Income Inequality in General Equilibrium

NBB Colloquium

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Motivation

- **Covid-19 has affected both firms and households**
  - forced closures
  - temporary unemployment
- **Leading to changes in welfare**
  - lower wages
  - workers moving across sectors
  - lower output
- **This paper: how do labor, wages and GDP change?**
  - in response to a labor-specific productivity shock
  - due to changes in labor mobility

Production and consumption

- **Households/consumers** identical homothetic preferences

$$\mathcal{Y} = D\left(\{c_i\}_{i\in\mathcal{N}}\right)$$

- **Production** $i \in \mathcal{N}$ sectors, with constant returns-to-scale

$$y_i = F_i(z_{il}, l_i, \{x_{ij}\}_{j\in\mathcal{N}})$$

Labor markets and the production network

- **Households supply labor** in sector i based on preferences and wages

$$l_i = \frac{\Phi_i w_i^\kappa}{\mathcal{W}^\kappa}$$

where the wage index $\mathcal{W} = \left(\sum_j \Phi_j w_j^\kappa\right)^{1/\kappa}$

- **Labor mobility**
  - $\kappa \to 0$, perfect immobility of workers across sectors
  - $\kappa \to \infty$, perfect mobility

- **Real GDP contribution** of a sector (λ_i) and its share of workers (Λ_i)

$$\lambda_i = \sum_j \Omega_{cj} \Psi_{ji} \;\; ; \qquad\qquad \Lambda_i = \Omega_{il} \lambda_i$$

with Leontief multiplier of input i for the production of j, Ψ_{ji},
final demand share of good j, Ω_{cj} , and the labor share in sector i, Ω_{il}

Comparative statics

- **Change in wage in sector i from labor productivity shock in s**

$$\frac{d\,log\,w_i}{d\,log\,z_{sl}} = \underbrace{\frac{d\,log\,\Lambda_i}{d\,log\,z_{sl}}}_{\text{labor centrality channel}} - \underbrace{\frac{d\,log\,l_i}{d\,log\,z_{sl}}}_{\text{labor supply channel}} + \underbrace{\frac{d\,log\,GDP}{d\,log\,z_{sl}}}_{\text{Aggregate channel}}$$

- **Wage inequality**
  - inequality-neutral result in Cobb-Douglas models
  - only change in real GDP, keeping wage gaps constant
  - more generally, inequality arises from two labor channels

A CES network economy

- **CES production and consumption** to take to the Belgian data

$$y_i = \left(\omega_{il}^{\frac{1}{\sigma_y}} \, l_i^{\frac{\sigma_y - 1}{\sigma_y}} + \sum_j \omega_{ij}^{\frac{1}{\sigma_y}} \, x_{ij}^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{\sigma_{y,i}}{\sigma_y - 1}} \quad ; \quad U = \left(\sum_j \omega_{cj}^{\frac{1}{\sigma_c}} \, c_j^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}}$$

- **Two exercises**
  - Impact of labor productivity shock on GDP and sector wages
  - Impact of change in labor mobility on GDP and sector wages

Data and calibration

- **Belgium IO tables** (64 sectors, 2015)
  - intermediate good matrix Ω_X
  - labor and capital shares Ω_L, Ω_K
  - employment L_i

sectoral elasticity	σ_y	0.5	[Oberfield and Raval, 2021]; [Atalay, 2017]
final demand elasticity	σ_c	0.9	[Herrendorf et al., 2013]; [Oberfield and Raval, 2021]
mobility	κ	1.4	[Galle and Lorentzen, 2021]
worker preferences	Φ	from κ	and employment and wages data

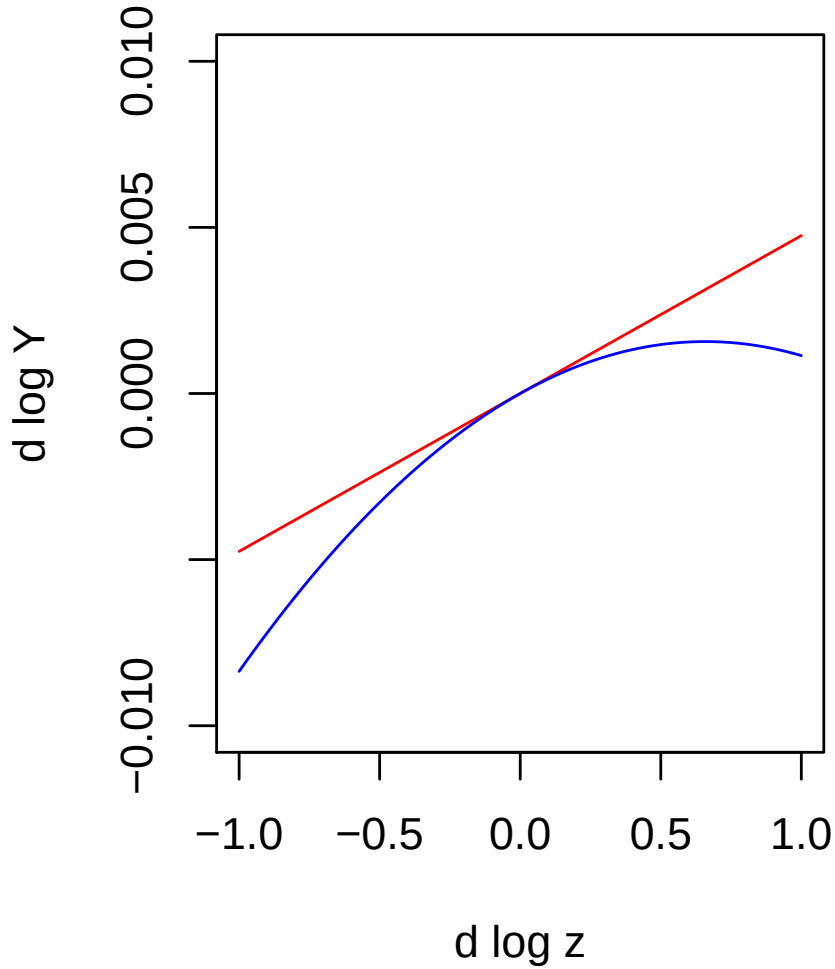
Results 1: A labor productivity shock in the energy sector

$$d\,log\,\mathcal{Y} = \frac{d\,log\,\mathcal{Y}}{d\,log\,z_{El}} d\,log\,z_{El} + \frac{1}{2}\frac{d^2\,log\,\mathcal{Y}}{d\,log\,z_{El}^2}\left(d\,log\,z_{El}\right)^2$$

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d\ \log\ \mathcal{Y}}{d\,log\,z_{El}}$	$\frac{d^2\ \log\ \mathcal{Y}}{d\,log\,z_{El}^2}$
(0.5, 0.9, 1.4)	0.005	-0.007

- 1st order: 1% increase in productivity $\rightarrow$ 0.005% increase in real GDP
- 2nd order: amplifies negative shocks, dampens positive shocks

Results 1: First- and second-order effects on GDP



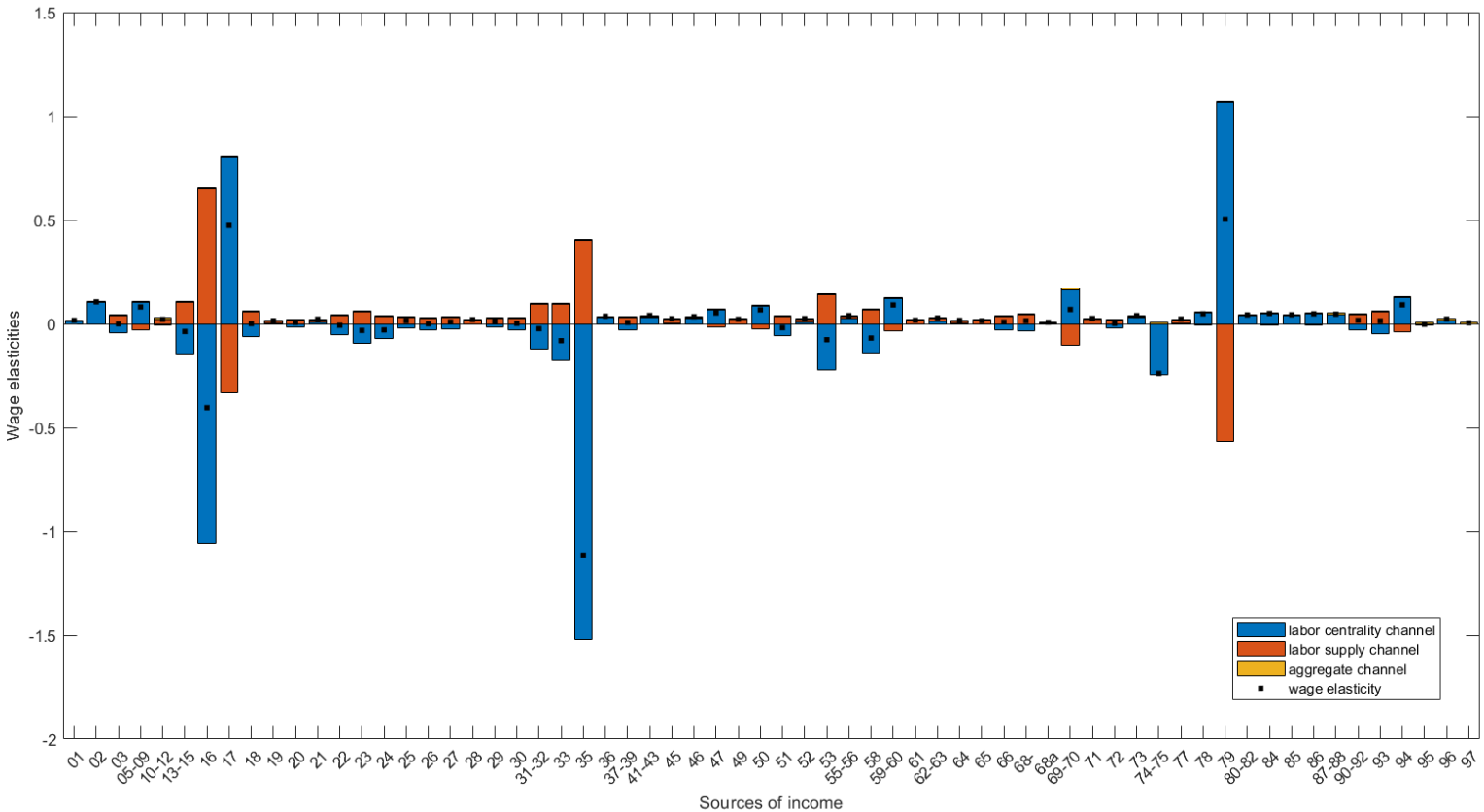
Red line: first-order effect. Blue line: total effect.

Results 1: Changes in sector wages

mean wage elasticities	s.d. wage elasticities	25th percentile	75th percentile
0.004	0.18	0.00006	0.04

- Wage changes around zero, some sectors with large changes

Results 1: Decomposition of wage elasticities



- Productivity shock energy sector (35) decreases wages in own sector
- Offsetting effects: labor centrality vs labor supply channels

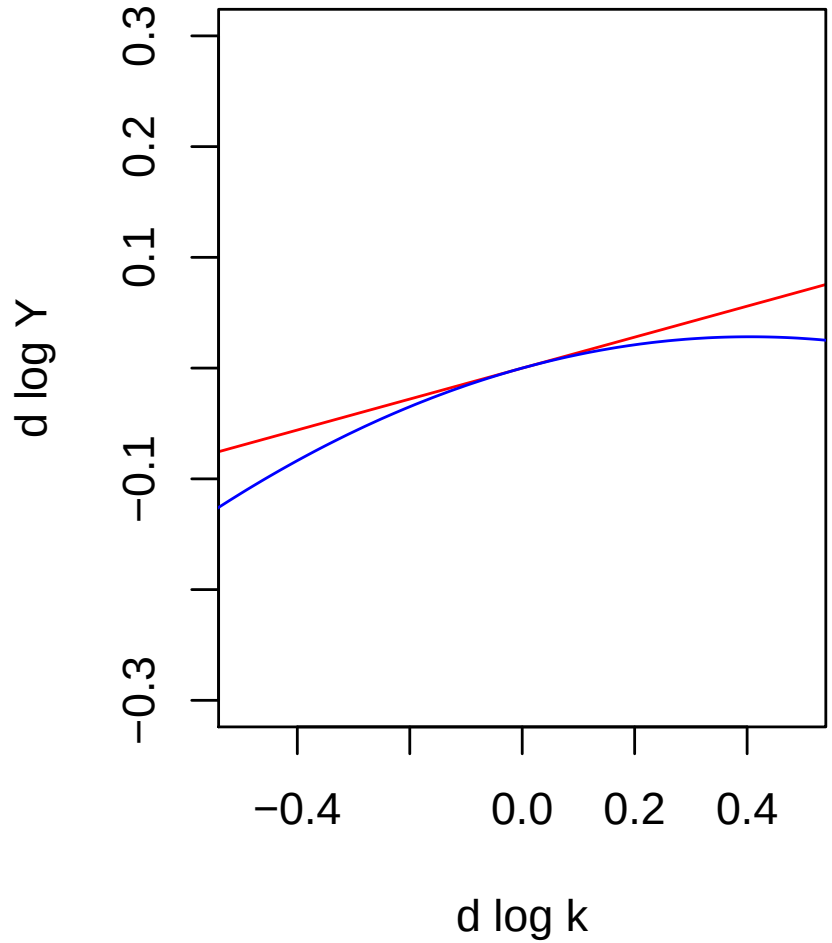
Results 2: A change in labor mobility

$$d\,log\,\mathcal{Y} = \frac{d\,log\,\mathcal{Y}}{d\,log\,\kappa} d\,log\,\kappa + \frac{1}{2}\frac{d^2\,log\,\mathcal{Y}}{d\,log\,\kappa^2}\,(d\,log\,\kappa)^2$$

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d\,log\,\mathcal{Y}}{d\,log\,\kappa}$	$\frac{d^2\,log\,\mathcal{Y}}{d\,log\,\kappa^2}$
(0.5, 0.9, 1.4)	0.14	-0.34

- Total effect on real GDP > 0 when workers are reallocated towards sectors that are more important for final demand

Results 2: First- and second-order effects on GDP



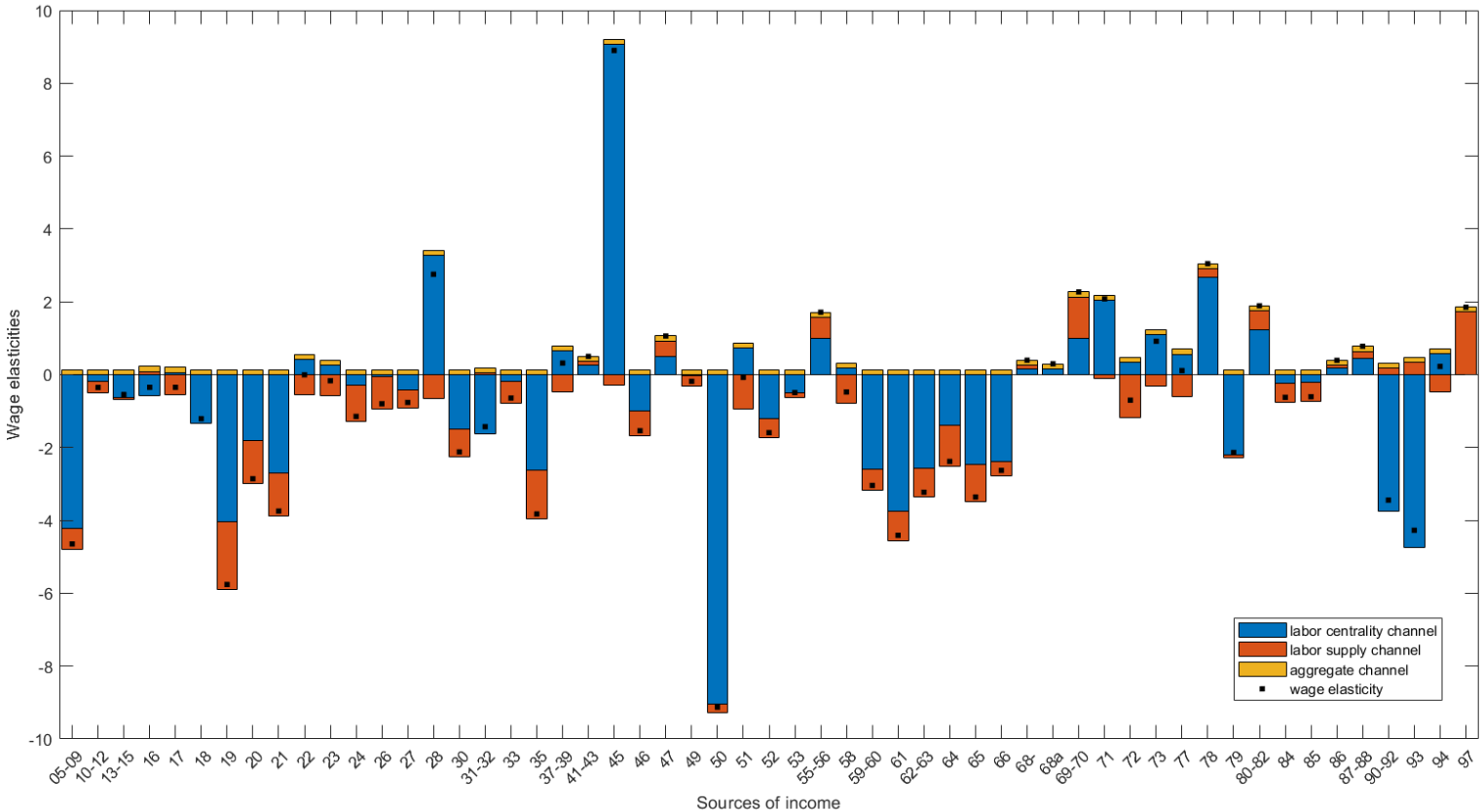
- 1st order effect: positive
- 2nd order effect: amplifies negative shocks, dampens positive shocks

Results 2: Changes in sectoral wages

mean wage elasticities	s.d. wage elasticities	25th percentile	75th percentile
-0.82	10.58	- 2.74	0.39

- Wages experience a negative change when mobility between sectors increases
- There is a large dispersion between wage changes

Results 2: Decomposition of wage elasticities



- The correlation between the labor centrality and the labor supply effects now depends on the initial situation

Conclusion

- **We demonstrate new sources of income inequality**
  - sector wages can change in response to shocks in other sectors
  - how much depends on IO structure of production and labor mobility
- **We provide a model of income inequality**
  - shocks to labor productivity
  - changes in labor mobility
  - shocks to one sector can affect wages and labor supply in others
  - affecting real GDP
- **Policy implications**
  - spillover effects from one sector to another: unintended consequences?
  - direct and indirect effects from increasing labor mobility
  - total effect mobility depends on initial equilibrium

Thank you

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Appendix: Sectoral employment in Belgium

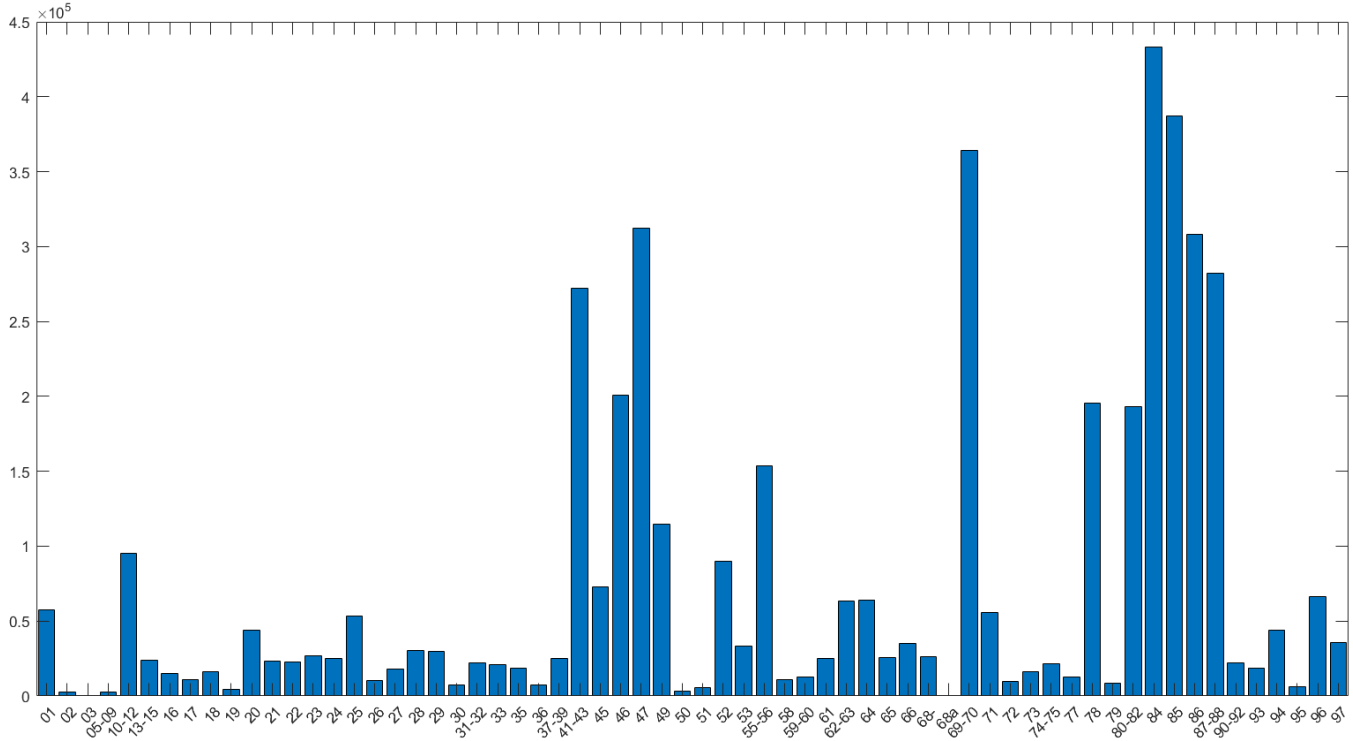

Figure: Sectoral employment in Belgium ▸ calibration

Appendix: Sectoral wage in Belgium

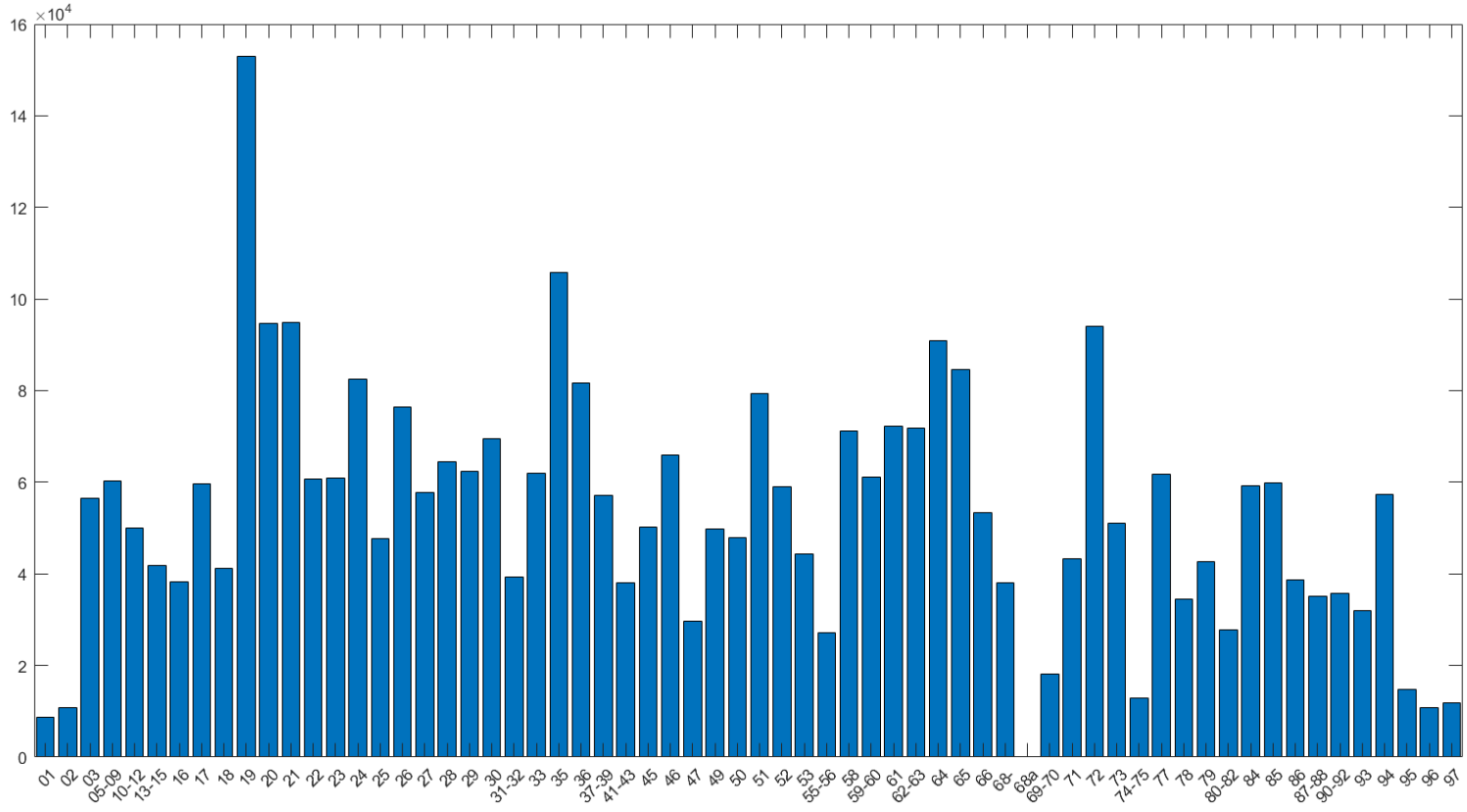

Figure: Sectoral wages in Belgium and capital incomes

▸ calibration

Appendix: Ω_X matrix

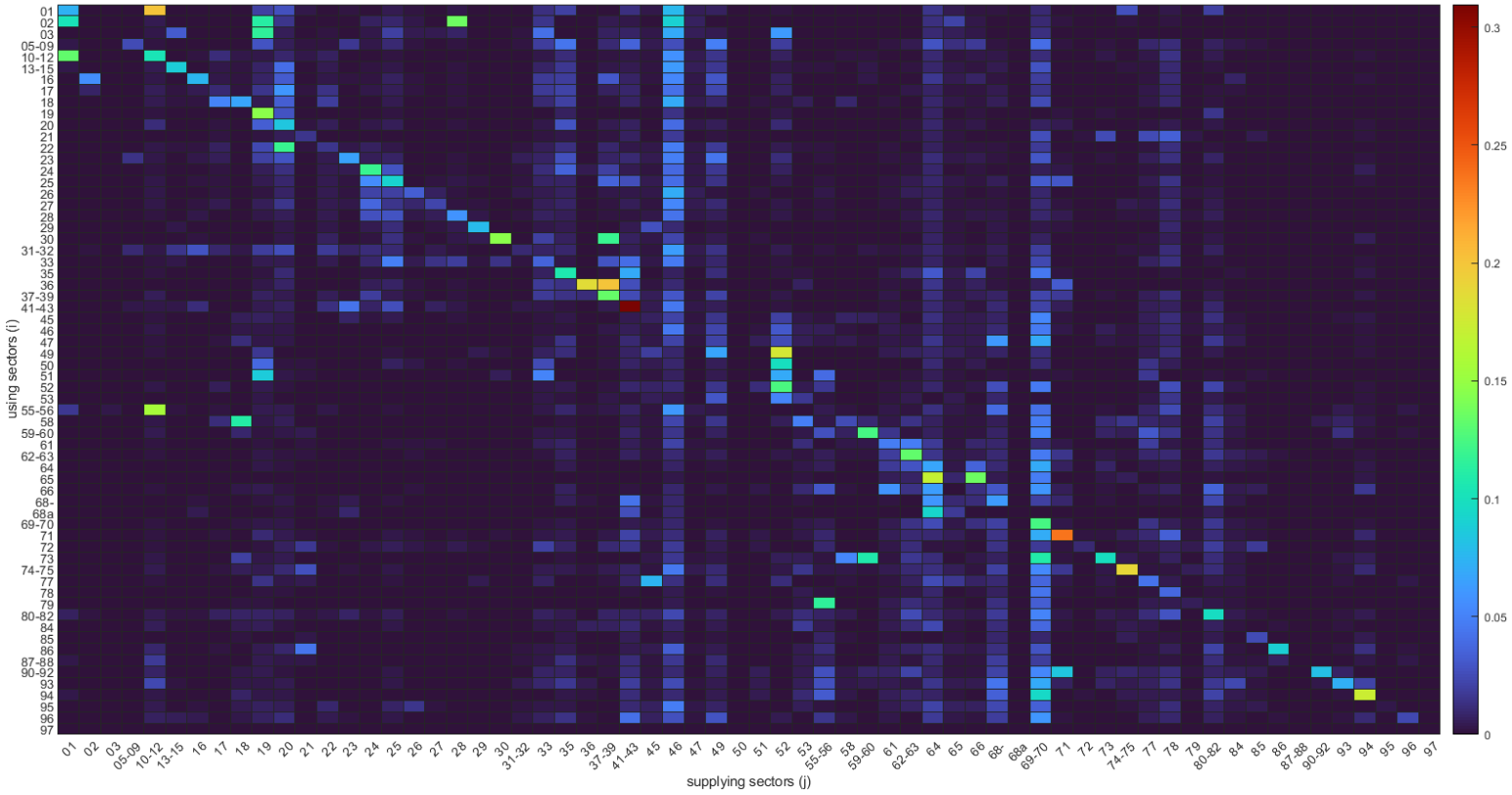

Figure: Ω_X matrix ▸ Calibration

Appendix: Ψ matrix

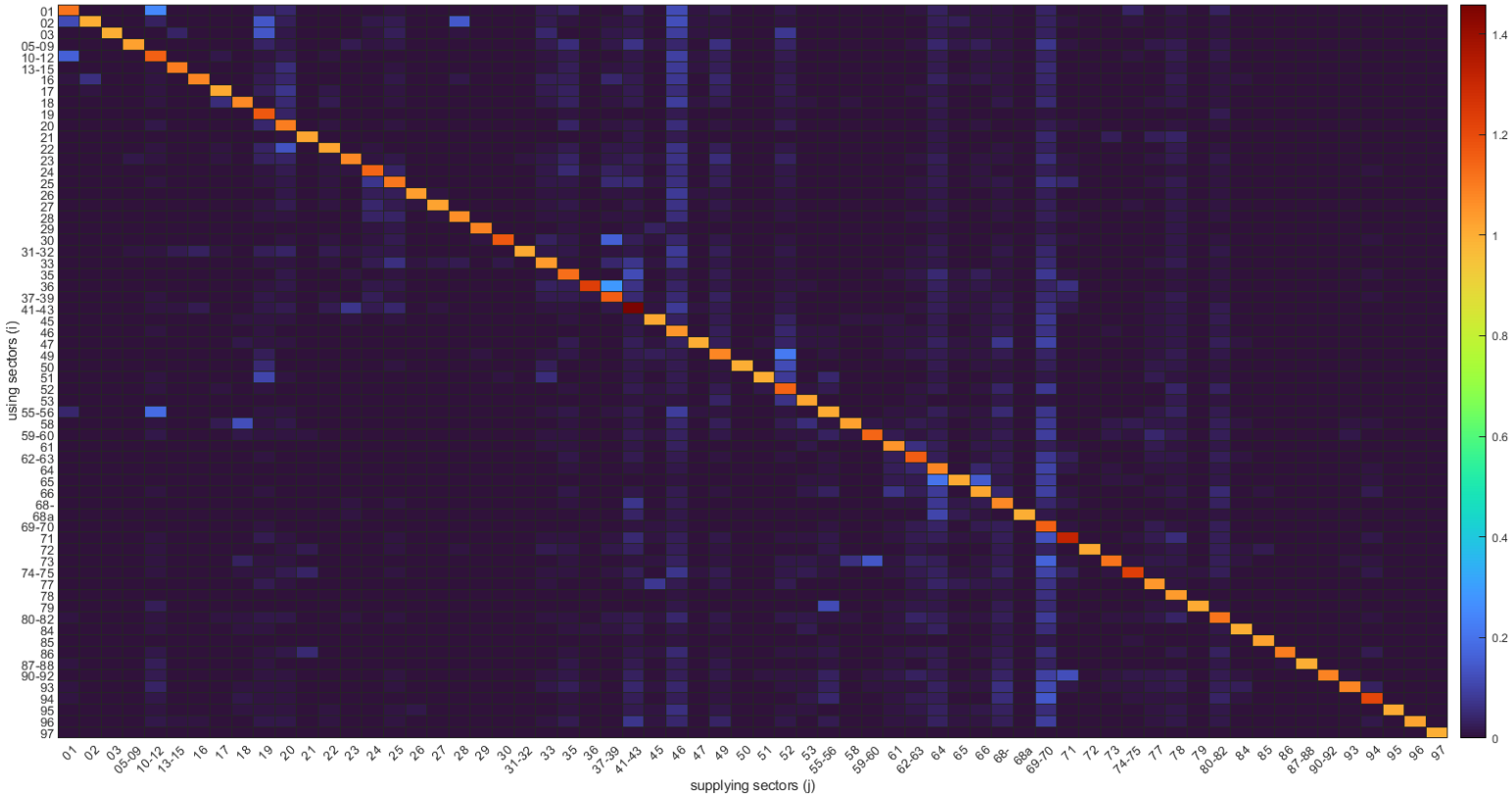

Figure: Ψ matrix

▸ Calibration

Appendix: Off-diagonal Ψ matrix

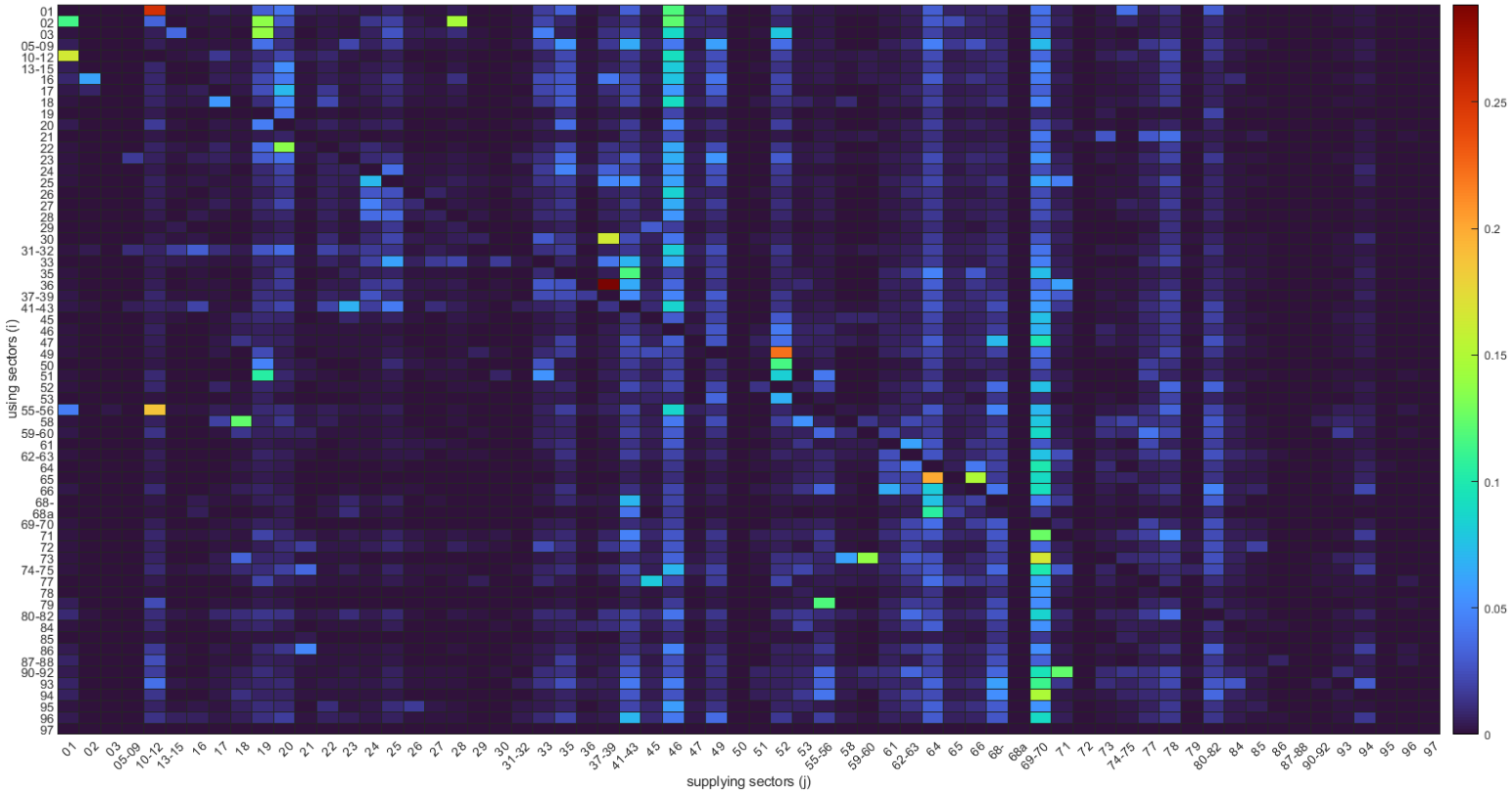

Figure: Off-diagonal Ψ matrix ▸ Calibration

Appendix: Ω_X matrix with imports/exports

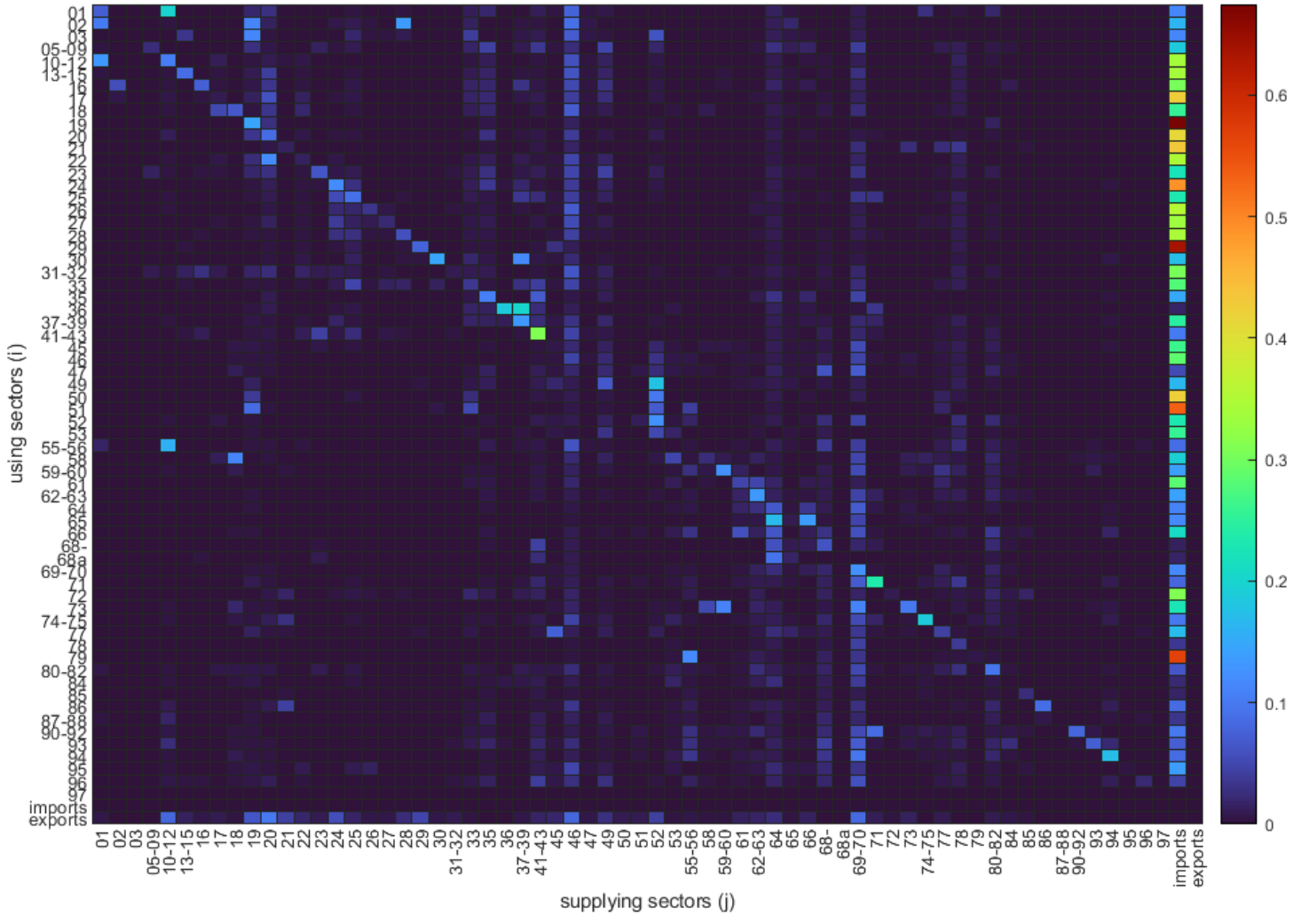

Figure: Ω_X matrix with imports/exports ▸ Calibration

Appendix: Ψ matrix with imports/exports

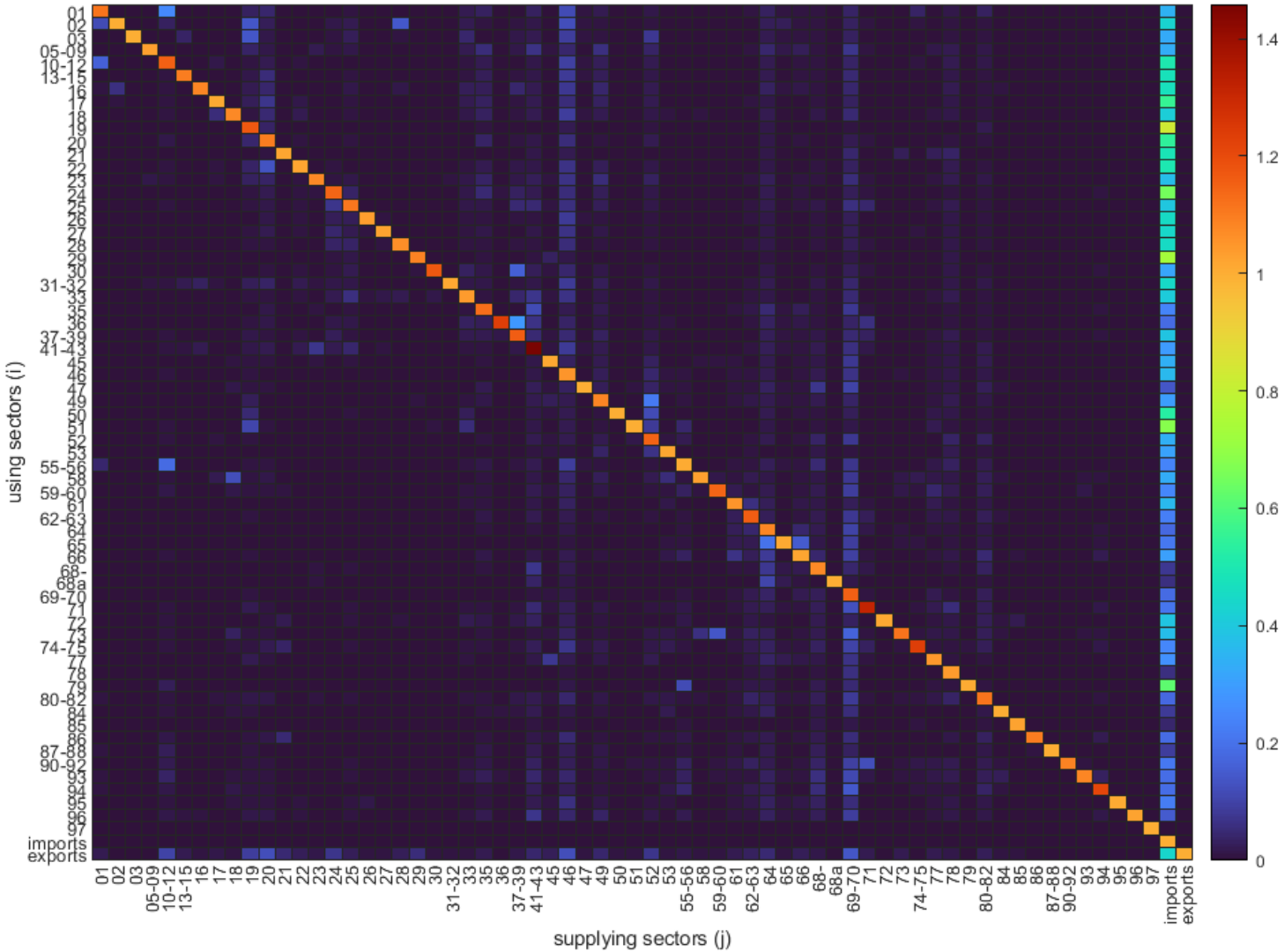

Figure: Ψ matrix with imports/exports

▸ Calibration

Appendix: Vectors Ω_L, Ω_K, λ and Λ

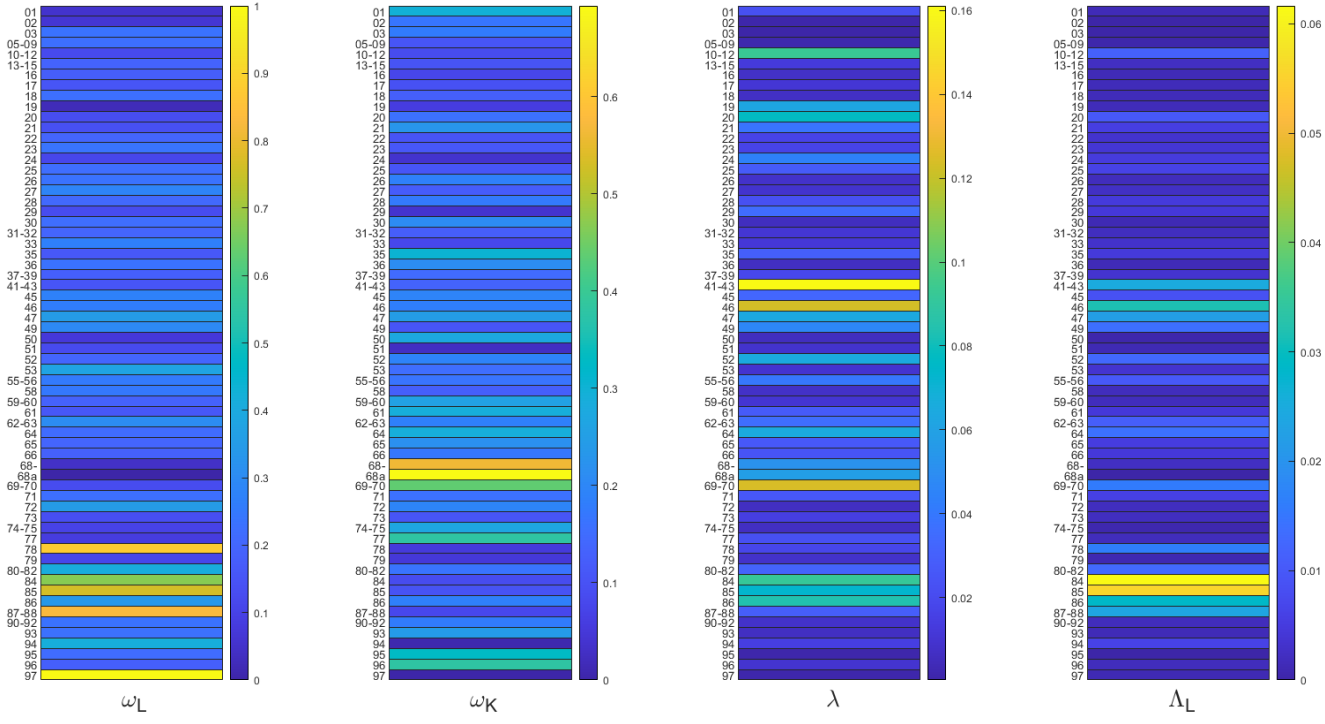

Figure: Vectors Ω_L, Ω_K, λ and Λ ▸ Calibration

A CES network economy: Labor-specific sectoral productivity shock

- Impact on real GDP:

$$d\log \mathcal{Y} = \underbrace{\Lambda_s d\log z_{sl}}_{\text{1st order effect}} + \underbrace{\frac{1}{2}\frac{d\Lambda_s}{d\log z_{sl}}(d\log z_{sl})^2}_{\text{2nd order effect}}$$

- Heterogeneous impact on wages:

$$\frac{d\log w_i}{d\log z_{sl}} = \underbrace{\hat{\sigma}_{y,i} d\log\left(\frac{w_i}{p_i}\right) + \frac{1}{\lambda_i}\hat{\sigma}_c \sum_o \Omega_{co}\Psi_{oi} d\log\left(\frac{p_o}{\mathcal{P}}\right) + \sum_k \frac{\lambda_k}{\lambda_i}\hat{\sigma}_{y,k}\sum_o \Omega_{ko}\Psi_{oi} d\log\left(\frac{p_o}{p_k}\right)}_{\text{labor centrality channel}}$$

$$\underbrace{-\kappa\frac{\Phi_i}{l_i}\, d\log\left(\frac{w_i}{\mathcal{W}}\right)}_{\text{labor supply channel}} + \underbrace{\Lambda_s}_{\text{aggregate channel}}$$

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A CES network economy: Shock to workers' mobility

$$d\log\mathcal{Y} = \underbrace{\left(\kappa\sum_j \Lambda_j\left(\log w_j - \sum_o l_o \log w_o\right)\right) d\log\kappa}_{\text{1st order effect}} +$$

$$\underbrace{\frac{1}{2}\left(\frac{d\log\mathcal{Y}}{d\log\kappa} + \kappa\sum_j \frac{d\Lambda_j}{d\log\kappa}\left(\log w_j - \sum_o l_o \log w_o\right) + \kappa\sum_j \Lambda_j\left(\frac{d\log w_j}{d\log\kappa} - \sum_o l_o \frac{d\log w_o}{d\log\kappa}\right)\right)}_{\text{2nd order effect}} \underbrace{\times (d\log\kappa)^2}_{\text{2nd order effect}}$$

$$\frac{d\log w_i}{d\log\kappa} = \underbrace{\hat{\sigma}_{y,i}\left(\frac{d\log w_i}{d\log\kappa} - \frac{d\log p_i}{d\log\kappa}\right) + \frac{1}{\lambda_i}\hat{\sigma}_c\sum_o \Omega_{co}\Psi_{oi}\left(\frac{d\log p_o}{d\log\kappa} - \frac{d\log\mathcal{P}}{d\log\kappa}\right)}_{\text{labor centrality channel}}$$

$$\underbrace{+\sum_k \frac{\lambda_k}{\lambda_i}\hat{\sigma}_{y,k}\sum_o \Omega_{ko}\Psi_{oi}\left(\frac{d\log p_o}{d\log\kappa} - \frac{d\log p_k}{d\log\kappa}\right)}_{\text{labor centrality channel}}$$

$$\underbrace{-\kappa\left(\log w_i - \sum_o l_o \log w_o\right)}_{\text{labor supply channel}} + \underbrace{\left(\kappa\sum_j \Lambda_j\left(\log w_j - \sum_o l_o \log w_o\right)\right)}_{\text{aggregate channel}}$$

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Results: 2nd order decomposition and nonlinearities for the shock to workers' mobility

$$\frac{d^2 log\, \mathcal{Y}}{d log\, \kappa^2} = \underbrace{\kappa \sum_j \Lambda_j \left(log\, w_j - \sum_o l_o log\, w_o \right)}_{\text{scale effect of } \kappa \text{ change}} + \underbrace{\kappa \sum_j \frac{d\, \Lambda_j}{d log\, \kappa} \left(log\, w_j - \sum_o l_o log\, w_o \right)}_{\text{change in final demand importance}}$$

$$+ \underbrace{\kappa \sum_j \Lambda_j \left(\frac{d log\, w_j}{d log\, \kappa} - \sum_o l_o \frac{d log\, w_o}{d log\, \kappa} \right)}_{\text{change in comparative wages}}$$

▸ back

Appendix: Different specifications for the shock to labor-specific productivity

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d\,log\,\mathcal{Y}}{d\,log\,z_{El}}$	$\frac{d^2\,log\,\mathcal{Y}}{d\,log\,z_{El}^2}$	mean wage elasticities	s.d. wage elasticities
(0.5, 0.9, 1.4)	0.005	-0.007	0.004	0.18
(0.1, 0.9, 1.4)	0.005	0.0008	0.002	9.39
(0.5, 0.9, 1.1)	0.005	-0.007	0.004	0.85
(0.5, 0.9, 2)	0.005	-0.007	-0.06	0.73

Table: Results of the real GDP elasticities to workers' mobility shocks

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Appendix: Different specifications for the shock to workers' mobility

$(\sigma_y, \sigma_c, \kappa)$	$\frac{d\,log\,\mathcal{Y}}{d\,log\,\kappa}$	$\frac{d^2\,log\,\mathcal{Y}}{d\,log\,\kappa^2}$	mean wage elasticities	s.d. wage elasticities
(0.5, 0.9, 1.4)	0.14	0.19	-1.57	26.45
(0.1, 0.9, 1.4)	0.14	-0.69	-0.23	16.78
(0.5, 0.9, 1.1)	0.11	-0.19	-0.64	8.31
(0.5, 0.9, 2)	0.20	-0.79	-1.17	15.12

Table: Results of the real GDP elasticities to workers' mobility shocks

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