METHODOLOGY FOR THE DECOMPOSITION OF A CHANGE IN WEIGHTED AVERAGES

PRESENTATION: DIFFERENT CAUSAL SITUATIONS

The statistics on the interest rates applied by monetary financial institutions (MFI Interest Rates) are made up of a series of elements – production volume, rates and specific components under this heading – some of which are more volatile than others. This volatility can be linked to various causes such as seasonal variations (during certain recurrent events such as trade fairs), publicity campaigns, changes in market share related to business moves or relocation, etc.

Since the MIR statistic is compiled by weighted averages and this weighting is done by using either monthly production figures (rates of new business) or stocks (rates on outstanding amounts)[1], it is possible for the variation in the average weighted rate to be caused by a change in the weighting, while the rates applied by each reporting institution have remained virtually unchanged. Thus, a situation may arise where the change in the rate can be explained exclusively by a change in the weighting; the term "structure effect" (or even “weight effect”) will then be used. Conversely, it is possible to envisage a variation that is only linked to rate changes because the weightings (i.e. the month's production volume or outstanding amounts) have not changed from the previous month. In this latter case, the expression "interest rate effect" is used or, more generally speaking, the "principal term". In most cases, there will be a – variable – combination of these two effects. To avoid too hasty an interpretation of the results, it is useful to isolate the two effects.

This distinction between effects is essential for the part devoted to new contracts under the MIR statistics. The part dedicated to amounts outstanding is in fact weighted by stocks which, of course, do not change much and therefore implies an insignificant structure effect.

The objective of the decomposition presented here is to put the emphasis on the different effects ("rates" and "structure") enabling an explanation of changes in a weighted average between two periods. Unlike price indices which are based on ratios, the proposed decomposition is based on differences and therefore needs to be treated carefully.

The formula used is the *Marshall-Edgeworth type* (sometimes referred to as the *Benet formula* by the ECB), the logical properties of which are deemed to be "superior" to other formulae. The different decomposition methods along with their properties are described in a European Central Bank methodological paper that is available on the NBB's MIP webpage as well as on the ECB's website:

"An application of index numbers theory to interest rates", by Javier Huerga and Lucia Steklacova (Working Paper series / N°939 / September 2008)

MIR link: http://www.nbb.be/doc/dq/mir/fr/presentation/MIR_mir.htm

ECB link: http://www.ecb.int/pub/scientific/wps/author/html/author1108.en.html

[1] See the methodological note on the presentation of MIR in Belgium: "L’enquête harmonisée sur les taux d’intérêt dans la zone euro : description du volet belge (The harmonised survey on interest rates in the euro area: description of the Belgian section - June 2011) § 5.1.

DECOMPOSITION OF RATE CHANGES

The difference between the two rates is referred to as[2] the "global effect" (ΔI). The global effect can be broken up into two components: the "principal term" (TP), also referred to as the "rate effect" and the "structure effect" (ES) also referred to as the "weighting effect". The principal term corresponds to the change in interest rate and is predominant when there is little variation in the weights for the two periods; the structure effect corresponds to the change in weight from one period to the other and is predominant if there is little change in individual interest rates while the weights vary.

In symbols, this gives us:

$\Delta I = TP + ES$

Two periods of time, *(t)* and *(t-1),* are considered. It should be noted that the two periods do not necessarily have to be adjacent, so *(t-1)* can therefore be replaced by *(t-n)*. We take into consideration two weighted averages of the "interest rate" variable (corresponding to these two periods): I_t and I_{t-1} and the difference of these averages: $\Delta I_{t,t-1}$, as well as the weights attached to each reporting agent for each of the two periods: $w(k)_t$ and $w(k)_{t-1}$., and the rates applied by each reporting institution for each of the two periods: $i(k)_t$ and $i(k)_{t-1}$., with each reporting agent being designated by an index *k.* With all these elements, the decomposition can be denoted by the following formula[3]:

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{[i(k)_t - I_t] + [i(k)_{t-1} - I_{t-1}]}{2} \right)$$

Where:

I_t = MIR aggregate rate at time *t.*

$\Delta I_{t,t-1} = I_t - I_{t-1}$

$i(k)_t$ = rate of reporting agent *k* at time *t* for the MIR heading.

$\Delta i(k)_{t,t-1} = i(k)_t - i(k)_{t-1}$

$w(k)_t$ = weight of reporting agent *k* at time *t* as a percentage of the total volume for the MIR heading.

$\Delta w(k)_{t,t-1} = w(k)_t - w(k)_{t-1}$

The principal term or "rate effect"

The principal term corresponds to the influence of the interest rate difference from one period to the next for each reporting agent and its overall effect on the aggregate. It is equal to:

[2] We use the ECB's symbols as well as Berthier's terminology (for their general aspects) and the ECB's terms (for their interest-rate-specific aspect). See references in the ECB paper.

[3] The formula given here is from the extended Marshall-Edgeworth decomposition. It enables a better analysis analysis of the individual contributions to the aggregate than the simple decomposition formula and it yields the same aggregate results as the simple formula. It is therefore particularly useful as a means of detecting individual influence. See the ECB's methodological note.

$$TP = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right)$$

This is usually where the main explanation can be found for the global effect; it is the "pure" part of the rate change in the overall change. This effect corresponds to the statistic's interest variable (here, interest rates).

The structure effect or "weighting effect"

The structure effect points up the change in weight for each individual component rate and its overall effect on the aggregate. It can be expressed as in the following formula:

$$ES = \sum_k \Delta w(k)_{t,t-1} \left(\frac{[i(k)_t - I_t] + [i(k)_{t-1} - I_{t-1}]}{2} \right)$$

Mathematic properties of this decomposition and comparison with other decomposition methods

See the ECB paper which provides a full comparative analysis.

TREATMENT OF NON-CYLINDRICAL DATA

When there is a data mismatch between the two periods, for instance when a reporting party is added or withdrawn, the principle chosen is to keep as much information as possible while respecting the calculation of the actual weighted average[4]. To this end, in the period under comparison where declarant *k* is absent – here, this is in time *t* – a weight equal to *zero [i.e.* $w(k)_t = 0$] will be attributed and the rate from the period when the declarant is present is used instead [*i.e.* $i(k)_t = i(k)_{t-1}$].

This zero-weighting makes it possible to calculate a structure effect related to the "presence" of the reporting agent [*i.e.* $\Delta w(k)_{t,t-1} = w(k)_t$] and not to take account of the rate in the calculation of the weighted average for the period when the declarant is not present. Through the use of the rate from the previous period, the principal term (*TP*) is cancelled out [*i.e.* $\Delta i(k)_{t,t-1} = 0$].

MIR SERIES USED FOR THE DECOMPOSITION

All the series released for new business are therefore accompanied by their decomposition into factors. Published series corresponding to the new business included in indicators 1 to 29 of Annex 2 of Regulation ECB/2001/18 and in indicators 32 to 85 of Annex 2 of Regulation ECB/2009/07 are therefore selected. Furthermore, some of the specifically Belgian series are also included (the cash bonds series and those on intra-monthly flows for non-financial corporations).

[4] In other words, we are **not** going to make the sample similar across time and it will **not** be adjusted (for example, by only retaining elements that are the same for the two periods).

SUMMARY BIBLIOGRAPHY

J.-P. BERTHIER "Contribution à l'écart entre deux moyennes pondérées", Courrier des statistiques (INSEE) n°100, December 2001, Paris.

J.-P. BERTHIER "Les comptes nationaux français en volume au prix d'une année fixe et aux prix de l'année précédente : écart entre les indices d'évolution dans les deux systèmes de prix" in E. ARCHAMBAULT and M. BOEDA (dir.) "Comptabilité Nationale: nouvelles frontières", Économica, Paris, 1999.

J.-P. BERTHIER "Calcul des contributions aux écarts entre évolution du PIB à prix 80 et évolution aux prix de l'année précédente", Note INSEE, June 1997, Paris.

J. HUERGA & L. STEKLACOVA "An application of index numbers theory to interest rates", Working Paper series, N°939, September 2008