1}$ is equal to 0.27 with a standard deviation of 0.024, what is significantly different from zero at all conventional significance levels\textsuperscript{57}.

It is also interesting to note that the explanatory power of the estimation is the square of the slope coefficient of the explanatory variable. Using the results reported for equation (A1), we see that more than 7 p.c. of the current monthly variation of the stock price is

\textsuperscript{57} Remember that the ratio between the estimated coefficient and its standard deviation, being equal here to 10.8, determines if the variable is significantly different from zero. By comparing this ratio to the critical value of the test, equal to 1.96 at 5 p.c. significance level, the explanatory variable is considered as significant if the value of the ratio is higher than the critical value, what is here the case.
explained by its variation of the previous month, what is inconsistent with the random walk hypothesis.

2. Tests for heteroskedasticity

As regards the non-heteroskedasticity condition, it requires a constant variance for the increments $\varepsilon_t$, i.e. the residuals must be homoskedastic. This condition is useful as well, since the standard deviation, i.e. the square of the variance, determines the significance level of each explanatory variable. Therefore, with unstable variance, the explanatory variable, $\Delta p_{t-1}$, for example may be significantly different from zero during some sub-periods, but not during other sub-periods.

The standard ARCH test strongly rejects the null hypothesis of non-heteroskedasticity. Indeed, the value of the test is $F(7, 1566)=29.088$ with a probability value, called p-value hereafter, of $0.0000$.

Furthermore, if the random walk hypothesis holds, the variance should be a linear function of the time interval. If that is the case, the "average variance" of $P_t$, denoted by

$$\frac{1}{T} \sum_{t=1}^{T} Var[p_t]$$

converges to a finite positive number decreasing linearly through time.

Fuller (1976) also shows that, under the random walk hypothesis, the distribution of the first-order autocorrelation coefficient has a normal distribution with mean 0 and standard deviation $\frac{1}{\sqrt{T}}$.

---

58 According to this, the sum of the variance of the stock price over two periods, denoted $\sigma_{p_t} + \sigma_{p_{t-1}}$, must be equal to twice the variance of $\sigma_{p_t}$.
Annex 2  Derivation of the Discount Dividend Model

Starting from the net simple return on a stock, we know that:

\[ h_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \]  
(A2)

where \( h_{t+1} \) denotes the return on stock held from period \( t \) to period \( t+1 \), \( P_{t+1} \) is the price of stock at the end of holding and \( D_{t+1} \) is the dividend per share. If we however postulate that the expected stock return is a constant \( h \), by rearranging equation (A2) we obtain:

\[
P_t = \left[ \frac{P^e_{t+1} + D^e_{t+1}}{1+h} \right]
\]  
(A3)

where \( P^e_{t+1} \) and \( D^e_{t+1} \) denotes the expectation of agents on the stock price and the dividends per share, based the information set at time \( t \). After solving equation (A3) forward \( N \) periods, we find the classical stock return specification:

\[
P_t = \left[ \sum_{i=1}^{N} \left( \frac{1}{1+h} \right)^i D^e_{t+i} \right] + \left[ \left( \frac{1}{1+h} \right)^N P^e_{t+N} \right]
\]  
(A4)

If we assume that \( N \to \infty \), which seems reasonable over a long period of time as explained in Campbell et al. (1997), the second term is equal to zero and then the current stock price is only equal to the present discount value of expected future dividend (per share) payments:

\[
P_t = \left[ \sum_{i=1}^{N} \left( \frac{1}{1+h} \right)^i D^e_{t+i} \right]
\]  
(A4')

Starting from this, if we now postulate that dividends per share are expected to grow at a constant rate \( g \), i.e. \( D^e_{t+i} = (1+g)D^e_{t+i-1} = (1+g)^i D_t \), and introduce this assumption into equation 5', we obtain the Gordon-Shapiro specification:

\[
P_t = \left[ \sum_{i=1}^{N} \left( \frac{1+g}{1+h} \right)^i \right] D_t
\]  
(A4'')

As the term \( \sum_{i=1}^{N} \left( \frac{1+g}{1+h} \right)^i \) is a sum of a geometric series, equation (A4'') then becomes simplified:

\[
P_t = \left[ \frac{D^e_{t+N}}{h-g} \right] = \frac{(1+g)D_t}{h-g}
\]  
(A5)
We easily see that this model makes the stock price extremely sensitive to a permanent change in the discount rate \( h^{59} \).

Equation (A5) induces two mechanical straightforward relationships:

\[
\frac{P_t}{D_t} = \frac{(1 + g)}{h - g} \quad (A5')
\]

and

\[
\frac{P_t}{E_t} = \delta \left( \frac{1 + g}{h - g} \right) \quad (A5'')
\]

where \( E_t \) is the earnings (per share) after taxes and since \( D_t = \delta^* (E_t) \) with \( \delta \) defines as the pay-out ratio\(^{60}\). In other words, the model predicts that stock prices will rise if investors expect higher future dividends (per share) and/or if they apply a lower discount rate, and that they will fall in the opposite case.

Note furthermore that the discount factor \( h \) can be decomposed as the return on a risk-free asset, \( \epsilon_t \), e.g. the long-term investment in government bonds, and a risk premium, \( \sigma \), related to the features of the asset in question\(^{61}\). Equation (A5) then becomes:

\[
P_t = \frac{D_t(1 + g)}{r + \sigma - g} \quad (A6)
\]

---

\(^{59}\) Note that equation (A5) holds if, and only if, \( R > G \). For more details, see Campbell et al. (1997), Chapter 7.

\(^{60}\) It is important to underline for the following that, by assuming that \( \delta \) is constant over time, equations (A5')- (A5'') have the same statistical properties. See Gordon and Shapiro (1956) and Gordon (1962).

\(^{61}\) An in-depth discussion on these transformations can also be found for example in Campbell et al. (1997, Chapter 7) and Balke and Wohar (2001).
Annex 3 - Episodes of extreme variations in the indicators value: breakdown according to changes in real price and real fundamental

Table A1 - Price - earnings

<table>
<thead>
<tr>
<th>Episode</th>
<th>Start</th>
<th>Initial P/E</th>
<th>End</th>
<th>Final P/E</th>
<th>Change in P/E</th>
<th>Change in real P</th>
<th>Change in real E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rise</td>
<td>November 1873</td>
<td>8.8</td>
<td>December 1894</td>
<td>26.9</td>
<td>204.3</td>
<td>92.8</td>
<td>-36.6</td>
</tr>
<tr>
<td>Fall</td>
<td>December 1894</td>
<td>26.9</td>
<td>December 1917</td>
<td>5.3</td>
<td>-80.2</td>
<td>-24.2</td>
<td>283.4</td>
</tr>
<tr>
<td>2 Rise</td>
<td>December 1917</td>
<td>5.3</td>
<td>December 1921</td>
<td>25.2</td>
<td>374.5</td>
<td>-14.9</td>
<td>-82.1</td>
</tr>
<tr>
<td>Fall</td>
<td>December 1921</td>
<td>25.2</td>
<td>October 1923</td>
<td>8.6</td>
<td>-65.8</td>
<td>9.8</td>
<td>221.3</td>
</tr>
<tr>
<td>3 Rise</td>
<td>October 1923</td>
<td>8.6</td>
<td>September 1929</td>
<td>20.2</td>
<td>134.0</td>
<td>289.8</td>
<td>66.6</td>
</tr>
<tr>
<td>Fall</td>
<td>September 1929</td>
<td>20.2</td>
<td>June 1932</td>
<td>9.4</td>
<td>-53.6</td>
<td>-80.6</td>
<td>-58.2</td>
</tr>
<tr>
<td>4 Rise</td>
<td>June 1932</td>
<td>9.4</td>
<td>July 1933</td>
<td>26.3</td>
<td>180.9</td>
<td>144.4</td>
<td>-13.0</td>
</tr>
<tr>
<td>Fall</td>
<td>July 1933</td>
<td>26.3</td>
<td>November 1937</td>
<td>9.7</td>
<td>-63.2</td>
<td>-9.9</td>
<td>145.1</td>
</tr>
<tr>
<td>5 Rise</td>
<td>November 1937</td>
<td>9.7</td>
<td>October 1938</td>
<td>20.8</td>
<td>115.8</td>
<td>20.8</td>
<td>-44.0</td>
</tr>
<tr>
<td>Fall</td>
<td>October 1938</td>
<td>20.8</td>
<td>December 1941</td>
<td>7.6</td>
<td>-63.8</td>
<td>-39.4</td>
<td>67.2</td>
</tr>
<tr>
<td>6 Rise</td>
<td>December 1941</td>
<td>7.6</td>
<td>June 1946</td>
<td>22.1</td>
<td>192.9</td>
<td>75.8</td>
<td>-40.0</td>
</tr>
<tr>
<td>Fall</td>
<td>June 1946</td>
<td>22.1</td>
<td>June 1949</td>
<td>5.8</td>
<td>-73.7</td>
<td>-41.2</td>
<td>123.6</td>
</tr>
<tr>
<td>7 Rise</td>
<td>June 1949</td>
<td>5.8</td>
<td>November 1961</td>
<td>22.6</td>
<td>288.5</td>
<td>305.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Fall</td>
<td>November 1961</td>
<td>22.6</td>
<td>April 1980</td>
<td>6.8</td>
<td>-70.0</td>
<td>-46.3</td>
<td>78.8</td>
</tr>
<tr>
<td>8 Rise</td>
<td>April 1980</td>
<td>6.8</td>
<td>March 2002</td>
<td>46.7</td>
<td>588.1</td>
<td>407.5</td>
<td>-26.3</td>
</tr>
<tr>
<td>Fall</td>
<td>March 2002</td>
<td>46.7</td>
<td>December 2002</td>
<td>28.5</td>
<td>-39.3</td>
<td>-23.0</td>
<td>26.3</td>
</tr>
</tbody>
</table>

Sources: BLS, Shiller's website (see footnote 5), S&P, Thomson Financial Datastream; own calculations.
Annex 3 - Episodes of extreme variations in the indicators of value: breakdown according to changes in real P and real F (following)

Table A2 - Q

<table>
<thead>
<tr>
<th>Episode</th>
<th>Start</th>
<th>Initial Q</th>
<th>End</th>
<th>Final Q</th>
<th>Change in Q</th>
<th>Change in real P</th>
<th>Change in real NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rise</td>
<td>June 1877</td>
<td>0.50</td>
<td>June 1901</td>
<td>1.84</td>
<td>268.3</td>
<td>317.8</td>
<td>13.4</td>
</tr>
<tr>
<td>Fall</td>
<td>June 1901</td>
<td>1.84</td>
<td>December 1920</td>
<td>0.41</td>
<td>-78.0</td>
<td>-69.0</td>
<td>41.2</td>
</tr>
<tr>
<td>2 Rise</td>
<td>December 1920</td>
<td>0.41</td>
<td>September 1929</td>
<td>1.95</td>
<td>381.8</td>
<td>415.4</td>
<td>7.0</td>
</tr>
<tr>
<td>Fall</td>
<td>September 1929</td>
<td>1.95</td>
<td>June 1932</td>
<td>0.38</td>
<td>-80.6</td>
<td>-80.6</td>
<td>-0.3</td>
</tr>
<tr>
<td>3 Rise</td>
<td>June 1932</td>
<td>0.38</td>
<td>February 1937</td>
<td>1.41</td>
<td>272.3</td>
<td>266.2</td>
<td>-1.6</td>
</tr>
<tr>
<td>Fall</td>
<td>February 1937</td>
<td>1.41</td>
<td>April 1942</td>
<td>0.52</td>
<td>-63.0</td>
<td>-62.1</td>
<td>2.3</td>
</tr>
<tr>
<td>4 Rise</td>
<td>April 1942</td>
<td>0.52</td>
<td>January 1966</td>
<td>2.02</td>
<td>285.8</td>
<td>502.6</td>
<td>56.2</td>
</tr>
<tr>
<td>Fall</td>
<td>January 1966</td>
<td>2.02</td>
<td>August 1982</td>
<td>0.50</td>
<td>-75.5</td>
<td>-61.7</td>
<td>55.9</td>
</tr>
<tr>
<td>5 Rise</td>
<td>August 1982</td>
<td>0.50</td>
<td>December 1999</td>
<td>2.65</td>
<td>434.6</td>
<td>656.0</td>
<td>41.4</td>
</tr>
<tr>
<td>Fall</td>
<td>December 1999</td>
<td>2.65</td>
<td>July 2002</td>
<td>1.32</td>
<td>-50.3</td>
<td>-40.9</td>
<td>18.9</td>
</tr>
</tbody>
</table>

Sources: Shiller's website (see footnote 5), Smithers and Co website (see footnote 30); own calculations.
Annex 4 Might an increase in earnings continue to contribute, from now on, to a downward adjustment in P/E?

The rise in earnings (per share), as observed between April and December 2002, might be expected to continue and, therefore, to contribute to the possible continuation of the downward correction in the P/E. This expectation is due mainly to two reasons. First, as reported earnings of the S&P 500 companies had fallen more than corporate profits of all companies recorded in the national accounts. Second, corporate profits are highly cyclical and reached a low point in the last quarter of 2001.

1. Earnings had fallen more than profits

As reported earnings\(^{62}\) had fallen more than corporate profits (chart A1). This exceptionally more pronounced fall in as reported earnings is mainly attributable to the revisions of accounts which were necessitated by the manipulations of the previous years (Nordhaus, 2002). These manipulations prevented the fall in earnings just after the corporate profits reached their maximum in the third quarter of 1997. Since the fourth quarter of 2000, this fall was however amplified because the delayed charges weighed on earnings at the time they were put under pressure due to the weak activity.

\(^{62}\) For a description of the differences between the as reported earnings from the company accounts and profits from the national accounts, see Petrick (2001).
Chart A1 - Earnings of the S&P500 companies and corporate profits
(billions of dollars, annualised figures)

Sources: BEA and S&P; own calculations.

2. Profits are highly cyclical and a low had been reached

A second reason why earnings might still be expected to recover is that they are highly cyclical and that they reached a low point in the last quarter of 2001.

On the basis of profits after tax derived from the national accounts, Chart A2 shows that:

- the profit cycles correspond, more often than not, to the business cycles approximated here by the development of the national income\(^6\), of which profits are one component. Thus, compared with the corresponding period of the previous year, the decline in profits during the last few quarters under review corresponds to a phase of mediocre growth in national income;

- profits show more marked variations than national income.

\(^6\) The national income is recorded here, because the profits after tax in the national accounts incorporate, as do those of the companies of the S&P 500, the profits which American companies derive from their activities abroad and exclude the profits of foreign companies made in the United States.
According to chart A3, the share of corporate profits in the national income, which had reached a high over nearly 20 years in the third quarter of 1997 at 8.6 p.c., declined almost constantly from then on and amounted to barely 5.2 p.c. in the fourth quarter of 2001. This share has increased somewhat since then and showed a value of 5.4 p.c. in the third quarter of 2002. This poor performance of American companies was however not as bad as that observed between the third quarter of 1984 and the fourth quarter of 1987.
Chart A3 - Share of corporate profits in national income (p.c.)

Source: BEA; own calculations.

The share of profits in the national income may also have fallen temporarily due to the emergence of a New Economy. Indeed, the productivity gains associated with it seem to have been reaped more by high skilled workers than by shareholders.
Annex 5 The VECM Model

Theoretical background

The starting point of a vector error correction model (VECM) is a vector autoregressive model (VAR), which is basically a multiple equation system where each variable depends on its own past values and the values of other variables included in the system. In our case here, there is an equation for the stock price, $P_t$, and the fundamental, $F_t$, and each equation contains the past values of the stock price, i.e. $P_{t-1}$, $P_{t-2}$, ..., $P_{t-n}$, and the fundamental, $F_{t-1}$, $F_{t-2}$, ..., $F_{t-n}$, as explanatory variables. Conceptually, this system can be written as follows:

$$P_t = \mu_1 + \alpha_{11}P_{t-1} + \alpha_{12}P_{t-2} + \ldots + \alpha_{1n}P_{t-n} + \beta_{11}F_{t-1} + \beta_{12}F_{t-2} + \ldots + \beta_{1n}F_{t-n} + \epsilon_{1t}$$

$$F_t = \mu_2 + \alpha_{21}F_{t-1} + \alpha_{22}F_{t-2} + \ldots + \alpha_{2n}F_{t-n} + \beta_{21}P_{t-1} + \beta_{22}P_{t-2} + \ldots + \beta_{2n}P_{t-n} + \epsilon_{2t}$$  (A7)

where $\mu_1$ and $\mu_2$ denote the constant term respectively in the equation of the stock price, $P_t$, and in the equation of the fundamental, $F_t$, while the terms $\epsilon_{1t}$ and $\epsilon_{2t}$ represent the increments.\(^{64}\)

This system of two equations can be written using the vector form. Equation (A7) then becomes:

$$X_t = \mu + \sum_{i=1}^{n} \Phi_i X_{t-i} + \epsilon_t$$  (A8)

where

$$X_t = \begin{pmatrix} P_t \\ F_t \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \quad \Phi_i = \begin{pmatrix} \beta_{11} \\ \beta_{21} \end{pmatrix}, \quad \epsilon_t = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

This VAR model can be put in an error correction form and, in this general framework, cointegration restrictions can be verified and imposed to the coefficients of the model.\(^{65}\) The implications of cointegration restrictions for forecasting are exposed in Clements and Hendry (1995). The structure of the reaction delays of each variable is not imposed arbitrarily.

\(^{64}\) For each dependent variable, $R$ and $F$, the increments contain the part that is not explained by the past (or lagged) values of the dependent variable and the lagged values of the other explanatory variable. The values that are taken by these increments give an impulse or shock to the system, so that they are also called shocks in the VAR structure.

Methodology

When the variables used for the estimation display non stationary processes, it may be interesting to impose a restriction on the VAR structure in order to take into account this characteristic. Indeed, we know from section 3.4 that, on the one hand, the stock price and the earnings, on the other hand, the stock price and the net worth, are cointegrated while each variable, in level, displays a \( I(1) \) process separately. By imposing the cointegration restriction in the VAR structure, this amounts to estimating a cointegrated vector autoregressive model.

Starting from equation (A8) where \( X_t \) is a 2-sized vector of \( I(1) \) variables, \( \Phi_l \) are 2 x 2 matrices and \( u_t \) a vector of white noise error terms, we can re-parameterise it as follows:

\[
X_t - X_{t-1} = \mu - \Phi(1)X_{t-1} + \sum_{\ell=1}^{n-1} A_{\ell} (X_{t-\ell} - X_{t-\ell-1}) + u_t
\]

(A9)

where

\[
\Phi(1) = I_2 - \sum_{\ell=1}^{n} \Phi_{\ell}
\]

and

\[
A_{\ell} = - \sum_{j=0}^{n-1} \Phi_{j}
\]

Unit roots imply that \( \Phi(1) \) is singular. When the VAR is cointegrated, which means that \( r \) cointegrating vectors exist, \( \Phi(1) \) can be written as \( -\alpha \beta' \) where \( \alpha \) and \( \beta \) are 2 x \( r \) matrices of rank \( r \). \( \beta'X_{t-1} \) is a stationary \( r \)-components vector of cointegrating relations, the fluctuations of which describe short-term departures from the long term relations. \( \alpha \) is a matrix of adjustment coefficients, describing the reaction of each \( I(1) \) variable to previous short term departures from each long term relation\(^{66}\).

As suggested by the cointegration tests in section 3.4, there is one rank of cointegration between the stock price, \( P \), and the fundamental, \( F \), i.e. \( r=1 \). By imposing this restriction, the cointegrating vector is normalised by constraining the coefficient of \( P \) to 1.\(^{67}\) Clements and Hendry (1995) show that the estimation of the VAR with the constraint \( r = 1 \), when

\(^{66}\) In this sense, the matrix \( \alpha \) contains as useful information as the error correction term in the error correction mechanism model (ECM). See Enders (1995).

\(^{67}\) For more details on the methodology, see Johansen (1992).
true, strongly improves the forecasts. Furthermore, given the results of the Wald test\textsuperscript{68} for which the hypothesis $\beta=(1,-1)$ is accepted for the models considered, we also estimate the VECM systems by imposing this restriction on the values of the coefficients in the cointegrating space. Finally, the estimation of the VECM requires a large number of years (Campbell and Perron, 1991), what is clearly the case in the previous exercise given our sample.

\textsuperscript{68} Over the sample period (February 1872 - July 2002), the results of this restriction test is $\chi^2(1)= 4.7163$ [with a p-value = 0.03] for the P/E model and $\chi^2(1)= 1.0246$ [with a p-value of 0.314] for the Q model.
References


Shiller R.J., http: \~\aida.econ.yale.edu\~shiller\data.html.


